

Wine Quality Analysis

2025-11-17

Libraries

```
library(readr)
library(naniar)

## Warning: package 'naniar' was built under R version 4.4.3
library(ggplot2)

## Warning: package 'ggplot2' was built under R version 4.4.3
library(tidymodels)

## Warning: package 'tidymodels' was built under R version 4.4.3

## -- Attaching packages ----- tidymodels 1.4.1 --
## v broom      1.0.11    v rsample     1.3.1
## v dials       1.4.2     v tailor      0.1.0
## v dplyr       1.1.4     v tidyverse   1.3.1
## v infer        1.0.9     v tune        2.0.1
## v modeldata   1.5.1     v workflows   1.3.0
## v parsnip      1.4.0     v workflowsets 1.1.1
## v purrr       1.2.0     v yardstick   1.3.2
## v recipes      1.3.1

## Warning: package 'broom' was built under R version 4.4.3
## Warning: package 'dials' was built under R version 4.4.3
## Warning: package 'scales' was built under R version 4.4.3
## Warning: package 'infer' was built under R version 4.4.3
## Warning: package 'modeldata' was built under R version 4.4.3
## Warning: package 'parsnip' was built under R version 4.4.3
## Warning: package 'purrr' was built under R version 4.4.3
## Warning: package 'recipes' was built under R version 4.4.3
## Warning: package 'rsample' was built under R version 4.4.3
## Warning: package 'tailor' was built under R version 4.4.3
## Warning: package 'tune' was built under R version 4.4.3
## Warning: package 'workflows' was built under R version 4.4.3
## Warning: package 'workflowsets' was built under R version 4.4.3
## Warning: package 'yardstick' was built under R version 4.4.3
```

```

## -- Conflicts ----- tidyverse_conflicts() --
## x purrr::discard() masks scales::discard()
## x dplyr::filter()  masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## x yardstick::spec() masks readr::spec()
## x recipes::step()  masks stats::step()

library(tidyr)
library(DescTools)

## Warning: package 'DescTools' was built under R version 4.4.3
library(tibble)
library(car)

## Loading required package: carData

##
## Attaching package: 'car'

## The following object is masked from 'package:DescTools':
##
##     Recode

## The following object is masked from 'package:purrr':
##
##     some

## The following object is masked from 'package:dplyr':
##
##     recode

library(MASS)

## Warning: package 'MASS' was built under R version 4.4.3
##
## Attaching package: 'MASS'

## The following object is masked from 'package:dplyr':
##
##     select

library(effects)

## Warning: package 'effects' was built under R version 4.4.3
## lattice theme set by effectsTheme()
## See ?effectsTheme for details.

library(dplyr)

```

Data Import

```

wine <- read_csv("winequalityN.csv")

## Rows: 6497 Columns: 14
## -- Column specification -----
## Delimiter: ","
## chr (1): type

```

```

## dbl (13): ID, fixed_acidity, volatile_acidity, citric_acid, residual_sugar, ...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.

```

Data Set Overview

Structure of the Data Set

Exploring the overall structure of the data set:

```
summary(wine)
```

```

##      ID      type      fixed_acidity      volatile_acidity
## Min.   : 1  Length:6497   Min.   : 3.800  Min.   :0.0800
## 1st Qu.:1625 Class  :character  1st Qu.: 6.400  1st Qu.:0.2300
## Median :3249 Mode   :character  Median : 7.000  Median :0.2900
## Mean   :3249                   Mean   : 7.217  Mean   :0.3397
## 3rd Qu.:4873                   3rd Qu.: 7.700  3rd Qu.:0.4000
## Max.   :6497                   Max.   :15.900  Max.   :1.5800
## 
## NA's   :10                    NA's   :8
##      citric_acid      residual_sugar      chlorides      free_sulfur_dioxide
## Min.   :0.0000  Min.   : 0.600  Min.   :0.00900  Min.   : 1.00
## 1st Qu.:0.2500 1st Qu.: 1.800  1st Qu.:0.03800  1st Qu.: 17.00
## Median :0.3100  Median : 3.000  Median :0.04700  Median : 29.00
## Mean   :0.3187  Mean   : 5.444  Mean   :0.05604  Mean   : 30.53
## 3rd Qu.:0.3900 3rd Qu.: 8.100  3rd Qu.:0.06500  3rd Qu.: 41.00
## Max.   :1.6600  Max.   :65.800  Max.   :0.61100  Max.   :289.00
## NA's   :3        NA's   :2        NA's   :2
##      total_sulfur_dioxide      density          pH      sulphates
## Min.   : 6.0      Min.   :0.9871  Min.   :2.720  Min.   :0.2200
## 1st Qu.: 77.0     1st Qu.:0.9923  1st Qu.:3.110  1st Qu.:0.4300
## Median :118.0     Median :0.9949  Median :3.210  Median :0.5100
## Mean   :115.7     Mean   :0.9947  Mean   :3.218  Mean   :0.5312
## 3rd Qu.:156.0     3rd Qu.:0.9970  3rd Qu.:3.320  3rd Qu.:0.6000
## Max.   :440.0      Max.   :1.0390  Max.   :4.010  Max.   :2.0000
## 
##      alcohol      quality
## Min.   : 8.00  Min.   :3.000
## 1st Qu.: 9.50  1st Qu.:5.000
## Median :10.30  Median :6.000
## Mean   :10.49  Mean   :5.818
## 3rd Qu.:11.30  3rd Qu.:6.000
## Max.   :14.90  Max.   :9.000
## 
```

```
head(wine)
```

```

## # A tibble: 6 x 14
##      ID type  fixed_acidity volatile_acidity citric_acid residual_sugar
##    <dbl> <chr>      <dbl>            <dbl>       <dbl>
## 1     1 white       7           0.27         0.36      20.7
## 2     2 white      6.3          0.3          0.34      1.6
## 3     3 white      8.1          0.28         0.4       6.9
## 4     4 white      7.2          0.23         0.32      8.5

```

```

## 5      5 white          7.2          0.23          0.32          8.5
## 6      6 white          8.1          0.28          0.4           6.9
## # i 8 more variables: chlorides <dbl>, free_sulfur_dioxide <dbl>,
## #   total_sulfur_dioxide <dbl>, density <dbl>, pH <dbl>, sulphates <dbl>,
## #   alcohol <dbl>, quality <dbl>

```

Checking if there are any duplicates in the data set:

```
wine |>
  group_by(ID) |>
  summarise(n = n()) |>
  filter(n > 1L)

## # A tibble: 0 x 2
## # i 2 variables: ID <dbl>, n <int>
```

Handling Missing Values

Calculating how many missing values this data set contains:

```
sum(is.na(wine))
```

```
## [1] 38
```

Investigating which rows contain missing values:

```
wine |>
  filter(if_any(everything(), is.na))

## # A tibble: 34 x 14
##       ID type fixed_acidity volatile_acidity citric_acid residual_sugar
##   <dbl> <chr>      <dbl>          <dbl>        <dbl>            <dbl>
## 1     18 white        NA          0.66         0.48            1.2
## 2     34 white        6.2          0.12         0.34            NA
## 3     55 white        6.8          0.2          0.59            0.9
## 4     87 white        7.2          NA          0.63            11
## 5     99 white        9.8          0.36         0.46           10.5
## 6    140 white        8.1          0.28         0.39            1.9
## 7    175 white        NA          0.27         0.31           17.7
## 8    225 white        6.3          0.495        0.22            1.8
## 9    250 white        NA          0.41         0.14           10.4
## 10   268 white        NA          0.58         0.07            6.9
## # i 24 more rows
## # i 8 more variables: chlorides <dbl>, free_sulfur_dioxide <dbl>,
## #   total_sulfur_dioxide <dbl>, density <dbl>, pH <dbl>, sulphates <dbl>,
## #   alcohol <dbl>, quality <dbl>
```

