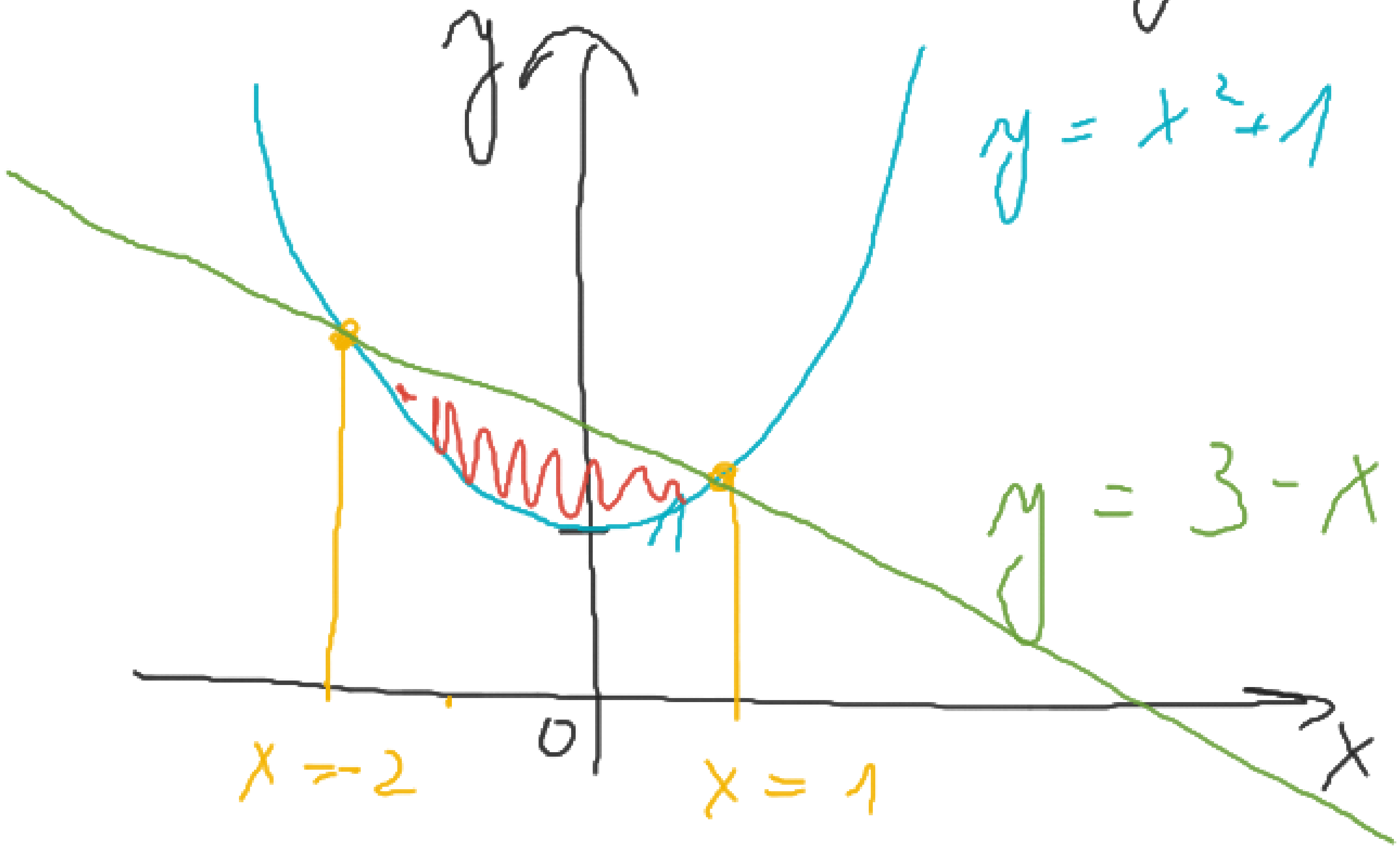
PR 12 VYPOČÍTAJTE VEĽKOSŤ OBLASTÍ OHRANIČENÝCH DANÝMI KRIVKAMI

b) $\boxed{y = x^2 + 1}$ ; $x + y = 3$

$\boxed{y = 3 - x}$




$\boxed{y = y}$

$x^2 + 1 = 3 - x$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$\boxed{x_1 = -2}$ $\boxed{x_2 = 1}$

$$P = \int_{-2}^{1} (3 - x) - (x^2 + 1)\, dx = \int_{-2}^{1} 3 - x - x^2 - 1\, dx =$$

$$= \int_{-2}^{1} 2 - x - x^2\, dx = \left[ 2x - \frac{x^2}{2} - \frac{x^3}{3} \right]_{-2}^{1} =$$

