$$\int 3^{2x}\cdot 5^x\,dx = \int (3^2)^x\cdot 5^x\,dx = \int 9^x 5^x\,dx = \int 45^x\,dx = \frac{45^x}{\ln 45} + C$$

$3^{2x} = (3^x)^2$, $3^{2x} = (3^2)^x$

$4^2\cdot 5^2 = 16\cdot 25 = 400$, $(20)^2 = 400$

$\ln 45 = \ln(5\cdot 9) = \ln 5 + \ln 9 = \ln 5 + \ln 3^2 = \ln 5 + 2\ln 3$

$$\int \frac{\ln^4 x}{x}\,dx = \int t^4\,dt \qquad \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x}\,dx \\ x\,dt = dx \end{array} \right.$$

$$= \int \frac{t^4}{x}\cdot x\,dt = \int t^4 = \frac{t^5}{5} = \frac{(\ln x)^5}{5} + C$$

$$\int \frac{\cos(\ln x)}{x}\,dx = \int \cos(t)\,dt \qquad \left| \begin{array}{l} t = \ln(x) \\ dt = \frac{1}{x}\,dx \end{array} \right.$$

$$= \sin(t) = \sin(\ln x) + C$$

$$\int x\cdot\ln(x)\,dx = \qquad \int uv' = uv - \int u'v$$

$$\left| \begin{array}{l} u' = x \to u = \frac{x^2}{2} \\ v = \ln x \to v' = \frac{1}{x} \end{array} \right| =$$

$$\int x\cdot\ln(x)\,dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{x^2}{2} + C$$

$$\int x\sin(3x)\,dx = \left| \begin{array}{l} u = x \quad u' = 1 \\ v' = \sin(3x) \to v = \int \sin(3x)\,dx = -\frac{\cos(3x)}{3} \\ v = \int \sin(3x)\,dx = \left| t = 3x, dx = \frac{dt}{3} \right| = \int \sin(t)\frac{dt}{3} = \frac{1}{3}(-\cos(3x)) \end{array} \right| = \qquad \int uv' = uv - \int u'v$$

$$= x\cdot\frac{-\cos 3x}{3} - \int 1\cdot\frac{-\cos 3x}{3}\,dx = \frac{-x}{3}\cos 3x + \frac{1}{3}\int \cos 3x =$$

$$= \frac{-x}{3}\cos(3x) + \frac{1}{3}\cdot\frac{\sin(3x)}{3} + C$$

$$\int (2x+1)\cos\left(\frac{\pi}{3} - 5x\right)dx = \left| \begin{array}{l} u = 2x+1 \to u' = 2 \\ v' = \cos\left(\frac{\pi}{3}-5x\right) \to v = \int \cos\left(\frac{\pi}{3}-5x\right)dx = \left| \begin{array}{l} t = \frac{\pi}{3} - 5x \\ dt = -5\,dx \\ dx = \frac{dt}{-5} \end{array} \right| \\ v = \int \cos(t)\frac{dt}{-5} = -\frac{1}{5}\int \cos(t)\,dt = -\frac{1}{5}\sin(t) = -\frac{1}{5}\sin\left(\frac{\pi}{3}-5x\right) \end{array} \right. \qquad \int uv' = uv - \int u'v$$

$$= (2x+1)\cdot\left(-\frac{1}{5}\right)\sin\left(\frac{\pi}{3}-5x\right) - \int 2\left(-\frac{1}{5}\right)\sin\left(\frac{\pi}{3}-5x\right)dx =$$

$$= -\frac{1}{5}(2x+1)\sin\left(\frac{\pi}{3}-5x\right) + \frac{2}{5}\int \sin\left(\frac{\pi}{3}-5x\right)dx = -\frac{1}{5}(2x+1)\sin\left(\frac{\pi}{3}-5x\right) + \frac{2}{5}\cdot\frac{-\cos\left(\frac{\pi}{3}-5x\right)}{-5} + C$$

$$\int x^2\cos x\,dx = \left| \begin{array}{l} u = x^2 \to u' = 2x \\ v' = \cos x \to v = \int \cos x = \sin x \end{array} \right| \qquad \int uv' = uv - \int u'v$$

$$= x^2\sin x - \int 2x\sin x\,dx =$$

$$= x^2\sin x - 2\int x\sin x\,dx = \left| \begin{array}{l} u = \sin x \to u' = \cos x \\ v' = x \to v = \frac{x^2}{2} \end{array} \right.$$

$$= x^2\sin x - 2\left(\sin x\,\frac{x^2}{2} - \int \cos x\cdot\frac{x^2}{2}\,dx\right)$$

$$\left| \begin{array}{l} u = x \to u' = 1 \\ v' = \sin x \to v = \int \sin x\,dx = -\cos x \end{array} \right.$$

$$= x^2\sin x - 2\left(x(-\cos x) - \int 1\cdot(-\cos x)\,dx\right)$$

$$= x^2\sin x + 2x\cos x - 2\int \cos x\,dx =$$

$$= x^2\sin x + 2x\cos x - 2\sin x + C$$

Pozor! Ďalší príklad:

$$\int \cos(\ln x)\,dx \qquad \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x}\,dx \\ dx = x\,dt \end{array} \right| = \int \cos(t)\,e^t\,dt \qquad \left| \begin{array}{l} u = \cos(t) \to u' = -\sin(t) \\ v' = e^t \to v = e^t \end{array} \right.$$

$t = \ln x \to x = e^t$

$dt = \frac{1}{x}\,dx \to dx = x\,dt$

$e^t = e^{\ln x} = x$

a teraz per partes: $\int uv' = uv - \int u'v$

$$= \cos(t)\,e^t - \int -\sin(t)\,e^t\,dt = \cos(t)\,e^t + \int \sin(t)\,e^t\,dt = \qquad \left| \begin{array}{l} u = \sin(t) \to u' = \cos(t) \\ v' = e^t \to v = e^t \end{array} \right.$$

$$I = \underbrace{\int \cos(t)\,e^t\,dt}_{} = \underbrace{\cos(t)\,e^t + \sin(t)\,e^t}_{a} - \underbrace{\int \cos t\;e^t\,dt}_{I} \qquad I = a - I \;/ + I$$

$2I = a$

$$I = \frac{1}{2}a \Rightarrow \int \cos(t)\,e^t\,dt = \frac{1}{2}\left(e^t\cos(t) + e^t\sin(t)\right) + C =$$

$$\int \cos(\ln x)\,dx = \frac{1}{2}e^t(\cos t + \sin t) + C = \frac{1}{2}x\left(\cos(\ln x) + \sin(\ln x)\right) + C$$

$e^{\ln x} = x$

Podobne riešte: $\int \sin(3x)\cdot\cos\left(\frac{x}{2}\right)dx$

a tiež: $\int \sin(2x)\cdot e^{-3x}\,dx$

Doma: $\int \arcsin\left(\frac{x}{2}\right)dx = \left| \begin{array}{l} u = \arcsin\left(\frac{x}{2}\right) \to u' = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\cdot\frac{1}{2} \\ v' = 1 \to v = \int 1\,dx = x \end{array} \right|$ dokončiť

$\int v'u = uv - \int u'v$