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Вариант №24

Найти производные:

(1) $y = \arccos^3\left(\sqrt{\ln\frac{\sqrt{8+x}}{x+2}}\right)$

$$y' = \left(\arccos^3\left(\sqrt{\ln\frac{\sqrt{8+x}}{x+2}}\right)\right)' = 3\arccos^2\left(\sqrt{\ln\frac{\sqrt{8+x}}{x+2}}\right)\cdot\left(\arccos\left(\sqrt{\ln\frac{\sqrt{8+x}}{x+2}}\right)\right)'$$

1) $$\left(\arccos\left(\sqrt{\ln\frac{\sqrt{8+x}}{x+2}}\right)\right)' = \left(\frac{-1}{\sqrt{1-\left(\sqrt{\ln\frac{\sqrt{x+8}}{x+2}}\right)^2}}\right)\cdot\left(\sqrt{\ln\frac{\sqrt{x+8}}{x+2}}\right)'$$

2) $$\left(\sqrt{\ln\frac{\sqrt{x+8}}{x+2}}\right)' = \frac{1}{2}\ln^{\frac{1}{2}-1}\left(\frac{\sqrt{x+8}}{x+2}\right)\cdot\left(\ln\left(\frac{\sqrt{x+8}}{x+2}\right)\right)'$$

3) $$\left(\ln\frac{\sqrt{x+8}}{x+2}\right)' = \frac{x+2}{\sqrt{x+8}}\cdot\left(\frac{\sqrt{x+8}}{x+2}\right)'$$

4) $$\left(\frac{\sqrt{x+8}}{x+2}\right)' = \frac{(\sqrt{x+8})'\cdot(x+2)-\sqrt{x+8}\cdot(x+2)'}{(x+2)^2} = \frac{\frac{1}{2}(x+8)^{\frac{1}{2}-1}\cdot(x+8)'\cdot(x+2)-\sqrt{x-8}\cdot 1}{(x+2)^2} = \frac{(x+2)-\sqrt{x-8}\,1}{2\sqrt{x-8}\cdot(x+2)^2} = -\frac{1}{2(x+2)}$$

5) $$\frac{x+2}{\sqrt{x+8}}\cdot\left(-\frac{1}{(x+2)}\right) = -\frac{x+2}{\sqrt{x+8}\cdot(x+2)} = -\frac{1}{\sqrt{x+8}}$$

6) $$\frac{1}{2}\ln^{-\frac{1}{2}}\left(\frac{\sqrt{x+8}}{x+2}\right)\cdot\left(-\frac{1}{\sqrt{x+8}}\right) = -\frac{1}{2\cdot\sqrt{\ln\left(\frac{\sqrt{x+8}}{x+2}\right)}\cdot\sqrt{x+8}}$$

7) $$\left(\frac{-1}{\sqrt{1-\left(\sqrt{\ln\frac{\sqrt{x+8}}{x+2}}\right)^2}}\right)\cdot\left(-\frac{1}{2\cdot\sqrt{\ln\left(\frac{\sqrt{x+8}}{x+2}\right)}\cdot\sqrt{x+8}}\right) = \frac{1}{2\cdot\sqrt{1-\ln\frac{\sqrt{x+8}}{x+2}}\cdot\sqrt{\ln\left(\frac{\sqrt{x+8}}{x+2}\right)}\cdot\sqrt{x+8}}$$

8) $$\frac{3\arccos^2\left(\sqrt{\ln\frac{\sqrt{x+8}}{x+2}}\right)}{2\sqrt{1-\ln\frac{\sqrt{x+8}}{x+2}}\cdot\sqrt{\ln\left(\frac{\sqrt{x+8}}{x+2}\right)}\cdot\sqrt{x+8}}$$

Ответ: $$\frac{3\arccos^2\left(\sqrt{\ln\frac{\sqrt{x+8}}{x+2}}\right)}{2\cdot\sqrt{1-\ln\frac{\sqrt{x+8}}{x+2}}\cdot\sqrt{\ln\frac{\sqrt{x+8}}{x+2}}\cdot\sqrt{x+8}}$$

② $y=\left(\sqrt{x^4-10x+6}\right)^{\arcsin(x+x^2)}$

$$y'=\left(\sqrt{x^4-10x+6}\right)^{\arcsin(x+x^2)}=(x^4-10x+6)^{\frac{\arcsin(x^2+x)}{2}}\cdot\left(\ln(x^4-10x+6)\cdot\frac{\arcsin(x^2+x)}{2}\right)'=$$

