(1)

POSTUPNOSTI

POSTUPNOSŤ JE FUNKCIA DEFINOVANÁ NA MNOŽINE PRIRODZENÝCH ČÍSEL

$N = \{1, 2, 3, \dots\}$

$1 \to a_1$

$2 \to a_2$

$3 \to a_3$

$\vdots$

$n \to a_n$

$\vdots$

$\{a_n\}_{n=1}^{\infty} = \{a_1, a_2, a_3, \dots\}$

$a_n = \frac{1}{n}$

(*) $\left\{\frac{1}{n}\right\}_{n=1}^{\infty} = \left\{1, \frac{1}{2}, \frac{1}{3}, \dots \frac{1}{n}, \dots\right\}$ $a_n \to 0$ KONVERGENTNÁ, AK $a < \infty$ konvergentná

a_i – členy postupnosti

AKÝM SPÔSOBOM MÔŽE BYŤ POSTUPNOSŤ DANÁ?

(**) $\{n^2\}_{n=1}^{\infty} = \{1, 4, 9, 25, 36, \dots\}$ $a_n \to \infty$ DIVERGENTNÁ POSTUPNOSŤ

$\{(-1)^n\}_{n=1}^{\infty} = \{-1, 1, -1, +1, \dots\}$

LIMITA NEEXISTUJE

a) PREDPISOM (*) $a_n = \frac{1}{n}$; (**) $a_n = n^2$;

$a_n = 2^n; \dots$

b) REKURENTNE a_n – tý člen je daný pomocou predchádzajúcich členov (min. jedného)

PRÍKLAD. FIBONACCIHO POSTUPNOSŤ $a_1 = 1; a_2 = 1; a_{n+1} = a_n + a_{n-1}$

$a_3 = a_2 + a_1 = 2$

PRÍKLAD.

$\{a_n\}_{n=1}^{\infty}$ $\quad a_{n+1} = 4a_n$, kde $a_1 = 4$. REKURENTNE

$a_n = ?$

$a_1 = 1;\ a_2 = 4;\ a_3 = 16;\ a_4 = 64;\ \dots$

$\underline{\underline{a_n = 4^{n-1}}};$ PREDPISOM

PRÍKLAD.

$\{a_n\}_{n=1}^{\infty}$, $\quad a_n = 3n - 4;$

AKO TO VYJADRIŤ REKURENTNE?

$a_1 = -1$ $\qquad$ DEF. $a_1 = -1;$

$a_2 = 2;$ $\qquad a_{n+1} = a_n + 3;$

$a_3 = 5$

$a_4 = 8;$

$\vdots$

(1)

2 TYPY POSTUPNOSTÍ

1) ROZDIEL DVOCH PO SEBE IDÚCICH ČLENOV JE KOŠTANTNÝ AP

2) PODIEL ——"—— ——"—— GP

ARITMETICKÁ POSTUPNOSŤ.

$a_{n+1} - a_n =$ konštanta (diferencia, d)

$a_{n+1} - a_n = d$

$\underline{\underline{a_{n+1} = a_n + d}}$ $\quad a_1, \underbrace{a_1+d}_{a_2}, \underbrace{a_2+d}_{a_3}, \dots$

$a_1 = a_1$

$a_2 = a_1 + d$

$a_3 = a_2 + d = a_1 + d + d = \underline{a_1 + 2d}$

$a_4 = a_3 + d = a_1 + 2d + d = a_1 + 3d$

$\vdots$

$a_n = a_{n-1} + d = \underline{\underline{a_1 + (n-1)d}}$

$\underline{\underline{a_r = a_s + (r-s)d}}$

PRÍKLAD

$a_{10} = a_1 + 9d$

$a_{10} = a_6 + 4d$

$a_{10} = a_3 + 7d$

$a_{999} = a_{900} + 99d$

$a_{999} = a_1 + 998d$

(4)

$s_n = a_1 + a_2 + a_3 + \dots + a_n$ n-tý čiastočný súčet

$s_1 = a_1$

$s_2 = a_1 + a_2 = s_1 + a_2$

$a_n - a_{n-1} = d$

$s_3 = a_1 + a_2 + a_3 = s_2 + a_3$

$\vdots$

$s_n = a_1 + a_2 + \dots + a_{n-1} + a_n$

$$\boxed{s_n = \frac{n}{2}(a_1 + a_n)}$$

GAUSS

$\boxed{1} + \boxed{2} + 3 + 4 + 5 + \dots + \underbrace{50 + 51} + \dots \boxed{99} + \boxed{100}$

$\left.\begin{array}{l} 101 \\ 101 \\ \vdots \\ 101 \end{array}\right\} 50\times$

$101 \cdot 50 = 5050$

V AP $a_1 = 3$, $d = 2$. Koľko členov postupnosti potrebujeme sčítať, aby súčet bol $s_n = 120$.

