

## **Лабораторная работа** **«Решение дифференциальных уравнений»**

Задание:

решить аналитически следующие дифференциальные уравнения по вариантам (в задаче 2 минимум одно уравнение;

решить численно эти же дифференциальные уравнения в заданном (самостоятельно) промежутке;

изобразить совместно на графике полученные решения.

Задача 1:

№	Задача Коши
1	$y' + xy = 0,5(x-1)e^x y^2, y(0) = 2; a = 0, b = 2.$
2	$y' - y \operatorname{tg} x = -2/3 y^4 \sin x, y(0) = 1; a = 0, b = 1,2.$
3	$y' + y^2 = x, y(0) = 1; a = 0, b = 2.$
4	$xy' + y = y^3 e^{-x}, y(1) = 1; a = 1, b = 2.$
5	$y' + xy = 0,5(x+1)e^x y^2, y(0) = 1; a = 0; b = 2.$
6	$xy' - y = -y^2(2 \ln x + \ln^2 x), y(1) = 2; a = 1, b = 2.$
7	$y' + 4x^3 y = 4y^2 e^{4x}(1 - x^3), y(1) = 1; a = 1, b = 2,8.$
8	$2y' + 3y \cos x = e^{2x}(2 + 3 \cos x) / y, y(1) = 2; a = 1, b = 1,6.$
9	$y' + 2xy = 2x^3 y^3, y(0) = 1; a = 0, b = 1.$
10	$xy' + y = y^2 \ln x, y(1) = 0,5; a = 1, b = 5.$
11	$2y' + 3y \cos x = (8 + 12 \cos x)e^{2x} / y, y(0) = 2; a = 0, b = 2.$
12	$4y' + x^3 y = (x^3 + 8)e^{-2x} y^2, y(0) = 0,5; a = 0, b = 2,4.$
13	$8xy' = 12y - (5x^2 + 3)y^3, y(1) = 1; a = 1, b = 3.$
14	$y' + y = 0,5xy^2, y(0) = 2; a = 0, b = 2.$
15	$y' + xy = (x-1)e^x y^2, y(0) = 1; a = 0, b = 2.$
16	$3y' - 3y \cos x = -e^{-2x}(2 + 3 \cos x) / y, y(0) = 1,1; a = 0, b = 0,8.$
17	$y' - y = xy^2, y(0) = 0,5; a = 0, b = 0,8.$
18	$xy' + y = y^2 \ln x, y(1) = 1; a = 1, b = 2,6.$
19	$y' + y = xy^2, y(0) = 1; a = 0, b = 2.$
20	$xy' + y = xy^2, y(1) = 1; a = 1, b = 2.$
21	$2y' + 3y \cos x = e^{2x}(2 + 3 \cos x) / y, y(0) = 1; a = 0, b = 1,6.$
22	$3(xy' + y) = xy^2, y(1) = 1; a = 1, b = 5.$
23	$y' - y = 2xy^2, y(-1) = 0,2; a = -1, b = 0,6.$
24	$2xy' - 3y = -(20x^2 + 12)y^3, y(1) = 0,25; a = 1, b = 5.$
25	$2y' + 3y \cos x = (8 + 12 \cos x)e^{2x} / y, y(0) = 3; a = 0, b = 3.$
26	$y' + xy = (1+x)e^x y^{-2}, y(0) = 1, a = 0, b = 1,6.$
27	$xy' + y = 2y^2 \ln x, y(1) = 0,5; a = 1, b = 5$
28	$2xy' + 2y = xy^2, y(1) = 2; a = 1, b = 1,8.$
29	$y' + 4x^3 y = 4(x^3 + 1)e^{-4x} y^2, y(0) = 0,5; a = 0, b = 1.$
30	$xy' - y = -y^2(2 \ln x + \ln^2 x), y(1) = 1; a = 1, b = 3.$

