Cv-02 LIMITY

1.1

1.1.

$$\lim_{x\to 1}\frac{x-1}{x^2+3x-4} \overset{\frac{0}{0}}{=} \lim_{x\to 1}\frac{\cancel{x-1}}{(x+4)\cancel{(x-1)}} = \lim_{x\to 1}\frac{1}{x+4} = \frac{1}{5}$$

1.2

1.2)

$$\lim_{x\to -4}\frac{x-1}{x^2+3x-4} = \lim_{x\to -4}\frac{x-1}{(x+4)(x-1)} = \lim_{x\to -4}\frac{1}{x+4}$$

$$\lim_{x\to -4^-}\frac{1}{x+4} = -\infty$$

$$\lim_{x\to -4^+}\frac{1}{(x+4)} = +\infty$$

$$\lim_{x\to -4}\frac{x-1}{x^2+3x-4} \text{ neexistuje } \nexists$$

1.3

1.3

1a)

$$\lim_{x\to 1}\frac{x^3-2x^2-5x+6}{x^2+2x-3} \overset{\frac{0}{0}}{=} \lim_{x\to 1}\frac{(x^2-x-6)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}} = \lim_{x\to 1}\frac{x^2-x-6}{x+3} = \frac{1-1-6}{4} = \frac{-6}{4} = \boxed{-\frac{3}{2}}$$

$$\begin{aligned}
&x^3-2x^2-5x+6 : (x-1) = x^2-x-6\\
&\underline{-(x^3-x^2)}\\
&\quad -x^2-5x+6\\
&\quad \underline{-(-x^2+x)}\\
&\qquad -6x+6\\
&\qquad \underline{-(-6x+6)}\\
&\qquad\quad 0
\end{aligned}$$

1.4-1.6

1.4 $$\lim_{x\to0}\left(\frac{3-2x}{2+5x}\right)^{\frac{\sqrt{x+1}-1}{x}} = \lim_{x\to0}\left(\frac{3-2x}{2+5x}\right)^{\frac{\sqrt{x+1}-1}{x}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}}$$

$\{(a-b)(a+b)=a^2-b^2\}$

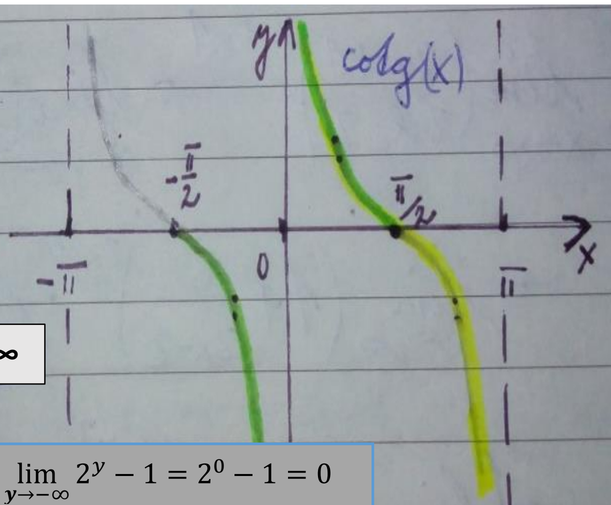
$$=\lim_{x\to0}\left(\frac{3-2x}{2+5x}\right)^{\frac{x+1-1}{x(\sqrt{x+1}+1)}} = \lim_{x\to0}\left(\frac{3-2x}{2+5x}\right)^{\frac{1}{\sqrt{x+1}+1}} = \left(\frac{3}{2}\right)^{\frac{1}{2}} = \sqrt{\frac{3}{2}}$$

1.5 $$\lim_{x\to\frac{\pi}{4}}\left(2^{\operatorname{cotg} x}-1\right) = 2^{\lim_{x\to\frac{\pi}{4}}\operatorname{cotg}(x)}-1 = 2^1-1 = 1$$

1.6 $$\lim_{x\to\frac{\pi}{2}}\left(2^{\operatorname{cotg} x}-1\right) = 2^{\lim_{x\to\frac{\pi}{2}}\operatorname{cotg} x}-1 = 2^0-1 = 0$$

