three point perspective

At this point it's customary to explore the capabilities of 2PP in a variety of specific drawing problems. I want to keep the momentum and look at **three point perspective**, which allows you to construct a form in any orientation (from any viewpoint).

Three point perspective is often illustrated with aerial views of Manhattan, looking down on a skyline bristling with skyscrapers. But artists will find 3PP equally useful in still life or figure paintings — where the view downward onto a table of objects or a piece of furniture can be just as steep — and in landscape views up toward soaring cliffs or a stand of tall trees.

The 3PP perspective problems and construction methods are complex, and it may seem we lose more in clarity than we gain in drawing power. Many artists have come to the same conclusion, and avoid 3PP for simpler approaches, including freehand modification of drawings blocked out in 2PP, or the expedient of **tracing photos**.

I won't disagree with those solutions; they can be convenient and effective. They fall short, however, if you must add new forms around the primary form — for example, if you have traced the photograph of an existing building, and want to insert new or different buildings around it — or if you want to show the building from a different point of view, or require more precision than freehand perspective can provide. For these common situations, 3PP is invaluable.

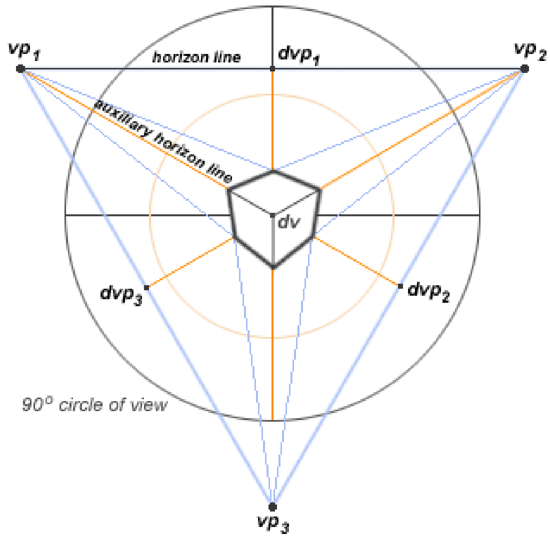
three point perspective

As we add vanishing points, we remove aspects of perspective that we can take for granted. In 1PP or central perspective, the relationship of the vanishing points and horizon line to the direction of view are taken for granted. In 3PP both the vanishing point locations and the relationship between in the direction of view and the ground plane (horizon line) must be specified.

Defining Features of Three Point Perspective. The diagram shows the simplest 3PP situation: a cube centered in view but first rotated 45° to one side and then downward until all front faces appear of equal size. In all **three point perspective** views there are *no faces or edges parallel with the picture plane.*

In particular, because the direction of view is still assumed to be perpendicular to the image plane, the direction of view is **no longer parallel to the ground plane** when the primary forms are constructed as buildings are, with walls perpendicular to the ground.

The canonical view places the three front edges of the cube in a 54.7° angle to the direction of view, so that all three vanishing points are outside the circle of view. The planes of the three front faces are at a 35.3° angle to the direction of view, with vanishing lines defined by the triangle of three vanishing points.



three point perspective: the basic geometry

The three **vanishing points** (**vp**$_1$, **vp**$_2$ and **vp**$_3$) control the recession of all *lines* parallel to the edges of the cube. This means the outline of each face is determined by two vanishing points, rather than one as in 2PP.

Connecting the vanishing points are three **vanishing lines**, which control recession of all *planes* parallel with each front and matching back face of the cube and all planes parallel to them. Each vanishing line also contains the vanishing points for all lines parallel to their respective planes, including the diagonal vanishing points (**dvp**$_1$, **dvp**$_2$ and **dvp**$_3$) for the planes.

A vanishing line perpendicular to the viewer's vertical orientation (parallel to the ground plane) is typically the **horizon line** in architectural or landscape uses of perspective. It is the vanishing line for all planes parallel to the ground plane, and contains all vanishing points for lines parallel to the ground plane (perspective **rules 13 and 14**).

Each vanishing line is connected to the vanishing point opposite to it by an **auxiliary horizon line** (shown in orange in the figure). These are the *vanishing lines for measure points* for each of the three dimensions of the cube. In 2PP, the horizon line was a vanishing line for both the vanishing points and measure points, but in 3PP these functions can be separated.

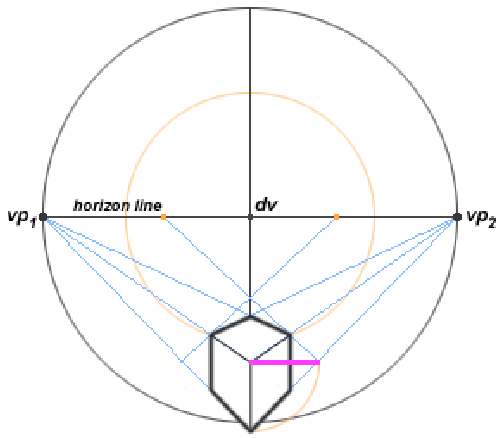
The auxiliary horizon lines always **intersect at the direction of view** (the principal point) — that is, they link the vanishing points of the object to the vanishing point of the viewer's central recession (perspective gradient). Therefore the **principal point is always inside the vp triangle** formed by the three vanishing lines: if it is not, then the primary form does not define right angled vanishing points (it is a pyramid or a lopsided cube).

The measure points become significantly more complex in the 3PP orientation: **two vanishing points define the edges of each face**, and each edge requires its own measure point. So we have in all six **measure points** (**mp**$_1$ to **mp**$_6$) — two for each vanishing point in relation to the two faces it governs.

Finally, with the visual ray method we had a simple way to **rotate the vanishing points** in 2PP, but this also becomes significantly more complex in 3PP. In 2PP we just had to rotate two faces joined in *one* right angle, which we could easily diagram in two dimensions as two lines joined in one angle. In 3PP we must rotate three faces joined in *three* right angles, and that complicates the visual ray approach to a perspective solution.

Direction of View & Horizon Line. A 3PP construction **allows the direction of view to be oblique to the ground plane**, so that we are *looking down or up* on objects rather than looking at them directly from one side. Consequently in 3PP it is necessary to distinguish between (1) the object geometry (the vanishing points defined by the edges of the primary form), (2) the central recession defined by the direction of view, and (3) recession on the ground plane, for example in the visual texture of forests, grassy plains, deserts or bodies of water.

For example, we can redraw the cube illustrated above in **two point perspective** so that it has exactly the same angular size in the field of view (using a measure bar), and is positioned below the direction of view so that we look down on its upper face at a 35° angle. This locates the top front corner on the 71° circle of view and the bottom front corner just in front of the ground line (diagram, below).



the 3PP canonical view in two point perspective

Because both the angular size of the cube and the angle of its faces to the direction of view are identical, we are viewing it from **exactly the same location in physical space**. All we have done is **shift our gaze** from the object itself to the horizon line behind it. This keeps the same visual angle between the front corner of the cube and the horizon line. But changing the direction of view in 3PP means that:

(1) the horizon line no longer must intersect the principal point, and in fact may no longer be within 90° circle of view; and

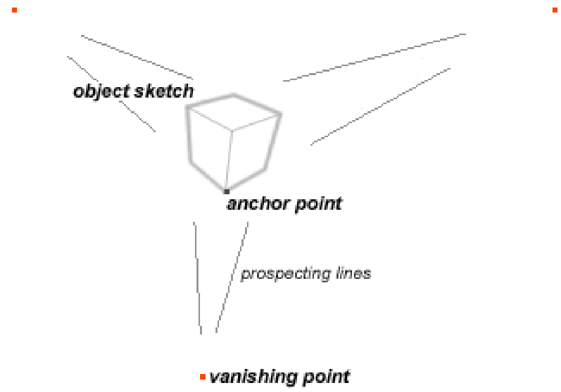
(2) the geometrical relationship between any two vanishing points (the size and shape of the triangle the vanishing points define on the image plane) depends on the location of the third vanishing point and the location of the direction of view (the orientation of the image plane to the perspective problem).

the perspective sketch construction method

The solution is basically to draw the form first, so you can locate the vanishing points and measure points which will produce that perspective view. You then use these to reconstruct the primary form in accurate perspective, and to add objects around the primary form within the same perspective space.

Why not draw the primary form by freehand perspective alone? Because, as we've already seen in 2PP, **inaccurate placement of vanishing points** results in a distorted perspective view; even small distortions can be obvious in a finished drawing. There is a better way.

You start with a **freehand perspective sketch** or scaled down perspective drawing at the center of a fairly large piece of paper (a 3' section from a roll of wrapping paper or white butcher paper is ideal).



three point perspective: perspective sketch of the primary form

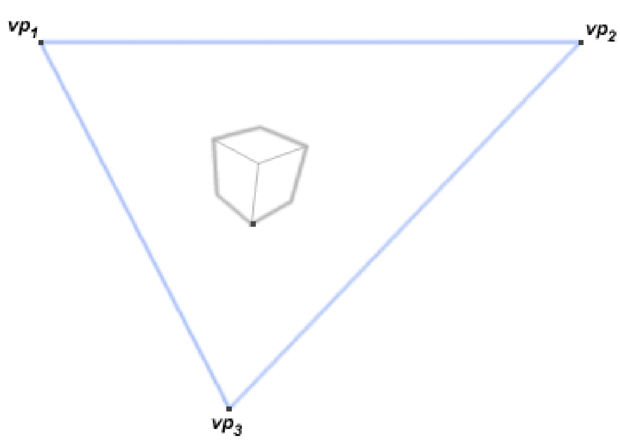
Your drawing or photograph of the primary form should be small enough to fit all perspective points on the sheet of paper, yet large enough to work with accurately — usually a drawing about 10cm or 4 inches on its longest side is practical.

Take your time with the freehand drawing, and try to capture the relative proportions of the dominant edge angles and faces as accurately as you can. Don't worry about extraneous features (such as doors, windows or domes): you want to capture the basic perspective shape as it recedes in three directions. Be sure to define the edges and corner points clearly.

You can also start with a drawing or photograph of a building or monument that presents clear vanishing lines in its edges or surfaces, in the perspective orientation you want to duplicate. This photograph is only used to **specify the approximate perspective view** of the primary form in the drawing, so it does not have to look anything like the primary form you actually want to draw.

Once the drawing is finished to your satisfaction, or you have taped your photograph to the sheet of paper, you draw **prospecting lines** from the edges of the front planes to find the three vanishing points. Using a ruler or yardstick, **extend the outer edges** of the form until these prospecting lines intersect at three separate points. In a cube you have three edges tending to each vanishing point; use these in combination to reconcile discrepancies and find the point that gives all three the best definition.

