FUNKCIE

$f: x \in A \quad y \in B \quad A, B \neq \emptyset$

NAJVIAC 1 y

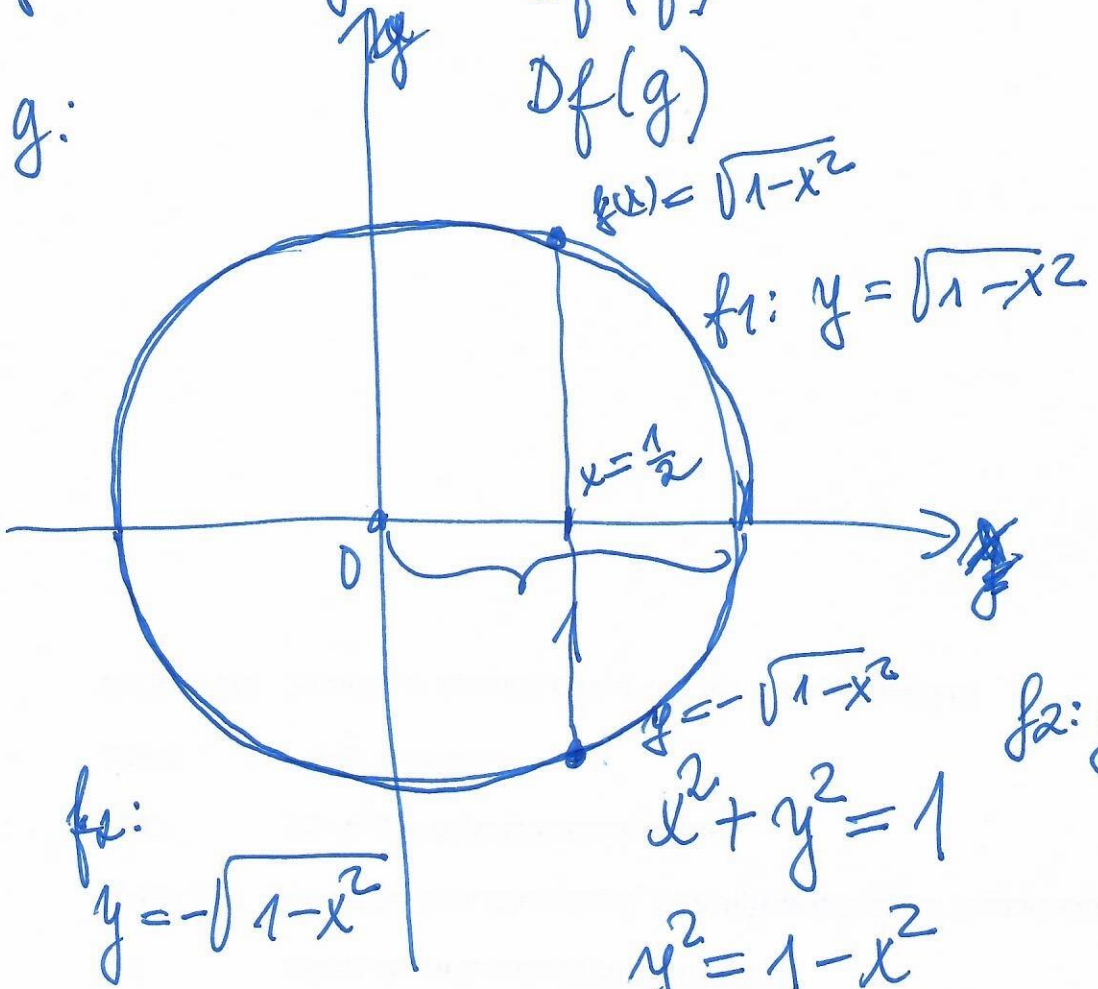
$A, B \subset R$

$f: x \to y$

$Df(f)$

$Df(g)$

$g:$



$f(x) = \sqrt{1-x^2}$

$f_1: y = \sqrt{1-x^2}$

$y = -\sqrt{1-x^2}$

$f_2: y = -\sqrt{1-x^2}$

$f_2:$

$y = -\sqrt{1-x^2}$

$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$

$f_1: y = \sqrt{1-x^2}$

$1 - x^2 \geq 0$

$Df(f_1): \langle -1, 1 \rangle$

$|x| \leq 1$

$H(f_1): \langle 0, 1 \rangle$

$x \in \langle -1, 1 \rangle$

ROVNOSŤ FUNKCIÍ

$f(x)$, $g(x)$ 1) $Df(f) = Df(g)$

2) $\forall x \in Df : f(x) = g(x)$

$f(x) = \frac{x^2-9}{x+3} = x-3$ pre $x \neq -3$ $\quad Df(f) = \mathbb{R} - \{-3\}$

