



DEPARTMENT OF ENGINEERING CYBERNETICS

TTK4550 - SPECIALIZATION PROJECT

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# Design and Control of a Spring-actuated Jumping Quadruped in Earth Gravity

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## Abstract

This project report presents our specialization project, which is the design and control of a quadruped, spring-actuated, etc.

Here I am trying to cite [7].

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# Abbreviations

Abbreviation	Description
AI	Artificial Intelligence
API	Application Programming Interface
CPU	Central Processing Unit
DRL	Deep Reinforcement Learning
EKF	Extended Kalman Filter
ESKF	Error State Kalman Filter
GNC	Guidance, Navigation, and Control
INS	Inertial Navigation System
ML	Machine Learning
MOOS	Mission Oriented Operating Suite
PPO	Proximal Policy Optimization
RL	Reinforcement Learning
USV	Unmanned Surface Vehicle

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## 1 Introduction

### 1.1 Motivation

The exploration of extraterrestrial environments represents one of the most demanding frontiers of robotic systems, requiring exceptional autonomy, resilience, and adaptability to navigate complex and unpredictable terrain. On Mars, wheeled rovers have proven their utility, with six successful deployments to date [8], robots like Axel [5] and Reachbot [6] have also been designed, tailored towards specific tasks. One such task that has received much attention in recent years, is the exploration of potential Martian and Lunar lava tubes [1]. These tubes are hollow caverns hypothesized to exist beneath the surface of Mars and the Moon, formed by ancient lava flows. They are of particular interest to astrobiologists and planetary scientists, as they could provide shelter from cosmic radiation and micrometeorites, as well as stable temperatures and access to subsurface water ice [1].

The exploration of such lava tubes present a unique challenge to robotic systems, as they are believed to be characterized by rough, uneven terrain, sharp rocks, and steep slopes. This could present a challenge to traditional wheeled rovers. Further, the motion of wheeled robots is limited to the ground plane, and thus, inherently, they do not utilize the lower gravity of extraterrestrial objects such as asteroids, the Moon and Mars. Jumping quadrupeds, on the other hand, inherently utilize the lower gravity of such objects, and in low earth gravity could potentially jump to heights of several meters TODO: CITE. This could allow them to traverse obstacles that would be insurmountable to wheeled rovers, such as steep slopes, large rocks, and gaps in the terrain.

While recent years have seen great progress in the development of quadruped robots, most quadrupeds still struggle with jumping in earth gravity TODO: CITE. Since, additionally, low gravity environments are very hard to replicate on earth, it is difficult to test hardware and control algorithms intended for low gravity jumping quadrupeds. Jumping also includes high velocity impacts, making damage to the often expensive hardware likely. This motivates the main goal of this project, which is to develop a design for a small, lightweight, and low-cost jumping quadruped robot. The robot’s low weight is intended to reduce the risk of damage during testing, and the low cost to make it more accessible to researchers, as well as reduce the cost of potential damage.

### 1.2 Scope

As described in the Motivation section, section 1.1, the main goal of this project is to develop a design for a small, lightweight, and low-cost jumping quadruped robot. The work presented in

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this report is part of a specialization project, TTK4550 - Engineering Cybernetics, Specialization Project TODO: CITE, pursued at the Norwegian University of Science and Technology (NTNU), as a preparation for a master's thesis. So while the scope of the specialization project is limited to the development of a design, the overall goal is for the design to be used as the basis for a master's thesis, where the robot will be built and tested. The master's thesis will also include the development of control algorithms for the robot, which is not included in this report.

More precisely, the scope of this project is limited to the following:

- Developing a simplified simulation for the robot in MATLAB/Simulink, to be used for verification and evaluation of various design choices.
- Choosing a specific method of actuation, such as motors, parallel torsional springs, parallel extension springs, or a combination of these.
- Identifying key hardware components, such as motors and springs.
- Designing a CAD model for a single leg of the robot. The leg must adhere to geometric and mechanical constraints such as:
  - Accommodating chosen springs and motors.
  - Being easily manufacturable using 3D printing and machining TODO: WHAT?
  - Sturdiness, ie. being able to withstand the forces and impacts of jumping.