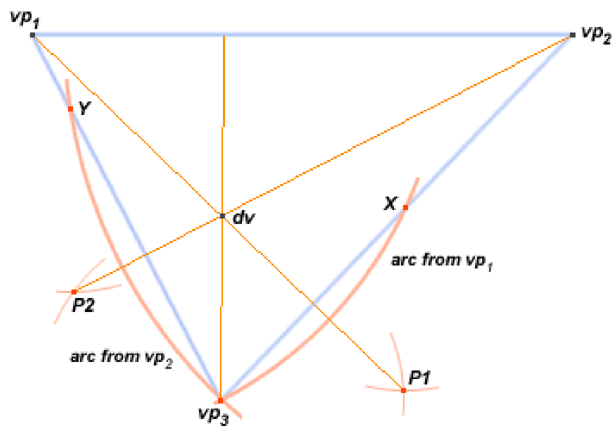
The Vanishing Line Triangle. Next, connect these three vanishing points with three vanishing lines. You have defined the *vanishing line triangle* that will define (and usually contain) the primary form.



three point perspective: the vanishing line triangle

This is the point to look at the overall placement of the vanishing points in relation to the primary form and the space around it that will appear in the finished drawing. You can block in the format outline, or sketch other large forms around the primary form, to make sure you will get the effect you want.

Constructing Auxiliary Horizon Lines. Next, **draw the three auxiliary horizon lines** through each vanishing point and perpendicular to the opposite vanishing line. There are two ways to do this. The quicker is to use a large carpenter's square, laying one side against each vanishing line and sliding it back and forth along the line until the other arm is exactly on the vanishing point. Then draw the line.



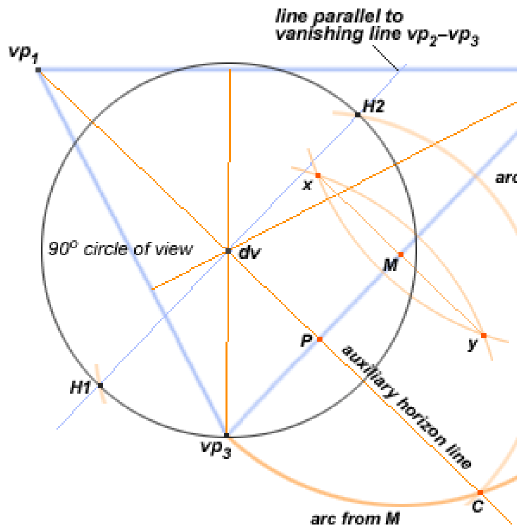
three point perspective: constructing the auxiliary horizon lines

A more accurate method in large drawings is to **construct the perpendiculars** using a long piece of fishing line, hemp (not stretchable cotton) string or strip of cardboard as a compass measure. With your thumb, a tack or a piece of tape, fix one end of the measure at the vanishing point, and with the other end scribe a wide pencil arc across the opposite vanishing line. (Put the tip of the pencil through a loop in the string or a small hole in the cardboard strip.) The arc must intersect the vanishing line at two widely spaced points. Then either scribe two intersecting arcs centered on each of these new points, or measure with a ruler 1/2 the distance between them.

In the figure, the two arcs have been scribed around $\mathbf{vp}_1$ and $\mathbf{vp}_2$, and through $\mathbf{vp}_3$, to define the new points **X** and **Y**. Intersecting arcs drawn from **X**, **Y** and $\mathbf{vp}_3$ create the new points **P1** and **P2**; lines to these points from the corresponding vanishing points create two auxiliary horizon lines. The direction of view (**dv**) is always at the intersection of all three auxiliary horizon lines, so the third line can simply be drawn from $\mathbf{vp}_3$ through **dv** to the opposite vanishing line. You end up with a **vanishing line triangle** similar to the one shown above.

Didn't I say elsewhere that the **freehand placement** of vanishing points leads to distortions? No: it's the *clumsy scaling* of **drawing size** in relation to the distance between the vanishing points that introduces distortions. If your three auxiliary horizon lines are at right angles (perpendicular) to their vanishing lines, if they meet in a single point (**dv**), and if this point is inside the vanishing line triangle, then the triangle defines a valid (physically possible) perspective space for a rectilinear solid.

Constructing the Circle of View. Now we insert the **90° circle of view**. This requires you to (1) find the midpoint of any of the three vanishing lines (connecting two vanishing points), (2) draw a **semicircle of Thales** over the vanishing line, (3) extend to the semicircle the auxiliary horizon line that intersects the vanishing line, (4) construct a line parallel to the vanishing line, and finally (5) draw a second arc back to this parallel line. The intersection of this arc with the parallel line defines the radius of the 90° circle of view around **dv**.



three point perspective: constructing the circle of view

In the traditional solution, the artist uses either a ruler or the method of **intersecting arcs** to find the midpoint **M** on the vanishing lines between two vanishing points. In the diagram, I've chosen the vanishing line between $\mathbf{vp}_2$ and $\mathbf{vp}_3$. When arcs of equal radius are inscribed across the vanishing line from the two vanishing points, they intersect at two points, **x** and **y**. (1) A line through these points defines the midpoint **M** of the vanishing line.

(2) From point **M** the artist constructs a *semicircle of Thales* between the two vanishing points, then (3) extends to the semicircle the auxiliary horizon line that intersects the inscribed vanishing line at **P**. This defines a new point **C**. (For visual clarity, the semicircle is shown outside the perspective triangle, but to save space it can just as well be drawn to intersect the interior auxiliary horizon line.)

(4) Next, the artist constructs a line through **dv** that is parallel to the vanishing line.

(5) Finally, the artist inscribes an arc from **P** with radius equal to **PC**, the extended segment of the auxiliary horizon line. This intersects the line parallel to the vanishing line at either **H1** or **H2**, depending on where it is more convenient to construct the arc.

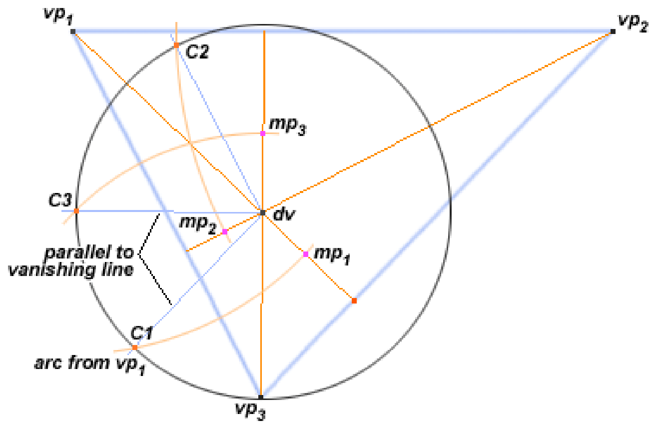
(6) The line segments **dv-H1** or **dv-H2** are equivalently the radius of the 90° circle of view. The artist draws this circle from **H1** (**H2**) with **dv** as its center.

It is often useful to include the 60° circle of view, which is a second circle with a radius equal to 0.58 (58%) of the radius of the 90° circle of view. This completes the perspective space.

Locating Measure Points. The last step is locating the measure points. Six are required if they are marked along the vanishing lines, but only three if you locate them on the auxiliary horizon lines.

Auxiliary Horizon Line Measure Points. To find the measure points on the auxiliary horizon lines, use a protractor or architect's triangle (or the traditional method for **constructing a perpendicular**) to construct finish perpendiculars on each auxiliary horizon line, from **dv** to the circle of view: the intersection with the circle of view defines three new points,

C1, **C2** and **C3**. Draw arcs from each of these **C** points back to the auxiliary horizon line perpendicular to it, using the vanishing point on that auxiliary horizon line as the center of the arc.



three point perspective: finding the measure points

This completes the perpective space at a reduced scale. I find that this entire procedure, starting with a blank sheet of paper and ending with the finished perspective space, **requires about 20 minutes to complete**. Once you understand how to do it, the work goes quickly and smoothly.

You must carefully make seven measurements on this drawing (using a metric ruler) to rescale it to full size: (1) the longest distance between any two vanishing points (in the example, $\mathbf{vp}_3$ to $\mathbf{vp}_2$), (2) the distance from one of these vanishing points to the intersection with the auxiliary horizon line ($\mathbf{vp}_3$ to **h**), (3) the length of this auxiliary horizon line (**h** to $\mathbf{vp}_1$), (4) the length to the direction of view (**h** to **dv**), and finally (5-7) the distance from **dv** to each of the three measure points.

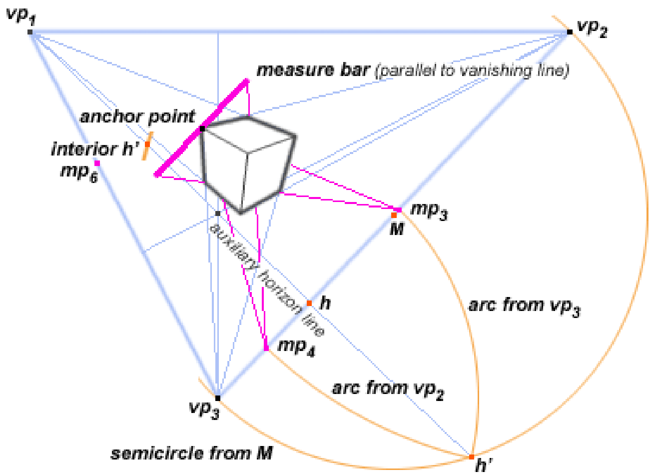
Divide the radius of the circle of view you want in the full sized drawing (say, 160cm) by the radius of the circle of view in your perspective sketch: multiply all the measurements by this number. This gives you the full scale perspective space. Your perspective work surface needs to be at least as long as the longest vanishing line and as wide as the 90° circle of view. In the example drawing, assuming a 3m circle of view, this would be roughly 5m by 3m.

On a surface large enough to accommodate these distances (a very large table, or a clean hardwood or linoleum floor, or a clean, flat patio, garage floor or driveway), measure out the longest vanishing line (in the figure, $\mathbf{vp}_2$ to $\mathbf{vp}_1$), and the auxiliary horizon line to $\mathbf{vp}_3$. Connect the three vanishing points to define the vanishing line triangle. Measure the distance from the vanishing line to **dv**, and draw the remaining two auxiliary horizon lines from the vanishing points through **dv**. Finally, mark the three **mp**'s on each auxiliary horizon line, measured from **dv**.

Use the *drawing scale* shown in the **distance to size** table to **compute the drawing scale** — the percentage of the actual object size (for a given viewing distance) that the drawing of the primary form should have. On a piece of paper, make a rough sketch of the primary form at this size, and lay the sketch on the format (size of support) you intend to use, to make sure the proportions work.

Vanishing Line Measure Points. The 3PP method of using three measure points is convenient, but it fails when the anchor point for measurements is close to the direction of view (**dv**). In this case, you may want to use the vanishing line points instead.