(5)

$$s_n = \frac{n}{2}(a_1 + a_n)$$

$$a_n = a_1 + (n-1)d$$

$$a_n = 3 + (n-1)2 = 3 + 2n - 2 = 1 + 2n$$

$$120 = \frac{n}{2}\left(3 + \underbrace{1+2n}_{a_n}\right) \Big/ \cdot 2$$

$$240 = n(4 + 2n)$$

$$\cancel{240}^{\,120} = \cancel{2}n\left(2 + n\right)$$

$$120 = 2n + n^2$$

$$n^2 + 2n - 120 = 0$$

$$n_{1,2} = \frac{-2 \pm \sqrt{4 + 480}}{2} = \frac{-2 \pm \sqrt{484}}{2} = \frac{-2 \pm 22}{2} = \begin{cases} \cancel{-12} \\ \underline{\underline{10}} \end{cases}$$

$$\boxed{n = 10}$$

$$a_{10} = 3 + 9 \cdot 2 = 21$$

$$120 \overset{?}{=} \frac{10}{2}(3 + 21)$$

$$120 \overset{?}{=} 5(24)$$

platí

GEOMETRICKÁ POSTUPNOSŤ

$$\frac{a_{n+1}}{a_n} = q \text{ (konštanta)}, \quad q \neq 1$$

$a_{n+1} = a_n \cdot q$

$a_2 = a_1 \cdot q$

$a_3 = a_2 \cdot q = a_1 \cdot q^2$

$a_4 = a_3 \cdot q = a_1 \cdot q^3$

$\vdots$

$$\boxed{\begin{aligned} a_n &= a_1 \cdot q^{n-1} \\ a_r &= a_s \cdot q^{r-s} \end{aligned}}$$

$s_n = a_1 + a_2 + a_3 + \dots + a_n$

(1) $s_n = a_1 + \cancel{a_1 \cdot q} + \cancel{a_1 \cdot q^2} + \dots + a_1 \cdot q^{n-1} \quad / \cdot q$

(2) $s_n \cdot q = \cancel{a_1 \cdot q} + \cancel{a_1 q^2} + a_1 q^3 + \dots + a_1 q^{n-1} + a_1 q^n$

(1) − (2)

$s_n - s_n \cdot q = a_1 - a_1 \cdot q^n$

$s_n (1 - q) = a_1 \cdot (1 - q^n)$

$$s_n = \frac{1 - q^n}{1 - q} \cdot a_1$$

$$\boxed{s_n = a_1 \cdot \frac{q^n - 1}{q - 1}} \quad q \neq 1$$

URČTE a_1, a_2, a_3 a koeficient q! (7)

$a_1 + a_3 = 20$

$a_1 + a_2 + a_3 = 26$

$a_1 + a_1 \cdot q^2 = 20$

$a_1 + a_1 \cdot q + a_1 \cdot q^2 = 26$

$a_1(1+q^2) = 20$ $\quad \frac{(1)}{(2)}$

$a_1(1+q+q^2) = 26$

$$\frac{1+q^2}{1+q+q^2} = \frac{20}{26} \quad / \cdot 26(1+q+q^2)$$

$26(1+q^2) = 20(1+q+q^2)$

$26 + 26q^2 = 20 + 20q + 20q^2$

$6q^2 - 20q + 6 = 0$

$3q^2 - 10q + 3 = 0$

$3q^2 - 10q + 3 = 0$

$$q_{1,2} = \frac{10 \pm \sqrt{100-36}}{6} = \frac{10 \pm 8}{6} = \begin{cases} 3 \\ \frac{1}{3} \end{cases}$$

$\underline{q = 3}$

(1) $a_1(1+9) = 20$

$\underline{\underline{a_1 = 2}}$

$\underline{\underline{a_2 = 6}}$

$\underline{\underline{a_3 = 18}}$

$q = \frac{1}{3}$

$a_1 \cdot \left(1 + \frac{1}{9}\right) = 20 \Rightarrow a_1 \cdot \frac{10}{9} = 20$

$\underline{\underline{a_1 = 18}}$

$\underline{\underline{a_2 = 6}}$

$\underline{\underline{a_3 = 2}}$

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$$s_n = a_1 \cdot \frac{q^n - 1}{q - 1}$$

$\{s_1, s_2, s_3, s_4 \dots s_n \dots\}$

(a) $s_n \to a$ ¿ KEDY?

