$\mathbb{C}=\{a+ib;\ a,b\in\mathbb{R}$
i je IMAG. JEDNOTKA$\}$

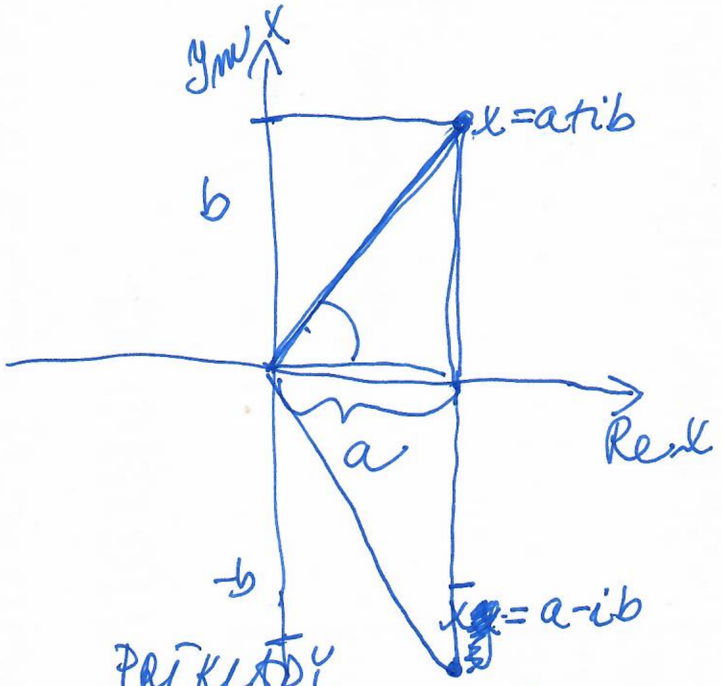
DEF. $i^2=-1$

$i=\sqrt{-1}$

$\mathbb{R}\subset\mathbb{C}$

$a+ib$; ak $b=0$

a



$x=a+ib$

$x=a+ib$
$\bar{x}=a-ib$ JE ZDRUŽENÉ KOMPLEXNÉ ČÍSLO

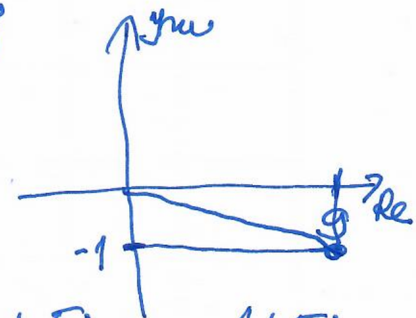
$|x|=\sqrt{a^2+b^2}$
VEĽKOSŤ

PRÍKLADY:

$(2+3i)+(-5-10i)=(2-5)+i(3-10)=\underline{\underline{-3-7i}}$;

$(1+i)(4-5i)=1\cdot4+4\cdot i-5i-5\underset{-1=i^2}{i^2}=4-5(-1)+$

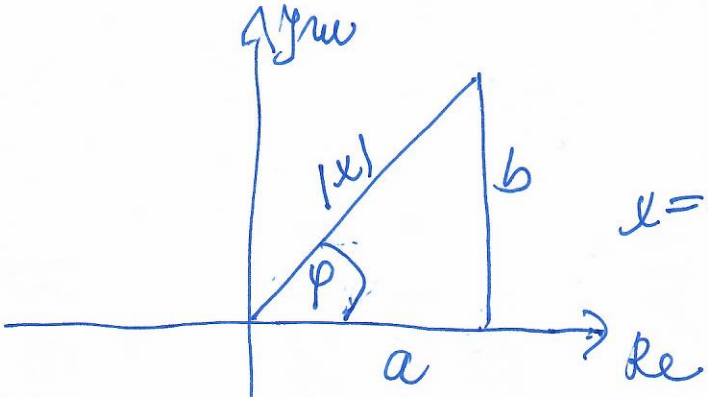
$+(-i)=9-i$;