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## 2 Theory

### 2.1 Actuator Modeling

#### 2.1.1 DC Motor Model

#### 2.1.2 BLDC Motor Model

#### 2.1.3 Gear Transmission Friction Model

### 2.2 Spring-Damper Systems

### 2.3 Kinematics, Jacobians, and Virtual Work

#### 2.3.1 Robot Kinematics

Consider a robotic link arm existing in  $\mathbb{R}^2$  consisting of  $n$  links, each with a length  $l_i$  and a joint angle  $q_i$ . The position of the end-effector is given by the vector  $\mathbf{x} = [x, y]^T$ , where  $x$  and  $y$  are the coordinates of the end-effector in the global coordinate system. Using simple trigonometry, the position of the end-effector can be expressed as a function of the joint angles and link lengths as seen in equation 1. Axes and joint angles corresponding to the expression in equation 1 can be seen in figure 1.

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n l_i \cos(q_i) \\ \sum_{i=1}^n l_i \sin(q_i) \end{bmatrix} \quad (1)$$

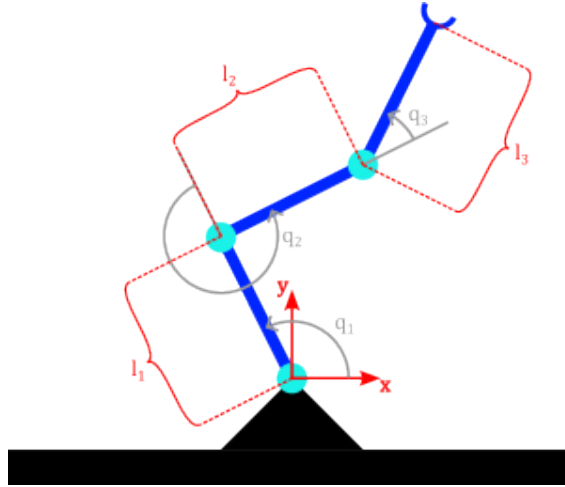


Figure 1: Illustration of a 3 link robotic link arm in  $\mathbb{R}^2$  with  $n$  links.

#### 2.3.2 Jacobian Matrix

As described in section 2.3.1, the position of the end-effector can be expressed as a function of the joint angles and link lengths. In robotics, it is often useful to express the relationship between infinitesimal changes in the joint angles and the resulting change in the end-effector position. As can be seen in equation 2, infinitesimal changes in variables  $\delta y$  and  $\delta x$  can be described by means of the partial derivative [2]. If this is compared to the definition of the jacobian in equation 3, it is clear that the jacobian matrix  $\mathbf{J}$  can be used to map infinitesimal changes in joint angles to changes in the end-effector position, as illustrated in equation 4. The limit of an infinitesimal change over

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an infinitesimal time interval is a derivative, and thus by dividing each side in equation 4 by  $\delta t$ , one arrives at the expression in equation 5, by which the jacobian can be used to map joint velocities to end-effector velocities.

$$\delta y = \frac{\partial y}{\partial x} \delta x \quad (2)$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \dots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \dots & \frac{\partial y}{\partial q_n} \end{bmatrix} \quad (3)$$

$$\delta \mathbf{x} = \mathbf{J} \delta \mathbf{q} \quad (4)$$

$$\dot{\mathbf{x}} = \mathbf{J} \dot{\mathbf{q}} \quad (5)$$

### 2.3.3 Force/Torque Mapping

Consider a general robotic manipulator, such as the one illustrated in figure 1, but with an arbitrary amount,  $n$ , of joints and links. Using the principle of conservation of power, one arrives at the formulation found in equation 6.

$$\text{power at the joints} = (\text{power to move the robot}) + (\text{power at the end-effector}) \quad (6)$$

As the power used to move the robot approaches zero,

Finally found northwestern book that says what I need: [4].