(b) $s_n \to \infty$ ak $q > 1$

$a_1, a_1 \cdot q, a_1 \cdot q^2, \dots$

$s = a_1 + a_1 \cdot q + a_1 \cdot q^2 + a_1 q^3 + \dots$

NEKONEČNÝ GEOM. RAD

MÁ SÚČET (JE KONVERGENTNÝ) $\iff |q| < 1$

$\underline{\underline{|q| < 1}}$

$-1 < q < 1$

Príkl.

$q = 2$

$\frac{2^n - 1}{2 - 1} \to \infty$

$q = \frac{1}{2}$ $\quad q^n = \left(\frac{1}{2}\right)^n \to 0$

$n \to \infty$

$q = \left(-\frac{1}{2}\right)$ $\quad q^n \to \left(-\frac{1}{2}\right)^n \to 0$

$q < -1$

$(-2)^n$ ~~...~~ nemá limitu

$$s_n = a_1 \cdot \frac{\overset{\to 0}{q^n} - 1}{q - 1} =$$

$\downarrow$

$$\underline{\underline{s}} = a_1 \cdot \frac{-1}{q-1} = \boxed{a_1 \cdot \frac{1}{1-q}}$$

RIEŠTE ROVNICU

(9)

$$\frac{5}{3} = x + 3x^2 + x^3 + 3x^4 + \dots$$

$$\frac{5}{3} = \underbrace{x(1+3x)} + x^3(1+3x) + x^5(1+3x) + \dots$$ JE GEOM. POSTUPNOSŤ

$$q = \frac{x^3(1+3x)}{x(1+3x)} = x^2$$

GEOM. RAD, AK JE NEK. POČET ČLENOV

$$q = \frac{x^5(1+3x)}{x^3(1+3x)} = x^2$$

$$\frac{5}{3} = (1+3x)\underbrace{\left[x + x^3 + x^5 + \dots\right]}$$

$$\frac{5}{3} = (1+3x) \cdot x \cdot \frac{1}{1-x^2}$$

$$\frac{5}{3} = \frac{x+3x^2}{1-x^2}$$

$$5(1-x^2) = 3(x+3x^2)$$

$$5 - 5x^2 = 3x + 9x^2$$

$$14x^2 + 3x - 5 = 0$$

$$x_{1,2} = \frac{-3 \pm \sqrt{9+280}}{28} = \frac{-3 \pm 17}{28} = \begin{cases} \frac{-20}{28} = \frac{-5}{7} \\ +\frac{1}{2} \end{cases}$$

$$\left(\frac{-5}{7}\right)^2 = q_1$$

$$\left(\frac{1}{2}\right)^2 = q_2$$

(10)

$x = 9{,}\overline{4}$

1. $9{,}\overline{4} = 9 + \frac{4}{10} + \frac{4}{10^2} + \frac{4}{10^3} + \frac{4}{10^4} + \dots = 9 + \frac{4}{10}\left(1 + \frac{1}{10} + \frac{1}{10^2} + \dots\right) =$

$$= 9 + \frac{4}{10} \cdot \frac{1}{1 - \frac{1}{10}} = 9 + \frac{4}{\cancel{10}} \cdot \frac{\cancel{10}}{9} = \qquad q = \frac{1}{10}$$

$$= 9 + \frac{4}{9} = \underline{\underline{\frac{85}{9}}};$$

2. (1) $x = 9{,}\overline{4} \quad / \cdot 10$

(2) $10x = 94{,}\overline{4}$

(2)−(1) $10x - x = 94{,}\overline{4} - 9{,}\overline{4}$

$9x = 85$

$x = \frac{85}{9}$

n. $9{,}\underline{\underline{\overline{35}}} = x \quad (1) \quad / \cdot 100$

(2) $100x = 935{,}\overline{35}$

$99x = 926 \quad (2)-(1)$

$x = \frac{926}{99}$

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V banke uložíme K – poč. kapitál na p-% úrok

? budeme mať po n-rokoch?

Po 1. roku $K_1 = K + \frac{p}{100} \cdot K = K \cdot \left(1 + \frac{p}{100}\right)$

$$K_2 = K_1 + \frac{p}{100} \cdot K_1 = K_1 \cdot \left(1 + \frac{p}{100}\right) = \underline{K} \cdot \left(1 + \frac{p}{100}\right)^2$$

$$K_3 = K_2 + \frac{p}{100} K_2 = \underline{\underline{K_2}} \left(1 + \frac{p}{100}\right) = K \cdot \left(1 + \frac{p}{100}\right)^3$$

$$\vdots$$

$$K_n = K \cdot \left(1 + \frac{p}{100}\right)^n$$

PERCENTO = STOTINA

PROMILE = TISÍCINA