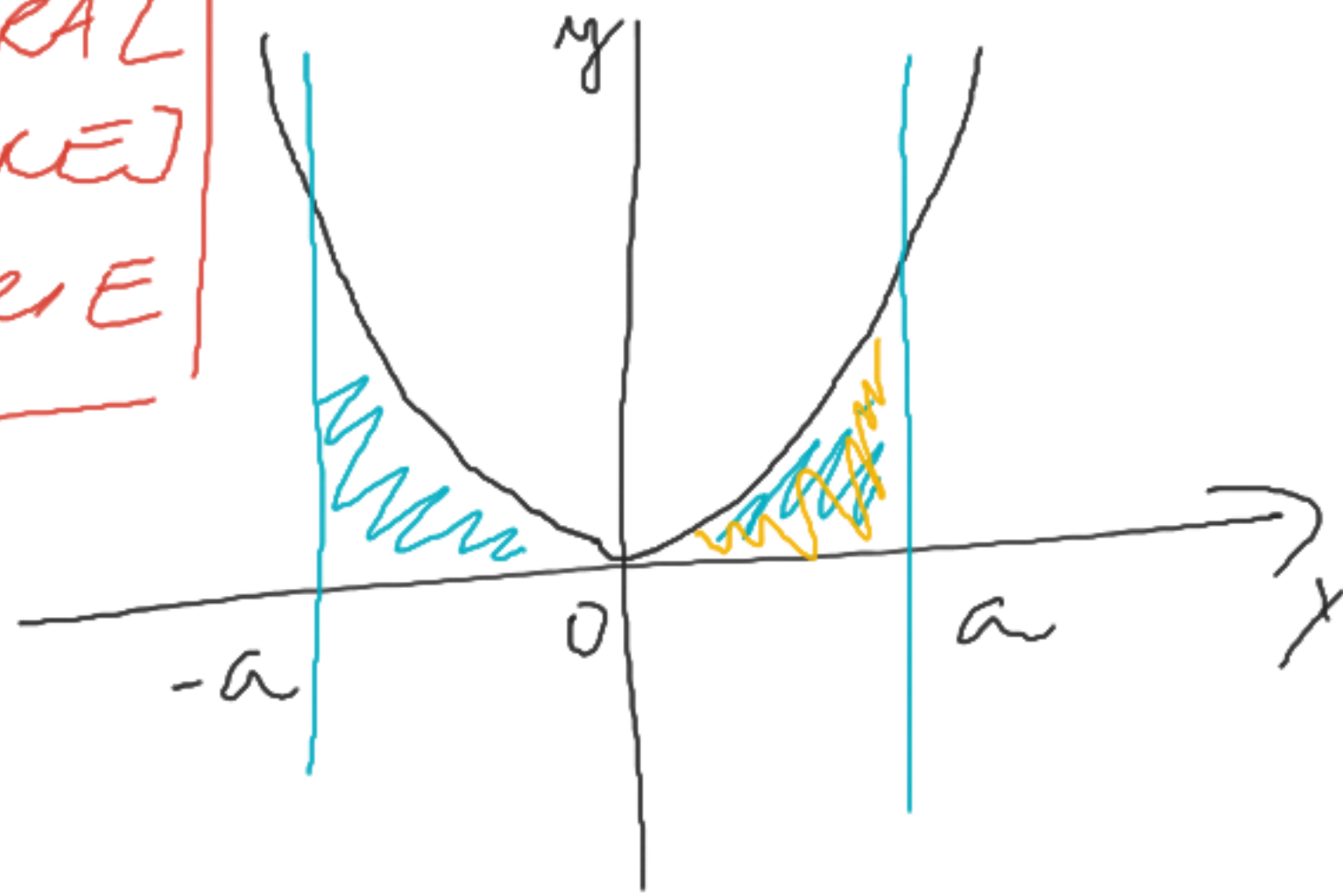