Investigating which fields contain missing values:

```
colSums(is.na(wine))
```

	ID	type	fixed_acidity
##	0	0	10
##	volatile_acidity	citric_acid	residual_sugar
##	8	3	2
##	chlorides	free_sulfur_dioxide	total_sulfur_dioxide
##	2	0	0
##	density	pH	sulphates
##	0	9	4

```

##           alcohol          quality
##             0                  0

Little's Missing Completely at Random (MCAR) Test:
mcar_test(wine |> dplyr::select(fixed_acidity:quality))

## # A tibble: 1 x 4
##   statistic    df p.value missing.patterns
##       <dbl>  <dbl>    <dbl>            <int>
## 1      117.    107     0.240              11

```

Is removing missing values going to affect my sample size?

```

sum(is.na(wine)) / nrow(wine) * 100

## [1] 0.5848853
# Less than 1% of all observations contain missing values.

```

Performing listwise deletion:

```
wine <- na.omit(wine)
```

Wine Quality

Identifying unique quality scores:

```

unique(wine$quality)

## [1] 6 5 7 8 4 3 9

```

Transforming quality variable to factor and renaming it to `quality_score`:

```

# Creating quality levels
quality_levels <- c("3", "4", "5", "6", "7", "8", "9")

wine <- wine |>
  mutate(quality = factor(quality, levels = quality_levels)) |>
  rename(quality_score = quality)

rm(quality_levels)

```

Creating colour scheme for quality scores:

```

clr_score <- c(
  "3" = "#8B0000",
  "4" = "#B22222",
  "5" = "#FF8C00",
  "6" = "#FFBF00",
  "7" = "#9ACD32",
  "8" = "#32CD32",
  "9" = "#008000"
)

```

Exploring distribution of wine quality scores:

```

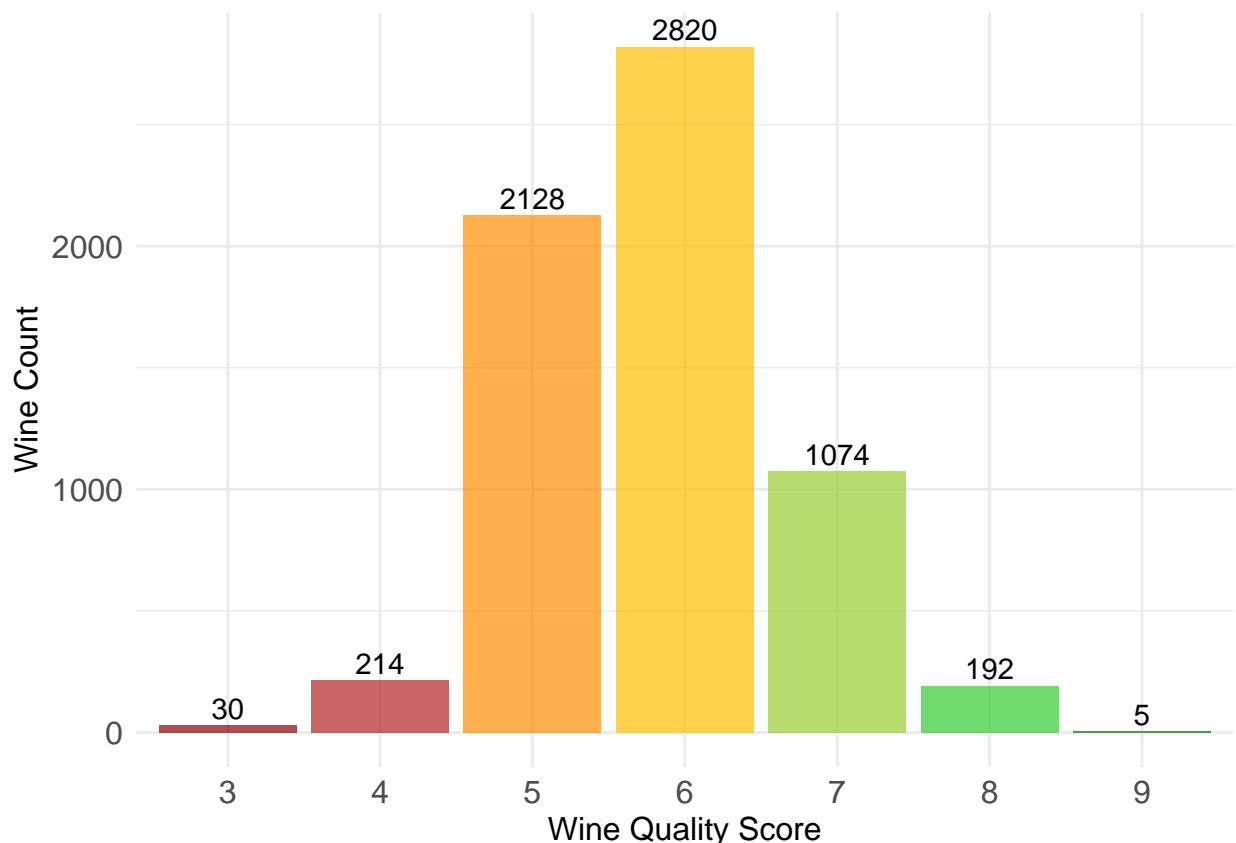
wine |>
  ggplot(aes(x = quality_score)) +
  geom_bar(
    aes(fill = quality_score),

```

```

alpha = 0.7,
show.legend = FALSE
) +
geom_text(
  stat = "count",
  aes(label = after_stat(count)),
  vjust = -0.3
) +
theme_minimal() +
labs(
  x = "Wine Quality Score",
  y = "Wine Count"
) +
scale_fill_manual(
  values = clr_score
) +
theme(
  axis.title.x = element_text(size = 12),
  axis.title.y = element_text(size = 12),
  axis.text    = element_text(size = 12)
)

```



Creating quality groups:

```

wine <- wine |>
  mutate(
    quality_category = case_when(

```

```

    quality_score %in% c("3", "4", "5") ~ "bad",
    quality_score %in% c("7", "8", "9") ~ "good",
    quality_score == "6" ~ "average",
),
quality_category = factor(
  quality_category,
  levels = c("bad", "average", "good")
)
)