$\frac{2+3i}{2}=1+\frac{3}{2}i$

$\frac{(2+3i)}{1-i}\cdot\frac{(1+i)}{1+i}=\frac{2+3i+3i^2+2i}{1^2-i^2}=\frac{-1+5i}{\underbrace{1-(-1)}_{=2}}=\underline{\underline{\frac{-1+5i}{2}}}$;

$(a-b)(a+b)=a^2-b^2$



$x = a + ib$

$\sin\varphi = \frac{b}{|x|}$ $\cos\varphi = \frac{a}{|x|}$

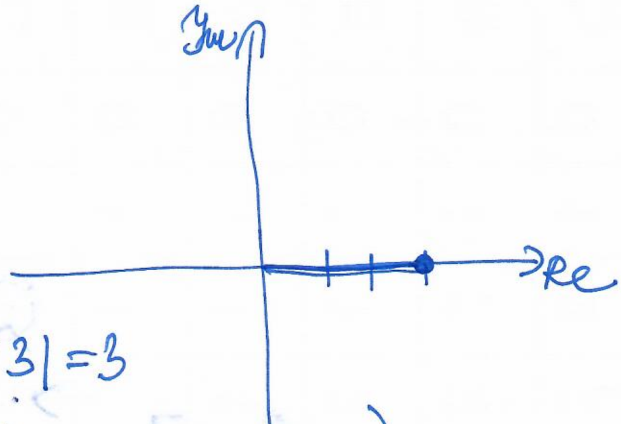
$tg\varphi = \frac{b}{a}$

$b = |x| \cdot \sin x$ $a = |x| \cdot \cos\varphi$

$\underline{x = a + ib} = |x| \cdot \cos\varphi + i|x| \cdot \sin\varphi =$

$= |x|(\cos\varphi + i\sin\varphi)$

ALGEBRICKÝ TVAR GONIOM. TVAR KOMPL. ČÍSLA



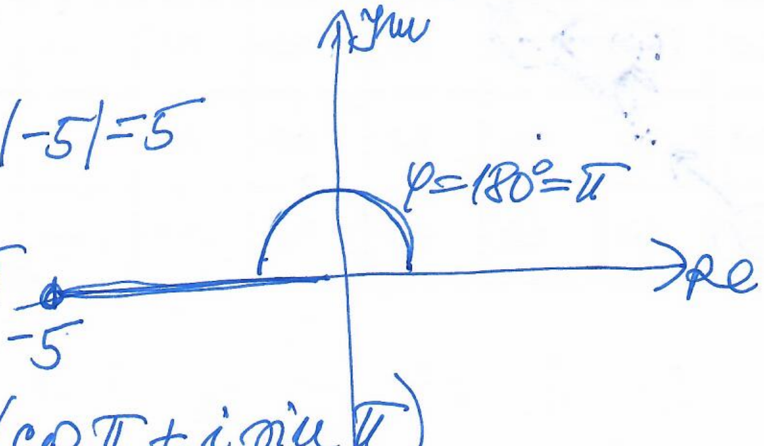
$x = 3$

$a = 3$

$b = 0$

$\varphi = 0$ $|a| = |3| = 3$

$3 = x = |3| \cdot (\cos 0 + i \sin 0)$



$x = -5$

$a = -5$ $|-5| = 5$

$b = 0$

$\varphi = 180^\circ = \pi$

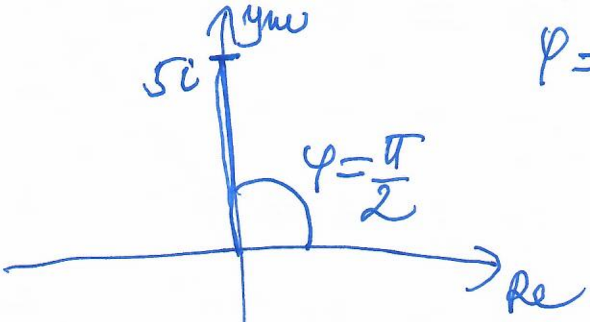
