1. $$\lim_{x \to 2} \frac{x-2}{\underbrace{x^2-3x+2}_{4-6+2=0}} \overset{0}{} \;\; \text{„}\frac{0}{0}\text{"} \;\; \lim_{x \to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}\underbrace{(x-1)}_{2-1}} = \frac{1}{1}$$

2. $$\lim_{x \to 0} \frac{3x + \frac{2}{x}}{x + \frac{4}{x}} = \lim_{x \to 0} \left[\frac{\frac{3x^2+2}{x}}{\frac{x^2+4}{x}}\right] =$$

$$= \lim_{x \to 0} \frac{\overbrace{(3x^2+2)}^{2} \cdot \cancel{x}}{\underbrace{(x^2+4)}_{4} \cdot \cancel{x}} = \frac{2}{4} = \frac{1}{2}$$

$$③\ \lim_{x\to\infty}\left(\frac{1-x}{x^2}+\frac{1-2x}{2-3x}\right)=\lim_{x\to\infty}\left(\frac{1-x}{x^2}\right)+$$

$$\text{"}\frac{\infty}{\infty}\text{"} \qquad \text{"}\frac{\infty}{\infty}\text{"}$$

$$+\lim_{x\to\infty}\frac{1-2x}{2-3x}=\lim_{x\to\infty}\left(\underbrace{\frac{1}{x^2}}_{0}-\underbrace{\frac{1}{x}}_{0}\right)=0\;+$$

$$+\lim_{x\to\infty}\frac{\cancel{x}\left(\underbrace{\frac{1}{x}}_{0}-2\right)}{\cancel{x}\left(\underbrace{\frac{2}{x}}_{0}-3\right)}=\frac{2}{3}$$

(4) $\lim\limits_{x \to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right) = \lim\limits_{x \to 1} \frac{1+x+x^2-3}{1-x^3} =$

$0 \qquad 0$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$= \lim\limits_{x \to 1} \frac{x^2+x-2}{1-x^3} \underset{=}{\overset{\text{"}\frac{0}{0}\text{"}}{=}} \lim\limits_{x \to 1} \frac{(x-1)(x+2)}{(1-x)(1+x+x^2)} =$

$= \lim\limits_{x \to 1} \frac{-\cancel{(1-x)}(x+2)}{\cancel{(1-x)}(1+x+x^2)} = \frac{-1-2}{1+1+1} = \underline{\underline{-1}}$

(5) $$\lim_{x\to\infty}\left(\frac{5x^2}{x^2-1}+2^{\frac{1}{x}}\right) = \lim_{x\to\infty}\frac{\cancel{x^2}\,5}{\cancel{x^2}\left(1-\frac{1}{x^2}\right)} +$$

$$\text{"}\frac{\infty}{\infty}\text{"} \qquad \frac{1}{\infty}=0$$

$$+\lim_{x\to\infty} 2^{\frac{1}{x}} = \frac{5}{1}+1=\underline{\underline{6}}$$

$$\text{"}2^{0}$$

(6) $$\lim_{x \to 1} \frac{1-3^x}{x} = \frac{1-3^1}{1} = -2$$

(7) $$\lim_{x \to 0} \frac{1-3^x}{x} = -\lim_{x \to 0} \frac{3^x-1}{x} = -\ln 3$$

$$\boxed{\lim_{x \to 0} \left(\frac{a^x-1}{x}\right) = \ln a}$$

(8) $\lim\limits_{x\to-\infty} \frac{1-3^x}{x} = \lim\limits_{x\to-\infty} \frac{1}{x} - \lim\limits_{x\to-\infty} \frac{3^x}{x}$

$\lim\limits_{x\to-\infty} \frac{1}{x} = 0$

$3^x \to$ "$3^{-\infty}$" $= \frac{1}{3^\infty}$ "

