HW 4

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4.8.3

Density function of normal distribution: $f(x) = \frac{1}{\sqrt{2\pi\sigma_k^2}} e^{\frac{-(x-\mu_k)^2}{2\sigma_k^2}}$
For a one dimensional feature $(p=1)$

Bayes Theorem

$$P(C_k|x) = P(x|C_k) \cdot P(C_k)/P(x)$$

$P(C_k|x) = P(C_{k+1}|x)$ Gives us the decision boundary

$$P(x|C_k) \cdot P(C_k)/P(x) = P(x|C_{k+1}) P(C_{k+1})/P(x)$$

$$P(x|C_k) P(C_k) = P(x|C_{k+1}) P(C_{k+1})$$

$$\frac{1}{\sqrt{2\pi\sigma_k^2}} e^{\frac{-(x-\mu_k)^2}{2\sigma_k^2}} \cdot P(C_k) = \frac{1}{2\pi\sigma_{k+1}^2} e^{\frac{-(x-\mu_{k+1})^2}{2\sigma_{k+1}^2}} P(C_{k+1})$$

$$\ln(P(C_k)) + \ln\left(\frac{1}{\sqrt{2\pi\sigma_k^2}}\right) - \frac{(x-\mu_k)^2}{2\sigma_k^2} = \ln(P(C_{k+1})) + \ln\left(\frac{1}{\sqrt{2\pi\sigma_{k+1}^2}}\right) - \frac{(x-\mu_{k+1})^2}{2\sigma_{k+1}}$$

$$\ln(P(C_k)) + -\frac{1}{2}\ln(2\pi\sigma_k^2) - \frac{(x-\mu_k)^2}{2\sigma_k^2} = \ln(P(C_{k+1})) - \frac{1}{2}\ln(2\pi\sigma_{k+1}^2) - \frac{(x-\mu_{k+1})^2}{2\sigma_{k+1}}$$

The x terms are: $\frac{-(x-\mu_k)^2}{2\sigma_k^2}$ and $\frac{-(x-\mu_{k+1})^2}{2\sigma_{k+1}}$

This shows that the Bayes classifier is not linear as both x terms are quadratic.

4.8.7 → Predict if stock will issue dividend

$\bar{x} = 10 \quad x = 4$

$\hat{\sigma}^2 = 36$

Prior = .8

$$Pr(y=1|x) = Pr(x|y=1)\underbrace{Pr(y=1)}_{\text{Prior} = .8}$$

For y = 1

$$Pr(y=1|x) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} \cdot e^{\frac{-(x-\mu_k)^2}{2\sigma^2}} \underbrace{Pr(y=1)}_{.8}$$

$$Pr(y=1|x=4) = \frac{1}{\sqrt{2\pi \cdot 36}} \cdot e^{\frac{-(4-10)^2}{2(36)}} (.8)$$

$$Pr(y=1|x=4) = \frac{1}{6\sqrt{2\pi}} \cdot e^{-\frac{1}{2}} \cdot (.8) = 0.03226$$

for y = 0 $\quad \bar{x} = 0 \quad \hat{\sigma}^2 = 36$

$$Pr(y=0|x) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} e^{\frac{-(\overset{4}{x} - \overset{0}{\mu_k})^2}{2\underset{36}{\sigma^2}}} \underbrace{Pr(y=0)}_{.2}$$

$$Pr(y=0|x) = \frac{1}{6\sqrt{2\pi}} e^{-2/9} \cdot (.2) = 0.01045$$

$$P(x=4) = P(x=4|y=1)P(y=1) + P(x=4|y=0)P(y=0)$$

$$P(x=4) = .03226 + 0.01605$$

$$Pr(y=1|x=4) = \frac{.03226}{.04291} = 75.2\%$$

The Probability that ~~the~~ company will issue a dividend is 75.2%