Admissible heuristic with proof of admissibility

1. **Heuristic for British Man-servant world cost function (Exp)**

   Heuristic:

```
if(height_difference>0){
    // need to go up first and go straight
    return (height_difference * Math.exp(1) + (chebyshev_distance - height_difference));
}else if(height_difference<0){
    // need to go down first and go straight
    return (-(height_difference) * Math.exp(-1) + (chebyshev_distance - (-height_difference)));
}else{
    // go straight
    return chebyshev_distance;
}
```

   Prove that this heuristic always underestimates the real cost:

   The cost function is: $e^{(height\ of\ point\ 2 - height\ of\ point\ 1)}$

   There are three cases:

   a. The end point is higher than the current point.

      Heuristic: moving toward the end point with 1 step up each time. The total steps it needs to take equals to the Chebyshev distance. The Chebyshev distance is the shortest distance between two points on a 2D plan when you are allowed to move 8-direction.

      Estimated cost = $height\ difference * e^{(1)} + (\text{Chebyshev distance} - \text{heigth difference})$

   b. The end pointer is lower than the current point.

      Use the first move to go down at the same height as the goal point first.

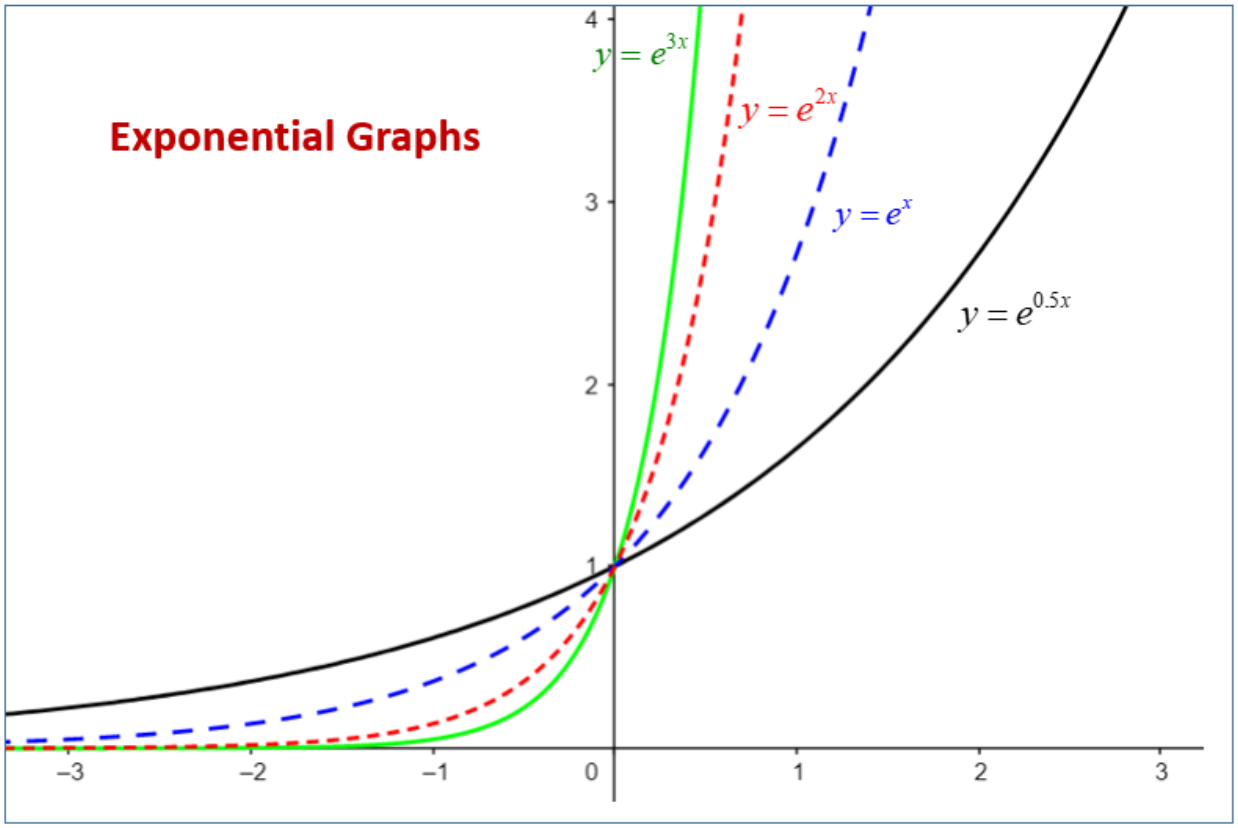
      Then, using the Chebyshev distance go straight to the end point

      Estimated cost = ($-height\ difference$) $* e^{(1)} + (\text{Chebyshev distance} - (-\text{heigth difference})$

   c. Two points are at the same height

      Estimated cost = $\text{Chebyshev distance}$

   Note: go straight mean using the Chebyshev distance, it is the shortest distance between two points on a 2D plan when you are allowed to move 8-direction.



