Literature review: Reinforcement Learning

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0.1 Markov Decision Processes

subfiles

Bellman (1957) introduced the concept of a Markov Decision Process (MDP) as an extension of the famous idea of Markov chains. Markov decision processes are a standard model for sequential decision making and control problems. An MDP is fully defined by the 5-tuple $(\mathcal{S}, \mathcal{A}, \mathcal{P}(\cdot|\cdot,\cdot), \mathcal{R}(\cdot,\cdot), \gamma)$. Whereby:

- $\mathcal{S}$ is the set of states $s \in \mathcal{S}$.
- $\mathcal{A}$ is the set of actions $a \in \mathcal{A}$.
- $\mathcal{P}(s'|s,a)$ where $s, s' \in \mathcal{S}$, $a \in \mathcal{A}$ is a transition kernel which states the probability of transitioning to state s' from state s after performig action a. $\mathcal{P} : \mathcal{S} \times \mathcal{A} \to \mathcal{S}$. If the environment is stochastic, as opposed to deterministic, the function $\mathcal{P}$ maps a state-action pair to a distribution over states in $\mathcal{S}$.
- $\mathcal{R}(s,a)$ where $s \in \mathcal{S}$, $a \in \mathcal{A}$; is the reward function, which returns the immediate reward (typically in the range $[-1, 1]$) of performing action a in state s. $\mathcal{R} : \mathcal{S} \times \mathcal{A} \to \mathcal{R}$. The reward at time step t can be interchangably written as r_t or $r(s_t, a_t)$.
- $\gamma \in [0, 1]$ is the discount factor, which represent the difference in importance between the current reward and future rewards. This is often used as a variance reduction method, and aids proofs in infinitely running environments. (links?)

From here we can introduce the notion of an agent. (link to control theory?), An agent is an entity that on every state $s_t \in \mathcal{S}$ it can take an action $a_t \in \mathcal{A}$ in an environment transforming the environment from s_t to s_{t+1}. The beheviour of an agent is fully defined by a policy π. A policy π is a mapping from states to actions, $\pi : \mathcal{S} \to \mathcal{A}$. The agent chooses which action a_t to take in every state s_t by querying its policy such that $a_t = \pi(s_t)$. If the policy is stochastic, π will map an action to a distribution over action $a_t \sim \pi(s_t)$. The objective for an agent is to find an *optimal* policy, which tries to maximize the cumulative sum of possibly discounted rewards.

There are two functions of special relevance in reinforcement learning, the *state value* function $V^\pi(s)$ and the *action value* function $Q^\pi(s,a)$:

- The state value function $V^\pi(s)$ under a policy π, where $s \in \mathcal{S}$, represents the expected sum of rewards obtained by starting in state s and following the policy π until termination. Formally defined as $V^\pi(s) = \mathbb{E}^\pi[\sum_{t=0}^{\infty} r(s_t, a_t)|s_0 = s]$
- The state-action value function $Q^\pi(s,a)$ under a policy π, where $s \in \mathcal{S}$, $a \in \mathcal{A}$, represents the expected sum of rewards obtained by performing action a in state s and then following policy π. Formally defined as: $Q^\pi(s,a) = \mathbb{E}^\pi[r(s_0, a_0) + \sum_{t=1}^{\infty} r(s_t, a_t)|s_0 = s, a_0 = a]$

The Bellman equations are the most straight forward, dynamic programming approach at solving MDPs (Bertsekas, 2007; Bellman, 1957).

0.2 Bellman equations and optimality principle

Note that in general it is not the case that all actions $a \in \mathcal{A}$ can be taken on every state $s_t \in \mathcal{S}$.

The optimality principle, found in Bellman (1957), states the following: An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. The optimality principle, coupled with the proof of the existance of a deterministic optimal policy for any MDP as outlined in (Borkar, 1988) give rise to the optimal state value function $V^*(s) = \text{argmax}_\pi V^\pi(s) = V^{\pi^*}(s)$ and the optimal action value function $Q^*(s,a) = \text{argmax}_\pi Q^\pi(s,a) = Q^{\pi^*}(s,a)$. The optimal value functions determine the best possible performance in a MDP. An MDP is considered *solved* once the optimal value functions are found.