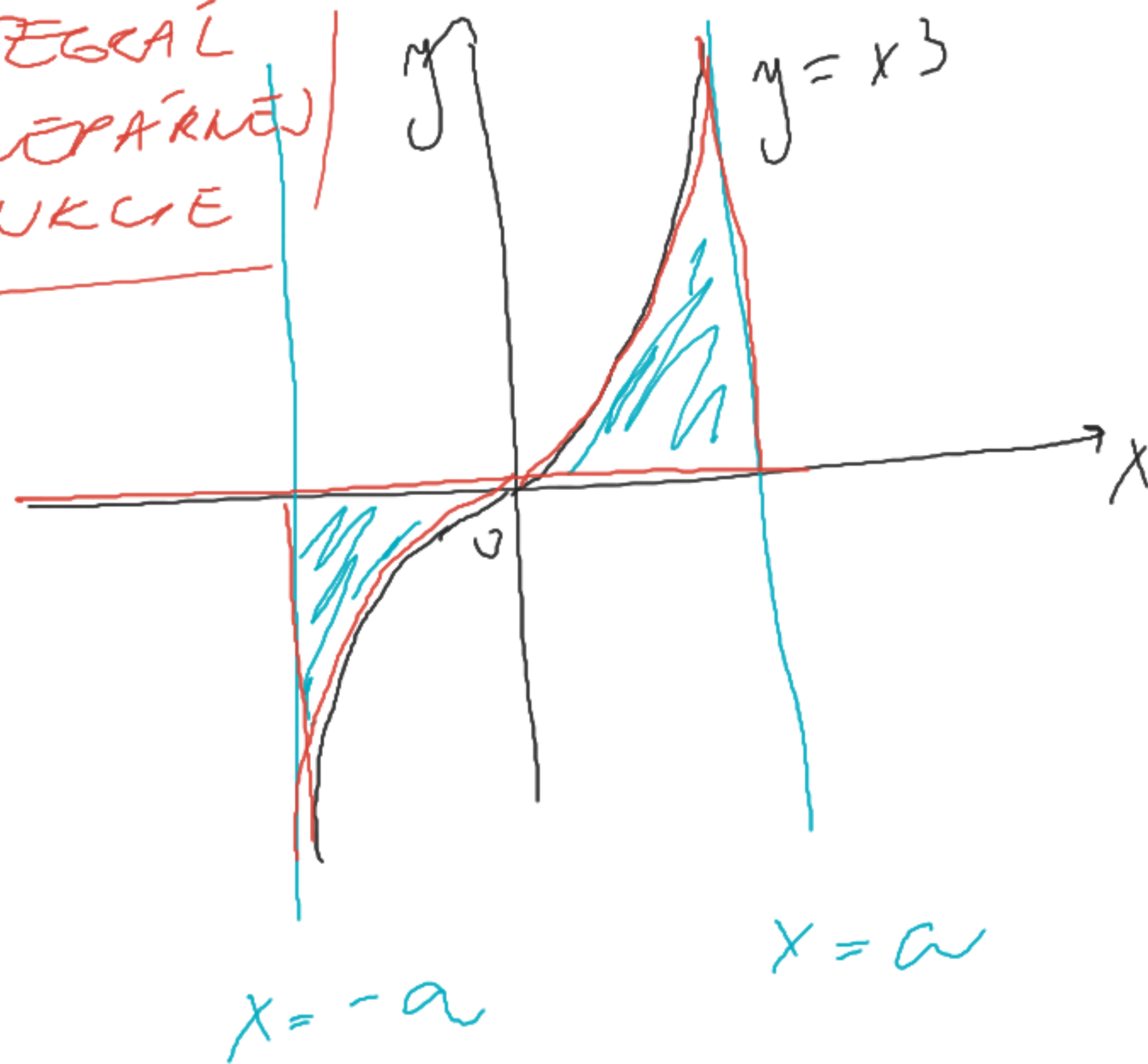


