Vyriešte rovnice:

1. $\sqrt{y+4}+3\sqrt{y}=7$

   Podmienky $(y+4)>0 \Rightarrow y>-4 \wedge y>0 \Rightarrow y>0$

   $$\left(\sqrt{y+7}+3\sqrt{y}\right)^2=7^2$$

   $$\left(y+7+6\sqrt{(y+7)y}+9y\right)=49$$

   $$6\sqrt{(y+7)y}=49-7-10y$$

   $$6\sqrt{(y+7)y}=42-10y \Rightarrow 42>10y \Rightarrow y<4.2$$

   $$\left(6\sqrt{(y+7)}y\right)^2=(42-10y)^2$$

   $$36y^2+36\bullet 7y=42^2-840y+100y^2$$

   $$64y^2-1092y+1764=0$$

   $$16y^2-273y+441$$

   $$y_1=\frac{273+\sqrt{273^2-4\bullet 16\bullet 441}}{32}\approx 15.3\notin(0,4.2) \quad NIE$$

   $$y_2=\frac{273-\sqrt{273^2-4\bullet 16\bullet 441}}{32}\approx 1.8\in(0,4.2) \quad OK$$

   ----

2. $2\sqrt{x-1}-\sqrt{x+4}=1$

   Podmienky $(x+4)>0 \Rightarrow x>-4 \wedge (x-1)>0 \Rightarrow x>1 \Rightarrow x>1$

   $$\left(2\sqrt{x-1}-\sqrt{x+4}\right)^2=1^2$$

   $$\left(4x-4-4\sqrt{(x+4)(x-1)}+x+4\right)=1$$

   $$5x-4\sqrt{(x+4)(x-1)}=1$$

   $$-4\sqrt{(x+4)(x-1)}=1-5x \Rightarrow 1-5x<0 \Rightarrow x>\frac{1}{5}$$

   $$\left(-4\sqrt{(x+4)(x-1)}\right)^2=(1-5x)^2$$

   $$\left(16x^2+48x-64\right)=1-10x+25x^2$$

   $$9x^2-58x+65=0$$

   $$D=\sqrt{58^2-4\bullet 9\bullet 65}=\sqrt{1024}=32$$

   $$x_1=\frac{58+32}{18}=\frac{90}{18}=5\in(1,\infty) \quad OK$$

   $$x_2=\frac{58-32}{18}=\frac{13}{9}\in(1,\infty) \quad OK$$

   Skúška

$$\left(2\sqrt{x_1-1}-\sqrt{x_1+4}\right)=\left(2\sqrt{5-1}-\sqrt{5+4}\right)=4-3=1$$

$$\left(2\sqrt{x_2-1}-\sqrt{x_2+4}\right)=\left(2\sqrt{\frac{13}{9}-1}-\sqrt{\frac{13}{9}+4}\right)=\left(2\sqrt{\frac{13-9}{9}}-\sqrt{\frac{13+36}{9}}\right)=\frac{4}{3}-\frac{7}{3}=-1$$

Vidíme, že vyhovuje iba x = 5.

----

3. $\sqrt{x+27}=2\sqrt{x}-5$

Podmienky $(x+27)>0 \Rightarrow x>-27 \wedge x>0 \Rightarrow x>0$

$$\left(\sqrt{x+27}-2\sqrt{x}\right)^2=(-5)^2$$

$$x+27-4\sqrt{x^2+27x}+4x=25$$

$$4\sqrt{x^2+27x}=5x+2 \Rightarrow x>-\frac{2}{5}$$

$$\left(4\sqrt{x^2+27x}\right)^2=(5x+2)^2$$

$$16x^2+432x=25x^2+20x+4$$

Podmienky $(x+27)>0 \Rightarrow x>-27 \wedge x>0 \Rightarrow x>0$

$$9x^2-412x+4=0$$

$$D=\sqrt{412^2-4\bullet4\bullet9}=\sqrt{169000}\approx 411$$

$$x_1=\frac{412+D}{18}\approx\frac{823}{18}\approx 45.7$$

$$x_2=\frac{412-D}{18}\approx\frac{1}{18}\approx 0.06$$

Skúška

$$\sqrt{x_1+27}-2\sqrt{x_1}\approx\sqrt{45.7+27}-2\sqrt{45.7}\approx\sqrt{72.7}-2\sqrt{45.7}\approx 8.5-2\bullet6.8\approx -5$$

$$\sqrt{x_2+27}-2\sqrt{x_2}\approx\sqrt{0.05+27}-2\sqrt{0.05}\approx\sqrt{27.05}-2\sqrt{0.05}\approx 5.2-0.4\approx 4.8$$

Vidíme, že vyhovuje iba x = 45.