```

Creating colour scheme for quality groups:

```

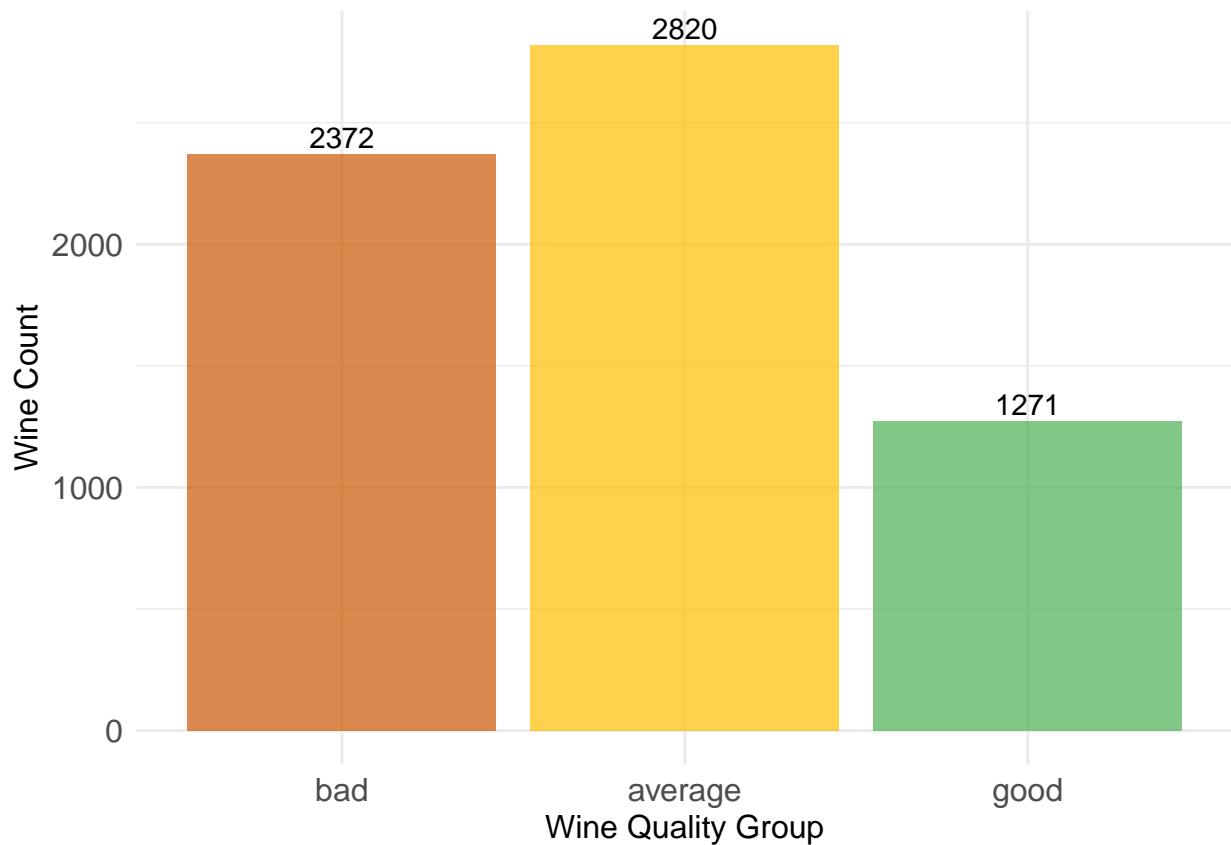
clr_category <- c(
  "bad"      = "#CC5500",
  "average"  = "#FFBF00",
  "good"     = "#4CAF50"
)

```

Visualising new quality categories:

```

wine |>
  ggplot(aes(x = quality_category)) +
  geom_bar(
    aes(fill = quality_category),
    alpha = 0.7,
    show.legend = FALSE
  ) +
  geom_text(
    stat = "count",
    aes(label = after_stat(count)),
    vjust = -0.3
  ) +
  theme_minimal() +
  labs(
    x = "Wine Quality Group",
    y = "Wine Count"
  ) +
  scale_fill_manual(
    values = clr_category
  ) +
  theme(
    axis.title.x = element_text(size = 12),
    axis.title.y = element_text(size = 12),
    axis.text    = element_text(size = 12)
  )
)
```



Wine Type

What are the unique types of wines within the data set?

```
unique(wine$type)
```

```
## [1] "white" "red"
```

Transforming type variable into the factor with two levels: “white” and “red”:

```
wine <- wine |>
  mutate(type = factor(type))
```

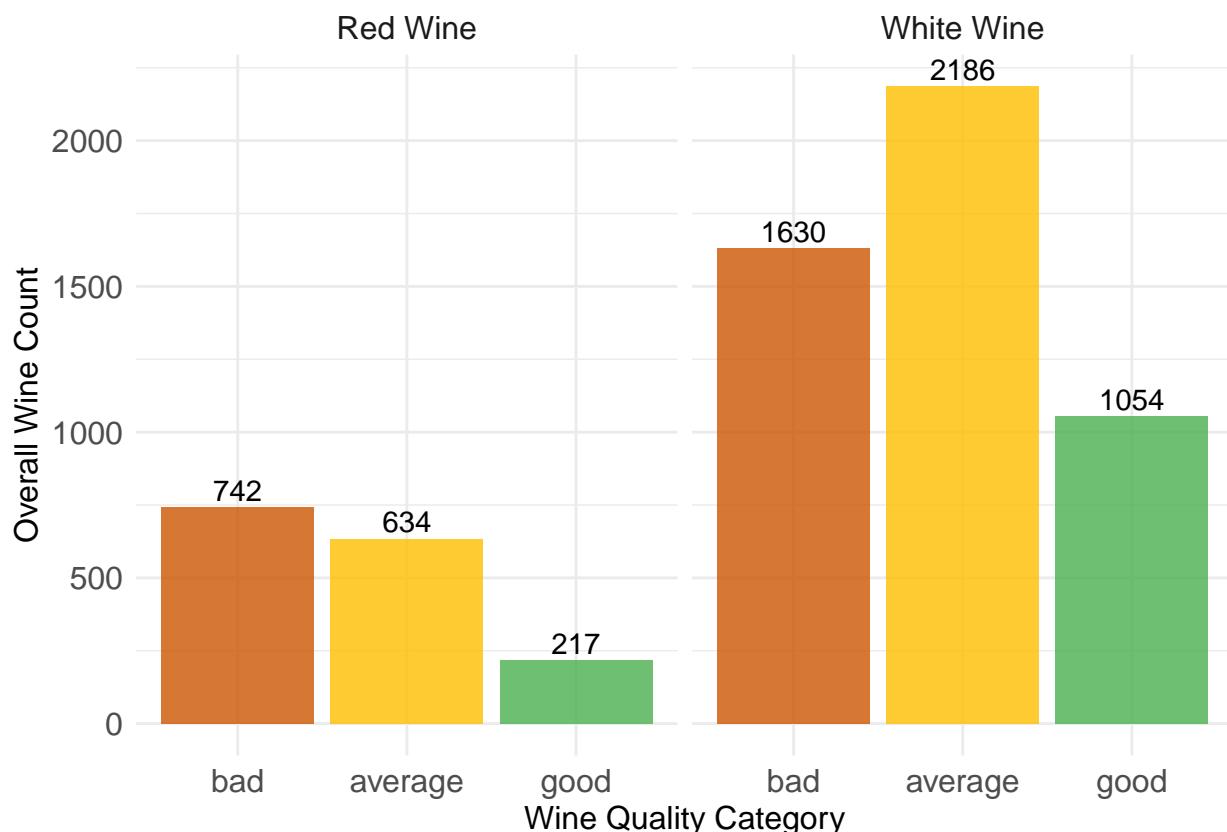
Counting the number of white and red wines within the data set:

```
wine |>
  group_by(type) |>
  summarise(
    wine_count = n(),
    wine_prop = wine_count/6497
  )

## # A tibble: 2 x 3
##   type  wine_count wine_prop
##   <fct>     <int>     <dbl>
## 1 red        1593     0.245
## 2 white      4870     0.750
```

Exploring quality group sizes for red and white wines:

```
wine |>
  mutate(type = if_else(type == "red", "Red Wine", "White Wine")) |>
  ggplot(aes(x = quality_category)) +
  geom_bar(
    aes(fill = quality_category),
    alpha = 0.8,
    show.legend = FALSE
  ) +
  facet_wrap(~type) +
  geom_text(
    stat = "count",
    aes(label = after_stat(count)),
    vjust = -0.3
  ) +
  theme_minimal() +
  scale_fill_manual(values = clr_category) +
  labs(
    x = "Wine Quality Category",
    y = "Overall Wine Count"
  ) +
  theme(
    axis.title.x = element_text(size = 12),
    axis.title.y = element_text(size = 12),
    axis.text = element_text(size = 12),
    strip.text = element_text(size = 12)
  )
)
```