INTEGRAL
Z PAARER
FUNKTIONE



$$\begin{aligned} y &= x^2 \\ \int_{-a}^a f(x) dx &= \\ &= 2 \int_0^a f(x) dx \end{aligned}$$

INTEGRÁL
Z NĚPÁRNĚJ
FUNKCE

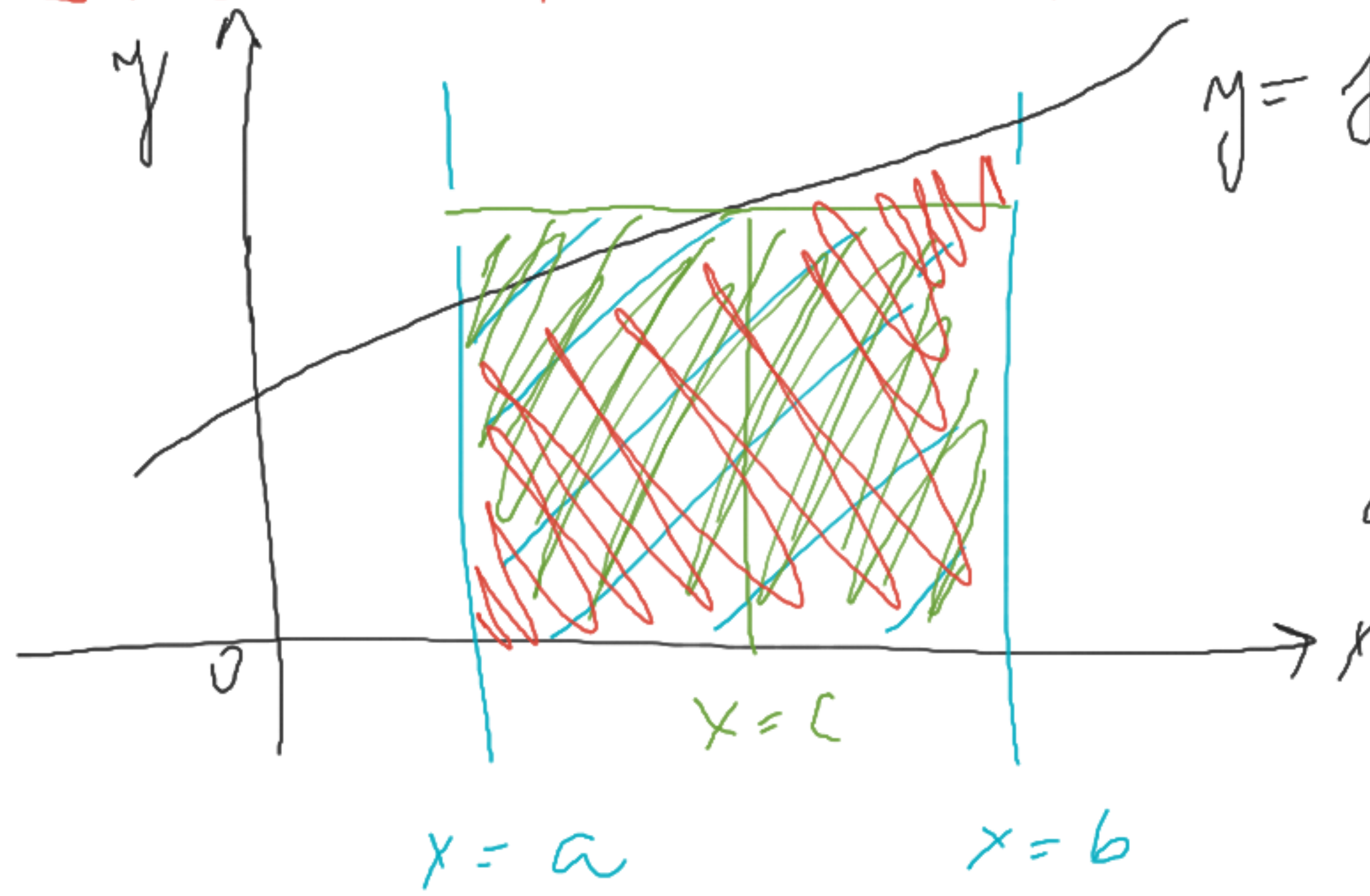


