Reconfigurable Intelligent Surface Empowered Multi-Hop Transmission over Generalized Fading

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***Abstract*—The use of multiple reconfigurable intelligent surfaces (RIS) between a source and destination can enhance the performance of wireless communications over severe shadowing environment by creating line-of-sight (LOS) connectivity. This paper analyzes the performance of a multiple RIS empowered multi-hop transmission for a wireless system. We develop an analytical framework to derive statistical results of the signal-to-noise ratio (SNR) of the multi-RIS communication by considering independent and non-identical double generalized gamma (dGG) fading channels in each hop. We analyze the performance of the considered multi-RIS system by deriving exact analytical expressions of the outage probability, average bit-error rate (BER), and ergodic capacity in terms of Fox's H-function. We present asymptotic analysis and diversity order of the outage probability in the high SNR regime to provide a better insight into the system performance. We use computer simulations to demonstrate the effect of multiple RIS modules, and fading parameters on the RIS-aided multi-hop transmissions for the considered communication system.**

***Index Terms*—Double generalized fading channels, multi-hop, performance analysis, reconfigurable intelligent surface (RIS).**

I. Introduction

Reconfigurable intelligent surface (RIS) is emerging as a disruptive technology for wireless communications [1]–[3]. RISs are artificial planar structures of metasurfaces, intelligently programmed to control electromagnetic waves in the desired direction. A single RIS module can contain hundreds of elements, each sub-wavelength size, to beamform the signal for enhanced performance without requiring expensive relaying procedures. The RIS can assist RF transmissions to resolve the problem of signal blockage for enhanced performance in fixed terrestrial and vehicular communication networks.

There has been an increased research interest to assess the applicability of RIS for heterogeneous wireless systems [4]–[8]. In the aforementioned and related research, a single RIS module has been employed for vehicular communications. However, the deployment of multiple RIS units is highly desirable for reliable connectivity. The authors in [9] developed algorithms for optimum placement of multiple RISs for a roadside deployment scenario considering the size and operating modes of RISs for vehicle-to everything (V2X) connectivity. To further substantiate, it is desirable to analyze the performance metrics such as outage probability, average BER, and ergodic capacity of multiple RIS enabled transmissions over fading channels. In [10], the authors proposed a deep reinforcement learning (DRL) based hybrid beamforming for multi-hop RIS-empowered terahertz system with Rician fading channel to improve the coverage range at terahertz-band frequencies. Recently, the authors in [11] analyzed the system performance of a multi-hop RIS assisted free space optics (FSO) system considering Gamma-Gamma atmospheric turbulence with pointing errors. They modeled the multi-RIS as a cascaded system, and used the mathematical induction method to derive the probability density function (PDF) and cumulative distribution function (CDF) of the Gamma-Gamma fading channel and zero-boresight pointing errors. However, the induction method may not be applicable readily for other atmospheric turbulence and RF fading distributions such as double generalized Gamma (dGG) [12], [13]. The dGG model is suitable for modeling non-homogeneous double-scattering radio propagation fading conditions over RF frequencies, and contains most of the existing statistical models such as double-Nakagami, double-Rayleigh, double Weibull, and Gamma-Gamma as special cases.

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In this paper, we analyze the performance of a RF system by employing multiple RISs for multi-hop transmissions for reliable connectivity between a source and destination considering independent and non-identical (i.ni.d) dGG fading channels in each hop. We develop an analytical framework to derive density and distribution functions of the end-to-end SNR of a multi-RIS communication system over the dGG fading model without any constraint on its parameters. It is noted that the existing representation of the PDF of the dGG model requires the ratio of shape parameters to be an integer for the performance analysis of FSO systems [14], [15] and equal shape parameters for the statistical analysis of vehicular communications [13]. We analyze the performance of the considered system by deriving exact analytical expressions of the outage probability, average bit-error-rate (BER), and ergodic capacity in terms of Fox's H-function. We present asymptotic analysis in the high SNR regime for outage probability and average BER in terms of simpler Gamma functions, and derive diversity order depicting the impact of fading parameters on the performance of the considered system. We use computer simulations to demonstrate the performance of the multi-hop system and validate the accuracy of derived analytical expressions through Monte-Carlo simulations for various fading scenarios.

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Notations: $\jmath$ denotes the imaginary number, $\mathbb{E}[.]$ denotes the expectation operator, $\exp(.)$ denotes the exponential function, while $\Gamma(a) = \int_0^\infty u^{a-1}e^{-u}\,du$ denotes the Gamma function. Finally, $G_{p,q}^{m,n}\left(x\middle| \begin{matrix}\{a_w\}_{w=1}^{p}\\ \{b_w\}_{w=1}^{q}\end{matrix}\right)$ and $H_{p,q}^{m,n}\left(x\middle| \begin{matrix}\{(a_w,A_w)\}_{w=1}^{p}\\ \{(b_w,B_w)\}_{w=1}^{q}\end{matrix}\right)$ denote Meijer-G and Fox's H functions respectively and a shortened notation $\{a_i\}_1^N = \{a_1,\cdots,a_N\}$ to represent their coefficients.

