Recommendation Systems

Matrix Factorization. Part II

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Matrix Factorization

Item

User	W	X	Y	Z
A		4.5	2.0	
B	4.0		3.5	
C		5.0		2.0
D		3.5	4.0	1.0

Rating Matrix

=

A	*1.2*	*0.8*
B	*1.4*	*0.9*
C	*1.5*	*1.0*
D	*1.2*	*0.8*

User Matrix

X

W	X	Y	Z
1.5	*1.2*	*1.0*	*0.8*
1.7	*0.6*	*1.1*	*0.4*

Item Matrix

X – *Rating matrix*

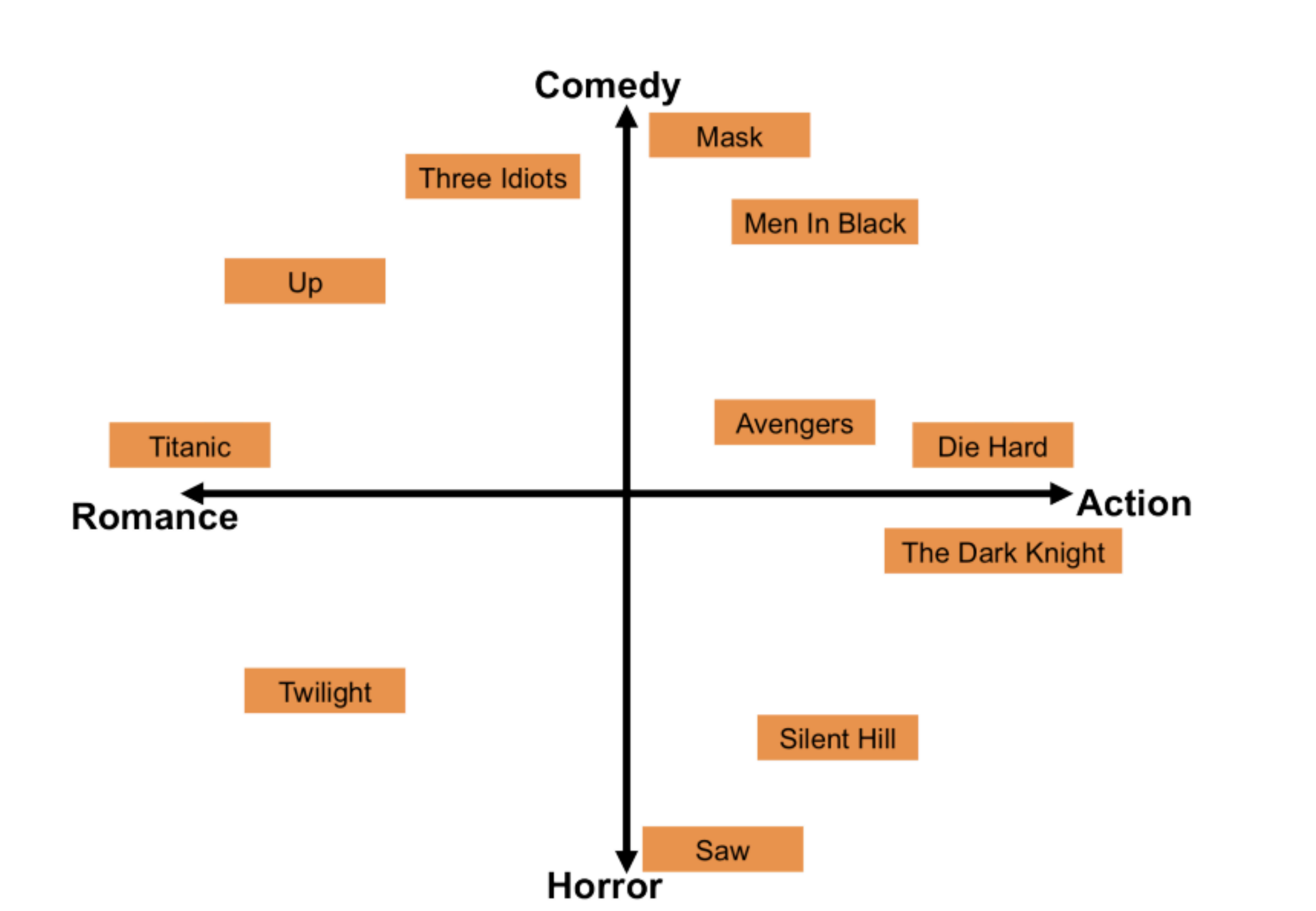
U – *User embedding matrix*

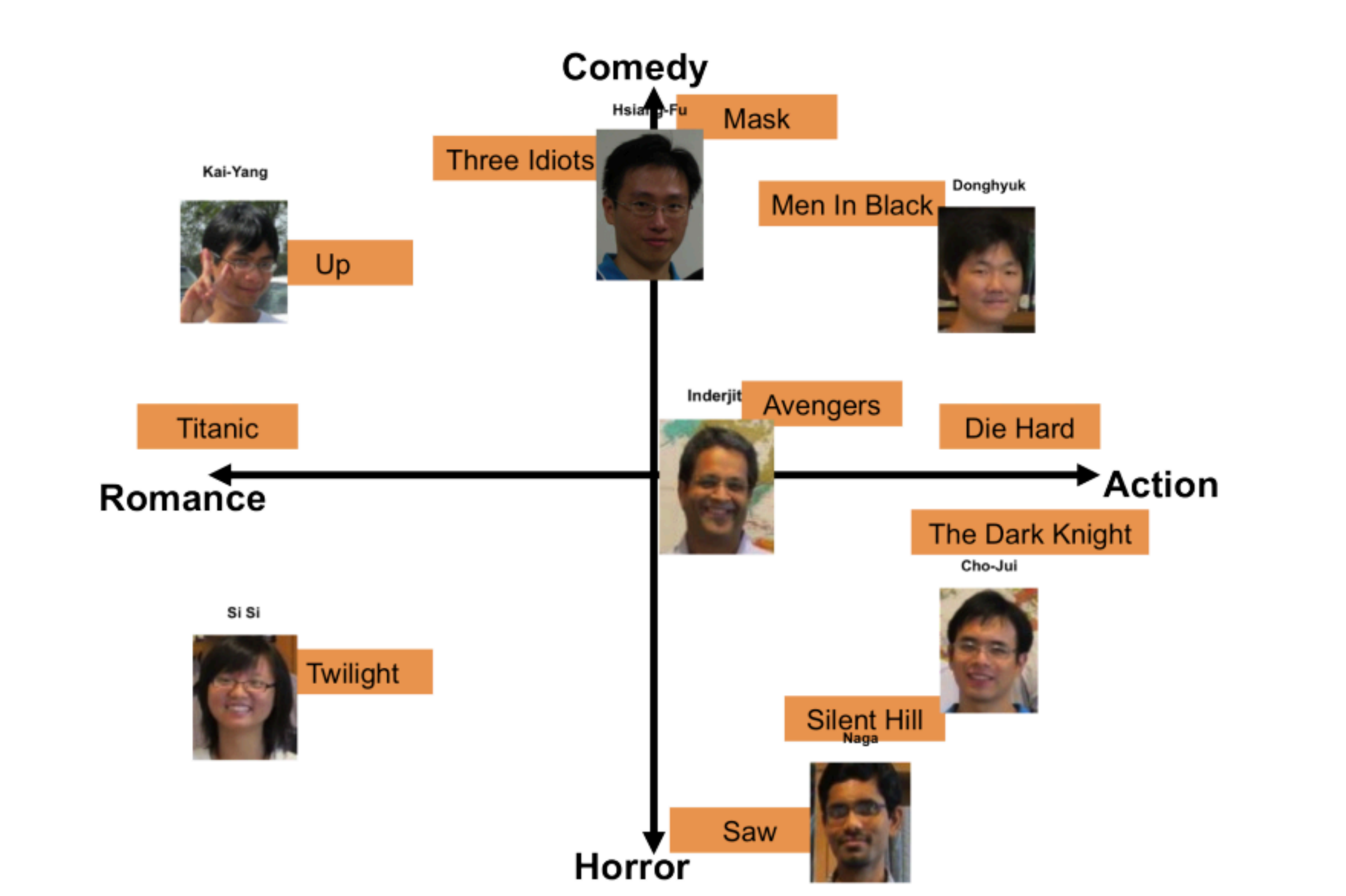
V – *Item embedding matrix*

k – *embedding size*

$$X_{l,n} \approx U_{l,k} \, . \, V^T_{k,n}$$

