PR 10 a $y = x^3$; $x \in \langle 0, 1 \rangle$

$$\boxed{S = 2\pi \int_a^b |f(x)| \cdot \sqrt{1 + (f'(x))^2}\, dx}$$

$y = x^3 = f(x)$

$f'(x) = 3x^2$

$$S = 2\pi \int_0^1 |x^3| \cdot \sqrt{1 + (3x^2)^2}\, dx =$$

$$= 2\pi \int_0^1 x^3 \sqrt{1 + 9x^4}\, dx =$$

$t = 1 + 9x^4$

$dt = 36x^3 dx \Rightarrow x^3 dx = \frac{dt}{36}$

$t_1 = 1$

$t_2 = 10$

$$= 2\pi \int_1^{10} t^{\frac{1}{2}} \frac{dt}{36} = \frac{\pi}{\cancel{18}\,9} \cdot \frac{\cancel{2}}{3} \left[t^{\frac{3}{2}} \right]_1^{10} = \underline{\underline{\frac{\pi}{27} (10\sqrt{10} - 1)}} \; j^2$$

(b) $y = \sqrt{x}\,, \; x \in \langle 0, 2 \rangle$

$$y' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$S = 2\pi \int_0^2 |\sqrt{x}| \cdot \sqrt{1 + \left(\frac{1}{2\sqrt{x}}\right)^2} \, dx =$$

$$= 2\pi \int_0^2 \sqrt{x} \sqrt{1 + \frac{1}{4x}} \, dx = 2\pi \int_0^2 \sqrt{x} \cdot \sqrt{\frac{4x+1}{4x}} \, dx = \cancel{2}\pi \int_0^2 \cancel{\sqrt{x}} \cdot \frac{\sqrt{4x+1}}{\cancel{2}\cancel{\sqrt{x}}}$$

$$= \pi \int_0^2 \sqrt{4x+1} \, dx =$$

$t = 4x + 1$

$dt = 4\,dx \Rightarrow dx = \frac{dt}{4}$

$t_1 = 1$

$t_2 = 9$

$$= \pi \int_1^9 t^{\frac{1}{2}} \frac{dt}{4} = \frac{\pi}{\cancel{4}\,2} \cdot \frac{\cancel{2}}{3} \left[t^{\frac{3}{2}} \right]_1^9 =$$

$$= \frac{\pi}{6} (9 \cdot 3 - 1) = \frac{\pi}{6} \cdot 26 = \frac{13}{3}\pi \quad J^{\cdot 2}$$

c) $y = x, \quad x \in \langle 0, 3 \rangle$

$y' = 1$

$$S = 2\pi \int_0^3 |x| \underbrace{\sqrt{1+1^2}}_{\sqrt{2}}\, dx = \cancel{2}\pi\sqrt{2}\left[\frac{x^2}{\cancel{2}}\right]_0^3 = \underline{\underline{9\sqrt{2}\pi\ j^2}}$$

a) $x = \cos t$ $\quad t \in \langle 0, 2\pi \rangle$

$y = 1 + \sin t$

$x = \varphi(t)$

$y = \psi(t)$; $t \in \langle \alpha, \beta \rangle$

$$S = 2\pi \int_\alpha^\beta |\psi(t)| \cdot \sqrt{(\varphi'(t))^2 + (\psi'(t))^2}\, dt$$