Data Set Split

Splitting data into training and test for red and white wines:

```
wine$split <- interaction(wine$type, wine$quality_score)

set.seed(123)
split <- initial_split(wine, prop = 0.8, strata = split)

train <- training(split)
test <- testing(split)

rm(split)
```

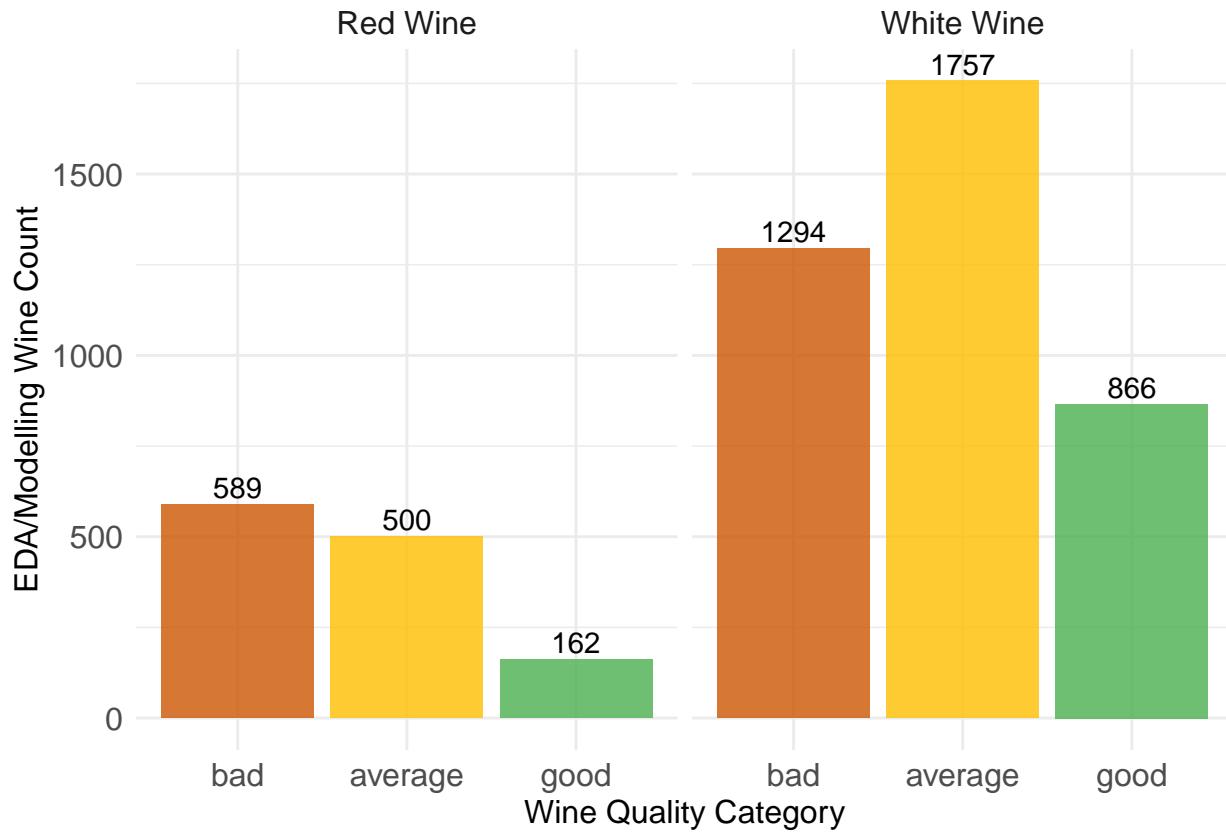
Creating separate data sets for each group:

```
train <- train |>
  dplyr::select(-quality_score, -split)

test <- test |>
  dplyr::select(-quality_score, -split)
```

Visualising final sizes of groups used for EDA/Modelling:

```
train |>
  mutate(type = if_else(type == "red", "Red Wine", "White Wine")) |>
  ggplot(aes(x = quality_category)) +
  geom_bar(
    aes(fill = quality_category),
    alpha = 0.8,
    show.legend = FALSE
  ) +
  facet_wrap(~type) +
  geom_text(
    stat = "count",
    aes(label = after_stat(count)),
    vjust = -0.3
  ) +
  theme_minimal() +
  scale_fill_manual(values = clr_category) +
  labs(
    x = "Wine Quality Category",
    y = "EDA/Modelling Wine Count"
  ) +
  theme(
    axis.title.x = element_text(size = 12),
    axis.title.y = element_text(size = 12),
    axis.text = element_text(size = 12),
    strip.text = element_text(size = 12)
  )
```



Wine Physicochemical Properties

Univariate Analysis

Data set transformation into tidy data, moving all columns that describe the wine's physicochemical properties into one column, with the respective values in another column:

```
tidy_train <- train |>
  pivot_longer(
    cols = fixed_acidity:alcohol,
    names_to = "property_name",
    values_to = "property_value"
  )
```

Renaming each property name:

```
# Creating new property labels:

property_labels <- c(
  alcohol = "Alcohol Percentage by Volume",
  chlorides = "Sodium Chloride (g/L)",
  citric_acid = "Citric Acid (g/L)",
  density = "Density (g/cm3)",
  fixed_acidity = "Fixed Acidity (g/L)",
  free_sulfur_dioxide = "Free Sulphur Dioxide (mg/L)",
  total_sulfur_dioxide = "Total Sulphur Dioxide (mg/L)",
  pH = "pH",
  residual_sugar = "Sugar (g/L)",
```

```

    sulphates = "Potassium Sulphate (g/L)",
    volatile_acidity = "Volatile Acidity (g/L)"
)

# Replacing old property labels with the new ones:
tidy_train <- tidy_train |>
  mutate(property_name = property_labels[property_name])

```

Transforming property_name column into a factor:

```

# Creating property levels:
# (I want them in specific order - e.g., variables that relate to
# acidity grouped together).

```

```

property_order <- c(
  "Volatile Acidity (g/L)",
  "Fixed Acidity (g/L)",
  "Citric Acid (g/L)",
  "pH",
  "Free Sulphur Dioxide (mg/L)",
  "Total Sulphur Dioxide (mg/L)",
  "Potassium Sulphate (g/L)",
  "Sodium Chloride (g/L)",
  "Sugar (g/L)",
  "Density (g/cm3)",
  "Alcohol Percentage by Volume"
)

tidy_train <- tidy_train |>
  mutate(
    property_name = factor(property_name, levels = property_order)
  )

