

$$m = 2417$$

The range of values mod 2417 is between -1208 and 1208

Step 0: We write  $m-1 = 2416 = 2^4 \cdot 151$ , hence  $b=4$  and  $t=151$

Iteration 1:

Step 1: Choose  $a=2$

Step 2: Compute the following sequence (mod 2417)

$$2^{151}, 2^{2 \cdot 151}, 2^{2^2 \cdot 151}, 2^{2^3 \cdot 151}, 2^{2^4 \cdot 151}$$

Step 3: First, we compute  $2^{151} \pmod{2417}$  using repeated squaring modular exponentiation.

$$151 = 2^0 + 2^1 + 2^2 + 2^4 + 2^7$$

$$2^{(2^0)} = 2 \pmod{2417}$$

$$2^{(2^1)} = 2^{(2^0)} \cdot 2^{(2^0)} = 2 \cdot 2 = 4 \pmod{2417}$$

$$2^{(2^2)} = 2^{(2^1)} \cdot 2^{(2^1)} = 4 \cdot 4 = 16 \pmod{2417}$$

$$2^{(2^3)} = 2^{(2^2)} \cdot 2^{(2^2)} = 16 \cdot 16 = 256 \pmod{2417}$$

$$2^{(2^4)} = 2^{(2^3)} \cdot 2^{(2^3)} = 256 \cdot 256 = 277 \pmod{2417}$$

$$2^{(2^5)} = 2^{(2^4)} \cdot 2^{(2^4)} = 277 \cdot 277 = 1802 \pmod{2417}$$

$$2^{(2^6)} = 2^{(2^5)} \cdot 2^{(2^5)} = 1802 \cdot 1802 = 1173 \pmod{2417}$$

$$2^{(2^7)} = 2^{(2^6)} \cdot 2^{(2^6)} = 1173 \cdot 1173 = 656 \pmod{2417}$$

$$\begin{aligned} 2^{151} &= 2^{2^0 + 2^1 + 2^2 + 2^4 + 2^7} = 2^{(2^0)} \cdot 2^{(2^1)} \cdot 2^{(2^2)} \cdot 2^{(2^4)} \cdot 2^{(2^7)} \\ &= 2 \cdot 4 \cdot 16 \cdot 277 \cdot 656 = 345 \pmod{2417} \end{aligned}$$

Continuing Step 3:

$$2^{151} = 345 \pmod{2417}$$

$$2^{2 \cdot 151} = (2^{151})^2 = 345^2 = 592 \pmod{2417}$$

$$2^{2^2 \cdot 151} = (2^{2 \cdot 151})^2 = 592^2 = -1 \pmod{2417}$$

$$2^{2^3 \cdot 151} = (2^{2^2 \cdot 151})^2 = (-1)^2 = 1 \pmod{2417}$$

Hence,  $m = 2417$  is possible to be prime (the sequence is: 345, 592, -1, 1, 1)

### Iteration 2:

Step 1: Choose  $a = 3$

Step 2: Compute the following sequence (mod 2417)

$$3^{151}, 3^{2 \cdot 151}, 3^{2^2 \cdot 151}, 3^{2^3 \cdot 151}, 3^{2^4 \cdot 151}$$

Step 3:

$$151 = 2^0 + 2^1 + 2^2 + 2^4 + 2^7$$

$$3^{(2^0)} = 3 \pmod{2417}$$

$$3^{(2^1)} = 3^{(2^0)} \cdot 3^{(2^0)} = 3 \cdot 3 = 9 \pmod{2417}$$

$$3^{(2^2)} = 3^{(2^1)} \cdot 3^{(2^1)} = 9 \cdot 9 = 81 \pmod{2417}$$

$$3^{(2^3)} = 3^{(2^2)} \cdot 3^{(2^2)} = 81 \cdot 81 = 1727 \pmod{2417}$$

$$3^{(2^4)} = 3^{(2^3)} \cdot 3^{(2^3)} = 1727 \cdot 1727 = 2368 \pmod{2417}$$

$$3^{(2^5)} = 3^{(2^4)} \cdot 3^{(2^4)} = 2368 \cdot 2368 = 2401 \pmod{2417}$$

$$3^{(2^6)} = 3^{(2^5)} \cdot 3^{(2^5)} = 2401 \cdot 2401 = 256 \pmod{2417}$$

$$3^{(2^7)} = 3^{(2^6)} \cdot 3^{(2^6)} = 256 \cdot 256 = 277 \pmod{2417}$$

$$3^{151} = 3^{2^0 + 2^1 + 2^2 + 2^4 + 2^7} = 3 \cdot 9 \cdot 81 \cdot 2368 \cdot 277 = 1443 \pmod{2417}$$

$$3^{151} = -974 \pmod{2417} \quad (-2417 + 1443 \text{ so the result is between } -1208 \text{ and } 1208)$$

$$3^{2 \cdot 151} = (3^{151})^2 = (-974)^2 = -1205 \pmod{2417}$$

$$3^{2^2 \cdot 151} = (3^{2 \cdot 151})^2 = (-1205)^2 = -592 \pmod{2417}$$

$$3^{2^3 \cdot 151} = (3^{2^2 \cdot 151})^2 = (-592)^2 = -1 \pmod{2417}$$

$$3^{2^4 \cdot 151} = (3^{2^3 \cdot 151})^2 = (-1)^2 = 1 \pmod{2417}$$

Hence,  $n = 2417$  is possible to be prime (seq: -974, -1205, -592, -1, 1)

### Iteration 3 :

Step 1 : Choose  $a = 5$ .

