(1) $$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

$$f'(\overset{0}{\overset{\shortparallel}{a}}) = \lim_{x \to \underset{0}{\underset{\shortparallel}{a}}} \frac{f(x) - f(a)}{x - a}$$

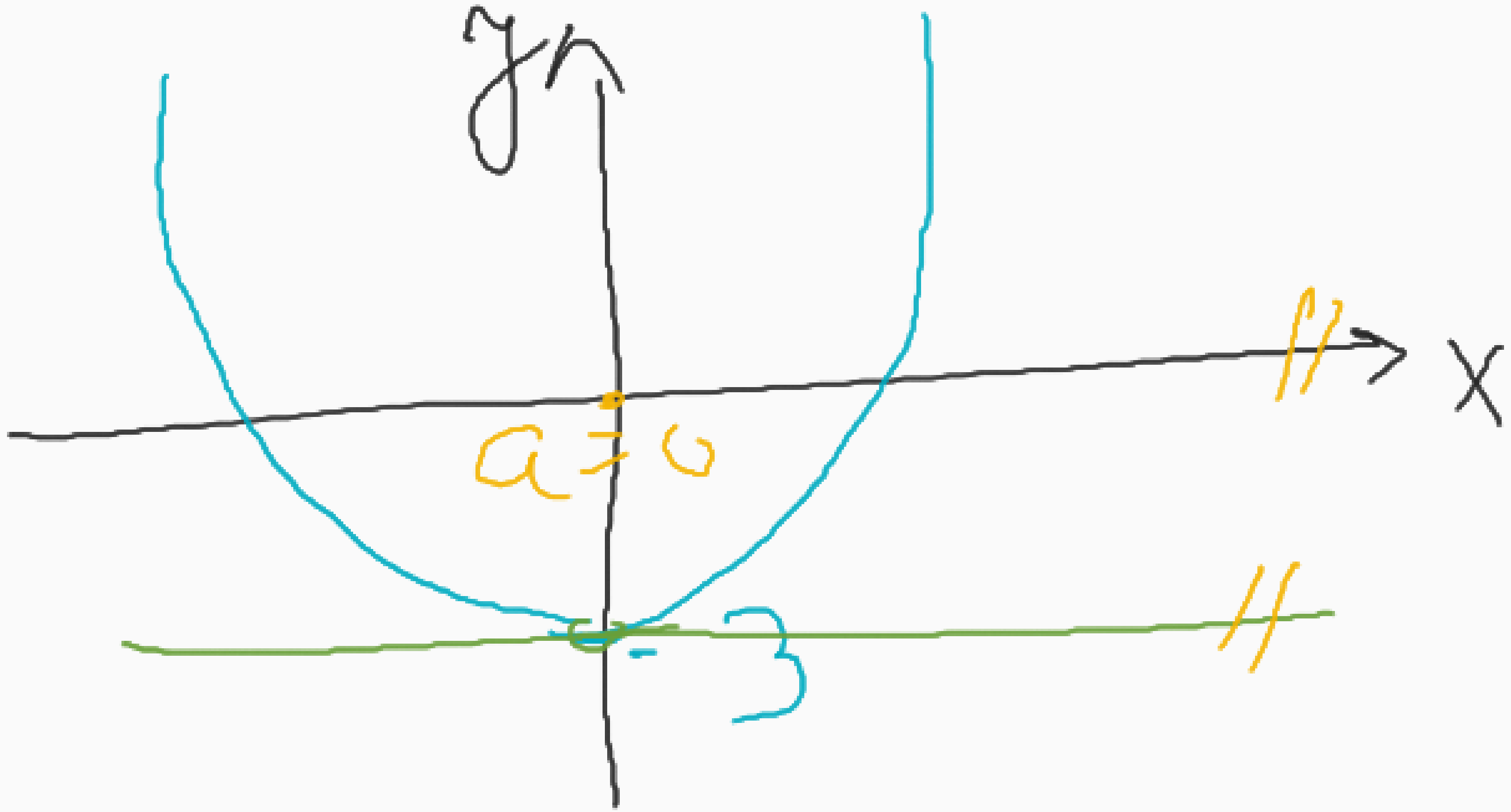
(a) $a = 0$

$$\boxed{f(x) = x^2 - 3}$$

$f(a) = 0 - 3 = -3$

$$f'(0) = \lim_{x \to 0} \frac{x^2 - 3 - (-3)}{x - 0} = \lim_{x \to 0} \frac{x^2 - 3 + 3}{x} =$$