Matrix Factorization





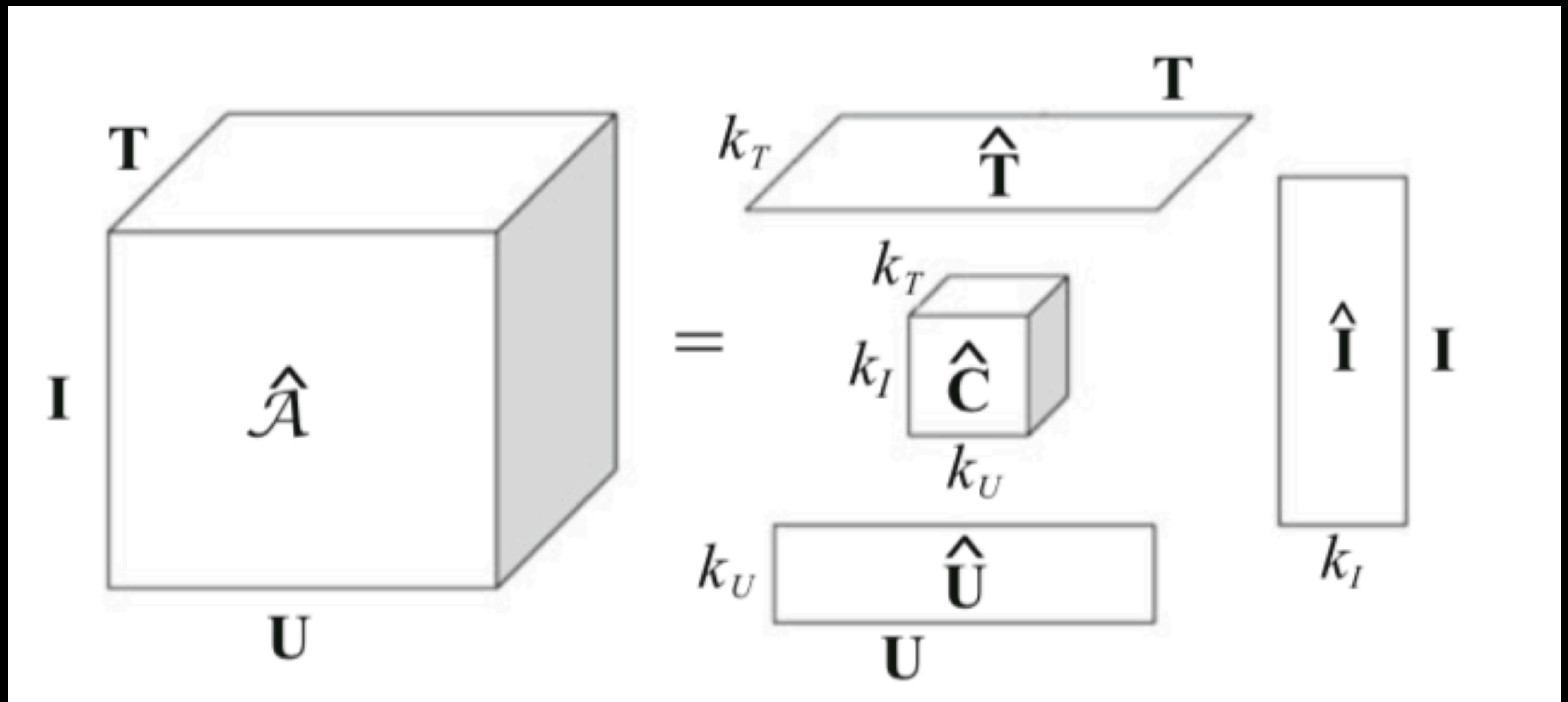
Matrix Factorization

$$Q = \sum_{i,j} (\langle u_i, v_j \rangle - x_{ij})^2 \to min$$

$$Q = \sum_{i,j} (\langle u_i, v_j \rangle - x_{ij})^2 + \alpha \sum_i \|u_i\|^2 + \beta \sum_j \|v_j\|^2 \to min$$

$$Q = \sum_{i,j} (\langle u_i, v_j \rangle + b_u + b_i + \mu - x_{ij})^2 + \alpha \sum_i \|u_i\|^2 + \beta \sum_j \|v_j\|^2 + \gamma \|b_u\| + \theta \|b_i\| \to min$$

Tensor Factorization (HOSVD)



Algorithm 5.1 AlsHOSVD

Require: The initial tensor $\mathcal{A}$ with user, tag, and item dimensions.