Step 2 : Compute the following sequence (mod 2417)

$$5^{151}, 5^{2 \cdot 151}, 5^{2^2 \cdot 151}, 5^{2^3 \cdot 151}, 5^{2^4 \cdot 151}$$

Step 3 :

$$151 = 2^0 + 2^1 + 2^2 + 2^4 + 2^7$$

$$5^{(2^0)} = 5 \pmod{2417}$$

$$5^{(2^1)} = 5^{(2^0)} \cdot 5^{(2^0)} = 5 \cdot 5 = 25 \pmod{2417}$$

$$5^{(2^2)} = 5^{(2^1)} \cdot 5^{(2^1)} = 25 \cdot 25 = 625 \pmod{2417}$$

$$5^{(2^3)} = 5^{(2^2)} \cdot 5^{(2^2)} = 625 \cdot 625 = 1488 \pmod{2417}$$

$$5^{(2^4)} = 5^{(2^3)} \cdot 5^{(2^3)} = 1488 \cdot 1488 = 172 \pmod{2417}$$

$$5^{(2^5)} = 5^{(2^4)} \cdot 5^{(2^4)} = 172 \cdot 172 = 580 \pmod{2417}$$

$$5^{(2^6)} = 5^{(2^5)} \cdot 5^{(2^5)} = 580 \cdot 580 = 437 \pmod{2417}$$

$$5^{(2^7)} = 5^{(2^6)} \cdot 5^{(2^6)} = 437 \cdot 437 = 26 \pmod{2417}$$

$$5^{151} = 5^{2^0 + 2^1 + 2^2 + 2^4 + 2^7} = 5 \cdot 25 \cdot 625 \cdot 172 \cdot 26 = 67 \pmod{2417}$$

$$5^{151} = 67 \pmod{2417}$$

$$5^{2 \cdot 151} = (5^{151})^2 = 67 \cdot 67 = -345 \pmod{2417}$$

$$5^{2^2 \cdot 151} = (5^{2 \cdot 151})^2 = (-345)^2 = 592 \pmod{2417}$$

$$5^{2^3 \cdot 151} = (5^{2^2 \cdot 151})^2 = 592^2 = -1 \pmod{2417}$$

$$5^{2^4 \cdot 151} = (5^{2^3 \cdot 151})^2 = (-1)^2 = 1 \pmod{2417}$$

Hence,  $m = 2417$  is possible to be prime (seq : 67, -345, 592, -1, 1)

According to the algorithm,  $m = 2417$  is probable prime. The probability is  $1 - 1/4^3$  (98.43 %)

$$n = 763$$

The range of values mod 763 is between -381 and 381

Step 0: We write  $n-1 = 762 = 2 \cdot 381$ , hence  $s=1$  and  $t=381$

Iteration 1:

Step 1: Choose  $a=2$ .

Step 2: Compute the following sequence (mod 763)

$$2^{381}, 2^{2 \cdot 381}$$

Step 3:

$$381 = 2^0 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^8$$

$$2^{(2^0)} = 2 \pmod{763}$$

$$2^{(2^1)} = 2^{(2^0)} \cdot 2^{(2^0)} = 2 \cdot 2 = 4 \pmod{763}$$

$$2^{(2^2)} = 2^{(2^1)} \cdot 2^{(2^1)} = 4 \cdot 4 = 16 \pmod{763}$$

$$2^{(2^3)} = 2^{(2^2)} \cdot 2^{(2^2)} = 16 \cdot 16 = 256 \pmod{763}$$

$$2^{(2^4)} = 2^{(2^3)} \cdot 2^{(2^3)} = 256 \cdot 256 = 681 \pmod{763}$$

$$2^{(2^5)} = 2^{(2^4)} \cdot 2^{(2^4)} = 681 \cdot 681 = 620 \pmod{763}$$

$$2^{(2^6)} = 2^{(2^5)} \cdot 2^{(2^5)} = 620 \cdot 620 = 611 \pmod{763}$$

$$2^{(2^7)} = 2^{(2^6)} \cdot 2^{(2^6)} = 611 \cdot 611 = 214 \pmod{763}$$

$$2^{(2^8)} = 2^{(2^7)} \cdot 2^{(2^7)} = 214 \cdot 214 = 16 \pmod{763}$$

$$2^{381} = 2^{2^0 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^8} = 2 \cdot 16 \cdot 256 \cdot 681 \cdot 620 \cdot 611 \cdot 16 =$$

$$= 744428 \pmod{763}$$

$$2^{381} = -335 \pmod{763}$$

$$2^{2 \cdot 381} = (2^{381})^2 = (-335)^2 = 64 \pmod{763}$$

Hence  $n = 763$  is surely composite (seg: -335, 64)

$$763 = 109 \cdot 7$$