SemMat1 cv1 – Úprava výrazov – riešenia

1) Pre $a \neq \pm b$ platí $\dfrac{\dfrac{a}{a+b}+\dfrac{b}{a-b}}{\dfrac{a}{a-b}-\dfrac{b}{a+b}} = \dfrac{\dfrac{a(a-b)+b(a+b)}{(a+b)(a-b)}}{\dfrac{a(a+b)-b(a-b)}{(a+b)(a-b)}} = \dfrac{a^2-ab+ab+b^2}{a^2+ab-ab+b^2} = \dfrac{a^2+b^2}{a^2+b^2} = 1$

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2) $$\left(\frac{\sqrt{10}+1}{3}\right)^{365}\left(\frac{\sqrt{10}-1}{3}\right)^{365} = \left[\left(\frac{\sqrt{10}+1}{3}\right)\left(\frac{\sqrt{10}-1}{3}\right)\right]^{365} = \left[\frac{\left(\sqrt{10}+1\right)\left(\sqrt{10}-1\right)}{9}\right]^{365} = \left[\frac{10-1}{9}\right]^{365} = 1$$

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3)

$$1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\sqrt{2}}}} = 1+\cfrac{1}{2+\cfrac{1}{\cfrac{2\left(1+\sqrt{2}\right)+1}{1+\sqrt{2}}}} = 1+\cfrac{1}{2+\cfrac{1}{\cfrac{3+2\sqrt{2}}{1+\sqrt{2}}}} = 1+\cfrac{1}{2+\cfrac{1+\sqrt{2}}{3+2\sqrt{2}}} =$$

$$= 1+\cfrac{1}{\cfrac{2\left(3+2\sqrt{2}\right)+1+\sqrt{2}}{3+2\sqrt{2}}} = 1+\cfrac{1}{\cfrac{7+5\sqrt{2}}{3+2\sqrt{2}}} = 1+\frac{3+2\sqrt{2}}{7+5\sqrt{2}} = \frac{7+5\sqrt{2}+3+2\sqrt{2}}{7+5\sqrt{2}} =$$

$$= \frac{10+7\sqrt{2}}{7+5\sqrt{2}} = \sqrt{2}\left(\frac{7+5\sqrt{2}}{7+5\sqrt{2}}\right) = \sqrt{2}$$

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4) Pre $a>0$ platí $\sqrt[5]{\left(\dfrac{\sqrt{a}\,a^{-1}}{\sqrt[3]{a}}\right)^{-3}} = \left(\dfrac{a^{\frac{1}{2}}a^{-1}}{a^{\frac{1}{3}}}\right)^{-\frac{3}{5}} = \left(a^{\frac{1}{2}-1-\frac{1}{3}}\right)^{-\frac{3}{5}} = \left(a^{-\frac{5}{6}}\right)^{-\frac{3}{5}} = a^{\left(-\frac{3}{5}\right)\left(-\frac{5}{6}\right)} = a^{\frac{1}{2}} = \sqrt{a}$

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5) Pre $a,b \geq 0, (a+b) > 0$ máme

$$\frac{(a-\sqrt{b})(b+\sqrt{a})+\sqrt{ab}\left(1-\sqrt{ab}\right)}{a+b+\sqrt{ab}} = \frac{ab+a\sqrt{a}-b\sqrt{b}-\sqrt{ab}+\sqrt{ab}-ab}{a+b+\sqrt{ab}} = \frac{a\sqrt{a}-b\sqrt{b}}{a+b+\sqrt{ab}} =$$

$$\frac{a\sqrt{a}-b\sqrt{b}}{a+b+\sqrt{ab}}\left(\frac{a+b-\sqrt{ab}}{a+b-\sqrt{ab}}\right) = \frac{\left(a\sqrt{a}-b\sqrt{b}\right)\left(a+b-\sqrt{ab}\right)}{\left(a+b\right)^2-ab} = \frac{a^2\sqrt{a}-ab\sqrt{b}+ab\sqrt{a}-b^2\sqrt{b}-a^2\sqrt{b}+b^2\sqrt{a}}{a^2+b^2+ab} =$$

$$= \frac{a^2\left(\sqrt{a}-\sqrt{b}\right)+ab\left(\sqrt{a}-\sqrt{b}\right)+b^2\left(\sqrt{a}-\sqrt{b}\right)}{a^2+b^2+ab} = \frac{\left(a^2+ab+b^2\right)\left(\sqrt{a}-\sqrt{b}\right)}{a^2+b^2+ab} = \sqrt{a}-\sqrt{b}$$