```

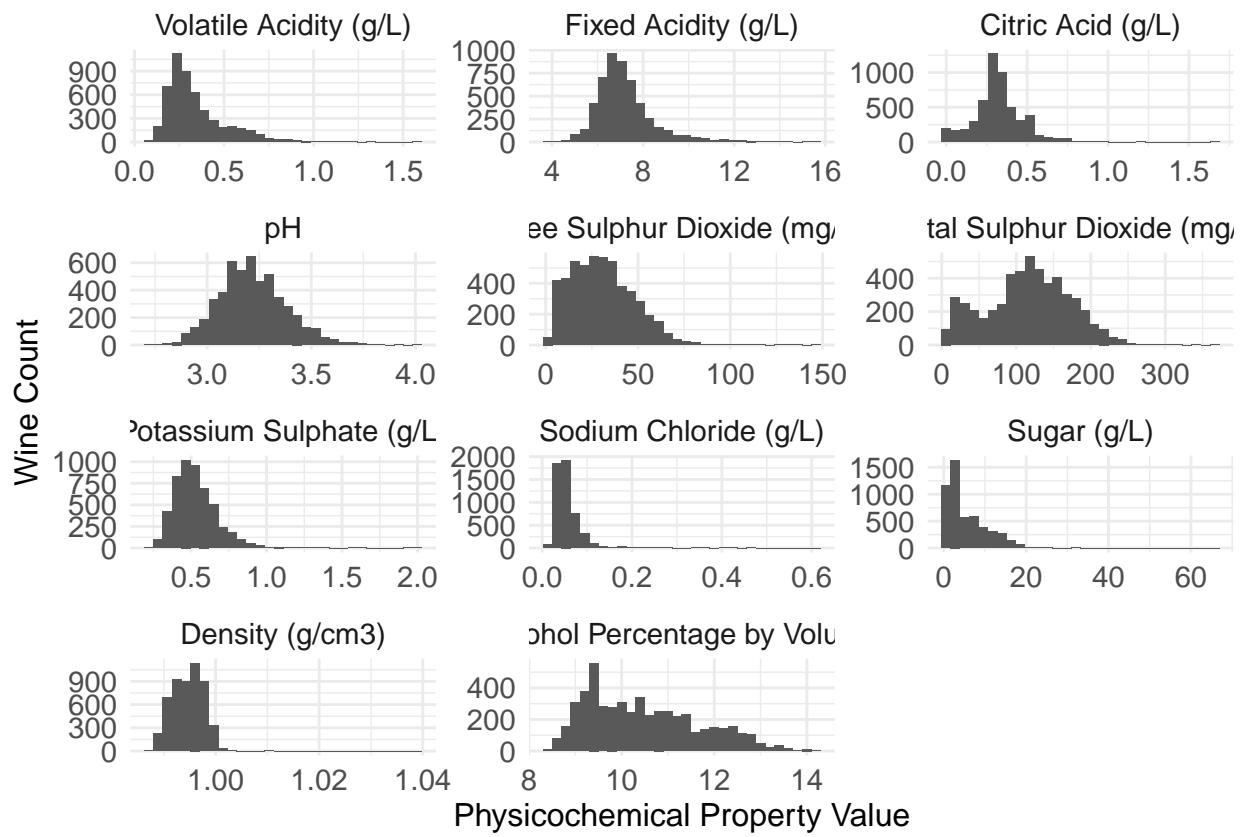
Visualising data distribution for each property (histograms):

```

tidy_train |>
  ggplot(aes(x = property_value)) +
  geom_histogram() +
  facet_wrap(~property_name,
            scales = "free",
            nrow = 4
  ) +
  labs(
    y = "Wine Count",
    x = "Physicochemical Property Value"
  ) +
  theme_minimal() +
  theme(
    axis.title.x = element_text(size = 12),
    axis.title.y = element_text(size = 12),
    axis.text = element_text(size = 11),
    strip.text = element_text(size = 11)
  )

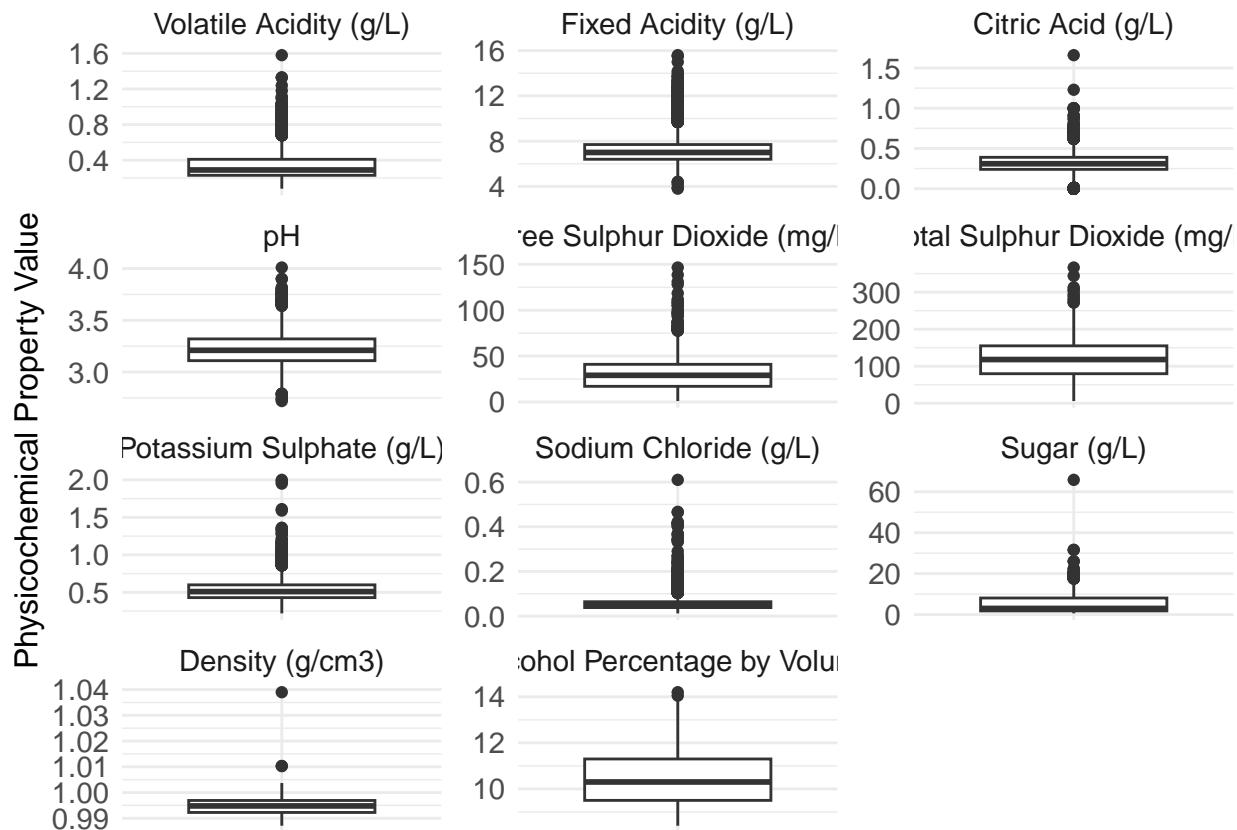
```

```
## `stat_bin()` using `bins = 30`. Pick better value `binwidth`.
```



Visualising data distribution for each property (boxplots):

```
tidy_train |>
  ggplot(aes(x = factor(1), y = property_value)) +
  geom_boxplot(width = 0.7) +
  facet_wrap(~property_name,
             scales = "free_y",
             nrow = 4
  ) +
  labs(
    y = "Physicochemical Property Value"
  ) +
  theme_minimal() +
  theme(
    axis.title.x = element_blank(),
    axis.title.y = element_text(size = 12),
    axis.text.y = element_text(size = 11),
    axis.text.x = element_blank(),
    strip.text = element_text(size = 11)
  )
```



Capping Outliers

I capped outliers for red and white wines separately:

```
ws_train <- train |>
  group_by(type) |>
  mutate(across(
    fixed_acidity:alcohol,
    ~Winsorize(., val = quantile(., probs = c(0.05, 0.95)))
  )) |>
  ungroup()
```

