PR2 VYPOČÍTAJTE ZADANÉ INTEGRÁLY

(a) $$\int \frac{x^3 - 2x^2 + 9}{x^2 - x - 2}\,dx = \int x - 1 + \frac{x+7}{(x-2)(x+1)}\,dx =$$

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$$(x^3 - 2x^2 + 9) : (x^2 - x - 2) = x - 1 + \frac{x+7}{x^2 - x - 2}$$

$$x^2 - x - 2 \to (x-2)(x+1)$$

$$\begin{array}{l} -(x^3 - x^2 - 2x) \\ \hline \quad -x^2 + 2x + 9 \\ -\quad(-x^2 + x + 2) \\ \hline \qquad x + 7 \end{array}$$

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$$= \frac{x^2}{2} - x + \int \frac{1x + 7}{(x-2)(x+1)}\,dx = \frac{x^2}{2} - x + \int \frac{3}{x-2} - \frac{2}{x+1}\,dx$$

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$$\frac{A}{x-2} + \frac{B}{x+1} = \frac{Ax + A + Bx - 2B}{(x-2)(x+1)} \Rightarrow$$

$$A + B = 1 \quad (-1) \Rightarrow \boxed{A = 3}$$

$$A - 2B = 7 \quad \swarrow \oplus$$

$$-3B = 6 \Rightarrow \boxed{B = -2}$$

$$= \frac{x^2}{2} - x + \int \frac{3\cdot 1}{x-2}\,dx - \int \frac{2}{x+1}\,dx =$$

$$= \frac{x^2}{2} - x + 3\ln|x-2| - 2\ln|x+1| + C$$

5) $\int \frac{x}{x^3-3x+2}\,dx =$

| | 1 | 0 | -3 | 2 |
|---|---|---|---|---|
| 1 | 1 | 1 | -2 | 0 |

$$(x^3-3x+2) = (x-1)(x^2+x-2) = (x-1)(x+2)(x-1)$$

$$= \int \frac{x}{(x-1)(x+2)(x-1)}\,dx = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

$$(x-1)^2(x+2)$$

$$= A(x-1)(x+2) + B(x+2) + C(x^2-2x+1) =$$

$$= A(x^2+x-2) + B(x+2) + C(x^2-2x+1) =$$

$$= x^2(A+C) + x(A+B-2C) + (-2A+2B+C) = x$$

$$A + C = 0 \Rightarrow A = -C \Rightarrow A = \frac{2}{9}$$

$$A + B - 2C = 1 \Rightarrow -C + B - 2C = 1$$

$$-2A + 2B + C = 0 \Rightarrow 2C + 2B + C = 0$$

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$$\left.\begin{aligned} -3C + B &= 1 \\ 3C + 2B &= 0 \end{aligned}\right\} (+) \quad \boxed{B = \frac{1}{3}}$$

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$$\boxed{C = \frac{B - 1}{3} = -\frac{2}{9}}$$

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$\Rightarrow$

$$= \int \frac{\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{-\frac{2}{9}}{x+2}\,dx =$$

$$(x-1)^{-2}$$

$$= \frac{2}{9}\ln|x-1| + \frac{1}{3}\int \frac{1}{(x-1)^2}\,dx - \frac{2}{9}\ln|x+2|$$

$$= \frac{2}{9}\ln|x-1| - \frac{1}{3}\frac{1}{(x-1)} - \frac{2}{9}\ln|x+2| + C$$

$$= \frac{2}{9}\ln\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C$$

c) $$\int \frac{x^2+x+12}{x^3+7x^2+11x+5}\,dx =$$

| | | | | |
|---|---|---|---|---|
| | 1 | 7 | 11 | 5 |
| -5 | 1 | 2 | 1 | 0 |

$$\Rightarrow (x+5)(x^2+2x+1) = \underline{\underline{(x+5)(x+1)^2}}$$

$$= \int \frac{1x^2 + 1x + 12}{(x+5)(x+1)^2} =$$

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$$\frac{A}{x+5} + \frac{B}{x+1} + \frac{C}{(x+1)^2} =$$

$$= A(x^2 + 2x + 1) + B(x+5)(x+1) + C(x+5) =$$

$$= A(x^2 + 2x + 1) + B(x^2 + 6x + 5) + C(x+5) =$$

$$= x^2(A+B) + x(2A + 6B + C) + (A + 5B + 5C)$$

$$A + B = 1 \Rightarrow A = 1 - B \Rightarrow \boxed{A = 1 - (-1) = 2}$$

$$2A + 6B + C = 1 \Rightarrow 2 - 2B + 6B + C = 1$$

$$A + 5B + 5C = 12 \Rightarrow 1 - B + 5B + 5C = 12$$

$$4B + 3 = -1 \Leftarrow 4B + C = -1 \quad (-1)$$

$$4B = -4 \qquad 4B + 5C = 11 \quad \oplus$$

$$\boxed{B = -1}$$

$$4C = 12$$

$$\boxed{C = 3}$$

$$= \int \frac{2}{x+5} - \frac{1}{x+1} + \frac{3}{(x+1)^2}\, dx =$$

$$= 2 \ln|x+5| - \ln|x+1| - \frac{3}{x+1} + C$$

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d $\displaystyle\int \frac{7-x}{x^3 - x^2 + 3x + 5}\, dx =$

| | 1 | -1 | 3 | 5 |
|---|---|---|---|---|
| -1 | 1 | -2 | 5 | 0 |

$$\Rightarrow (x+1)\underbrace{(x^2-2x+5)}$$

$$D = b^2 - 4ac$$
$$= 4 - 4 \cdot 5 \cdot 1 < 0$$

$$= \int \frac{7 - 1x + 0x^2}{(x+1)(x^2-2x+5)}\,dx$$

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$$\frac{A}{x+1} + \frac{Bx+C}{x^2-2x+5} = \frac{Ax^2 - 2Ax + 5A + Bx^2 + Bx + Cx + C}{(x+1)(x^2-2x+5)}$$

$$\Rightarrow \quad x^2: \quad A + B = 0 \quad \Rightarrow \quad A = -B$$

$$x^1: \quad -2A + B + C = -1$$

$$x^0: \quad 5A + C = 7$$

$$3B + C = -1 \quad (-1)$$

$$-5B + C = 7$$

$$-8B = 8$$

$$\boxed{B = -1}$$

$$\boxed{A = 1}$$

$$\boxed{C} = 7 - 5 = \boxed{2}$$

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$$= \int \frac{1}{x+1} + \frac{-x+2}{x^2-2x+5}\,dx =$$

$$= \ln|x+1| - \frac{1}{2}\int \frac{2(x-2)}{x^2-2x+5}\,dx =$$

$$-4 = -2 - 2$$

$$= \ln|x+1| - \frac{1}{2}\int \frac{2x-4}{x^2-2x+5}\,dx =$$

$$= \ln|x+1| - \frac{1}{2}\int \frac{2x-2}{x^2-2x+5}\,dx - \frac{1}{2}\int \frac{-2}{x^2-2x+5}\,dx$$

$$= \ln|x+1| - \frac{1}{2}\ln|x^2-2x+5| + \int \frac{1}{(x-1)^2+\underbrace{4}_{a^2}}\,dx =$$

$$= \ln|x+1| - \frac{1}{2}\ln|x^2-2x+5| + \frac{1}{2}\operatorname{arctg}\frac{x-1}{2} + C$$

$$\boxed{\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + c}$$