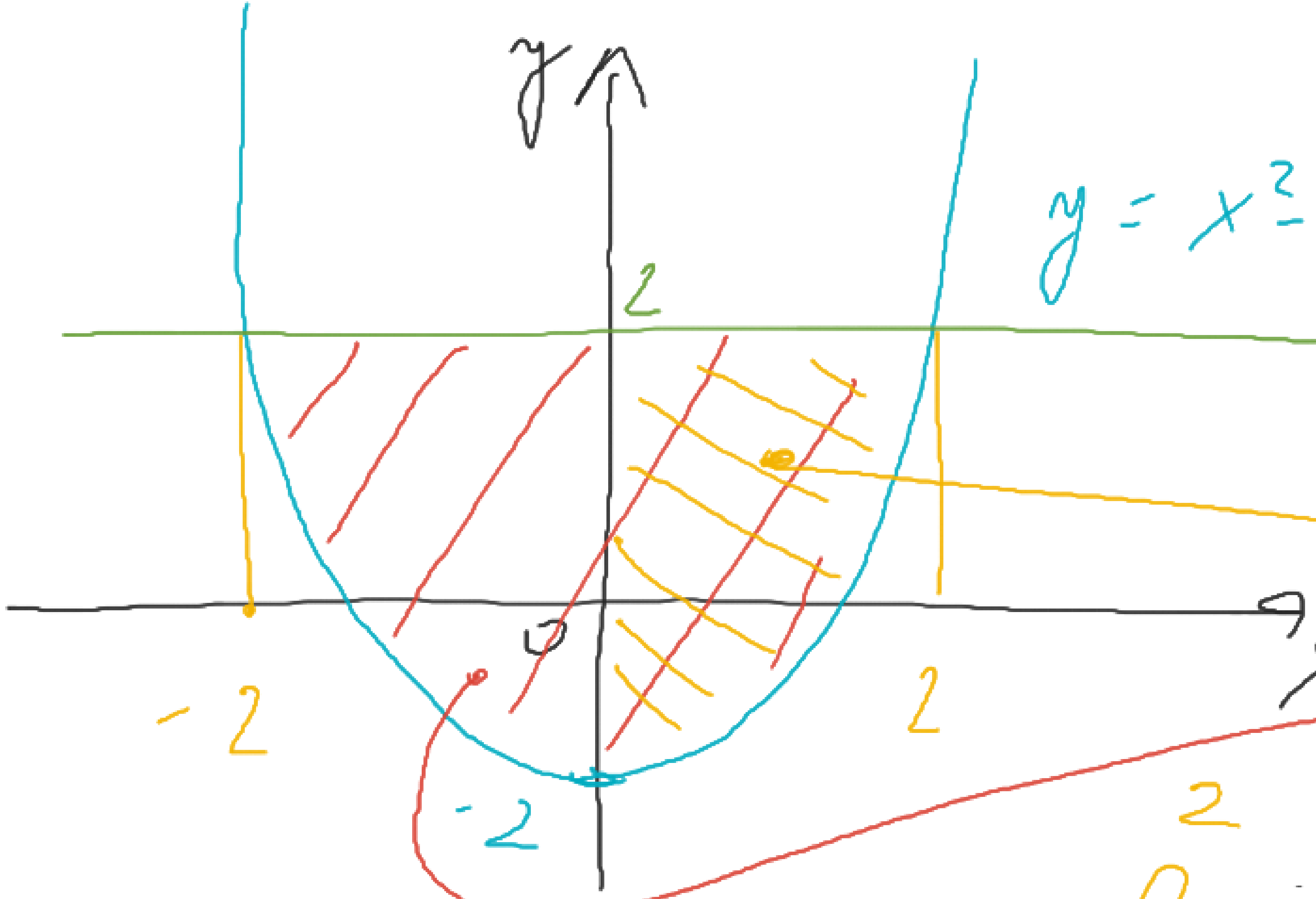
$$= \left( 2 - \frac{1}{2} - \frac{1}{3} \right) - \left( -4 - \overset{2}{\frac{4}{2}} + \frac{8}{3} \right) =$$

