Tomographic Reconstruction

Fernando S. Furusato

Processamento Digital de Imagens
Faculdade de Engenharia Elétrica e Computação - Unicamp

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Agenda

- Introduction
  - Variations
  - Motivation
- Equations
  - Feature function
  - Radon Transform
  - Inversion
  - Filtered backprojection
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- References

Introduction I

What is Tomography?

Production of cross-sectional images of an object by the observation of the effects on the passage of waves of energy through such object, from many different directions.

Introduction II

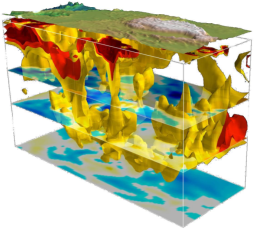
Computerized Tomography

The use of computational power to aid on the data acquisition and reconstruction.

Inverse Problem

We know how to generate the result. Can we take an inverse path to get to the cause?

Variations I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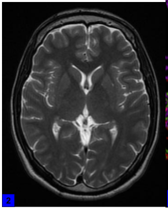

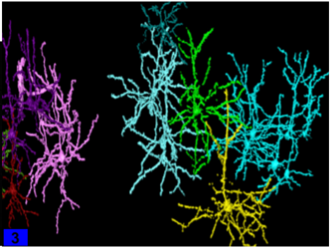

Types of Tomography

1. *Seismic tomography*
2. *Medical imaging*
3. *Micro and nano tomographies*

Variations II

Diffraction

- *Diffracting*
- **Nondiffracting**

Algorithm

- *Iterative*
- **Analytical**

Motivation

- Why do we need tomographic reconstruction?

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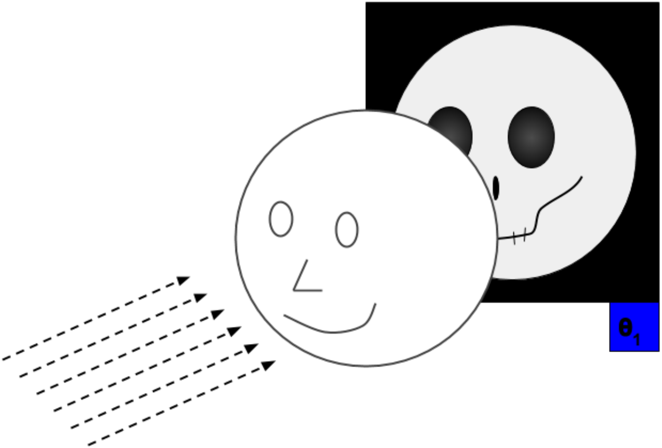

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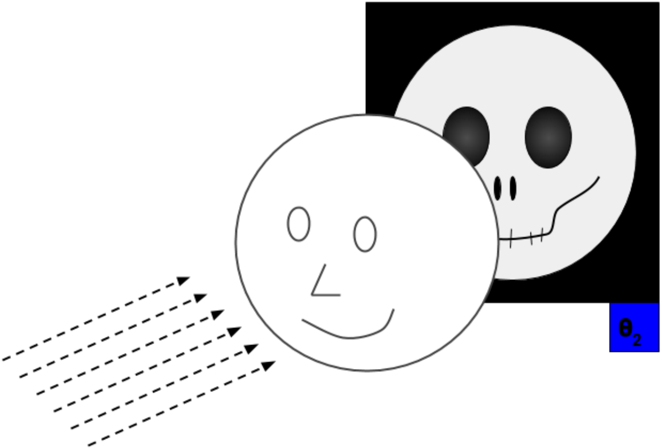

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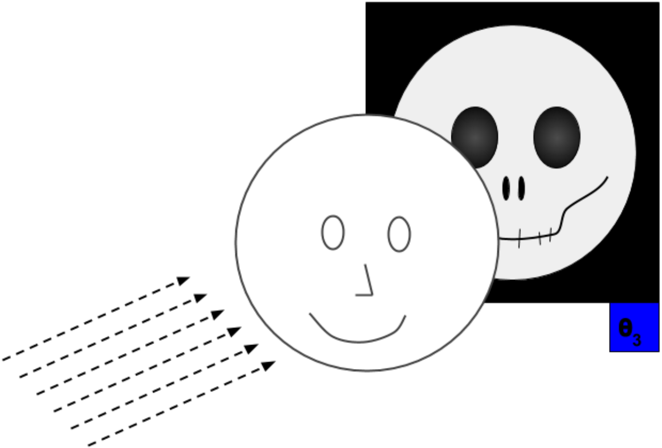