1.7

$$\lim_{x\to0}\left(2^{\operatorname{cotg} x}-1\right):$$



$$\lim_{x\to0+}\left(2^{\operatorname{cotg}(x)}-1\right) = 2^{\lim_{x\to0+}\operatorname{cotg}(x)}-1 = \boxed{+\infty}$$

$$\lim_{x\to0-}\left(2^{\operatorname{cotg}(x)}-1\right) = 2^{\lim_{x\to0-}\operatorname{cotg}(x)}-1 = \boxed{\lim_{y\to-\infty} 2^y - 1 = 2^0 - 1 = 0}$$

Lebo $\lim_{x\to0^-} cotg(x) = -\infty$ a $\lim_{x\to-\infty} 2^x = 0$

$$\lim_{x\to0-}\left(2^{\operatorname{cotg}(x)}-1\right) \neq \lim_{x\to0+}\left(2^{\operatorname{cotg}(x)}-1\right) \Rightarrow \lim_{x\to0}\left(2^{\operatorname{cotg}(x)}-1\right)\text{ NEExistuje}$$

Pozn. k pr.1.7: **nakreslite si graf funkcie $y = 2^x$.**

1.8 – 1.10:

1.8

$$\lim_{x\to-\infty}\frac{4x^3-2x^2+7}{7x^3-3x^2+x-1}\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=}\lim_{x\to-\infty}\frac{4x^3-2x^2+7}{7x^3-3x^2+x-1}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}=$$

$$=\lim_{x\to-\infty}\frac{4-\frac{2}{x}+\frac{7}{x^3}}{7-\frac{3}{x}+\frac{1}{x^2}-\frac{1}{x^3}}=\frac{4}{7}\ \checkmark$$

1.10

$$\lim_{x\to-\infty}\frac{|x|}{\sqrt{x^2-1}}\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=}\lim_{x\to-\infty}\frac{\sqrt{x^2}\cdot\sqrt{x^2-1}}{\sqrt{x^2-1}\cdot\sqrt{x^2-1}}=\lim_{x\to-\infty}\frac{\sqrt{x^2(x^2-1)}}{x^2-1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}=$$

{poznámka: $|x|=\sqrt{x^2}$}

$$=\lim_{x\to-\infty}\frac{\frac{\sqrt{x^4-x^2}}{x^2}}{1-\frac{1}{x^2}}=\lim_{x\to-\infty}\frac{\sqrt{\frac{x^4-x^2}{x^4}}}{1-\frac{1}{x^2}}=$$

$$=\lim_{x\to-\infty}\frac{\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x^2}}=\frac{\sqrt{\lim\limits_{x\to-\infty}1-\frac{1}{x^2}}}{\lim\limits_{x\to-\infty}\left(1-\frac{1}{x^2}\right)}=\frac{\sqrt{1}}{1}=1$$

1.9

$$\lim_{x\to\infty}\frac{x^2+3x-5}{2x^3-4x+1}\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=}\lim_{x\to\infty}\frac{\frac{x^2}{x^3}+\frac{3x}{x^3}-\frac{5}{x^3}}{\frac{2x^3}{x^3}-\frac{4x}{x^3}+\frac{1}{x^3}}=\lim_{x\to\infty}\frac{\frac{1}{x}+\frac{3}{x^2}-\frac{5}{x^3}}{2-\frac{4}{x^2}+\frac{1}{x^3}}=\frac{0}{2}=0$$

1.11

$$\lim_{x\to\infty}x\left(\sqrt{x^2+9}-x\right)=\lim_{x\to\infty}x\cdot\left(\sqrt{x^2+9}-x\right)\cdot\frac{\sqrt{x^2+9}+x}{\sqrt{x^2+9}+x}=$$

$$=\lim_{x\to\infty}\frac{x\left(x^2+9-x^2\right)}{\sqrt{x^2+9}+x}=\lim_{x\to\infty}\frac{9x}{x\left(\sqrt{1+\frac{9}{x^2}}+1\right)}=$$

$$\sqrt{x^2+9}+x\ \to\ \sqrt{x^2\left(1+\frac{9}{x^2}\right)}+x\ \to\ \sqrt{x^2}\cdot\sqrt{1+\frac{9}{x^2}}+x$$