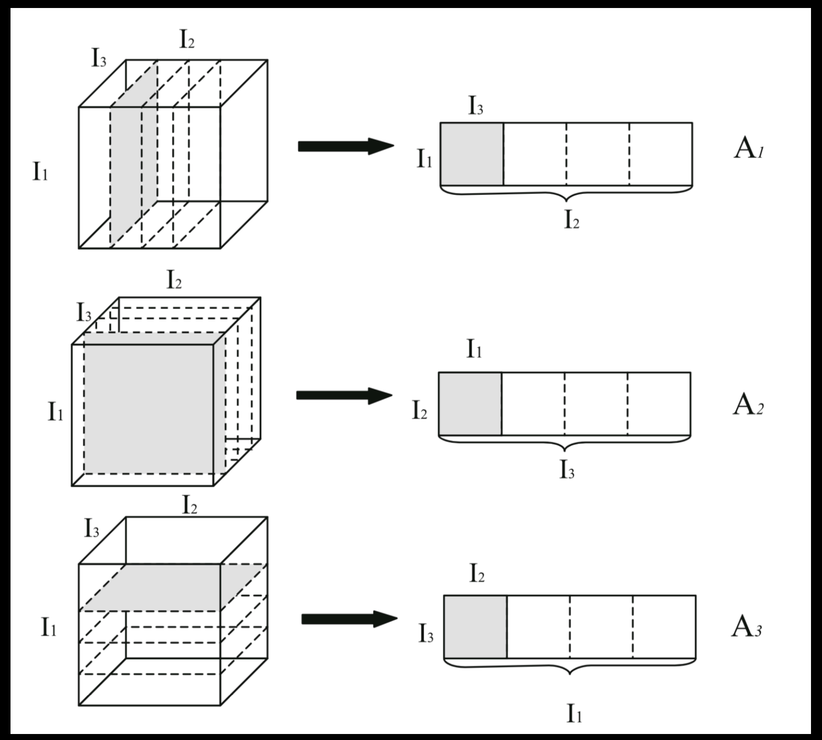
Ensure: The approximate tensor $\hat{\mathcal{A}}$ with k_U, k_I and k_T left leading eigenvectors of each dimension, respectively.

1: Initialize core tensor $\mathcal{C}$ and left singular vectors $U^{(1)}$, $U^{(2)}$, $U^{(3)}$ of A_1, A_2, and A_3, respectively.
2: **repeat**
3: $\quad \mathcal{C} = \mathcal{A} \times_1 {U_{k_U}^{(1)}}^T \times_2 {U_{k_I}^{(2)}}^T \times_3 {U_{k_T}^{(3)}}^T$
4: $\quad \hat{\mathcal{A}} = \mathcal{C} \times_1 U_{k_U}^{(1)} \times_2 U_{k_I}^{(2)} \times_3 U_{k_T}^{(3)}$
5: $\quad U_{k_U}^{(1)} \leftarrow k_U$ leading left singular vectors of A_1
6: $\quad U_{k_I}^{(2)} \leftarrow k_I$ leading left singular vectors of A_2
7: $\quad U_{k_T}^{(3)} \leftarrow k_T$ leading left singular vectors of A_3
8: **until** $\|\mathcal{A} - \hat{\mathcal{A}}\|^2$ ceases to improve **OR** maximum iterations reached
9: **return** $\mathcal{C}$, $U_{k_U}^{(1)}$, $U_{k_I}^{(2)}$, and $U_{k_T}^{(3)}$

$$\hat{\mathcal{A}} := \hat{C} \times_u \hat{U} \times_i \hat{I} \times_t \hat{T}$$

$$\underset{\hat{\theta}}{\operatorname{argmin}} \sum_{(u,i,t)\in Y} \left(\hat{a}_{u,i,t} - a_{u,i,t}\right)^2$$

$$\hat{a}(u,i,t) := \sum_{\tilde{u}=1}^{k_U} \sum_{\tilde{i}=1}^{k_I} \sum_{\tilde{t}=1}^{k_T} \hat{c}_{\tilde{u},\tilde{i},\tilde{t}} \cdot \hat{u}_{u,\tilde{u}} \cdot \hat{i}_{i,\tilde{i}} \cdot \hat{t}_{t,\tilde{t}}$$



$$A_1 = U^{(1)} \cdot S_1 \cdot V_1^T A_2 = U^{(2)} \cdot S_2 \cdot V_2^T A_3 = U^{(3)} \cdot S_3 \cdot V_3^T$$

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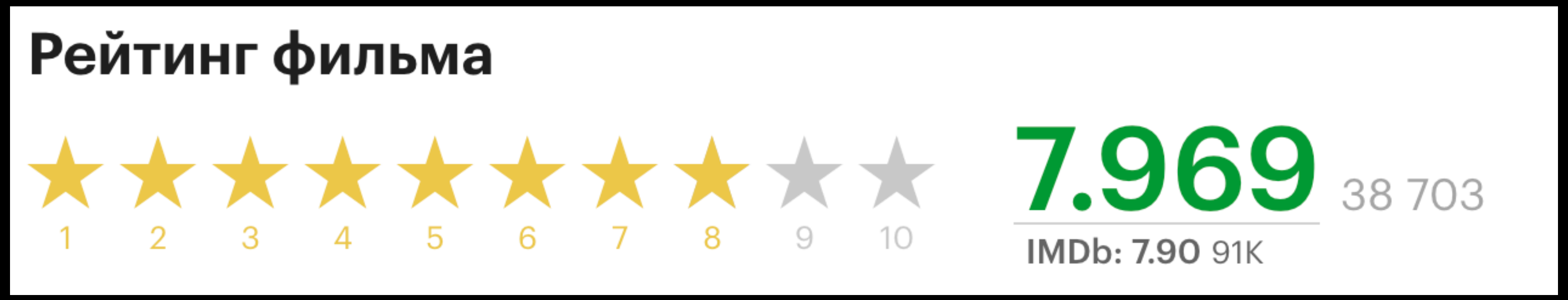
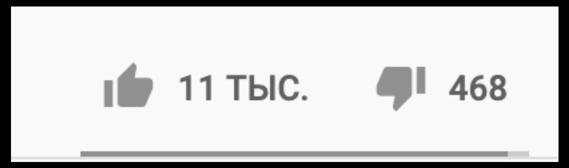
Explicit

1		5			5		3
5						4	
	3		3	4			
	2					4	5
2		4	1		3		
	3			2		5	

Ratings



Likes/Dislikes



Timespent on item (intervals)