The construction of the circle of view required a semicircle of Thales drawn around one of the vanishing lines, centered on **M** and intersecting the vanishing points at either end of the vanishing line, then extended the auxiliary horizon line to intersect the semicircle in a point **h'**. This is all you need to define the measure points on that vanishing line. (Note that you can save steps and work space by intersecting the auxiliary horizon line inside the perspective triangle, to define **interior h'**, and construct the measure points from there.)



three point perspective: alternate method to define measure points

The point **h'** will always define a 90° angle with the two vanishing points on the vanishing line. That is, it is equivalent to the viewpoint in a 2PP rotation of vanishing points. So you can draw two arcs from this point back to the vanishing line, using each vanishing point as the center of an arc, to define the measure points for the vanishing line — just as you would in **two point perspective**.

Confusion about the choice of vanishing line measure points is usually dispelled by the following two criteria:

• The controlling vanishing point is the **vanishing point for the convergence of the edges** that are being sized by the measure bar. Thus, edges converging to the right side vanishing point (**vp$_2$**) are controlled by that vanishing point.

• The measure point to use was **defined by an arc from the controlling vanishing point**. Thus, **mp$_4$** was defined by an arc centered on **vp$_2$**, so **mp$_4$** is the measure point to use when sizing edges that recede to that vanishing point. The height dimension is controlled by the vertical vanishing point (**vp$_3$**), which was the center of the arc used to define **mp$_3$**.

Measure bars to the vanishing line measure points **always must be parallel to the vanishing line** containing the measure point being used, not to any auxiliary horizon line as before. Note that two measure points are always available for each dimension. In the example, **mp$_6$** can be used to size the vertical edges receding to **vp$_3$**, if for some reason **mp$_3$** is inconvenient to use — but in that case, the measure bar must be parallel to the vanishing line containing **mp$_6$**.

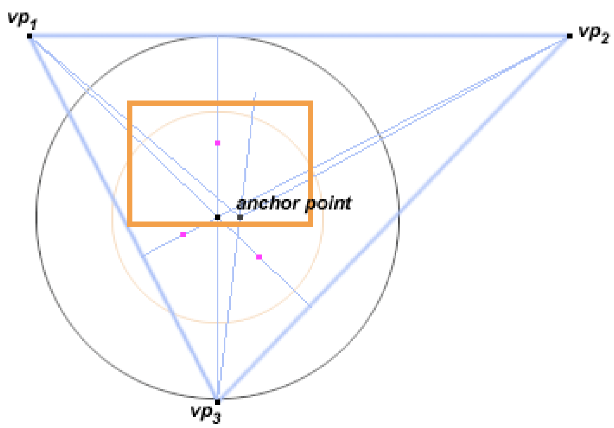
The measure bars in the illustration are the same length as those used previously, and as you can see, they define the same reduction in perspective depth. You do not need to rescale or recompute the measure bars you already have; just align them parallel with the appropriate vanishing line.

Because the semicircle on **M** is part of the circle of view procedure, and any vanishing line can be used to define the circle of view, you should consider the location of your anchor points in the perspective space, and place the semicircle of Thales around the vanishing line where measure points will be most convenient.

For example: I had originally put the anchor point at the front bottom corner of the cube; in that location **mp**$_3$ worked fine, but the other two points created badly slanting measure lines that would introduce inaccuracies. The best alternative points would be found on the top vanishing line (between **vp**$_1$ and **vp**$_2$), so I should have started building the circle of view by putting the first semicircle on that side.

constructing a 3PP cube (perspective sketch method)

Once you have constructed the 3PP space, you can begin construction of the cube or primary form. This explanation excludes the procedures necessary to scale the drawing, which are **developed below**.

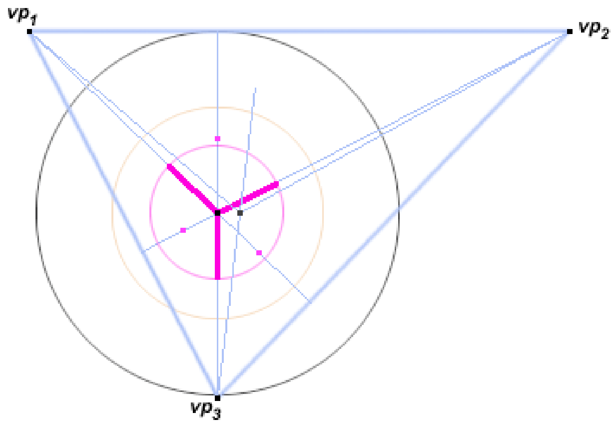


three point perspective: locating the primary form

Measure out the perspective space on the perspective drawing surface (floor, driveway, patio), and **tape or tack the support to the surface**, oriented with the top edge parallel to one of the vanishing lines (or to none, if the perspective view is tilted), and the **dv** in the correct location within the drawing. If you do not want to work directly on your watercolor paper, reconstruct the drawing on a large sheet of butcher paper or wrapping paper, and then trace or **square the drawing** to the format when you are done.

Mark the **dv** and draw the auxiliary horizon lines, the measure points, the anchor point, and the base vanishing lines through the anchor point.

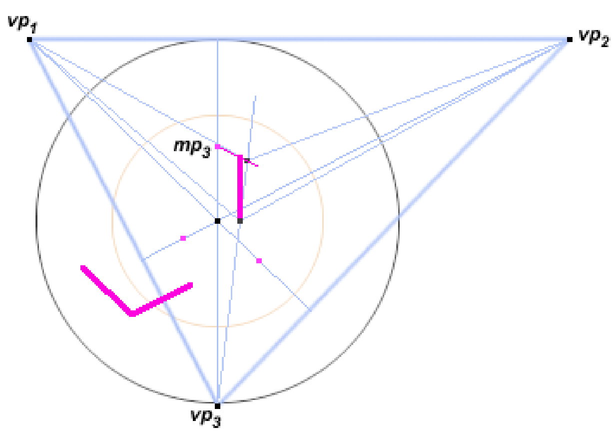
The diagram shows this done on an emperor sheet, 40" x 60", located with the **dv** near the bottom. (If you are going to all this trouble, you may as well make the painting spectacular!) For clarity, the support outline is omitted in the next several illustrations, although it is assumed you are working with the support in place.



three point perspective: constructing measure bars

The last preparatory step is constructing the measure bars. Do this from the center of the space (**dv**), because each measure bar must be parallel with its corresponding auxiliary horizon line. This is easiest to do by simply drawing the measure bar on a separate sheet of paper, directly over the auxiliary horizon lines.

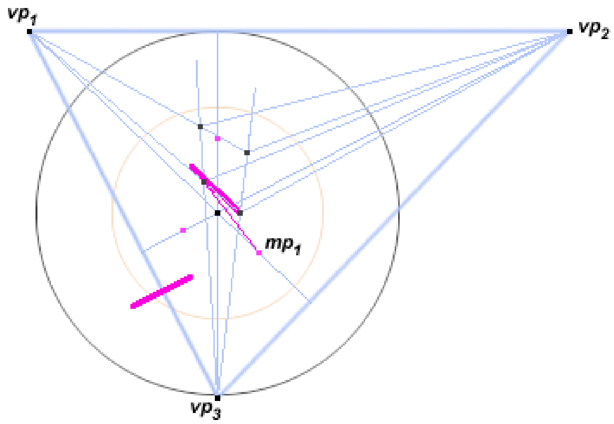
Draw the measure bars to the perspective length they have in space, so that you can line them up with one end against the anchor point. The length of the measure bars determines the *drawing size* of the primary form, so you want these to be accurate. For example, if the dimensions of a building are 150 feet long, 75 feet wide and 36 feet high, and you used the length of the building to scale the drawing size, then the proportions between the measure bars are 1.00 to 0.50 and 1.00 to 0.24. Since we are drawing a cube, all three measure bars will be of equal length, so we define them by drawing a circle around **dv** (shown above).



three point perspective: constructing front vertical

The rest is a piece of cake. First, align the vertical measure bar parallel to the vertical auxiliary horizon, with the bottom end on the anchor point. Draw a line from **mp**$_3$ through the top end of the measure bar to the vertical vanishing line (that is, the line parallel to the measure bar you are using). This defines the front height of the cube.

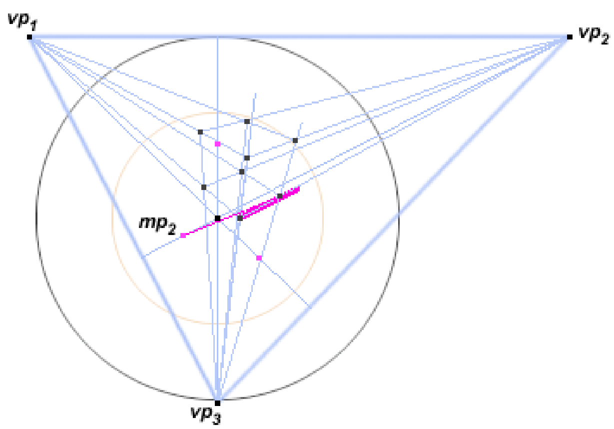
Using a yardstick, string or cardboard strip aligned with the bottom vanishing point, draw a line from the anchor point to the line from the vertical measure bar to **mp**$_3$. This is the front vertical. Use the yardstick, string or cardboard strip to connect the ends of this vertical to the two side vanishing points, and draw the front top and bottom edges of the form.



three point perspective: constructing left side

Next, use the second measure bar to define the depth dimension on one side (to **mp**$_1$, the measure point on the auxiliary horizon line parallel to the measure bar you are using). When one end of the measure bar is aligned with the anchor point, the back corner of the cube is located where the line from the other end of the measure bar to **mp**$_1$ crosses the bottom left edge of the figure. Mark this point.

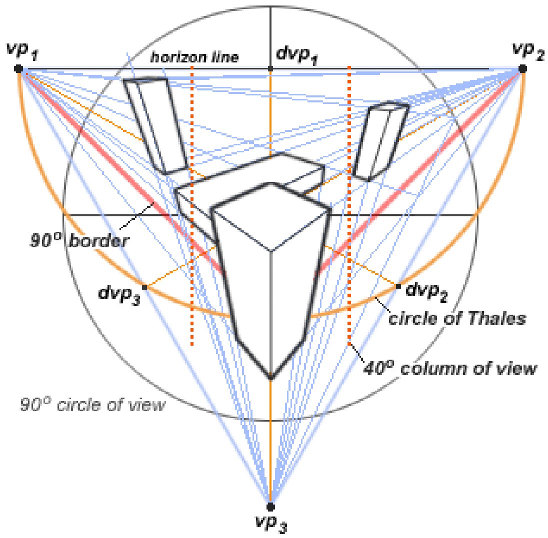
Again aligning your straight edge with **vp**$_3$, draw a line from this point to the top left edge line: this is the back vertical of the cube. Connect the ends of this vertical along vanishing lines to **vp**$_2$. These lines define the back left upper and lower edges of the figure.



three point perspective: constructing right side

With the third measure bar, construct the opposite side, define the corners, and connect to the vanishing points as before.