$$= 8 - \frac{1}{2} - 3 = 5 - \frac{1}{2} = \underline{\underline{\frac{9}{2}\, j^2}}$$

c) $\boxed{y = x^2 - 2}$, $\boxed{y = 2}$ $\Rightarrow$ $\boxed{y = y}$

$x^2 - 2 = 2$

$x^2 = 4 \Rightarrow \boxed{x_{1,2} = \pm 2}$




VYUŽIJEM SYMETRIU

$\boxed{P = 2P_1}$

$$P_1 = \int_0^2 \underbrace{2 - x^2 + 2}_{-x^2+4}\, dx =$$

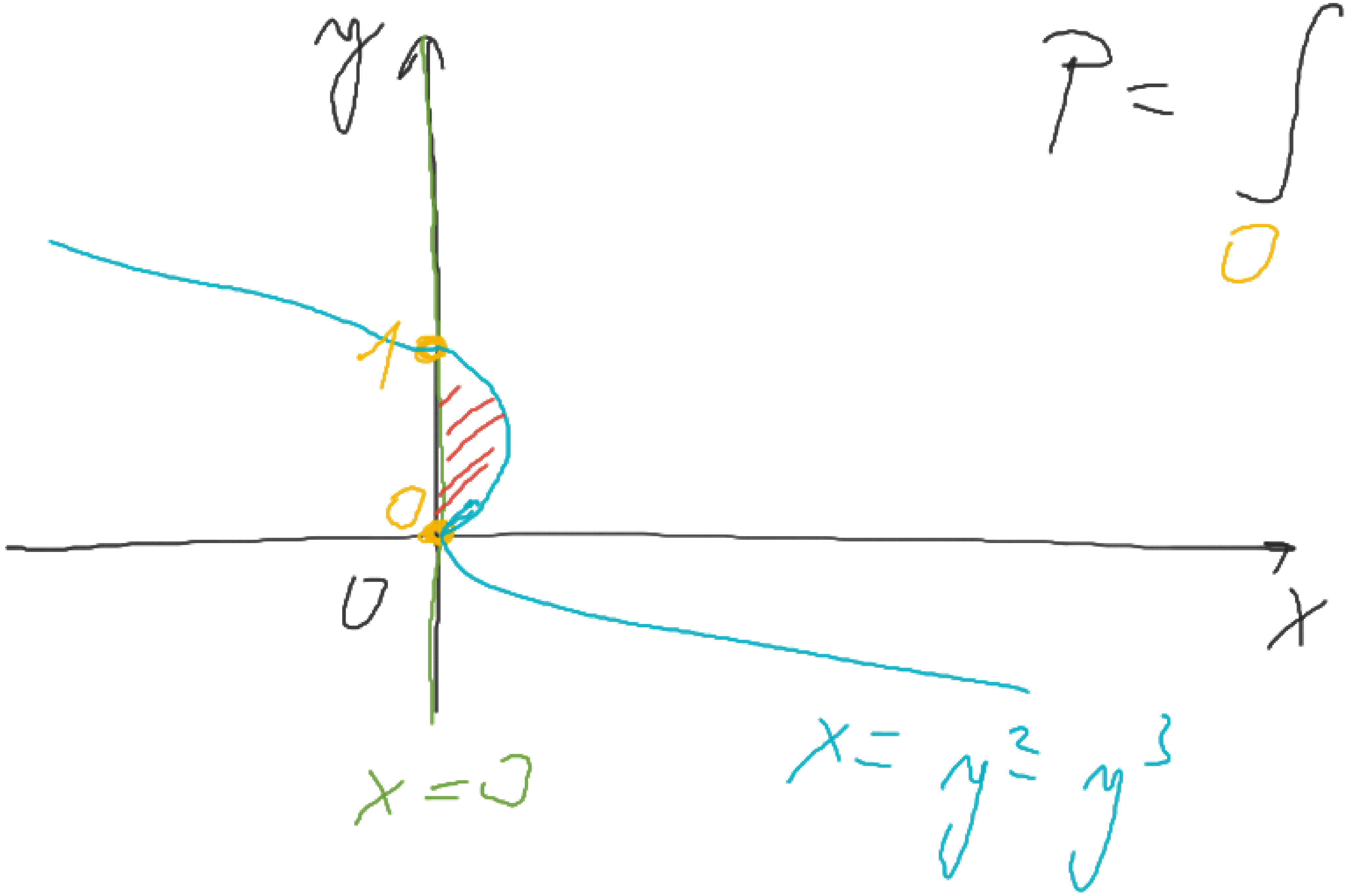
$$= \left[ -\frac{x^3}{3} + 4x \right]_0^2 = \left( -\frac{8}{3} + 8 \right) - 0 = \frac{16}{3} j^2$$