$$\int_{-a}^a f(x) dx = 0$$

PLOCHA MEZI

$f(x)$ a $y=0$
NEBŮDE KULOVÁ

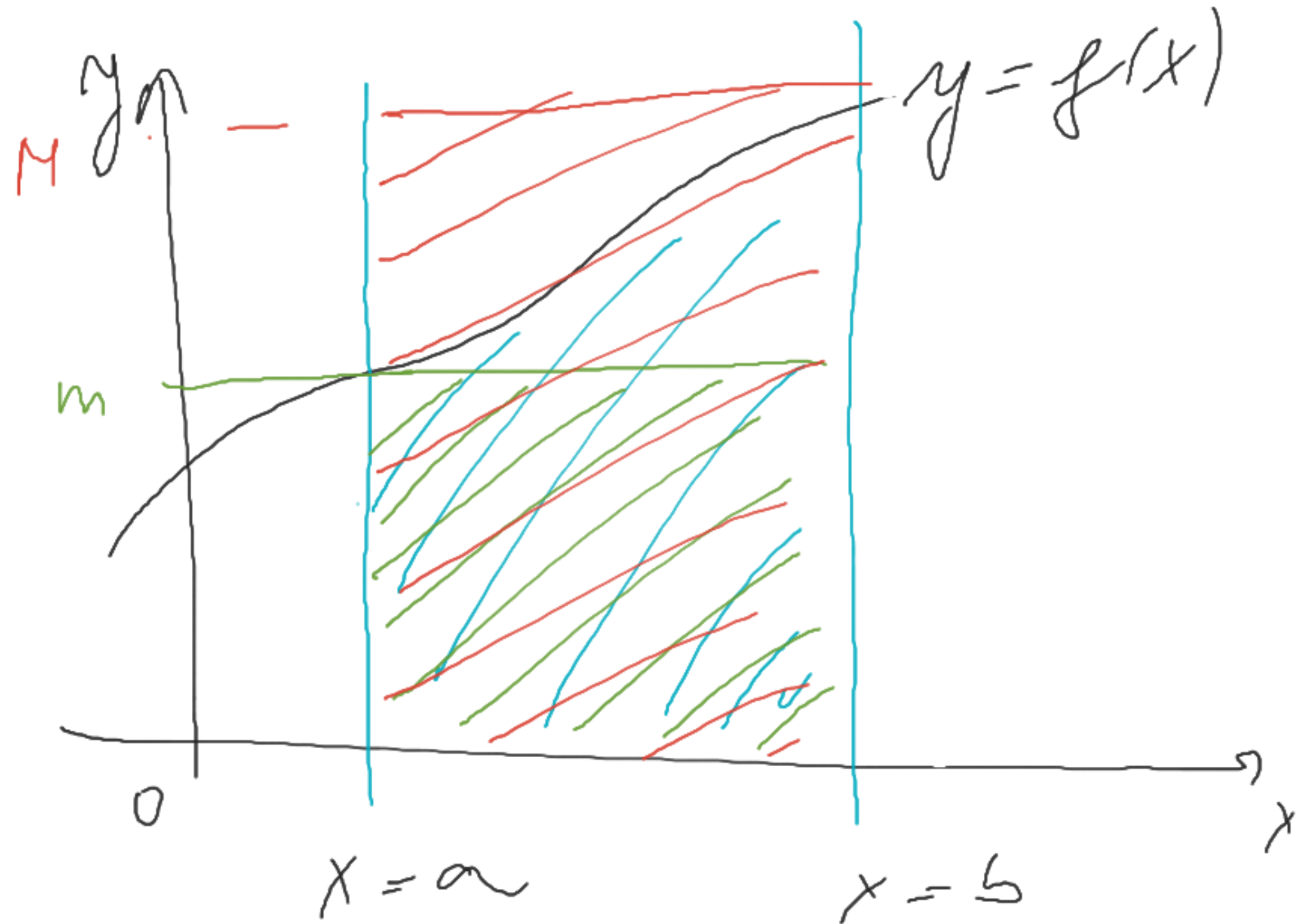
VĚTA O STŘEDNĚJ HODNOTĚ PRO URČITÝ INTEGRÁL



