

$\dot{\theta}_1 = -\frac{(\theta_2 + \pi)}{2}$   
 $\dot{\theta}_2 = -2\dot{\theta}_1$   
*manipulator.png* The manipulator corresponding to a vertical one leg jump.

$x_{end} = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2)$   
 $y_{end} = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2)$   
 (1)  $\frac{1}{L_2} \sin(\theta_1) - \sin(\theta_1 + \theta_2) - \frac{1}{L_2} \sin(\theta_1 + \theta_2)$   
 $\frac{1}{L_2} \cos(\theta_1) + \cos(\theta_1 + \theta_2) - \frac{1}{L_2} \cos(\theta_1 + \theta_2)$   
*jacobian\_velocity.eps* Vertical Paw velocity as a function of knee angle.

# Robot Jumping Controller Logic

**Friction Scaling:**

$$N_\mu \leftarrow 0.8 N_\mu$$

**Angle Conversions (Degrees to Radians):**

$$\begin{aligned}\theta_1 &= \frac{\pi}{180} \theta_{\text{hip\_deg}}, & \theta_2 &= \frac{\pi}{180} \theta_{\text{knee\_deg}} \\ w_1 &= \frac{\pi}{180} w_{\text{hip\_deg}}, & w_2 &= \frac{\pi}{180} w_{\text{knee\_deg}}\end{aligned}$$

**Link Lengths:**

$$l_1 = \text{thigh\_l\_m}, \quad l_2 = \text{calf\_l\_m}$$

**Capping Angular Velocities:**

$$w_{1,\text{abs}} = \min(|w_1|, w_{\text{hip\_max}}), \quad w_{2,\text{abs}} = \min(|w_2|, w_{\text{knee\_max}})$$

**Maximum Torque Based on Speed:**

$$\begin{aligned}\tau_{1,\text{max\_abs}} &= \tau_{\text{hip\_stall}} \left( 1 - \frac{w_{1,\text{abs}}}{w_{\text{hip\_max}}} \right) \\ \tau_{2,\text{max\_abs}} &= \tau_{\text{knee\_stall}} \left( 1 - \frac{w_{2,\text{abs}}}{w_{\text{knee\_max}}} \right)\end{aligned}$$

**Assigning Torques:**

$$\tau_2 = \tau_{2,\text{max\_abs}}, \quad \tau_{1,\text{motor\_dyn\_limit}} = -\tau_{1,\text{max\_abs}}$$

**Friction Cone Constraint:**

$$\tau_{1,\text{friction\_cone\_limit}} = \frac{-l_1 l_2 \sin(\theta_2) N_\mu + (l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)) \tau_2}{l_2 \sin(\theta_1 + \theta_2)}$$

**Final Hip Torque Selection:**

$$\tau_1 = \max(\tau_{1,\text{motor\_dyn\_limit}}, \tau_{1,\text{friction\_cone\_limit}})$$

**Direction-Dependent Torque Output:**

$$\tau_{\text{hip}} = (\text{front1\_backneg1}) \tau_1, \quad \tau_{\text{knee}} = (\text{front1\_backneg1}) \tau_2$$