$$P = 2 \cdot \frac{16}{3} = \frac{32}{3} j^2$$

d) $\sigma_y \Rightarrow x = 0 \quad ; \quad x = y^2 - y^3 \quad \Rightarrow y = \frac{1}{2} \Rightarrow \frac{1}{4} - \frac{1}{8} > 0$

$$0 = y^2 - y^3 \Rightarrow y^2(1 - y) = 0$$

$$\boxed{y_2 = 0} \quad \boxed{y_1 = 1}$$




$$P = \int_0^1 y^2 - y^3 - 0 \, dy =$$

$$= \left[ \frac{y^3}{3} - \frac{y^4}{4} \right]_0^1 =$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \, j^2$$

(e) $\boxed{y = 2x^2 + 10}$ , $\boxed{y = 4x + 16}$ , $\boxed{x = -2}$ , $\boxed{x = 5}$

$$2x^2 + 10 = 4x + 16$$

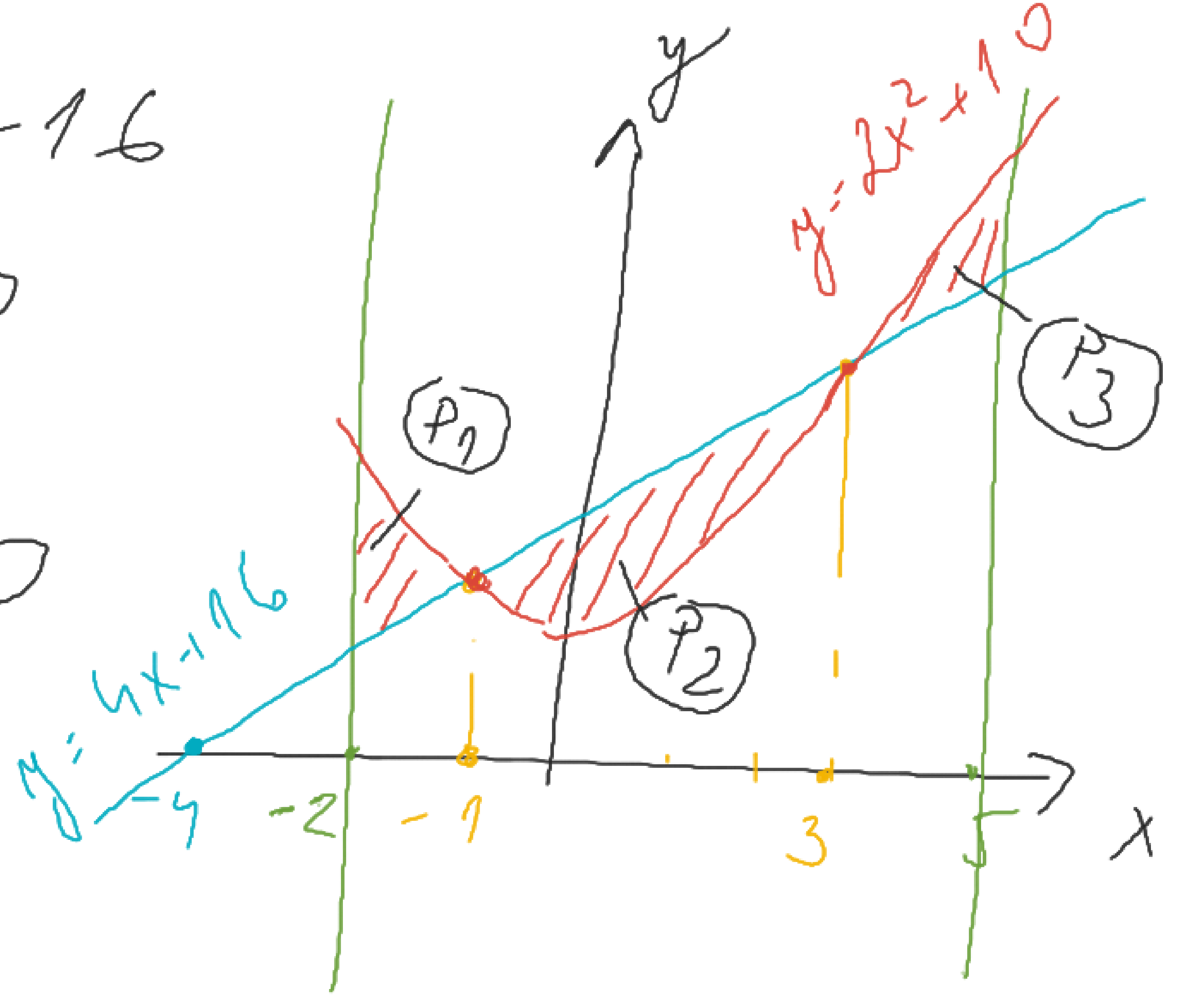
$$2x^2 - 4x - 6 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$\Downarrow \qquad \Downarrow$$