$$= \lim_{x \to 0} \frac{x^2}{x} = \lim_{x \to 0} x = \underline{\underline{0}}$$



$$f(x) = x^2 - 3$$

b) $a = 3$

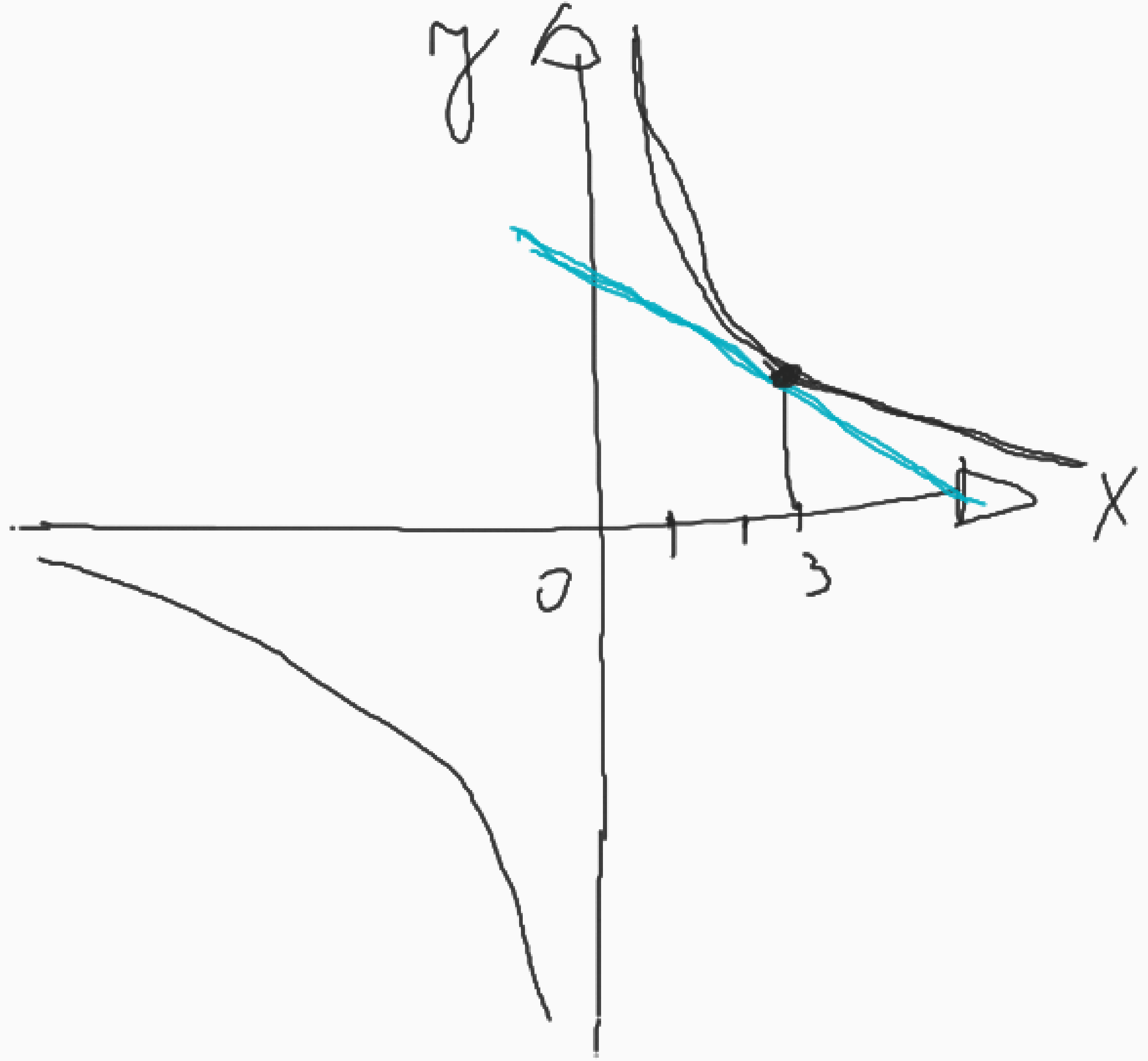
$f(x) = \frac{1}{x}$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$f(3) = \frac{1}{3}$