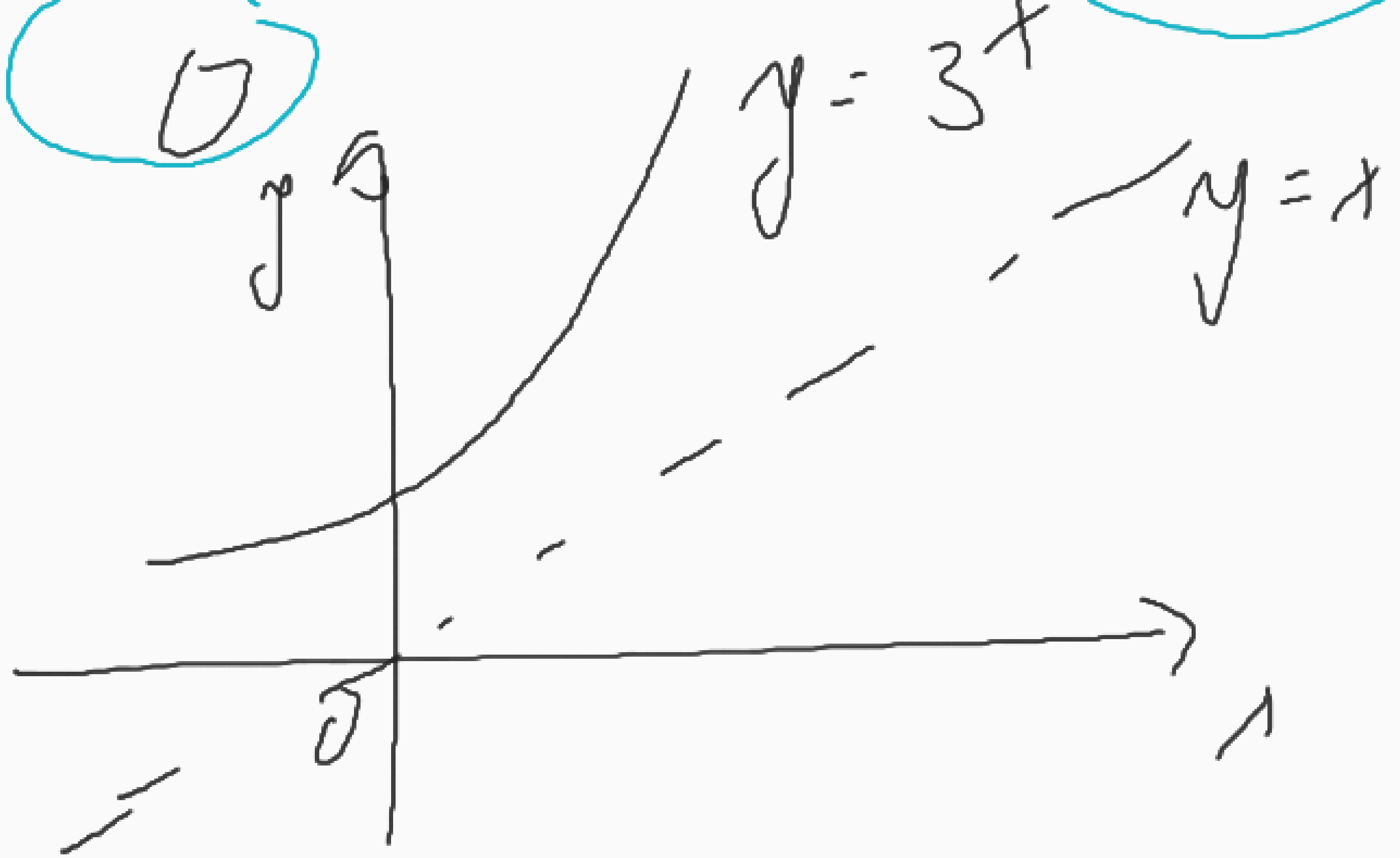
$3^\infty = \infty$, $\frac{1}{\infty} = 0$

$\lim\limits_{x\to-\infty} \frac{3^x}{x} = 0$

$= 0$

(9) $\lim\limits_{x\to\infty} \dfrac{1-3^x}{x} = \underbrace{\lim\limits_{x\to\infty} \dfrac{1}{x}}_{=0} - \underbrace{\lim\limits_{x\to\infty} \dfrac{3^x}{x}}_{\infty}$

$= -\infty$



(10) $$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x} = \lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x}$$

$$= \underline{\underline{e}}$$

(11) $$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x} = \lim_{x\to\infty}\left(\frac{3x\overbrace{+1-1}^{=0}-2}{3x+1}\right)^{x} =$$

1

$$= \lim_{x \to \infty} \left(\frac{3x+1}{3x+1} + \frac{-1-2}{3x+1} \right)^x = \lim_{x \to \infty} \left(1 + \frac{-3}{3x+1} \right)^x$$

$t = 3x+1$

$x = \frac{t-1}{3}$

$x \to \infty \Rightarrow t \to \infty$

$$= \lim_{t \to \infty} \left(1 + \frac{-3}{t} \right)^{\frac{t-1}{3}} =$$

$$= \lim_{t\to\infty}\left(1+\frac{-3}{t}\right)^{\frac{t}{3}} \cdot \lim_{t\to\infty}\underbrace{\left(1+\underbrace{\frac{-3}{t}}_{0}\right)}_{1}^{-\frac{1}{3}} = 1 =$$

$$= \lim_{t\to\infty}\left[\underbrace{\left(1+\frac{-3}{t}\right)^{t}}_{e^{-3}}\right]^{\frac{1}{3}} = \left(e^{-3}\right)^{\frac{1}{3}} = e^{-1}$$

(12) $\lim\limits_{x \to 0} \dfrac{1-\sqrt{1-x}}{x} = \text{"}\dfrac{0}{0}\text{"} =$

