FASIT OPPGAVER UKE 6-7, STA-1001 2024

3.14

a) $P(\text{mindre enn } 12) = P(X < 12/60) = F(0.2) = 0.551$

b) $P(\text{mindre enn } 12) = \int_0^{0.2} 4e^{-4x} dx = 0.551$

3.30

a) $k = \frac{3}{16}$

b) $P\left(X < \frac{1}{2}\right) = \frac{99}{128}$

c) $P(|X| > 0.8) = P(X < -0.8) + P(X > 0.8) = 0.164$

3.41

a) $P(X + Y < 0.5) = \int_0^{0.5} \int_0^{0.5-y} 24xy\, dxdy = \int_0^{0.5} (3y - 12y^2 + 12y^3) dy = \frac{1}{16}$

b) $g(x) = \int_0^{1-x} 24xy\, dx = 12x(1-x)^2, \quad 0 \leq x \leq 1$

c) $f(y \mid x) = \frac{f(x,y)}{g(x)} = \frac{24xy}{12x(1-x)^2} = \frac{2y}{(1-x)^2}, \quad 0 \leq y \leq 1$

$P\left(Y < \frac{1}{8} \mid X = \frac{3}{4}\right) = \int_0^{1/8} f(y \mid x = 0.75) dy = \int_0^{1/8} \frac{2y}{(1-0.75)^2} dy = \frac{1}{4}$

4.1

$E(X) = \sum_x x f(x) = \sum_{x=1}^{4} x f(x) = 0.88$

4.13

$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_0^2 x f(x)\, dx = \int_0^1 x \cdot x\, dx + \int_1^2 x(2-x)\, dx = 1$

4.17

$E[g(X)] = E[(2X+1)^2] = \sum_x (2x+1)^2 f(x) = 209$

4.21

Fortjeneste er her $g(X) = 5000 \cdot X^2$.

$E[g(X)] = E\left(5000 \cdot X^2\right) = \int_0^1 5000 \cdot x^2 \cdot 2(1-x)\, dx = 833.33$

A3

For å finne sannsynlighetene kan vi integrere $f(x, y)$ over det aktuelle området.

1. $P(X < 1) = \int_0^{\infty}\int_0^{1} f(x, y)dxdy = \int_0^{\infty}\int_0^{1} 2e^{-2y}e^{-x}dxdy = \int_0^{\infty} 2e^{-2y}(1-e^{-1})dy = 1-e^{-1} = 0.632$
2. $P(X > Y) = \int_0^{\infty}\int_0^{x} f(x, y)dydx = \int_0^{\infty}\int_0^{x} e^{-x}2e^{-2y}dydx = \int_0^{\infty} e^{-x}\left(1-e^{-2x}\right)dx = \frac{1}{2}$

A4

a)

t	0	1	2
$P(T = t)$	0.20	0.35	0.45

e	0	1	2
$P(E = e)$	0.20	0.35	0.45

Betinga fordeling for E når $T = 2$:

t	0	1	2
$P(E = e \mid T = 2)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{6}{9}$

b) Prøver med $E = 0$ og $T = 0$:
$P(E = 0, T = 0) = 0.1$ $\qquad P(E = 0) \cdot P(T = 0) = 0.20 \cdot 0.20 = 0.04$
Avhengige.

c) Fordelinga for antall solge krabber, S:

s	0	1	2
$P(S = s)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{3}{10}$