Creating a tidy version of the winsorized data set:

```
ws_tidy_train <- ws_train |>
  pivot_longer(
    cols = fixed_acidity:alcohol,
    names_to = "property_name",
    values_to = "property_value"
  )

ws_tidy_train <- ws_tidy_train |>
  mutate(property_name = property_labels[property_name])

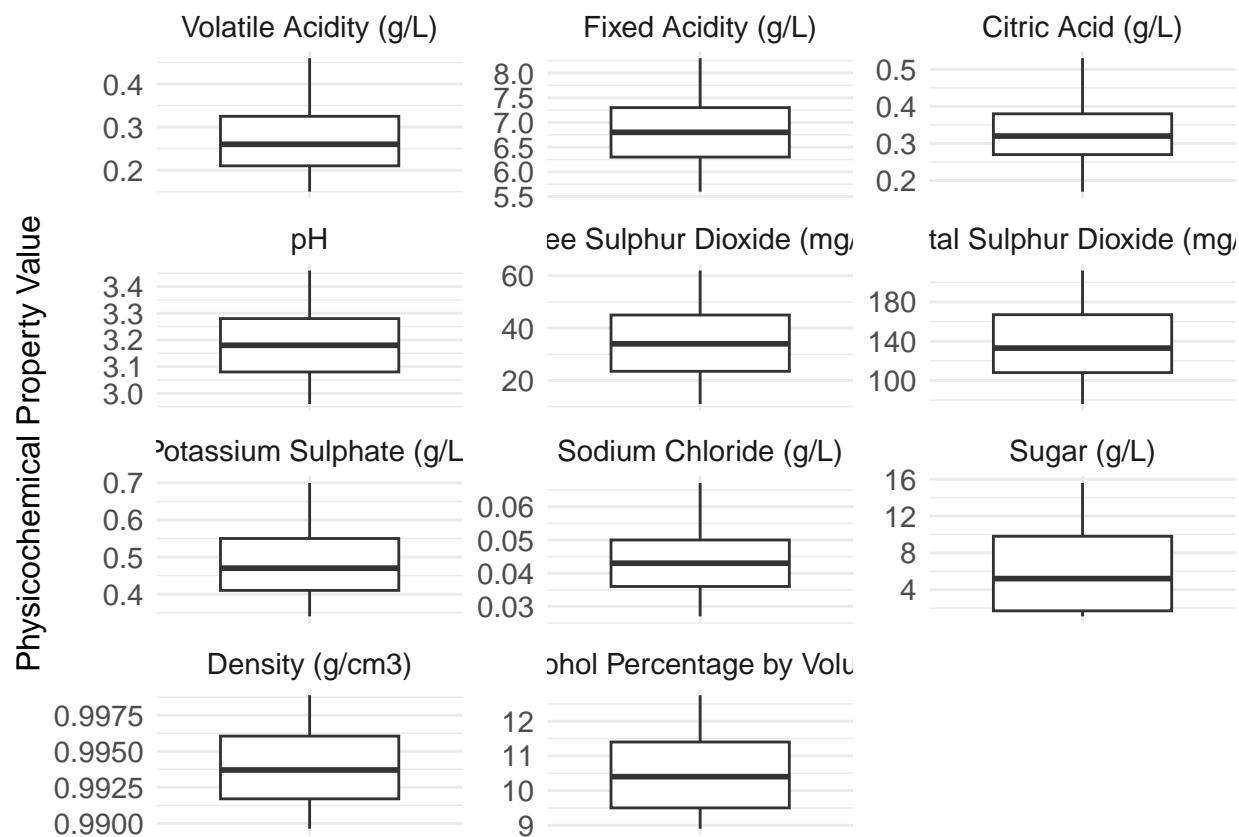
ws_tidy_train <- ws_tidy_train |>
  mutate(
```

```
    property_name = factor(property_name, levels = property_order)
)
```

Investigating how winsorization affected outliers:

```
ws_tidy_train |>

  # Select which group of wine - "red" or "white" - to investigate:
  filter(type == "white") |>
  ggplot(aes(x = factor(1), y = property_value)) +
  geom_boxplot(width = 0.7) +
  facet_wrap(
    ~property_name,
    scales = "free_y",
    nrow = 4
  ) +
  labs(
    y = "Physicochemical Property Value"
  ) +
  theme_minimal() +
  theme(
    axis.title.x = element_blank(),
    axis.title.y = element_text(size = 12),
    axis.text.y = element_text(size = 11),
    axis.text.x = element_blank(),
    strip.text = element_text(size = 11)
)
```



Multivariate Analysis

```
# Select if you want to visualise tidy data set (i.e., "tidy_train")
# or winsorized data set (i.e., "ws_tidy_train")

tidy_train |>

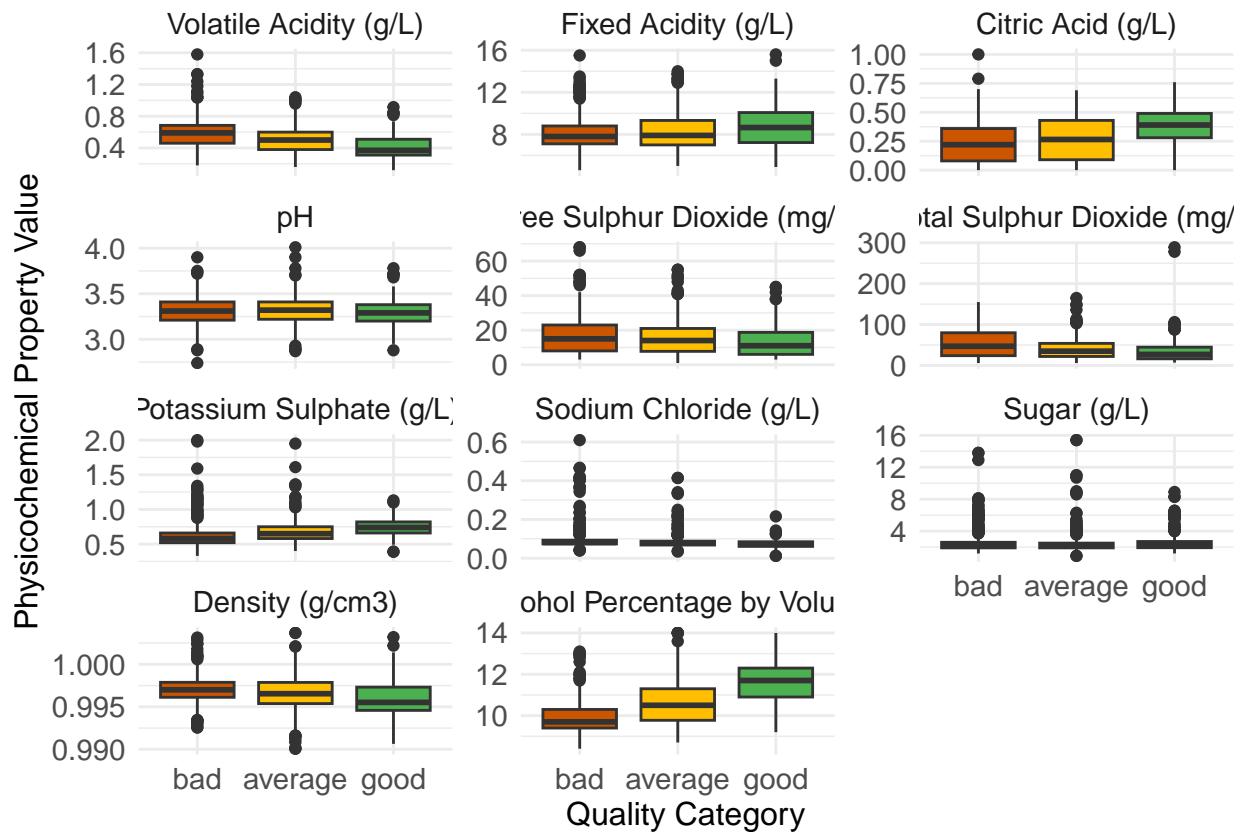
  # Select which group of wine - "red" or "white" - to investigate:
  filter(type == "red") |>

    ggplot(aes(x = quality_category, y = property_value)) +
    geom_boxplot(
      aes(fill = quality_category),
      show.legend = FALSE
    ) +
    facet_wrap(~property_name,
              scales = "free_y",
              nrow = 4
    ) +
    labs(
      y = "Physicochemical Property Value",
      x = "Quality Category"
    ) +
    theme_minimal() +
```

```

scale_fill_manual(values = clr_category) +
theme(
  axis.title.x = element_text(size = 12),
  axis.title.y = element_text(size = 12),
  axis.text    = element_text(size = 11),
  strip.text   = element_text(size = 11)
)