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6) Pre $x>0$ platí

$$\left[\frac{\left(\sqrt{7}+1\right)^2-\frac{7-\sqrt{7x}}{\sqrt{7}-\sqrt{x}}}{\left(\sqrt{7}+1\right)^3-7\sqrt{7}+2}\right]=\left[\frac{\left(\sqrt{7}+1\right)^2-\frac{7-\sqrt{7x}}{\sqrt{7}-\sqrt{x}}\left(\frac{\sqrt{7}+\sqrt{x}}{\sqrt{7}+\sqrt{x}}\right)}{\left(\sqrt{7}+1\right)^3-7\sqrt{7}+2}\right]=\left[\frac{\left(\sqrt{7}+1\right)^2-\frac{7\sqrt{7}-7\sqrt{x}+7\sqrt{x}-x\sqrt{7}}{7-x}}{7\sqrt{7}+3\cdot 7+3\sqrt{7}+1-7\sqrt{7}+2}\right]=$$

$$\left[\frac{8+2\sqrt{7}-\frac{\sqrt{7}(7-x)}{7-x}}{3\sqrt{7}+24}\right]=\left[\frac{8+2\sqrt{7}-\sqrt{7}}{3\sqrt{7}+24}\right]=\left[\frac{8+\sqrt{7}}{3\left(8+\sqrt{7}\right)}\right]=\frac{1}{3}=3^{-1}$$

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7) Ak $x\neq 1,2,3$ máme

$$\frac{5}{x-2}+\frac{3}{x-3}-\frac{7}{x-1}=\frac{5\cdot(x-3)\cdot(x-1)+3\cdot(x-2)\cdot(x-1)-7\cdot(x-2)\cdot(x-3)}{(x-2)\cdot(x-3)\cdot(x-1)}=$$

$$\frac{5\left(x^2-4x+3\right)+3\left(x^2-3x+2\right)-7\left(x^2-5x+6\right)}{(x-1)\cdot(x-2)\cdot(x-3)}=\frac{x^2(5+3-7)+x(-20-9+35)+(15+6-42)}{(x-1)\cdot(x-2)\cdot(x-3)}=$$

$$\frac{x^2+6x-21}{(x-1)\cdot(x-2)\cdot(x-3)}$$

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8) Pre $a\neq 3,4,-1/3,-1/4$ máme

$$\left(\frac{a-3}{1+3a}-\frac{a-4}{1+4a}\right)\cdot\left(1+\frac{a-3}{1+3a}\cdot\frac{a-4}{1+4a}\right)^{-2}=\left(\frac{(a-3)\cdot(1+4a)-(a-4)\cdot(1+3a)}{(1+3a)\cdot(1+4a)}\right)\cdot\left(\frac{(1+3a)\cdot(1+4a)+(a-3)\cdot(a-4)}{(1+3a)\cdot(1+4a)}\right)^{-2}=$$

$$\left(\frac{\left(a-3+4a^2-12a\right)-\left(a-4+3a^2-12a\right)}{(1+3a)\cdot(1+4a)}\right)\cdot\left(\frac{\left(1+4a+3a+12a^2\right)+\left(a^2-4a-3a+12\right)}{(1+3a)\cdot(1+4a)}\right)^{-2}=$$

$$\left(\frac{a^2+1}{(1+3a)\cdot(1+4a)}\right)\cdot\left(\frac{13a^2+13}{(1+3a)\cdot(1+4a)}\right)^{-2}=\left(\frac{a^2+1}{(1+3a)\cdot(1+4a)}\right)\cdot\left(\frac{13\cdot\left(a^2+1\right)}{(1+3a)\cdot(1+4a)}\right)^{-2}=$$

$$\left(\frac{a^2+1}{(1+3a)\cdot(1+4a)}\right)\cdot\left(\frac{(1+3a)\cdot(1+4a)}{13\cdot\left(a^2+1\right)}\right)^{2}=\frac{\left(a^2+1\right)}{((1+3a)\cdot(1+4a))}.\frac{((1+3a)\cdot(1+4a))^2}{13^2\cdot\left(a^2+1\right)^2}=\frac{(1+3a)\cdot(1+4a)}{13^2\cdot\left(a^2+1\right)}$$