Clean up the drawing as much as necessary to visually confirm the final perspective outline meets your expectations. Then go on to add any other objects in the environment around the primary form, or perspective details on its surface (doors, windows, etc.).



three point perspective: perspective distortions

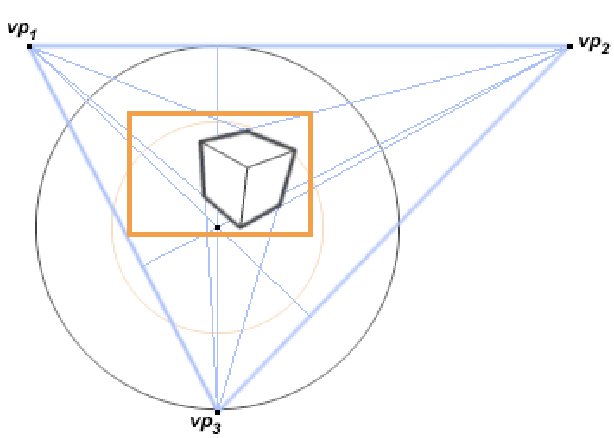
Familiarity with the 3PP mechanism will help you understand how to use it effectively. The diagram (above) gives some clues about the scale, placement and cropping of forms:

• In general, distortions toward the side vanishing points are much more objectionable than those toward the bottom vanishing point: **choose a vertical or square format** whenever feasible.

• Forms can be placed below the **90° border** — a 90° angle placed to intersect the two side vanishing points (red line) — to emphasize height or vertical scale, but forms should not be placed near the border on either side. (All possible locations of the right angled corner of this border are defined by a **circle of Thales** constructed below the horizon line.)

• The same circle of view rules apply in order to reduce **perspective distortions**, but the circle can be displaced downward from the direction of view, as if pulled away from the horizon line by the vertical depth. It is better to think in terms of a **column of view** centered on the principal point and extending from below the 90° border to above the horizon line (where cloud layers in perspective can enhance distance depth to balance the vertical depth). Any format that fits within a 40° to 60° column will produce a handsome image.

When you have finished with the perspective elements, carefully release the drawing surface from the table, floor or patio, and lay it out on your painting surface to erase the guidelines, measure points, and other extraneous elements, or to transfer the perspective outline to the actual painting surface. When the drawing is fully cleaned, add by freehand any additional outlines or guidelines necessary before you begin to paint.



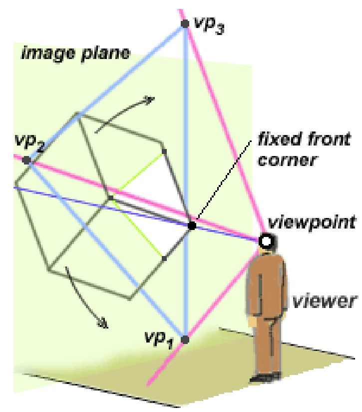
three point perspective: finished drawing

The diagram shows the finished perspective form, once again within the monumental 40"x60" emperor format. In this reduced diagram, the primary form appears to be little changed from the original perspective sketch. But in practice, despite all the work invested, you will be quite pleased with the increased perspective accuracy and "weight" of the finished drawing in comparison to anything you could manage by freehand methods alone.

the horizon line construction method

Two significant problems with the perspective sketch method are that it establishes the angles of the the primary form to the viewpoint approximately, through a sketch, and that it cuts the 3PP methods loose from the procedures for **scaling the drawing** within the circle of view. The actual perspective angles and scale of the circle of view are derived from the drawing, rather than given at the start. An alternative method is to start with the circle of view, and from there construct the vanishing points. This method starts by specifying the location of the horizon line (a horizontal vanishing line above or below the direction of view), so I refer to it as the **horizon line method** of 3PP construction, though the *circle of view method* is also apt.

A discussion of the 3PP geometry will clarify how this method works. Because all parallel lines converge to the same (single) vanishing point (perspective **rule 6**), and the 3PP vanishing points define visual rays at right angles to each other, the 3PP vanishing points are equivalently defined by the three right angled edges of a cube that can be turned or rotated around a front corner fixed on the direction of view (diagram, right).

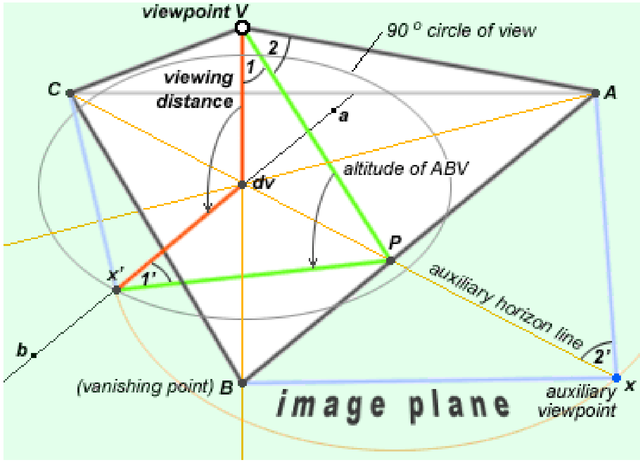


the 3PP vanishing points defined by three edges of a cube

These edges converge to the three right angle vanishing points at the vanishing lines for the three planes defined by the three front faces of the cube (perspective **rule 14**). Therefore the vanishing lines between the pairs of vanishing points will be parallel to the line intersections of the three front faces of this cube with the image plane (green, corollary to perspective **rule 11**). As a result, we have reduced the geometry of the 3PP vanishing points to the geometry of a three sided pyramid thrust through the image plane in any arbitrary angle and rotation.

As explained earlier, the circle of view framework provides a method to specify exactly the location of any vanishing point as a line rotated to the required angle around the viewpoint **folded into** the image plane. What we require is a way to perform this folding for elements of the 3PP "pyramid".

This is done by **moving the fixed corner of the cube forward until it coincides with the viewpoint**. In that position its three edges define three visual rays to the vanishing points (magenta lines, diagram above right). More important: the altitude of the pyramid is now equal to the viewing distance and therefore to the radius of the 90° circle of view (diagram, below).

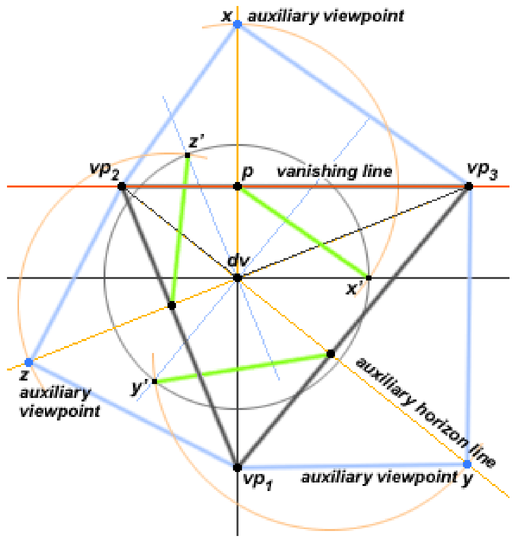


folding a pyramid right triangle into the image plane

Two kinds of folding operations are possible in this 3PP geometry. First are the **auxiliary line folds** that define the *interior angle* between a pyramid edge, or the pyramid face perpendicular to it, and the direction of view. These are found by folding into the image plane a *vertical section* of the pyramid defined by an auxiliary horizon line, for example the interior triangle **PVC** defined by the auxiliary horizon line **PC** in the diagram above. This triangle contains the two triangles **VdvC** and **VdvP**, each containing a right angle at **dv**. The fold brings line **Vdv** into the image plane as **x'dv**. Because the edge **Cdv** is continuous with edge **Pdv**, the right angle at **dv** is preserved. And the image edges **Cx'** = **CV** and **Px'** = **PV**. Therefore, by triangular equalities, the image angle **1'** equals the interior angle **1**, the angle between the direction of view and the face **ABV**.

This fold also identifies (at **Cx'dv**) the angle between the vanishing point **C** and the direction of view, so this folding down of an interior section of the perspective pyramid is geometrically identical to the folding of the viewpoint into the circle of view that is used to **rotate vanishing points** to the direction of view.

The second kind of folding operations are the **vanishing line folds** that define an *exterior angle* of one face of the perspective pyramid (angle **2**) as a "plan view" of the angle in the image plane (angle **2'**). This is the angle, on the face of the 3PP pyramid, between the edge of triangle **ABV** and its altitude **PV**. The fold is achieved by constructing a line (**ab**) that **intersects the direction of view parallel to the vanishing line** (**AB**). This line intersects the circle of view at **x'**. Because **Vdv** equals **x'dv**, the line **Px'** equals line **PV**, the altitude of **ABV**. Therefore an arc constructed on **P** with radius **Px'** intersects the auxiliary horizon line at **x**, and **Px** = **PV**. Therefore the right triangle **ABx** is the perpendicular view of the foreshortened triangle **ABV**, and **x** is the **auxiliary viewpoint** for the horizon line **AB**.

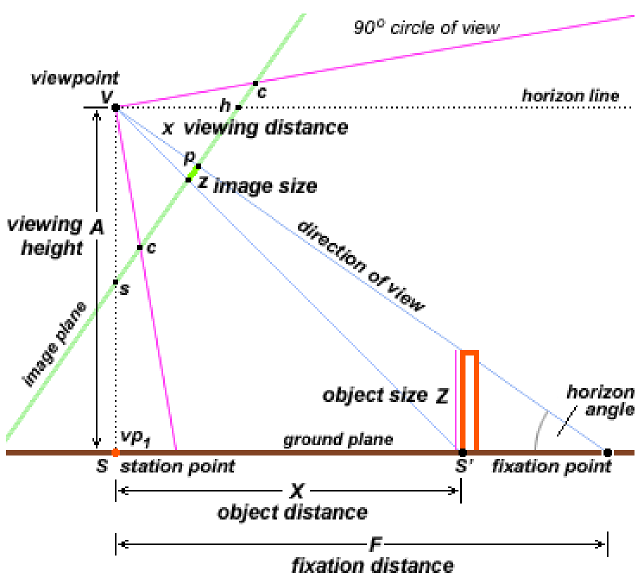


three right triangles folded out of the 3PP pyramid

The diagram (above) shows the three possible vanishing line folds and auxiliary viewpoints (**x**, **y** and **z**) constructed from a 3PP vanishing line triangle. Study this diagram carefully until you understand how each fold has been done.

