Enviromental statistics homework

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The mean squared error as a function of ρ

Exercise:

obtain (and store) k = 1000 simulation from Moran's I and APLE sampling distribution for "some" values of ρ ranging from 0 to 0.95 and obtain the plot of MSE vs rho

Solution

Before the solution let's run some global objects and variables from previous lectures.

```
library(sf)
library(tmap)
library(spdep)
library(tidyverse)

#read the italian provinces map
map <- read_sf("map-folder/map.shp")

map_nblist <- poly2nb(map)#the adjacency matrix

wlist <- nb2listw(map_nblist, style = "B")#adjacency matrix  in list form

wlist_til <- nb2listw(map_nblist, style = "W")#row-standardised adj. matrixin list form

n <- nrow(map)
w <- matrix(0, nrow = n, ncol = n)
for(k in 1:n){
  w[k, wlist$neighbours[[k]] ]<- 1
}

w_til <- w/rowSums(w)
```

Well, firstly, let's generate a vector of ρ, where $0 \leqslant \rho \leqslant 0.95$. Let's pick 20 points to get a more smooth graph.

```
n_draw = 20
rho <- seq(from = 0, to =  0.95, length.out = n_draw)
```

Then I create two numeric vectors to store results of simulation.

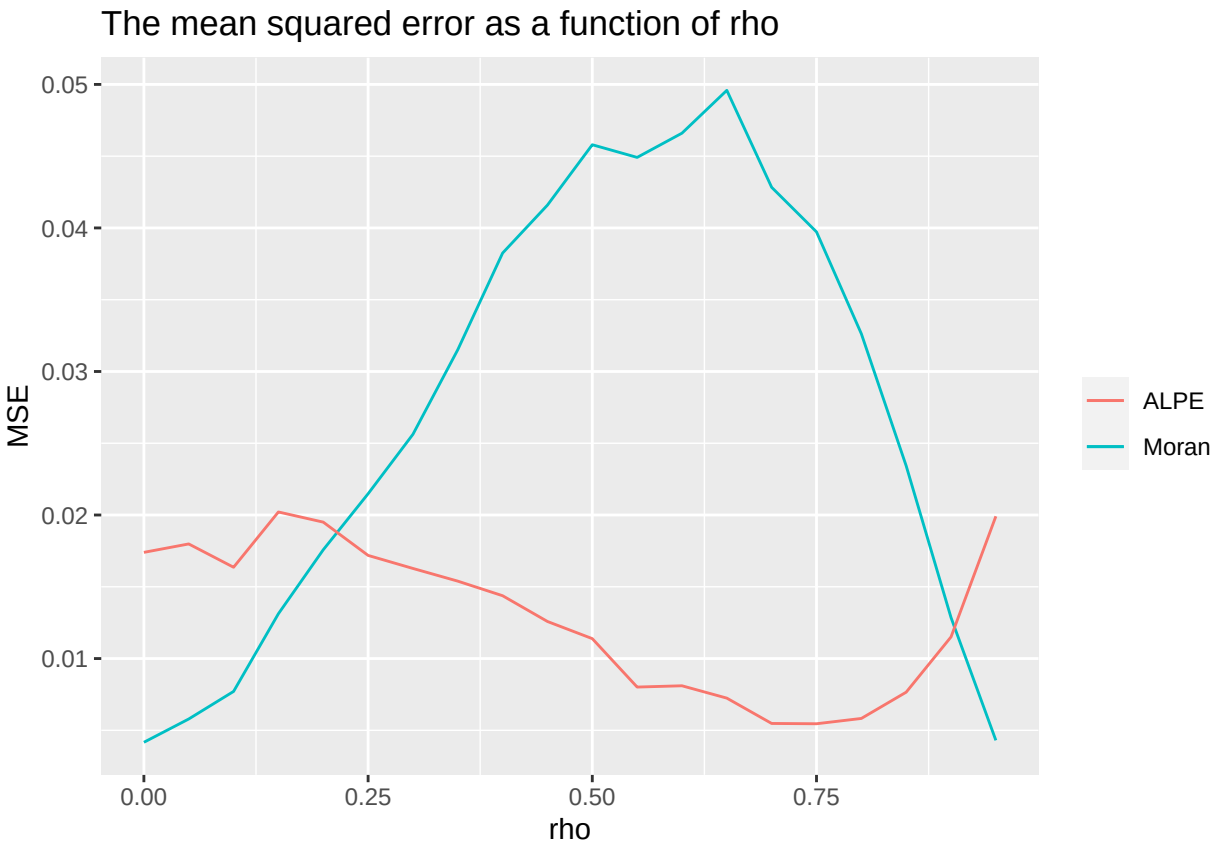
```
mse_moran_sim <- numeric(n_draw)#vector of moran's I
mse_aple_sim <- numeric(n_draw)#vector of aple
```

Then make the simulation with $K = 1000$

```
library(spatialreg)
library(mvtnorm)

k = 1000

for(j in 1:n_draw){

  In <- diag(n)

  sigma_sar <- solve(
    t(In - rho[j] * w_til) %*% (In - rho[j] * w_til)
  )

  # simulate the moran's I and aple

  moran_sim <- numeric(k)
  aple_sim <- numeric(k)

  for(i in 1:k){
    Y <- c(mvtnorm::rmvnorm(1, sigma = sigma_sar) )
    Y <- Y - mean(Y)
    moran_sim[i] <- moran(Y, wlist_til, n = n, S0 = n)$I
    aple_sim[i] <- aple(Y, wlist_til)
  }

  #store the result of simulation
  mse_moran_sim[j] <- mean((moran_sim - rho[j])^2)
  mse_aple_sim[j] <- mean((aple_sim - rho[j])^2)


}
```

Final plot

Finally, let's draw the graph using ggplot to compare MSEs of Moran's I and APLE


The mean squared error as a function of rho
MSE
0.05
0.04
0.03
0.02
0.01
0.00
0.25
0.50
0.75
rho
ALPE
Moran