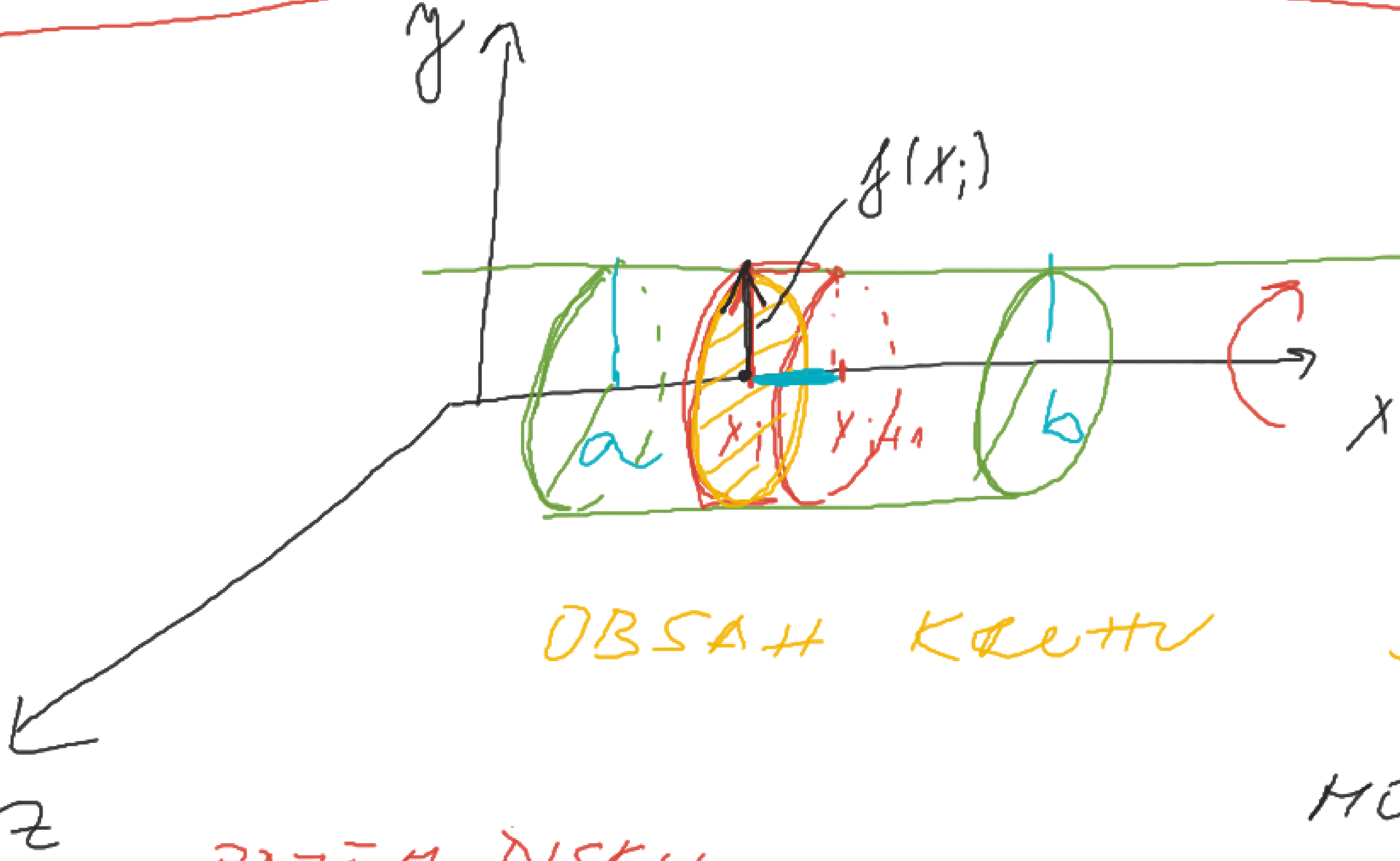
OBJEM ROTAČNÉHO TELESA – ROTÁCIA OKOLO o_x



OBSAH KRUHU $S = \pi r^2$

⇓

MOJE "r" JE $f(x_i)$

OBJEM DISKU (KOLIESKA SALÁMY) ⇒ (OBSAH KRUHU) × (HRÚBKA DISKU)