The paper is organized as follows: system model is summarized in section II followed by performance analysis through exact and asymptotic expressions in section III. The numerical and simulation results are discussed in section IV. Finally, the paper concludes with section V.

II. System Model

We consider a multi-hop RF system assisted by multiple RISs in the transmission link as shown in Fig. 1. The source (S) intends to transmit signals to the destination (D). We employ multiple RIS (denoted by $K-1$ with $K \geq 2$) since a single RIS may not provide sufficient LOS communications due to the presence of obstacles, as shown in Fig. 1. Thus, the received signal at destination connected through $(K-1)$-th RIS is given by

$$y = \prod_{i=1}^{K} g_i s + w_D \tag{1}$$

where s is the transmit signal with power P, w_D is the additive white Gaussian noise (AWGN) at the destination with variance σ_D^2, g_1 is the channel coefficient from the source to the first RIS, g_i is the channel from $(i-1)$-RIS to i-th RIS, and g_K is the channel coefficient of the last $(K-1)$-RIS to the destination. Here, $g_i = g_i^{(l)} g_i^{(f)}$ is the combined channel of path gain $g_i^{(l)}$ and short term fading $g_i^{(f)}$.

We denote SNR of the multi-RIS system as $\gamma = \bar{\gamma}|g|^2$, where $\bar{\gamma} = \frac{P|g_l|^2}{\sigma_D^2}$ is the average SNR. Here, $g_l = \prod_{i=1}^{K} g_i^{(l)}$ and $g = \prod_{i=1}^{K} g_i^{(f)}$ are the overall path gain and short term fading channel of the multi-hop transmission.

We consider the dGG distribution to model the double multi-path fading for the RF link as the product of two Gamma random variables $g_i^{(f)} = \chi_1\chi_2$, where $\chi_1 \sim \mathcal{GG}(\alpha_1,\beta_1,\Omega_1)$ and $\chi_2 \sim \mathcal{GG}(\alpha_2,\beta_2,\Omega_2)$. The PDF of a single generalized Gamma random variable, $\chi \sim \mathcal{GG}(\alpha,\beta,\Omega)$ is given as

$$f_\chi(x) = \frac{\alpha x^{\alpha\beta-1}}{(\frac{\Omega}{\beta})^{\beta}\Gamma(\beta)} \exp\left(-\frac{\beta}{\Omega}x^{\alpha}\right) \tag{2}$$

where parameters α, β are Gamma distribution shaping parameters and the parameter $\Omega = \left(\frac{\mathbb{E}[\chi^2]\Gamma(\beta)}{\Gamma(\beta+2/\alpha)}\right)^{\alpha/2}$ is the α-root mean value.

III. Performance Analysis

To facilitate the performance analysis, we require density and distribution functions of the multi-RIS channel $g = \prod_{i=1}^{K} g_i^{(f)}$. In the following proposition, we express the PDF of dGG distributed $g_i^{(f)}$ in terms of Fox-H function to remove the constraint of integer-valued fading parameter manifestation of

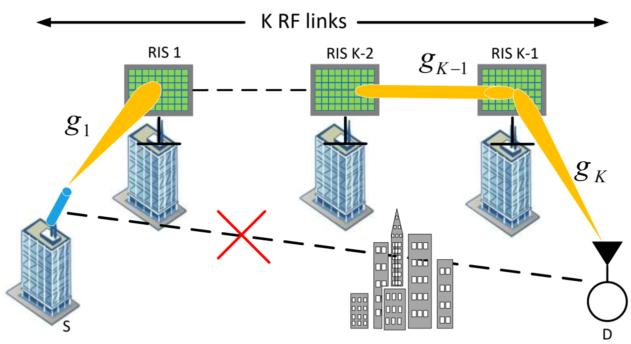

Fig. 1: Schematic diagram of a multiple RIS empowered communication system.

Meijer-G representation [12], [14] and equal shape parameters for the statistical analysis of vehicular communications [13].

Proposition 1. *The PDF of double generalized-gamma $g_i^{(f)} = \chi_1\chi_2$, where $\chi_1 \sim \mathcal{GG}(\alpha_1,\beta_1,\Omega_1)$ and $\chi_2 \sim \mathcal{GG}(\alpha_2,\beta_2,\Omega_2)$ is given by*

$$f_{g_i^{(f)}}(x) = \frac{x^{\alpha_2\beta_2-1}}{(\frac{\Omega_1}{\beta_1})^{\frac{\alpha_2\beta_2}{\alpha_1}}(\frac{\Omega_2}{\beta_2})^{\beta_2}\Gamma(\beta_1)\Gamma(\beta_2)} H_{0,2}^{2,0}\left[\zeta x \,\middle|\, \begin{matrix} - \\ (0,\frac{1}{\alpha_2}),(\frac{\alpha_1\beta_1-\alpha_2\beta_2}{\alpha_1},\frac{1}{\alpha_1})\end{matrix}\right] \tag{3}$$

where $\zeta = \left(\frac{\beta_2}{\Omega_2}\right)^{\frac{1}{\alpha_2}}\left(\frac{\beta_1}{\Omega_1}\right)^{\frac{1}{\alpha_1}}$.