$-5 = 5 \cdot (\cos\pi + i\sin\pi)$

$x = 5i \qquad a = 0 \qquad x = a + ib$

$b = 5$

$|x| = \sqrt{a^2 + b^2} = \sqrt{0^2 + 5^2} = 5$

$\varphi = \frac{\pi}{2} = 90^\circ$

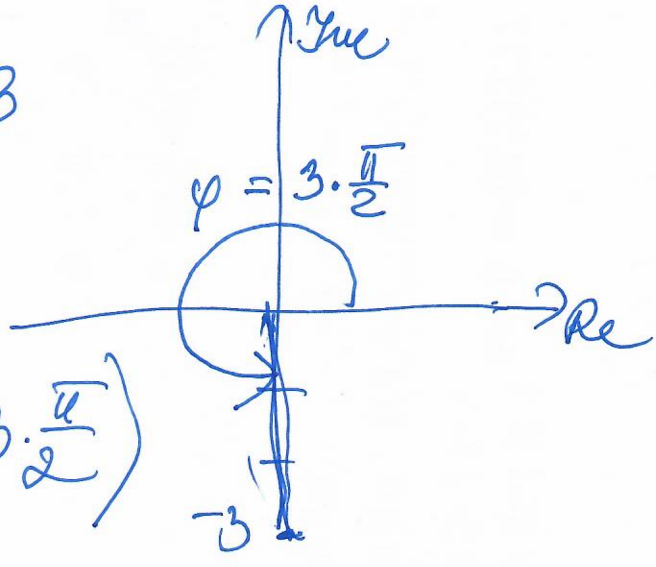


$x = 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$x = -3i \qquad a = 0 \quad b = -3$

$\varphi = 3\cdot\frac{\pi}{2}$

$|x| = 3$



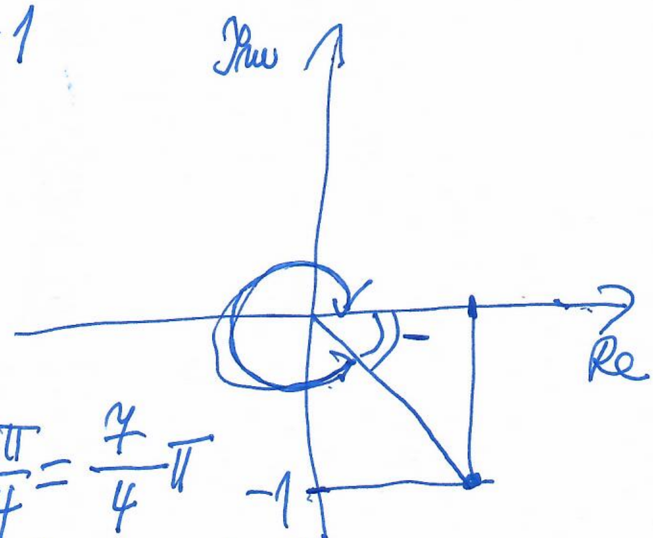
$x = 3\cdot\left(\cos 3\cdot\frac{\pi}{2} + i\sin 3\cdot\frac{\pi}{2}\right)$

$x = 1 - i \qquad a = 1 \quad b = -1$

$|x| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\operatorname{tg}\varphi = \frac{-1}{1} = -1$

$\varphi = -\frac{\pi}{4} \qquad \varphi = 3\cdot\frac{\pi}{2} + \frac{\pi}{4} = \frac{7}{4}\pi$



$x = \sqrt{2}\cdot\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

PRÍKLAD.