**Задача 2.**

№	Уравнение, начальные условия
1	1) $y'' + y' - 2y = -4x^2 - 12x + 16,$ $y(0) = -1, y'(0) = 0;$ 2) $y'' + 16y' + 68y = e^{-9x} \cdot ((10x^2 + 18x) \cdot \cos x + (10x^2 + 22x - 20) \cdot \sin x),$ $y(0) = 0, y'(0) = 1$
2	1) $y'' - y = -5x^2 + 8x + 12,$ $y(0) = 0, y'(0) = 0;$ 2) $y'' - 4y' + 8y = e^{3x} \cdot ((20x^2 + 14x - 18) \cdot \cos x + (2x + 28) \cdot \sin x),$ $y(0) = 0, y'(0) = 0$
3	1) $y'' - y' - 2y = -2x^2 - 16x + 5,$ $y(0) = -1, y'(0) = 0;$ 2) $y'' + 12y' + 40y = e^{-7x} \cdot ((-2x^2 - 20x - 14) \cdot \cos x + (24x^2 - 24x) \cdot \sin x),$ $y(0) = 0, y'(0) = 0$
4	1) $y'' + 3y' + 2y = 4x^2 + 14x - 9,$ $y(0) = -3, y'(0) = 0;$ 2) $y'' + 4y' + 5y = e^{-x} \cdot ((4x^2 + 14x - 8) \cdot \cos 2x + (-22x^2 + 18x - 2) \cdot \sin 2x),$ $y(0) = 0, y'(0) = 0$
5	1) $y'' - 2y' - 3y = -6x^2 + 7x - 13,$ $y(0) = 0, y'(0) = 0;$ 2) $y'' - 2y' + 2y = e^{2x} \cdot ((6x^2 + 24x - 12) \cdot \cos 2x + (-28x^2 - 20x - 6) \cdot \sin 2x),$ $y(0) = 0, y'(0) = 0$
6	1) $y'' - 5y' + 4y = 4x^2 + 2x - 5,$ $y(0) = 2, y'(0) = 0;$ 2) $y'' - 6y' + 18y = e^{4x} \cdot ((26x^2 + 2x + 12) \cdot \cos 2x + (26x^2 + 14x - 6) \cdot \sin 2x),$ $y(0) = 1, y'(0) = -1$
7	1) $y'' - 3y' + 2y = 10x^2 - 18x - 12,$ $y(0) = -1, y'(0) = 0;$ 2) $y'' + 10y' + 29y = e^{-6x} \cdot ((4x^2 + 20) \cdot \cos x + (12x^2 - 26x + 24) \cdot \sin x),$ $y(0) = 1, y'(0) = 0$
8	1) $y'' + 4y' - 5y = -10x^2 - 4x + 15,$ $y(0) = 3, y'(0) = 0;$ 2) $y'' + 10y' + 34y = e^{-4x} \cdot ((28x^2 - 6x - 6) \cdot \cos 2x + (16x^2 - 4x - 8) \cdot \sin 2x),$ $y(0) = 2, y'(0) = 0$

9	1) $y'' + 2y' - 3y = -12x^2 + 4x - 8,$ $y(0) = 0, y'(0) = 0;$ 2) $y'' + 8y' + 17y = e^{-3x} \cdot ((-2x^2 + 2x + 18) \cdot \cos x + (-14x^2 - 16x + 12) \cdot \sin 2x),$ $y(0) = 1, y'(0) = 0$
10	1) $y'' + 5y' - 6y = -18x^2 - 11,$ $y(0) = -3, y'(0) = 2;$ 2) $y'' + 2y' + 2y = e^{-2x} \cdot ((-22x^2 + 6x - 22) \cdot \cos 2x + (-6x^2 + 2x - 26) \cdot \sin 2x),$ $y(0) = 0, y'(0) = 3$
11	1) $y'' - 4y' - 5y = -5x^2 + 7x - 11,$ $y(0) = -4, y'(0) = 0;$ 2) $y'' - 14y' + 50y = e^{8x} \cdot ((-2x^2 + 26x - 18) \cdot \cos 2x + (-14x^2 - 28x + 4) \cdot \sin 2x),$ $y(0) = 0, y'(0) = -1$
12	1) $y'' - 4y' + 3y = 9x^2 - 6x + 6,$ $y(0) = 0, y'(0) = 0;$ 2) $y'' + 6y' + 13y = e^{-2x} \cdot ((22x^2 + 28x - 24) \cdot \cos x + (24x^2 - 6x - 6) \cdot \sin x),$ $y(0) = 1, y'(0) = 0$
13	1) $y'' - 4y = -12x^2 - 16x + 10,$ $y(0) = -1, y'(0) = 0;$ 2) $y'' + 16y' + 65y = e^{-7x} \cdot ((2x^2 + 14x - 4) \cdot \cos 2x + (-16x^2 - 14x - 26) \cdot \sin 2x),$ $y(0) = 0, y'(0) = 0$
14	1) $y'' + 4y' + 3y = 3x^2 + 2x,$ $y(0) = 2, y'(0) = 0;$ 2) $y'' + 2y' + 5y = e^{-2x} \cdot ((16x^2 + 22x + 20) \cdot \cos x + (18x^2 - 16x + 14) \cdot \sin x),$ $y(0) = 0, y'(0) = 0$
15	1) $y'' - 3y' - 4y = -4x^2 - 14x - 16,$ $y(0) = 0, y'(0) = 0;$ 2) $y'' - 12y' + 37y = e^{5x} \cdot ((-14x^2 - 24x - 20) \cdot \cos 2x + (8x^2 - 6x) \cdot \sin 2x),$ $y(0) = 0, y'(0) = 0$

