POLYNÓMY.

$a_n \neq 0$ (1)

$P_n(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, kde $a_i \in R$ $\boxed{\cancel{\text{alebo } a_i \in C}}$ nebudeme brať teraz

a_i koeficienty polynómu

n – je stupeň polynómu

polynóm n-tého stupňa

$C = \{a+ib;\ a,b \in R;\ i$ je imag. jednotka$\}$

Def. $i^2 = -1$

$i = \sqrt{-1}$

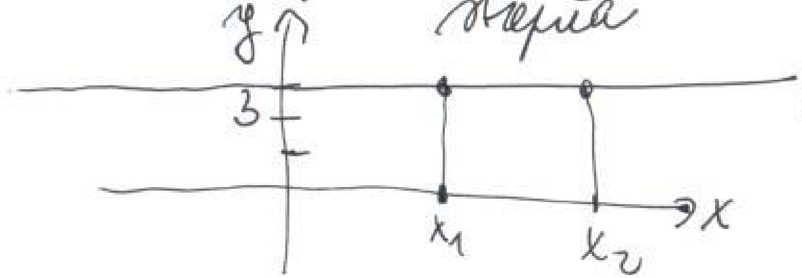
$R \subset C$

Príklady:

$P_0(x) = \underline{\underline{3}}$ $\boxed{a_0 = 3}$ $\boxed{a_i = 0}$ $\forall i$