$(x_{i+1} - x_i)$ ⇐

OBJEM "CELEJ SALÁMY" ⇒ SUMA OBJEMOV
KOLIESOK

KONŠT.

$$V = \sum_{i=1}^{n} \pi \, f^2(x_i) \cdot \underbrace{(x_{i+1} - x_i)}_{dx}$$

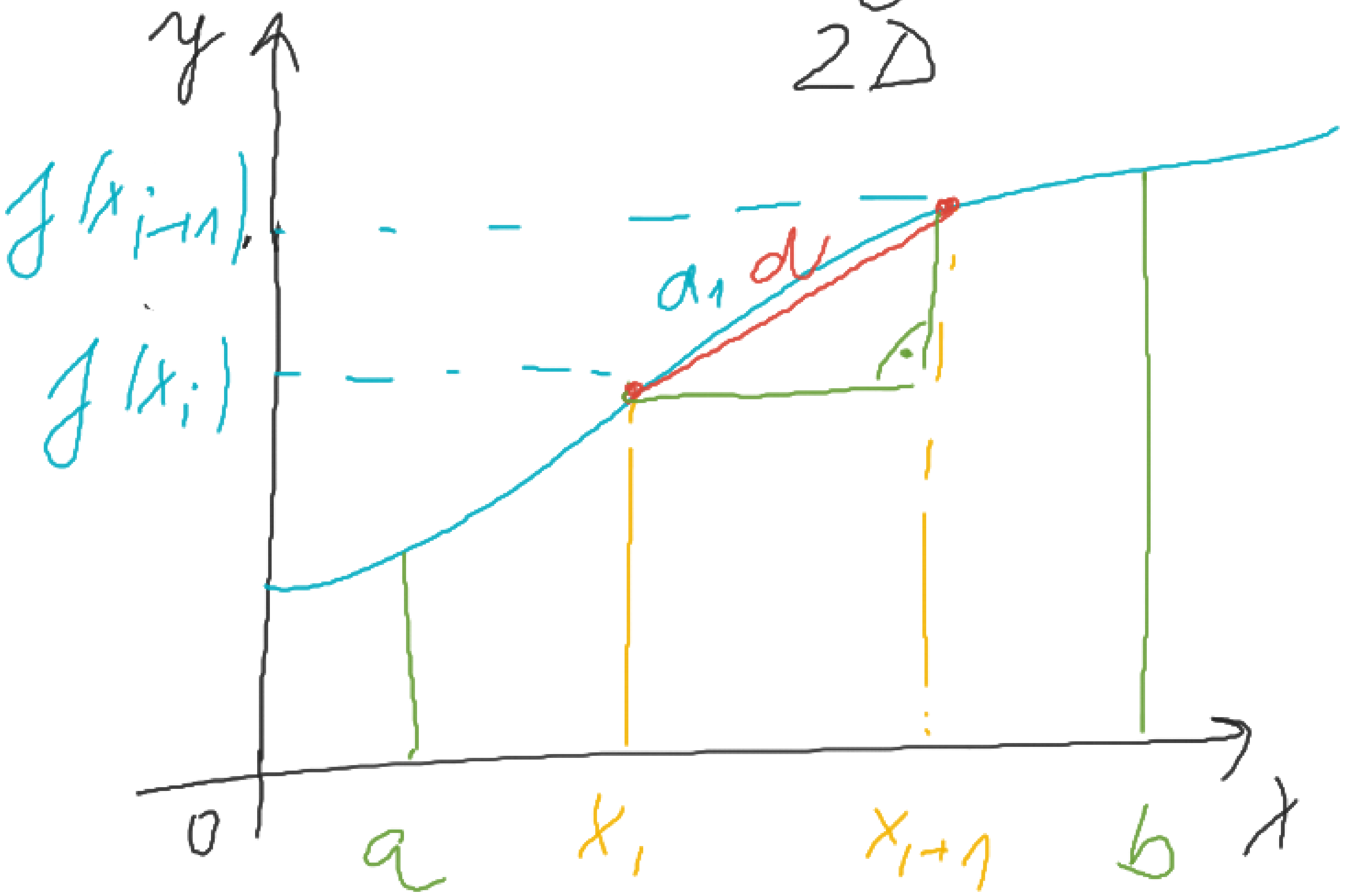
$$\boxed{V = \pi \int_a^b f^2(x) \, dx}$$

