PR7 | OPAKOVANIE NEURČITÝCH INTEGRÁLOV

a) SUBST

$$\int e^{x+e^x}\,dx = \int e^x \cdot e^{e^x}\,dx = \left| \begin{array}{l} t = e^x \\ dt = e^x dx \end{array} \right|$$

$$= \int e^t\,dt = e^t + C = \underline{\underline{e^{e^x} + C}}$$

b) SUBST.

$$\int x\sqrt[3]{x+4}\,dx = \left| \begin{array}{l} t = \sqrt[3]{x+4} \\ t^3 = x+4 \Rightarrow x = t^3 - 4 \end{array} \right.$$

$$dx = 3t^2\,dt$$

$$= \int (t^3 - 4) \cdot t \cdot 3t^2 \, dt = \int 3t^6 - 12t^3 \, dt =$$

$$= \frac{3t^7}{7} - \frac{12t^4}{4} + C = \frac{3}{7}(x+4)^{\frac{7}{3}} - 3(x+4)^{\frac{4}{3}} + C$$

c) $\int x^2 \cdot \operatorname{arccotg} \frac{1}{x} \, dx =$ $\left| \cancel{\begin{array}{l} t = x^{-1} \\ dt = -\frac{1}{x^2} dx \end{array}} \right|$

"$x^2$" SA NEVYKRÁTI $\Rightarrow$

$\Rightarrow$ MUSÍM CEZ PER PARTES

PER PARTES

$u = \operatorname{arccotg} \frac{1}{x}$ $\quad u' = \frac{1}{1 + \frac{1}{x^2}} \cdot \left(-\frac{1}{x^2}\right) = \frac{1}{\frac{x^2+1}{\cancel{x^2}}} \cdot \left(-\frac{1}{\cancel{x^2}}\right)$

$v' = x^2$ $\quad v = \frac{x^3}{3}$

$$= \frac{x^3}{3} \operatorname{arctg} \frac{1}{x} + \int \frac{1}{x^2+1} \cdot \frac{x^3}{3} \, dx =$$

$$= \frac{x^3}{3} \operatorname{arctg} \frac{1}{x} + \frac{1}{3} \int \frac{x^3}{x^2+1} \, dx =$$

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$$x^3 : (x^2+1) = x - \frac{x}{x^2+1}$$

$$-(x^3+x)$$

$$-x$$

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$$= \frac{x^3}{3} \operatorname{arccotg} \frac{1}{x} + \frac{1}{3} \int x - \frac{x}{x^2+1} \, dx = \frac{x^3}{3} \operatorname{arccotg} \frac{1}{x} +$$

$$+ \frac{1}{3} \frac{x^2}{2} - \frac{1}{3} \cdot \frac{1}{2} \int \frac{2x}{x^2+1} \, dx = \frac{x^3}{3} \operatorname{arccotg} \frac{1}{x} +$$

$$+ \frac{x^2}{6} - \frac{1}{6} \ln |x^2+1| + c$$

a) $$\int \frac{8x - \arcsin^2 x}{\sqrt{1-x^2}} \, dx = \int \underbrace{\frac{8x}{\sqrt{1-x^2}}}_{I_1} - \underbrace{\frac{\arcsin^2 x}{\sqrt{1-x^2}}}_{I_2} \, dx$$

ROZDELIŤ NA 2 INT. + SUBSTITÚCIA

$I_1$: $$\int \frac{8x}{\sqrt{1-x^2}}\,dx = \left|\begin{aligned} t &= 1-x^2 \\ dt &= -2x\,dx \\ x\,dx &= \frac{dt}{-2} \end{aligned}\right.$$

$$= \int \frac{8}{\sqrt{t}}\,\frac{dt}{-2} = -4\int t^{-\frac{1}{2}}\,dt = -\frac{4t^{\frac{1}{2}}}{\frac{1}{2}} + C =$$

$$= -8\sqrt{1-x^2} + C$$

$I_2$: $\int \frac{\arcsin^2 x}{\sqrt{1-x^2}}\,dx = \left| \begin{array}{l} t = \arcsin x \\ dt = \frac{1}{\sqrt{1-x^2}}\,dx \end{array} \right.$

$= \int t^2\,dt = \frac{t^3}{3} + C = \frac{\arcsin^3 x}{3} + C$

$= -8\sqrt{1-x^2} - \frac{\arcsin^3 x}{3} + C$

$$\text{e)} \quad \int (x^2+3x)\, e^{-5x}\, dx = \left| \begin{array}{ll} u = x^2+3x & u' = 2x+3 \\ v' = e^{-5x} & v = \dfrac{e^{-5x}}{-5} \end{array} \right.$$

2x PER PARTES

$$= \frac{(x^2+3x)\, e^{-5x}}{-5} + \int \frac{(2x+3)}{5}\, e^{-5x}\, dx = \left| \begin{array}{ll} u = \dfrac{2x+3}{5} & u' = \dfrac{2}{5} \\ v' = e^{-5x} & v = \dfrac{e^{-5x}}{-5} \end{array} \right.$$

$$= \underbrace{-\frac{(x^2+3x)}{5}\, e^{-5x} - \frac{(2x+3)}{25}\, e^{-5x}} + \int \frac{2e^{-5x}}{25}\, dx =$$

$$= \text{————} \;\|\; \text{————} - \frac{2}{125}\, e^{-5x} + C$$

7. $\displaystyle\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx =$

SUBST.

