PR 4

(1) $$\lim_{x \to 2} \frac{\overbrace{x^3 - 2x - 4}^{8-4-4=0}}{\underbrace{x^2 - x - 2}_{4-2-2=0}} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x \to 2} \frac{\overbrace{3x^2 - 2}^{3 \cdot 4 - 2 = 10}}{\underbrace{2x - 1}_{4-1=3}} = \frac{10}{3}$$

(2) $$\lim_{x \to \infty} \frac{e^{2x}}{x^3} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x \to \infty} \frac{e^{2x} \cdot 2}{3x^2} =$$

$$\lim_{x \to \infty} \frac{e^{2x}}{x^3} = \lim_{x \to \infty} \frac{\frac{1}{x^3}}{\frac{1}{e^{2x}}} = \lim_{x \to \infty} \frac{e^{2x}}{x^3}$$

$$= \frac{2}{3} \lim_{x \to \infty} \frac{e^{2x}}{x^2} \text{ ,,}\frac{\infty}{\infty}\text{''} \overset{L'H}{=} \frac{2}{3} \lim_{x \to \infty} \frac{e^{2x} \cdot \cancel{2}}{\cancel{2}x} \text{ ,,}\frac{\infty}{\infty}\text{''} \overset{L'H}{=}$$

$$= \frac{2}{3} \lim_{x \to \infty} \frac{e^{2x} \cdot 2}{1} = \frac{4}{3} \cdot \lim_{x \to \infty} e^{2x} = \infty$$

$$= \infty$$

$$3 \quad \lim_{x \to 1} \left(\frac{1}{2\ln x} - \frac{1}{x^2 - 1} \right) = \lim_{x \to 1} \frac{\overbrace{x^2 - 1 - 2\ln x}^{1-1=0}}{\underbrace{2\ln x \, (x^2 - 1)}_{0}}$$

$$= \text{,,}\frac{0}{0}\text{''}$$

L'H

$$= \lim_{x \to 1} \frac{2x - 2 \cdot \frac{1}{x}}{2 \cdot \frac{1}{x} \cdot (x^2 - 1) + 2 \ln x \cdot 2x} = \lim_{x \to 1} \frac{\cancel{2}\left(x - \frac{1}{x}\right)}{\cancel{2}\left(\cancel{x} - \frac{1}{x} + \ln x \, 2x\right)}$$

$$= \lim_{x \to 1} \frac{\frac{x^2 - 1}{\cancel{x}}}{\frac{x^2 - 1 + 2x^2 \ln x}{\cancel{x}}} = \lim_{x \to 1} \frac{\overbrace{x^2 - 1}^{0}}{\underbrace{x^2 - 1 + 2x^2 \ln x}_{0}}$$

L'H

$$= \lim_{x \to 1} \frac{2x}{2x + 4x \ln x + 2x^{\cancel{2}} \cdot \frac{1}{\cancel{x}}} =$$

$$= \lim_{x \to 1} \frac{\cancel{2}(x)}{\cancel{2}(x + 2x\ln x + x)} = \frac{1}{1+0+1} = \underline{\underline{\frac{1}{2}}}$$

$$0 \cdot (-\infty)$$

4 $$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{x^{-1}} \overset{L'H}{=}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -\lim_{x \to 0^+} \frac{x^{\cancel{2}}}{\cancel{x}} = \underline{\underline{0}}$$

(5) $\lim\limits_{x \to 0^+} \sqrt{x} \cdot \ln x \quad {}^{0 \cdot (-\infty)} = \lim\limits_{x \to 0^+} \frac{\ln x}{x^{-\frac{1}{2}}} \overset{L'H}{=}$

$= \lim\limits_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{2} x^{-\frac{3}{2}}} = -2 \lim\limits_{x \to 0^+} x^{\frac{3}{2}-1} = \underline{\underline{0}}$

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(6) $\lim\limits_{x \to 0} \cos x^{\cot^2 x} \quad {}^{1^{\infty}}$