⇐ VZŤAH NA VÝPOČET OBJEMU ROTAČNÉHO TELESA - ROTÁCIA OKOLO O_X

DĹŽKA ROVINNEJ KRIVKY

⇓

2D



• PYTAGOROVA VETA

$d_1 \approx d$

$$d = \sqrt{(x_{i+1} - x_i)^2 + (f(x_{i+1}) - f(x_i))^2}$$

- $\boxed{\text{DERIVÁCIA } f'(x_i):}$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

$$f(x_{i+1}) - f(x_i) = f'(x_i) \cdot (x_{i+1} - x_i)$$

→ dosadíme do "d"

$$d = \sqrt{(x_{i+1}-x_i)^2 + f'^2(x_i)(x_{i+1}-x_i)^2} =$$

$$= \underbrace{(x_{i+1}-x_i)}_{dx}\sqrt{1+f'^2(x_i)}$$

$\boxed{\text{CELÁ KRIVKA :}}$

$$D = \sum_{i=1}^{n} \sqrt{1+f'^2(x_i)}\,\overbrace{(x_{i+1}-x_i)}^{dx}$$

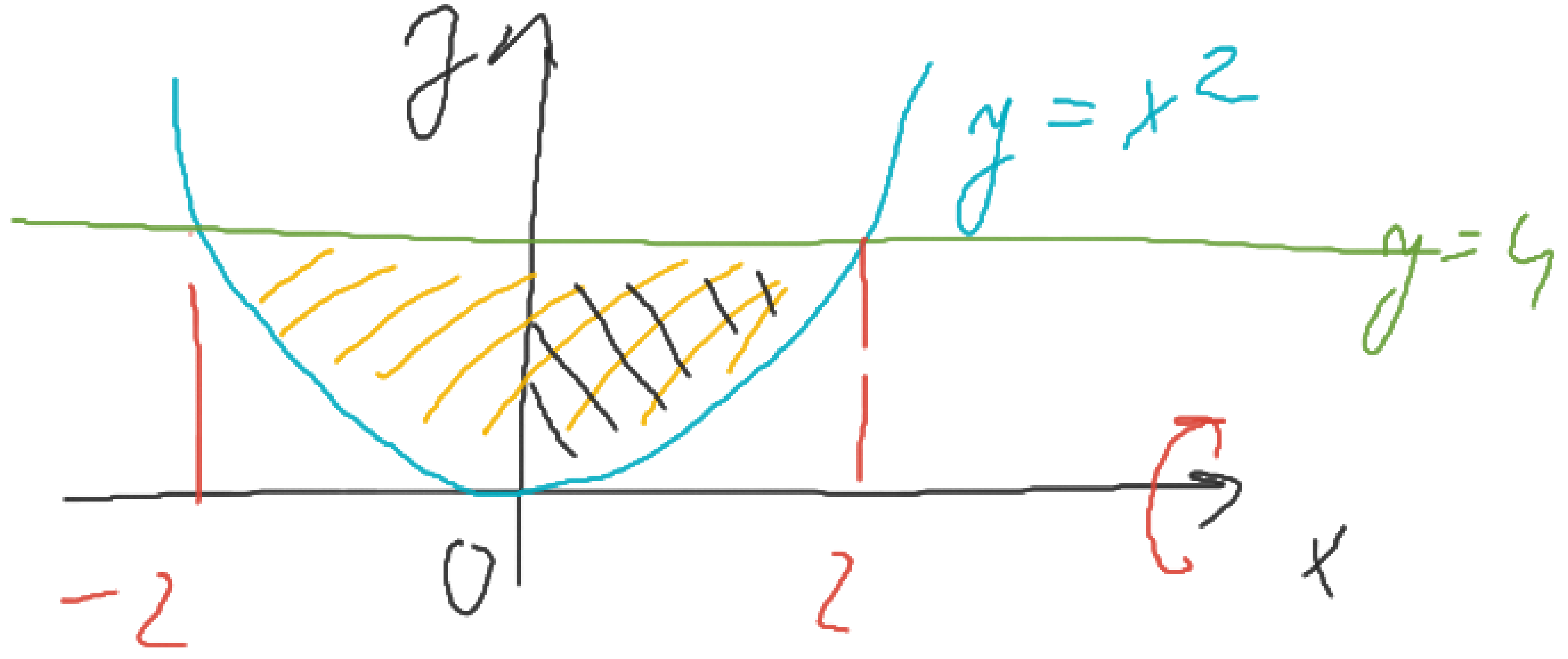
$$\boxed{D = \int_a^b \sqrt{1+f'^2(x)}\,dx}$$

