

Seminár z mat. 1 – cv11 – Sústavy rovníc

1.

$$x^2 + y^2 = 25 \quad 2x^2 - 3y^2 = 5$$

$$s + t = 25 \quad 2s - 3t = 5$$

$$2s + 2t = 50 \quad 2s - 3t = 5$$

$$2s + 2t - (2s - 3t) = 50 - 5 = 45$$

$$5t = 45 \quad t = 9 \quad y = 3 \text{ alebo } y = -3$$

$$s = 25 - t = 25 - 9 = 16 \quad x = 4 \text{ alebo } y = -4$$

Riešenie: $(-3, -4), (-3, 4), (3, -4), (3, 4)$

2.

$$2x^2 + y^2 = 86 \quad 4y^2 - 5x^2 = 19$$

$$2s + t = 86 \quad 4t - 5s = 19$$

$$4s + 4t = 172 \quad 4t - 5s = 19$$

$$4s + 4t - (4t - 5s) = 172 - 19 = 153$$

$$9s = 153 \quad s = 17 \quad x = \sqrt{17} \text{ alebo } x = -\sqrt{17}$$

$$t = 86 - 2s = 86 - 34 = 52 \quad x = 2\sqrt{13} \text{ alebo } x = -2\sqrt{13}$$

Riešenie:

$(-2\sqrt{13}, -\sqrt{17}), (-2\sqrt{13}, \sqrt{17}), (2\sqrt{13}, -\sqrt{17}), (2\sqrt{13}, \sqrt{17})$

3.

$$x^2 + y^2 = 125 \quad x^2 - y^2 = 25$$

$$s + t = 125 \quad s - t = 25$$

$$s + t - (t - s) = 125 - 25 = 100$$

$$2s = 100 \quad s = 50 \quad x = \sqrt{50} \text{ alebo } x = -\sqrt{50}$$

$$t = 125 - s = 125 - 50 = 75 \quad y = 5\sqrt{3} \text{ alebo } x = -5\sqrt{3}$$

Riešenie:

$(-\sqrt{50}, -5\sqrt{3}), (-\sqrt{50}, 5\sqrt{3}), (\sqrt{50}, -5\sqrt{3}), (\sqrt{50}, 5\sqrt{3})$

4.

$$x^2 + y^2 = 25 \quad xy = 12$$

$$(x + y)^2 = 25 + 2 \cdot 12 \quad (x - y)^2 = 25 - 2 \cdot 12$$

$$(x + y)^2 = 49 \quad (x - y)^2 = 1$$

$$x + y = 7 \text{ alebo } x + y = -7$$

$$x - y = 1 \text{ alebo } x - y = -1$$

Riešenie: (4, 3), (-3,-4), (3,4),(-4,-3)

5.

$$x^2 + y^2 = 2a^2 \quad xy = a^2$$

$$(x + y)^2 = 4a^2 \quad (x - y)^2 = 0$$

$$x + y = 2a \text{ alebo } x + y = -2a$$

$$x - y = 0$$

Riešenie: (a,a), (-a,-a)

6.

$$x^2 + y^2 = 74 \quad 3x - 2y = 1$$

$$y = (3x - 1)/2$$

$$x^2 + (3x - 1)^2/4 = 74$$

$$4x^2 + (3x - 1)^2 = 74 * 4$$

$$4x^2 + 9x^2 - 6x + 1 = 296$$

$$13x^2 - 6x - 295 = 0$$

Riešenie:

$$x = 5 \quad y = 7 \text{ alebo } x = -\frac{59}{13} \quad y = -95/13$$

7.

$$x^2 = 40 - y^2 \quad x = 3y$$

$$9y^2 = 40 - y^2$$

$$10y^2 = 40$$

$$y^2 = 4$$

Riešenie:

$$y = 2 \quad x = 6 \text{ alebo } y = -2 \quad x = -6$$

8.

$$5x^2 + y = 3xy \quad 2x - y = 0$$

$$y = 2x$$

$$5x^2 + 2x = 3x * (2x)$$

$$5x^2 + 2x - 6x^2 = 0$$

$$x^2 - 2x = x(x - 2) = 0$$

Riešenie:

$$x = 0 \quad y = 0 \quad \text{alebo} \quad x = 2 \quad y = 4$$

9.

$$4x(x - 1) = y^2 \quad 2x + y - 1 = 0$$

$$y = 1 - 2x$$

$$4x(x - 1) = (1 - 2x)^2$$

$$4x^2 - 4x = 1 - 4x + 4x^2$$

$$0 = 1$$

Riešenie neexistuje.

10.

$$x^2 + y^2 = 1 \quad 3x + y = m$$

$$y = m - 3x$$

$$x^2 + (m - 3x)^2 = 1$$

$$x^2 + m^2 - 6mx + 9x^2 = 1$$

$$10x^2 - 6mx + m^2 - 1 = 0$$

$$D = 36m * m - 4 * (m * m - 1) * 10$$

$$D = 40 - 4m * m$$

Pre $m > \sqrt{10}$ a $m < \sqrt{10}$ riešenie neexistuje.

Pre $-\sqrt{10} < m < \sqrt{10}$ a $m < \sqrt{10}$

$$\text{riešenie } x = \frac{6m \pm \sqrt{40 - m * m}}{20}$$

$$y = \frac{2m \mp 3\sqrt{40 - m * m}}{20}$$