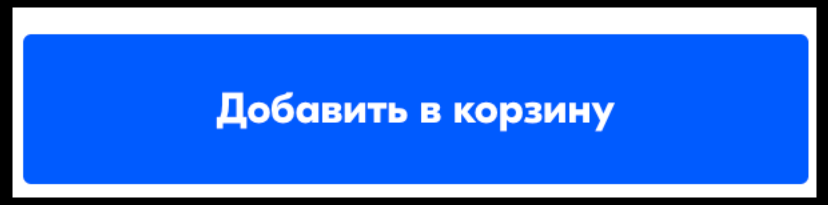
etc

Implicit

1		1			1		1
1						1	
	1		1	1			
	1					1	1
1		1	1		1		
	1			1		1	

- Likes

- Buy



- Watch full movie

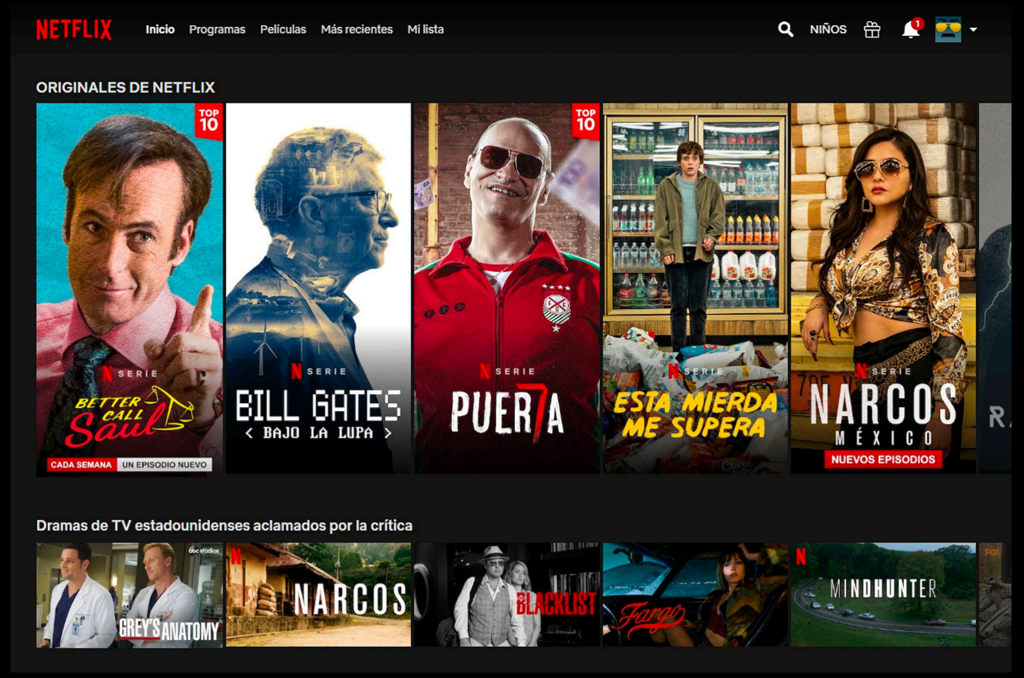
- etc

5 - 4 = 3 -2 ?

- RMSE -> approximate number

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (Predicted_i - Actual_i)^2}{N}}$$