VZŤAH NA VÝPOČET DĹŽKY ROVINNEJ KRIVKY

PR3 VYPOČÍTAJTE OBJEM ROTAČNÉHO TELESA

a) $\underbrace{\boxed{y = x^2} \quad \boxed{y = 4}}$

$x^2 = 4 \Rightarrow x = \pm 2$



$$V = 2\cdot\pi \int_0^2 4^2 - x^4 \, dx = 2\pi \left[16x - \frac{x^5}{5} \right]_0^2 =$$

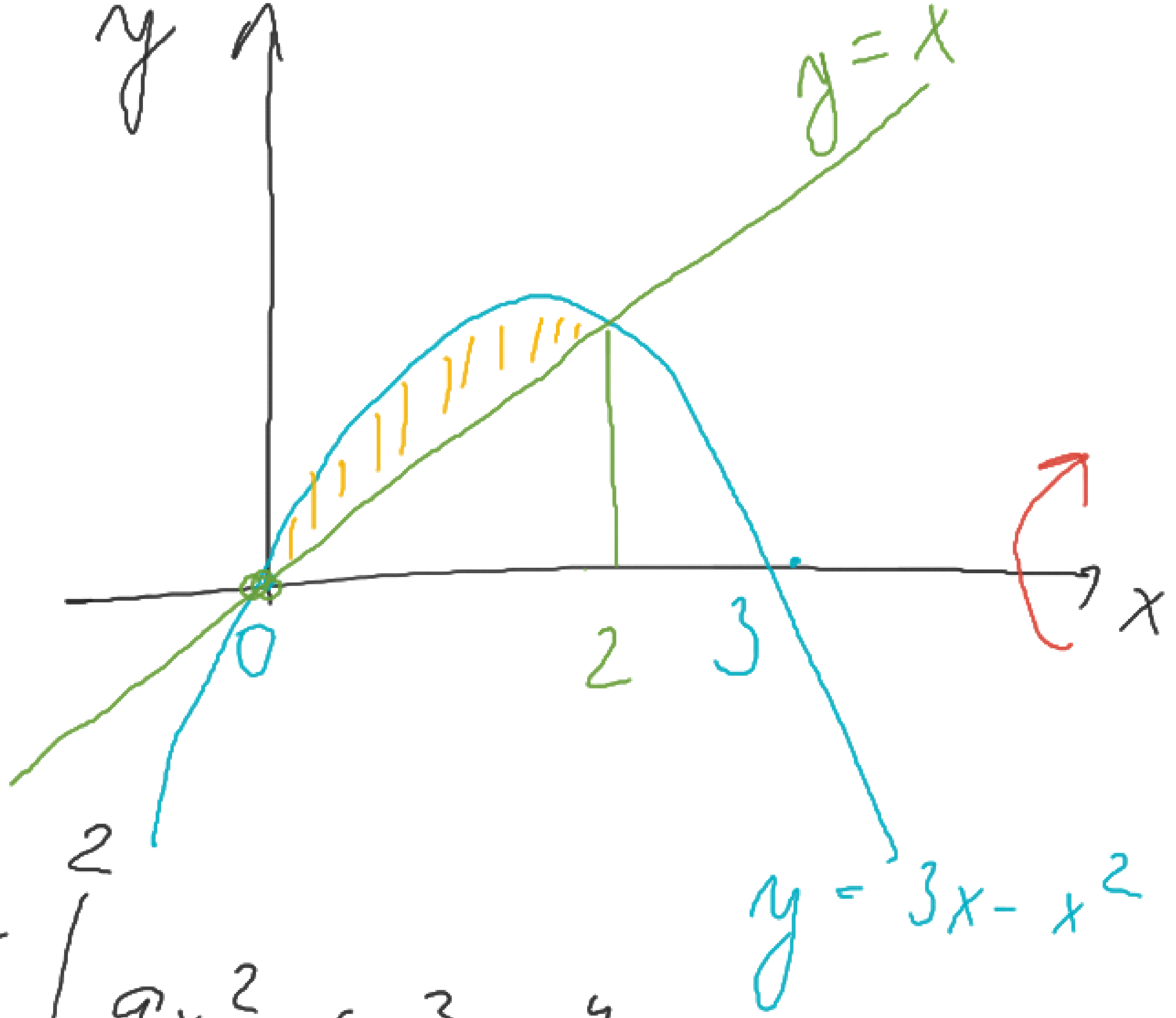
$$= 2\pi \left[32 - \frac{32}{5} \right] = 2\pi \cdot \frac{4\cdot 32}{5} = \frac{256}{5}\pi \, j^3$$

