(PR4) POMOCOU ZÁKL. VZORCOV SPOČÍTAJTE

(a) $\int x^2+1\,dx = \int x^2\,dx + \int 1\,dx = \frac{x^3}{3} + x + C$

(b) $\int \frac{(x^2+1)^2}{x^3}\,dx = \int \frac{x^4+2x^2+1}{x^3}\,dx =$

$x^{-3}$

$= \int x + \frac{2}{x} + \frac{1}{x^3}\,dx = \frac{x^2}{2} + 2\ln|x| - \frac{1}{2x^2} + C$

(c) $\int tg\,x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{-\sin x}{\cos x}\,dx =$

$= \underline{\underline{-\ln|\cos x| + C}}$

(d) $\int cotg^2 x\,dx = \int \frac{\cos^2 x}{\sin^2 x}\,dx = \int \frac{1 - \cancel{\sin^2 x}}{\cancel{\sin^2 x}}\,d$

$= \int \frac{1}{\sin^2 x} - 1\,dx = \underline{\underline{-cotg\,x - x + C}}$

(e) $\int e^{x}\left(1-\frac{e^{-x}}{x^{2}}\right)dx = \int e^{x} - \frac{\cancel{e^{x}}}{\cancel{e^{x}}\,x^{2}}\,dx =$

$= \int e^{x} - x^{-2}\,dx = e^{x} - \frac{x^{-1}}{-1} + C = \underline{\underline{e^{x} + \frac{1}{x} + C}}$

(f) $\int \sqrt{x\sqrt{x\cdot\sqrt{x}}} + \frac{1}{\sqrt{x}}\,dx = \int x^{\frac{1}{2}}\cdot x^{\frac{1}{4}}\cdot x^{\frac{1}{8}} + x^{-\frac{1}{2}}\,dx$

$= \int x^{\frac{7}{8}} + x^{-\frac{1}{2}}\,dx = \frac{x^{\frac{15}{8}}}{\frac{15}{8}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C =$

$$= \frac{8}{15} x \sqrt[8]{x^7} + 2\sqrt{x} + C$$

(g) $$\int \frac{dx}{x^2 + 9_{3^2}} = \left| \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \right.$$

$$= \frac{1}{3} \operatorname{arctg} \frac{x}{3} + C$$

(h) $\int \frac{1 . dx}{\sin^2 x \cos^2 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} dx =$

$= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx = \underline{\underline{\operatorname{tg} x - \operatorname{cotg} x + C}}$

(i) $\int 3^x \cdot 5^{2x} dx = \int 3^x \cdot 25^x dx = \int 75^x dx$

$= \underline{\frac{75^x}{\ln 75} + C}$

PŘÍKLAD č. 6) SUBSTITUČNÍ METODA

(a) $$\int \overset{dt}{(2x+5)} \cdot (\underbrace{x^2+5x}_{t})^7 \, dx =$$

$$t = x^2 + 5x$$

$$1\,dt = (2x+5)\,dx$$

$$= \int t^7\,dt = \frac{t^8}{8} + C = \frac{(2x+5)^8}{8} + C$$

(b) $\int (x+3)\sqrt{x^2+6x+1}\,dx =$

$t$

$t = x^2+6x+1$

$dt = (2x+6)\,dx$

$dt = 2(x+3)\,dx$

$\boxed{dx} = \dfrac{dt}{2(x+3)}$

$= \int \cancel{(x+3)}\sqrt{t}\,\dfrac{dt}{2\cancel{(x+3)}}$

$= \dfrac{1}{2}\int t^{\frac{1}{2}}\,dt =$

$= \dfrac{1}{\cancel{2}}\,\dfrac{t^{\frac{3}{2}}}{\frac{3}{\cancel{2}}} + C = \boxed{\dfrac{(x^2+6x+1)^{\frac{3}{2}}}{3} + C}$

(c) $\displaystyle\int \frac{\sin(\overset{t}{\ln x})}{x}\,\overset{dt}{dx} = \quad\Big|\; t = \ln x \;\Big|\; dt = \frac{1}{x}\,dx$

$$= \int \sin t \, dt = -\cos t + C =$$

$$= \underline{\underline{-\cos(\ln x) + C}}$$

a) $$\int x \cdot e^{1-x^2}\,dx = \left| \begin{array}{l} t = 1-x^2 \\ dt = -2x\,dx \\ x\,dx = \dfrac{dt}{-2} \end{array} \right| =$$

$$= \int e^t \frac{dt}{-2} = -\frac{1}{2}e^t + C = \underline{\underline{-\frac{1}{2}e^{1-x^2} + C}}$$

e) $\int \frac{x}{x+16}\,dx = \int \frac{x+16-16}{x+16}\,dx =$

$= \int 1 - \frac{16}{x+16}\,dx = x - 16\int \frac{1}{x+16}\,dx =$

$$= x - 16\ln|x+16| + C$$

f) $\displaystyle\int \frac{x}{x^2+16}\,dx = \left| \begin{array}{l} t = x^2+16 \\ dt = 2x\,dx \\ x\,dx = \dfrac{dt}{2} \end{array} \right| =$

$$= \int \frac{dt}{2t} = \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln|t| + C =$$

$$= \underline{\underline{\frac{1}{2}\ln(x^2+16) + C}}$$

g) $\displaystyle\int \frac{x}{x^4+16}\,dx = \left| \begin{aligned} t &= x^2 \\ dt &= 2x\,dx \\ x\,dx &= \frac{dt}{2} \end{aligned} \right| =$

