Discrete Mathematics

Lecture 11

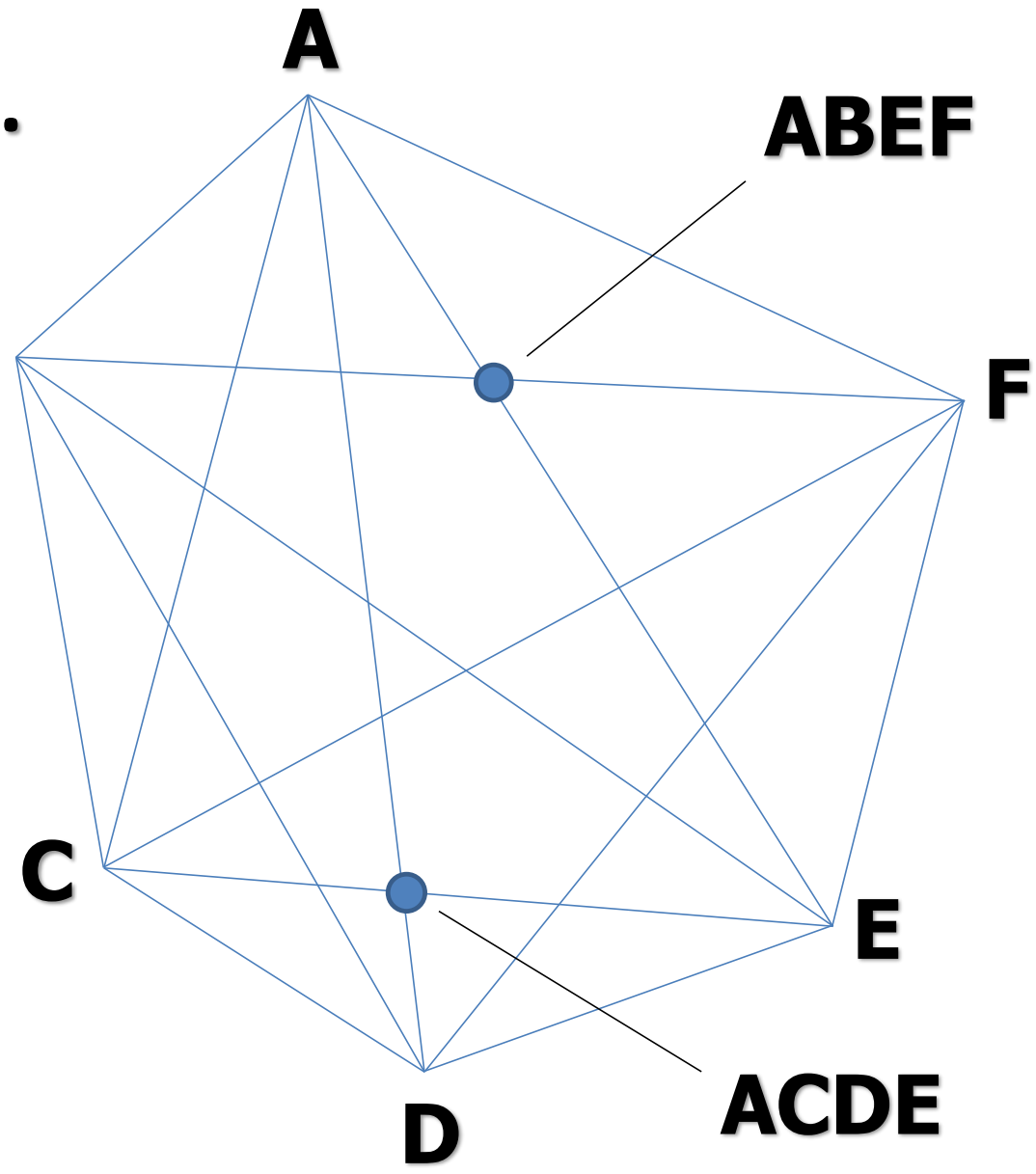
Combinatorics in Geometry

11.1 Intersections of Diagonals

A polygon is convex if every angle of it is convex, i.e., less than π.

Consider a convex polygon with *n* vertices. Assume that it has no 3 diagonals going through the same point. How many intersection points do the diagonals have?