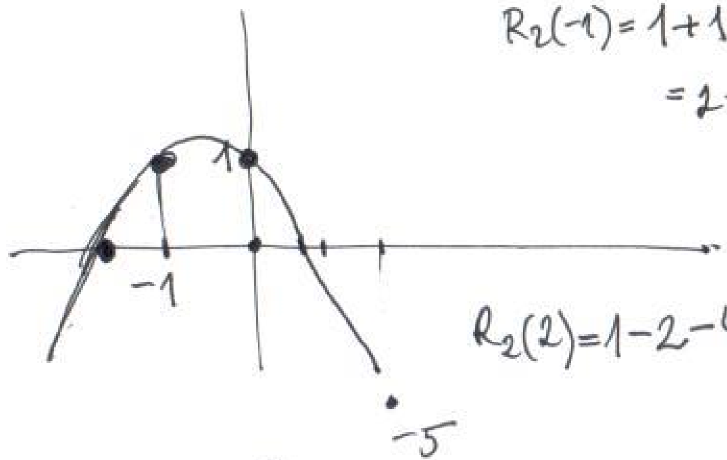
polynóm nultého stupňa

je funkcia premennej x

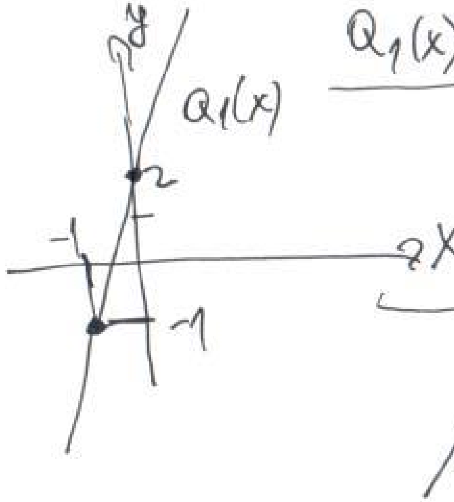


$R_2(x) = 1 - x - x^2$ $\quad a_0 = 1;\ a_1 = -1;\ a_2 = -1;$

$R_2(-1) = 1 + 1 - (1)^2 =$
$= 2 - 1 = 1$



$R_2(2) = 1 - 2 - 4 = -5$

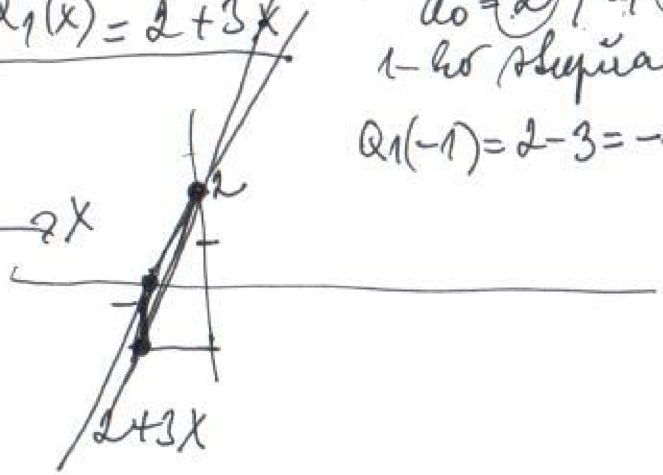


$Q_1(x) = 2 + 3x$

$a_0 = 2;\ a_1 = 3;$

1-ho stupňa

$Q_1(-1) = 2 - 3 = -1$



$R_2(x) = -x^2 - x + 1$

$x_{1,2} = \frac{1 \pm \sqrt{1+4}}{-2} = \frac{1 \pm \sqrt{5}}{-2} \approx$ $\approx -1{,}6$ / $\approx 0{,}6$

polynóm n-tého stupňa

(n+1) - údajov

$P_n(x) = a_0 + a_1 x + \dots + a_n x^n$, $a_n \neq 0$

$x \xmapsto{(n+1)} P_n(x)$ $\boxed{\text{evaluácia polynómu}}$

(n+1) bodov
a v nich hodnoty
a chceme určiť polynóm, ktorý je nimi daný

$P_n(x) = 0$;
x - koreň polynómu

$P_3(x) = 6x - 5x^2 + x^3 = x(6 - 5x + x^2) = x(x-2)(x-3)$

$P_3(0) = 0$

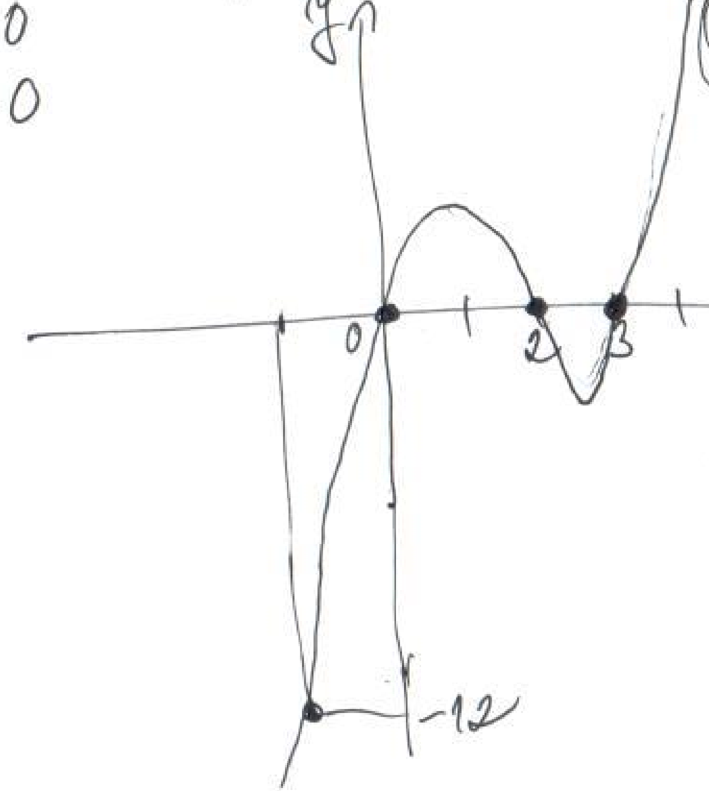
$P_3(2) = 0$

$P_3(3) = 0$

$\boxed{[0; 0]; [2; 0]; [3; 0]}$

$P_3(-1) = (-1)(-3)(-4) = -12$

$[-1; -12]$



$P_3(x)$ $\boxed{P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3}$

$\boxed{\text{interpolácia}}$

4 rovnice o 4 neznámych

(1) $a_0 \neq 0$

(2) $a_0 + a_1 \cdot 2 + a_2 4 + a_3 \cdot 8 = 0$

(3) $a_0 + 3a_1 + a_2 \cdot 9 + a_3 \cdot 27 = 0$

(4) $a_0 - a_1 + a_2 - a_3 = -12$

$a_0 = 0$; $a_1 = 6$; $a_2 = -5$; $a_3 = 1$;