$g(x) = x-3 \qquad Df(g) = \mathbb{R}$

$f(x) \neq g(x)$

$f(x) = x$

$g(x) = (\sqrt{x})^2$

$Df(f) = \mathbb{R}$

$Df(g) = \langle 0, \infty)$

x

$f(x) \neq g(x)$

OPERACE S FUNKCIAMI

$f(x) \pm g(x)$

$\frac{f(x)}{g(x)}$, $f(x) \cdot g(x)$

$g(x) \neq 0$

ZLOŽENÍ

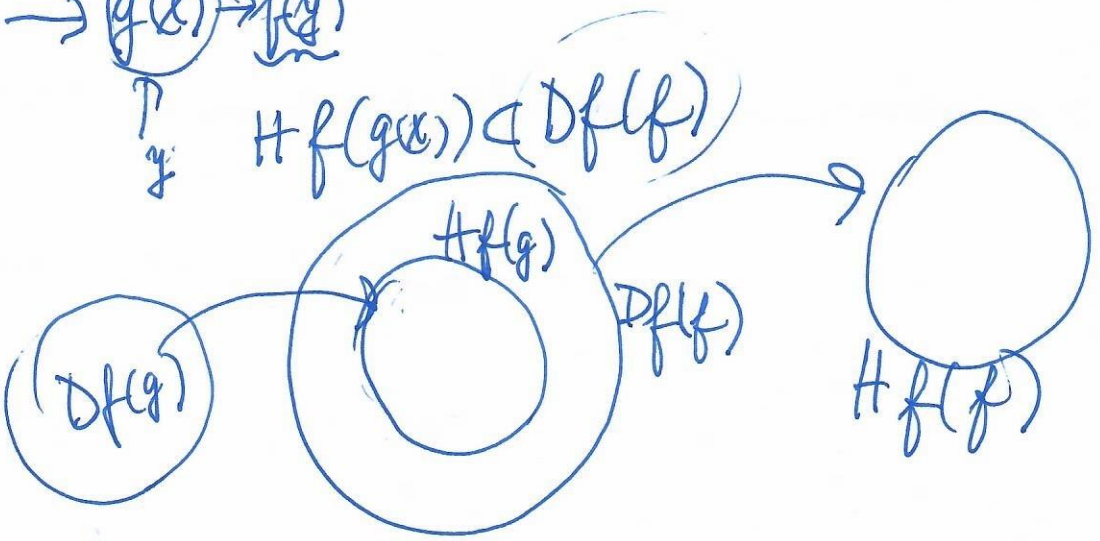
$f(g(x)) \neq g(f(x))$

$h(x) = f(g(x))$

$x \to g(x) \to f(y)$

y

$Hf(g(x)) \subset Df(f)$



$f(x) = \frac{1}{1+x}$ $g(x) = \sqrt{1-x^2}$

$Df(f) = R - \{-1\}$

$Df(g) = \langle -1, 1 \rangle$

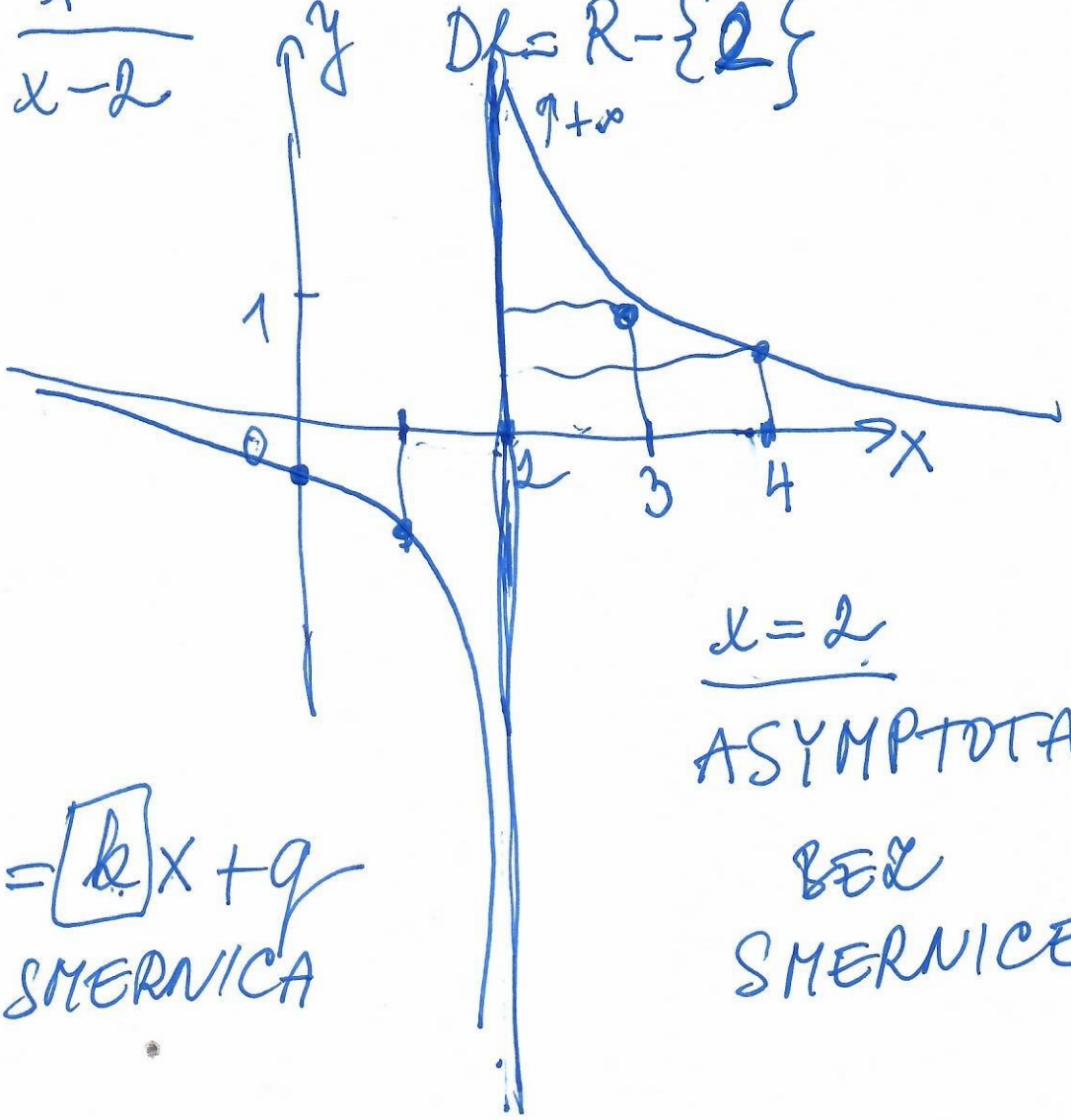
$h(x) = f(g(x)) = \frac{1}{1+\sqrt{1-x^2}}$