----

4. $3\sqrt{\frac{x+1}{x-1}}+2\sqrt{\frac{x-1}{x+1}}=7$

Podmienky

$$\frac{x+1}{x-1}>0 \Rightarrow x>1 \vee (x<1 \wedge x<-1) \Rightarrow x>1 \vee x<-1$$

$$\left(3\sqrt{\frac{x+1}{x-1}}+2\sqrt{\frac{x-1}{x+1}}\right)^2=7^2$$
$$9\frac{x+1}{x-1}+4\frac{x-1}{x+1}+12\sqrt{\frac{x+1}{x-1}\frac{x-1}{x+1}}=49$$
$$\frac{9(x+1)^2}{(x+1)(x-1)}+\frac{4(x-1)^2}{(x+1)(x-1)}=49-12$$
$$9x^2+18x+9+4x^2-8x+4=37\left(x^2-1\right)$$
$$13x^2+10x+13=37x^2-37$$
$$24x^2-10x-50=0$$
$$12x^2-5x-25=0$$
$$D=\sqrt{5^2+4.12.25}=35$$
$$x_1\approx\frac{5+35}{24}=\frac{5}{3}\in(1,\infty)\quad OK$$
$$x_2\approx\frac{5-35}{24}=-\frac{5}{4}\in(-\infty,-1)\quad OK$$

Skúška

$$\left(3\sqrt{\frac{x_1+1}{x_1-1}}+2\sqrt{\frac{x_1-1}{x_1+1}}\right)=\left(3\sqrt{\frac{5/3+1}{5/3-1}}+2\sqrt{\frac{5/3-1}{5/3+1}}\right)=3\sqrt{8/2}+2\sqrt{2/8}=3.2+2/2=7\quad OK$$
$$\left(3\sqrt{\frac{x_1+1}{x_1-1}}+2\sqrt{\frac{x_1-1}{x_1+1}}\right)=\left(3\sqrt{\frac{-5/4+1}{-5/4-1}}+2\sqrt{\frac{-5/4-1}{-5/4+1}}\right)=3\sqrt{-1/-9}+2\sqrt{-9/-1}=3/3+2.3=7\quad OK$$

Vidíme, že obe riešenia vyhovujú.

----

5. $4\frac{1}{2}-\frac{1}{2}\sqrt{2x+6}=1\frac{1}{2}$

Podmienka: x>-3

$$4\frac{1}{2}-\frac{1}{2}\sqrt{2x+6}=1\frac{1}{2}$$
$$4\frac{1}{2}-1\frac{1}{2}=\frac{1}{2}\sqrt{2x+6}$$
$$\frac{9-3}{2}=\frac{1}{2}\sqrt{2x+6}$$
$$\sqrt{2x+6}=6$$
$$2x+6=36$$
$$2x=36-6$$
$$x=15$$

Skúška

$$4\frac{1}{2}-\frac{1}{2}\sqrt{2\cdot 15+6}=4.5-0.5\bullet 6=4.5-3=1.5 \quad OK$$

Vidíme, že riešenie vyhovuje.

----

6. $\sqrt{5+x}+\sqrt{5-x}=\sqrt{10}$

   Podmienky

$$5+x>0 \wedge 5-x>0$$
$$x>-5 \wedge x<5$$
$$x\in(-5,5)$$
$$\left(\sqrt{5+x}+\sqrt{5-x}\right)^2=\left(\sqrt{10}\right)^2$$
$$5+x+2\sqrt{\left(25-x^2\right)}+5-x=10$$
$$10+2\sqrt{\left(25-x^2\right)}=10$$
$$\sqrt{\left(25-x^2\right)}=0$$

Vidíme, že riešenie neexistuje.

----

7. $\sqrt{5+x}-\sqrt{x^2-7}=0$

$$\sqrt{5+x}-\sqrt{x^2-7}=0$$
$$\left(\sqrt{5+x}\right)^2=\left(\sqrt{x^2-7}\right)^2$$
$$5+x=x^2-7$$
$$x^2-x-2=0$$
$$x_1=2$$
$$x_2=-1$$

Skúška

$$\sqrt{5+x_1}-\sqrt{x_1^{\,2}-7}=\sqrt{5+2}-\sqrt{4-7}=\sqrt{7}-\sqrt{-3} \quad NIE$$
$$\sqrt{5+x_2}-\sqrt{x_2^{\,2}-7}=\sqrt{5-1}-\sqrt{1-7}=\sqrt{4}-\sqrt{-6} \quad NIE$$

Vidíme, že riešenie neexistuje.