$(a-b)\,(a+b) = a^2-b^2$

$= \lim\limits_{x \to 0} \dfrac{1-\sqrt{1-x}}{x} \cdot \dfrac{1+\sqrt{1-x}}{1+\sqrt{1-x}} =$

$= \lim\limits_{x \to 0} \dfrac{\cancel{1}-\cancel{1}+\cancel{x}}{\cancel{x}\left(1+\sqrt{1-x}\right)} = \dfrac{1}{1+1} = \dfrac{1}{2}$

(13) $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{1+x} - \sqrt{1-x}} = \dfrac{0}{0}$

$= \lim\limits_{x \to 0} \dfrac{x}{\sqrt{1+x} - \sqrt{1-x}} \cdot \dfrac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} =$

$= \lim\limits_{x \to 0} \dfrac{\cancel{x}\left(\overset{\to 1}{\sqrt{1+x}} + \overset{\to 1}{\sqrt{1-x}}\right)}{\underbrace{\cancel{1} + x - \cancel{1} + x}_{2\cancel{x}}} = \frac{2}{2} = 1$

(14) $$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2-1}}{x} +$$

$$+\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to\infty} \sqrt{\frac{x^2-1}{x^2}} +$$

$$+\lim_{x\to\infty} \sqrt{\frac{x^2+1}{x^2}} = \sqrt{\lim_{x\to\infty} \frac{x^2-1}{x^2}} +$$

$$+\sqrt{\lim_{x\to\infty} \frac{x^2+1}{x^2}} = \sqrt{\underbrace{\lim_{x\to\infty} \frac{1-\overset{\to 0}{\frac{1}{x^2}}}{1}}_{=1}} +$$

$$+\sqrt{\underbrace{\lim \frac{1+\frac{1}{x^2}}{1}}_{=1}} = 1+1 = \underline{\underline{2}}$$

(15) $\lim\limits_{x\to\infty}\left(x-\sqrt{x^2-1}\right) \underline{\underline{\text{“}\infty-\infty\text{”}}}$

$$= \lim_{x\to\infty}\left(x-\sqrt{x^2-1}\right)\cdot\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} =$$

$$= \lim_{x\to\infty}\frac{\overbrace{\cancel{x^2}-\cancel{x^2}+1}^{1}}{\underbrace{x+\sqrt{x^2-1}}_{\infty+\infty=\infty}} = \underline{\underline{0}}$$

(16) $$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1} = \text{"}\frac{0}{0}\text{"}$$

$$\left| \begin{array}{l} t = \sqrt[6]{x} \\ x = t^6 \\ x \to 1 \Rightarrow t \to 1 \end{array} \right|$$

$$= \lim_{t\to 1} \frac{t^3-1}{t^2-1} = \lim_{t\to 1} \frac{\cancel{(t-1)}(t^2+t+1)}{\cancel{(t-1)}(t+1)} = \underline{\underline{\frac{3}{2}}}$$

(17) $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$ "$\dfrac{0}{0}$" $\quad\Big|\quad x = t^{12}$

$x \to 1 \Rightarrow t \to 1$

$= \lim\limits_{t\to 1} \dfrac{t^4-1}{t^3-1} = \lim\limits_{t\to 1} \dfrac{(t^2-1)(t^2+1)}{(t-1)(t^2+t+1)} =$

$= \lim\limits_{t\to 1} \dfrac{\cancel{(t-1)}(t+1)(t^2+1)}{\cancel{(t-1)}(t^2+t+1)} = \dfrac{2\cdot 2}{3} = \underline{\underline{\dfrac{4}{3}}}$

(18)

$$\lim_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1} \quad \infty-\infty \quad = \lim_{x\to\infty} \frac{\cancel{x}\left(\frac{\sqrt{x}}{x}-6\right)}{\cancel{x}\left(3+\frac{1}{x}\right)} =$$

$$= \lim_{x\to\infty} \left(\frac{\overset{0}{\frac{1}{\sqrt{x}}}-6}{3+\overset{0}{\frac{1}{x}}} \right) = -2$$

(19) $\lim_{x \to 0} \frac{\sin(5x)}{x} = "\frac{0}{0}" =$

$= \lim_{x \to 0} \frac{\sin 5x \cdot 5}{5x}$

$$\boxed{\lim_{x \to 0} \frac{\sin \alpha x}{\alpha x} = 1}$$

$= 5 \underbrace{\lim_{x \to 0} \frac{\sin 5x}{5x}}_{1}$

$= \underline{\underline{5}}$

(21) $\lim\limits_{x \to 0} \dfrac{\sin(mx)}{\sin(nx)} = "\dfrac{0}{0}"$

$= \lim\limits_{x \to 0} \dfrac{\sin(mx)}{mx} \cdot m\cancel{x} \cdot \lim\limits_{x \to 0} \dfrac{nx}{\sin(nx) \cdot n\cancel{x}}$ $= 1$ $= 1$

$= \dfrac{m}{n}$