- Recommended systems is about ranking, not about numbers

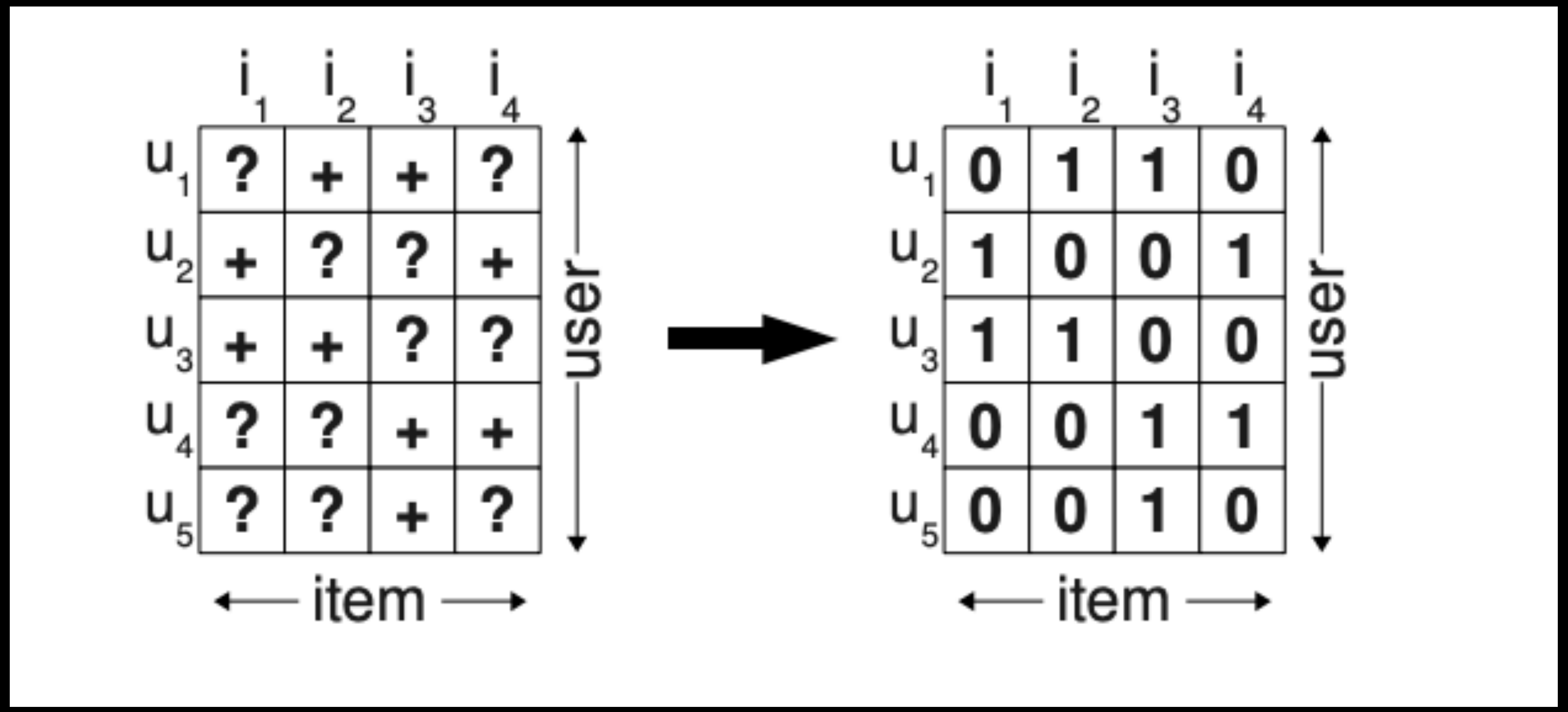


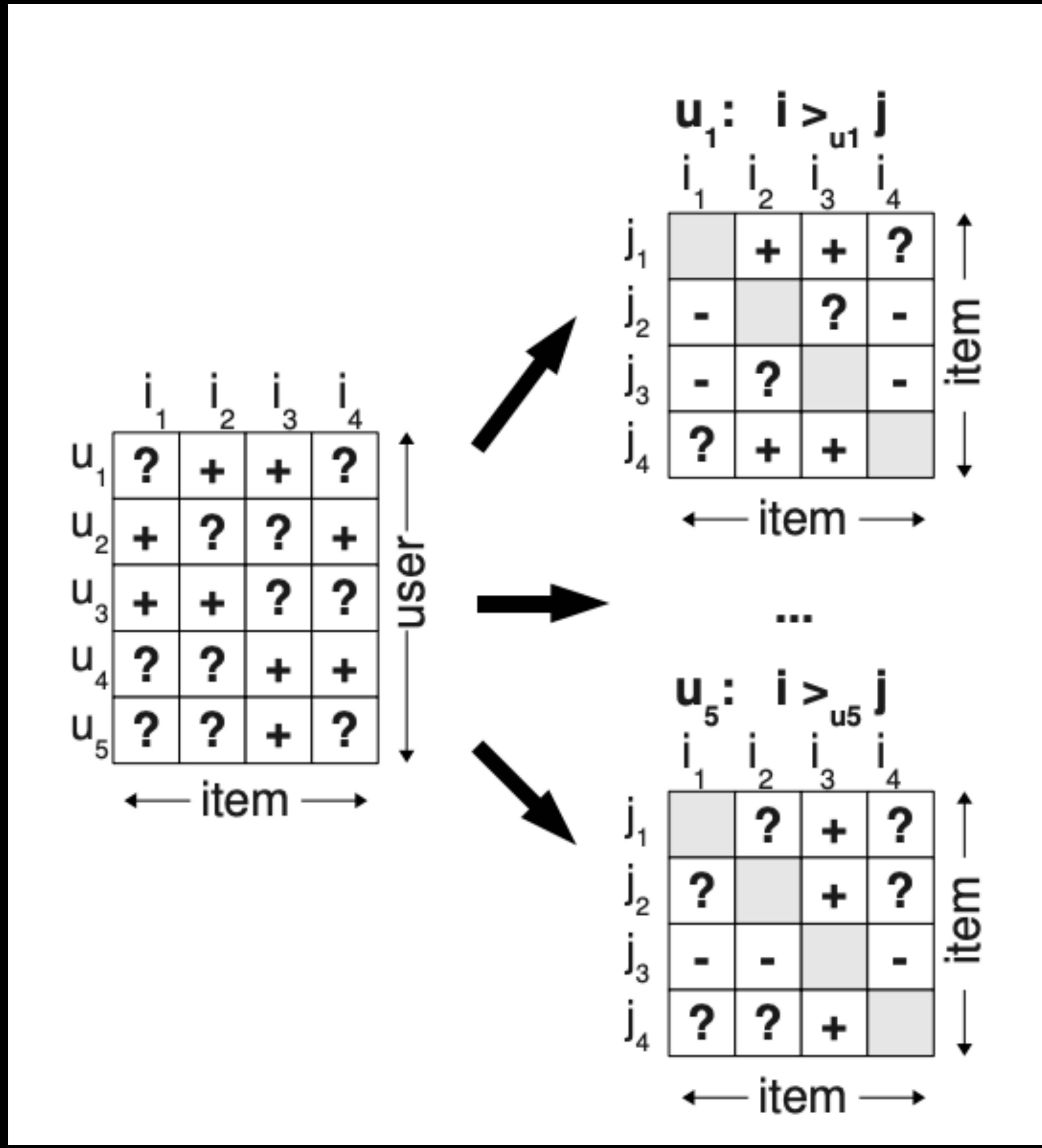
$$5 > 4 > 3 > 2 > 1 > 0$$

$$Positives > Negatives$$

BPR

- BPR: Bayesian Personalized Ranking from Implicit Feedback





https://arxiv.org/pdf/1205.2618.pdf

BPR

$$p\left(\Theta \mid >_u\right) \propto p\left(>_u \mid \Theta\right) p(\Theta)$$

$$\prod_{u \in U} p(> u \mid \Theta) = \prod_{(u,i,j) \in D_s} p\left(i >_u j \mid \Theta\right)$$

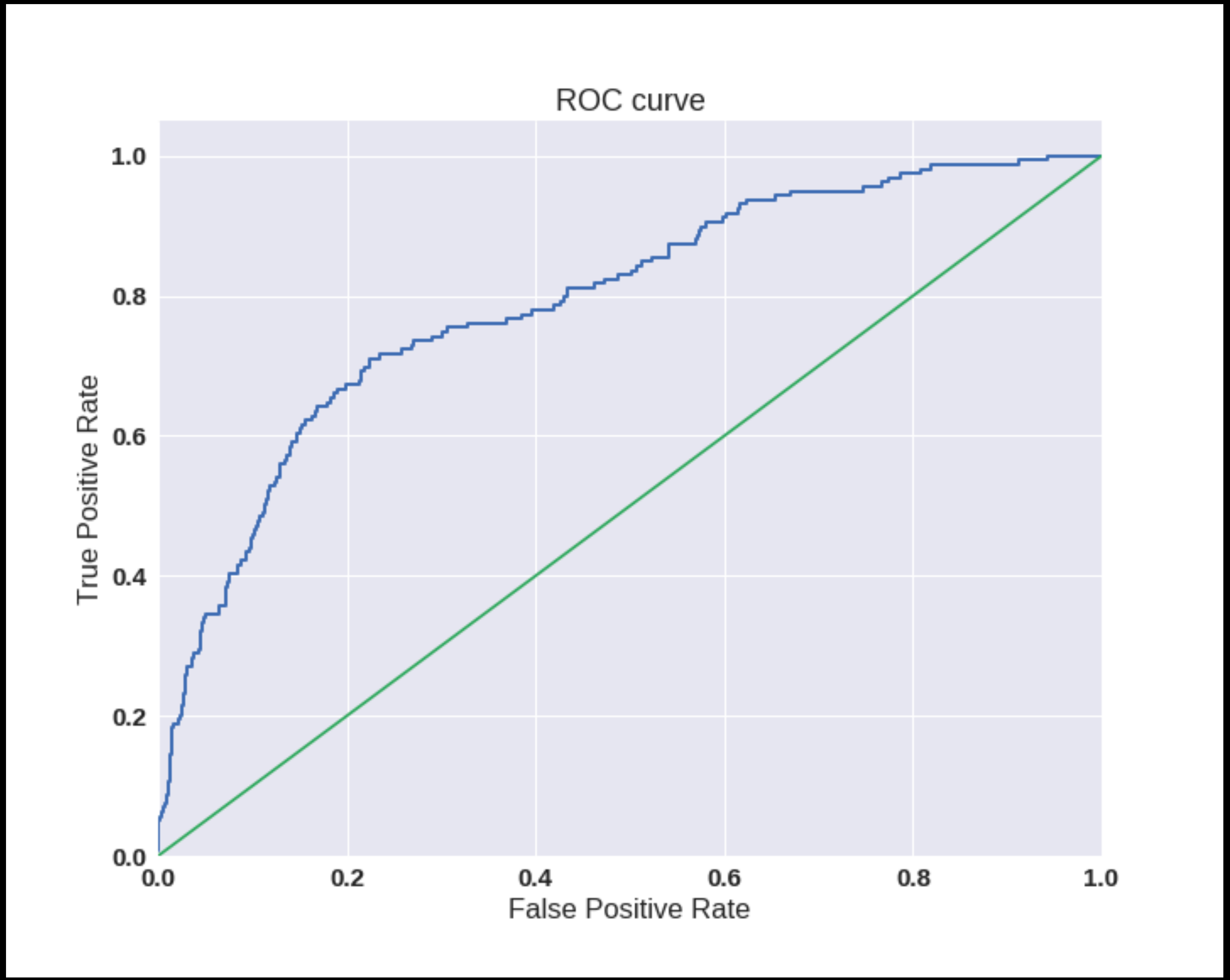
$$\begin{aligned} BPR - O_{PT} &:= \ln p\left(\Theta \mid >_u\right) \\ &= p(> u \mid \Theta) p(\Theta) \\ &= \sum_{uij} \ln \sigma\left(\widehat{x}_{uij}\right) + \ln p(\Theta) \\ &= \sum_{uij} \ln \sigma\left(\widehat{x}_{uij}\right) + \lambda_\Theta \|\Theta\|^2 \end{aligned}$$

