Домашняя работа №10

$M\xi = 2$; $D\xi = 4/3$

$$\begin{cases} \frac{a+b}{2} = 2 \quad \Rightarrow \quad a = 4 - b \\ \frac{b^2 - 2ab + a^2}{12} = 4/3 \end{cases}$$

$$\frac{b^2 - 2b(4-b) + (4-b)^2}{12} = 4/3$$

$$\frac{b^2 - 8b + 2b^2 + 16 - 8b + b^2}{12} = 4/3$$

$$\frac{4b^2 - 16b + 16}{12} = 4/3$$

$$\frac{b^2 - 4b + 4}{3} = 4/3$$

$$b^2 - 4b = 0$$

$$b(b-4) = 0$$

$$b = 0 \lor b = 4$$

$[a, b] = [0; 4]$

$$E\xi = \int_{-\infty}^{+\infty} x f(x)\,dx$$

$$M\xi = \int_{-\infty}^{+\infty} x \cdot \frac{1}{4}\,dx =$$

$$= \frac{1}{4}\int_0^4 x\,dx = \frac{1}{8}x^2\Big|_0^4 =$$

$$= \frac{16}{8} = 2 \checkmark$$

$$D\xi = M\xi^2 - (M\xi)^2$$

$$M\xi^2 = \frac{1}{4}\int_0^4 x^2\,dx = \frac{1}{12}$$

<u>№ 8.17</u>

1) $P(0 < \xi < 1) = 2/3$, $P(1 < \xi < 2) = 1/6$

$$\begin{cases} \frac{1}{b-a} = 2/3 \\ \frac{1}{b-a} = 1/6 \end{cases}$$

$2/3 + 1/6 =$

$? = 5/6$

$$\begin{cases} 2(b-a) = 3 \\ (b-a) = 6 \end{cases}$$

$\cancel{\frac{2(b-a)}{3}}$

$P(\xi < x) = \frac{x-a}{\cancel{b-a}}$

$P(\eta < x) = \boxed{823}$

$M\xi = \int\limits_{-\infty}^{+\infty} x\,dx =$

$= \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$

$M\eta = \int\limits_{-\infty}^{+\infty} (1-x)\,dx =$

$= -\frac{(1-x)^2}{2}\Big|_0^1 = 0 + 1/2 = 1/2$

$M\xi^2 = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$

$D\xi = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

$M\eta^2 = -\frac{(1-x)^3}{3}\Big|_0^1 = \frac{1}{3}$

$D\eta = 1/3 - 1/4 = \frac{1}{12}$

$D\eta = D\xi$, $E\eta = E\xi$

следовательно $\eta = 1 - \xi \sim R(0, 1)$.

Обратное тоже верно

№8.38

1) $P(|\xi - M\xi| < 1) = 6/7$

$P(\xi < x) = \lambda \int_0^{+\infty} e^{-\lambda x} dx$

$P(|\xi - M\xi| < 1) =$

$= \lambda \int_0^{1} e^{-\lambda\left(x - \frac{1}{\lambda}\right)} dx =$

$= \lambda \int_0^{1} e^{-\lambda x - 1} dx =$

$= \frac{\lambda}{e} \int_0^{1} e^{-\lambda x} dx =$

$= -\frac{\lambda^2}{e} e^{-\lambda x} \Big|_0^1 =$

$= -\frac{\lambda^2}{e} e^{-\lambda} + \frac{\lambda^2}{e} = 6/7$

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$$\cancel{P(|\xi - M\xi| < 1) = \lambda \int_0^{1} \left(x - \frac{1}{\lambda}\right) e^{-\lambda x} dx}$$

$$P(|\xi - M\xi| < 1) =$$

$$= P(1 < (\xi - 1/\lambda) < 1) =$$

$$= P\left(\frac{1-\lambda}{\lambda} < \xi < \frac{\lambda+1}{\lambda}\right) =$$

$$= \lambda \int_{\frac{1-\lambda}{\lambda}}^{\frac{\lambda+1}{\lambda}} e^{-\lambda x} dx =$$

$$= \cancel{\lambda \cdot \left(\lambda\right)} - e^{-\lambda x} \Big|_{\frac{1-\lambda}{\lambda}}^{\frac{\lambda+1}{\lambda}} =$$

$$= -e^{-\lambda - 1} + e^{-1+\lambda} =$$

$$= \frac{e^{\lambda}}{e} - \frac{e^{-\lambda}}{e} = \frac{2\,\mathrm{sh}(\lambda)}{e} = 6/7$$

$2\,sh(\lambda) = \frac{6e}{7}$

$sh(\lambda) = \frac{3e}{7}$

$\lambda = arcsh\left(\frac{3e}{7}\right)$

$M\xi = \frac{1}{arcsh\left(\frac{3e}{7}\right)}$

2) $P(|\xi - M\xi| > 1) = 1/8$

$P(|\xi - M\xi| > 1) =$

$= P(1 < |\xi - M\xi| < \infty) =$

№ 8.401

$$M(\xi - c) = \lambda \int_{-\infty}^{+\infty} (x-c) e^{-\lambda x} dx =$$

$$= \lambda \left(\int_{-\infty}^{+\infty} x e^{-\lambda x} - \int_{-\infty}^{+\infty} c e^{-\lambda x} dx \right) =$$

$$= M\xi - c = 0$$

$$\frac{1}{\lambda} - c = 0$$

$$c = 1/\lambda$$