$= \lim\limits_{x \to 0} e^{\ln \cos x \cdot \cot^2 x} =$

$$= e^{\boxed{\lim\limits_{x \to 0} \cot g^2 x \ln(\cos x)}}$$

$$\Rightarrow \lim_{x \to 0} \cot g^2 x \ln(\cos x) = \lim_{x \to 0} \frac{\overset{=0}{\ln(\cos x)}}{\underset{=0}{tg^2 x}} \quad \text{"}\frac{0}{0}\text{"}$$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{\frac{1}{\cos x} \cdot (-\sin x)}{2 \underbrace{tg x}_{\frac{\sin x}{\cos x}} \cdot \frac{1}{\cos^2 x}} = \lim_{x \to 0} \frac{\frac{-\cancel{\sin x}}{\cancel{\cos x}}}{\frac{2 \cancel{\sin x}}{\cos^{\cancel{3}\,2} x}} =$$

$$\lim_{x\to 0} \frac{-\cos^2 x}{2} = -\frac{1}{2}$$

$$\Rightarrow e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

(7) $$\lim_{x\to 0} \frac{\overset{0}{x-\sin x}}{\underset{0}{x^3}} \overset{\text{"}\frac{0}{0}\text{"}}{\underset{L'H}{=}} \lim_{x\to 0} \frac{\overset{1-1=0}{1-\cos x}}{\underset{0}{3x^2}} \overset{\text{"}\frac{0}{0}\text{"}}{\underset{L'H}{=}}$$

$$= \lim_{x \to 0} \frac{\sin x}{6x} \overset{L'H}{=} \lim_{x \to 0} \frac{\cos x}{6} = \frac{1}{6}$$

$$"\frac{0}{0}"$$

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8 $$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right) = \lim_{x \to 0} \frac{\sin x - x}{x \sin x} \overset{L'H}{=}$$

$$"\frac{0}{0}"$$

$$\lim_{x \to 0} \frac{\cos x - 1}{\sin x + x \cdot \cos x} \overset{L'H}{=}$$

$$= \lim_{x \to 0} \frac{-\sin x}{\underbrace{\cos x}_{1} + \underbrace{\cos x}_{1} + \underbrace{x(-\sin x)}_{0}} = \frac{0}{2} = \underline{\underline{0}}$$

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$$\lim_{x \to \infty} \sqrt[x]{x} = \lim_{x \to \infty} e^{\ln x^{\frac{1}{x}}} =$$

$$= \lim_{x \to \infty} e^{\frac{1}{x} \ln x} = e^{\boxed{\lim_{x \to \infty} \frac{1}{x} \ln x}}$$

$$\lim_{x \to \infty} \frac{\overset{\infty}{\ln x}}{\underset{\infty}{x}} \overset{L'H}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{1} =$$

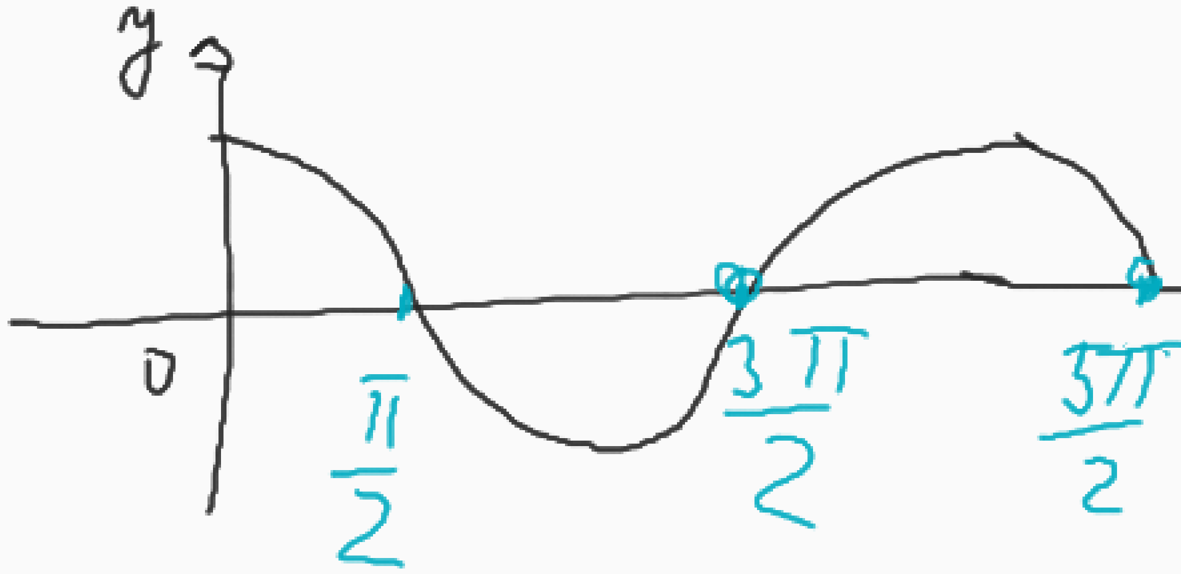
$$= \lim_{x \to \infty} \frac{1}{x} = 0$$

