PR 1

a) $f(x) = \dfrac{x^4}{(1+x)^3}$

$(1+x)^3 \neq 0$

$1+x \neq 0$

$x \neq -1$

$\boxed{D(f) = R - \{-1\}}$

$$f'(x) = \frac{4x^3 \cdot (1+x)^3 - x^4 \cdot 3(1+x)^2 \cdot 1}{(1+x)^6} =$$

$$= \frac{\cancel{(1+x)^2}\,(4x^3 + 4x^4 - 3x^4)}{(1+x)^{\cancel{6}\,4}} = \frac{4x^3 + x^4}{(1+x)^4} = \frac{x^3(4+x)}{(1+x)^4}$$

$x = 0$ $\quad$ $x = -4$

NULOVÉ BODY 1. DERIVÁCIE 0

	$(-\infty,-4)$	$(-4,-1)$	$(-1,0)$	$(0,\infty)$
x^3	$-$	$-$	$-$	$+$
$4+x$	$-$	$+$	$+$	$+$
$(1+x)^4$	$+$	$+$	$+$	$+$
	$+$ $\nearrow$	$-$ $\searrow$	$-$ $\searrow$	$+$ $\nearrow$

FUNKCIA JE RÝDZO RASTÚCA NA INTERVALOCH

$(-\infty,-4\rangle$ a $\langle 0,\infty)$.

FUNKCIA JE RÝDZO KLESAJÚCA NA $\langle -4,-1)$ a $(-1,0\rangle$

(b) $f(x) = x + 2\,\mathrm{arccotg}\,x$ $\qquad D(f) = \mathbb{R}$

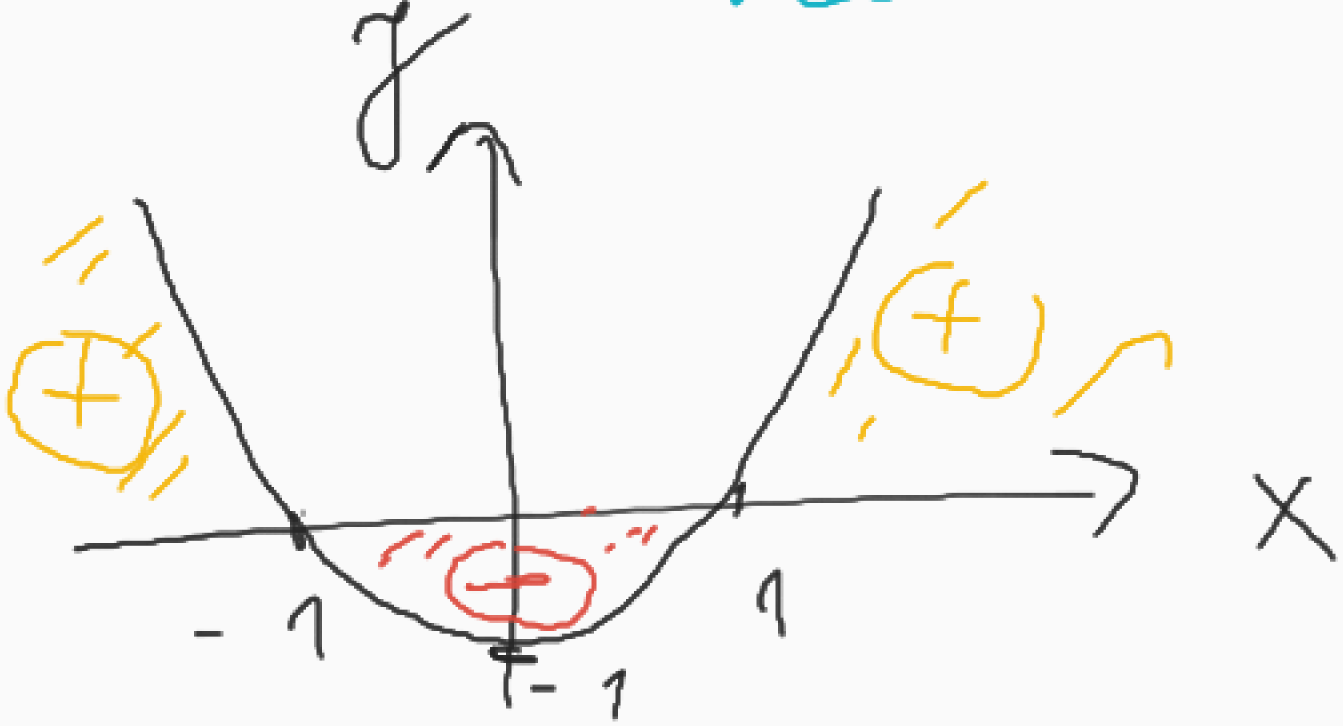
$$f'(x) = 1 + 2\cdot(-1)\cdot\frac{1}{1+x^2} = 1 - \frac{2}{1+x^2} =$$

