PR2

$$\boxed{df(x_0, x) = f'(x_0)(x - x_0)}$$

a) $f(x) = \ln(\sin x) \quad ; \quad x_0 = \frac{\pi}{4}$

$$f'(x) = \frac{1}{\sin x} \cdot \cos x \Rightarrow f'\left(\frac{\pi}{4}\right) = \frac{\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \frac{\cancel{\frac{\sqrt{2}}{2}}}{\cancel{\frac{\sqrt{2}}{2}}} = 1$$

$$df\left(\frac{\pi}{4}, x\right) = 1 \cdot \left(x - \frac{\pi}{4}\right)$$

(b) $f(x) = \operatorname{arctg}\left(\frac{x}{2}\right)$; $x_0 = 2$

$$f'(x) = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2}$$

$$f'(2) = \frac{1}{1+\left(\frac{2}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\underline{\underline{df(2,x) = \frac{1}{4}(x-2)}}$$

(c) $f(x) = \sqrt{x^2+1}$; $x_0 = 1$

$$f'(x) = \frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}} \cdot \cancel{2}x$$

$$f'(1) = (1+1)^{-\frac{1}{2}} \cdot 1 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$df(1,x) = \frac{\sqrt{2}}{2}(x-1)$$

PR4

$\sqrt{98}$ x

1

$f(x) = \sqrt{x}$ $x_0 = 100$

$f(x_0) = f(100) = \sqrt{100} = 10$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(x_0) = f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20}$

$$\boxed{f(x) \approx f(x_0) + f'(x_0)(x - x_0)}$$

$$\sqrt{98} \approx 10 + \frac{1}{20}(\underbrace{98-100}_{-2}) = 10 - \frac{1}{10} = \underline{\underline{9.9}}$$

② $(2{,}03)^3$ $\quad$ x

$$f(x) = x^3 \Rightarrow f'(x) = 3x^2$$

$$x_0 = 2 \qquad f'(2) = 3 \cdot 2^2 = 12$$

$$f(x_0) = 8$$

$$(2{,}03)^3 \approx 8 + 12(2{,}03 - 2) = 8 + 12 \cdot 0{,}03 =$$

$$= 8 + 0{,}36 = \underline{\underline{8{,}36}}$$

(3) $3^{1.95}$ x

$$f(x) = 3^x \quad ; \quad x_0 = 2$$

$$f(x_0) = 3^2 = 9$$

$$f'(x) = 3^x \ln 3$$

$$f'(x_0) = 3^2 \ln 3 = 9 \ln 3$$

$$3^{1,95} \approx 9 + 9 \overset{=1}{\ln 3} (1,95 - 2) =$$

$$= 9 - 9 \cdot 0,05 = 9 - 0,45 = \underline{\underline{8,55}}$$

(4) $\text{arctg}\,1.1$ $\quad x = 1.1 \;;\; x_0 = 1$

$$f(x) = \text{arctg}\,x \Rightarrow f(1) = \text{arctg}\,1 = \frac{\pi}{4}$$

$$f'(x) = \frac{1}{1+x^2}$$

$$f'(x_0) = \frac{1}{2}$$

$$\text{arctg}\,1.1 \approx \frac{\pi}{4} + \frac{1}{2}(1.1-1) = \frac{\pi}{4} + \frac{1}{2}\cdot 0{,}1 =$$

$$= \frac{\pi}{4} + 0.05$$

(5) $\sin(-0.2)$ $\quad X = -0.2 \; ; \; x_0 = 0$

X

$$f(x) = \sin x$$

$$f(x_0) = \sin 0 = 0$$

$$f'(x) = \cos x$$

$$f'(0) = \cos 0 = 1$$

$$\sin(-0.2) = 0 + 1 \cdot (-0.2 - 0) = \underline{\underline{-0.2}}$$

⑥ $\ln(1,3)$

$x = 1.3$ $\quad x_0 = 1$

X

$f(x) = \ln x \qquad f(x_0) = \ln 1 = 0$

$f'(x) = \frac{1}{x} \qquad f'(x_0) = \frac{1}{1} = 1$

$\ln(1.3) \approx 0 + 1(1.3 - 1) = \underline{\underline{0.3}}$

PR7

$$M_n(f, x) = f(x_0) + \frac{f'(x_0)}{1!}x^1 + \frac{f''(x_0)}{2!}x^2 + \frac{f'''(x_0)}{3!}x^3 + \dots + \frac{f^{(n)}(x_0)}{n!}x^n$$

$$f(x) = \frac{1}{2^x} = 2^{-x} \; ; \; x_0 = 0 \; ; \; n = 3$$

$$f(0) = \frac{1}{2^0} = \frac{1}{1} = 1$$

$$f'(x) = 2^{-x}\ln 2 \cdot (-1) = -2^{-x}\ln 2 \Rightarrow f'(0) = -2^0 \ln 2 = -\ln 2$$

$$f''(x) = 2^{-x} \ln 2 \cdot \ln 2$$

$$f''(0) = \ln^2 2$$

$$f'''(x) = 2^{-x} \cdot \ln 2 \cdot \ln^2 2 \cdot (-1)$$

$$f'''(0) = -\ln^3 2$$

$$M_3(f, x) = 1 + \frac{-\ln 2}{1!} x^1 + \frac{\ln^2 2}{2!} x^2 + \frac{-\ln^3 2}{3!} x^3$$