1) $\left(\ln(x^4-10x+6)\cdot\frac{\arcsin(x^2+x)}{2}\right)'=\frac{1}{2}\left(\arcsin(x^2+x)\cdot\ln(x^4-10x+6)\right)'$

2) $\left(\arcsin(x^2+x)\cdot\ln(x^4-10x+6)\right)'=(\arcsin(x^2+x))'\cdot\ln(x^4-10x+6)+\arcsin(x^2+x)\cdot$
$\cdot(\ln(x^4-10x+6))'$

$$y'=\frac{(x^4-10x+6)^{\frac{\arcsin(x^2+x)}{2}}\cdot\left(\frac{1}{\sqrt{1-(x^2+x)^2}}\cdot(x^2+x)'\cdot\ln(x^4-10x+6)+\arcsin(x^2+x)\cdot\frac{1}{x^4-10x+6}\cdot(x^4-10x+6)'\right)}{2}=$$

$$=\frac{(x^4-10x+6)^{\frac{\arcsin(x^2+x)}{2}}\left(\frac{(2x+1)\ln(x^4-10x+6)}{\sqrt{1-(x^2+x)^2}}+\frac{\arcsin(x^2+x)(4x^3-10)}{x^4-10x+6}\right)}{2}=$$

Ответ: $$\frac{(x^4-10x+6)^{\frac{\arcsin(x^2+x)}{2}}\left(\frac{(2x+1)\ln(x^4-10x+6)}{\sqrt{1-(x^2+x)^2}}+\frac{(4x^3-10)\arcsin(x^2+x)}{x^4-10x+6}\right)}{2}$$

③ $\sin(14+3xy-3y)+\frac{7x^4y-xy+11}{5y^2}=y^2-15x^3$

$$\left(\sin(14+3xy-3y)+\frac{7x^4y-xy+11}{5y^2}\right)'=(y^2-15x^3)'$$

$$(\sin(14+3xy-3y))'+\frac{1}{5}\left(\frac{7x^4-xy+11}{y^2}\right)'=(y^2)'-15(x^3)'$$

$$\cos(14+3xy-3y)\cdot(14+3xy-3y)'+\frac{1}{5}\left(\frac{(7x^4-xy+11)'\cdot y^2-(7x^4-xy+11)\cdot(y^2)'}{y^4}\right)=$$
$5y^3$
$$=2y\cdot y'-15\cdot3x^2$$

$$\cos(14+3xy-3y)\cdot(3(y'x+y)-3y')+\frac{y^2(28x^3-y'x-y)-2yy'(7x^4-xy+11)}{5y^4}=$$
$$=2yy'-45x^2$$

$$\frac{\cos(14+3xy-3y)\cdot(y^3y'(15x-15)+15y^4)}{5y^3}+\frac{y(y(28x^3-y'x-y)-2y'(7x^4-xy+11))}{5y^{4}}=2yy'-45$$

$$\frac{\cos(14+3xy-3y)\cdot(y^3y'(15x-15)+15y^4)+28x^3y-xyy'-y^2-14x^4y'+2xyy'-22y'}{5y^3}=2yy'-45x^2$$

Найти интегралы:

(1) $\int \frac{x\,dx}{(11-7x^2)^9} = \left[\begin{matrix} \text{подстановка:} & \frac{dt}{dx} = t' \\ t = 11-7x^2; & dt = (11-7x^2)'dx = 14x\,dx; \quad dx = -\frac{1}{14x}dt \end{matrix} \right] =$

$= \int \left(-\frac{1}{14x}\right) \cdot \frac{x}{t^9}\,dt = -\frac{1}{14}\int t^{-9} \cdot dt = -\frac{1}{14} \cdot \left(-\frac{1}{8t^8}\right) + C = \frac{1}{112t^8} + C = \left[\begin{matrix} \text{обратная} \\ \text{замена} \\ t = 11-7x^2 \end{matrix} \right] =$

$= \frac{1}{112(11-7x^2)^8} + C$

Ответ: $\frac{1}{112(11-7x^2)^8} + C$

(2) $\int (x^5+9x)\ln x\,dx = \left[\begin{matrix} \text{метод стрелок:} & u = \ln x;\ u' = \frac{1}{x} \\ \int uv' = uv - \int u'v; & v = (x^5+9x)'; \int v = \int (x^5+9x)dx = \end{matrix} \right.$