```



Checking for multicollinearity using Variance Inflation Factor (VIF):

```

lm_model <- lm(
  fixed_acidity ~ .,
  
  # Select if you want to visualise regular or winsorized data set
  # And which group of wine - "red" or "white" - to investigate:
  data = dplyr::select(ws_train |> filter(type == "white"), fixed_acidity:alcohol)
)

```

```

vif_values <- vif(lm_model)
vif_values

```

##	volatile_acidity	citric_acid	residual_sugar
##	1.118959	1.140817	7.156704
##	chlorides	free_sulfur_dioxide	total_sulfur_dioxide
##	1.514900	1.837052	2.336314
##	density	pH	sulphates
##	16.181059	1.220517	1.100082

```

##           alcohol
##      5.784256
rm(list = c("lm_model", "vif_values"))

```

Wine Type Selection

To avoid code repetition select which group of wine – “red” or “white” – to investigate further:

```

# input either "red" or "white"

tp_wine <- "red"

```

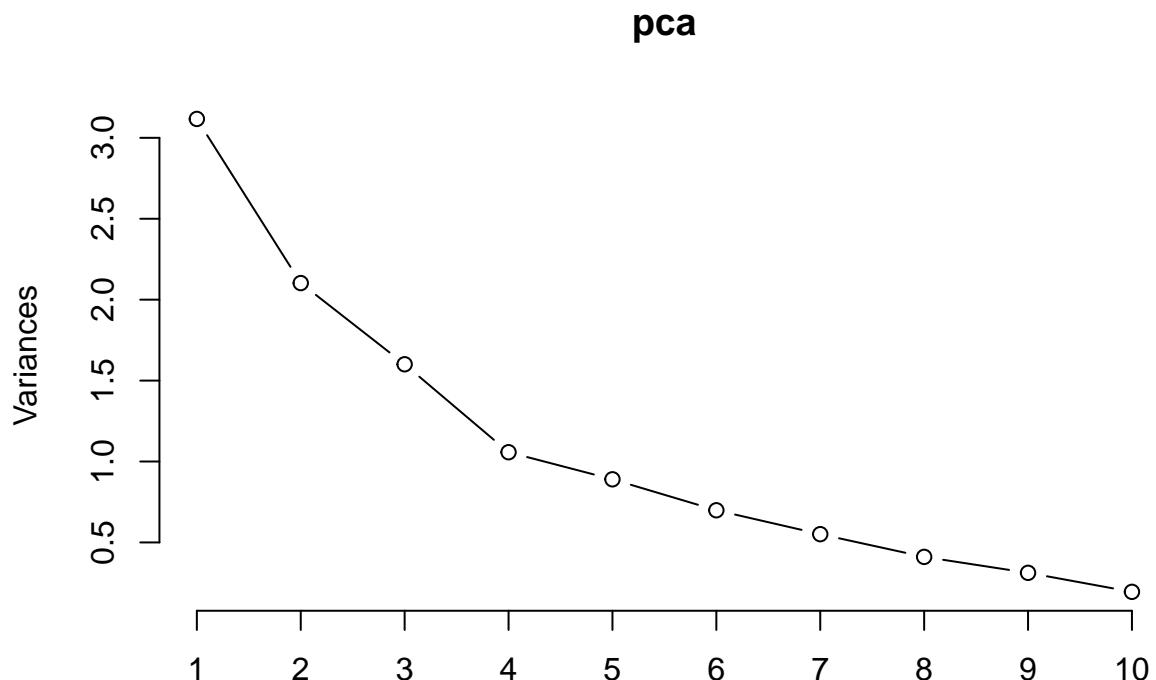
The code below first runs principal component analysis and then fits ordinal logistic regression on the specified wine type.

Principal Component Analysis

```

pca <- prcomp(
  dplyr::select(ws_train |> filter(type == tp_wine), fixed_acidity:alcohol),
  scale. = TRUE
)
plot(pca, type = 'l')

```



```

summary(pca)

## Importance of components:
##          PC1    PC2    PC3    PC4    PC5    PC6    PC7
## Standard deviation   1.7654 1.4500 1.2652 1.02831 0.94342 0.83587 0.7423
## Proportion of Variance 0.2833 0.1911 0.1455 0.09613 0.08091 0.06352 0.0501
## Cumulative Proportion 0.2833 0.4745 0.6200 0.71613 0.79705 0.86056 0.9107
##          PC8    PC9    PC10   PC11
## Standard deviation   0.64085 0.55875 0.4413 0.25524
## Proportion of Variance 0.03734 0.02838 0.0177 0.00592
## Cumulative Proportion 0.94800 0.97638 0.9941 1.00000

loadings <-
  pca$rotation |>
  as.data.frame() |>
  rownames_to_column("property_name")

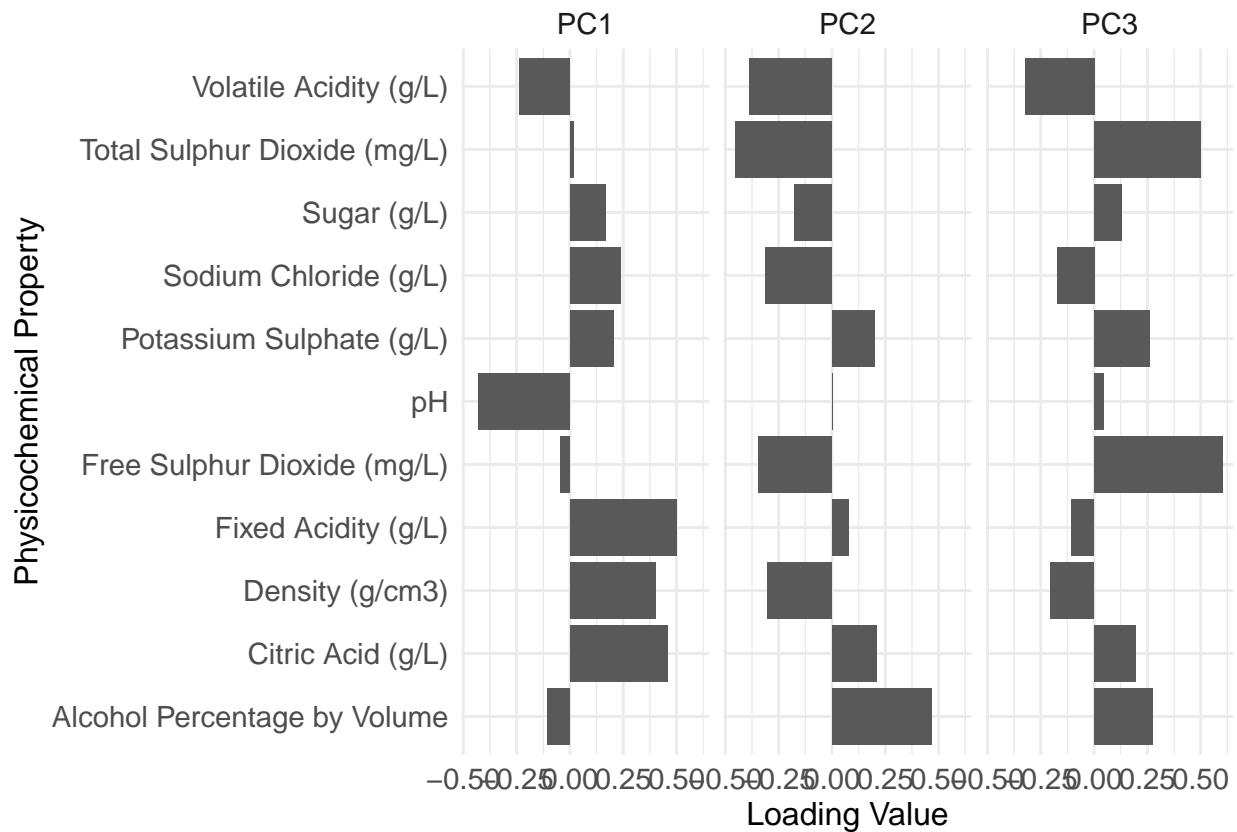
```