Proof: Using the PDF of the product of two random variables $f_{g_i^{(f)}}(x) = \int_0^\infty \frac{1}{u} f_{\chi_1}(\frac{x}{u}) f_{\chi_2}(u)\,du$

$$f_{g_i^{(f)}}(x) = \frac{\alpha_1\alpha_2 x^{\alpha_1\beta_1-1}}{(\frac{\Omega_1}{\beta_1})^{\beta_1}(\frac{\Omega_2}{\beta_2})^{\beta_2}\Gamma(\beta_1)\Gamma(\beta_2)} \int_0^\infty u^{\alpha_2\beta_2-\alpha_1\beta_1-1} \exp\left(-\frac{\beta_1 x^{\alpha_1}}{\Omega_1}u^{-\alpha_1}\right)\exp\left(-\frac{\beta_2}{\Omega_2}u^{\alpha_2}\right)du \tag{4}$$

Using $u^{-\alpha_1} = t$, and representing the exponential function using Meijer-G, we get

$$f_{g_i^{(f)}}(x) = \frac{\alpha_2 x^{\alpha_1\beta_1-1}}{(\frac{\Omega_1}{\beta_1})^{\beta_1}(\frac{\Omega_2}{\beta_2})^{\beta_2}\Gamma(\beta_1)\Gamma(\beta_2)} \int_0^\infty t^{\frac{\alpha_1\beta_1-\alpha_2\beta_2}{\alpha_1}-1} G_{0,1}^{1,0}\left[\frac{\beta_1 x^{\alpha_1}}{\Omega_1}t \,\middle|\, \begin{matrix}-\\0\end{matrix}\right] G_{1,0}^{0,1}\left[\frac{\Omega_2}{\beta_2}t^{\frac{\alpha_2}{\alpha_1}} \,\middle|\, \begin{matrix}1\\-\end{matrix}\right]dt \tag{5}$$

Applying identity [16, 07.34.21.0012.01] in (5), we get (3). ∎

The derived PDF in (3) does not have any constraint on the parameters of dGG as present in the Meijer's-G representation [12]. It should be mentioned that the use of Fox's-H function for performance analysis is becoming popular similar to the Meijer's-G function. Recently MATHEMATICA introduced Fox-H function for the computation of single-variate Fox's-H function.

We develop a unifying framework to derive the PDF and CDF of the multi-RIS channel g in the following Theorem:

Theorem 1. *If X_i, $i = 1\cdots K$ are i.ni.d random variables with a PDF of the form*

$$f_{X_i}(x) = \psi_i x^{\phi_i-1} H_{p,q}^{m,n}\left[\zeta_i x \,\middle|\, \begin{matrix}\{(a_{i,j},A_{i,j})\}_{j=1}^{p}\\ \{(b_{i,j},B_{i,j})\}_{j=1}^{q}\end{matrix}\right] \tag{6}$$

then the PDF and CDF of $X = \prod_{i=1}^{K} X_i$ are given by

$$f_X(x) = \frac{1}{x}\prod_{i=1}^{K}\psi_i\zeta_i^{-\phi_i}$$

$$H_{Kp,Kq}^{Km,Kn}\left[\ \prod_{i=1}^{K}\zeta_i x \ \middle|\ \begin{matrix}\{\{(a_{i,j}+A_{i,j}\phi_i, A_{i,j})\}_{j=1}^{p}\}_{i=1}^{K}\\ \{\{(b_{i,j}+B_{i,j}\phi_i, B_{i,j})\}_{j=1}^{q}\}_{i=1}^{K}\end{matrix}\ \right] \tag{7}$$

$$F_X(x) = \prod_{i=1}^{K}\psi_i\zeta_i^{-\phi_i} H_{Kp+1,Kq+1}^{Km,Kn+1}$$

$$\left[\ \prod_{i=1}^{K}\zeta_i x \ \middle|\ \begin{matrix}(1,1),\{\{(a_{i,j}+A_{i,j}\phi_i, A_{i,j})\}_{j=1}^{p}\}_{i=1}^{K}\\ \{\{(b_{i,j}+B_{i,j}\phi_i, B_{i,j})\}_{j=1}^{q}\}_{i=1}^{K},(0,1)\end{matrix}\ \right] \tag{8}$$

Proof: See Appendix A. ■

In what follows, we capitalize Theorem 1 to present the PDF and CDF of the SNR of the multiple RIS empowered system:

Corollary 1. *The PDF and CDF of the SNR for the multi-hop RIS system are given by*

$$f_\gamma(\gamma) = \frac{1}{2\gamma}\psi H_{0,2K}^{2K,0}\left[\ U\sqrt{\tfrac{\gamma}{\bar{\gamma}}}\ \middle|\ \begin{matrix}-\\ V\end{matrix}\ \right] \tag{9}$$

$$F_\gamma(\gamma) = \psi H_{1,2K+1}^{2K,1}\left[\ U\sqrt{\tfrac{\gamma}{\bar{\gamma}}}\ \middle|\ \begin{matrix}(1,1)\\ V,(0,1)\end{matrix}\ \right] \tag{10}$$

where $\psi = \prod_{i=1}^{K}\frac{1}{\Gamma(\beta_{i,1})\Gamma(\beta_{i,2})}$, $U = \prod_{i=1}^{K}\left(\frac{\beta_{i,2}}{\Omega_{i,2}}\right)^{\frac{1}{\alpha_{i,2}}}\left(\frac{\beta_{i,1}}{\Omega_{i,1}}\right)^{\frac{1}{\alpha_{i,1}}}$ *and* $V = \{(\beta_{i,1},\frac{1}{\alpha_{i,1}}),(\beta_{i,2},\frac{1}{\alpha_{i,2}})\}_1^K$.

Proof: A straight forward application of Theorem 1 with simple transformation of random variables $f_\gamma(\gamma) = \frac{1}{2\sqrt{\bar{\gamma}\gamma}}f_g(\sqrt{\frac{\gamma}{\bar{\gamma}}}), F_\gamma(\gamma) = F_g(\sqrt{\frac{\gamma}{\bar{\gamma}}})$ completes the proof. ■

A. Outage Probability

An exact outage probability for the considered system can be obtained as $P_{\text{out}} = F_\gamma(\gamma_{\text{th}})$. We use the asymptotic result of a single-variate Fox's H-function in [17, Th. 1.11] to express the outage probability in the high SNR regime, as given in (11). Using the dominant term, we express (11) as $P_{\text{out}} \propto \bar{\gamma}^{-G_{\text{out}}}$ to get the outage diversity order, $G_{\text{out}} = \min\{\{\frac{\beta_{i,1}\alpha_{i,1}}{2},\frac{\beta_{i,2}\alpha_{i,2}}{2}\}_1^K\}$.

$$P_{\text{out}}^{\infty} = \psi\Big[\sum_{i=1}^{K}\frac{1}{\beta_{i,1}}\prod_{j=1,j\neq i}^{K}\Gamma(\beta_{j,1}-\beta_{i,1}\tfrac{\alpha_{i,1}}{\alpha_{j,1}})$$
$$\prod_{j=1}^{K}\Gamma(\beta_{j,2}-\beta_{i,1}\tfrac{\alpha_{i,1}}{\alpha_{j,2}})\left(U\sqrt{\tfrac{\gamma_{th}}{\bar{\gamma}}}\right)^{\beta_{i,1}\alpha_{i,1}}$$
$$+\sum_{i=1}^{K}\frac{1}{\beta_{i,2}}\prod_{j=1}^{K}\Gamma(\beta_{j,1}-\beta_{i,2}\tfrac{\alpha_{i,2}}{\alpha_{j,1}})$$
$$\prod_{j=1,j\neq i}^{K}\Gamma(\beta_{j,2}-\beta_{i,2}\tfrac{\alpha_{i,2}}{\alpha_{j,2}})\left(U\sqrt{\tfrac{\gamma_{th}}{\bar{\gamma}}}\right)^{\beta_{i,2}\alpha_{i,2}}\Big] \tag{11}$$

B. Average BER

The average BER of a communication system using the CDF of SNR can be represented as

$$\bar{P}_e = \frac{q^p}{2\Gamma(p)}\int_0^\infty \gamma^{p-1}e^{-q\gamma}F_\gamma(\gamma)d\gamma \tag{12}$$

where p and q are modulation specific parameters.

To derive $\bar{P}_e$, we substitute $F_\gamma(\gamma)$ in (12), expand the definition of Fox-H function and then interchange the order of integration to get

$$\bar{P}_e = \frac{q^p}{2\Gamma(p)}\psi\frac{1}{2\pi\jmath}\int_{\mathcal{L}}\prod_{i=1}^{K}\Gamma(\beta_{i,2}-\frac{s}{\alpha_{i,2}})\Gamma(\beta_{i,1}-\frac{s}{\alpha_{i,1}})$$
$$\left(U\sqrt{\frac{1}{\bar{\gamma}}}\right)^s\frac{\Gamma(s)}{\Gamma(s+1)}\left(\int_0^\infty\gamma^{p+\frac{s}{2}-1}e^{-q\gamma}d\gamma\right)ds \tag{13}$$