$$f'(3) = \lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3} = \lim_{x \to 3} \frac{\frac{3 - x}{3x}}{\frac{x - 3}{1}} =$$

$$= \lim_{x \to 3} \frac{(3 - x) \cdot 1}{3x(x - 3)} = \lim_{x \to 3} \frac{-\cancel{(x-3)}}{3x\cancel{(x-3)}} = -\frac{1}{9}$$

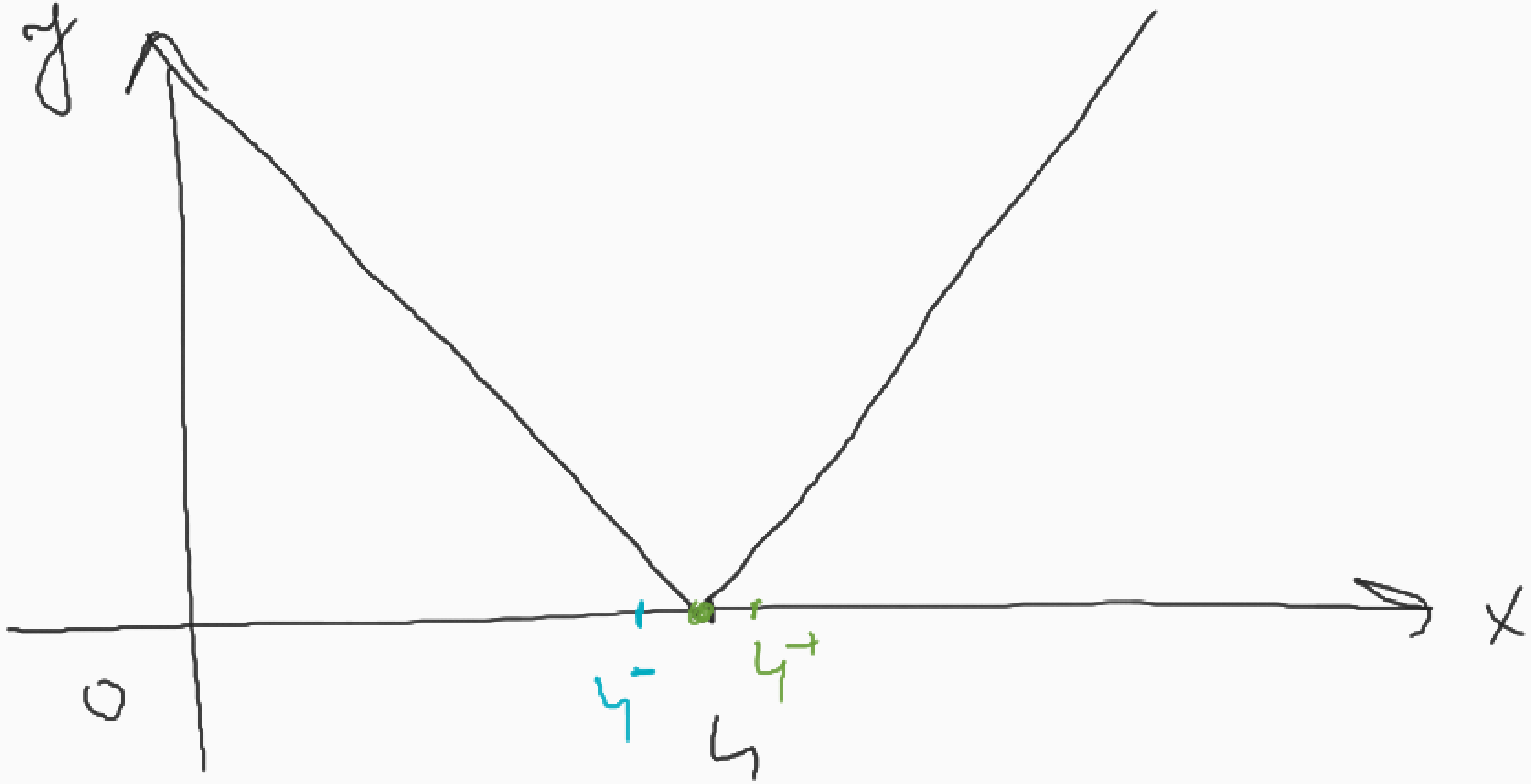


$$f(x) = \frac{1}{x}$$

(c) $a = 4$; $f(x) = |2x-8|$; $f(a) = f(4) = 0$

$$f'(4) = \lim_{x \to 4} \frac{|2x-8| - 0}{x-4} = 2 \lim_{x \to 4} \frac{|x-4|}{x-4}$$

$$\begin{cases} 2 \lim\limits_{x \to 4^-} \dfrac{-x+4}{x-4} = 2 \cdot (-1) = \underline{\underline{-2}} \\[2ex] 2 \lim\limits_{x \to 4^+} \dfrac{x-4}{x-4} = 2 \cdot 1 = \underline{\underline{2}} \end{cases} \quad \nexists$$


y
x
0
4⁻
4⁺
4

(a) $a = 0$ $\quad f(x) = x \cdot \cos x \quad x \leq 0$

$f(x) = x^2 \quad x > 0$

$$\lim_{x \to 0^-} \frac{x\cos x - 0}{x - 0} = \lim_{x \to 0^-} \frac{\cancel{x}\cos x}{\cancel{x}} = \underline{\underline{1}}$$

$$\lim_{x \to 0^+} \frac{x^2 - 0}{x - 0} = \lim_{x \to 0^+} \frac{x^{\cancel{2}}}{\cancel{x}} = \underline{\underline{0}}$$

$\nexists$

(3) a) $f(x) = 5x^{4} - 2x^3 + 3x^2 - 2$

$f'(x) = 5 \cdot 4x^3 - 6x^2 + 6x - 0$

$= 20x^3 - 6x^2 + 6x$

(b) $f(x) = x\sqrt{x} = x^1 \cdot x^{\frac{1}{2}} = x^{\frac{3}{2}}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$