$$= \frac{1+x^2-2}{1+x^2} = \frac{x^2-1}{1+x^2}$$

VŽDY KLADNÉ

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$



FUNKCIA $f(x)$ JE RÝDZO RASTÚCA NA INTERVALOCH $(-\infty,-1\rangle$ a $\langle 1,\infty)$

FUNKCIA JE RÝDZO KLES. NA $\langle -1,1\rangle$

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c) $f(x)=\cos(x)+\ln(\underbrace{\cos x})$

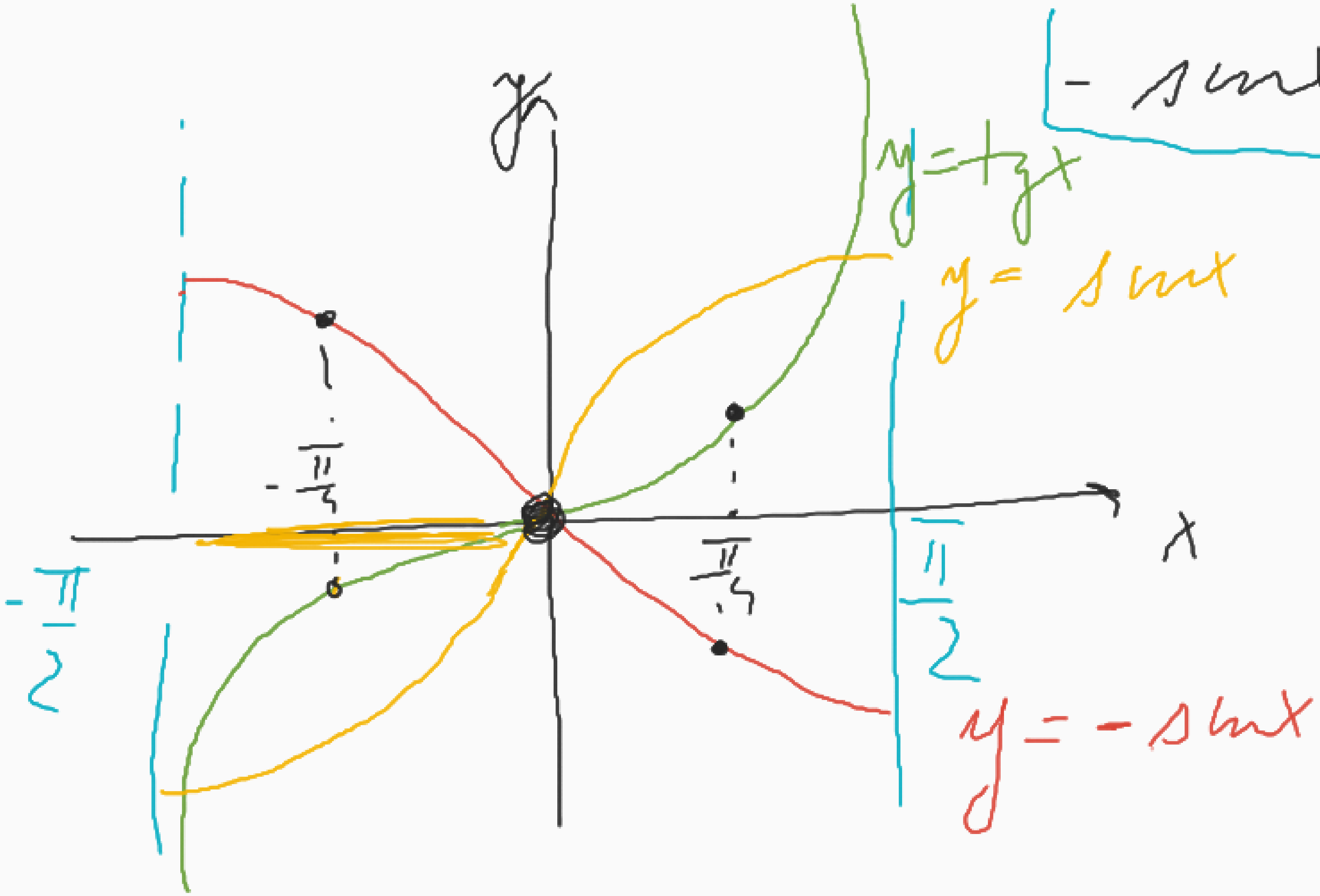
$D(f): \left\{ \left(-\frac{\pi}{2}+2k\pi, \frac{\pi}{2}+2k\pi\right); k\in\mathbb{Z} \right\}$