$Hg = \langle 0, 1 \rangle$

$Hg \subset Df(f)$

$Df(h) \subset \langle -1, 1 \rangle$

$y = \frac{1}{x-2}$



$x = 2$

ASYMPTOTA BEZ SMERNICE

$y = kx + q$

SMERNICA

$Hf(f) = \{ y \in B : \exists x \in Df(f) : y = f(x) \}$

$f: y = \frac{3}{x^2+1}$

$Df(f) = R$

$Hf(f) = \langle 3, \infty)$

MONOTÓNNOSŤ FUNKCIE

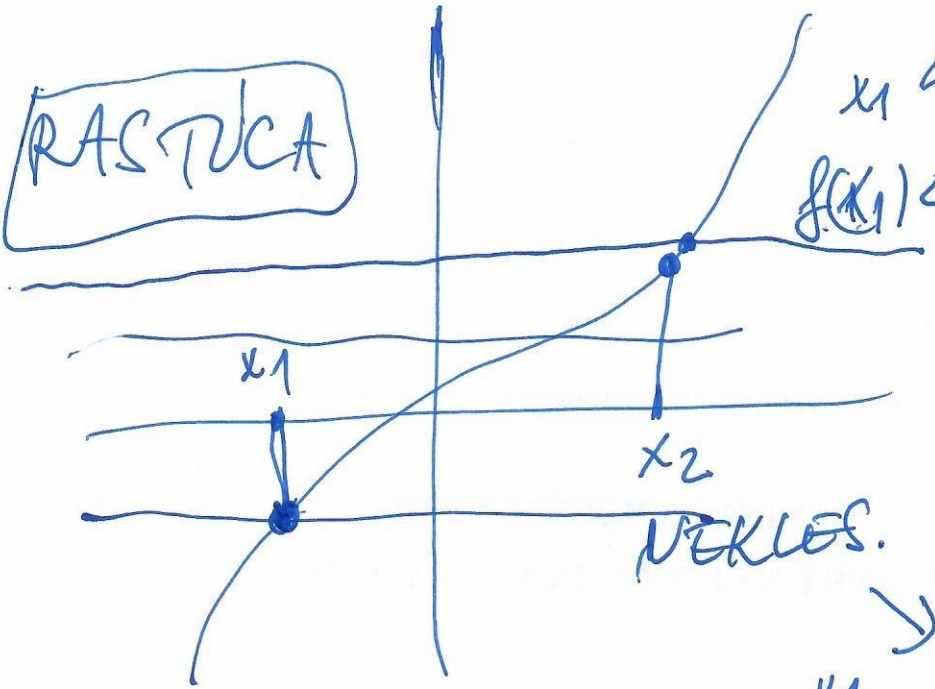
$f(x)$: $x_1, x_2 \in D_f$ $\boxed{x_1 < x_2}$

