0.1 Original NEB

Nudged elastic band (NEB) approaches to locating transition states are efficient alternatives to evaluating the PES on a uniform grid over some coordinates of interest. The implementation in autodE follows that in [J. Chem. Phys., 2000, 113, 9978]

For an image i in the nudged elastic band

$$\tau_{i} = \begin{cases}
\tau_{i}^{+} & \text{if } V_{i-1} < V_{i} < V_{i+1} \\
\tau_{i}^{-} & \text{if } V_{i+1} < V_{i} < V_{i-1} \\
\tau_{i}^{+} \Delta V_{i}^{max} + \tau_{i}^{-} \Delta V_{i}^{min} & \text{if } V_{i-1} < V_{i+1} \\
\tau_{i}^{+} \Delta V_{i}^{min} + \tau_{i}^{-} \Delta V_{i}^{max} & \text{if } V_{i+1} < V_{i-1}
\end{cases}$$
(1)

where

$$\tau_i^+ = x_{i+1} - x_i$$

$$\tau_i^- = x_i - x_{i-1}$$
(2)

and

$$\Delta V_i^{max} = \max(|V_{i+1} - V_i|, |V_{i-1} - V_i|)$$

$$\Delta V_i^{min} = \min(|V_{i+1} - V_i|, |V_{i-1} - V_i|)$$
(3)

and x_i are the coordinates of image i. The spring force is

$$\mathbf{F}_{i}^{s}|_{\parallel} = (k_{i}|\mathbf{x}_{i+1} - \mathbf{x}_{i}| - k_{i-1}|\mathbf{x}_{i} - \mathbf{x}_{i-1}|)\hat{\boldsymbol{\tau}}_{i}$$
(4)

and the total force on the image

$$\boldsymbol{F}_i = \boldsymbol{F}_i^s|_{\parallel} - \nabla V(\boldsymbol{x}_i)|_{\perp} \tag{5}$$

where

$$\nabla V(\boldsymbol{x}_i)|_{\perp} = \nabla V(\boldsymbol{x}_i) - \nabla V(\boldsymbol{x}_i) \cdot \hat{\boldsymbol{\tau}}_i \hat{\boldsymbol{\tau}}_i$$
 (6)

and finally $\hat{\boldsymbol{\tau}} = \boldsymbol{\tau}_i/|\boldsymbol{\tau}_i|$.

0.2 CI-NEB

The climbing image (CI) NEB implementation follows that in $[J.\ Chem.\ Phys., 2000, 113, 9901]$ where after a few iterations the force on the maximum energy image (m) is given by

$$\mathbf{F}_m = -\nabla V(\mathbf{x}_m) + 2\nabla V(\mathbf{x}_m) \cdot \hat{\boldsymbol{\tau}}_i \hat{\boldsymbol{\tau}}_i \tag{7}$$

which is the force due to the potential along the band being inverted.