1: **procedure** LEARNBPR(D_S, Θ)
2: initialize Θ
3: **repeat**
4: draw (u, i, j) from D_S
5: $\Theta \leftarrow \Theta + \alpha\left(\frac{e^{-\hat{x}_{uij}}}{1+e^{-\hat{x}_{uij}}} \cdot \frac{\partial}{\partial\Theta}\hat{x}_{uij} + \lambda_\Theta \cdot \Theta\right)$
6: **until** convergence
7: **return** $\hat{\Theta}$
8: **end procedure**

$$\begin{aligned} \frac{\partial BPR - O_{PT}}{\partial \Theta} &:= \sum_{(u,i,j) \in D_S} \frac{\partial}{\partial \Theta} \ln \sigma\left(\widehat{x}_{uij}\right) + \lambda_\Theta \frac{\partial}{\partial \Theta} \|\Theta\|^2 \\ &\propto \sum_{(u,i,j) \in D_S} \frac{-e^{\hat{x}_{uij}}}{1 + -e^{\hat{x}_{uij}} \cdot \frac{\partial}{\partial \Theta} \widehat{x}_{uij} - \lambda_\Theta \Theta} \end{aligned}$$

https://arxiv.org/pdf/1205.2618.pdf

BPR -> AUC



$$\mathrm{AUC}(u) = \frac{1}{\left|I_u^+\right| \left|I \backslash I_u^+\right|} \sum_{i \in I_u^+} \sum_{j \in \left|I \backslash I_u^+\right|} \delta\left(\hat{x}_{uij} > 0\right)$$

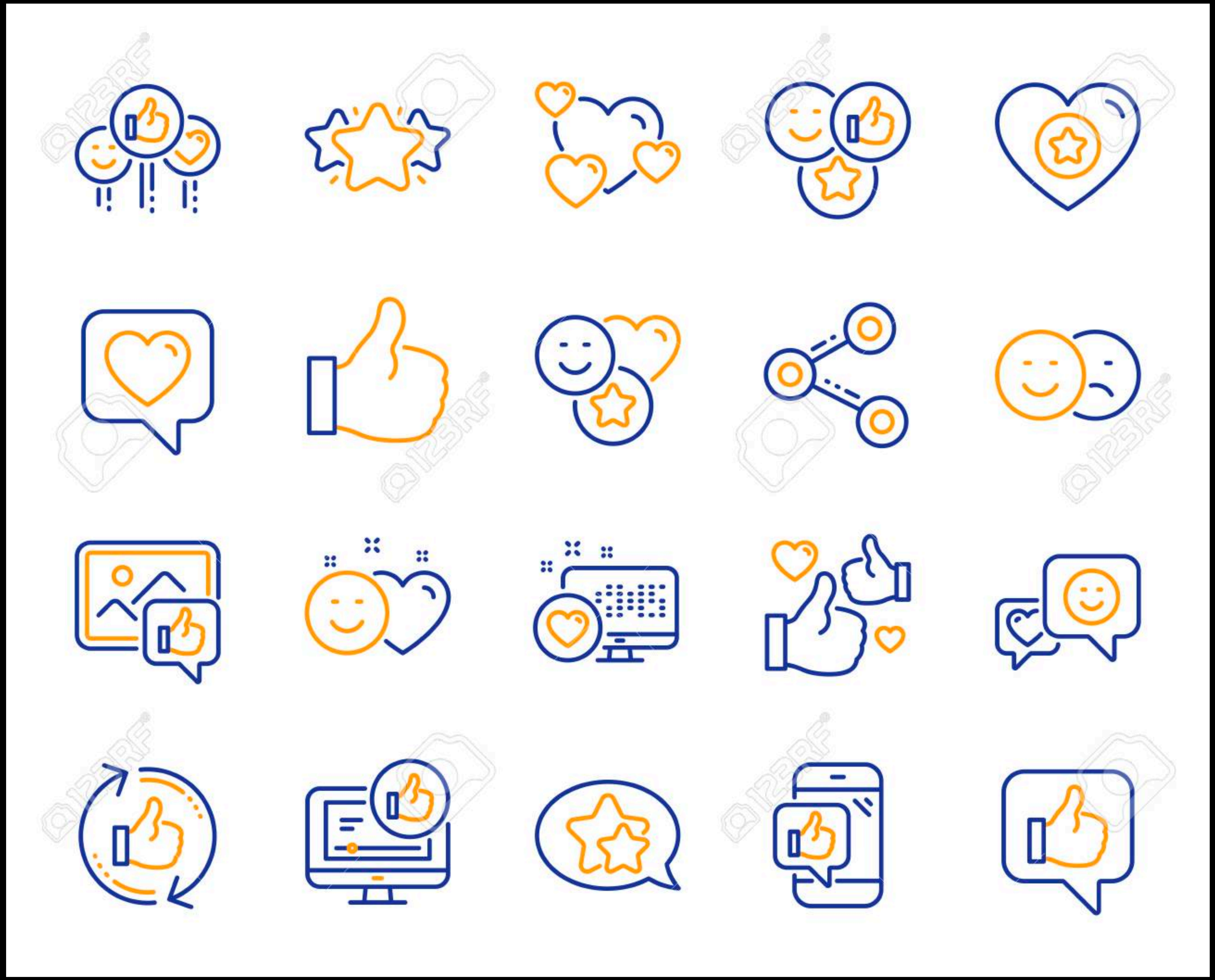
$$\mathrm{AUC} = \frac{1}{|U|} \sum_{u \in U} AUC(u)$$

$$\mathrm{AUC}(u) = \sum_{(u,i,j) \in D_S} z_u \delta\left(\hat{x}_{uij} > 0\right)$$