(1)

$$P_4(x) = x^4 + 4x^3 + 3x^2 - 4x - 4$$

$$P_4(x) = a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4$$

$$P_4(x) = x\Big(x\big(x(x+4)+3\big)-4\Big)-4$$

$$a_0(x-x_1)(x-x_2)(x-x_3)(x-x_4)$$

ale $x_i \in R$

$b_0 = 1 \quad (b_0 = a_0)$

$b_1 = b_0x + 4 \quad (b_0 \cdot x + a_1)$

$b_2 = x \cdot b_1 + 3 \quad (b_1x + a_2)$

$b_3 = x \cdot b_2 - 4 \quad (b_2x + a_3)$

$b_4 = x\, b_3 - 4 \quad (b_3x + a_4)$

$\underline{b_4 = P_4(x)}$

	a_0	a_1	a_2	a_3	a_4
x		xa_0	xb_1	xb_2	xb_3
	a_0	$\underbrace{a_1+xa_0}_{b_1}$	$\underbrace{xb_1+a_2}_{b_2}$	$\underbrace{xb_2+a_3}_{b_3}$	$\underbrace{xb_3+a_4}_{b_4}$

HORNEROVA SCHÉMA

	1	4	3	−4	−4	
1		1	5	8	4	
	1	5	8	4	0	$= P_4(1)$
−1		−1	−4	−4		
	1	4	4	0		

$P_4(1) = 0$

je kořenem polynomu $P_4(x)$

$P_4(-1) = 0$

$$P_4(x) = (x-1)(\underbrace{x^3+5x^2+8x+4}) =$$

$$= (x-1)(x+1)(x^2+4x+4) =$$

$$= (x-1)(x+1)(x+2)^2 =$$

$$= (x-1)(x+1)(x+2)(x+2)$$

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$$P_4(x) = x^4 + 4x^3 + 3x^2 - 4x - 4 = (x-1)(b_0x^3 + b_1x^2 + b_2x + b_3)$$

$$(x^4 + 4x^3 + 3x^2 - 4x - 4) : (x-1) = \underline{\underline{x^3 + 5x^2 + 8x + 4}}$$

$$\begin{array}{l}
\underline{-\cancel{x^4} + x^3} \\
\quad \cancel{5x^3} + 3x^2 - 4x - 4 \\
\quad \underline{-\cancel{5x^3} + 5x^2} \\
\qquad \cancel{8x^2} - 4x - 4 \\
\qquad \underline{-\cancel{8x^2} + 8x} \\
\qquad\quad \cancel{4x} - \cancel{4} \\
\qquad\quad \underline{-\cancel{4x} + \cancel{4}} \\
\qquad\qquad \cancel{0}
\end{array}$$

$$(x^3 + 5x^2 + 8x + 4) : (x+2) = x^2 + 3x + 2$$

$$\begin{array}{l}
\underline{-\cancel{x^3} \mp 2x^2} \\
\quad 3\cancel{x^2} + 8x + 4 \\
\quad \underline{-\cancel{3x^2} \mp 6x} \\
\qquad \cancel{2x} + \cancel{4} \\
\qquad \underline{-\cancel{2x} \mp \cancel{4}} \\
\qquad\quad \cancel{0}
\end{array}$$

$$(x^3 + 5x^2 + 8x + 4) = (x^2 + 3x + 2)(x+2)$$

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$P_3(x) = 6x - 5x^2 + x^3$; vidite či $x=2$ je korenina!

	1	-5	6	0
2		2	-6	0
	$\underline{\underline{1}}$	$\underline{\underline{-3}}$	$\cancel{0}$	$\cancel{0}$

$$(x^3 - 5x^2 + 6x) = (x-2)(\underline{\underline{x^2}} - \underline{\underline{3}}x \overset{+0}{}) = x(x-2)(x-3)$$

$$(x^3 - 5x^2 + 6x) : (x-2) = x^2 - 3x$$
$$\underline{-\cancel{x^3} + 2x^2}$$
$$-3\cancel{x^2} + 6x$$
$$\underline{+\cancel{3x^2} \mp 6x}$$
$$\cancel{0}$$

KANONICKÝ ROZKLAD POLYNÓMU

$$a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n = a_n (x - x_1)^{\alpha_1} (x - x_2)^{\alpha_2} \dots (x - x_k)^{\alpha_k} \cdot (x^2 + p_1 x + q_1)^{\beta_1} \cdot$$

$$\dots (x^2 + p_s x + q_s)^{\beta_s}$$

α_i, β_i – násobnosti koreňov $\qquad p_i^2 - 4q_i < 0, \forall i$

$$\alpha_1 + \alpha_2 + \dots + \alpha_k + 2(\beta_1 + \beta_2 + \dots \beta_s) = n$$