(c) $f(x) = 2e^x$

$f'(x) = 0 \cdot e^x + 2e^x = 2e^x$

(a) $f(x) = \sqrt[3]{\frac{1}{x^2}} = \left(\frac{1}{x^2}\right)^{\frac{1}{3}} = \left(x^{-2}\right)^{\frac{1}{3}} = x^{-\frac{2}{3}}$

$f'(x) = -\frac{2}{3} x^{-\frac{2}{3}-\frac{3}{3}} = -\frac{2}{3} x^{-\frac{5}{3}}$

(e) $f(x) = \sqrt{x\sqrt{x\sqrt{x}}} = x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{8}} = x^{\frac{4+2+1}{8}}$

$= x^{\frac{7}{8}}$

$f'(x) = \frac{7}{8} x^{-\frac{1}{8}}$

f

$$f(x) = \frac{1}{x^2-1}$$

$$f'(x) = \frac{\cancel{0\cdot(x^2-1)} - 1\cdot(2x-0)}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

g

$$f(x) = \frac{x-1}{x+1}$$

$$f'(x) = \frac{1\cdot(x+1) - (x-1)\cdot 1}{(x+1)^2} = \frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2} = \frac{2}{(x+1)^2}$$

h $\quad f(x) = (x^3+8)(x-2)$

$$f'(x) = 3x^2(x-2) + (x^3+8)\cdot 1 =$$

$$= 3x^3 - 6x^2 + x^3 + 8 = \underline{\underline{4x^3 - 6x^2 + 8}}$$

i $\quad f(x) = 3.4^x + 2\log(x)$

$$f'(x) = 3\cdot 4^x \ln 4 + 2\frac{1}{x\ln 10}$$

g) $f(x) = \frac{e^x + \sin x}{2} = \frac{1}{2}\left(e^x + \sin x\right)$

$f'(x) = \frac{1}{2}\left(e^x + \cos x\right)$

h) $f(x) = x \ln x - 4x$

$f'(x) = 1 \cdot \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}} - 4 = \ln x + 1 - 4 =$