Consider a robotic manipulator with  $n$  joints, each with a joint angle  $q_i$  and a joint torque  $\tau_i$ . The position of the end effector for such a system is given by equation 1, and thus the formula in equation 5 can be used to map joint velocities to end effector velocities.

## 3 Modeling and Simulation

### 3.1 Simscape

Her forklares Simscape og hvordan vi lagde en simscape simulering. Her forklares \*ikke\* hvordan vi valgte ut link-lengths, spring, motor, etc. Altså dimensjonering forklares ikke her. Kun hvordan vi lagde simuleringen som etter hverdt skal brukes til dimensjonering. Det at den brukes til dimensjonering, forklares trolig i hardware/design seksjonen...

## 4 Robot Design

This is where we explain "overall" design. It's motivation, etc. Also where we cover dimensioning, link-lengths, etc. We explain here our parameter sweep to find optimal design. We also explain here our kinematics-script that we used to derive the required stall-torque for our motor given springs and link-lengths.



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**4.1**

**4.2**

## **5 Robot Hardware**

This is where we discuss actual hardware, ie. component selection, materials, CAD, etc.

So here we specify motors and how they match specs, while referring to the design section to explain why we want these motors. We should here have a list of existing motors. The design section can link to this list to motivate its own choice. It will be circular, but that's okay.

## **6 Robot Control**

### **6.1 RL Problem Description?**

This is not where we explain the goal of the thesis, ie. "we want to jump". This is just where we explain the RL problem. That we want to \* jump \* land \* etc. is something we describe earlier, for instance in Introduction/Scope/Problem Description.

## **7 Results**

## **8 Discussion**

## **9 Conclusion**

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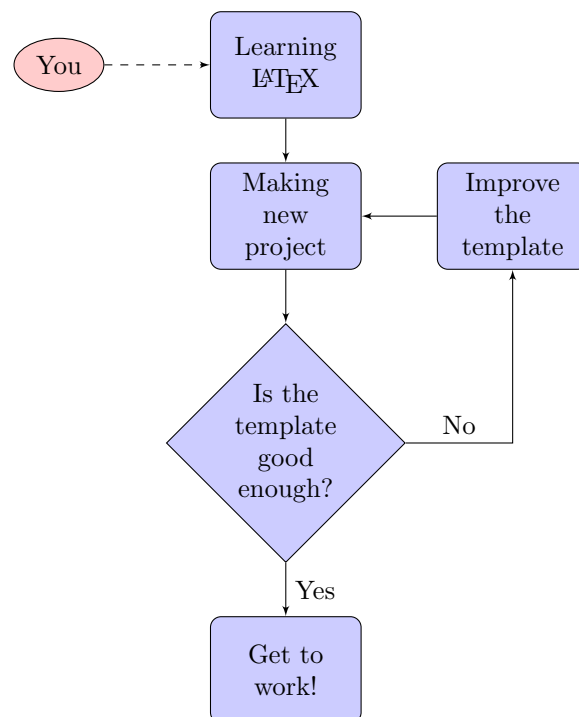
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## Appendix

### A Hello World Example

```
int main {  
    // This is a comment  
    std::cout << "Hello World from C++!" << std::endl;  
    std::cout << "I am using the default style to print this code in beautiful colors. Since the t  
    return 0;  
}  
  
:  
:  
  
# This is a comment  
print('Hello world from Python!')  
print('I am using the "rrt" style to print this code in beautiful colors')  
  
:  
:  
  
% Content of HelloWorld.m  
disp('Hello World from Matlab!')
```

