Štvrtok 11.00, M.L., 15.4.2021

Odvodzovacie pravidlá Gentzenovho systému logiky

Axiómy

$\overline{A \vdash A}$ (I)

Pravidlo cut

$\dfrac{\Gamma \vdash A, \Delta \qquad \Sigma, A \vdash \Pi}{\Gamma, \Sigma \vdash \Delta, \Pi}$ (cut)

Ľavé logické pravidlá; Pravé logické pravidlá

$\dfrac{\Gamma, A \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}$ ($\wedge L_1$) $\qquad \dfrac{\Gamma \vdash A, \Delta}{\Gamma \vdash A \vee B, \Delta}$ ($\vee R_1$)

$\dfrac{\Gamma, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}$ ($\wedge L_2$) $\qquad \dfrac{\Gamma \vdash B, \Delta}{\Gamma \vdash A \vee B, \Delta}$ ($\vee R_2$)

$\dfrac{\Gamma, A \vdash \Delta \qquad \Sigma, B \vdash \Pi}{\Gamma, \Sigma, A \vee B \vdash \Delta, \Pi}$ ($\vee L$) $\qquad \dfrac{\Gamma \vdash A, \Delta \qquad \Sigma \vdash B, \Pi}{\Gamma, \Sigma \vdash A \wedge B, \Delta, \Pi}$ ($\wedge R$)

rozdeľujúce sa predpoklady (L5 a R5)

$\dfrac{\Gamma \vdash A, \Delta \qquad \Sigma, B \vdash \Pi}{\Gamma, \Sigma, A \Rightarrow B \vdash \Delta, \Pi}$ ($\Rightarrow L$) $\qquad \dfrac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \Rightarrow B, \Delta}$ ($\Rightarrow R$) → pravidlo o dedukcii

$\dfrac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}$ ($\neg L$) $\qquad \dfrac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}$ ($\neg R$)

rovná A

Ľavé štrukturálne pravidlá

$\dfrac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta}$ (WL) → pravidlo oslabenia → vymazanie $\qquad \dfrac{\Gamma \vdash \Delta}{\Gamma \vdash A, \Delta}$ (WR) → pravidlo oslabenia → vymazania

$\dfrac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta}$ (CL) → rozmnoženie, zopakovanie $\qquad \dfrac{\Gamma \vdash A, A, \Delta}{\Gamma \vdash A, \Delta}$ (CR) → nakopírovanie, rozmnoženie

$\dfrac{\Gamma, A, B, \Sigma \vdash \Delta}{\Gamma, B, A, \Sigma \vdash \Delta}$ (PL) → prehodenie premennej $\qquad \dfrac{\Gamma \vdash \Delta, A, B, \Pi}{\Gamma \vdash \Delta, B, A, \Pi}$ (PR) → prehodenie premennej

Cv. 9.1

a) $\vdash A \Rightarrow (B \Rightarrow A)$

axióm (I)
$A \vdash A$ (WL)
$A, B \vdash A$ ($\Rightarrow R$)
$A \vdash B \Rightarrow A$ ($\Rightarrow R$)
$\vdash A \Rightarrow (B \Rightarrow A)$

b) $\vdash (\neg A \Rightarrow \neg B) \Rightarrow ((\neg A \Rightarrow B) \Rightarrow A)$

$A \vdash A$ (I) ($\neg R$)
$\vdash \neg A, A$; $B \vdash B$ (I), $\neg B, B \vdash$ ($\neg L$)
$\neg A \Rightarrow \neg B, B \vdash A$ ($\Rightarrow L$)
$\emptyset, \neg A \Rightarrow \neg B, \neg A \Rightarrow B \vdash A, A$ ($\Rightarrow L$) (CR)
$\neg A \Rightarrow \neg B, \neg A \Rightarrow B \vdash A$ ($\Rightarrow R$)
$(\neg A \Rightarrow \neg B) \vdash (\neg A \Rightarrow B) \Rightarrow A$ ($\Rightarrow R$)
$\vdash (\neg A \Rightarrow \neg B) \Rightarrow [(\neg A \Rightarrow B) \Rightarrow A]$

$\dfrac{\Gamma \vdash A, \Delta \qquad \Sigma, B \vdash \Pi}{\Gamma, \Sigma, A \Rightarrow B \vdash \Delta, \Pi}$

9.2 b) $A \Rightarrow (B \Rightarrow C) \vdash B \Rightarrow (A \Rightarrow C)$

axióm (I) $A \vdash A$; axióm (I) $B \vdash B$; axióm (I) $C \vdash C$
$B, B \Rightarrow C \vdash C$ ($\Rightarrow L$)
$B, A, A \Rightarrow (B \Rightarrow C) \vdash C$ ($\Rightarrow L$)
$A \Rightarrow (B \Rightarrow C), B, A \vdash C$ (PL)
$A \Rightarrow (B \Rightarrow C), B \vdash (A \Rightarrow C)$ ($\Rightarrow R$)
$A \Rightarrow (B \Rightarrow C) \vdash B \Rightarrow (A \Rightarrow C)$ ($\Rightarrow R$)