$(x^2)^2$

$$= \frac{1}{2}\int \frac{dt}{t^2+16} = \frac{1}{2}\cdot\frac{1}{4}\,\mathrm{arctg}\,\frac{t}{4} + C =$$

$4^2$

$$= \underline{\underline{\frac{1}{8}\,\mathrm{arctg}\,\frac{x^2}{4} + C}}$$

h) $\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}} dx = \left| \begin{array}{l} t = \sqrt{x+1} \\ dt = \frac{1}{2\sqrt{x+1}} dx \\ \frac{dx}{\sqrt{x+1}} = 2dt \end{array} \right|$

$= 2\int e^t dt =$

$= 2e^t + C = \underline{\underline{2e^{\sqrt{x+1}} + C}}$

# $\boxed{\text{PŘÍKLAD č. 9}}$ PER PARTES

$$\boxed{\int u'(x)\,v(x)\,dx = u(x)\,v(x) - \int u(x)\,v'(x)\,dx}$$

(a) $\displaystyle\int x \sin x \, dx = \left| \begin{matrix} u' = \sin x & u = -\cos x \\ v = x & v' = 1 \end{matrix} \right| =$

$= -x\cos x - \displaystyle\int -\cos x \, dx = -x\cos x + \int \cos x \, dx =$

$= -x\cos x + \sin x + C$

2x PP

6 $\int x^2 e^{3x}\,dx = \left| \begin{array}{ll} u' = e^{3x} & u = \frac{e^{3x}}{3} \\ v = x^2 & v' = 2x \end{array} \right|$

ZINTEGRUJAM POMOCNU SUBSTITUCIE

$t = 3x$

$= \frac{x^2 e^{3x}}{3} - \int \frac{2x e^{3x}}{3}\,dx = \left| \begin{array}{ll} u' = e^{3x} & u = \frac{e^{3x}}{3} \\ v = \frac{2x}{3} & v' = \frac{2}{3} \end{array} \right|$

$$= \frac{x^2 e^{3x}}{3} - \left[\frac{2x e^{3x}}{9} - \int \frac{2e^{3x}}{9} dx\right] =$$

$$= \frac{x^2 e^{3x}}{3} - \frac{2x e^{3x}}{9} + \frac{2}{9}\frac{e^{3x}}{3} + C$$

27

c) $\int x \ln(x^2)\,dx = \quad \Big| \; u' = x \qquad u = \frac{x^2}{2}$

$\Big| \; v = \ln(x^2) \qquad v' = \frac{1}{x^2}\cdot 2x$

$= \frac{x^2}{2}\ln(x^2) - \int \frac{\cancel{x^2}}{\cancel{2}} \cdot \frac{\cancel{2x}}{\cancel{x^2}}\,dx = \frac{x^2}{2}\ln(x^2) - \frac{x^2}{2} + C$

d) $\int 1 \cdot \operatorname{arctg} x \, dx = \left| \begin{array}{ll} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \dfrac{1}{1+x^2} \end{array} \right|$

$= x \operatorname{arctg} x - \dfrac{1}{2} \int \dfrac{2x}{1+x^2} \, dx = x \operatorname{arctg} x -$

$- \dfrac{1}{2} \ln(1+x^2) + C$

(e) $\int 1 \cdot \ln(x^2+1)\,dx = \left| \begin{array}{ll} u' = 1 & u = x \\ v = \ln(x^2+1) & v' = \frac{1 \cdot 2x}{x^2+1} \end{array} \right.$

$$= x\ln(x^2+1) - \int \frac{2x^2}{x^2+1}\,dx = x\ln(x^2+1) -$$

$$-2\int \frac{x^2+1-1}{x^2+1}\,dx = x\ln(x^2+1) - 2\int 1 - \frac{1}{x^2+1}\,dx$$

$$= \underline{\underline{x\ln(x^2+1) - 2x + 2\,\mathrm{arctg}\,x + C}}$$

(f) $1 \int e^{2x} \cos x \, dx = \left| \begin{array}{ll} u' = \cos x & u = \sin x \\ v = e^{2x} & v' = e^{2x} \cdot 2 \end{array} \right|$

2x PP + RÓVNICA

$= e^{2x} \sin x - \int 2e^{2x} \sin x \, dx = \left| \begin{array}{ll} u' = \sin x & u = -\cos x \\ v = 2e^{2x} & v' = 4e^{2x} \end{array} \right|$

$= e^{2x} \sin x + 2e^{2x} \cos x - \int 4 e^{2x} \cos x \, dx$

$\Rightarrow 5 \int e^{2x} \cos x \, dx = e^{2x} \sin x + 2e^{2x} \cos x$

$$\boxed{\int e^{2x} \cos x \, dx = \frac{1}{5} \left( e^{2x} \sin x + 2e^{2x} \cos x \right) + C}$$