$$f(x) = tg\,x \quad ; \quad x_0 = 0 \quad ; \quad n = 3$$

$$f(0) = tg\,0 = 0$$

$$f'(x) = \frac{1}{\cos^2 x} = \cos^{-2} x \Rightarrow f'(0) = \underline{1}$$

$$f''(x) = +2\cos^{-3} x \cdot \sin x \Rightarrow f''(0) = \underline{0}$$

$$f'''(x) = +6\cos^{-4} x \cdot \sin x \cdot \sin x + 2\cos^{-3} x \cdot \cos x$$

$$f'''(x) = \underline{2}$$

$$M_3(f, x) = 0 + 1x + 0 + \frac{2}{3!}x^3 = \underline{x + \frac{x^3}{3}}$$

$$f(x) = \cos x \quad ; \quad x_0 = 0 \quad ; \quad n = N$$

$$f(0) = \cos 0 = \underline{1}$$

$$f'(x) = -\sin x \Rightarrow f'(0) = \underline{0}$$

$$f''(x) = -\cos x \Rightarrow f''(0) = \underline{-1}$$

$$f'''(x) = \sin x \Rightarrow f'''(0) = \underline{0}$$

$$f^{4}(x) = \cos x \Rightarrow f^{4}(0) = \underline{1}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

$$M_N(f, x) = 1 + 0 + \frac{-1}{2!} x^2 + 0 + \frac{1}{4!} x^4 - \frac{1}{6!} x^6 + \frac{1}{8!} x^8 =$$

PR 8

a) $f(x) = x \ln x \; ; \; x_0 = 1 \; ; \; n = 4$

$f(1) = 1 \cdot \ln 1 = 0$

$f'(x) = \ln x + x \frac{1}{x} = \ln x + 1 \Rightarrow f'(1) = 0 + 1 = 1$

$f''(x) = \frac{1}{x} = x^{-1} \Rightarrow f''(1) = 1$

$f'''(x) = -1x^{-2} \Rightarrow f'''(1) = -1$

$f^{4}(x) = 2x^{-3} \Rightarrow f^{4}(1) = 2$

$$T_4(x\ln x, 1, x) = 0 + \frac{1}{1!}(x-1)^1 +$$

$$\frac{1}{2!}(x-1)^2 - \frac{1}{3!}(x-1)^3 + \frac{2}{4!}(x-1)^4 =$$

$$= x-1 + \frac{(x-1)^2}{2} - \frac{(x-1)^3}{6} + \frac{(x-1)^4}{12}$$

(b) $f(x) = x^x \; ; \; x_0 = 1 \; ; \; n = 2$

$$f(1) = 1^1 = 1$$

$$f'(x) = x^x \left(\ln x^x\right)' = x^x \left(x \ln x\right)' =$$

$$= x^x \left(\ln x + x \cdot \frac{1}{x}\right) = x^x \left(\ln x + 1\right)$$

$$f'(1) = 1^1 \left(\ln 1 + 1\right) = 1 \qquad \boxed{\Rightarrow 1 \cdot 1 \cdot 1 + 1} = 2$$

$$f''(x) = \underbrace{x^x}_{1} \cdot \underbrace{\left(\ln x + 1\right)}_{1} \cdot \underbrace{\left(\ln x + 1\right)}_{1} + \underbrace{x^x \cdot \frac{1}{x}}_{1}$$

$$T_2(x^x, 1, x) = 1 + \frac{1}{1!}(x-1)^1 + \frac{2}{2!}(x-1)^2 =$$

$$= \cancel{1} + x - \cancel{1} + x^2 - 2x + 1 = \underline{\underline{x^2 - x + 1}}$$

(c) $f(x) = \operatorname{arctg} x$; $x_0 = 1$; $n = 2$

$$f(1) = \operatorname{arctg} 1 = \frac{\pi}{4}$$

$$f'(x) = \frac{1}{1+x^2} \Rightarrow f'(1) = \frac{1}{2}$$

$$= (1+x^2)^{-1}$$

$$f''(x) = -1\,(1+x^2)^{-2} \cdot 2x$$

$$f''(1) = -1\,(1+1)^{-2} \cdot 2 = -\frac{2}{4} = -\frac{1}{2}$$

$$T_2(\operatorname{arctg} x, 1, x) = \frac{\pi}{4} + \frac{x-1}{2} + \frac{-\frac{1}{2}}{\cancel{2!}_{\,1}}(x-1)^2 =$$

$$= \frac{\pi}{4} + \frac{x-1}{2} - \frac{1}{4}(x-1)^2$$

$$f''(x) = -2x(1+x^2)^{-2}$$

$$f'''(x) = -2\cdot(1+x^2)^{-2} - 2x\cdot(-2)\cdot(1+x^2)^{-3}\cdot 2x$$

$$= \frac{-2}{(1+x^2)^2} + \frac{8x^2}{(1+x^2)^3} = \frac{-2-2x^2+8x^2}{(1+x^2)^3} = \frac{6x^2-2}{(1+x^2)^3}$$

NEVYČÍSLUJEM

LEN $x \to \xi$

$$\frac{6\xi^2-2}{(1+\xi^2)^3}$$

$$\operatorname{arctg} x = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{1}{3!}\cdot\frac{6\xi^2-2}{(1+\xi^2)^3}(x-1)^3$$

$T_2(f,1,x)$

KDE ξ LEŽÍ MEDZI

x a 1

$R_2(x)$