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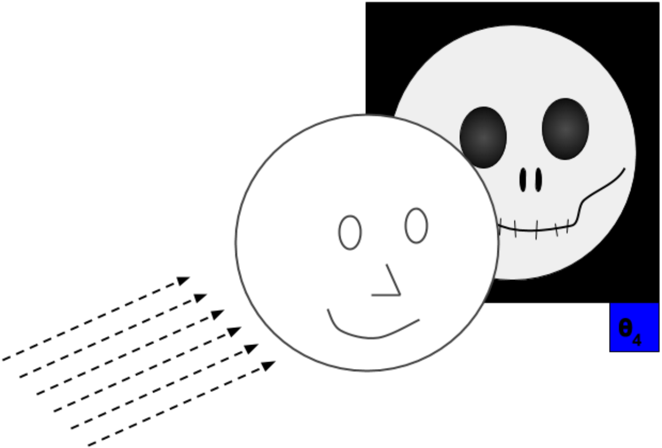

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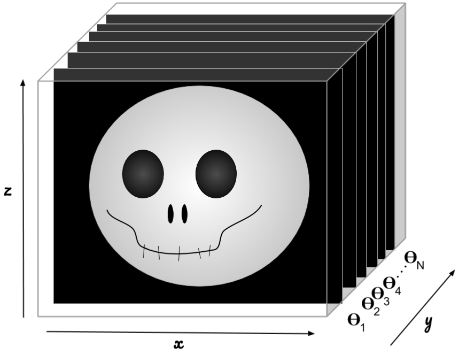

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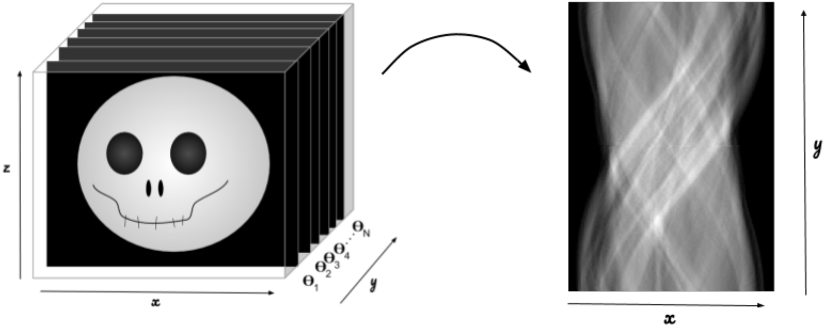
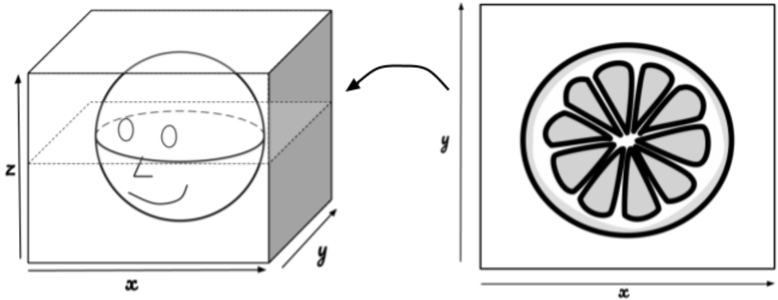
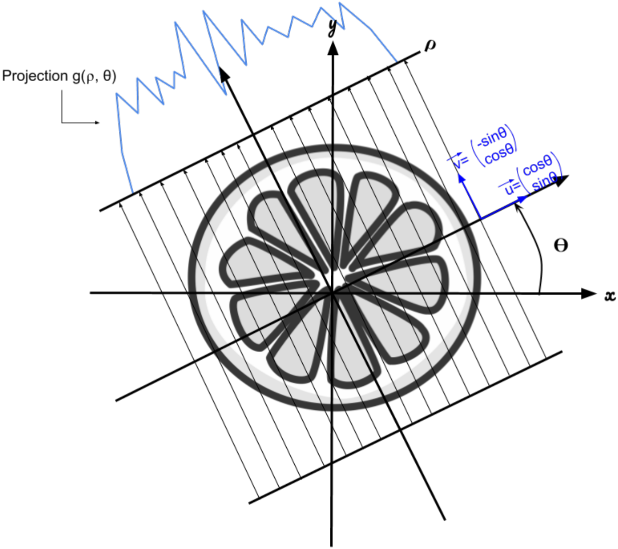


