2.9 $$f''(x_1) = \frac{f(x_1-h) - 2f(x_1) + f(x_1+h)}{h^2} - \frac{h^2}{12}\cdot f^{(4)}(\xi)$$

$|f^{(4)}(x)| \le M$ $\quad e(x_i) \le \varepsilon$ – выч. погрешность

$$\overset{\text{точное знач.}}{f}(x_1-h) = \overset{\text{факт.}}{\tilde{f}}(x_1-h) + \overset{\text{погрешность}}{e}(x_1-h), \quad f(x_1+h) = \tilde{f}(x_1+h) + e(x_1+h)$$

$$\overset{\text{ошибка}}{E} = \left| f''(x_1) - \frac{\tilde{f}(x_1-h) - 2\tilde{f}(x_1) + \tilde{f}(x_1+h)}{h^2} \right| =$$

$$= \left| \frac{e(x_1-h) - 2e(x_1) + e(x_1+h)}{h^2} - \frac{h^2}{12}\cdot f^{(4)}(\xi) \right|$$

$$E \le \frac{|e(x_1-h)| + |2e(x_1)| + |e(x_1+h)|}{h^2} + \frac{h^2}{12}\cdot |f^{(4)}(\xi)|$$

$$E \le \frac{\varepsilon + 2\varepsilon + \varepsilon}{h^2} + \frac{h^2}{12} M$$

$E \le \frac{4\varepsilon}{h^2} + \frac{h^2}{12} M$ – оценка вычисл. погрешности

$$\lim_{h\to 0}\left(\frac{4\varepsilon}{h^2} + \frac{h^2}{12}M\right) = \lim_{h\to 0}\left(\frac{4\varepsilon}{h^2}\right) + \lim_{h\to 0}\left(\frac{h^2 M}{12}\right) = +\infty + 0 = +\infty$$

$h \to 0,\ E \to \infty \Rightarrow$ вычислительная неустойчивость

$E = \min$ при $h_{opt} = \left(\frac{48\varepsilon}{M}\right)^{\frac{1}{4}}$

$$f(h) = \frac{4\varepsilon}{h^2} + \frac{h^2 M}{12}$$

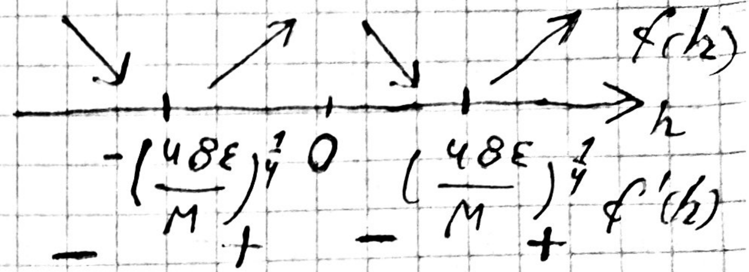
$$f'(h) = -\frac{8\varepsilon}{h^3} + \frac{hM}{6}$$

$$f'(h) = 0$$

$$\frac{hM}{6} - \frac{8\varepsilon}{h^3} = 0$$

$$\frac{h^4 M - 48\varepsilon}{6h^3} = 0$$

$$h^4 = \frac{48\varepsilon}{M}$$



$f(h) = \min$

при $h = \pm\left(\frac{48\varepsilon}{M}\right)^{\frac{1}{4}} \Rightarrow$

$$h_{opt} = \left(\frac{48\varepsilon}{M}\right)^{\frac{1}{4}}$$