6 $\boxed{y = 3x - x^2}$, $\boxed{y = x}$

$$3x - x^2 = x$$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$



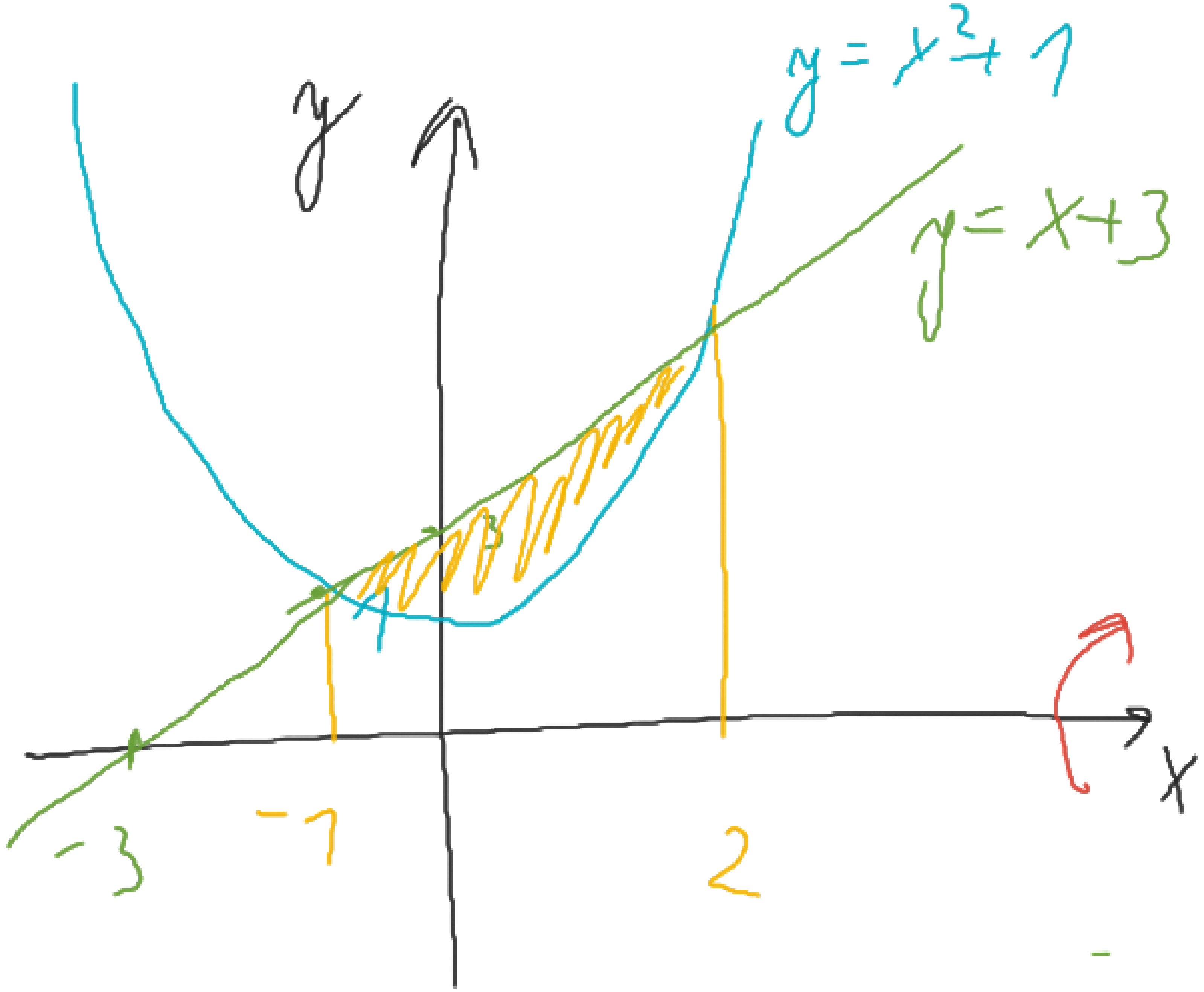
$$V = \pi \int_0^2 (3x - x^2)^2 - x^2\,dx = \pi \int_0^2 9x^2 - 6x^3 + x^4 - x^2\,dx =$$