Feature Function



Objective: feature image $f(\boldsymbol{x}, \boldsymbol{y})$

Parametrization



Radon Transform I

$$g(\rho,\theta) = \int_{-\infty}^{\infty} f(\rho\vec{u}_\theta + s\vec{v}_\theta)ds \quad (1)$$

$$= \int_{-\infty}^{\infty} f(\rho\cos\theta - s\sin\theta, \rho\sin\theta + s\cos\theta)ds \quad (2)$$

with

$$\begin{cases} \vec{u} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \\ \vec{v} = \vec{u}^{\perp} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \end{cases}$$

Radon Transform II

Discrete:

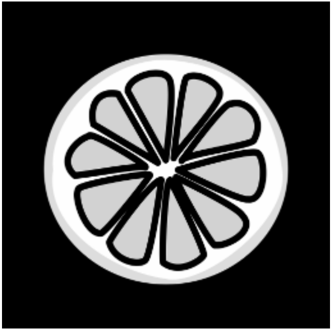
$$g(\rho,\theta) = \sum_{j=0}^{N-1} f(\rho\vec{u}_\theta + s_j\vec{v}_\theta)\Delta s \tag{3}$$

$$= \Delta s \sum_{j=0}^{N-1} f(\rho\cos\theta - s_j\sin\theta, \rho\sin\theta + s_j\cos\theta) \tag{4}$$

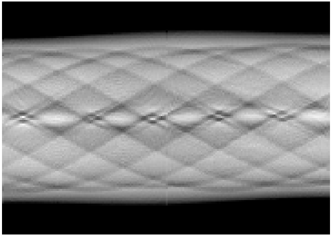
with

$$\begin{cases} \Delta s = \frac{2a}{N}, & \text{"length(object)"} \leq 2a \\ N & \text{Number of pixels at detector} \end{cases}$$

Example



Original feature image

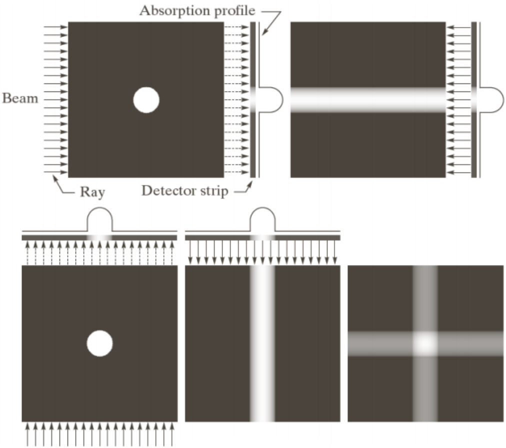


Radon transform of the feature image (sinogram)

Inversion

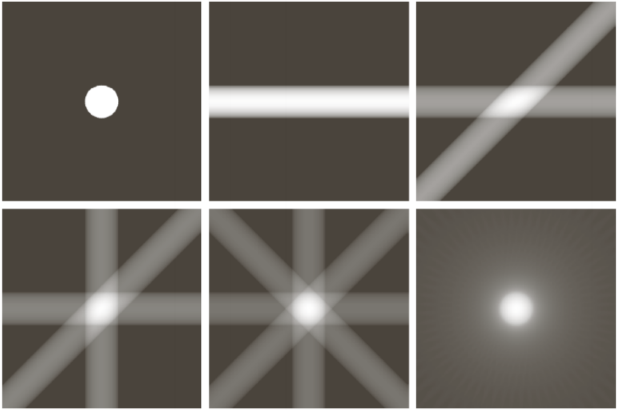
- Backprojection
- Fourier Slice Theorem
- Filtered Backprojection

Backprojection I



What is backprojection?

Backprojection II



Fourier Slice Theorem

- The one-dimensional Fourier transform of a projection (signal) at an angle θ of a feature image is the slice, at the same angle θ, of the 2 dimensional Fourier transform of the same feature image.