Using the inner integral $\int_0^\infty\gamma^{p+\frac{s}{2}-1}e^{-q\gamma}d\gamma = \frac{\Gamma(p+\frac{s}{2})}{q^{p+\frac{s}{2}}}$ in (13), and applying the definition of Fox-H function to get

$$\bar{P}_e = \frac{\psi}{2\Gamma(p)}H_{2,2K+1}^{2K,2}\left[\ \frac{U}{\sqrt{q\bar{\gamma}}}\ \middle|\ \begin{matrix}(1,1),(1-p,\frac{1}{2})\\ V,(0,1)\end{matrix}\ \right] \tag{14}$$

Since the average BER has a similar mathematical functional representation to the outage probability, we can use [17, Th. 1.11] to express average BER in the high SNR (15) to get the BER diversity order $G_{\text{BER}} = \min\{\{\frac{\beta_{i,1}\alpha_{i,1}}{2},\frac{\beta_{i,2}\alpha_{i,2}}{2}\}_1^K\}$. It can be seen that the diversity order using the average BER and outage probability has similar dependence on the channel and system parameters.

$$P_e^{\infty} = \frac{\psi}{2\Gamma(p)}\Big[\sum_{i=1}^{K}\frac{1}{\beta_{i,1}}\prod_{j=1,j\neq i}^{K}\Gamma(\beta_{j,1}-\beta_{i,1}\tfrac{\alpha_{i,1}}{\alpha_{j,1}})$$
$$\prod_{j=1}^{K}\Gamma(\beta_{j,2}-\beta_{i,1}\tfrac{\alpha_{i,1}}{\alpha_{j,2}})\Gamma(p+\tfrac{\beta_{i,1}\alpha_{i,1}}{2})\left(U\sqrt{\tfrac{1}{q\bar{\gamma}}}\right)^{\beta_{i,1}\alpha_{i,1}}$$
$$+\sum_{i=1}^{K}\frac{1}{\beta_{i,2}}\prod_{j=1,j\neq i}^{K}\Gamma(\beta_{j,2}-\beta_{i,2}\tfrac{\alpha_{i,2}}{\alpha_{j,2}})$$
$$\prod_{j=1}^{K}\Gamma(\beta_{j,1}-\beta_{i,2}\tfrac{\alpha_{i,2}}{\alpha_{j,1}})\Gamma(p+\tfrac{\beta_{i,2}\alpha_{i,2}}{2})\left(U\sqrt{\tfrac{1}{q\bar{\gamma}}}\right)^{\beta_{i,2}\alpha_{i,2}}\Big] \tag{15}$$

C. Ergodic Capacity

The ergodic capacity of a communication link over fading channels has the following representation:

$$\bar{\eta} = \int_0^\infty\log_2(1+\gamma)f_\gamma(\gamma)d\gamma \tag{16}$$

We substitute $f_\gamma(\gamma)$ in (16), expand the definition of Fox-H function and then interchange the order of integration to get

$$\bar{\eta} = \frac{\log_2(e)}{2}\psi\frac{1}{2\pi\jmath}\int_{\mathcal{L}}\prod_{i=1}^{K}\Gamma(\beta_{i,2}-\frac{s}{\alpha_{i,2}})\Gamma(\beta_{i,1}-\frac{s}{\alpha_{i,1}})$$
$$\left(U\sqrt{\frac{1}{\bar{\gamma}}}\right)^s\left(\int_0^\infty\gamma^{\frac{s}{2}-1}ln(1+\gamma)d\gamma\right)ds \tag{17}$$

To solve the inner integral, we express $ln(1+\gamma)$ in terms of Meijer-G function and use the identity [16, 07.34.21.0009.01], $I = \frac{\Gamma(\frac{s}{2})\Gamma(1-\frac{s}{2})\Gamma(\frac{s}{2})}{\Gamma(1+\frac{s}{2})}$. with the definition of Fox-H function to get (18).

$$\bar{\eta} = \frac{\log_2(e)}{2}\psi H_{2,2K+2}^{2K+1,2}\left[\ \frac{U}{\sqrt{\bar{\gamma}}}\ \middle|\ \begin{matrix}(1,\frac{1}{2}),(1,\frac{1}{2})\\ V,(1,\frac{1}{2}),(0,\frac{1}{2})\end{matrix}\ \right] \tag{18}$$

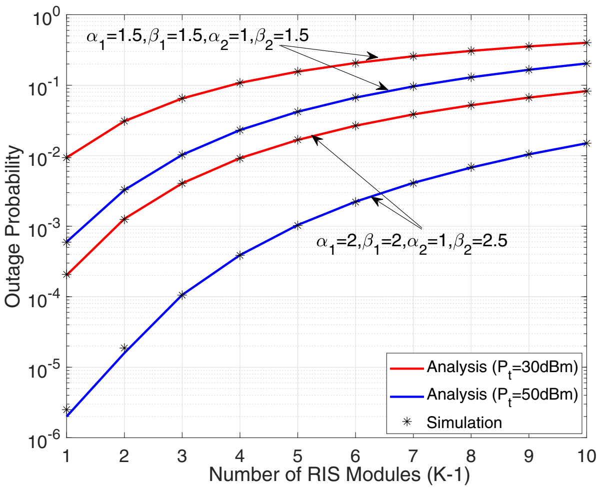

Fig. 2: Outage performance of multi-RIS system.