$= \ln x - 3$

(l) $f(x) = \frac{\ln x}{x}$

$$f'(x) = \frac{\frac{1}{\cancel{x}} \cdot \cancel{x} - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$$

(m) $f(x) = \frac{\sin x}{\cos x}$

$\cos^2 x + \sin^2 x = 1$

$$f'(x) = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x}$$

(n) $f(x) = \frac{\cos x}{\sin x}$

$= -\sin^2 x - \cos^2 x = -1$

$$f'(x) = \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x} = \frac{-1}{\sin^2 x}$$

(o) $f(x) = \operatorname{tg} x - 3x \log_4 x$

$$f'(x) = \frac{1}{\cos^2 x} - 3\left(1 \log_4 x + x \cdot \frac{1}{x \ln 4}\right)$$

(n) $f(x) = \sqrt[5]{x^9} = x^{\frac{9}{5}}$

$$f'(x) = \frac{9}{5} x^{\frac{4}{5}}$$

(r) $f(x) = 4^x + x^4$

$$f'(x) = 4^x \ln 4 + 4x^3$$

(1) $f(x) = \arcsin x - \dfrac{\operatorname{arctg} x}{\sqrt{x}}$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{\frac{1}{1+x^2}\cdot\sqrt{x} - \operatorname{arctg} x \cdot \frac{1}{2} x^{-\frac{1}{2}}}{x}$$

(t) $f(x) = \operatorname{ctg} x - \dfrac{3^x}{\sqrt[3]{\pi}}$

$$f'(x) = -\frac{1}{\sin^2 x} - \frac{1}{\sqrt[3]{\pi}} 3^x \ln 3$$

(14) $f(x) = (2+3x)^{17}$ $f'(x) = 17(2+3x)^{16} \cdot 3 =$

$= 51(2+3x)^{16}$

$f(x) = \sin(x^{-5}) \Rightarrow f'(x) = \cos(x^{-5}) \cdot (-5)x^{-6} =$

$= \frac{-5\cos(x^{-5})}{x^6}$

$f(x) = e^{3x} \Rightarrow f'(x) = e^{3x} \cdot 3$

$$f(x) = x^2 (x^3 - 1)^2$$

$$f'(x) = 2x(x^3-1)^2 + x^2 \cdot 2(x^3-1) \cdot 3x^2 =$$

$$= 2x(x^3-1)(x^3 - 1 + 3x^3) = 2x(x^3-1)(4x^3-1)$$

$$f(x) = \sqrt[3]{(2x+3)^2} = (2x+3)^{\frac{2}{3}}$$

$$f'(x) = \frac{2}{3}(2x+3)^{-\frac{1}{3}} \cdot 2 = \frac{4}{3\sqrt[3]{2x+3}}$$

$$f(x) = 4^{3x} + 3^{x^3}$$

$$f'(x) = 4^{3x} \cdot \ln 4 \cdot 3 + 3^{x^3} \cdot \ln 3 \cdot 3x^2$$

$$f(x) = \log(x + 2x^2)$$

$$f'(x) = \frac{1}{(x+2x^2)\ln 10} \cdot (1 + 4x)$$

$$f(x) = \sqrt{\ln\left(\frac{2x}{3}\right)} \Rightarrow f'(x) = \frac{1}{\cancel{2}}\left(\ln\left(\frac{2x}{3}\right)\right)^{-\frac{1}{2}} \cdot \frac{\cancel{2}}{3}$$

$$f(x) = \ln\sqrt{\frac{x-2}{x+2}} \Rightarrow f(x) = \frac{1}{\sqrt{\frac{x-2}{x+2}}} \cdot \frac{1}{2}\left(\frac{x-2}{x+2}\right)^{-\frac{1}{2}} \cdot \frac{1\cdot(x-2)-(x-2)\cdot 1}{(x+2)}$$

$$f(x) = tg^{2}(x^3) \Rightarrow$$

$$\Rightarrow f'(x) = 2\,tg(x^3) \cdot \frac{1}{\cos^2(x^3)} \cdot 3x^2$$