The geometry of triangles is efficient: defining any one side with its two adjacent angles, or any two sides with their common angle, defines the rest of the triangle. Therefore **only two folding operations are necessary** to define the image of a 3PP vanishing line triangle: one auxiliary horizon line fold and one vanishing line fold. This is sufficient to define the location of all three vanishing points and vanishing lines in relation to the direction of view and circle of view.

Finally, the **3PP construction** releases the direction of view from its parallel position to the ground plane, and this creates several novel features in the perspective geometry which affect in particular the scaling of the 3PP drawing. For now I only want to describe this geometry and define a few new terms (diagram, below).



elevation view of 3PP geometry

In this example we assume the perspective view is *downward* in relation to the ground plane: it can just as well be *upward* (as the top of a skyscraper viewed from the ground) or *tilted* (as a city viewed from a turning airplane), a problem I leave for the reader. In the downward view case:

• The **image plane is oblique to the ground plane**, as is the direction of view. As a result the direction of view does not terminate in a vanishing point, but in a **fixation point**, some physical point on the ground. This **fixation distance** is typically different from the **object distance** from the station point to the primary form.

• The station point **S** is still directly under the viewpoint, but now the station point appears on the image plane, where it is the image **s** equivalent to the *vertical vanishing point* ($\mathbf{vp_1}$).

• The horizon line is now located *above* the direction of view in the circle of view, which means the principal point, the vanishing point for the viewer's central recession (at **p**), is no longer the same as the orthogonal vanishing point (at **h**), the vanishing point of ground plane recession.

• The primary form appears in **rotation foreshortening** — the vertical and horizontal dimensions are in a different scale. Foreshortening is corrected by using the measure points; **measure bars parallel to the image plane may be rotated** in the image plane to any other angle. However, it is sometimes useful to estimate the amount of *vertical* foreshortening, for example when planning the image layout. This is found by a **cosine correction** for foreshortening:

$$(10)\ \textbf{vertical unit = horizontal unit * cosine}(\theta)$$

where θ is the **horizon angle**. Because the angle of view to the ground plane is only equal to the horizon angle at the fixation point, a measure bar established at any other point must be calculated with the correct angle of view to that point on the ground plane.

• An object's angular size or **image size is determined by the sight line distance from the viewpoint**, which is simply the hypotenuse of the right triangle formed by the object distance and viewing height.

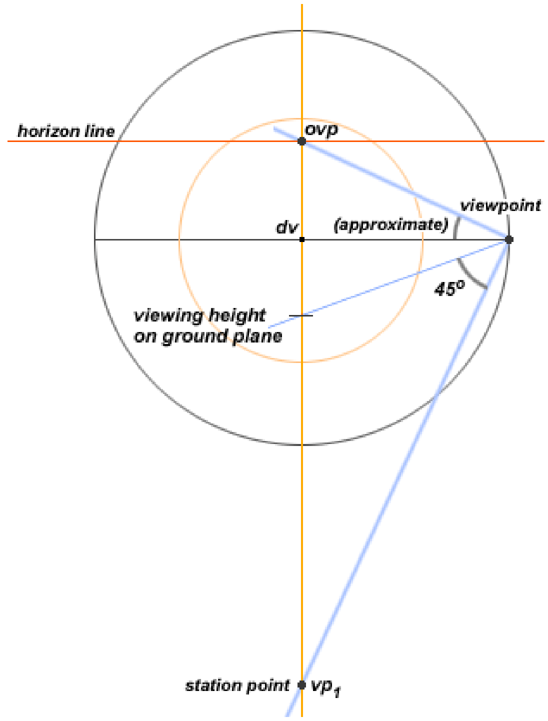
These points need to be understood in order to apply the correct **distance & size** calculations when **scaling the drawing** in a 3PP construction.

constructing a 3PP drawing (horizon line method)

The horizon line method builds on the assumption that most three dimensional perspective problems concern a viewer whose line of sight is not parallel to the ground plane. Either the viewer is looking upward, toward the top of a tower, building, mountain or cliff; or the viewer is looking downward, from a vantage at the top of a tower, building, mountain or cliff.

Approximate Horizon Line Method. In this approach the artist places the horizon line and vanishing points by judgment or whim, but uses the **pyramid folds** to make these landmarks consistent with each other.

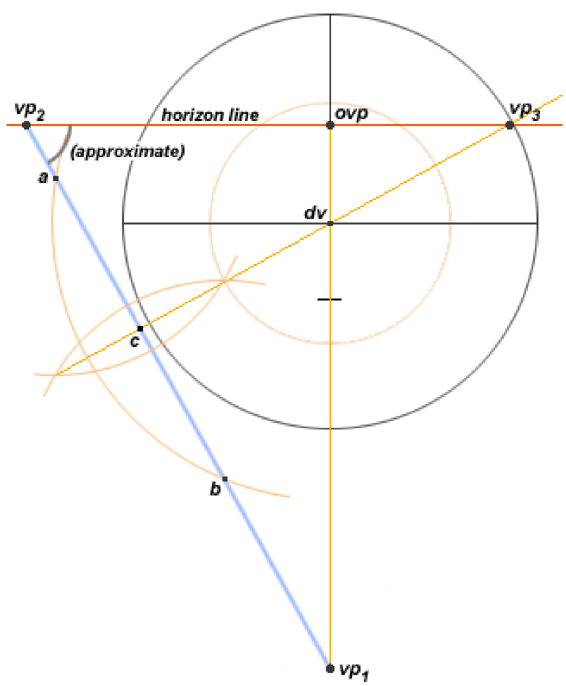
The first step is the placement of the horizon line in relation to the principal point: either above (for a downward direction of view) or below (for an upward direction of view). Then, using a drafting triangle, the artist finds the 90° angle at one of the diagonal vanishing points, and extends this line until it meets the median line below the circle of view: this is the vertical vanishing point ($\mathbf{vp_1}$).



three point perspective: rotating the horizon line

It is useful to bisect this angle to **find the diagonal view** (45° from either the horizon line or $\mathbf{vp_1}$ visual rays), as this projects in depth the viewing height above the ground plane.

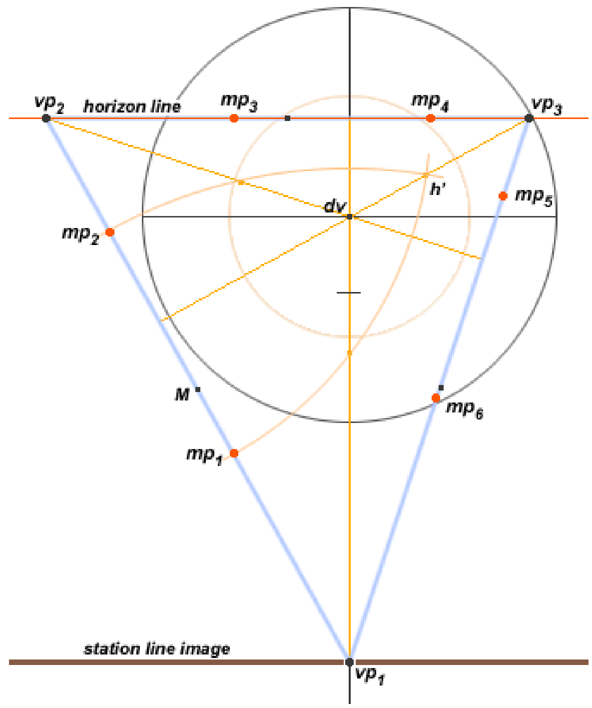
Next the second vanishing point ($\mathbf{vp_2}$) is located on the horizon line somewhere to the left of the median line. A ruler laid from **dv** to this point will show the angle to view of a cubic form in perspective space at the direction of view.



three point perspective: approximately placing $\mathbf{vp_2}$

Once the location of the point is completed, draw the *vanishing line* between the two vanishing points. Then you must construct a perpendicular line from this vanishing line through the direction of view (**dv**), as **described here**.

The steps are: (1) draw a circular arc around **dv** that intersects the vanishing line at two widely spaced points, **a** and **b**; (2) draw an arc from each point with a radius greater than half the segment length between them; (3) draw a line through the double intersection of the arcs to define the normal point **c**; (4) draw a line from **c** through **dv** until it intersects the horizon line on the opposite side of the circle of view. This is the auxiliary horizon line for the constructed vanishing line; it locates **vp$_3$**.



three point perspective: completed "approximate" perspective triangle

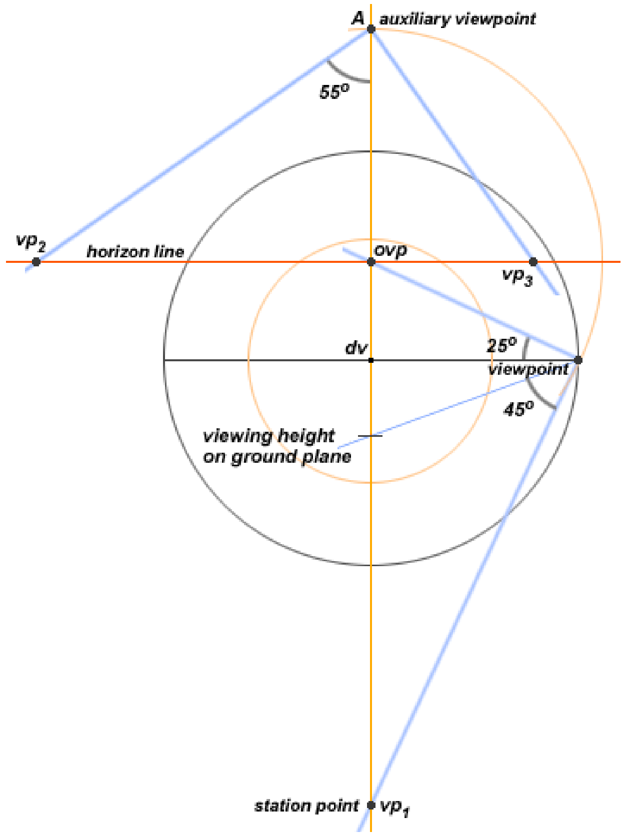
Construct the third vanishing line and its auxiliary horizon line from **vp$_2$** through **dv**.

Find the internal or external altitude points on the auxiliary horizon lines, and from these locate the six measure points on the vanishing lines. (The diagram above shows one internal altitude point, **h'** and the two measure points constructed from it.) This completes the three point perspective triangle.

Exact Horizon Line Method. In some cases (illustrated below) it is desirable to locate the three vanishing points precisely. In this case the pyramid folds are precisely defined with a protractor or using the **tangent ratio** for the required angle, applied to the radius length of the circle of view that is perpendicular to the viewpoint.