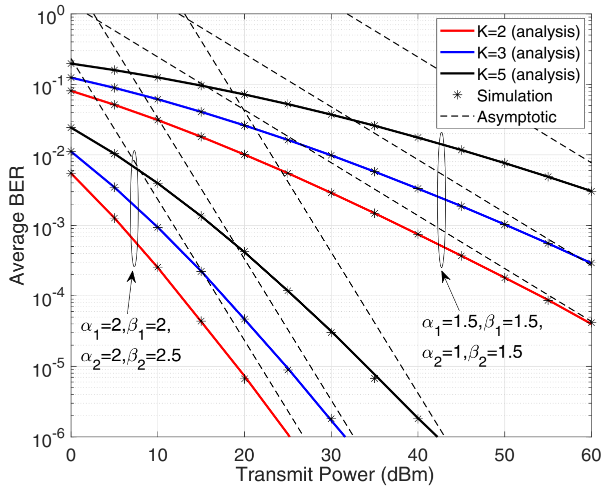

Fig. 3: Average BER of multi-RIS system.

IV. Simulation and Numerical Results

In this section, we demonstrate the performance of multiple RIS empowered RF communications and validate the derived analytical expressions through numerical and Monte-Carlo simulations. We consider varying fading parameters to demonstrate the effect of the diversity order on the system performance. We deploy $K-1$ RIS at an equal distance to establish reliable connection. The distance between the source and the last RF RIS is fixed at 100 m. We consider independent but identical distributed fading parameters in all the links i.e., $\alpha_{i,j} = \alpha_j$, $\beta_{i,j} = \beta_j$ and $\Omega_{i,j} = \Omega_j$ $\forall i$ and $j \in \{1, 2\}$. We use RF dGG parameters $\alpha_1 = 1.5$,$\alpha_2 = 1$,$\beta_1 = 1.5$,$\beta_2 = 1.5$,$\Omega_1 = 1.5793$,$\Omega_2 = 0.9671$ to model the RF link [13]. We consider transmit and receive antenna gains as $G_t = G_r = 25$ dB and a carrier frequency of 800 MHz for RF transmissions. A noise floor of -104.4 dBm is considered at the receiver.

We demonstrate the effectiveness of multiple RISs by plotting the outage probability in Fig. 2. Figure shows the performance of multi-RIS system vs number of RISs ($K-1$ RISs constituting K hops) at a transmit power of $P_t = 30$ dBm and $P_t = 50$ dBm. We consider two fading scenarios i.e., two sets of dGG parameters: scenario 1 ($\alpha_1 = 1.5$,$\alpha_2 = 1$,$\beta_1 = 1.5$,$\beta_2 = 1.5$,$\Omega_1 = 1.5793$,$\Omega_2 = 0.9671$) and scenario 2 ($\alpha_1 = 2$,$\alpha_2 = 1$,$\beta_1 = 2$,$\beta_2 = 2.5$,$\Omega_1 = 1.5793$,$\Omega_2 = 0.9671$). Fig. 2 shows that for scenario 2, we can deploy 2 RIS (for a transmit power 30 dBm) and 5 RIS (for a transmit power 50 dBm) modules to achieve the outage performance of 10^{-3} for reliable communications. Moreover, a desirable performance can be achieved even at low transmit powers with a proper selection of the number of RISs. For example, an outage probability of 10^{-4} is achievable with 1-RIS modules at $P_t = 30$ dBm for scenario 1 compared to atmost 4 RISs at $P_t = 50$ dBm for scenario 2.

In Fig. 3, we demonstrate the impact of multiple RIS modules on the average BER performance for DBPSK modulation ($p = 1$, $q = 1$). We consider two different fading parameters and the average BER is plotted for different number of hops with fixed distance from source to destination. It can be seen that the performance degrades with an increase in the number of RIS modules due to the double fading effect of RISs at the benefit of providing reliable LOS connection which might not be possible without RIS structure in the presence of multiple obstacles. Moreover, the slope of the plots clearly demonstrates the diversity order of the system. For example, the diversity order $G_{\text{out}} = 0.75, 2$ for the considered two different dGG parameters respectively. The figure also depicts the convergence of asymptotic expressions to the numerical and simulation results in the high SNR regime. Finally, we demonstrate the ergodic capacity in Fig. 4 for different fading parameters. The performance degrades with the increase in the number of RIS modules but is relatively less significant compared to outage and BER performance. It can be seen that there is a loss of about 3 bits/sec/Hz with an increase of $K = 2$ to $K = 5$.