### B Flow Chart Example



## C Sub-figures Example

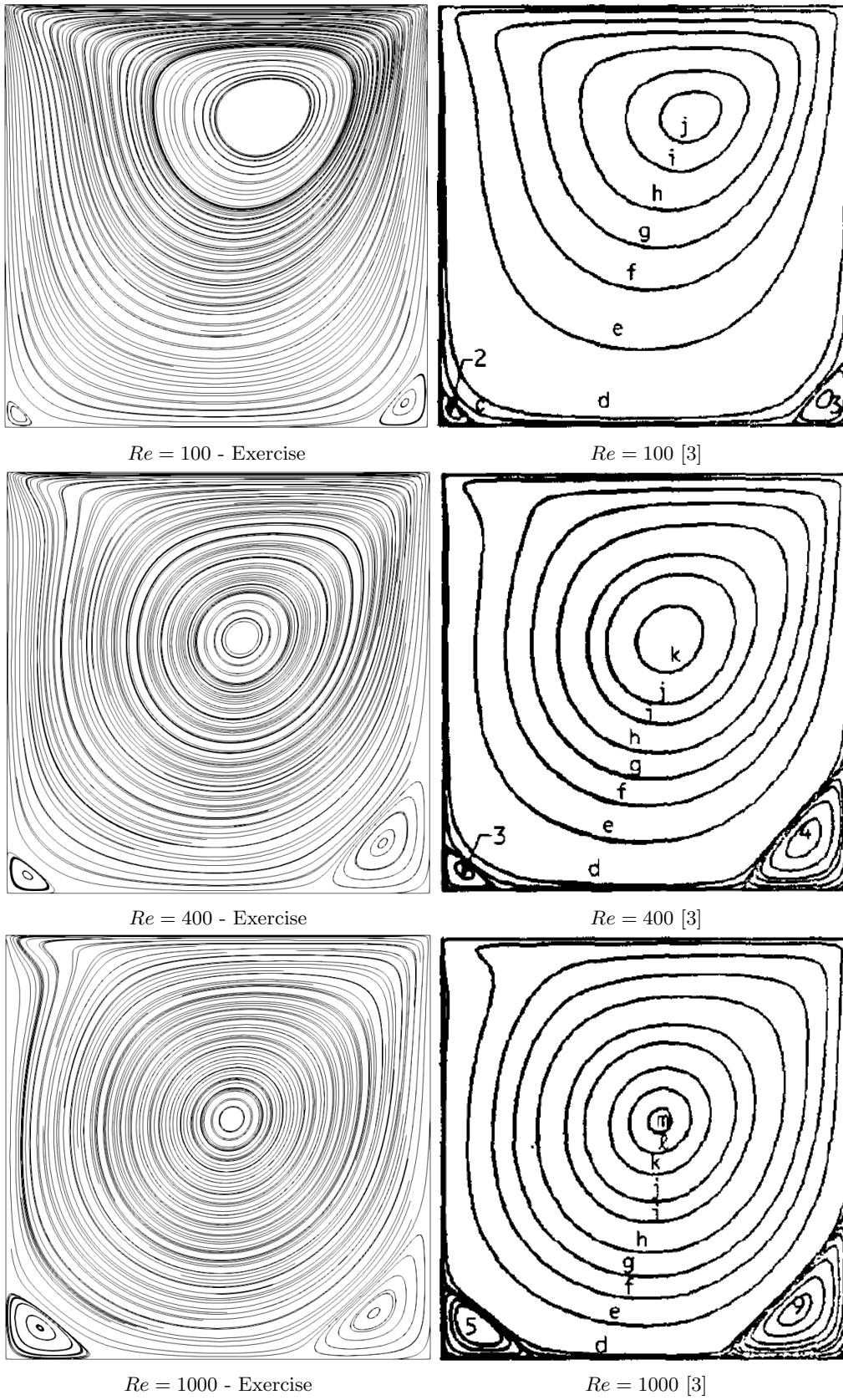


Figure 2: Streamlines for the problem of a lid-driven cavity.