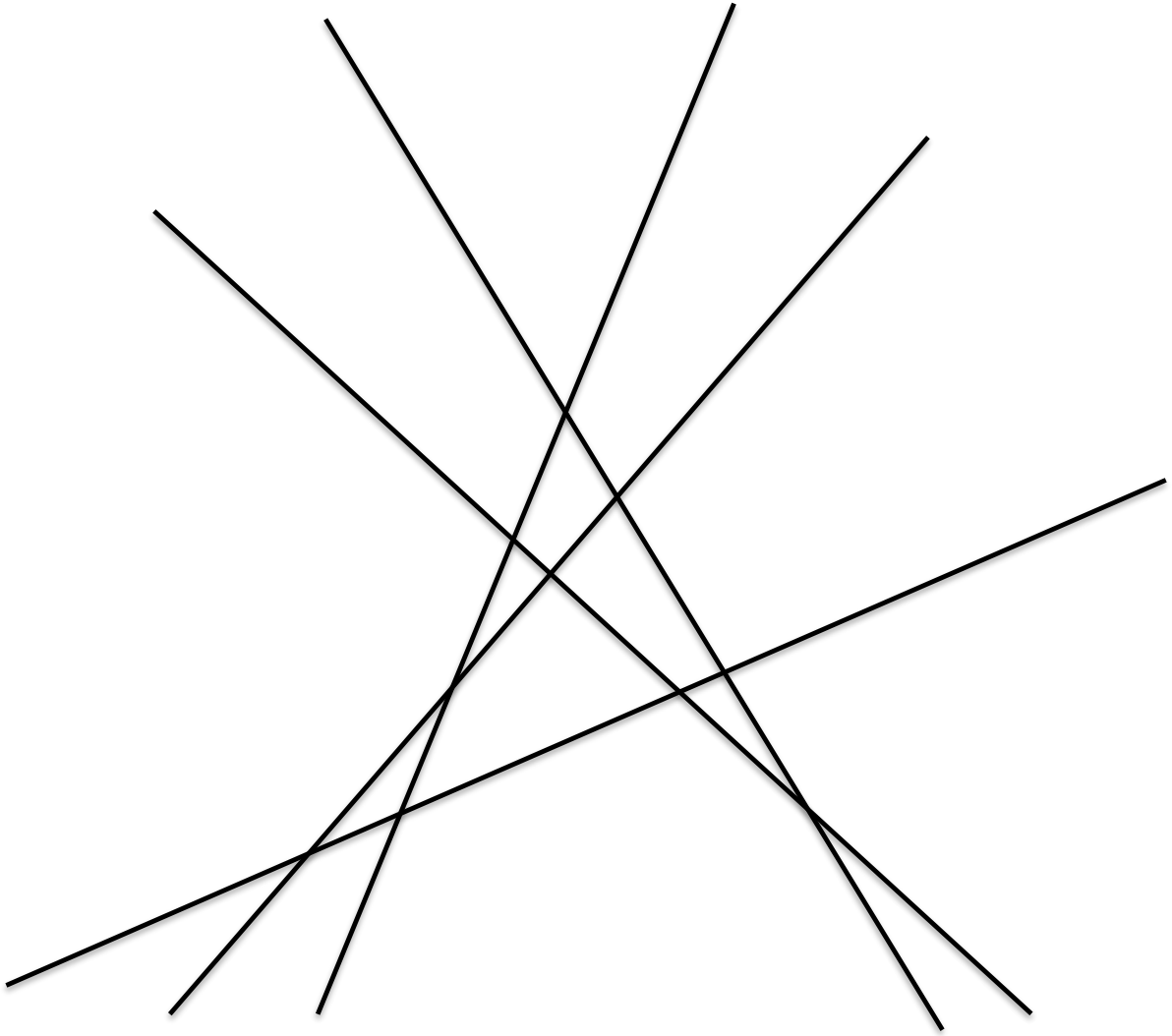
C(*n*,4).