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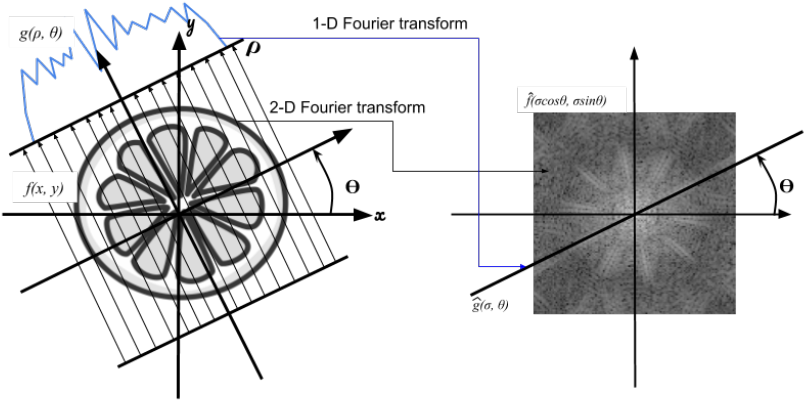
Theorem: For all $\sigma \geq 0$ and $\theta \in [0, \pi)$

$$\hat{f}(\sigma \cos\theta, \sigma \sin\theta) = \hat{g}(\sigma, \theta)$$

Fourier Slice Theorem



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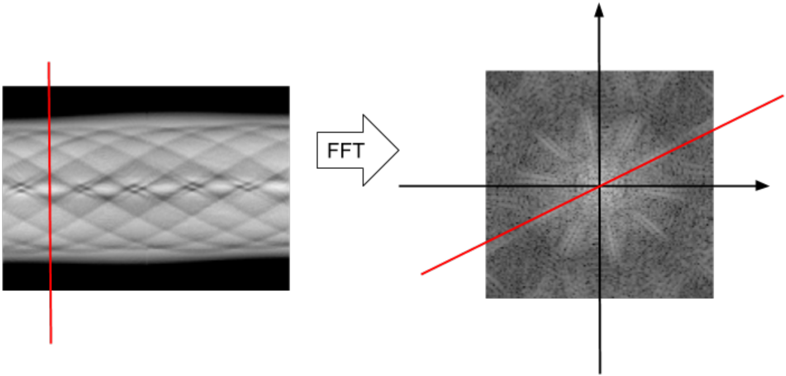

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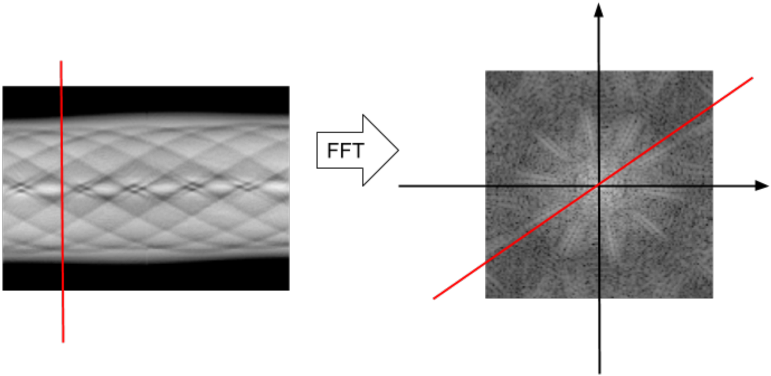

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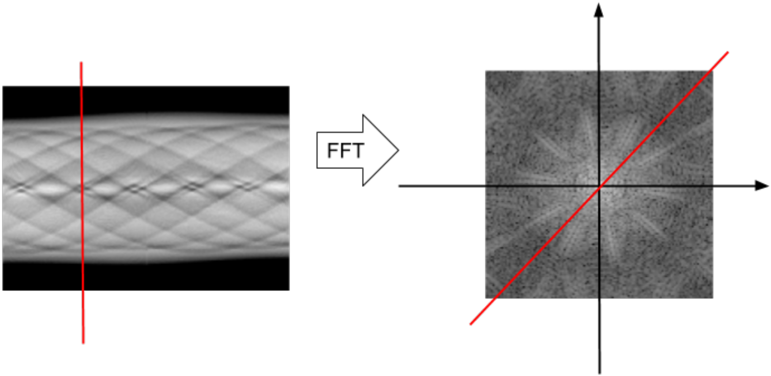