PRÍKLAD

$$P_{18}(x) = (x-2)^2 (x+1)^2 (x^2 + x + 1)^5 (x^2 + 2x + 3)(x^2 + 2x + 3)$$

$x = 2$ je dvojnás. koreň $\qquad 1 - 4 = -3 < 0$

$x = -1$ je dvoj. koreň $\qquad$ združené kompl. korene

$$2 + 2 + 2(5+2) = 4 + 14 = 18$$

$$Q_{19}(x) = (x-2)^2 (x+1)^2 (x^2 + x + 1)^5 (x^2 + 2x + 7)^2 (x - 5)$$

$$2 + 2 + 1 + 2(5+2) = 19$$

Primjer.

$$\underbrace{p(x) = (x+1)(x-1)(x^2+x+1)(x^2+2x+5)}$$

kanonski rastav polinoma

$\boxed{x^2+x+1=0}$

$$x_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

$x^2+2x+5=0$

$$x_{1,2} = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm 4i}{2} = \underline{-1 \pm 2i}$$

$\underline{x_1=-1}$

$\underline{x_2=1}$

$(x+1)(x+1)(x+1)$

$$q(x) = (x+1)^{3}(x-1)^{1}(x^2+x+1)^{3}$$

1 korijen jednostruk

korijen $x=-1$ je trostruk

stupanj $q(x)$ $3+1+2\cdot 3 = \underline{\underline{10}}$

$$(x^2+x+1)(x^2+x+1)(x^2+x+1)$$

x^6

$$(x^2)^k = \underline{\underline{x^{2k}}}$$

Hľadanie racionálnych koreňov polynómov.

Veta.

Nech $\frac{r}{s}$, kde $r \in \mathbb{Z}$, $s \in \mathbb{N}$ je koreňom polynómu $a_0 + a_1x + \dots + a_nx^n$,
$a_i \in \mathbb{Z}$, $a_n \neq 0$

Potom platí: $r \mid a_0 \wedge s \mid a_n$

Naviac platí: $\forall m \in \mathbb{Z} \quad (r - ms) \mid f(m)$

Špeciálne pre $m = 1 \quad (r - s) \mid f(1)$

$m = -1 \quad (r + s) \mid f(-1)$

Príklad.

Nájdite všetky racionálne korene polynómu

$$f_3(x) = 4x^3 - 8x^2 - 11x - 3$$

$\frac{r}{s}$, $r \in \mathbb{Z}$, $s \in \mathbb{N}$ $\qquad r \mid -3$, $s \mid 4$ $\qquad a_0 = -3$, $r \in \mathbb{Z}$, $s \in \mathbb{N}$ $\qquad r \in \{-3, -1, 1, 3\}$, $s \in \{1, 2, 4\}$

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$p_3(x) = 4x^3 - 8x^2 - 11x - 3$

$r \in \{-3, -1, 1, 3\}$

$s \in \{1, 2, 4\}$

$r+s \mid f(-1) = -4 - 8 + 11 - 3 = -4$

$r-s \mid f(1) = 4 - 8 - 11 - 3 = -18$

$\frac{r}{s}$	$\frac{-3}{1}$	$\frac{-3}{2}$	$\frac{-3}{4}$	$\frac{-1}{1}$	$\frac{-1}{2}$	$\frac{-1}{4}$	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{4}$	
$r+s$	-2	-1	1	$\cancel{0}$	1	$\cancel{3}$	2	$\cancel{3}$	$\cancel{5}$	4	$\cancel{5}$	$\cancel{7}$	-4
$r-s$	$\cancel{4}$	$\cancel{5}$	$\cancel{7}$	$\cancel{2}$	-3	$\cancel{5}$	$\cancel{0}$	$\cancel{1}$	$\cancel{3}$	2	$\cancel{1}$	$\cancel{1}$	-18

$x_1 = -\frac{1}{2}$ i $x_2 = 3$? su to korijeni

	4	−8	−11	−3
$-\frac{1}{2}$		−2	5	3
	4	−10	−6	0
$-\frac{1}{2}$		−2	6	
3	4	−12	0	
		0		

$x_1 = -\frac{1}{2}$ je korijen

višestrukost mu je dvostruka?

$(4x^3 - 8x^2 - 11x - 3) = \left(x + \frac{1}{2}\right)^2 \cdot (4x - 12) =$

$= \left(x + \frac{1}{2}\right)^2 4(x - 3)$