Required is one *auxiliary horizon line fold* along the vertical auxiliary horizon line (median line) to establish the tilt of the horizon line and the location of the vertical vanishing point (**vp$_1$**), and one *vanishing line fold* along the horizon line to establish the left/right placement of **vp$_2$** and **vp$_2$**.

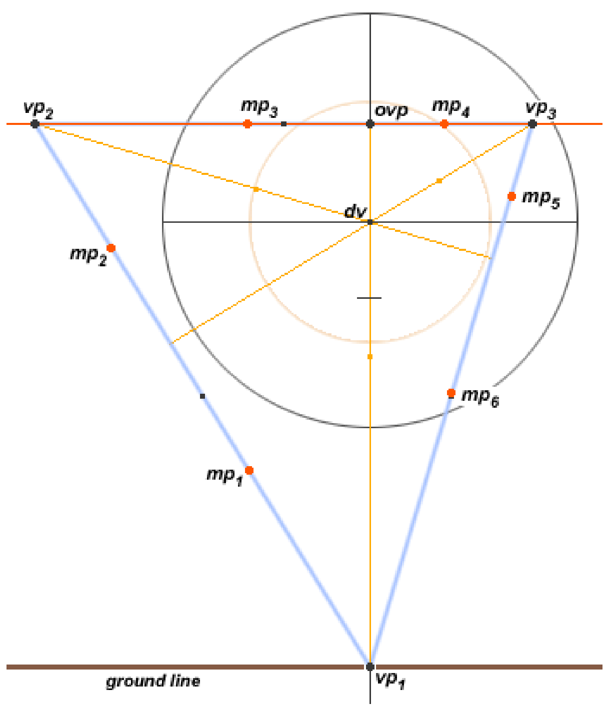
The diagram (below) shows these operations to provide an exact 25° downward angle of view to the ground plane (upward horizon angle of 25°), and a placement of **vp$_2$** 55° to the left of the median line. This places **vp$_3$** 35° to the right of the median line.



three point perspective: exact rotation of vanishing points

The vanishing lines are added as before; the auxiliary vanishing lines can be drawn directly, as lines from the vanishing points through **dv** to the opposite vanishing line, because the vanishing points have already been precisely located.

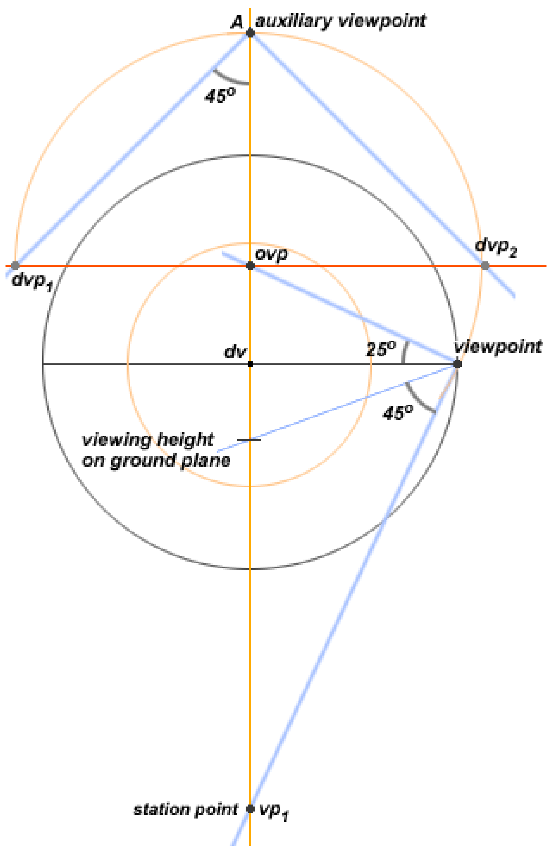
Measure points have been added using the "alternative" horizon line method **described above**.



three point perspective: completed "exact" perspective triangle

Although this 3PP triangle is very similar to the one constructed from an approximate judgment of the correct angles, here all the perspective landmarks are exactly placed from given values established in advance. This is especially important when the goal is a 3PP view of a specific primary form from a specific location — as is typical in architectural renderings or historical reconstructions — or when a certain arrangement of key forms within the image is required.

Diagonal Vanishing Points. It is usually very useful to take the extra step and establish the diagonal vanishing points on the horizon line. Once this is done a unit dimension on the station line can be projected across the ground plane, using the method of projecting a **unit dimension in depth** from the diagonal vanishing points.



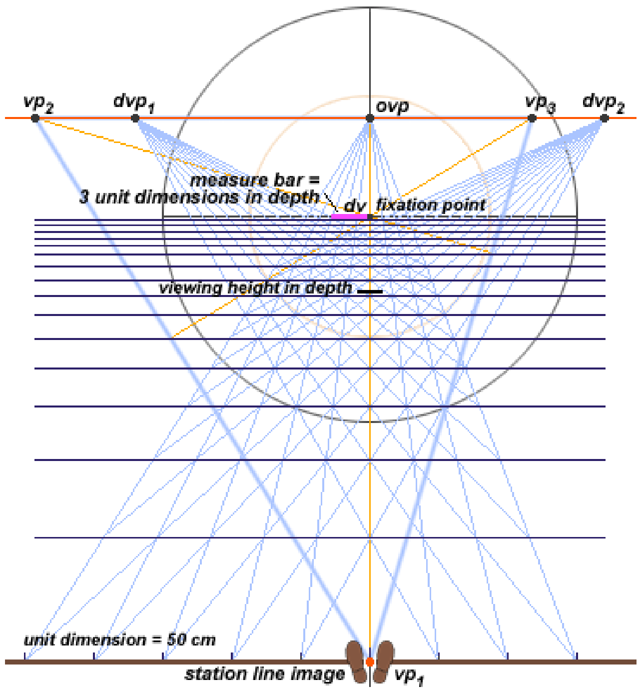
three point perspective: locating the central dvp's

In fact, no separate rotation is required to define the diagonal vanishing points from the auxiliary viewpoint: they are already located at the intersection of the arc used to define the auxiliary viewpoint with the horizon line (diagram, above). The method of rotating the diagonal vanishing points around the auxiliary viewpoint **A**, so that a 90° angle is bisected by the vertical auxiliary horizon line (median line), is shown simply to confirm this.

Once these diagonal vanishing points have been established, **ovp** serves as the *orthogonal vanishing point,* the convergence for recession in depth parallel to the ground plane (the *ground plane* central recession); but **dv** remains the *principal point,* the convergence for recession parallel to the direction of view (the *viewer's* central recession). The depth of transversals across orthogonals to **ovp** are found by diagonals to **dvp$_1$** and **dvp$_2$**; the depth of transversals across orthogonals to **dv** are found by vanishing lines to points on the circle of view.

Scaling the 3PP Drawing. This task is more complex than it is in one or two point perspective, but I outline it here because I have not seen it discussed in any other source. A minimal reliance on trigonometry is required, both to validate the basic principles and to provide calculation shortcuts or remedies to complex construction problems.

Construction Methods. Three drawing scale guides are already available: (1) the circle of view and the many **visual angles** that can be computed within it; (2) the **viewing height in depth**, added when the horizon line was rotated; and (3) a **ground line scale**, which is used in combination with the orthogonal vanishing point (**ovp**) to project a **unit dimension in depth** to approximately locate objects in depth and scale their image size. Provided image scale and perspective accuracy are not critically important, these are almost always adequate to scale the 3PP drawing.

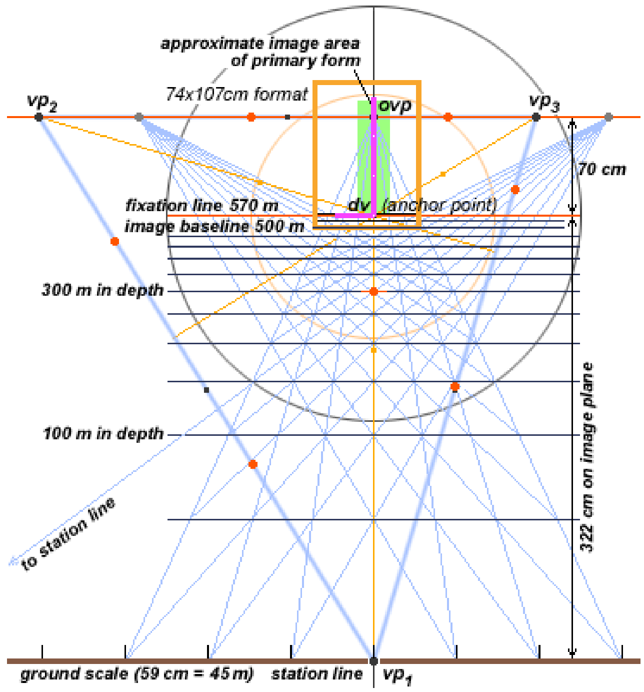


three point perspective: ground plane recession

Two scaling approaches can be used. In the first example (above), an arbitrary unit dimension of 50 cm measured along the station line image is projected into perspective space by orthogonals drawn to the orthogonal vanishing point (**ovp**). These indicate that the viewing height in depth is about 7.2 times the viewing height (measured from the station line) and that the fixation point is about 16 units away.

Dividing the viewing height by the depth units yields the ground plane width of the unit dimension. If the viewing height is 300 meters, then the unit dimension represents about 300/7.2 = 42 meters, and the fixation point is about 672 meters from the station line — all distances measured *on the ground plane* rather than along the line of sight. If the viewing height is 3 feet, then the unit dimension represents 36/7.2 = 5 inches and the fixation point is 80 inches away.

The orthogonals define this unit dimension at any depth; a transversal established at the base of the primary form creates a measure bar in unit dimensions at that distance. In the diagram (above), a measure bar is shown at the fixation point that is 3 unit dimensions long. If the viewing height is 300 meters, then the measure bar defines a width of 126 meters at a distance of 672 meters. This image bar is used to measure out the size of the primary form image along and above the base transversal.



three point perspective: key scaling dimensions

The second method is to establish an exact unit dimension. This is done by (1) drawing a line from one of the **dvp's** through the *viewing height in depth* (**vhd**) and extending this line until it intersects the station line image, then (2) dividing the distance from this intersection to the station point (**vp_1**) by an appropriate number of units. In the example this procedure yields a station line length of 354 cm, which is conveniently divided into six 59 cm units. If the viewing height is 270 meters, then this unit dimension represents exactly 45 meters on the ground plane.

