

Literature review: Reinforcement Learning

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0.1 Markov Decision Processes

subfiles

Bellman (1957) introduced the concept of a Markov Decision Process (MDP) as an extension of the famous idea of Markov chains. Markov decision processes are a standard model for sequential decision making and control problems. An MDP is fully defined by the 5-tuple $(\mathcal{S}, \mathcal{A}, \mathcal{P}(\cdot|\cdot, \cdot), \mathcal{R}(\cdot, \cdot), \gamma)$. Whereby:

- \mathcal{S} is the set of states $s \in \mathcal{S}$.
- \mathcal{A} is the set of actions $a \in \mathcal{A}$.
- $\mathcal{P}(s'|s, a)$ where $s, s' \in \mathcal{S}$, $a \in \mathcal{A}$ is a transition kernel which states the probability of transitioning to state s' from state s after performing action a . $\mathcal{P} : \mathcal{S} \times \mathcal{A} \rightarrow \mathcal{S}$. If the environment is stochastic, as opposed to deterministic, the function \mathcal{P} maps a state-action pair to a distribution over states in \mathcal{S} .
- $\mathcal{R}(s, a)$ where $s \in \mathcal{S}$, $a \in \mathcal{A}$; is the reward function, which returns the immediate reward (typically in the range $[-1, 1]$) of performing action a in state s . $\mathcal{R} : \mathcal{S} \times \mathcal{A} \rightarrow \mathcal{R}$. The reward at time step t can be interchangeably written as r_t or $r(s_t, a_t)$.
- $\gamma \in [0, 1]$ is the discount factor, which represent the difference in importance between the current reward and future rewards. This is often used as a variance reduction method, and aids proofs in infinitely running environments. (links?)

From here we can introduce the notion of an agent. (link to control theory?), An agent is an entity that on every state $s_t \in \mathcal{S}$ it can take an action $a_t \in \mathcal{A}$ in an environment transforming the environment from s_t to s_{t+1} . The behaviour of an agent is fully defined by a policy π . A policy π is a mapping from states to actions, $\pi : \mathcal{S} \rightarrow \mathcal{A}$. The agent chooses which action a_t to take in every state s_t by querying its policy such that $a_t = \pi(s_t)$. If the policy is stochastic, π will map an action to a distribution over action $a_t \sim \pi(s_t)$. The objective for an agent is to find an *optimal* policy, which tries to maximize the cumulative sum of possibly discounted rewards.

There are two functions of special relevance in reinforcement learning, the *state value* function $V^\pi(s)$ and the *action value* function $Q^\pi(s, a)$:

- The state value function $V^\pi(s)$ under a policy π , where $s \in \mathcal{S}$, represents the expected sum of rewards obtained by starting in state s and following the policy π until termination. Formally defined as $V^\pi(s) = \mathbb{E}^\pi[\sum_{t=0}^{\infty} r(s_t, a_t) | s_0 = s]$
- The state-action value function $Q^\pi(s, a)$ under a policy π , where $s \in \mathcal{S}$, $a \in \mathcal{A}$, represents the expected sum of rewards obtained by performing action a in state s and then following policy π . Formally defined as: $Q^\pi(s, a) = \mathbb{E}^\pi[r(s_0, a_0) + \sum_{t=1}^{\infty} r(s_t, a_t) | s_0 = s, a_0 = a]$

The Bellman equations are the most straight forward, dynamic programming approach at solving MDPs (Bertsekas, 2007; Bellman, 1957).

0.2 Bellman equations and optimality principle

Note that in general it is not the case that all actions $a \in \mathcal{A}$ can be taken on every state $s_t \in \mathcal{S}$.

The optimality principle, found in Bellman (1957), states the following: An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. The optimality principle, coupled with the proof of the existence of a deterministic optimal policy for any MDP as outlined in (Borkar, 1988) give rise to the optimal state value function $V^*(s) = \operatorname{argmax}_\pi V^\pi(s) = V^{\pi^*}(s)$ and the optimal action value function $Q^*(s, a) = \operatorname{argmax}_\pi Q^\pi(s, a) = Q^{\pi^*}(s, a)$. The optimal value functions determine the best possible performance in a MDP. An MDP is considered *solved* once the optimal value functions are found.

Most of the field of reinforcement learning research focuses on approximating these two equations (Tamar et al., 2017) (Watkins and Dayan, 1992) (Mnih et al., 2013). (cite many more)

Bellman (1957) outlined two analytical equations for the state value and action value function:

$$V^\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s) * (r(s, a) + \sum_{s' \in \mathcal{S}} \mathcal{P}(s'|s, a) * V^\pi(s')) \quad (1)$$

$$Q^\pi(s, a) = r(s, a) + \sum_{s' \in \mathcal{S}} \mathcal{P}(s'|s, a) * (\sum_{a' \in \mathcal{A}} \pi(a'|s') Q^\pi(s', a')) \quad (2)$$

Most RL algorithms can be divided into the following categories: Policy based Value based actor critic

0.3 Q-learning

subfiles The Q-learning algorithm was first introduced by Watkins and Dayan (1992)

References

- Bellman, R. (1957). *Dynamic Programming*, volume 70.
- Bertsekas, D. P. (2007). *Dynamic Programming and Optimal Control, Vol. II*, volume 2.
- Borkar, V. S. (1988). A convex analytic approach to Markov decision processes. *Probability Theory and Related Fields*, 78(4):583–602.
- Mnih, V., Kavukcuoglu, K., Silver, D., Graves, A., Antonoglou, I., Wierstra, D., and Riedmiller, M. (2013). Playing Atari with Deep Reinforcement Learning. pages 1–9.
- Tamar, A., Wu, Y., Thomas, G., Levine, S., and Abbeel, P. (2017). Value iteration networks. *IJCAI International Joint Conference on Artificial Intelligence*, (Nips):4949–4953.
- Watkins, C. J. and Dayan, P. (1992). Technical Note: Q-Learning. *Machine Learning*, 8(3):279–292.