$(1-i\sqrt{3})^{12}$

MOIVREOVA VETA

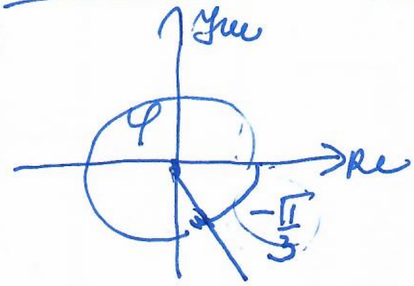
$z=|z|\cdot(\cos\varphi+i\sin\varphi)$

$z^{n}=|z|^{n}(\cos\varphi+i\sin\varphi)^{n}=$

$=z^{n}(\cos n\varphi+i\sin n\varphi)$

$z=1-i\sqrt{3}$

$|z|=\sqrt{1^{2}+(-\sqrt{3})^{2}}=\sqrt{1+3}=\sqrt{4}=\underline{\underline{2}}$



$tg\,\varphi=\frac{b}{a}=\frac{-\sqrt{3}}{1}=-\sqrt{3}$

$\varphi=60^{\circ}=\frac{\pi}{3}$

$\boxed{z=2\cdot\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)}$

$\underline{\varphi=-\frac{\pi}{3}}$

$2\pi-\frac{\pi}{3}=\frac{6\pi-\pi}{3}=\frac{5\pi}{3}$

$\cos\left(-\frac{\pi}{3}\right)=\cos\frac{\pi}{3}$

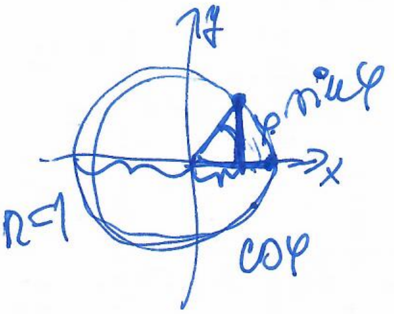
$\sin\left(-\frac{\pi}{3}\right)=-\sin\frac{\pi}{3}$

$z^{12}=2^{12}\cdot\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)^{12}=$

$=2^{12}\cdot\left(\cos\frac{12\pi}{3}-i\sin\frac{12\pi}{3}\right)=$

$=2^{12}\cdot(\underbrace{\cos 4\pi}_{=1}-i\underbrace{\sin 4\pi}_{=0})=$

$=\underline{\underline{2^{12}}}$



cos φ – x-ová súr. bodu na jed.

sin φ (y-ová súradnica bodu na jednot. kružnici

VYNÁSOBTE:

$$z=\left[2\cdot\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]\cdot\left[6\cdot\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)\right]=$$

$$=12\cdot\left(\cos\left(\frac{\pi}{4}+\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{4}+\frac{\pi}{12}\right)\right)=$$

$$=12\left(\cos\frac{3\pi+\pi}{12}+i\sin\frac{4\pi}{12}\right)=$$

$$=12\cdot\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$$

PREVEĎTE NA ALGEBR. TVAR

$$\frac{\pi}{3}=60^\circ$$

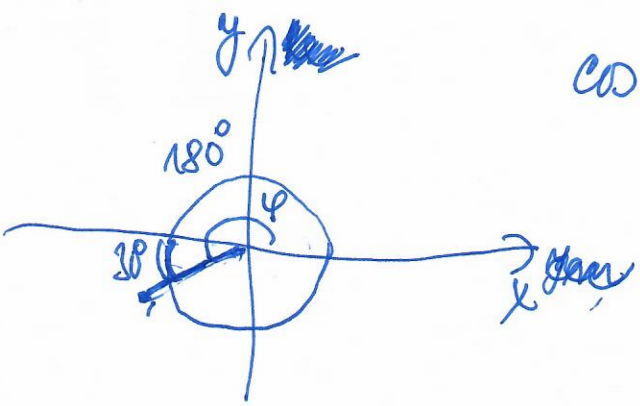
$$\cos 60^\circ=\frac{1}{2}$$

$$\sin 60^\circ=\frac{\sqrt{3}}{2}$$

$$z=12\cdot\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=\underline{\underline{6+i6\cdot\sqrt{3}}}$$