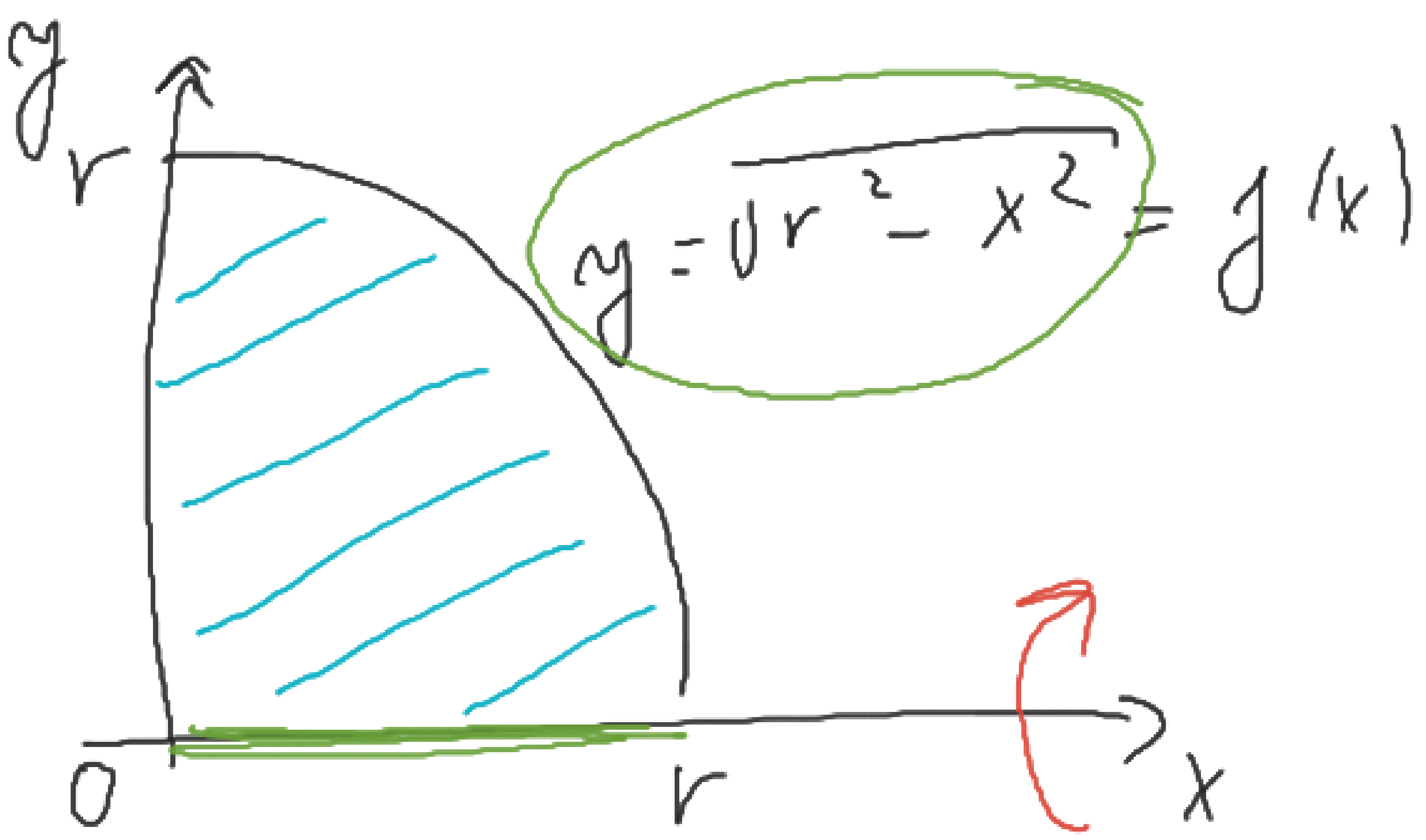
$$S = 2\pi \int_0^{2\pi} |1+\sin t| \cdot \sqrt{\underbrace{(-\sin t)^2 + \cos^2 t}_{=1}}\, dt =$$

$$= 2\pi \int_0^{2\pi} 1+\sin t \, dt = 2\pi \Big[t - \cos t \Big]_0^{2\pi} =$$

$$= 2\pi \Big[2\pi - 1 - (0-1) \Big] = \underline{\underline{4\pi^2 \, j^2}}$$

PR ODVOĎTE VZŤAH NA VÝPOČET OBJEMU GULE, DĹŽKY KRUŽNICE A POVRCHU GULE S POLOMEROM r.