$$= \pi \int_0^2 8x^2 - 6x^3 + x^4 \, dx = \pi \left[\frac{8x^3}{3} - \frac{6x^4}{4} + \frac{x^5}{5} \right]_0^2 =$$

$$= \pi \left[\frac{2^6}{3} - \frac{3 \cdot 2^{\cancel{4}\,3}}{\cancel{2}} + \frac{2^5}{5} \right] = \frac{\pi \cdot 2^3 \left(5 \cdot 2^3 - 3 \cdot 15 + 3 \cdot 2^2\right)}{15} =$$

$$= \frac{\pi \cdot 8 \left(40 - 45 + 12\right)}{15} = \frac{56}{15} \pi \, j^3$$

c) $\underbrace{\boxed{y=x^2+1};\quad \boxed{y=x+3}}$



$$x^2+1 = x+3$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$x_1 = 2 \qquad x_2 = -1$$

$$V = \pi \int_{-1}^{2} (x+3)^2 - (x^2+1)^2 \, dx =$$

$$= \pi \int_{-1}^{2} x^2 + 6x + 9 - (x^4 + 2x^2 + 1)\, dx = \pi \int_{-1}^{2} -x^4 - x^2 + 6x + 8\, dx =$$

$$= \pi \left[-\frac{x^5}{5} - \frac{x^3}{3} + 3x^2 + 8x \right]_{-1}^{2} = \pi \Big[-\frac{32}{5} - \frac{8}{3} \overbrace{+12+16}^{28} -$$

$$- \Big(\frac{1}{5} + \frac{1}{3} + \underbrace{(3-8)}_{-5} \Big) \Big] = \pi \left[-\frac{33}{5} - 3 + 28 + 5 \right] =$$

$$= \pi \left[-\frac{33}{5} + 30 \right] = \pi \frac{-33+150}{5} = \frac{117}{5}\pi \quad \checkmark^{3}$$

