SemMat1 – cv1– úprava výrazov

Ekvivalentnými úpravami zjednodušte nasledujúce výrazy a určte podmienky ich existencie:

1. a) $\left(\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{4}+\frac{1}{3}}\right)^{-1}$ b) $\dfrac{\frac{\sqrt{3}}{2}}{\frac{2}{\frac{\sqrt{3}}{4}}}$ c) $\dfrac{\sqrt{63}-\sqrt{28}}{\sqrt{7}}$ d) $1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\sqrt{2}}}}}$

2. a) $\dfrac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}$ b) $\sqrt[6]{\dfrac{3\sqrt[3]{3}}{6}}\cdot\sqrt[3]{\dfrac{2}{\sqrt{8^3}}}\cdot\sqrt{\dfrac{8}{\sqrt[3]{2}}}:\sqrt[6]{\dfrac{3^{-1}\sqrt{3}}{6^{-1}\sqrt{3}}}$ c) $5^{\frac{5}{4}}.125.25^{-0,4}\cdot\left(\dfrac{1}{5}\right)^2$

3. a) $\dfrac{\frac{a}{a+b}+\frac{b}{a-b}}{\frac{a}{a-b}-\frac{b}{a+b}}$ b) $\left(\dfrac{a-3}{1+3a}-\dfrac{a-4}{1+4a}\right):\left(1+\dfrac{a-3}{1+3a}\cdot\dfrac{a-4}{1+4a}\right)$

4. a) $\left(\dfrac{\sqrt{10}+1}{3}\right)^{365}\cdot\left(\dfrac{\sqrt{10}-1}{3}\right)^{365}$ b) $\left(a+\dfrac{1}{b}\right)^{-2}\cdot\left(b-\dfrac{1}{a}\right)^{-3}\cdot\left(ab-\dfrac{1}{ab}\right)^{2}$

5. a) $\sqrt[5]{a\cdot\sqrt[4]{a\cdot\sqrt[3]{a\cdot\sqrt{a}}}}$ b) $\left(\dfrac{a\sqrt{a}+b\sqrt{b}}{\sqrt{a}+\sqrt{b}}-\sqrt{ab}\right):(a-b)+\dfrac{2\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

6. a) $\left(\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}}\right)^2$ b) $\dfrac{2+\sqrt{3}}{2-\sqrt{3}}-2\sqrt{12}$ c) $\dfrac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{5}}\left(\sqrt{15}+4\right)$

7. a) $\sqrt[5]{\left(\dfrac{\sqrt{a}\cdot a^{-1}}{\sqrt[3]{a}}\right)^{-3}}$ b) $\dfrac{\sqrt{a^3}\cdot\sqrt{b}}{\sqrt{a}\cdot\sqrt{b^5}}$ c) $\dfrac{\sqrt[3]{x^2}\cdot x^{0,75}\cdot\sqrt{x\cdot\sqrt[3]{x^2\cdot\sqrt[4]{x^3}}}}{\sqrt{x\cdot\sqrt[3]{x}}\cdot x^{\frac{-2}{3}}}$

8. a) $\left[\dfrac{5-\sqrt{5x}}{\sqrt{5}-\sqrt{x}}\right]^4$ b) $\dfrac{\left(a-\sqrt{b}\right)\left(b+\sqrt{a}\right)+\sqrt{ab}\left(1-\sqrt{ab}\right)}{a+b+\sqrt{ab}}$

9. a) $\left[\dfrac{\left(\sqrt{7}+1\right)^2-\dfrac{7-\sqrt{7x}}{\sqrt{7}-\sqrt{x}}}{\left(\sqrt{7}+1\right)^3-7\sqrt{7}+2}\right]^{-3}$ b) $\left[\dfrac{(x-y)\left(x^4-y^4\right)}{\left(x^2-y^2\right)\left(x^3-y^3\right)}\right]^{-1}$

10. $\left(\dfrac{\sqrt{a}+\sqrt{x}}{\sqrt{a+x}}-\dfrac{\sqrt{a+x}}{\sqrt{a}+\sqrt{x}}\right)^{-2}-\left(\dfrac{\sqrt{a}-\sqrt{x}}{\sqrt{a+x}}-\dfrac{\sqrt{a+x}}{\sqrt{a}-\sqrt{x}}\right)^{-2}$

11. $\left(\dfrac{x}{\sqrt{xy}+y}+\dfrac{y}{\sqrt{xy}}-\dfrac{x+y}{\sqrt{xy}}\right)^{-2}\left(\sqrt{x}+\dfrac{y-\sqrt{xy}}{\sqrt{x}+\sqrt{y}}\right)^{2}$

12. $\dfrac{\left(\sqrt[4]{u}+\sqrt[4]{v}\right)^2+\left(\sqrt[4]{u}-\sqrt[4]{v}\right)^2}{u-v}:\dfrac{2}{\sqrt{u}-\sqrt{v}}$

13. $\left(\dfrac{4+a^2}{4a-1}\right)^{-1}\left[\left(\dfrac{2+a\sqrt{a}}{2a+\sqrt{a}}-\sqrt{a}\right):\left(\dfrac{2a-\sqrt{a}}{2+a\sqrt{a}-a}\right)\right]$

VZORCE

$(a+b)^2=a^2+2ab+b^2$ $(a-b)^2=a^2-2ab+b^2$

$a^2-b^2=(a+b)(a-b)$

$a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$

ZLOMKY

$\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{ad}{bd}+\dfrac{cb}{db}=\dfrac{ad+cb}{bd}$ $\dfrac{a}{b}\cdot\dfrac{c}{d}=\dfrac{ac}{bd}$ $\dfrac{\frac{a}{b}}{\frac{c}{d}}=\dfrac{ad}{bc}$

$b\neq 0, d\neq 0$ $b\neq 0, d\neq 0$ $b\neq 0, c\neq 0, d\neq 0$

MOCNINY

$a^0=1$ $a^r=\dfrac{1}{a^{-r}}$ $a^{-r}=\dfrac{1}{a^r}$ $a\neq 0, r\in N$

$a^r a^s=a^{r+s}$ $\dfrac{a^r}{a^s}=a^{r-s}$ $(a^r)^s=a^{rs}$ $r,s\in N\cup\{0\}$

$(ab)^r=a^r b^r$ $\left(\dfrac{a}{b}\right)^r=\dfrac{a^r}{b^r}$ $\sqrt[s]{a^r}=a^{\frac{r}{s}}$ $b\neq 0,\ s\neq 0$