$$=\lim_{x\to\infty}\frac{9}{\sqrt{1+\frac{9}{x^2}}+1}=$$

$$=\frac{9}{\lim\limits_{x\to\infty}\sqrt{1+\frac{9}{x^2}}+1}=\frac{9}{2}$$

1.12

$$L=\lim_{x\to\infty} 3^{-x}\cdot\cos(5x) = \lim_{x\to\infty}\left(3^{-1}\right)^{x}\cdot\cos(5x) = \underbrace{\lim_{x\to\infty}\left(\frac{1}{3}\right)^{x}}_{\to 0}\cdot\cos(5x)=0$$

lebo $g(x)=\cos(5x)$ je ohraničená: $-1\le g(x)\le 1$ pre $\forall x\to\infty$.

1.13

ALE: $K=\lim_{x\to-\infty} 3^{-x}\cdot\cos(5x) = \left\{\text{subst.: } t=-x \Rightarrow \lim_{x\to-\infty} 3^{-x} = \lim_{t\to+\infty} 3^{t}=\infty\right\}$

a potom: $\lim_{x\to-\infty} 3^{-x}\cdot\cos(5x)\ \nexists$, lebo hodnoty $3^{-x}\cdot\cos(5x)$ pre $x\to\infty$ budú oscilovať $\begin{matrix}+\infty\\-\infty\end{matrix}$!

2.1

$$\lim_{x\to0}\frac{\sin x}{x}=1$$

1e) $$\lim_{x\to0}\frac{tg(5x)}{tg(6x)} \overset{\frac{0}{0}}{=} \lim_{x\to0}\frac{\frac{\sin 5x}{\cos 5x}}{\frac{\sin 6x}{\cos 6x}} = \lim_{x\to0}\frac{\sin(5x)\cdot\cos(6x)}{\sin(6x)\cos(5x)} =$$

$$=\lim_{x\to0}\frac{\frac{\sin(5x)}{5x}\cdot 5x\cdot\cos(6x)}{\frac{\sin 6x}{6x}\cdot 6x\cdot\cos(5x)} = \frac{\overbrace{\lim_{x\to0}\frac{\sin 5x}{5x}}^{=1}}{\underbrace{\lim_{x\to0}\frac{\sin 6x}{6x}}_{=1}}\cdot\lim_{x\to0}\frac{5x\cdot\cos(6x)}{6x\cdot\cos(5x)} =$$

$$=\lim_{x\to0}\frac{5\cancel{x}\cdot\cos(6x)}{6\cancel{x}\cos(5x)} = 1\cdot\lim_{x\to0}\frac{5\cdot\cos 0}{6\cdot\cos 0} = \frac{5}{6}$$

2.2

$$\lim_{x\to0}\left[\frac{\sqrt{x+4}-2}{\sin(2x)}+\ln(1-x^2)\right] = \lim_{x\to0}\frac{\sqrt{x+4}-2}{\sin(2x)}+\underbrace{\lim\ln(1-x^2)}_{0} =$$

$$=\lim_{x\to0}\frac{\sqrt{x+4}-2}{\sin(2x)} \overset{\frac{0}{0}}{=} \lim_{x\to0}\frac{(\sqrt{x+4}-2)(\sqrt{x+4}+2)}{\sin(2x)(\sqrt{x+4}+2)} =$$

$$=\lim_{x\to0}\frac{x+4-4}{\sin(2x)(\sqrt{x+4}+2)} = \lim_{x\to0}\frac{x}{\sin(2x)(\sqrt{x+4}+2)} =$$

$$=\lim_{x\to0}\frac{x}{\sin(2x)}\cdot\underbrace{\lim_{x\to0}\frac{1}{\sqrt{x+4}+2}}_{=\frac{1}{4}} = \lim_{x\to0}\frac{2x}{2\sin(2x)}\cdot\frac{1}{4} =$$

$$=\frac{1}{2}\cdot\frac{1}{4}\cdot\lim_{x\to0}\frac{2x}{\sin(2x)} = \frac{1}{8}$$