d) $\boxed{y=\sqrt{x}}$; $\boxed{y=\dfrac{x^2}{8}}$

$$\underbrace{\quad\quad\quad\quad\quad}$$

$$\sqrt{x}=\frac{x^2}{8} \Rightarrow x=\frac{x^4}{64}$$

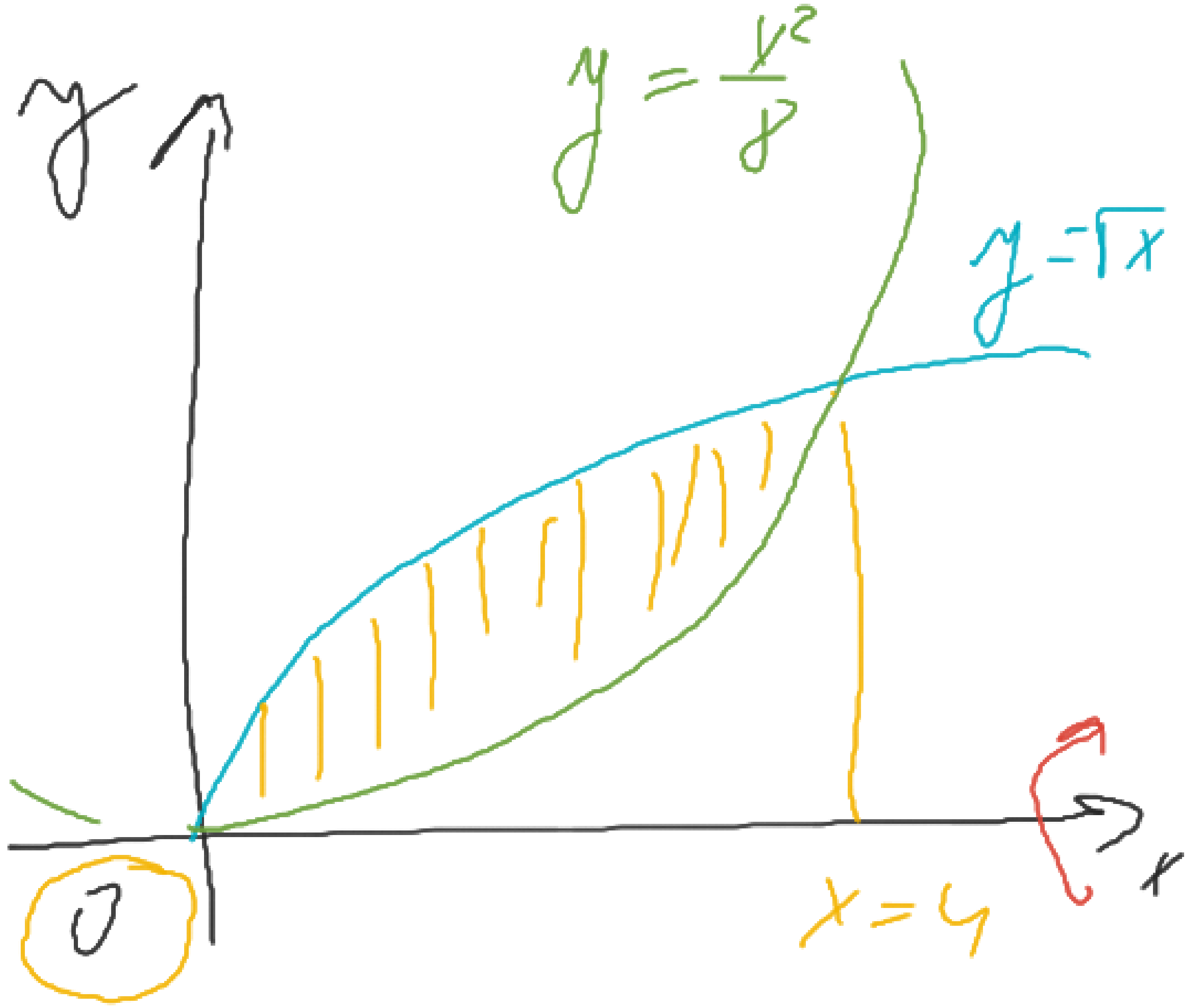
$$64x = x^4$$

$$x^4-64x=0$$

$$x(x^3-64)=0$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$x=0 \qquad\qquad x=4$$



$$V = \pi \int_0^4 x - \frac{x^4}{64}\, dx = \pi \left[\frac{x^2}{2} - \frac{x^5}{5 \cdot 64} \right]_0^4 =$$

$$4^3$$

$$= \pi \left[8 - \frac{16}{5} \right] = \underline{\underline{\frac{24}{5} \pi \, j^3}}$$