NAPÍŠTE V ALGEBR. TVARE

$$5\cdot(\cos 210^\circ+i\sin 210^\circ)=5\cdot\left(-\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)=\underline{\underline{-\frac{5\cdot\sqrt{3}}{2}-i\frac{5}{2}}}$$



$$\cos 210^\circ=-\cos 30^\circ=-\frac{\sqrt{3}}{2}$$

$$\sin 210^\circ=-\sin 30^\circ=-\frac{1}{2}$$

PŘÍKLAD.

$$\frac{\frac{3}{4}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}{4\cdot\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)}=\frac{\frac{3}{4}}{\frac{4}{1}}\cdot\left[\cos\left(\frac{\pi}{3}-\frac{3\pi}{4}\right)+i\sin\left(\frac{\pi}{3}-\frac{3\pi}{4}\right)\right]=$$

$$=\frac{3}{16}\cdot\left[\cos\left(-\frac{5\pi}{12}\right)+i\sin\left(-\frac{5\pi}{12}\right)\right]=$$

$$\frac{\pi}{3}-\frac{3\pi}{4}=\frac{4\pi-9\pi}{12}=-\frac{5\pi}{12}\qquad=\underline{\underline{\frac{3}{16}\cdot\left[\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right]}}$$

PŘÍKLAD.

DOKAŽTE, ŽE $\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ JE KOMPLEXNOU JEDNOTKOU.

$$|x|=1\qquad x=\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}$$

$$|x|=\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2+\left(-\frac{\sqrt{2}}{2}\right)^2}=\sqrt{\frac{2}{4}+\frac{2}{4}}=\underline{\underline{1}}$$

$$x^n = z$$

$z \in \mathbb{C}$

$$z = |z|(\cos\varphi + i\sin\varphi)$$

$$x_k = \sqrt[n]{|z|}\left[\cos\left(\frac{\varphi+2k\pi}{n}\right) + i\sin\left(\frac{\varphi+2k\pi}{n}\right)\right],$$

$$\boxed{k = 0, 1, \dots n-1}$$

$x^2 = 1$

$x_1 = 1$

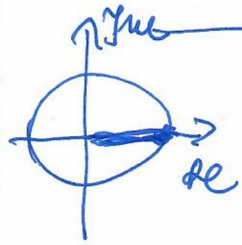
$x_2 = -1$

$(i)^n = i^{n \bmod 4}$

$i^{27} = i^{27 \bmod 4} = i^3 = i^2 \cdot i = -i$

$z = 1$

AK $z=1$, POČÍTAME KORENE Z 1



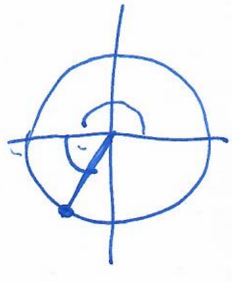
$x^3 = 1$

$1 = |1|(\cos 0 + i\sin 0)$

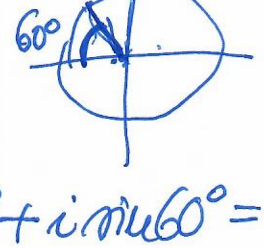
$\varphi = 0 \quad |1| = 1$

$$x_k = \sqrt[3]{1}\left[\cos\left(\frac{0+2\pi k}{3}\right) + i\sin\left(\frac{0+2k\pi}{3}\right)\right],$$

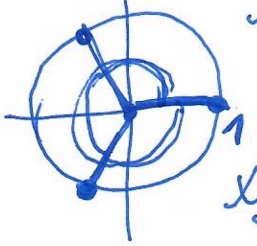
$\sqrt[3]{1} = 1$



$\boxed{k = 0, 1, 2}$



$x_0 = \cos 0 + i\sin 0 = \underline{\underline{1}}$



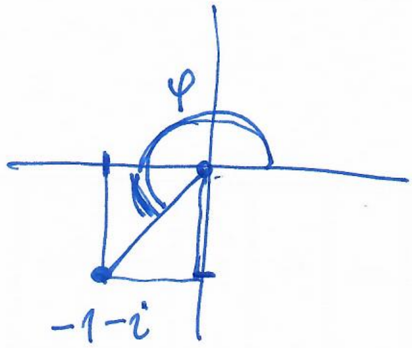
$x_1 = \cos\left(\frac{0+2\pi}{3}\right) + i\sin\left(\frac{0+2\pi}{3}\right) = -\cos 60° + i\sin 60° = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