8. $\sqrt{2x-5}-\sqrt{2x+2}=1$

Podmienky

$$2x-5>0 \wedge 2x+2>0$$
$$x>2.5 \wedge x>-1 \Rightarrow x>2.5$$

$$\left(\sqrt{2x-5}-\sqrt{2x+2}\right)^2=(1)^2$$
$$2x-5-2\sqrt{2x-5}\sqrt{2x+2}+2x+2=1$$
$$4x-3-2\sqrt{2x-5}\sqrt{2x+2}=1$$
$$2\sqrt{2x-5}\sqrt{2x+2}=4x-4$$
$$\left(2\sqrt{2x-5}\sqrt{2x+2}\right)^2=(4x-4)^2$$
$$4(2x-5)(2x+2)=16x^2-16x+16$$
$$16x^2-24x-40=16x^2-16x+16$$
$$8x=56$$
$$x=7$$

Skúška

$$\left(\sqrt{2x-5}-\sqrt{2x+2}\right)=\left(\sqrt{14-5}-\sqrt{14+2}\right)=\sqrt{9}-\sqrt{16}=3-4=-1$$

Vidíme, že riešenie neexistuje.

---

9. $-\sqrt{2x-5}+\sqrt{2x+2}=1$

Podmienky

$$2x-5>0 \wedge 2x+2>0$$
$$x>2.5 \wedge x>-1 \Rightarrow x>2.5$$

$$\left(-\sqrt{2x-5}+\sqrt{2x+2}\right)^2=(1)^2$$
$$2x-5-2\sqrt{2x-5}\sqrt{2x+2}+2x+2=1$$
$$4x-3-2\sqrt{2x-5}\sqrt{2x+2}=1$$
$$2\sqrt{2x-5}\sqrt{2x+2}=4x-4$$
$$\left(2\sqrt{2x-5}\sqrt{2x+2}\right)^2=(4x-4)^2$$
$$4(2x-5)(2x+2)=16x^2-16x+16$$
$$16x^2-24x-40=16x^2-16x+16$$
$$8x=56$$
$$x=7$$

Skúška

$$\left(-\sqrt{2x-5}+\sqrt{2x+2}\right)=\left(-\sqrt{14-5}+\sqrt{14+2}\right)\ =-\sqrt{9}+\sqrt{16}=-3+4=1$$

Vidíme, že riešenie x=7 vyhovuje.

----

10. $\sqrt{2x-5}+\sqrt{2x+2}=1$

Podmienky

$$2x-5>0\wedge 2x+2>0$$
$$x>2.5\wedge x>-1\Rightarrow x>2.5$$

$$\left(\sqrt{2x-5}+\sqrt{2x+2}\right)^2=(1)^2$$
$$2x-5+2\sqrt{2x-5}\sqrt{2x+2}+2x+2=1$$
$$4x-3+2\sqrt{2x-5}\sqrt{2x+2}=1$$
$$2\sqrt{2x-5}\sqrt{2x+2}=4-4x$$
$$\left(2\sqrt{2x-5}\sqrt{2x+2}\right)^2=(4-4x)^2$$
$$4(2x-5)(2x+2)=16x^2-16x+16$$
$$16x^2-24x-40=16x^2-16x+16$$
$$8x=56$$
$$x=7$$

Skúška

$$\left(\sqrt{2x-5}+\sqrt{2x+2}\right)=\left(\sqrt{14-5}+\sqrt{14+2}\right)\ =\sqrt{9}+\sqrt{16}=3+4=7$$

Vidíme, že riešenie neexistuje.

----

11. $-\sqrt{2x-5}-\sqrt{2x+2}=1$

$$\left(-\sqrt{2x-5}-\sqrt{2x+2}\right)^2=(1)^2$$

$$2x-5+2\sqrt{2x-5}\sqrt{2x+2}+2x+2=1$$

$$4x-3+2\sqrt{2x-5}\sqrt{2x+2}=1$$

$$2\sqrt{2x-5}\sqrt{2x+2}=4-4x$$

$$\left(2\sqrt{2x-5}\sqrt{2x+2}\right)^2=(4-4x)^2$$

$$4(2x-5)(2x+2)=16x^2-16x+16$$

$$16x^2-24x-40=16x^2-16x+16$$

$$8x=56$$

$$x=7$$

Skúška

$$\left(-\sqrt{2x-5}-\sqrt{2x+2}\right)=\left(-\sqrt{14-5}-\sqrt{14+2}\right)\ =-\sqrt{9}-\sqrt{16}=-3-4=-7$$

Vidíme, že riešenie neexistuje.