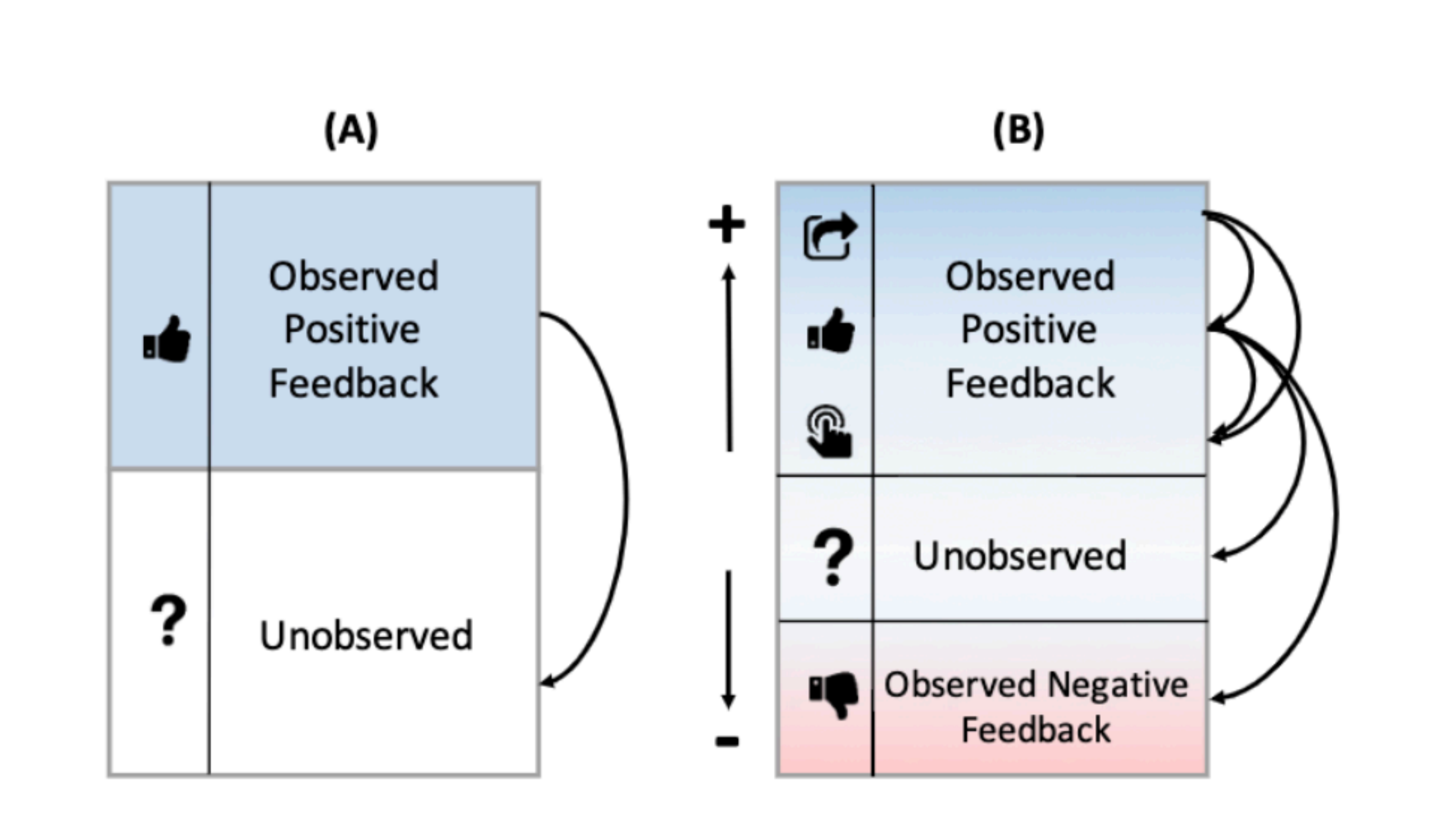
$$z_u = \frac{1}{|U| \left|I_u^+\right| \left|I \backslash I_u^+\right|}$$

https://arxiv.org/pdf/1205.2618.pdf

Learn on likes or shares?



MultiChannel BPR



procedure LEARN MF-BPR($S, \beta, W, \mathbb{L}$)
  initialize Θ
  repeat
    draw (u, i, L) from $p(u, i, L)$
    draw N from $p(N|u, L)$
    draw j from $p(j|u, L, N)$
    update Θ with BPR update rule [8]
  until convergence
  return Θ
end procedure

$$[Like, ?, Dislike, ?, Dislike, Like, Like]$$

$$[Like, Like, Like, ?, ?, Dislike, Dislike]$$

WARP

$$\sum_{i \in D_u} L\left(rank_i(f_u)\right)$$

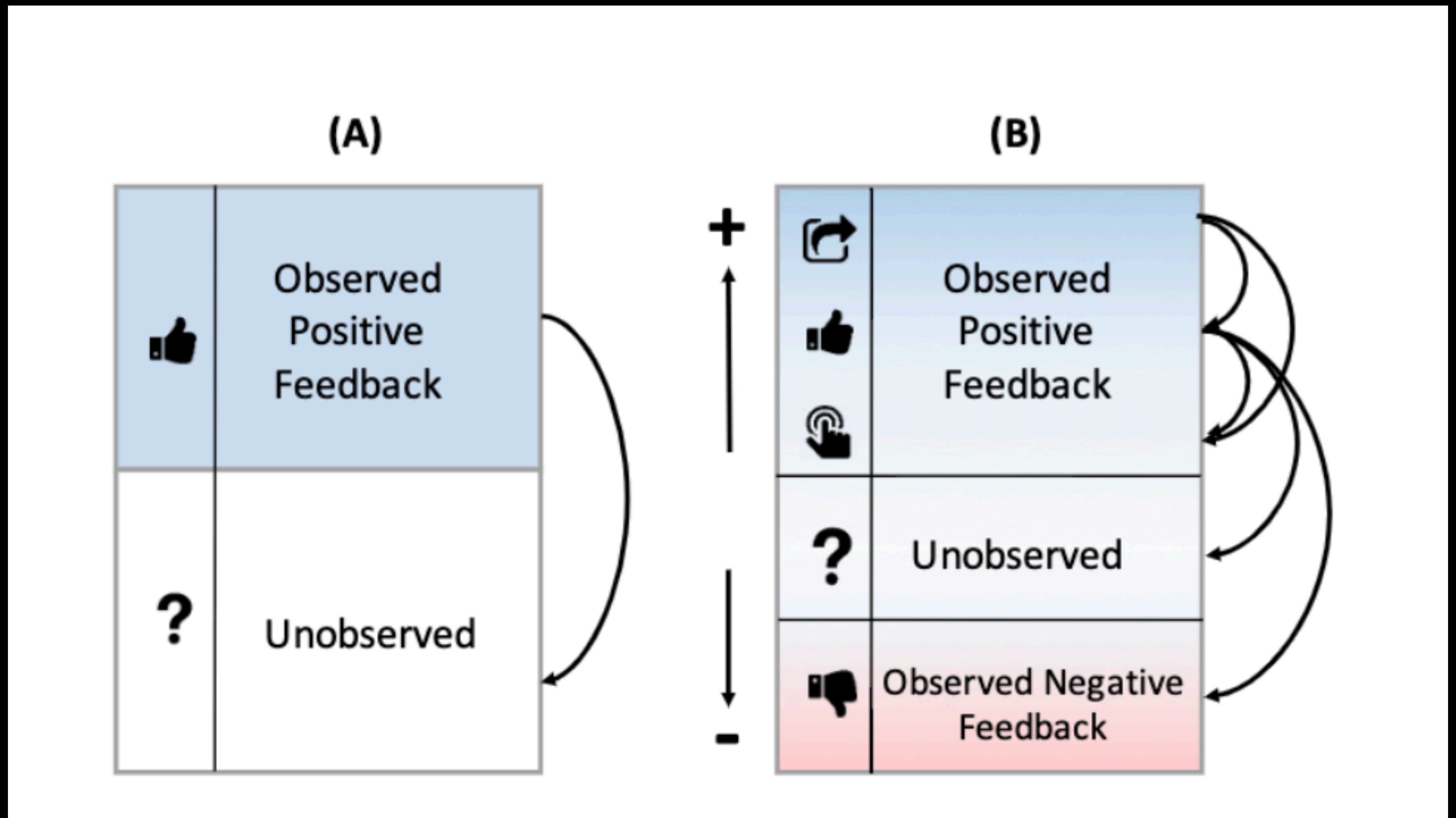
$$rank_i(f_u) = \sum_{j \notin D_u} I\left(f_u(j) \geq f_u(i)\right)$$