$\left. = \frac{x^6+27x^2}{6} \right] = \left(\frac{x^6+27x^2}{6}\right) \cdot \ln x - \int \frac{1}{x} \cdot \frac{(x^6+27x^2)}{6} = \frac{\ln x(x^6+27x^2)}{6} -$

$- \int \frac{x(x^5+27x)}{x \cdot 6} = \frac{\ln x(x^6+27x^2)}{6} - \frac{1}{6}\int x^5 - \frac{27}{6}\int x = \frac{\ln x(x^6+27x^2)}{6} - \frac{x^6}{36} - \frac{9x^2}{4} + C =$

$= \frac{\ln x(x^6+27x^2)}{6} - \frac{x^6+81x^2}{36} + C = \frac{6\ln x(x^6+27x^2) - x^6 + 81x^2}{36} + C =$

$= \frac{6 \cdot x^2 \cdot \ln x(x^4+27) - x^2(x^4+81)}{36} = \frac{x^2(6\ln x(x^4+27) - x^4 - 81)}{36} + C$

Ответ: $\frac{x^2(6\ln x(x^4+27) - x^4 - 81)}{36} + C$

(4) $\int \frac{dx}{1+\sqrt{2x+1}} = \left[\begin{matrix} \text{метод подстановки} & \\ t = \sqrt{2x+1} & x = \frac{1}{2}(t^2-1) \\ t^2 = 2x+1 & dx = x'dt = \frac{1}{2} \cdot 2t\,dt = t\,dt \end{matrix} \right] =$

$= \int \frac{t\,dt}{1+t} = \int \frac{t+1-1}{t+1}\,dt = \int \frac{t+1}{t+1}dt - \int \frac{dt}{t+1} = t - \ln|t+1| + C = \sqrt{2x+1} - \ln|\sqrt{2x+1}+1| +$

$+ C$

Ответ: $\sqrt{2x+1} - \ln|\sqrt{2x+1}+1| + C$

(5) $$\int \frac{dx}{3\cos^2 x + 5\sin^2 x} = \left[\begin{array}{l}\text{подстановка:} \\ tg\, x = t;\ dx = \frac{dt}{1+t^2};\ \sin x = \frac{t}{\sqrt{1+t^2}};\ \cos x = \frac{1}{\sqrt{1+t^2}}\end{array}\right] =$$

$$= \int \frac{1}{\left(\frac{5t^2}{1+t^2} + \frac{3}{1+t^2}\right)(t^2+1)}\, dt = \int \frac{1}{\frac{(5t^2+3)}{\cancel{1+t^2}} \cdot \cancel{(t^2+1)}}\, dt = \int \frac{1}{5t^2+3}\, dt =$$

$$= \frac{\sqrt{15}\, arctg\left(\sqrt{15}\cdot\frac{t}{3}\right)}{\cancel{15}\cdot\sqrt{15}} + C = \frac{arctg\left(\sqrt{15}\cdot\frac{t}{3}\right)}{\sqrt{15}} + C$$

Ответ: $\frac{arctg\left(\sqrt{15}\cdot\frac{t}{3}\right)}{\sqrt{15}} + C$

(6) $$\int x \arcsin(5x)\, dx = \left[\begin{array}{l}\text{по частям:} \\ u = \arcsin 5x;\ u' = \frac{5}{\sqrt{1-25x^2}} \\ v' = x;\ \int v' = \frac{x^2}{2}\end{array} \middle| \int uv' = uv - \int u'v \right] =$$

$$= \frac{x^2}{2}\cdot \arcsin 5x - \int \frac{5x^2}{2\sqrt{1-25x^2}}\, dx = \frac{x^2}{2}\cdot\arcsin 5x - \frac{5}{2}\int \frac{x^2}{\sqrt{1-25x^2}}\, dx =$$

$$= \left[\begin{array}{l}\text{подстановка} \\ x = \frac{\sin t}{5};\ dx = \frac{\cos t}{25}\end{array}\right] = \frac{x^2}{2}\cdot\arcsin 5x - \frac{5}{2}\int \frac{\sin^2 t \cdot \frac{\cos t}{25}}{\cos t} = \frac{x^2}{2}\cdot\arcsin 5x -$$