$$\Rightarrow e^{0} = \underline{\underline{1}}$$

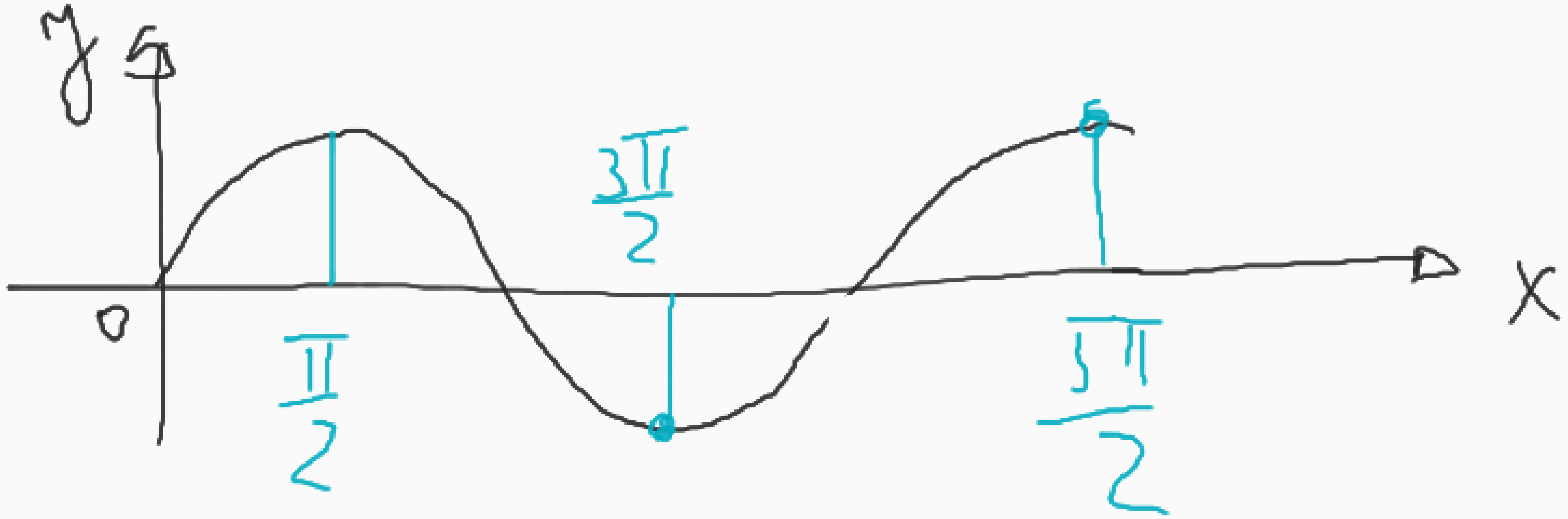
(10) $$\lim_{x\to\infty} x\cdot e^{-x} = \lim_{x\to\infty} \frac{\overset{\infty}{x}}{\underset{\infty}{e^{x}}} \overset{L'H}{=}$$

$$= \lim_{x\to\infty} \frac{1}{e^{x}} = \underline{\underline{0}}$$

(11) $$\lim_{x\to\frac{\pi}{2}} \frac{\overset{0}{\cos(5x)}}{\underset{0}{\cos(3x)}} \quad „\frac{0}{0}" \overset{L'H}{=}$$



$$\lim_{x \to \frac{\pi}{2}} \frac{+\overbrace{\sin(5x)}^{1}\cdot 5}{+\underbrace{\sin(3x)}_{-1}\cdot 3} = \frac{5}{3}\cdot\frac{1}{(-1)} = -\frac{5}{3}$$



$$\lim_{x\to\frac{\pi}{2}}\left(\operatorname{tg}x-\frac{1}{\cos x}\right)=\lim_{x\to\frac{\pi}{2}}\left(\frac{\sin x}{\cos x}-\frac{1}{\cos x}\right)$$

$$=\lim_{x\to\frac{\pi}{2}}\frac{\overset{0}{\sin x-1}}{\underset{0}{\cos x}}\ \underset{L'H}{\overset{\text{"}\frac{0}{0}\text{"}}{=}}\ \lim_{x\to\frac{\pi}{2}}\frac{\overset{=0}{\cos x}}{\underset{=-1}{-\sin x}}$$

$$=\frac{0}{-1}=\underline{\underline{0}}$$

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$$\lim_{x \to \infty} \frac{\overset{\infty}{\ln x}}{\underset{\infty}{x}} \overset{L'H}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = \underline{\underline{0}}$$