Cv. 9.3 a)

a) $\vdash (A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$

$A \vdash A$ (I) ($\neg R$) ; $B \vdash B$ (I) ($\neg L$)
$\vdash A, \neg A$; $\neg B, B \vdash$
$\neg B, A \Rightarrow B \vdash \neg A$ ($\Rightarrow L$)
$A \Rightarrow B, \neg B \vdash \neg A$ (PL)
$A \Rightarrow B \vdash \neg B \Rightarrow \neg A$ ($\Rightarrow R$)
$\vdash (A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$ ($\Rightarrow R$)

b) $\vdash (A \Rightarrow \neg B) \Rightarrow (B \Rightarrow \neg A)$

axióm (I) $A \vdash A$ ($\neg R$) ; $B \vdash B$ (I) ($\neg L$)
$\vdash A, \neg A$; $B, \neg B \vdash$
$B, A \Rightarrow \neg B \vdash \neg A$ ($\Rightarrow L$)
$A \Rightarrow \neg B, B \vdash \neg A$ (PL)
$(A \Rightarrow \neg B) \vdash B \Rightarrow \neg A$ ($\Rightarrow R$)
$\vdash (A \Rightarrow \neg B) \Rightarrow (B \Rightarrow \neg A)$ ($\Rightarrow R$)

Cv. 9.6 a)

a) $\vdash (\neg A \Rightarrow B) \Rightarrow [(A \Rightarrow B) \Rightarrow B]$

$A \vdash A$ (I) ($\neg R$) ; $B \vdash B$ (I) ; axióm (I) $B \vdash B$
$\vdash A, \neg A$
$\neg A \Rightarrow B \vdash A, B$ ($\Rightarrow L$)
$\neg A \Rightarrow B, A \Rightarrow B \vdash B, B$ ($\Rightarrow L$)
$\neg A \Rightarrow B, A \Rightarrow B \vdash B$ (CR)
$\neg A \Rightarrow B \vdash [(A \Rightarrow B) \Rightarrow B]$ ($\Rightarrow R$)
$\vdash (\neg A \Rightarrow B) \Rightarrow [(A \Rightarrow B) \Rightarrow B]$ ($\Rightarrow R$)

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predik. Gent. logika

$P(z) \vdash P(z)$ ($\forall L$)
$(\forall x) P(x) \vdash P(z)$ ($\forall R$)
$(\forall x) P(x) \vdash \forall y\, P(y)$

$P(z) \vdash P(z)$ ($\forall L$)
$\forall x\, P x \vdash P(z)$ ($\exists R$)
$\forall x\, P x \vdash \exists x\, P(x)$

$P(z) \vdash P(z)$ ($\exists L$)
$\exists x\, P x \vdash P(z)$ ($\forall R$)
$\exists x\, P x \vdash \forall y\, P(y)$

$P(z) \vdash P(z)$ ($\exists R$)
$P(z) \vdash \exists y\, P y$ ($\exists L$)
$\exists x\, P x \vdash \exists y\, P y$

Poznámka: $\forall R$, $\exists L$ } nová konšt.

$\forall R$, $\exists L$ } nová konšt.

Cv. 9.10 A

a) $\vdash (\exists x)(A \Rightarrow B) \Rightarrow [(\forall x) A \Rightarrow (\exists x) B]$

($\forall L$) aj ($\exists R$) môžeme $t = y_0$

$A_x[y_0] \vdash A_x[y_0]$ (I) ($\forall L$) ; $B_x[y_0] \vdash B_x[y_0]$ (I) ($\exists R$)
$(\forall x) A \vdash A_x[y_0]$; $B_x[y_0] \vdash (\exists x) B$
$(\forall x) A, A_x[y_0] \Rightarrow B_x[y_0] \vdash (\exists x) B$ ($\Rightarrow L$) ($\exists L$)
$(\forall x) A, (\exists x)(A \Rightarrow B) \vdash (\exists x) B$ (PL)
$(\exists x)(A \Rightarrow B), (\forall x) A \vdash (\exists x) B$ ($\Rightarrow R$)
$(\exists x)(A \Rightarrow B) \vdash (\forall x) A \Rightarrow (\exists x) B$ ($\Rightarrow R$)
$\vdash (\exists x)(A \Rightarrow B) \Rightarrow [(\forall x) A \Rightarrow (\exists x) B]$

$(\forall x) A, (\exists x)(A \Rightarrow B) \vdash (\exists x) B$ … $\exists L$

9.8.

a) $\vdash (\forall x) P(x) \Rightarrow (\forall y) P(y)$

(I)
$P(t) \vdash P(t)$ ($\forall L$)
$(\forall x) P x \vdash P(t)$ ($\forall R$)
$(\forall x) P(x) \vdash (\forall y) P(y)$ ($\Rightarrow R$)
$\vdash (\forall x) P(x) \Rightarrow (\forall y)(P y)$

Cv. 9.11 a) $\vdash (\exists x)(A \Rightarrow B) \Rightarrow [(\forall x) A \Rightarrow B]$

→ môžem zvoliť $t = y_0$

$A_x[y_0] \vdash A_x[y_0]$ (I) ($\forall L$) ; $B \vdash B$ (I)
$\forall x(A) \vdash A_x[y_0]$ ($\exists L$)
$(\forall x) A, (\exists x)(A \Rightarrow B) \vdash B$ (PL)
$(\exists x)(A \Rightarrow B), (\forall x) A \vdash B$ ($\Rightarrow R$)
$(\exists x)(A \Rightarrow B) \vdash (\forall x) A \Rightarrow B$ ($\Rightarrow R$)
$\vdash (\exists x)(A \Rightarrow B) \Rightarrow [(\forall x) A \Rightarrow B]$