

(PR 1) ROZLOŽTE NA PARCIÁLNÉ ZLOMKY

$$(a) \quad \frac{1}{x^2 - 4} = \frac{1}{(x-2)(x+2)} \quad \frac{A = \frac{1}{4}}{x-2} + \frac{B = -\frac{1}{4}}{x+2} =$$

$$= \frac{A(x+2) + B(x-2)}{(x-2)(x+2)} = \frac{\boxed{Ax} + \boxed{2A} + \boxed{Bx} - \boxed{2B}}{(x-2)(x+2)}$$

$$0x + 1 = x(A+B) + 2A - 2B$$

$$0 = A + B \Rightarrow A = -B$$

$$1 = 2A - 2B$$

$$\boxed{A = \frac{1}{4}}$$

$$-4B = 1 \Rightarrow \boxed{B = -\frac{1}{4}}$$

$$\textcircled{b} \quad \frac{5x^2 - 17x + 12}{x^3 - 4x^2 + 4x} = \frac{5x^2 - 17x + 12}{x(x^2 - 4x + 4)} = \frac{5x^2 - 17x + 12}{x(x-2)^2}$$

$$\frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} = \frac{A(x-2)^2 + Bx(x-2) + Cx}{x(x-2)^2}$$

$$= \frac{Ax^2 - 4Ax + 4A + Bx^2 - 2Bx + Cx}{x(x-2)^2} = \frac{5x^2 - 17x + 12}{x(x-2)^2}$$

$$\Rightarrow x^2(A+B) + x(-4A-2B+C) + 4A = 5x^2 - 17x + 12$$

$$A+B=5 \Rightarrow \boxed{B=2}$$

$$-4A-2B+C=-17 \Rightarrow -12-4+C=-17$$

$$4A=12 \Rightarrow \boxed{A=3}$$

$$\underline{\underline{C=-1}}$$

$$\Rightarrow \frac{3}{x} + \frac{2}{x-2} - \frac{1}{(x-2)^2}$$

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$$\textcircled{C} \quad \frac{\textcircled{2}x \textcircled{-3} + 0.x^2}{1x^3 + \textcircled{2}x^2 \textcircled{-1}x \textcircled{-2}}$$

$$x = \pm 1 \pm 2$$

	1	2	-1	-2
1	1	3	2	0

$$\Rightarrow (x-1)(x^2+3x+2) = \underline{\underline{(x-1)(x+2)(x+1)}}$$

$$\begin{aligned} \Rightarrow x^3 + 2x^2 - x - 2 &= \\ &= x^2(x+2) - 1(x+2) \\ &= (x+2)(x^2-1) = \\ &= \underline{\underline{(x+2)(x-1)(x+1)}} \end{aligned}$$



$$\Rightarrow \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+1} = \frac{A(x^2+3x+2) + B(x^2-1) + C(x^2+x-2)}{(x-1)(x+2)(x+1)}$$

$$= \frac{x^2(A+B+C) + x(3A+C) + (2A-B-2C)}{(x-1)(x+2)(x+1)}$$

$$A+B+C=0$$

$$3A+C=2$$

$$2A-B-2C=-3$$

$$\begin{array}{l} 3A+C=2 \quad (+) \\ 3A-C=-3 \quad (+) \end{array}$$

$$\Rightarrow 6A = -1$$

$$A = -\frac{1}{6}$$

$$\begin{aligned} B-A-C &= \\ &= \frac{1}{6} - \frac{5}{2} = \\ &= -\frac{7}{3} \end{aligned}$$

$$C = 2 - 3A = 2 + 3 \cdot \frac{1}{6} = \frac{5}{2}$$

$$= \frac{-\frac{1}{6}}{x-1} + \frac{-\frac{7}{3}}{x+2} + \frac{\frac{5}{2}}{x+1}$$


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(a)  $\frac{1x^3 - 3x^2 - 3x - 10}{(x-1)^2(x^2+4)} = \frac{A=0}{x-1} + \frac{B=-3}{(x-1)^2} + \frac{Cx+D}{x^2+4}$

$$A(x-1)(x^2+4) + B(x^2+4) + (Cx+D)(x^2-2x+1) =$$

$$\begin{aligned}
 &= A \left( \overset{\checkmark}{x^3} + \overset{\checkmark}{4x} - \overset{\checkmark}{x^2} - \overset{\checkmark}{4} \right) + \overset{\checkmark}{B}x^2 + \overset{\checkmark}{4B} + \overset{\checkmark}{C}x^3 - 2\overset{\checkmark}{C}x^2 + \overset{\checkmark}{C}x + \\
 &+ \overset{\checkmark}{D}x^2 - 2\overset{\checkmark}{D}x + \overset{\checkmark}{D} = x^3 (A+C) + x^2 (-A+B-2C+D) \\
 &+ x (4A+C-2D) + (-4A+4B+D)
 \end{aligned}$$


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$$A+C=1 \Rightarrow \underline{\underline{A=1-C}}$$

$$-A+B-2C+D=-3$$

$$4A+C-2D=-3$$

$$-4A+4B+D=-10$$

$$\left( \begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ -1 & 1 & -2 & 1 & -3 \\ 4 & 0 & 1 & -2 & -3 \\ -4 & 4 & 0 & 1 & -10 \end{array} \right) \xrightarrow{\substack{r_2 \oplus r_1 \\ r_3 \oplus (-4)r_1 \\ r_4 \oplus 4r_1}} \left( \begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 0 & -3 & -2 & -7 \\ 0 & 4 & 4 & 1 & -6 \end{array} \right) \xrightarrow{r_4 \oplus (-4)r_2}$$

$$\sim \left( \begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 0 & -3 & -2 & -7 \\ 0 & 0 & 8 & -3 & 2 \end{array} \right) \xrightarrow{\substack{(2) \sim (3) \\ (3) \sim (2)}} \sim \left( \begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 0 & -24 & -16 & -56 \\ 0 & 0 & 24 & -9 & 6 \end{array} \right) \xrightarrow{r_4 \oplus r_3}$$

$$\sim \left( \begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 & -2 \\ 0 & 0 & -24 & -16 & -56 \\ 0 & 0 & 0 & -25 & -50 \end{array} \right)$$

$$\Rightarrow \boxed{D=2}$$

$$-24C = -56 + 32 = -24 \Rightarrow \boxed{C=1}$$

$$\boxed{B} = -2 + 1 - 2 = \boxed{-3}$$

$$\boxed{A=0}$$