As explained in the discussion of **scaling the drawing** in the 1PP context, the location of the format requires the artist to decide the appropriate size and location of the primary form image. The principal scaling restriction is the **horizon line rule**: the horizon line intersects all forms at a height above their intersection with the ground plane that is equal to the viewing height, or causes the forms to appear below the horizon line by an equivalent added proportion of their total height. Following the procedure explained for central perspective, the artist **finds the ratio** between the viewing height and object height, then places the object so that the horizon line divides or stands above the object image by this ratio. This rule holds regardless of the angle of the direction of view to the ground plane.

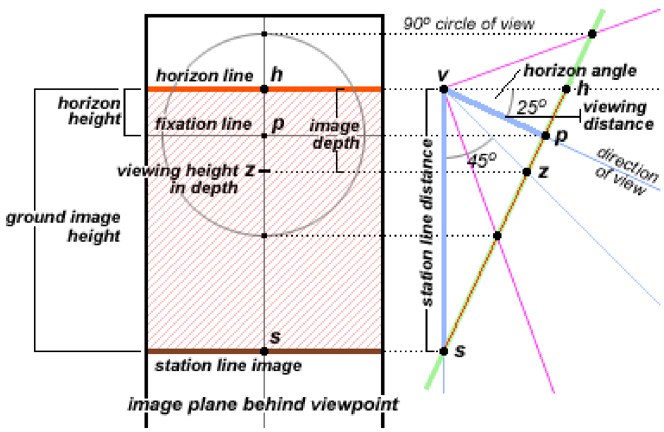
In the example I will develop, I want to render a primary form that is 300 meters high and 125 meters wide. I want the upper portion of the form to cut the horizon line, so that its top portion is silhouetted against the sky. Therefore I plan on approximately 30 meters of the form appearing above the horizon line and the remaining 270 meters below the horizon line. This means the **viewing height** will also be 270 meters (according to the **horizon height rule**), and 30/300 or 10% of the primary form image will be above the horizon line.

To minimize perspective distortion and create a "long view" image of the primary form, I decide to use the fixation point as the anchor point. The 13th unit transversal is just behind the fixation line (orange), which indicates an object distance (on the ground plane) of less than 585 meters. Now the *horizontal* unit dimension can be derived directly from the orthogonals along that transversal, then rotated 90° to provide the *vertical* image dimensions. Then the approximate image area of the primary form can be defined within the circle of view (green rectangle).

Finally, the format dimensions are positioned around the primary form area, horizon line, direction of view, or any other important composition elements. This can be done first, and the primary form fitted within the format, or done after the primary form is located within the circle of view. (Either way, the format dimensions are established as a proportion of the circle of view radius, as **explained here**.) Given my angle of view and the monumental size of the primary form, I decide on a large format. The example shown below is the 29"x42" (74 cm x 107 cm) double elephant (USA) format, in "portrait" orientation and positioned to accommodate the primary form above and below the horizon line (yellow rectangle).

Comment: if you compare the perspective gradients in the previous two diagrams to the perspective gradient in **central perspective**, the recession appears more gentle in 3PP — the squares at the base of the circle of view are still vertically elongated in 3PP, but are horizontally elongated in 1PP. This is because the ground plane is viewed at a more oblique angle and is therefore less foreshortened (at the station point or $\mathbf{vp_1}$, the view is perpendicular to the ground plane, as indicated by the shoe prints). But in the visual area above the location of the viewing height in depth, the two gradients become equivalent.

Calculation Methods. The alternative scaling method uses calculation rather than approximate construction. This method is more precise and robust. The diagram below identifies the key scaling terms in relation to the elevation view of the image plane and visual rays from the viewpoint, and the image plane as it appears in the perspective diagrams.



three point perspective: key scaling dimensions

__vp__ = viewing distance; __vs__ = station line distance; __hvp__ = horizon angle; __h__ = orthogonal vanishing point (horizon line); __p__ = principal point (fixation line); __z__ = viewing height in depth; __s__ = image station point (station line image); __hp__ = horizon height; __hs__ = ground image height; __hz__ = image depth

The only preparation necessary for these calculations is specification of (1) the viewing height, (2) viewing distance perpendicular to the image plane, and (3) horizon angle. Continuing the example above, I set the viewing height at 270 meters, the viewing distance at 1.5 m (150 cm) and the horizon angle at 25°.

In addition, you will need a pocket calculator that can provide the three **trigonometric functions** (sine, cosine and tangent) for any **horizon angle**. In the example, the horizon angle is 25°, therefore:

sine(25°) = 0.423
cosine(25°) = 0.906
tangent(25°) = 0.466

Format Dimensions. The procedure for establishing the format dimensions is **explained here**. It is useful to do this first, if an appropriate format size can be decided in advance, as this provides a frame of reference for other scaling decisions. I will continue with the double elephant example illustrated **above**.

Ground Scale. The second step is to establish the **ground scale**, the scale of the station line **S** on image plane at **s** (refer to the **diagram above**):

• The "level line" role of the horizon line is taken by a **fixation line** through the direction of view and parallel to the horizon line above or below it. This defines the vanishing line for all planes parallel to the direction of view and to the horizon line (perspective **rule 15**), and the "actual size" image scale (at a viewing distance of 150 cm, the circle of view radius along the fixation line is 150 cm).

• The **horizon height**, the distance of the horizon line above the fixation line as measured on the image plane, is equal to the viewing distance *multiplied by* the tangent of the horizon angle **θ**. The tangent of a 25° horizon angle is 0.466, so the horizon line is 150 cm*0.466 = 70 cm above the direction of view.

• The *station line image* is exactly below the viewpoint **v**, at point **s**, which defines the right triangle **vps** (because the direction of view is perpendicular to the image plane). Therefore the **station line distance** (the distance of image plane below the viewpoint, or **vs**) is the hypotenuse of the right triangle **vps**. This is equal to the viewing distance *divided by* the sine of the horizon angle, or 150 cm/0.423 = 355 cm.

• The **ground image height** is the extent of the image plane between the horizon line and station line image, or the hypotenuse of the right triangle **hvs**. This is found as the station line distance *divided by* the cosine of the horizon angle: 355 cm/0.906 = 392 cm.

• The station line image is below the fixation line at a distance *on the image plane* equal to **ps**, or the ground image height (**hs**) minus the horizon height (**hp**): 392 cm – 70 cm = 322 cm.

• Finally, **formula 5** provides the image ratio for the station line scale. If the viewing height is 270 meters and the station line distance is 355 cm, then the station line *image scale* is 3.55 m/270cm = 1.31%; or equivalently, a 1 centimeter unit dimension in the station line equals 270/3.55 = 76 cm on the ground plane. To obtain the 45 meter unit dimension in the ground plane as units of the image plane at the station line: 45*0.0131 m = 59 cm unit dimension in the station line. This is the **ground scale**, as summarized in the following formulas:

$$(11)\ \textbf{horizon height} = \textbf{viewing distance} * \textbf{tangent}(\theta)$$

$$(12)\ \textbf{station line distance} = \frac{\textbf{viewing distance}}{\textbf{sine}(\theta)}$$

$$(13)\ \textbf{ground scale} = \textbf{ground unit} * \frac{\textbf{station line distance}}{\textbf{viewing height}}$$

where the *ground unit* is whatever measurement unit is convenient for mapping objects on the ground plane (e.g., 1 meter or 100 meters), and the viewing distance, station line distance and viewing height are all measured in the same units.

• Finally, the diagonal vanishing points can be used to project the ground scale unit dimension in depth. The diagram (above) shows that the 270 meter line established by the projecting the 45 meter unit dimension in depth exactly coincides with the location of the viewing height projected in depth by the horizon plane rotation.

Fixation Line Scale. The third and last step is to determine the image depth **z** on the image plane for an object on the ground plane at distance

X from the station line, or the object distance **X** on the ground plane of a particular image depth **z** on the image plane. These relationships are defined in the following formulas:

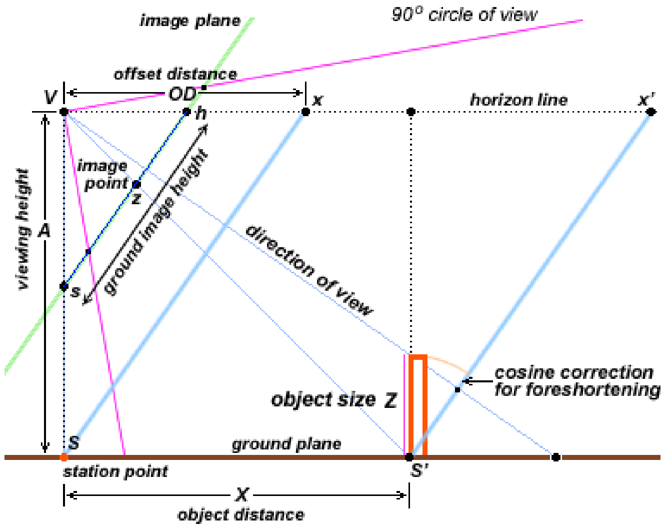
$$(14)\ \textbf{offset distance (OD)} = \textbf{viewing height} * \textbf{tangent}(\theta)$$

$$(15)\ \textbf{ground image height (GIH)} = \frac{\textbf{station line distance}}{\textbf{cosine}(\theta)}$$

$$(16)\ \textbf{image depth} = \textbf{GIH} * \frac{\textbf{OD}}{\textbf{(object distance + OD)}}$$

$$(17)\ \textbf{object distance} = \left[\textbf{OD} * \frac{\textbf{GIH}}{\textbf{image depth}}\right] - \textbf{OD}$$

$$(18)\ \textbf{image scale} = \frac{\textbf{viewing distance}}{\sqrt{\textbf{object distance}^2 + \textbf{viewing height}^2}}$$



three point perspective: anchor point and anchor line

The image plane extends from the image station line (**s**) to the horizon line (**h**), defining the **horizon height** (**sh**). This image plane can be duplicated by a secondary image plane **Sx**, some distance in front of and parallel to it. The secondary plane intersects the horizon line at **x**, removed from the viewpoint **V** by the **offset distance O**, and intersects the ground plane at the station line (**S**), which is a zero horizontal distance from the viewpoint. These intersections are identical with the image points **h** and **s**, so the horizon height **sh** is the image of the physical distance **Sx** when the object distance (on the ground plane) is zero.

Moving the secondary plane forward by the object distance **X** relocates the ground intersection to **S'** and the horizon line intersection at **x'**, so that the image of **S'x'** is now **zh** — the image point **z** is at an **image depth** (**zh**) below the horizon line. In this new arrangement, the triangular proportions define the proportional equality:

zh/sh = xV/x'V = OD/(X+OD)

which solves either for (16) the image depth (**zh**) or (17) the ground plane object distance (**X**) by algebraic rearrangement.