$$\int_a^b f(x) dx = f(c)(b-a)$$

PLOŠKA
OBDEŽENÍ

ODHAD VRČITEHO INTEGRALU



$$\int \frac{\sin x}{x} dx$$

$$\int e^{-x^2} dx$$

(PR 7)

a

$$\int_{x_1}^{x_2} \frac{dx}{(2x+1)^3}$$

$$= \left| \begin{array}{l} t = 2x+1 \\ dt = 2dx \Rightarrow dx = \frac{dt}{2} \end{array} \right.$$

$$x_1 = 0 \Rightarrow t_1 = 2 \cdot 0 + 1 = 1$$

$$x_2 = 1 \Rightarrow t_2 = 2 \cdot 1 + 1 = 3$$

$$= \int_1^3 \frac{dt}{2 \cdot t^3} = \left[\frac{1}{2} \cdot \frac{t^{-2}}{-2} \right]_1^3 = -\frac{1}{4} \left[\frac{1}{t^2} \right]_1^3 =$$

$$= -\frac{1}{4} \left[\frac{1}{9} - \frac{1}{1} \right] = -\frac{1}{4} \left(-\frac{8}{9} \right) = \underline{\underline{\frac{2}{9}}}$$

$$\textcircled{b} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x} dx =$$

$$t = \sin x$$

$$dt = \cos x dx$$

$$x_1 = \frac{\pi}{4} \Rightarrow t_1 = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$x_2 = \frac{\pi}{2} \Rightarrow t_2 = \sin \frac{\pi}{2} = 1$$

$$= \int_{\frac{\sqrt{2}}{2}}^1 \frac{dt}{t^2} = - \left[\frac{1}{t} \right]_{\frac{\sqrt{2}}{2}}^1 = - \left(1 - \frac{2}{\sqrt{2}} \right) =$$

$$= -1 + \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \underline{\underline{-1 + \sqrt{2}}}$$

$$\textcircled{c} \int_0^{\ln 2} x e^x dx = \left| \begin{array}{ll} u = x & u' = 1 \\ v' = e^x & v = e^x \end{array} \right| =$$

$$= \left[x e^x \right]_0^{\ln 2} - \int_0^{\ln 2} e^x dx = \ln 2 e^{\ln 2} - 0 -$$

$$- \left[e^x \right]_0^{\ln 2} = 2 \ln 2 - e^{\ln 2} + e^0 =$$

$$= 2 \ln 2 - 2 + 1 = \underline{\underline{2 \ln 2 - 1}}$$

(a) $\int_9^4 \frac{1-\sqrt{x}}{\sqrt{x}} dx = \left| \begin{array}{l} t = \sqrt{x} \\ t^2 = x \\ 2t dt = dx \end{array} \right.$

$x_1 = 9 \Rightarrow t_1 = 3$
 $x_2 = 4 \Rightarrow t_2 = 2$

$$= \int_3^2 \frac{1-t}{\cancel{t}} \cdot \cancel{2t} dt = [2t - t^2]_3^2 = \cancel{4} - \cancel{9} -$$

$$- (6 - 9) = \underline{\underline{3}}$$

(PR 12)

(a)