$$\boxed{\begin{aligned} t &= \sqrt{1-x^2} \\ t^2 &= 1-x^2 \\ \cancel{2}t\,dt &= -\cancel{2}x\,dx \\ x\,dx &= -t\,dt \end{aligned}}$$

$=$

$$= \int \frac{-t\,dt}{t^2+t} = \int \frac{-1}{t+1}\,dt = -\ln\left|\sqrt{1-x^2}+1\right| + C$$

9 $\int \cot x \cdot \ln(\sin x)\,dx = \left|\begin{array}{l} t = \ln(\sin x) \\ dt = \frac{1}{\sin x} \cdot \cos x\,dx \\ dt = \cot x\,dx \end{array}\right.$

SUBST.

$$= \int t\,dt = \frac{t^2}{2} + C = \frac{\ln^2(\sin x)}{2} + C$$

h) $$\int \frac{1}{\cos^2 x \sqrt{5+\operatorname{tg} x}}\,dx = \left| \begin{array}{l} t = 5+\operatorname{tg} x \\ dt = \frac{1}{\cos^2 x}\,dx \end{array} \right| =$$

SUBST

$$= \int \frac{dt}{\sqrt{t}} = \int t^{-\frac{1}{2}}\,dt = 2\sqrt{t} + C = 2\sqrt{5+\operatorname{tg} x} + C$$

i) $$\int 1 \cdot \ln(x^2-2x-3)\,dx = \left| \cancel{u = \ln(x^2-2x-3)} \right.$$

PER PARTES + ROZKLAD NA P. Z.

$u = \ln(x^2 - 2x - 3)$ $\qquad u' = \frac{1}{x^2 - 2x - 3} \cdot (2x - 2)$

$v' = 1$ $\qquad v = x$

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$= x \ln(x^2 - 2x - 3) - \int \frac{2x^2 - 2x}{x^2 - 2x - 3}\, dx =$

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$(2x^2 - 2x) : (x^2 - 2x - 3) = 2 + \frac{2x + 6}{x^2 - 2x - 3}$

$-(2x^2 - 4x - 6)$

$2x + 6$

$D > 0$

$(x - 3)(x + 1)$

MÔŽEM ROZLOŽIŤ NA SÚČIN

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$$= x\ln(x^2-2x-3) - \int 2 + \frac{2x+6}{(x-3)(x+1)}\,dx =$$

$$= x\ln(x^2-2x-3) - 2x - \int \frac{2x+6}{(x-3)(x+1)}\,dx =$$

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$$\frac{A}{x-3} + \frac{B}{x+1} = \frac{Ax+A+Bx-3B}{(x-3)(x+1)} \Rightarrow$$

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$$\left.\begin{array}{l} A+B=2 \;(-1) \\ A-3B=6 \end{array}\right\} \oplus \quad -4B=4$$

$$\boxed{B=-1} \Rightarrow \boxed{A=3}$$

$$= x\ln(x^2-2x-3) - 2x - \int \frac{3}{x-3} - \frac{1}{x+1}\,dx =$$

$$= x\ln(x^2-2x-3) - 2x - 3\ln|x-3| + \ln|x+1| + C$$

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j) $\int 1\cdot\ln(x^2+2x+3)\,dx =$ $\left|\begin{array}{ll} u = \ln(x^2+2x+3) & u' = \dfrac{2x+2}{x^2+2x+3} \\ v' = 1 & v = x \end{array}\right|$

PER PARTES + ÚPRAVA NA ÚPLNÝ ČTVEREC

$$= x\ln(x^2+2x+3) - \int \frac{2x^2+2x}{x^2+2x+3}\,dx =$$

$$(2x^2+2x):(x^2+2x+3) = 2 + \frac{-2x-6}{x^2+2x+3}$$

$$\frac{-(2x^2+4x+6)}{-2x-6}$$

$$= x\ln(x^2+2x+3) - \int 2 + \frac{-2x-6}{x^2+2x+3}\,dx =$$

$$= x\ln(x^2+2x+3) - 2x + \int \frac{2x+6}{x^2+2x+3}\,dx =$$

ROZPÍŠEM AKO $2+4$

$D < 0$

NEMÔŽEM ROZLOŽIŤ

MUSÍM UPRAVIŤ NA ÚPLNÝ ŠTVOREC

$$(x+1)^2 - 1 + 3 = (x+1)^2 + 2$$

$$= \underbrace{x \ln(x^2+2x+3) - 2x + \ln(x^2+2x+3)} + \int \frac{4}{(x+1)^2+2}\,dx$$

$$= \quad — \quad '' \quad — \quad + \frac{4}{\sqrt{2}} \operatorname{arctg} \frac{x+1}{\sqrt{2}} + C$$