Questions PCA

1. Why does the below hold?

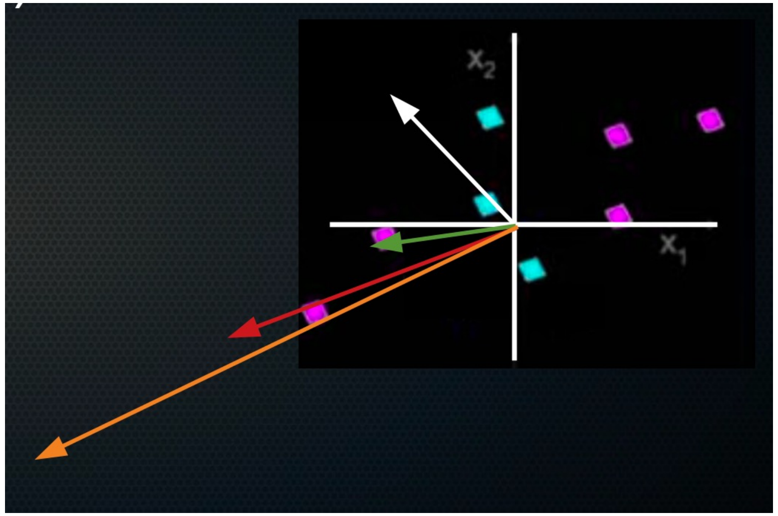
$$A \cdot v = \lambda \cdot v$$

- Rewrite:
  $A \cdot v - \lambda \cdot I \cdot v = 0 \rightarrow (A - \lambda I) \cdot v = 0$
- Only way this can be true (if v != 0): $det(A - \lambda I) = 0$

- Click

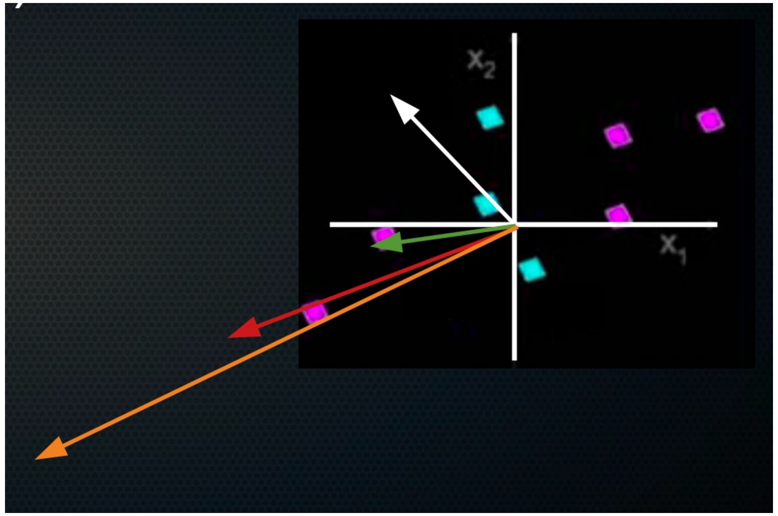
2. Okay, I showed that it turns in the direction of the eigenvector with largest eigenvalue (largest variance), what about the others?

- Repeated application of a matrix to a vector causes convergence to eigenvector with largest eigenvalue. Will not go through proof. However, can extract all eigenvectors and eigenvalues, and order by eigenvalue magnitude → new components in decreasing order of variance explained.



3. How do we get to this eigenvector-eigenvalue stuff?

- It turns out that asking for new directions through the data that maximise variance is exactly equal to getting the eigenvectors and eigenvalues of the covariance matrix (I ask you to believe this, we won’t cover the derivation).
- It is here if you are ~~crazy~~ very willing to learn



4. Why is the eigenvalue the variance of the corresponding eigenvector?

- I tell you that this is so. If you want to know: click.

5. Are eigenvectors always perpendicular?

- No, but they are for symmetric matrices. The covariance matrix is a symmetric matrix so we are in luck.