In all the above figures (Fig. 2, Fig. 3, Fig. 4), the performance degradation with an increase in the RIS modules should not be deterrent. As such, there may not be any connectivity if

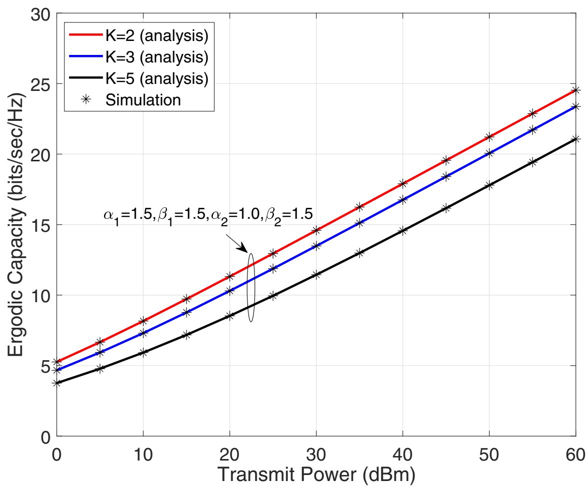

Fig. 4: Ergodic capacity of multi-RIS system.

a specified number of RIS modules are not deployed between the source and destination.

V. Conclusions

In this paper, we investigated the performance of a multiple RIS empowered wireless communication system. We developed PDF and CDF of the resultant SNR for multi-RIS transmission considering i.ni.d dGG fading model in each hop. We analyzed the system performance by deriving analytical expressions for the outage probability, average BER, and ergodic capacity using the Fox-H function. To provide insights on the system performance, we also presented asymptotic analysis in the high SNR regime for the outage probability, average BER and demonstrated the impact of channel parameters on the diversity order of the system. We presented simulation results to show that the performance of the multiple RISs modules can provide reliable connection if there is a lack of LOS propagation. However, an excessive use of multiple RIS should be avoided since an increase in the number of RIS modules between a source and destination decreases the performance caused by the dual fading effect of the RIS. We envision that the proposed work can be suitable for seamless connectivity for scenarios where a single RIS may not create a LOS connection. The proposed analysis can be extended by joint optimization of placement for multiple RIS and throughput of the system to achieve a higher quality of service.

Appendix A

We use Mellin transform to find the PDF of the product of K random variables as

$$f_X(x) = \frac{1}{x}\frac{1}{2\pi \jmath}\int_{\mathcal{L}} \prod_{i=1}^{K} \mathbb{E}[X_i^r] x^{-r}\, dr \tag{19}$$

Substituting (6) in $\mathbb{E}[X_i^n] = \int_0^\infty x^r f_{X_i}(x)dx$ and using the identity [18, 2.8], the r-th moment can be computed as

$$\mathbb{E}[X_i^r] = \psi_i \int_0^\infty x^{r+\phi_i-1} H_{p,q}^{m,n}\left[\ \zeta_i x \ \middle|\ \begin{matrix}\{(a_{i,j}, A_{i,j})\}_{j=1}^{p} \\ \{(b_{i,j}, B_{i,j})\}_{j=1}^{q}\end{matrix}\ \right] dx$$
$$= \psi_i \zeta_i^{-r-\phi_i} \frac{\prod_{j=1}^{m}\Gamma(b_{i,j}+B_{i,j}(r+\phi_i))}{\prod_{j=n+1}^{p}\Gamma(a_{i,j}+A_{i,j}(r+\phi_i))}$$
$$\frac{\prod_{j=1}^{n}\Gamma(1-a_{i,j}-A_{i,j}(r+\phi_i))}{\prod_{j=m+1}^{q}\Gamma(1-b_{i,j}-B_{i,j}(r+\phi_i))} \tag{20}$$

We use (20) in (19) and apply the definition of Fox-H function, to get (7). Using (7) in $F_X(x) = \int_0^x f_X(t)dt$, an expression for the CDF:

$$F_X(x) = \prod_{i=1}^{K} \psi_i \zeta_i^{-\phi_i} \frac{1}{2\pi \jmath}\int_{\mathcal{L}} (\prod_{i=1}^{K}\zeta_i)^r (\int_0^x t^{-1+r} dt)$$
$$\prod_{i=1}^{K} \frac{\prod_{j=1}^{m}\Gamma(b_{i,j}+B_{i,j}(-r+\phi_i))}{\prod_{j=n+1}^{p}\Gamma(a_{i,j}+A_{i,j}(-r+\phi_i))}$$
$$\frac{\prod_{j=1}^{n}\Gamma(1-a_{i,j}-A_{i,j}(-r+\phi_i))}{\prod_{j=m+1}^{q}\Gamma(1-b_{i,j}-B_{i,j}(-r+\phi_i))}\, dr \tag{21}$$

Using the inner integral $\int_0^x t^{-1+r}dt = \frac{x^r}{r} = x^r \frac{\Gamma(r)}{\Gamma(1+r)}$ in (21), and applying the definition of Fox-H function, we get (8).