$$y = 4x - x^2$$

$$0x \rightarrow y = 0$$

}

\Rightarrow

$$4x - x^2 = 0$$

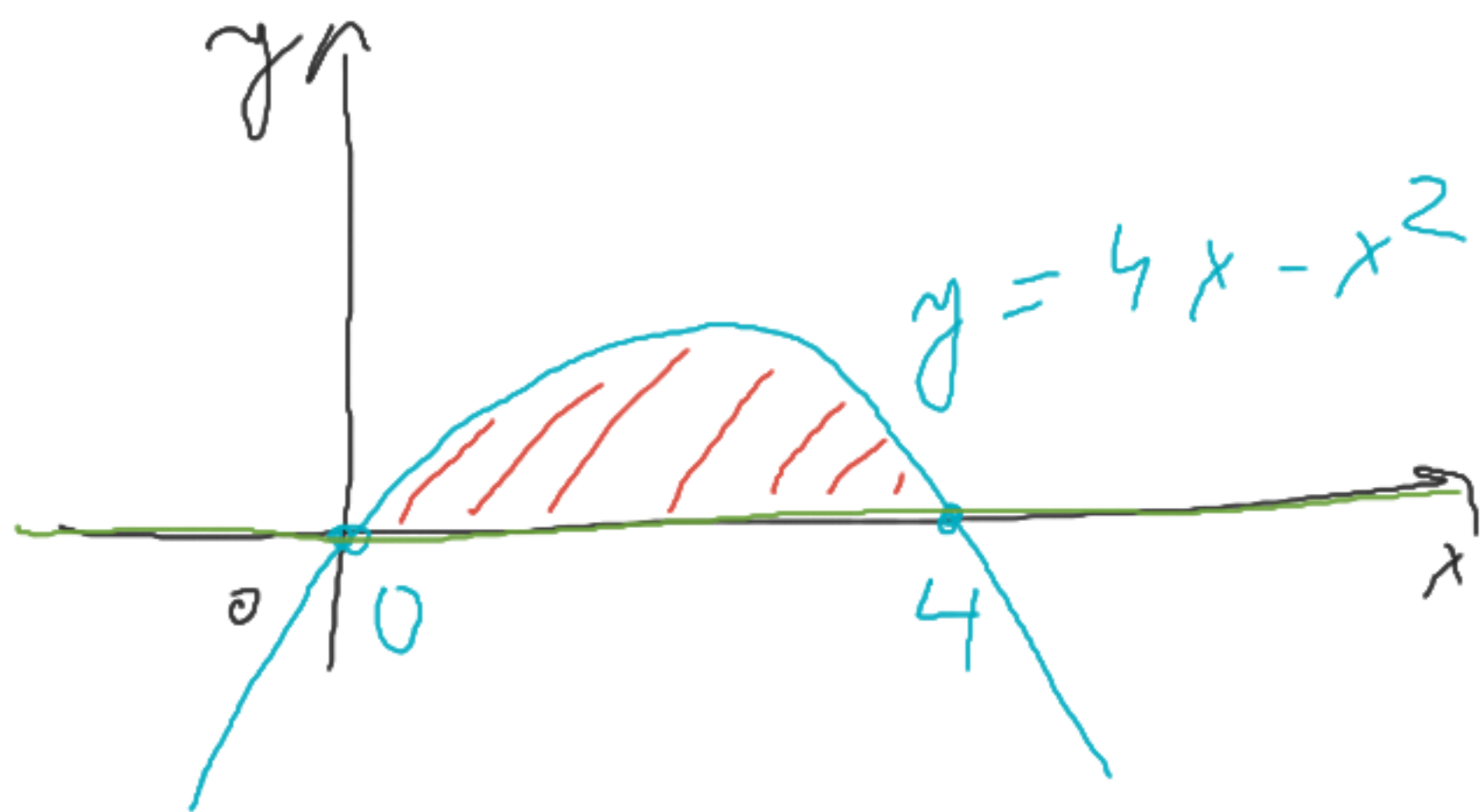
$$x(4 - x) = 0$$

\Downarrow

$$x_1 = 0$$

\Downarrow

$$x_2 = 4$$



$$y = 0$$

$$P = \int_0^4 (4x - x^2 - 0) dx =$$

$$= \left[\frac{2x^2}{2} - \frac{x^3}{3} \right]_0^4 = 2 \cdot 16 - \frac{4 \cdot 16}{3} = \frac{6 \cdot 16 - 4 \cdot 16}{3}$$

$$= \frac{2 \cdot 16}{3} = \frac{32}{\underline{\underline{3}}} \text{ J}^2$$