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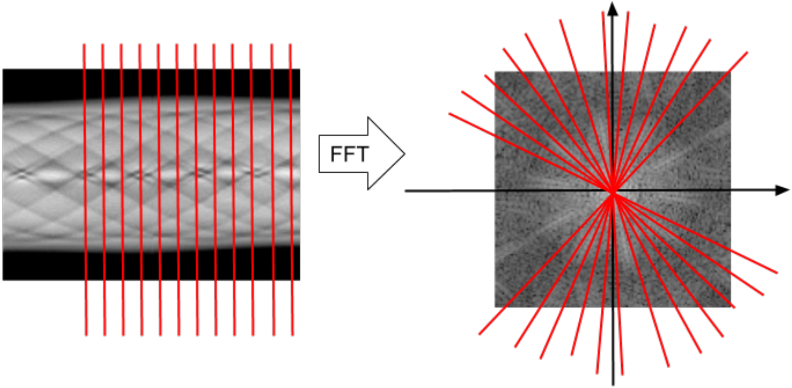

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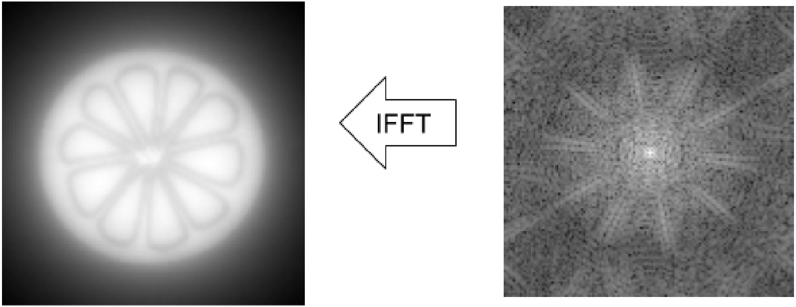
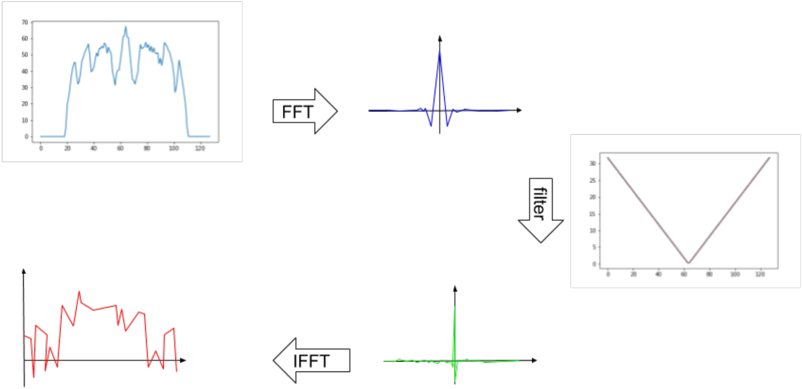


Filtered Backprojection

Filter before backprojection:

Filter before backprojection:



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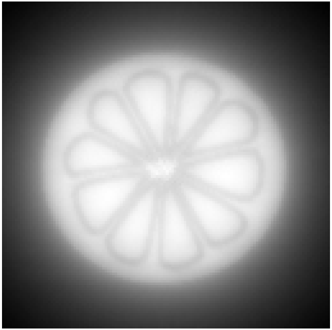
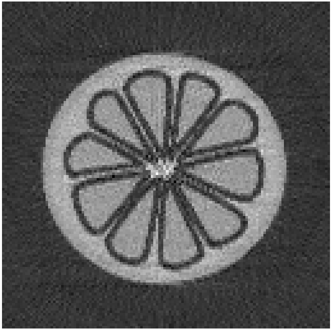


Backprojection



Filtered backprojection

Conclusion

- Fourier slice vs. backprojection loop.
- High frequency data loss resulting in loss of border details.

References

- A. C. Kak, and Malcolm Slaney. *Principles of Computerized Tomographic Imaging*, IEEE Press, 1999.
- Müller, Paul & Schürmann, Mirjam Guck, Jochen. (2015). The Theory of Diffraction Tomography.
- Rafael C. Gonzalez and Richard E. Woods. 2006. Digital Image Processing (3rd Edition). Prentice-Hall, Inc., Upper Saddle River, NJ, USA.