Investigating the first 3 principal components:

```

loadings |>
  pivot_longer(
    cols = PC1:PC11,
    names_to = "PC_number",
    values_to = "PC_value"
  ) |>
  mutate(property_name = property_labels[property_name]) |>
  filter(PC_number %in% c("PC1", "PC2", "PC3")) |>
  ggplot(aes(y = PC_value, x = property_name)) +
  geom_col() +
  coord_flip() +
  theme_minimal() +
  facet_wrap(~PC_number) +
  labs(
    y = "Loading Value",
    x = "Physicochemical Property"
  ) +
  theme(
    axis.title.x = element_text(size = 12),
    axis.title.y = element_text(size = 12),
    axis.text = element_text(size = 11),
    strip.text = element_text(size = 11)
  )

```



```
scores_train <-
  pca$x |>
  as.data.frame() |>
  cbind(train |> filter(type == tp_wine))
```

Ordinal Logistic Regression

Building Models

```
model = polr(
  quality_category ~ PC1 + PC2 + PC3 ,
  data = scores_train,
  Hess = TRUE
)

summary(model)

## Call:
## polr(formula = quality_category ~ PC1 + PC2 + PC3, data = scores_train,
##       Hess = TRUE)
##
## Coefficients:
##             Value Std. Error t value
## PC1 0.1114    0.03282   3.396
## PC2 0.8620    0.04908  17.566
## PC3 0.4147    0.04834   8.580
```

```

## 
## Intercepts:
##           Value   Std. Error t value
## bad|average -0.2315  0.0661   -3.5039
## average|good  2.5284  0.1068   23.6638
##
## Residual Deviance: 1999.315
## AIC: 2009.315

Estimating OR and confidence intervals:
exp(cbind(OR = coef(model), confint(model)))

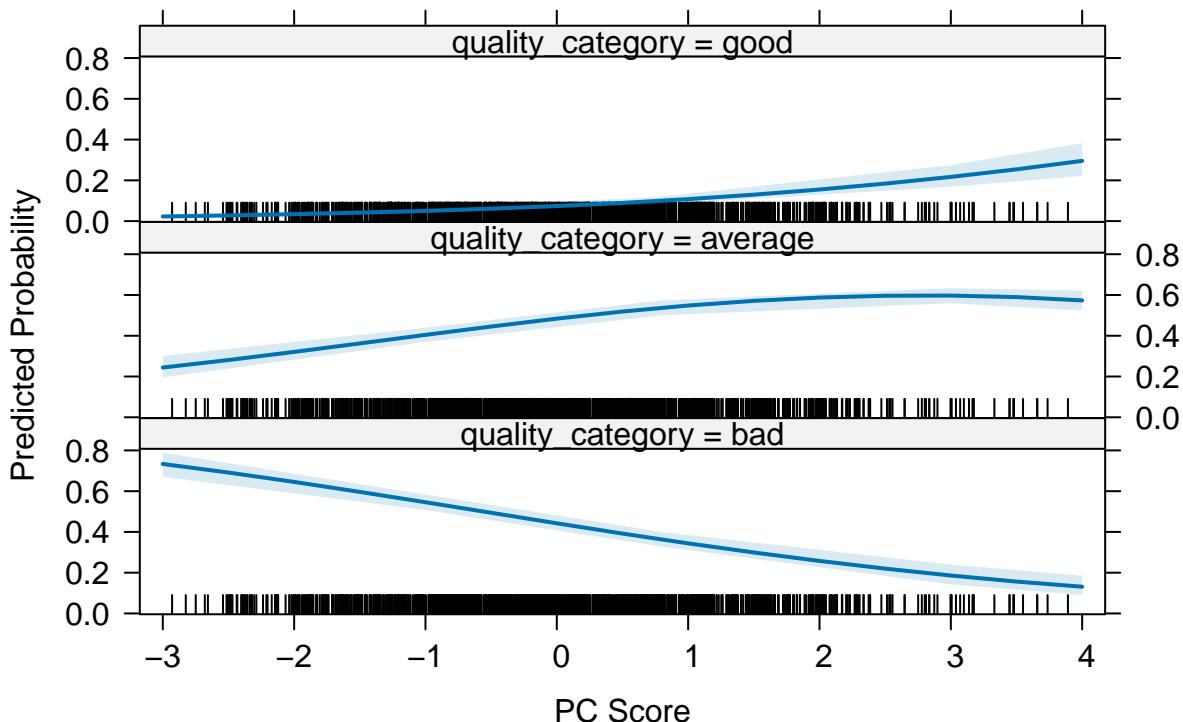
## Waiting for profiling to be done...

##          OR      2.5 %    97.5 %
## PC1 1.117877 1.048332 1.192310
## PC2 2.367988 2.154051 2.611206
## PC3 1.513991 1.377936 1.665568

plot(
  Effect(focal.predictors = "PC3", model),
  xlab = "PC Score",
  ylab = "Predicted Probability"
)

```

PC3 effect plot



Testing Models

Calculating 5th and 95th percentile cutoffs:

```
winsor_cutoffs <- apply(
  dplyr::select(train |> filter(type == tp_wine), fixed_acidity:alcohol),
  2,
  function(x) quantile(x, probs = c(0.05, 0.95))
)
```

Applying winsorization to the test data:

```
ws_test <- test |>
  dplyr::select(fixed_acidity:alcohol) |>
  mutate(across(
    fixed_acidity:alcohol,
    ~pmin(
      pmax(
        .x,
        winsor_cutoffs[1, cur_column()]
      ),
      winsor_cutoffs[2, cur_column()]
    )))

```

Transforming the test data using PCA:

```
scores_test <- as.data.frame(predict(pca, newdata = ws_test)[, 1:3])

scores_test$quality_category <- test$quality_category
```

Testing model predictions:

```
pred_test <- predict(model, newdata = scores_test, type = "class")

table(pred_test, scores_test$quality_category)

##
## pred_test bad average good
##   bad     257     151    31
##   average 230     398   183
##   good     2      14    29

mean(pred_test == scores_test$quality_category)

## [1] 0.5281853
```