$$\boxed{\lim_{\alpha\to0}\frac{\sin\alpha}{\alpha}=1}$$

3.1

$$\lim_{v\to\infty}\left(1+\frac{1}{v}\right)^{v}=e$$

$$\lim_{x\to\infty}\left(\frac{2x+1}{2x-2}\right)^{2x-1}=\lim_{x\to\infty}\left(\frac{2x-2+3}{2x-2}\right)^{2x-1}=$$

$$=\lim_{x\to\infty}\left(1+\frac{3}{2x-2}\right)^{2x-1}=\lim_{x\to\infty}\left(1+\frac{1}{\frac{2x-2}{3}}\right)^{\frac{2x-2}{3}\cdot 3+1}=$$

$$v\to\infty$$

$$=\left(\lim_{x\to\infty}\left(1+\frac{1}{\frac{2x-2}{3}}\right)^{\frac{2x-2}{3}}\right)^{3}\cdot\lim_{x\to\infty}\left(1+\frac{1}{\frac{2x-2}{3}}\right)^{1}=e^{3}\cdot 1=\underline{\underline{e^{3}}}$$

$$\frac{2x-2}{3}\to\infty$$

3.2

$$\lim_{x\to\infty}x\left[\ln(x+1)-\ln x\right]=\lim_{x\to\infty}x\cdot\ln\frac{x+1}{x}=$$

$$=\lim_{x\to\infty}x\cdot\ln\left(1+\frac{1}{x}\right)=\lim_{x\to\infty}\ln\left(1+\frac{1}{x}\right)^{x}=$$

$$=\ln\left(\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right)=\ln e=\underline{\underline{1}}$$

3.3

Spec.: „1^{∞}"

$$\left|\lim_{v\to\infty}\left(1+\frac{1}{v}\right)^{v}=e\right|\quad\lim_{v\to\infty}\left(1-\frac{1}{v}\right)^{v}=\frac{1}{e}\Bigg|$$

3\. 2.1d) $$\lim_{x\to\infty}\left(\frac{x+1}{x-2}\right)^{2x-1}\overset{„1^{\infty}"}{=}\lim_{x\to\infty}\left(\frac{x+1-3+3}{x-2}\right)^{2x-1}=$$

$$=\lim_{x\to\infty}\left(\frac{x-2+3}{x-2}\right)^{2x-1}=\lim_{x\to\infty}\left(1+\frac{3}{x-2}\right)^{2x-1}=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x-2}{3}}\right)^{2x-1}=$$

$$=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x-2}{3}}\right)^{\frac{2(x-2)+3}{3}\cdot 3}=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x-2}{3}}\right)^{\frac{6(x-2)+9}{3}}=$$

$$=\lim_{x\to\infty}\left(\left(1+\frac{1}{\frac{x-2}{3}}\right)^{\frac{x-2}{3}}\right)^{6}\cdot\lim_{x\to\infty}\left(1+\frac{1}{\frac{x-2}{3}}\right)^{3}=\left\{\begin{array}{l}\text{resp. subst.:}\\ v=\frac{x-2}{3}\\ x\to\infty\Rightarrow v\to\infty\end{array}\right\}$$

$$=\lim_{v\to\infty}\left(\left(1+\frac{1}{v}\right)^{v}\right)^{6}\cdot\lim_{x\to\infty}\left(1+\frac{3}{x-2}\right)^{3}=e^{6}\cdot 1^{3}=\underline{\underline{e^{6}}}$$

3.4 – 3.5

$$\lim_{x\to 0}\frac{2^{2x+1}-2}{x} \overset{\text{„}\frac{0}{0}\text{"}}{=} \lim_{x\to 0}\frac{(2^2)^x\cdot 2^1-2}{x}=\lim_{x\to 0}\frac{2\cdot 4^x-2}{x}=$$

$$=2\cdot\lim_{x\to 0}\frac{4^x-1}{x}=\underline{\underline{2\cdot\ln 4}}\qquad \boxed{\lim_{x\to 0}\frac{a^x-1}{x}=\ln a}$$

$$\lim_{x\to 0}\frac{2^{2x+3}-8}{x}=\lim_{x\to 0}\frac{4^x\cdot 2^3-8}{x}=8\cdot\lim_{x\to 0}\frac{4^x-1}{x}=\underline{\underline{8\cdot\ln 4}}$$