$\cos x>0$

$$f'(x) = -\sin x + \frac{1}{\cos x}\cdot(-\sin x) = -\sin x - \operatorname{tg} x$$

$$f'(x) = 0 \iff \boxed{-\sin x - \operatorname{tg} x} = 0$$

$$\boxed{-\sin x = \operatorname{tg} x} \Rightarrow x = 0 + 2k\pi;\ k \in \mathbb{Z}$$



$R\left(-\frac{\pi}{2}, 0\right) \Rightarrow f'\left(-\frac{\pi}{4}\right) = -\sin\left(-\frac{\pi}{4}\right) - \operatorname{tg}\left(-\frac{\pi}{4}\right) =$

$= \frac{\sqrt{2}}{2} - (-1) = \frac{\sqrt{2}}{2} + 1 > 0$

$K\left\langle 0, \frac{\pi}{2}\right)$

$\Rightarrow f'\left(\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} - \operatorname{tg}\frac{\pi}{4} = -\frac{\sqrt{2}}{2} - 1 < 0$

ZÁVER: FUNKCIA JE RÝDZO RASTÚCA NA INTERVALE $\left(-\frac{\pi}{2} + 2k\pi, 0 + 2k\pi\right\rangle$ a RÝDZO KLESAJÚCA NA $\left\langle 0 + 2k\pi, \frac{\pi}{2} + 2k\pi\right)$; $k \in \mathbb{Z}$

KONVEXNOSŤ A KONKÁVNOSŤ

(a) $f(x) = \arcsin\left(\frac{1-x}{1+x}\right)$

$1+x \neq 0$

$x \neq -1$

$$-1 \leq \frac{1-x}{1+x} \leq 1$$

$$-1 - \frac{1-x}{1+x} \leq 0 \quad \wedge \quad \frac{1-x}{1+x} - 1 \leq 0$$

$$\frac{-1-\cancel{x}-1+\cancel{x}}{1+x} \leq 0$$

VŽDY ZÁPORNÉ

$$\frac{-2}{1+x} \leq 0$$

$$1+x>0$$

$$\boxed{x>-1}$$

$\wedge$

$$\frac{\cancel{1}-x-\cancel{1}-x}{1+x} \leq 0$$

$$\frac{-2x}{1+x} \leq 0$$

$$(-2x \leq 0 \wedge 1+x>0) \vee$$

$$(-2x \geq 0 \wedge 1+x<0) \Rightarrow$$

$$\Rightarrow \underbrace{(x \geq 0 \wedge x>-1)}\cup\underbrace{(x \leq 0 \wedge x<-1)}$$