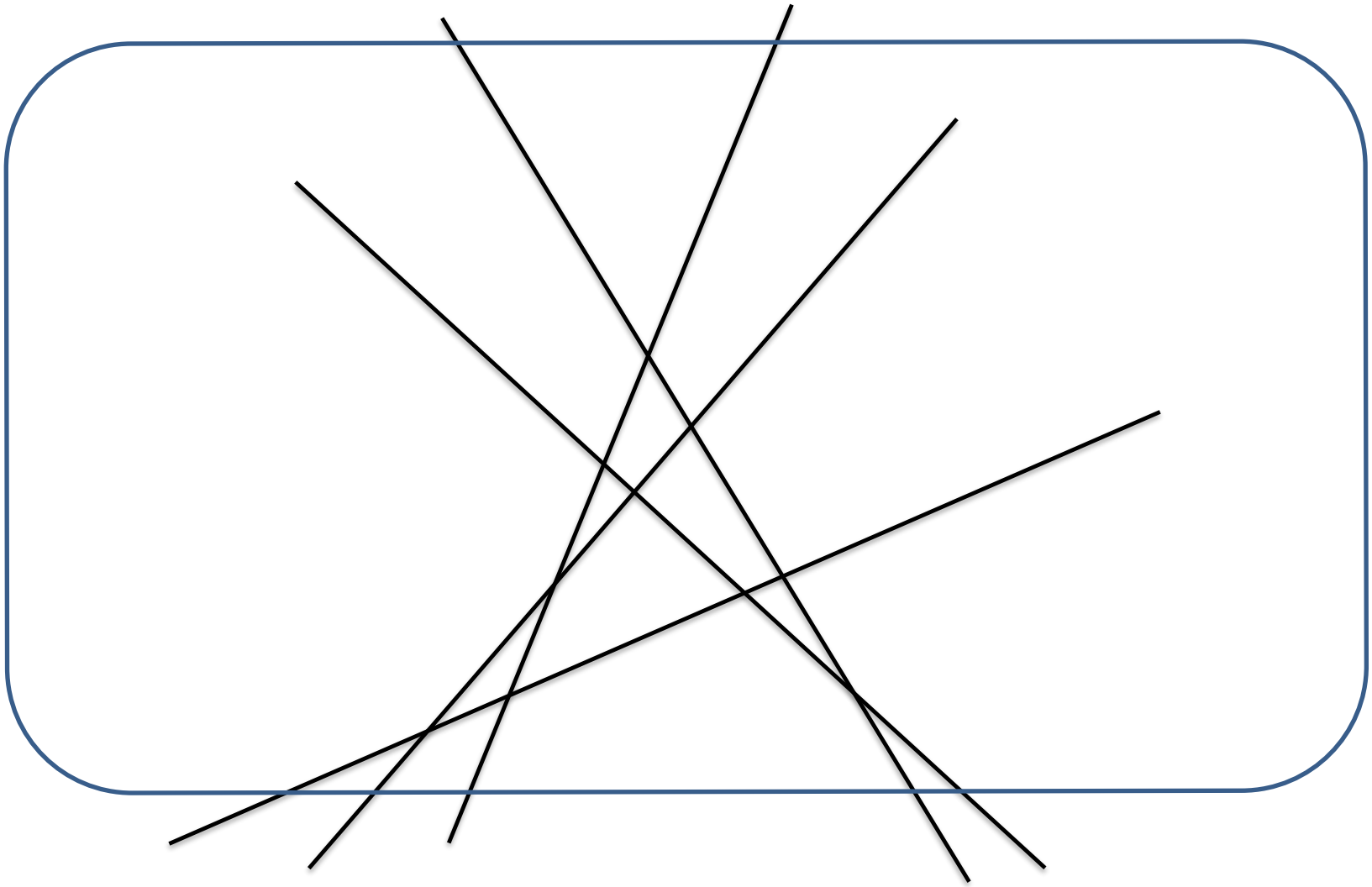
11.2 Counting regions

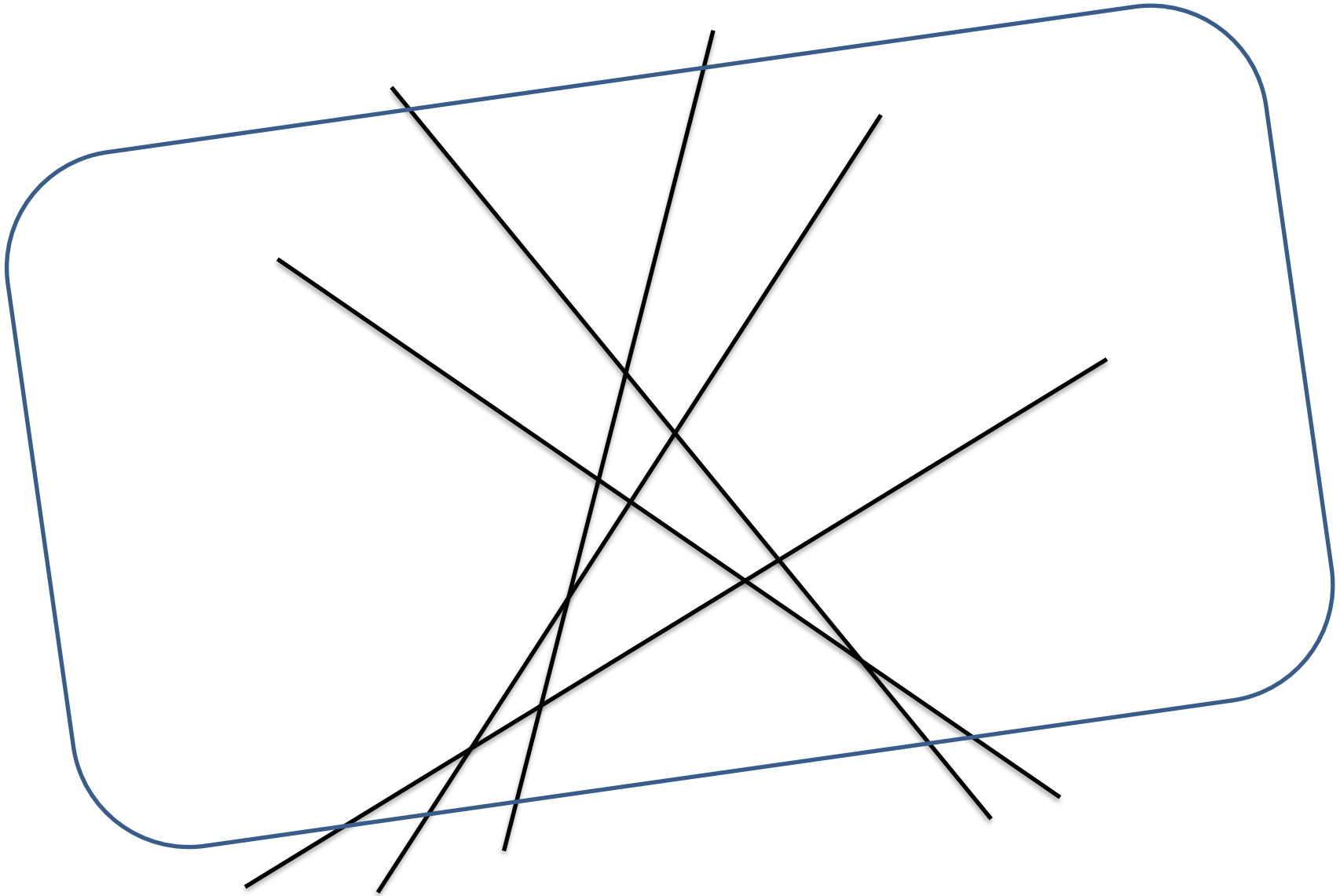
A set of lines in the plane such that no two are parallel and no three go through the same point is said to be in general position.

Theorem 11.2.1 A set of n lines in general position in the plane divides the plane into $1+n(n+1)/2$ regions.