$$x = 3 \qquad x = -1$$



$$P = P_1 + P_2 + P_3 = \int_{-2}^{-1} (2x^2+10) - (4x+16)\,dx +$$

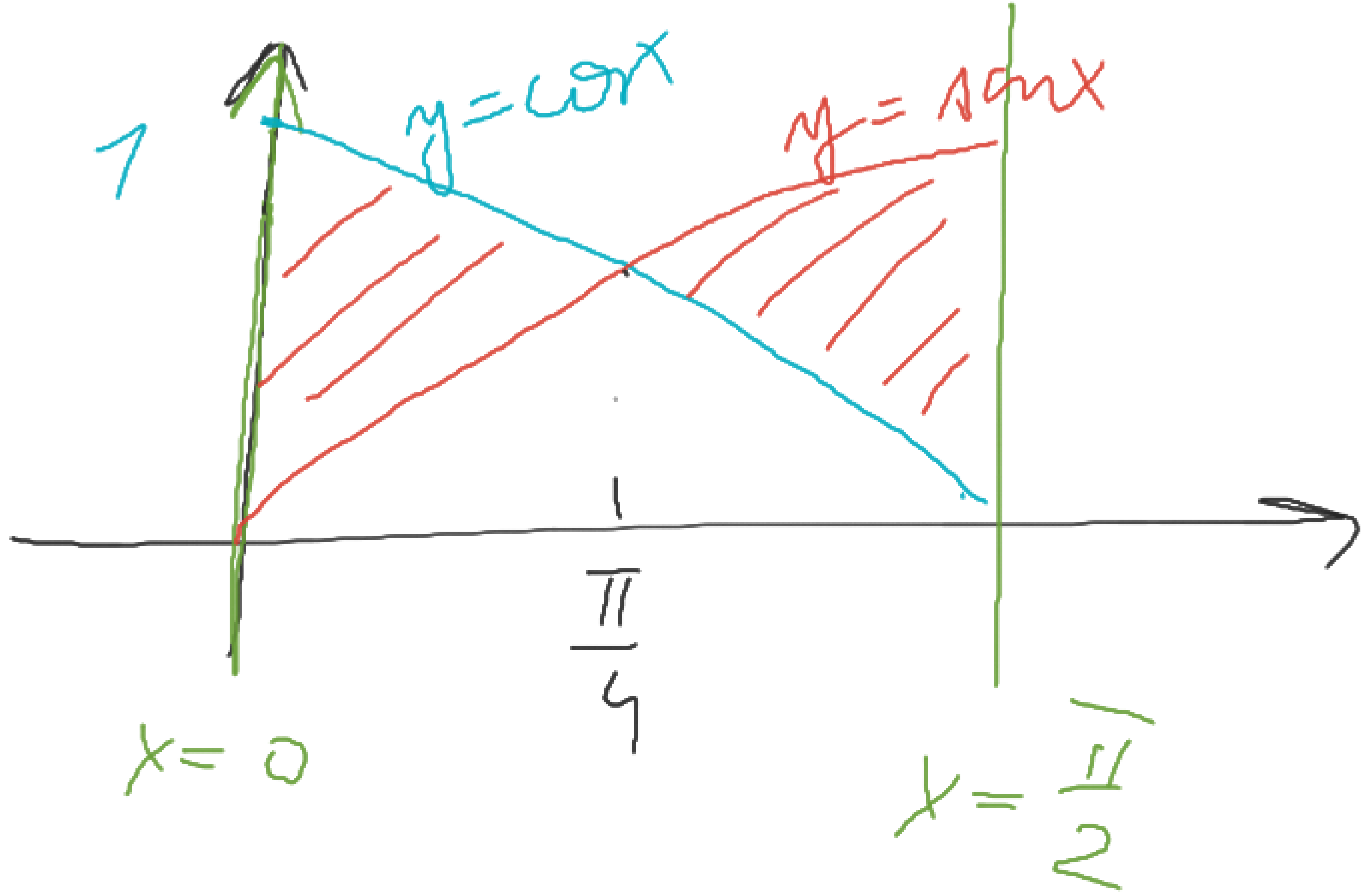
$$+ \int_{-1}^{3} (4x+16) - (2x^2+10)\,dx + \int_{3}^{5} 2x^2+10 - (4x+16)\,dx =$$

$$= \int_{-2}^{-1} 2x^2 - 4x - 6\,dx + \int_{-1}^{3} -2x^2 + 4x + 6\,dx + \int_{3}^{5} 2x^2 - 4x - 6\,dx$$

$$= \left[\frac{2x^3}{3} - \frac{4x^2}{2} - 6x\right]_{-2}^{-1} + \left[\frac{-2x^3}{3} + \frac{4x^2}{2} + 6x\right]_{-1}^{3} + \left[\frac{2x^3}{3} - \frac{4x^2}{2} - 6x\right]_{3}^{5} =$$

# VYČÍSLIT DOMA

j) $\boxed{O_y}$, $\boxed{x=\frac{\pi}{2}}$, $\boxed{y=\cos x}$, $\boxed{y=\sin x}$