a) OBJEM GULE



$$V = \pi \int_a^b f^2(x)\,dx$$

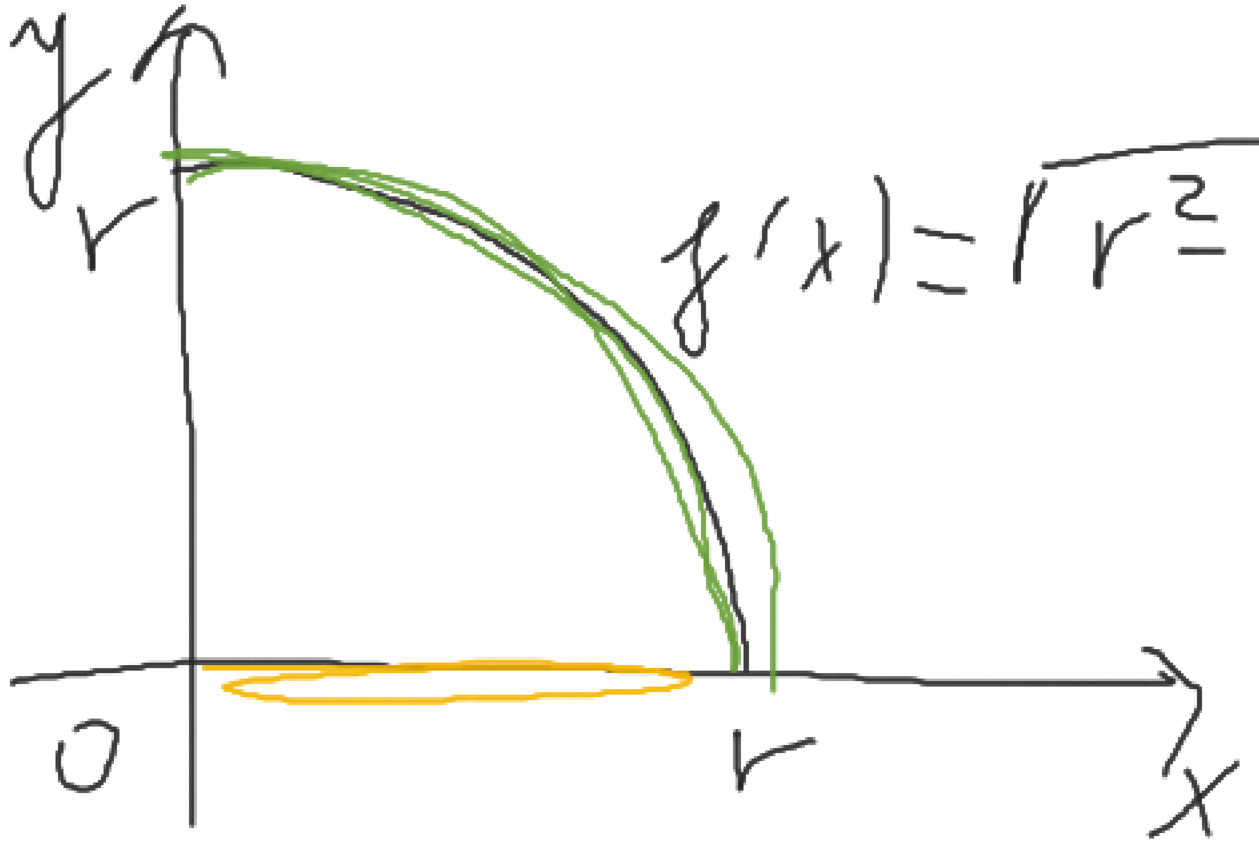
$$V = 2V_1$$

$$V = 2 \cdot \pi \int_0^r \left(\sqrt{r^2 - x^2}\right)^2 dx = 2\pi \int_0^r r^2 - x^2 \, dx =$$

$$= 2\pi \left[r^2 x - \frac{x^3}{3} \right]_0^r = 2\pi \left[r^3 - \frac{r^3}{3} \right] = 2\pi \cdot \frac{2r^3}{3} =$$

$$= \underline{\underline{\frac{4}{3} \pi r^3 \; j^3}}$$

(b) DÉLKA KRUŽNICE



$$f(x)=\sqrt{r^2-x^2}$$

$$D = 4 \cdot d$$

$$d \Downarrow \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

$$D = 4 \cdot \int_0^r \sqrt{1+\left(\frac{-x}{\sqrt{r^2-x^2}}\right)^2}\,dx =$$

$$= 4\int_0^r \sqrt{1+\frac{x^2}{r^2-x^2}}\,dx =$$

$$f(x)=\sqrt{r^2-x^2}$$

$$f'(x)=\frac{1}{\cancel{2}}(r^2-x^2)^{\frac{1}{2}}(-\cancel{2}x)$$

$$f'(x)=\frac{-x}{\sqrt{r^2-x^2}}$$

$$= \boxed{4\int_0^r \sqrt{\frac{r^2 - \cancel{x^2} + \cancel{x^2}}{r^2 - x^2}}\,dx} = 4r\int_0^r \frac{1}{\sqrt{r^2 - x^2}}\,dx =$$

$$= \frac{4\cancel{r}}{\cancel{r}}\int_0^r \frac{1}{\sqrt{1 - \left(\frac{x}{r}\right)^2}}\,dx =$$

$$t = \frac{x}{r}$$

$$dt = \frac{dx}{r} \Rightarrow dx = r\,dt$$

$$x_1 = 0 \Rightarrow t_1 = 0$$

$$x_2 = r \Rightarrow t_2 = 1$$

$$= 4\int_0^1 \frac{1}{\sqrt{1 - t^2}}\, r\,dt =$$

$$= 4r \cdot \left[\arcsin t\right]_0^1 = 2\cancel{4}r \cdot \left[\underbrace{\arcsin 1}_{\frac{\pi}{\cancel{2}}} - \underbrace{\arcsin 0}_{=0}\right] = \underline{\underline{2\pi r}}$$

C POVRCH GULE

$$f(x)=\sqrt{r^2-x^2}$$

$$S = 2 S_y$$

$$\Downarrow$$

$$2\pi \int_a^b |f(x)| \cdot \sqrt{1+(f'(x))^2}\, dx$$

$$*$$

$$= 2 \cdot 2\pi \int_0^r \cancel{\sqrt{r^2-x^2}} \cdot \frac{1 \cdot r}{\cancel{\sqrt{r^2-x^2}}}\, dx = 4\pi r [x]_0^r =$$

$$= 4\pi \cdot r \cdot r = \underline{\underline{4\pi r^2 \; j^2}}$$