$$a(n)=a(n-1)+n=1+(1+2+\cdots+n)$$
$$=1+n(n+1)/2.$$



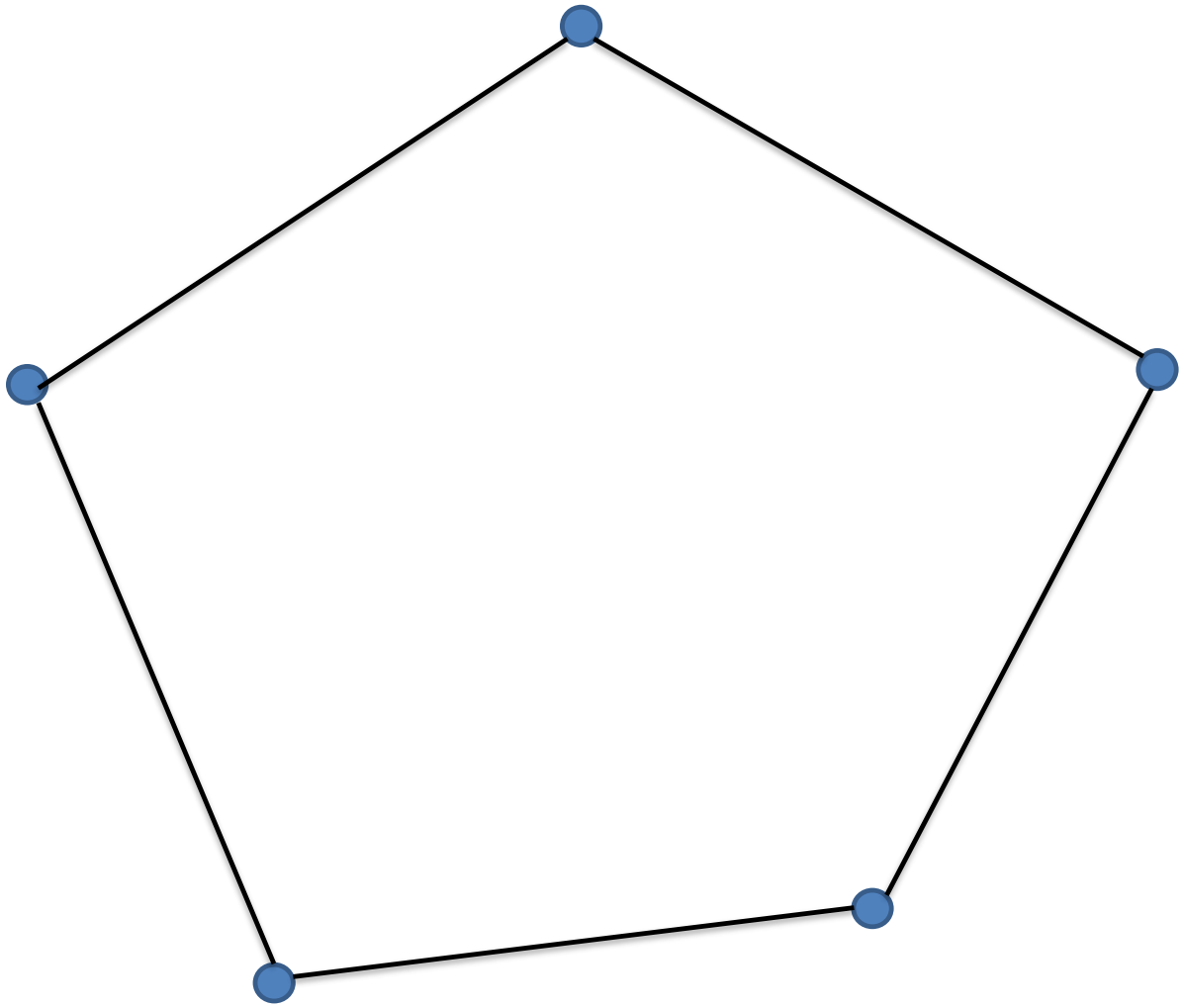


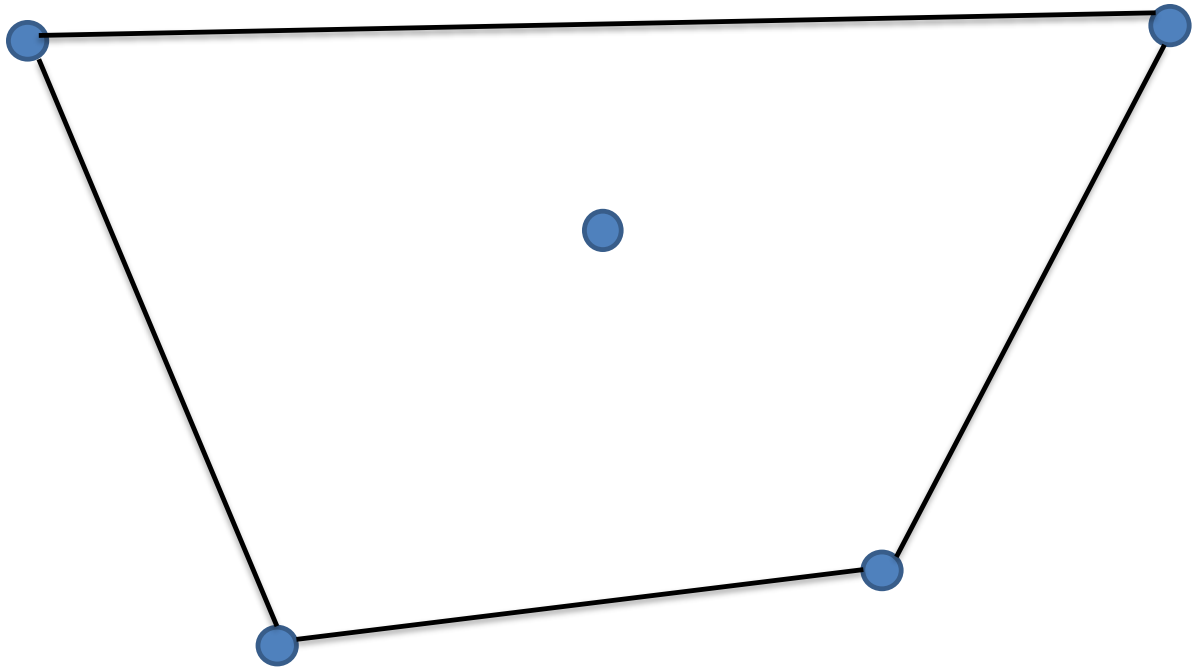
$$a(n)=1+n+C(n,2).$$

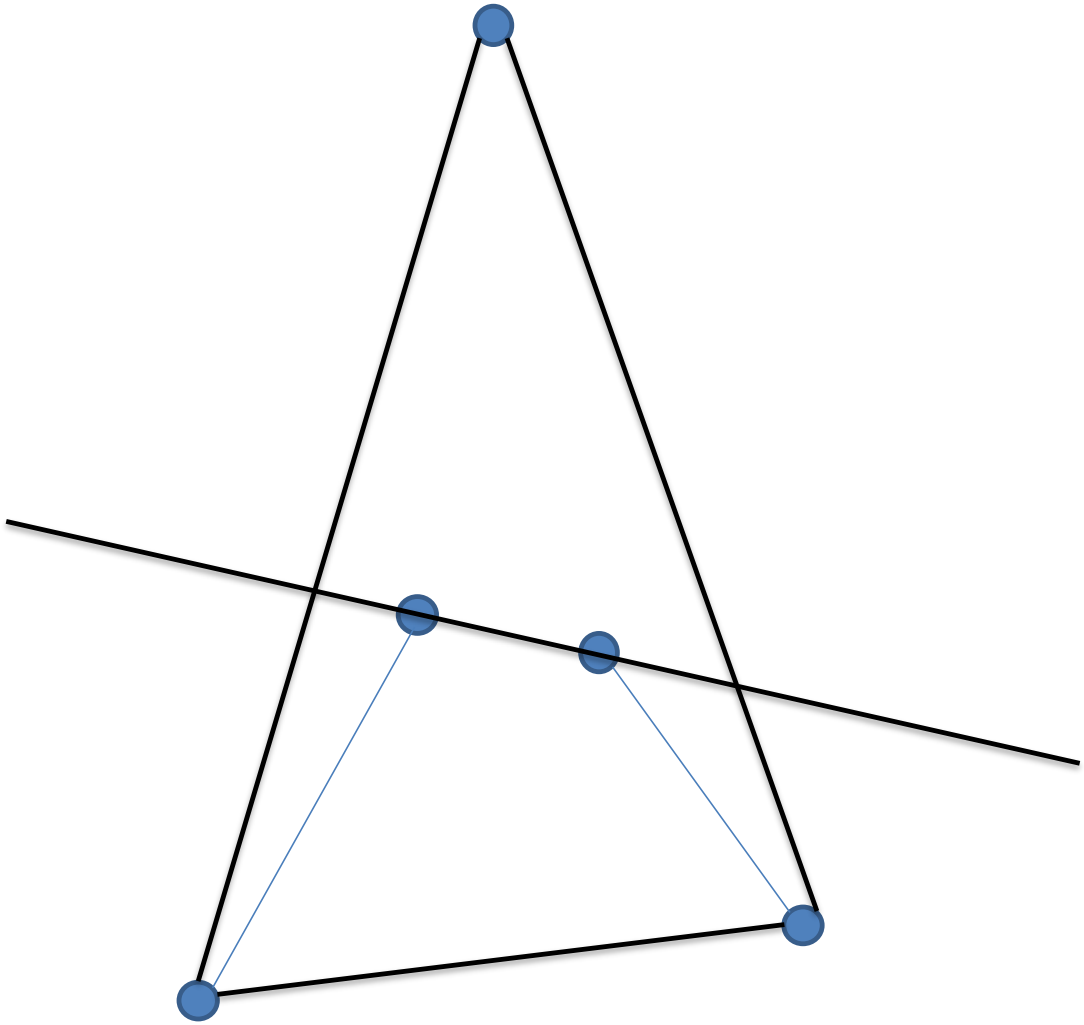
11.3 Convex Polygons

Happy End Problem

- If we are given five points in the plane such that no three of them are on a line, then we can always find four points among them that form the vertices of a convex quadrilateral.







- If we have **nine** points in the plane in general position, then one can choose **five** points among them that are the vertices of a convex **pentagon**.
- What is the maximum number of points in the plane, in general position, that do not contain the vertices of a convex *n*-gon?

n	2	3	4	5	6
	1	2	4	8	16?

Conjecture 2^{n-2}.