$x_3 = \cos\left(\frac{0+2\pi 2}{3}\right) + i\sin\left(\frac{0+2\pi \cdot 3}{3}\right)$

$= \cos 240° + i\sin 240° = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

$$x^4+(1+i)=0$$

$$x^4=\underbrace{-1-i}_{z}$$



$$|z|=\sqrt{1^2+1^2}=\sqrt{2}$$

$$a=-1 \quad b=-i$$

$$x_K=\sqrt[4]{\sqrt{2}}\cdot\left[\cos\left(\frac{\frac{5\pi}{4}+2k\pi}{4}\right)+i\sin\left(\frac{\frac{5\pi}{4}+2k\pi}{4}\right)\right] \qquad tg\,\varphi=\frac{b}{a}=1$$

$$4=n$$

$$k=0,1,2,3$$

$$\varphi=45^\circ=\frac{\pi}{4} \qquad \frac{\pi}{4}+\pi=\frac{5\pi}{4}$$

$$z=\sqrt{2}\cdot\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$$

$$x_0=\sqrt[8]{2}\cdot\left[\cos\left(\frac{\frac{5\pi}{4}+0\cdot 2\pi}{4}\right)+i\sin\left(\frac{\frac{5\pi}{4}}{4}\right)\right]=$$

$$=\sqrt[8]{2}\cdot\left(\cos\frac{5\pi}{16}+i\sin\frac{5\pi}{16}\right);$$

$$x_1=\sqrt[8]{2}\cdot\left[\cos\frac{\frac{5\pi}{4}+2\pi}{4}+i\sin\frac{\frac{5\pi}{4}+2\pi}{4}\right]=$$

$$=\sqrt[8]{2}\cdot\left[\cos\frac{13\pi}{16}+i\sin\frac{13\pi}{16}\right]$$

$$x_2=\sqrt[8]{2}\left[\cos\frac{\frac{5\pi}{4}+2\pi\cdot 2}{4}+i\sin\frac{\frac{5\pi}{4}+2\pi\cdot 2}{4}\right]=$$

$$x_2 = \sqrt[8]{2}\left[\cos\frac{21\pi}{16} + i\sin\frac{21\pi}{16}\right]$$

$$x_3 = \sqrt[8]{2}\cdot\left(\cos\frac{\frac{5\pi}{4} + 2\pi\cdot 3}{4} + i\sin\frac{\frac{5\pi}{4} + 2\pi\cdot 3}{4}\right) =$$

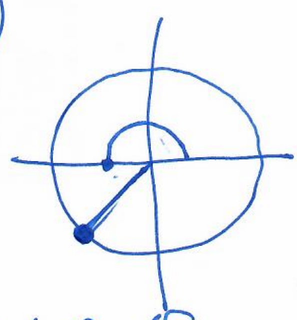
$$= \sqrt[8]{2}\cdot\left[\cos\frac{29\pi}{16} + i\sin\frac{29\pi}{16}\right]$$