$$D(f): \langle 0, \infty)$$

$$f(x) = \arcsin\left(\frac{1-x}{1+x}\right)$$

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{1-x}{1+x}\right)^2}} \cdot \frac{-1(1+x)-(1-x)\cdot 1}{(1+x)^2} =$$

$$= \frac{1}{\sqrt{\frac{\overbrace{\cancel{1}+2x+\cancel{x^2}-\cancel{1}+2x-\cancel{x^2}}^{4x}}{(1+x)^2}}} \cdot \frac{\overbrace{-1-\cancel{x}-1+\cancel{x}}^{-2}}{(1+x)^2} =$$

$$= \frac{\cancel{1+x}}{\cancel{2}\sqrt{x}} \cdot \frac{\cancel{-2}}{(1+x)^{\cancel{2}}} = \frac{-1}{\sqrt{x}\cdot(1+x)}$$

$$= -x^{-\frac{1}{2}} \cdot (1+x)^{-1}$$

$$f''(x) = -\left(-\frac{1}{2}\right) \cdot x^{-\frac{3}{2}} \cdot (1+x)^{-1} - x^{-\frac{1}{2}} \cdot (-1)(1+x)^{-2} =$$

$$= \frac{1}{2x\sqrt{x} \cdot (1+x)} + \frac{1}{\sqrt{x} \cdot (1+x)^2} = \frac{1+x+2x}{2x\sqrt{x} \cdot (1+x)^2} =$$

$$= \frac{1+3x}{2x\sqrt{x}(1+x)^2} > 0 \Rightarrow$$

KONVEXNÁ NA CELOM $D(f)$

(b) $y(x) = \dfrac{2x-1}{(x-1)^2}$

$x - 1 \neq 0$

$x \neq 1$

$D(y) = \mathbb{R} - \{1\}$

$$y'(x) = \frac{2 \cdot (x-1)^2 - (2x-1) \cdot 2(x-1)}{(x-1)^4} =$$

$$= \frac{\cancel{(x-1)}(2x - \cancel{2} - 4x + \cancel{2})}{(x-1)^{\cancel{4}\,3}} = \underline{\underline{\frac{-2x}{(x-1)^3}}}$$

$$f''(x) = \frac{-2(x-1)^3 + 2x \cdot 3(x-1)^2}{(x-1)^6} =$$

$$= \frac{\cancel{(x-1)^2}\,(-2x+2+6x)}{(x-1)^{\cancel{6}\,4}} = \frac{4x+2}{(x-1)^3}$$

$x = -\frac{1}{2}$

vždy $+$

	$(-\infty, -\frac{1}{2}\rangle$	$\langle -\frac{1}{2}, 1)$	$(1, \infty)$
$4y+2$	$-$ $\cap$	$+$ $\cup$	$+$ $\cup$

NA

KONKÁVNA NA $(-\infty, -\frac{1}{2}\rangle$ A KONVEXNÁ

(c) $f(x) = 1 + (x^2 - 1)^3 \qquad D(f) = \mathbb{R}$

$$f'(x) = 3(x^2-1)^2 \cdot 2x = 6x(x^2-1)^2$$

$$f''(x) = 6(x^2-1)^2 + 6x \cdot 2(x^2-1)^1 \cdot 2x =$$

$$= 6(x^2-1)(x^2-1+4x^2) = 6\underbrace{(x^2-1)}_{x=\pm 1}\underbrace{(5x^2-1)}_{x=\pm\frac{1}{\sqrt{5}}}$$

	$(-\infty, -1)$	$\langle -1, -\frac{1}{\sqrt{5}} \rangle$	$\langle -\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}} \rangle$	$\langle \frac{1}{\sqrt{5}}, 1 \rangle$	$\langle 1, \infty)$
x^2-1	+	−	−	−	+
$5x^2-1$	+	+	−	+	+
$(x^2-1)\cdot(5x^2-1)$	+ $\cup$	− $\cap$	+ $\cup$	− $\cap$	+ $\cup$

IF   IF   IF   IF

IF = INFLEXNÍ BOD