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9) Pre $a, x > 0, a \neq x$ máme

$$\left(\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a+x}}-\frac{\sqrt{a+x}}{\sqrt{a}+\sqrt{x}}\right)^{-2}-\left(\frac{\sqrt{a}-\sqrt{x}}{\sqrt{a+x}}-\frac{\sqrt{a+x}}{\sqrt{a}-\sqrt{x}}\right)^{-2}=\left(\frac{\left(\sqrt{a}+\sqrt{x}\right)^2-\left(\sqrt{a+x}\right)^2}{\sqrt{a+x}\left(\sqrt{a}+\sqrt{x}\right)}\right)^{-2}-\left(\frac{\left(\sqrt{a}-\sqrt{x}\right)^2-\left(\sqrt{a+x}\right)^2}{\sqrt{a+x}\left(\sqrt{a}-\sqrt{x}\right)}\right)^{-2}=$$

$$\left(\frac{a+x+2\sqrt{ax}-a-x}{\sqrt{a+x}\left(\sqrt{a}+\sqrt{x}\right)}\right)^{-2}-\left(\frac{a+x-2\sqrt{ax}-a-x}{\sqrt{a+x}\left(\sqrt{a}-\sqrt{x}\right)}\right)^{-2}=\left(\frac{2\sqrt{ax}}{\sqrt{a+x}\left(\sqrt{a}+\sqrt{x}\right)}\right)^{-2}-\left(\frac{-2\sqrt{ax}}{\sqrt{a+x}\left(\sqrt{a}-\sqrt{x}\right)}\right)^{-2}=$$

$$\left(\frac{\sqrt{a+x}\left(\sqrt{a}+\sqrt{x}\right)}{2\sqrt{ax}}\right)^{2}-\left(\frac{\sqrt{a+x}\left(\sqrt{a}-\sqrt{x}\right)}{-2\sqrt{ax}}\right)^{2}=\left(\frac{(a+x)\left(a+x+2\sqrt{ax}\right)}{4ax}\right)-\left(\frac{(a+x)\left(a+x-2\sqrt{ax}\right)}{4ax}\right)=$$

$$\frac{(a+x)\left(a+x+2\sqrt{ax}\right)-(a+x)\left(a+x-2\sqrt{ax}\right)}{4ax}=\frac{(a+x)\left(4\sqrt{ax}\right)}{4ax}=\frac{(a+x)\sqrt{ax}}{ax}=\frac{(a+x)}{\sqrt{ax}}$$

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10) Pre $u, v \geq 0, u \neq v$ máme

$$\frac{\left(\sqrt[4]{u}+\sqrt[4]{v}\right)^2+\left(\sqrt[4]{u}-\sqrt[4]{v}\right)^2}{u-v}:\frac{2}{\sqrt{u}-\sqrt{v}}=\frac{\left(\sqrt{u}+2\sqrt[4]{uv}+\sqrt{v}\right)+\left(\sqrt{u}-2\sqrt[4]{uv}+\sqrt{v}\right)}{u-v}\cdot\frac{\sqrt{u}-\sqrt{v}}{2}=$$

$$\frac{2\left(\sqrt{u}+\sqrt{v}\right)}{u-v}\cdot\frac{\sqrt{u}-\sqrt{v}}{2}=\frac{2\left(\sqrt{u}+\sqrt{v}\right)\left(\sqrt{u}-\sqrt{v}\right)}{2(u-v)}=\frac{2(u-v)}{2(u-v)}=1$$