(1) $x^2 + xy + y^2 - 3 = 0$

$x \cdot y(x) \Rightarrow$ MUSÍM DERIVOVAŤ AKO SÚČIN

$$2x + 1 \cdot y + x\frac{dy}{dx} + 2y \cdot \frac{dy}{dx} - 0 = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx}(x + 2y) = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

(2) $x^2 - 3xy + 4y^2 - 2x + 3y = 0$

$$2x - 3y - 3x\frac{dy}{dx} + 8y\frac{dy}{dx} - 2 + 3\frac{dy}{dx} = 0$$

$$-3x\frac{dy}{dx} + 8y\frac{dy}{dx} + 3\frac{dy}{dx} = -2x + 3y + 2$$

$$\frac{dy}{dx}(-3x + 8y + 3) = -2x + 3y + 2$$

$$\frac{dy}{dx} = \frac{-2x + 3y + 2}{-3x + 8y + 3}$$

(3) $x^2y^3 - \sin(xy) = 0$

$$2xy^3 + x^2 \cdot 3y^2 \cdot \frac{dy}{dx} - \cos(xy) \cdot \left[1y + x\frac{dy}{dx}\right] = 0$$

$$2xy^3 + x^2 3y^2 \frac{dy}{dx} - y\cos(xy) - x\cos(xy)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(3x^2y^2 - x \cdot \cos(xy)\right) = y\cos(xy) - 2xy^3$$

$$\frac{dy}{dx} = \frac{y\cos(xy) - 2xy^3}{3x^2y^2 - x\cos(xy)}$$

(4) PARAMETRICKY ZADANÁ FUNKCIA

$$x = te^t$$
$$y = t^3 + 6t \quad ; \quad t \in (0, \infty)$$

$$\frac{dy}{dx} = \frac{(t^3+6t)'}{(te^t)'} = \frac{3t^2+6}{e^t + te^t} = \underline{\underline{\frac{3(t^2+2)}{e^t(1+t)}}}$$

5

$$x = \sqrt{t^3}$$

$$y = t^2 \qquad t \in \langle 0, \infty)$$

$$\frac{dy}{dx} = \frac{(t^2)'}{(\sqrt{t^3})'} = \frac{2t}{(t^{\frac{3}{2}})'} = \frac{2t}{\frac{3}{2} t^{\frac{1}{2}}} = \frac{4t}{3\sqrt{t}} =$$

$$= \frac{4\sqrt{t}\,\cancel{\sqrt{t}}}{3\cancel{\sqrt{t}}} = \underline{\underline{\frac{4}{3}\sqrt{t}}}$$

DERIVÁCIE VYŠŠÍCH RÁDOV

1. $f^{4}(x)$, AK $f(x) = x^6 + 5x^4 + 2x^3 - x^2$

$$f'(x) = 6x^5 + 20x^3 + 6x^2 - 2x$$
$$f''(x) = 30x^4 + 60x^2 + 12x - 2$$
$$f'''(x) = 120x^3 + 120x + 12$$
$$f^{4}(x) = 360x^2 + 120$$

2. $f^{4}(x)$, ak $f(x) = \frac{2}{x} = 2x^{-1}$

$f'(x) = -2x^{-2}$

$f''(x) = 4x^{-3}$

$f'''(x) = -12x^{-4}$

$f^{4}(x) = 48x^{-5}$

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3. $f''(x)$, ak $f(x) = \operatorname{tg} x$

$f'(x) = \frac{1}{\cos^2 x} = \cos^{-2} x$

$f''(x) = -2\cos^{-3} x \cdot (-\sin x) = \frac{2\sin x}{\cos^3 x}$

4 $f'''(x)$, ak $f(x) = \operatorname{arctg} x$

$$f'(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$$

$$f''(x) = -1 \cdot (1+x^2)^{-2} \cdot 2x = -2x \cdot (1+x^2)^{-2}$$

$$f'''(x) = -2 \cdot (1+x^2)^{-2} - 2x \cdot (-2)(1+x^2)^{-3} \cdot 2x =$$

$$= \frac{-2}{(1+x^2)^2} + \frac{8x^2}{(1+x^2)^3} = \frac{-2-2x^2+8x^2}{(1+x^2)^3} = \frac{6x^2-2}{(1+x^2)^3}$$

(5) $f^{(5)}(x)$, Ak $f(x) = x^4 \cdot \ln x$

$$f'(x) = 4x^3 \ln x + x^4 \cdot \frac{1}{x} = 4x^3 \ln x + x^3$$

$$f''(x) = 12x^2 \ln x + \underbrace{4x^{\cancel{3}\,2} \cdot \frac{1}{\cancel{x}} + 3x^2}_{7x^2} =$$

$$= 12x^2 \ln x + 7x^2$$

$$f'''(x) = 24x \cdot \ln x + 12x^{\cancel{2}} \cdot \frac{1}{\cancel{x}} + 14x =$$

$$= 24x \ln x + 26x$$

$$f^{4}(x) = 24 \ln x + 24\cancel{x} \cdot \frac{1}{\cancel{x}} + 26 \quad 50$$

$$f^5(x) = \frac{24}{x}$$

DOTYČNICE A NORMÁLY KU GRAFU FUNKCIE

PR1 $x_0 = \frac{\pi}{4}$; $f(x) = tg\,x$

k_t: $f'(x) = \frac{1}{\cos^2 x}$

$f(x_0) = tg\left(\frac{\pi}{4}\right) = 1$

t: $y - y_0 = k_t (x - x_0)$

$y - 1 = 2 \cdot \left(x - \frac{\pi}{4}\right)$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{4}{2} = 2$

n: $y - 1 = -\frac{1}{2}\left(x - \frac{\pi}{4}\right)$

PR2 $\quad f(x) = x^2 - 3x + 5$; $t \parallel p$; $p: x - y + 1 = 0$

$y = kx + q$

$f'(x) = 2x - 3$ $\qquad p: y = 1x + 1$

$k_p = 1$

$2x_0 - 3 = 1$

$2x_0 = 4$

$x_0 = 2$

$y_0 = f(x_0) = 2^2 - 3.2 + 5 = \underline{3}$

$t: y - 3 = 1(x - 2)$

$n: y - 3 = -1(x - 2)$

PR3 $\quad f(x) = \ln(x-2)$; $\boxed{t \perp p}$; $p:\ x+y=0$

$y = -x \rightarrow k_p = -1$

$$\boxed{f'(x) = \frac{1}{x-2}}$$

$$\boxed{k_t = 1}$$

$$\frac{1}{x_0 - 2} = 1$$

$$1 = x_0 - 2$$

$$\boxed{x_0 = 3}$$

$$t:\ y - 0 = 1(x-3)$$

$$n:\ y - 0 = -1(x-3)$$

$$f(x_0) = \ln(3-2) = \ln 1 = \underline{\underline{0}}$$

$f(x) = \operatorname{arctg} x \qquad \alpha = 45^\circ$

$k_t = \operatorname{tg} \alpha = \operatorname{tg} 45^\circ = \operatorname{tg}\left(\frac{\pi}{4}\right) = 1$

$f'(x) = \frac{1}{1+x^2}$

$\frac{1}{1+x_0^2} = 1$

$1 = 1 + x_0^2$

$\boxed{x_0 = 0}$

$f(x_0) = f(0) = \operatorname{arctg} 0 = 0$

$t: \ y - 0 = 1(x - 0)$

$n: \ y - 0 = -1(x - 0)$