References

[1] M. D. Renzo, M. Debbah, D. Phan-Huy *et al.*, "Smart radio environments empowered by reconfigurable AI meta-surfaces: An idea whose time has come," *EURASIP J. Wireless Commun. Netw.*, vol. 2019, no. 129, May 2019.

[2] E. Basar, M. D. Renzo, J. D. Rosny, M. Debbah, M. S. Alouini, and R. Zhang, "Wireless communications through reconfigurable intelligent surfaces," *IEEE Access*, vol. 7, pp. 116 753–116 773, Aug. 2019.

[3] Q. Wu, S. Zhang, B. Zheng, C. You, and R. Zhang, "Intelligent reflecting surface aided wireless communications: A tutorial," *IEEE Trans. Commun.*, vol. 69, no. 5, pp. 3313–3351, Jan. 2021.

[4] Y. Zhu, B. Mao, Y. Kawamoto, and N. Kato, "Intelligent reflecting surface-aided vehicular networks toward 6G: Vision, proposal, and future directions," *IEEE Veh. Technol. Mag.*, vol. 16, no. 4, pp. 2–10, Dec. 2021.

[5] V. Jamali, H. Ajam, M. Najafi, B. Schmauss, R. Schober, and H. V. Poor, "Intelligent reflecting surface assisted free-space optical communications," *IEEE Commun. Magazine*, vol. 59, no. 10, pp. 57–63, Nov. 2021.

[6] M. Najafi and R. Schober, "Intelligent reflecting surfaces for free space optical communications," in *2019 IEEE Global Commun. Conference (GLOBECOM)*, Waikoloa, HI, USA, Dec. 2019, pp. 1–7.

[7] W. Tang, M. Z. Chen, X. Chen *et al.*, "Wireless communications with reconfigurable intelligent surface: Path loss modeling and experimental measurement," *IEEE Trans. Wireless Commun.*, vol. 20, no. 1, pp. 421–439, Jan 2021.

[8] A. R. Ndjiongue, T. M. N. Ngatched, O. A. Dobre *et al.*, "Performance analysis of RIS-based nT-FSO link over $\mathcal{G}$-$\mathcal{G}$ turbulence with pointing errors," arXiv: 2102.03654, 2021.

[9] Y. U. Ozcan, O. Ozdemir, and G. K. Kurt, "Reconfigurable intelligent surfaces for the connectivity of autonomous vehicles," *IEEE Trans. Vehi. Technol.*, vol. 70, no. 3, pp. 2508–2513, March 2021.

[10] C. Huang, Z. Yang, G. C. Alexandropoulos *et al.*, "Multi-hop RIS-empowered terahertz communications: A DRL-based hybrid beamforming design," *IEEE J. Sel. Areas Commun.*, vol. 39, no. 6, pp. 1663–1677, June 2021.

[11] A.-A. A. Boulogeorgos, N. Chatzidiamantis, H. G. Sandalidis, A. Alexiou, and M. D Renzo, "Cascaded composite turbulence and misalignment: Statistical characterization and applications to reconfigurable intelligent surface-empowered wireless systems," *IEEE Trans. Vehi. Technol.*, pp. 1–1, 2022.

[12] M. A. Kashani, M. Uysal, and M. Kavehrad, "A novel statistical channel model for turbulence-induced fading in free-space optical systems," *Journal of Lightwave Technology*, vol. 33, no. 11, pp. 2303–2312, June 2015.

[13] P. S. Bithas, A. G. Kanatas, D. B. da Costa, P. K. Upadhyay, and U. S. Dias, "On the double-generalized gamma statistics and their application to the performance analysis of V2V communications," *IEEE Trans. Commun.*, vol. 66, no. 1, pp. 448–460, Jan. 2018.

[14] H. AlQuwaiee, I. S. Ansari, and M.-S. Alouini, "On the performance of free-space optical communication systems over double Generalized Gamma channel," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 9, pp. 1829–1840, Sept. 2015.

[15] B. Ashrafzadeh, A. Zaimbashi, E. Soleimani-Nasab, and M. Uysal, "Unified performance analysis of multi-hop FSO systems over double generalized gamma turbulence channels with pointing errors," *IEEE Trans. Wirel. Commun.*, vol. 19, no. 11, pp. 7732–7746, Nov. 2020.

[16] *The Wolfram function Site*, Accessed May 2021: https://functions.wolfram.com/.

[17] A. Kilbas and M. Saigo, *H-Transforms: Theory and Applications*. CRC Press., Mar. 2004.

[18] A. Mathai, R. K. Saxena, and H. J. Haubold, *The H-Function: Theory and Applications*. Springer New York, 2009.