DĹŽKA KRIVKY PR7

a) $y = \frac{1}{3}(x^2+2)^{\frac{3}{2}}$; $x \in \langle 0, 3 \rangle$

$$y' = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}} (x^2+2)^{\frac{1}{2}} \cdot \cancel{2}x = x\sqrt{x^2+2}$$

$$D = \int_0^3 \sqrt{1 + x^2(x^2+2)}\,dx = \int_0^3 \sqrt{1 + x^4 + 2x^2}\,dx$$

$$= \int_0^3 \sqrt{(1+x^2)^2}\,dx = \int_0^3 1 + x^2\,dx = \left[x + \frac{x^3}{3}\right]_0^3 = 3 + 9 =$$

$$= 12\ j$$

5

$$y = \frac{2}{3}x\sqrt{x}\,, \quad x \in \langle 0, 1 \rangle$$

$$y' = \left[\frac{2}{3}x^{\frac{3}{2}}\right]' = \cancel{\frac{2}{3}} \cdot \cancel{\frac{3}{2}}\, x^{\frac{1}{2}}$$

$$D = \int_0^1 \sqrt{1+x}\,dx = \left| \begin{array}{l} t = 1+x \\ dt = dx \\ t_1 = 1 \\ t_2 = 2 \end{array} \right| = \int_1^2 t^{\frac{1}{2}}\,dt =$$

$$= \frac{2}{3}\left[t^{\frac{3}{2}}\right]_1^2 = \underline{\underline{\frac{2}{3}\left(2\sqrt{2}-1\right)}};$$

C $y = \ln(\sin x)$; $x \in \left\langle \frac{\pi}{3}, \frac{\pi}{2} \right\rangle$

$$y' = \frac{1}{\sin x} \cos x$$

$$D = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{1 + \frac{\cos^2 x}{\sin^2 x}}\, dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{\frac{\sin^2 x + \cos^2 x}{\sin^2 x}}\, dx =$$

1

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\sin x}\, dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{\sin^2 x}\, dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{1 - \cos^2 x}\, dx$$

$t = \cos x$

$dt = -\sin x \, dx$

$x_1 = \frac{\pi}{3} \Rightarrow t_1 = \frac{1}{2}$

$x_2 = \frac{\pi}{2} \Rightarrow t_2 = 0$

$$= \int_{\frac{1}{2}}^{0} \frac{-dt}{1-t^2} = -\int_{\frac{1}{2}}^{0} \frac{dt}{1-t^2} =$$

$$= \left[-\frac{1}{2} \ln \left| \frac{1+t}{1-t} \right| \right]_{\frac{1}{2}}^{0}$$

$$\boxed{\int \frac{1}{1-x} dx = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + C}$$

$$= \underbrace{-\frac{1}{2} \ln 1}_{=0} + \frac{1}{2} \ln \frac{\frac{3}{\cancel{2}}}{\frac{1}{\cancel{2}}} = \underline{\underline{\frac{1}{2} \ln 3}} \; ;$$

(d) $y = \frac{x^2}{4} - \frac{1}{2}\ln x \; ; \; x \in \langle 1, 2 \rangle$

$$y' = \frac{2x}{4} - \frac{1}{2x} = \frac{x}{2} - \frac{1}{2x}$$

$$D = \int_1^2 \sqrt{1 + \left(\frac{x}{2} - \frac{1}{2x}\right)^2}\,dx = \int_1^2 \sqrt{1 + \frac{x^2}{4} - \frac{\cancel{2}\cancel{x}}{2\cancel{4}\cancel{x}} + \frac{1}{4x^2}}\,dx$$

$$= \int_1^2 \sqrt{\frac{1}{2} + \frac{x^2}{4} + \frac{1}{4x^2}}\,dx = \int_1^2 \sqrt{\frac{2x^2 + x^4 + 1}{4x^2}}\,dx =$$

$$= \int_1^2 \sqrt{\frac{(1+x^2)^2}{(2x)^2}}\,dx = \int_1^2 \frac{1+x^2}{2x}\,dx = \int_1^2 \frac{1}{2x} + \frac{x}{2}\,dx$$

$$= \left[\frac{1}{2}\ln|x| + \frac{x^2}{4}\right]_1^2 = \frac{1}{2}\ln 2 + \underline{\frac{4}{4}} - \Big(\underbrace{\frac{1}{2}\ln 1}_{=0} +$$

$$+ \underline{\frac{1}{4}}\Big) = \frac{1}{2}\ln 2 + \frac{3}{4} \quad J$$