$$- \frac{\cancel{5}\,1}{2\cdot\cancel{25}\,5}\int \frac{\sin^2 t \cdot \cancel{\cos t}}{\cancel{\cos t}} = \frac{x^2}{2}\cdot\arcsin 5x - \frac{1}{10}\int \sin^2 t\, dt = \frac{x^2}{2}\cdot\arcsin 5x +$$

$$+ \frac{\cos t\cdot\sin t}{10\cdot 2} + \frac{t}{2} + C = \frac{x^2}{2}\cdot\arcsin 5x + \frac{\sqrt{1-25x^2}\cdot 5x}{20} + \frac{\arcsin x}{2} + C$$

Ответ: $\frac{x^2}{2}\cdot\arcsin 5x + \frac{\sqrt{1-25x^2}\cdot 5x}{20} + \frac{\arcsin x}{2} + C$

(7) $$\int_{-5}^{-2} \frac{dx}{\sqrt{5-4x-x^2}}$$

$$\int \frac{1}{\sqrt{5-4x-x^2}}\, dx = \int \frac{1}{\sqrt{-(x+2)^2+9}}\, dx = \left[\begin{array}{l}\text{подстановка} \\ t = \frac{(x+2)}{3} \\ dt = d\left(\frac{x+2}{3}\right) = \frac{1}{3};\ dx = 3dt\end{array}\right] = \int \frac{3}{\sqrt{9-9t^2}}\, dt =$$

$$= \int \frac{1}{\sqrt{1-t^2}}\, dt = \arcsin t + C = \arcsin\left(\frac{x+2}{3}\right) + C$$

$$\int_{-5}^{-2} \frac{1}{\sqrt{5-4x-x^2}} = [F(b) - F(a)] = \arcsin\left(\frac{-2+2}{3}\right) - \arcsin\left(\frac{-5+2}{3}\right) = \arcsin 0 - \arcsin(-1) =$$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Ответ: $\frac{\pi}{2}$

Решить Дифф. ур:

① Найти частное решение ДУ: $y' = -2y$, удовлетворяющее условию $y(0)=25$

$$y' = -2y$$

$$\frac{dy}{dx} = -2y$$

$$dy = -2y\,dx$$

$$\frac{dy}{y} = -2\,dx$$

$$\int \frac{dy}{y} = -2\int dx$$

$$\ln y = -2x + C$$

$$y = e^{-2x+C}$$

$$y = e^{-2x} \cdot e^{C}$$

$$y = e^{-2x} \cdot C \quad , y(0) = 25$$

$$25 = e^{-2\cdot 0} \cdot C$$

$$C = 25$$

Частное решение:

$$y = e^{-2x} \cdot 25$$

② Найти общий интеграл для ДУ:

$$xy' = 3\sqrt{3x^2+y^2}+y$$

$$y' = \frac{3\sqrt{3x^2+y^2}+y}{x}$$

$$y' = 3\sqrt{\frac{3x^2}{x^2}+\frac{y^2}{x^2}}+\frac{y}{x}$$

$$y' = 3\sqrt{3+\left(\frac{y}{x}\right)^2}+\frac{y}{x}$$

Замена:

$$t=\frac{y}{x}\ ;\ y=x\cdot t\ ;\ y'=x'\cdot t+x\cdot t'=t+xt'$$

$$t+x\cdot t' = 3\sqrt{3+t^2}+t$$

$$xt' = 3\sqrt{3+t^2}$$

$$x\frac{dt}{dx} = 3\sqrt{3+t^2}$$

$$x\,dt = 3\sqrt{3+t^2}\,dx$$

$$\frac{dt}{3\sqrt{3+t^2}} = \frac{dx}{x}$$

$$\int\frac{dt}{3\sqrt{3+t^2}} = \int\frac{dx}{x}$$

$$\frac{1}{3}\ln|t+\sqrt{t^2+3}| = \ln x + C \quad |\cdot 3$$

$$\ln|t+\sqrt{t^2+3}| = 3\ln x + \ln C$$

$$t+\sqrt{t^2+3} = x^3\cdot C$$

$$\frac{y}{x}+\sqrt{3+\left(\frac{y}{x}\right)^2} = x^3C$$

Ответ: $\frac{y}{x}+\sqrt{3+\left(\frac{y}{x}\right)^3} = x^3C$