$(x_3)^4 = -1-i$ OVERENIE

$$x_3^4 = \left(\sqrt[8]{2}\right)^4\cdot\left(\cos\frac{29\pi}{16}\cdot 4 + i\sin\frac{29\pi}{16}\cdot 4\right) =$$

$$= \sqrt{2}\cdot\left(\cos\frac{29\pi}{4} + i\sin\frac{29\pi}{4}\right)$$

$$\frac{29\pi}{4} = \frac{(24+5)\pi}{4} = \left(6\pi + \frac{5}{4}\pi\right)$$



$$= \sqrt{2}\cdot\left(\cos\frac{5\pi}{4} + i\sin\frac{5}{4}\pi\right) =$$

$\cos 45^\circ = \frac{\sqrt{2}}{2}$

$$= \sqrt{2}\left(-\frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{2}}{2}\right)i\right) =$$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$

$$= -\frac{2}{2} - i\,\frac{2}{2} = -1-i$$

10

$$\left|\frac{3+2i}{3-2i}\right| = \frac{|3+2i|}{|3-2i|} = 1$$

$$|3+2i| = \sqrt{3^2+4} = \sqrt{13}$$
$$|3-2i| = \sqrt{9+(-2)^2} = \sqrt{13}$$

$$\left|\frac{3+2i}{3-2i}\right| = \left|\frac{3+2i}{3-2i}\cdot\frac{3+2i}{3+2i}\right| = \left|\frac{9+12i+4i^2}{9+4}\right| =$$

$$= \left|\frac{5+12i}{13}\right| = \frac{1}{13}\cdot\underbrace{|5+12i|}_{13} = \underline{1}$$

$$|5+12i| = \sqrt{5^2+12^2} = \sqrt{25+144} = \sqrt{169} = 13$$

$$\left|\frac{i^{10}-i}{2i+1}\right| = \left|\frac{i^{10 \bmod 4}-i}{2i+1}\right| = \left|\frac{i^2-i}{2i+1}\right| = \left|\frac{-1-i}{2i+1}\right| =$$

$$= \frac{|1+i|}{|2i+1|} = \frac{\sqrt{1^2+1^2}}{\sqrt{2^2+1^2}} = \frac{\sqrt{2}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} \cancel{=}$$

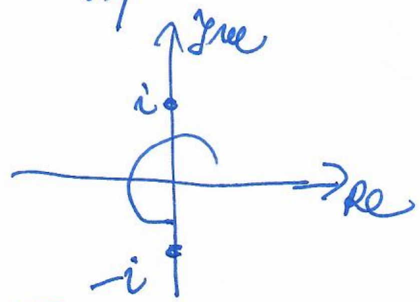
$$= \underline{\underline{\frac{\sqrt{10}}{5}}}$$

$$1+i \neq -1-i$$

$$|1+i| = |-1-i|$$

$x^4 + i = 0$

$x^4 = -i$ $\qquad -i = \underbrace{|-i|}_{=1} \cdot \left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$



$\varphi = \frac{3\pi}{2}$

$$z_k = \sqrt[4]{1} \cdot \left[\cos\left(\frac{\varphi + 2k\pi}{4}\right) + i\sin\left(\frac{\varphi + 2k\pi}{4}\right)\right],$$

$k = 0, 1, 2, 3$

$k=0$

$$z_0 = \cos\left(\frac{\frac{3\pi}{2}+0}{4}\right) + i\sin\left(\frac{\frac{3\pi}{2}+0}{4}\right) = \underline{\underline{\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}}}$$

$k=1$

$$z_1 = \cos\left(\frac{\frac{3\pi}{2}+2\cdot1\cdot\pi}{4}\right) + i\sin\left(\frac{\frac{3\pi}{2}+2\pi}{4}\right) =$$

$$= \underline{\cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}};$$

$k=2$

$$x_2 = \cos\left(\frac{\frac{3\pi}{2}+2\cdot2\cdot\pi}{4}\right) + i\sin\left(\frac{\frac{3\pi}{2}+4\pi}{4}\right) = \underline{\underline{\cos\frac{11\pi}{8} + i\,\frac{11\pi}{8}}}$$

$k=3$

$$x_3 = \cos\left(\frac{\frac{3\pi}{2}+2\cdot3\cdot\pi}{4}\right) + i\sin\left(\frac{\frac{3\pi}{2}+6\pi}{4}\right) = \underline{\cos\frac{15\pi}{8} + i\sin\frac{15\pi}{8}}$$