Most of the field of reinforcement learning research focuses on approximating these two equations (Tamar et al., 2017) (Watkins and Dayan, 1992) (Mnih et al., 2013). (cite many more)

Bellman (1957) outlined two analytical equations for the state value and action value function:

$$V^\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s) * (r(s,a) + \sum_{s' \in \mathcal{S}} \mathcal{P}(s'|s,a) * V^\pi(s')) \tag{1}$$

$$Q^\pi(s,a) = r(s,a) + \sum_{s' \in \mathcal{S}} \mathcal{P}(s'|s,a) * (\sum_{a' \in \mathcal{A}} \pi(a'|s') Q^\pi(s',a')) \tag{2}$$

Most RL algorithms can be devided into the following categories: Policy based Value based actor critic

A further categorization of algorithms is the notion of *model free* and *model based* algorithms. Consider a *model* of an environment to be the transition function $\mathcal{P}$ and reward function $\mathcal{R}$. Model free algorithms aim to approximate an optimal policy without them. Model based algorithms are either given a

prior model that they can use for planning (Browne et al., 2012; Soemers, 2014), or they learn a representation via their own interaction with the environment (Sutton, 1991; Guzdial et al., 2017). Note that an advantage of learning your own model is that you can choose a representation of the environment that is relevant to the agent's actions, which can have the advantage of modelling uninteresting (but perhaps complicated) environment behaviour (Pathak et al., 2017).

0.3 Q-learning

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The Q-learning algorithm was first introduced by Watkins (1989), and is arguably one of the most famous and widely implemented methods in the entire field. Given an MDP, Q-learning aims to calculate the corresponding optimal action value function Q^*, following the principle of optimality and proof of the existence of an optimal policy as described in Section 0.1. It is model free, learning via interaction with the environment, and it is an offline algorithm. The latter is because, even though we are learning the optimal action value function Q^*, we can choose any to gather experience from the environment with any policy of our choosing. This policy is often named an *exploratory* policy. Researchers like Tijsma et al. (2017) benchmarked the efficiency of using various exploratory policies in grid world stochastic maze environments.

Q-learning has been proven to converge to the optimal solution for an MDP under some assumptions:

1. Each state-action pair is visited an infinite number of times. (Watkins, 1989)
2. The sequence of updates of Q-values has to be monotonically increasing $Q(s_i, a_i) \leq Q(s_{i+1}, a_{i+1})$. (Thrun and Schwartz, 1993).
3. The learning rate α must decay over time, and such decay must be slow enough so that the agent can learn the optimal Q values. Expressed formally: $\sum_t \alpha_t = \infty$ and $\sum_t (\alpha_t)^2 < \infty$. (Watkins, 1989)

Algorithm 1: Q-learning

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repeat
    Select action a from policy ;
    Observe successor state s' and reward r after taking action a ;
    Update Q(s,a) ← Q(s,a) + α[r + argmax_a' Q(s',a') − Q(s,a)] ;
until done;
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Q-learning features its own share of imperfections, Watkins and Dayan (1992) tell us that the algorithm converges to optimality with probability 1 if each state-action pair is represented discretely, that is, if it is implemented in tabular form.

If there is a function approximator[1] in place, Thrun and Schwartz (1993) shows that if the approximation error is greater than a threshold which depends on the discount factor γ and episode length, then a systematic overestimation effect, which happens mainly due to the joint effort of function approximation methods and the max operator. On top of this, Kaisers and Tuyls (2010) introduces the concept of *Policy bias*, which states that state-action pairs that are favoured by the policy are chosen more often, biasing the updates. Ideally all state-action pairs are updated on every step. However, because agent's actions modify the environment, this is generally not possible in absence of an environment model. Frequency Adjusted Q-learning (FAQL) proposes scaling the update rule of Q-learning inversely proportional to the likelihood of choosing the action taken at that step (Kaisers and Tuyls, 2010). Abdallah et al. (2016) introduces Repeated Update Q-learning (RUQL), a more promising Q-learning spin off that proposes running the update equation *multiple times*, where the number of times is inversely proportional to the probability of the action selected given the policy being followed.

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[1]With neural networks being the most famous function approximators in reinforcement learning at the time of writing.

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