Proof:

(a)We can see that from the graph, when x > 0, the bigger the x is, the faster the y grows. Therefore, when the end point is at a higher position than the current point, going up 1 step at a time would minimize the value of the exponential cost function. After we get to the same level as the goal point, we add the remaining path cost (Chebyshev distance − differnce of height).

The Chebyshev distance is the minimum steps for the start point go to the end point. Going up 1 step at a time minimize the exponential function. Therefore, the estimated cost is the minimum steps times the minimum cost of step.

Therefore, it would never overestimate the real cost.

(b) We can see that from the graph again, when x < 0, y is less than 1. To minizine the minimize the cost, we can go down 1 step each time. When we go down one at a time, it could minimize the average cost of move. If we only use one step to go down, it will only minimize the cost of the first move. Therefore, the best case is to go down 1 step at a time and when it at the same height as the end point. we add the remaining path cost (Chebyshev distance − (−differnce of height)).

(c) When two points have equal height, Chebyshev distance never overestimates the real cost since $e^0 = 1$.

2. Heuristic for President-Biden world cost function (div)

Cost function: $\frac{new\ height}{old\ height+1}$

Heuristic:

```
// heuristic funciton for Div cost function
private double getHeuristic(final TerrainMap map, final Point startingPoint) {

    Point endPoint = map.getEndPoint();
    double chebyshev_distance = Math.max(Math.abs(endPoint.x - startingPoint.x), Math.abs(endPoint.y - startingPoint.y));
    return chebyshev_distance*(0.5);
}
```

Case 1: (going up or same height→ new height – old height >= 0)

$$\frac{new\ height}{old\ height+1} \geq \frac{1}{2}$$

New height would be not 0. If new height is 0, old height would be negative since we are going up. However, the range of height is 0 to 255.

We know that new height could not be 0 and both heights are positive number, then $\frac{new\ height}{old\ height+1}$ $never\ less\ than$ $\frac{1}{2}$.

Case 2: (going down → new height – old height < 0)

If we go down to height 0, the cost of the step would be 0. Since 0 divided by everything is 0. Every time we go down to height 0, it takes 0 cost, and the point is 1 step (Chebyshev distance-1) closer to the goal. After we reach 0, we need to go up as there is no two adjacent tiles are at height 0. The least cost of going up is going one step up. The cost of doing this is 1. We can keep doing this until we reach the goal point. The average cost of each step would be ½. The minimum number of steps it takes to get to the goal point is their Chebyshev distance. Therefore, the minimum cost would be **Chebyshev distance * ½**.

Cost of shortest path

AStarDiv

-seed 1

PathCost, 198.59165501141644, Uncovered, 36316, TimeTaken, 289

-seed 2

PathCost, 198.56141550095256, Uncovered, 36310, TimeTaken, 254

-seed 3

PathCost, 198.4864317411443, Uncovered, 36229, TimeTaken, 256

-seed 4

PathCost, 198.70066424826052, Uncovered, 36206, TimeTaken, 242

-seed 5

PathCost, 198.25499446810397, Uncovered, 36068, TimeTaken, 261

AStarExp

-seed 1

PathCost, 533.4482191461119, Uncovered, 71649, TimeTaken, 324

-seed 2

PathCost, 549.5036346739352, Uncovered, 82020, TimeTaken, 355

-seed 3

PathCost, 510.97825243663607, Uncovered, 74036, TimeTaken, 321

-seed 4

PathCost, 560.6570436319696, Uncovered, 66383, TimeTaken, 289

-seed 5

PathCost, 479.5879215923168, Uncovered, 67837, TimeTaken, 428

Part 4 improvement of A*

1. Don’t use stack for getting the original path. Use one for loop to make it faster to reconstruct the path.
2. Initialize the size of the HashMap and the priority queue, make the size bigger than the number of it would need to expand.