$$L(k) = \sum_{t=1}^{k} a_t, a_1 \geq a_2 \geq \ldots \geq 0$$

$$\begin{aligned} L\left(rank_i(f_u)\right) &= log\left(rank_i(f_u)\right) \frac{rank_i(f_u)}{rank_i(f_u)} \\ &= log\left(rank_i(f_u)\right) \frac{\sum_{j \notin D_u} I\left(f_u(j) \geq f_u(i)\right)}{rank_i(f_u)} \\ &= \sum_{j \notin D_u} log\left(rank_i(f_u)\right) \frac{I\left(f_u(j) \geq f_u(i)\right)}{rank_i(f_u)} \\ &= \sum_{j \notin D_u} log\left(rank_i(f_u)\right) \frac{\left|1 - f_u(i) + f_u(j)\right|_+}{rank_i(f_u)} \end{aligned}$$

$$L\left(rank_i(f_u)\right) = \sum_{j \notin D_u} log\left(rank_i(f_u)\right) \left|1 - f_u(i) + f_u(j)\right|_+$$

MultiChannel WARP



$[Like, Dislike, Share, Comment, Dislike, Like, Like]$

$[Like, Like, Like, Share, Comment, Dislike, Dislike]$

How to prepare data

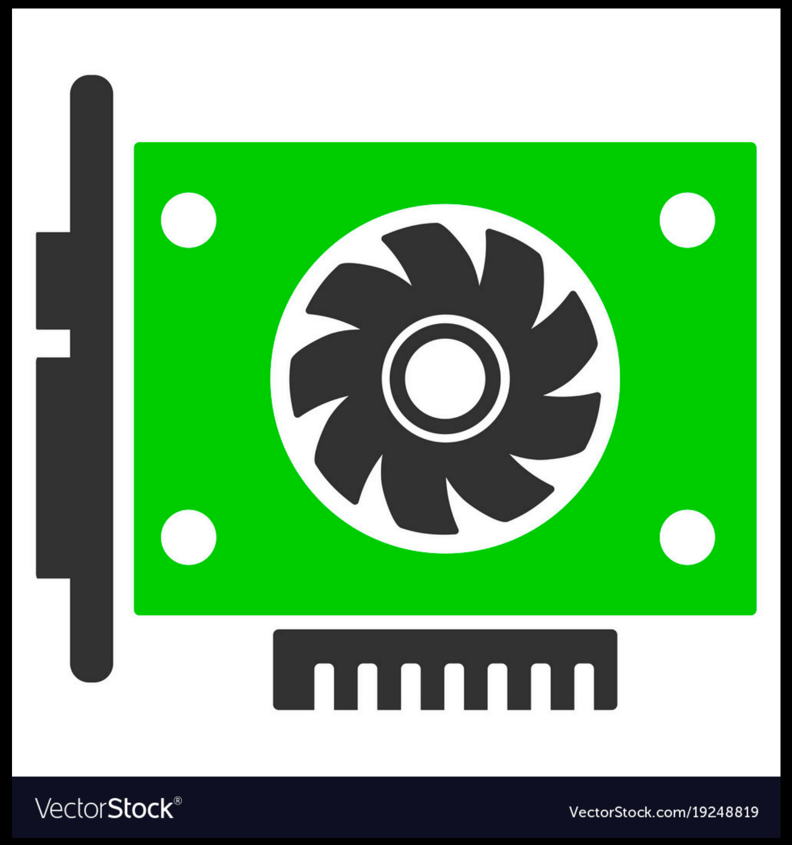
- Few user interactions -> bad recommendations for user | noise in data
- Few item interactions -> bad similars for item | noise in data
- Session analysis
- Detect strong negatives

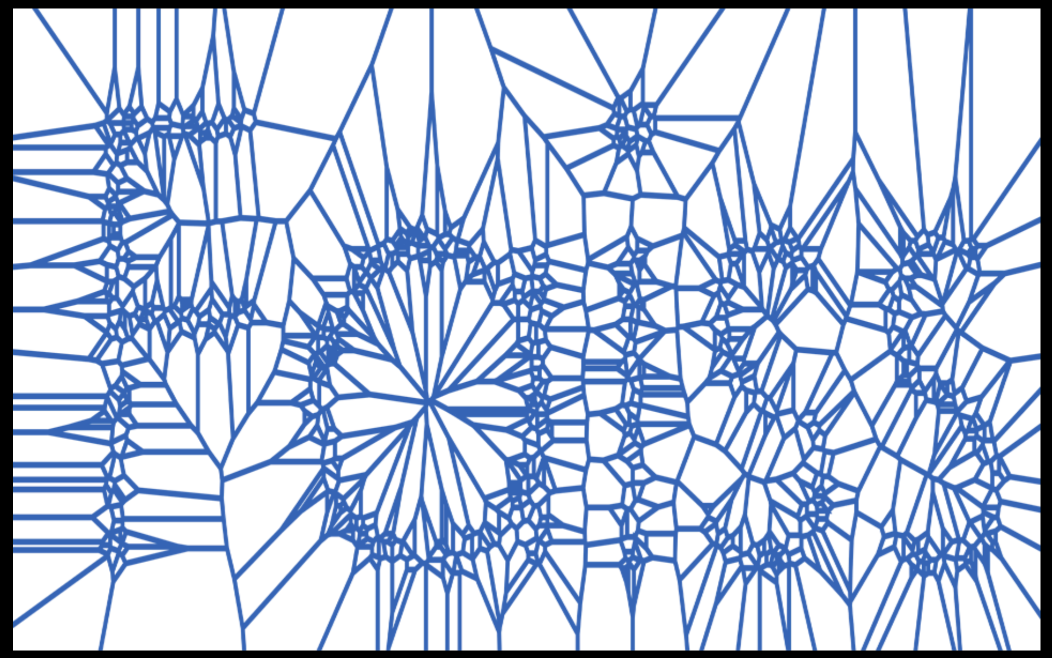
From .ipynb to production

- For each user construct recommendations in ipynb and send it to production and return it on each user's request

- Upload user's and item's embeddings to storage and scores each pair of user-item on user's request

- For each item construct similar items and send it to production and return it on each item's request





MF in business

- Mail.Ru | OK | VK
- Yandex
- Ivy
- Megogo
- etc

Problems

- Context features
- No Sessions
- No information about sequences
- Cold start

Conclusion

- MF - one of the easiest way to construct recommendations and similar users/items
- There is much more implicit feedback than explicit
- Recommendations is not about number approximation, it is about ranking (Use BPR or WARP)

Sources

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- https://github.com/benfred/implicit

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- http://www.machinelearning.ru/wiki/images/9/95/Voron-ML-CF.pdf