Using these formulas, we establish (for a horizon angle of 25°, a viewing height of 270 m and a viewing distance of 1.5 m) that:

(8) **format scale** = 25%(W), 36%(H)
(11) horizon height = 70 cm
(11) station line distance = 355 cm
(13) ground scale: 66 cm = 50 meters

(14) offset distance = 126 m
(15) ground image height = 392 cm
(16) image depth (of *fixation line*) = 70 cm
(16) image depth (of *viewing distance in depth*) = 125 cm
(16) image depth (of *format baseline @ 500 m*) = 79 cm
(17) object distance (at fixation line) = 579 m.

Anchor Line and Anchor Point. Given the fixation point as the anchor point, the ground plane object distance is 579 meters at a viewing height of 270 meters, which is a *diagonal object distance* of 639 meters to the base of the primary form. So the **fixation line scale** (derived from **formula 5**) is:

(18) image scale (at *fixation line*) = $1.5/[579^2+270^2]^{1/2}$
= 1.5/639 m
= 0.00235 (0.235%)

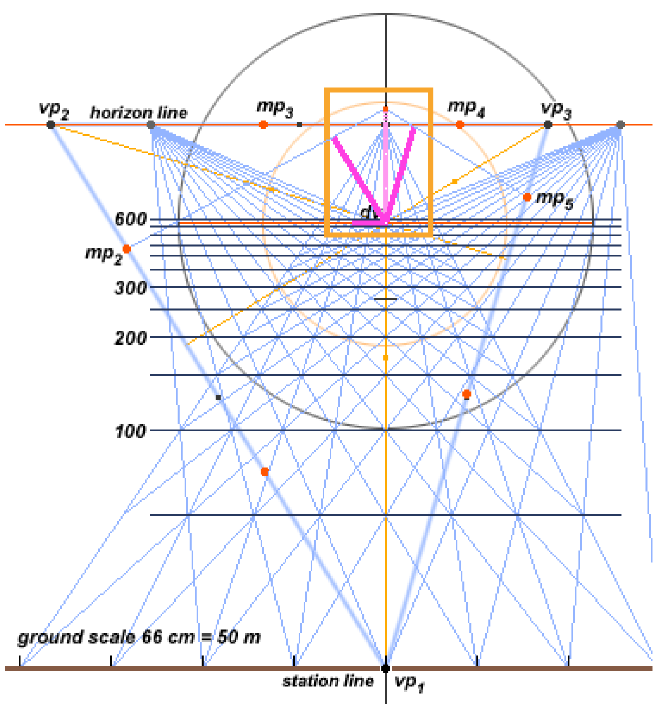
To go from image plane units to ground plane units (at the fixation line), you divide by the image scale factor:

1 cm = 1 cm/0.00235 = 4.26 m.

To go from ground plane units to image plane units, you multiply by the image scale factor. Thus, an object 125 meters wide and 301 meters tall, oriented parallel to the image plane, creates the image dimensions

125 m*0.00235 = 0.294 m = 29.4 cm
301 m*0.00235 = 0.707 m = 70.7 cm.

These are the **measure bar dimensions** for the image at the anchor point (established as the fixation point), as shown in the diagram (below).



three point perspective: anchor point and anchor line

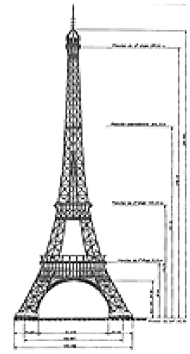
Confusion about the choice of vanishing line measure points and the orientation of measure bars (for cubic forms) is usually dispelled by the following three criteria:

• The measure point to use for any edge is the measure point **defined by an arc from the controlling vanishing point**. Thus, the height dimension is controlled by the vertical vanishing point (**vp_1**), which was the center of the arc used to define **mp_2** and **mp_5**.

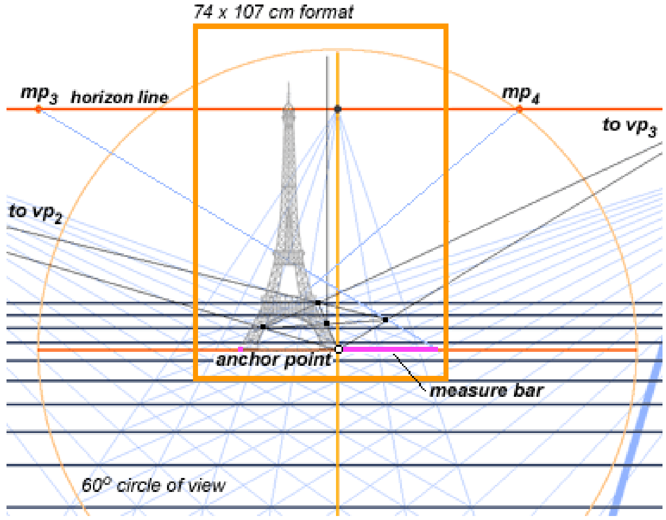
• The controlling vanishing point is the **vanishing point for the convergence of the edges** that are being sized by the measure bar. Thus, the left side edges of a plan in the ground plane are defined by the lefthand horizon vanishing point.

• The measure bar is **always oriented parallel to the vanishing line** containing the measure point.

With a plan and elevation of the primary form (diagram, right), the artist is ready to construct the perspective drawing.



elevation of the primary form



three point perspective: constructing the primary form

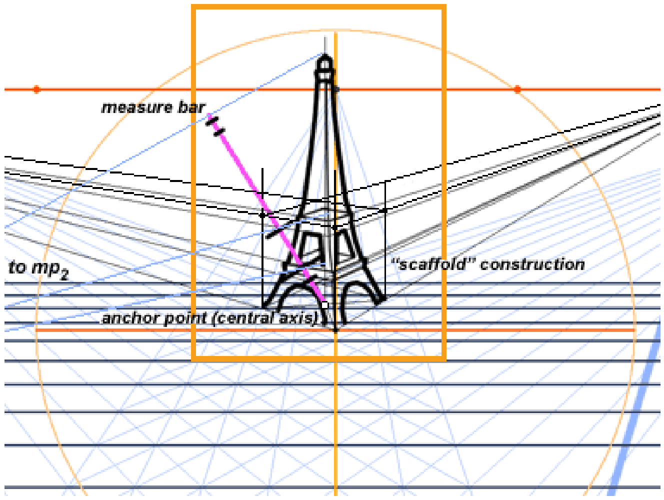
diagram enlarged to 60° circle of view for clarity

It is usually convenient to establish the plan or "street map" dimensions of the drawing first, as the plan outlines do not intersect one another and clearly establishe the front to back ordering of large forms. Here the drawing is being constructed from the elevation only, as the base is square. Only the 60° circle of view is shown for clarity.

Using the measure point guidelines (above), the controlling vanishing point for the right side of the base of the tower is **vp_3**; and this vanishing point defined the arc for **mp_3** (refer to the **diagram above**). The controlling vanishing point for the height of the tower is the vertical vanishing point, **vp_1**; and this vanishing point defined the arc for **mp_2**.

The fixation line measure bar is used to establish the base width of 125 meters, and this dimension is projected in depth by the lines to the opposite measure points **mp_3** and **mp_4**. The measure bar is parallel to the vanishing line containing the measure points, Then vanishing lines from the anchor point to **vp_2** and **vp_3** establish the sides of the plan.

The tower is symmetrical on its four sides, but the sides are not vertical: they define an exponential function designed to maximize the tower's strength against strong winds. To facilitate the perspective construction we have to find the central axis, which is simply the intersection of the diagonals of the plan.



three point perspective: finished drawing

diagram enlarged to 60° circle of view for clarity

The major stages of the tower are marked off on a vertical measure bar, this bar is rotated to be parallel with the vanishing line of the appropriate measure point, and the tower stages are projected onto a vertical axis.

Two strategies are available. The existing elevation drawing can be used to create the measure bar: this drawing is in the image scale defined by the anchor point on the fixation line, 579 meters from the viewpoint. Elevation points are projected onto a vertical line constructed from the anchor point. These elevation points are the front corners of new squares, of equal size as the base of the tower, containing the tower platforms. These are constructed "scaffold style", by vertical lines from the four corners of the plan. At each level the scaffold squares are recessed to the side vanishing points from the front corner, and the diagonal found as before. Then the plan of the tower platform is constructed within this square, its four corners along the diagonals.

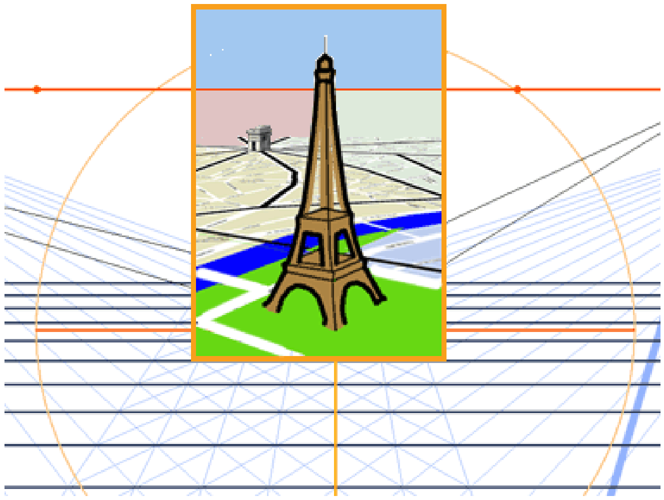
The alternative method is to project the elevation points onto the central axis, and project the tower platforms out from these central points. This method is also shown in the diagram: the measure bar must be anchored at the base of the central axis. Note however that this point is over 80 meters farther away from the viewpoint than the front corner (as shown by the 50 meter distance transversals in the ground plan), therefore the measure bar must be sized, using **formula 18**, to the new image distance. First the added distance is derived from the whole diagonal, which is then aligned to the direction of view by the cosine correction:

base diagonal = $[125^2+125^2]^{1/2}$ = 177 m
half diagonal = 177 m/2 = 88.5 m
ground plane distance = 88.5 m * cosine(10°) = 80.2

and the new ground plane distance (579+80 = 659) is used to compute the new image scale:

(18) image scale (at *central axis*) = $1.5/[659^2+270^2]^{1/2}$
= 1.5/712 m
= 0.00211 (0.211%)

(Note that the central axis distance could be estimated from its position just beyond the 650 m transversal distance line.) Once the major external points of the tower profile are established, the outside curves of the tower can be drawn with a French curve or freehand, and details of the tower filled in as appropriate.



three point perspective: finished drawing

diagram enlarged to 60° circle of view for clarity

And here is the finished drawing. The point of using the exact rotation method is that the Arc de Triomphe could be precisely positioned behind the Tour Eiffel, and both positioned in relation to the direction of view and horizon line, to produce a specific effect.

The plan of the distant streets is taken from a Michelin map of Paris, projected onto the ground plane using the foreshortened and recessed orthogonal squares and plotting the major streets, square by square, as far back as useful.

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