$$P = \int_0^{\frac{\pi}{4}} \cos - \sin x \, dx + $$

$$+ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x - \cos x \, dx =$$

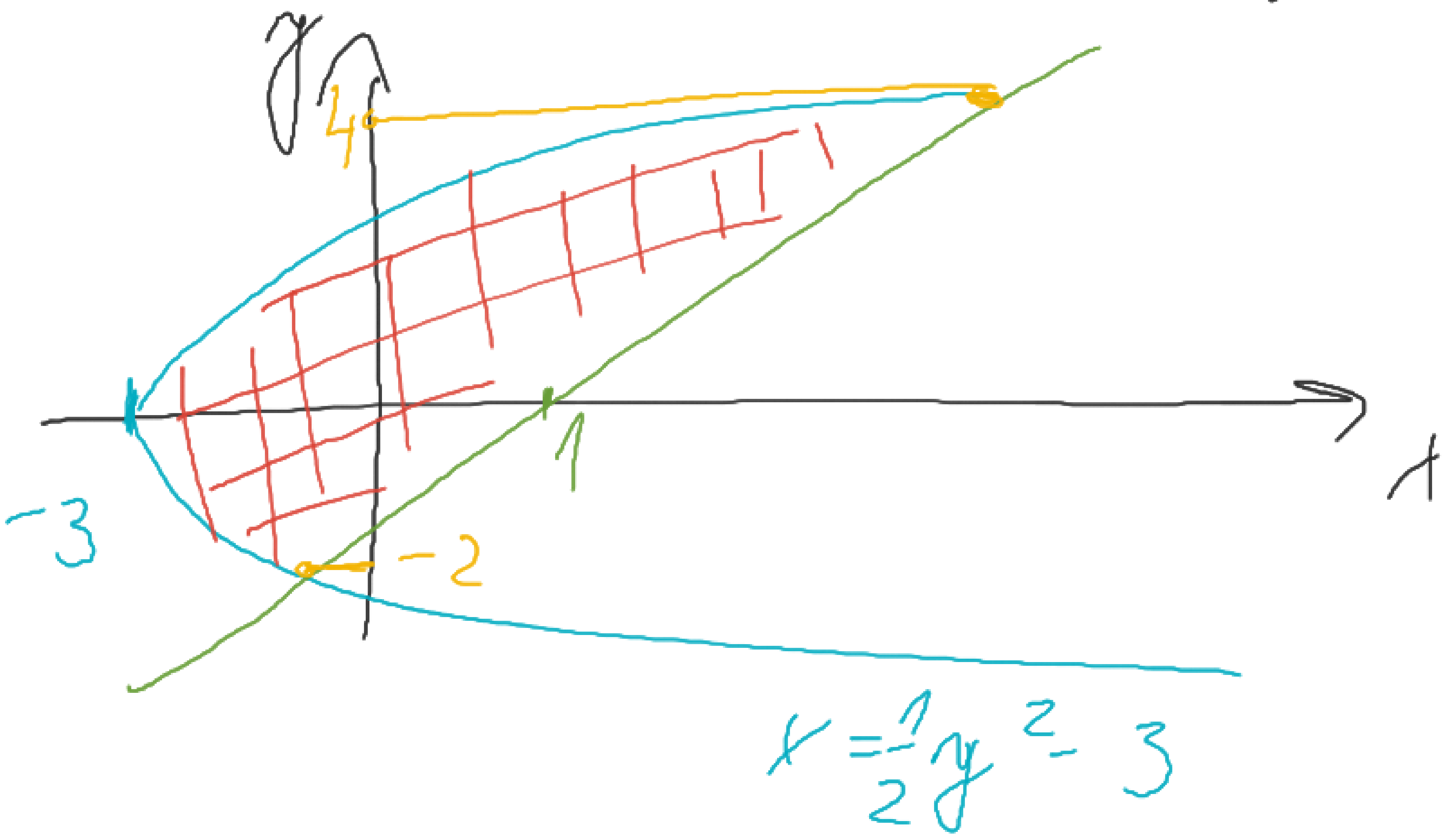
$$= \left[\sin x + \cos x\right]_0^{\frac{\pi}{4}} + \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 0 - 1 + 0 - 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} =$$

$$= 2\sqrt{2} - 2 \quad j^2$$

g) $\boxed{x = \frac{1}{2}y^2 - 3}$ , $\boxed{y = x - 1}$

$x = y + 1$




$\frac{1}{2}y^2 - 3 = y + 1$

$\frac{1}{2}y^2 - y - 4 = 0 \quad / \cdot 2$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$\Downarrow \qquad \Downarrow$

$y = 4 \qquad y = -2$

$$P = \int_{-2}^{4} y + 1 - \frac{1}{2}y^2 + 3 \, dy = \int_{-2}^{4} -\frac{1}{2}y^2 + y + 4 \, dy =$$

$$= \left[ -\frac{y^3}{6} + \frac{y^2}{2} + 4y \right]_{-2}^{4} = \left( -\frac{\cancel{4}^{\,2} \cdot 16}{\cancel{6}\,3} + \underbrace{8 + 16}_{24} \right) -$$

$$- \left( \frac{\cancel{2} \cdot 4}{\cancel{6}\,3} + \underbrace{2 - 8}_{-6} \right) = -\frac{36}{3} + 30 = -12 + 30 =$$

$$= \underline{\underline{18 \, j^2}}$$