16	1) $y'' + 3y' - 4y = -12x^2 + 14x + 13$ , $y(0) = 3, y'(0) = 0$ ; 2) $y'' + 18y' + 85y = e^{-8x} \cdot ((26x^2 + 20x - 24) \cdot \cos x + (2x^2 - 12x + 14) \cdot \sin x)$ , $y(0) = 1, y'(0) = 0$
17	1) $y'' + 7y' + 6y = 12x^2 + 10x + 7$ , $y(0) = -4, y'(0) = 0$ ; 2) $y'' - 8y' + 17y = e^{5x} \cdot ((6x^2 + 20x + 2) \cdot \cos 2x + (-28x^2 - 18x + 8) \cdot \sin 2x)$ , $y(0) = 0, y'(0) = 0$
18	1) $y'' + 5y' + 4y = 12x^2 + 14x + 2$ , $y(0) = -3, y'(0) = 0$ ; 2) $y'' - 16y' + 68y = e^{7x} \cdot ((8x^2 + 14x + 2) \cdot \cos x + (14x^2 + 14x + 10) \cdot \sin x)$ , $y(0) = 0, y'(0) = 0$
19	1) $y'' - 5y' + 6y = 6x^2 - 4x - 15$ , $y(0) = 0, y'(0) = 0$ ; 2) $y'' - 18y' + 90y = e^{8x} \cdot ((26x^2 - 26x - 2) \cdot \cos 2x + (26x^2 - 10x - 6) \cdot \sin 2x)$ , $y(0) = 0, y'(0) = 1$

#### Литература:

1. <http://www.math24.ru/%D1%83%D1%80%D0%B0%D0%B2%D0%BD%D0%B5%D0%BD%D0%B8%D0%B5-%D0%B1%D0%B5%D1%80%D0%BD%D1%83%D0%BB%D0%BB%D0%B8.html>
2. <http://www.math24.ru/%D0%BE%D0%B4%D0%BD%D0%BE%D1%80%D0%BE%D0%B4%D0%BD%D1%8B%D0%B5-%D0%B4%D0%B8%D1%84%D1%84%D0%B5%D1%80%D0%B5%D0%BD%D1%86%D0%B8%D0%B0%D0%BB%D1%8C%D0%BD%D1%8B%D0%B5-%D1%83%D1%80%D0%B0%D0%B2%D0%BD%D0%B5%D0%BD%D0%B8%D1%8F-%D0%B2%D1%82%D0%BE%D1%80%D0%BE%D0%B3%D0%BE-%D0%BF%D0%BE%D1%80%D1%8F%D0%B4%D0%BA%D0%B0-%D1%81-%D0%BF%D0%BE%D1%81%D1%82%D0%BE%D1%8F%D0%BD%D0%BD%D1%8B%D0%BC%D0%B8-%D0%BA%D0%BE%D1%8D%D1%84%D1%84%D0%B8%D1%86%D0%B8%D0%B5%D0%BD%D1%82%D0%B0%D0%BC%D0%B8.html>
3. <http://www.math24.ru/%D0%BD%D0%B5%D0%BE%D0%B4%D0%BD%D0%BE%D1%80%D0%BE%D0%B4%D0%BD%D1%8B%D0%B5-%D0%B4%D0%B8%D1%84%D1%84%D0%B5%D1%80%D0%B5%D0%BD%D1%86%D0%B8%D0%B0%D0%BB%D1%8C%D0%BD%D1%8B%D0%B5-%D1%83%D1%80%D0%B0%D0%B2%D0%BD%D0%B5%D0%BD%D0%B8%D1%8F-%D0%B2%D1%82%D0%BE%D1%80%D0%BE%D0%B3%D0%BE-%D0%BF%D0%BE%D1%80%D1%8F%D0%B4%D0%BA%D0%B0-%D1%81-%D0%BF%D0%BE%D1%81%D1%82%D0%BE%D1%8F%D0%BD%D0%BD%D1%8B%D0%BC%D0%B8-%D0%BA%D0%BE%D1%8D%D1%84%D1%84%D0%B8%D1%86%D0%B8%D0%B5%D0%BD%D1%82%D0%B0%D0%BC%D0%B8.html>