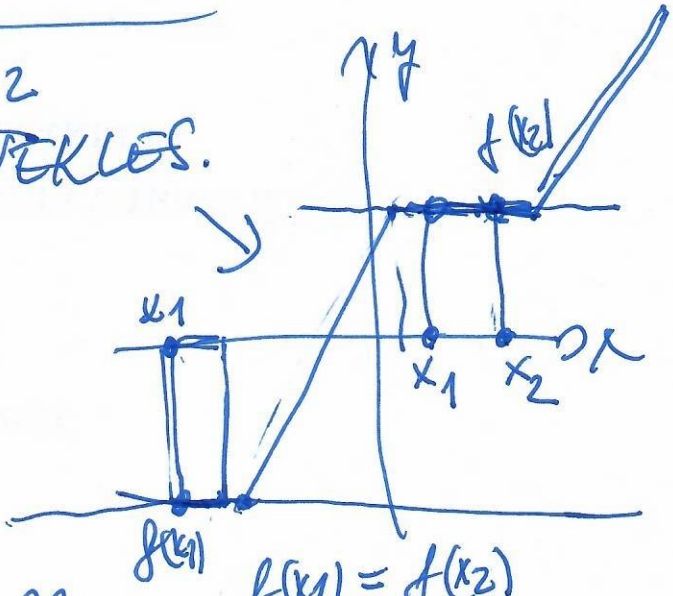
$f(x_1) < f(x_2)$ RASTÚCA

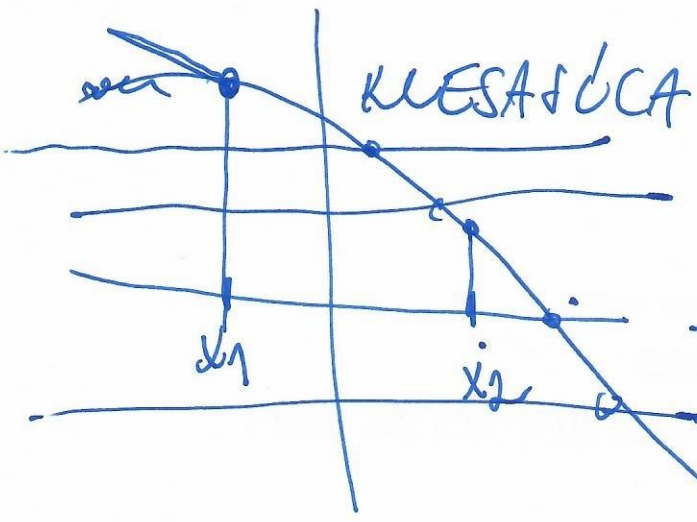
$f(x_1) \leq f(x_2)$ NEKLESAJÚCA

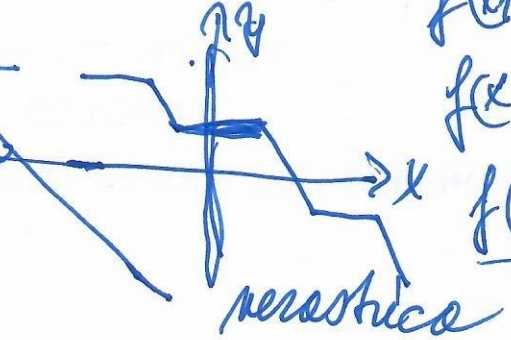
$f(x_1) > f(x_2)$ KLESAJÚCA

$f(x_1) \geq f(x_2)$ NERASTÚCA









$f(x_1) = f(x_2)$

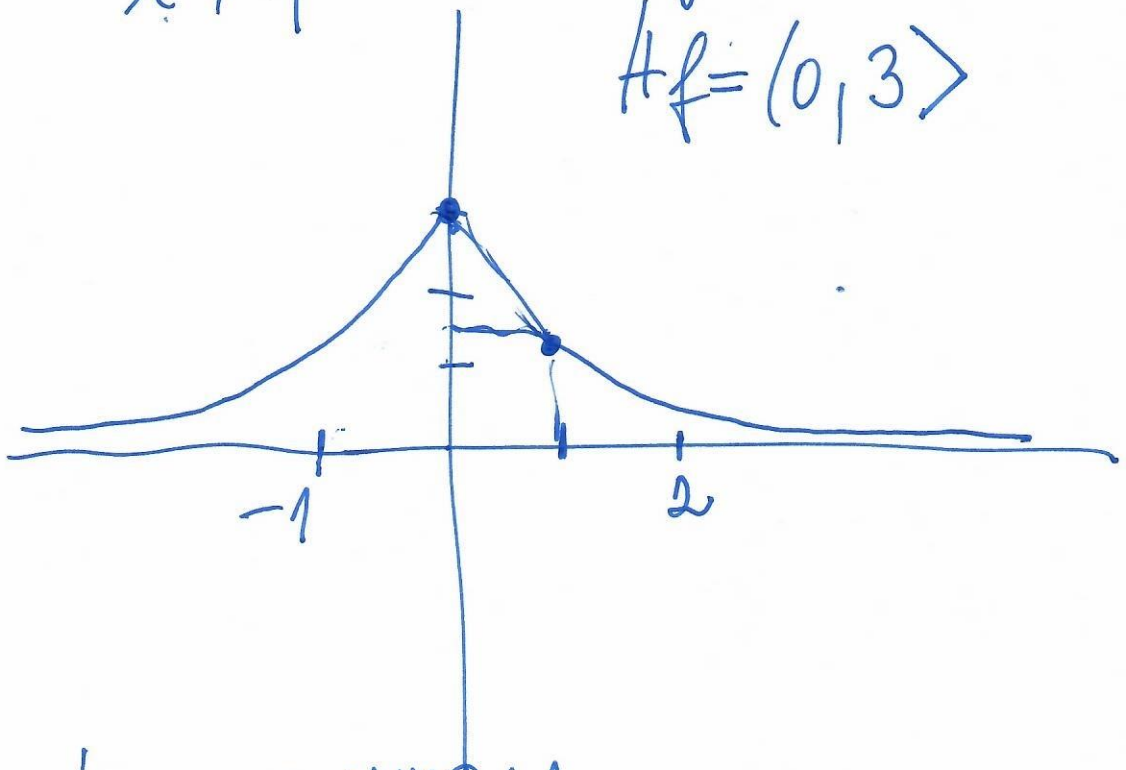
$f(x_1) < f(x_2)$

$f(x_1) \leq f(x_2)$

$y = \frac{3}{x^2+1}$

$D_f = R$

$H_f = (0, 3\rangle$



PÁRNA FUNKCIA

NEPÁRNA FUNKCIA

$\forall x \in D_f: f(x) = f(-x)$ PÁRNA

$\forall x \in D_f: f(x) = -f(-x)$ NEPÁRNA

GRAF PÁRNEJ FUNKCIE JE SÚMETR. PODĽA OSI Y

→ NEPÁRNE FUNKCIE JE SÚM. PODĽA POČIATKU SÚR. SYSTÉMU

PROSTA'

$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$

$y = 3x + 2$

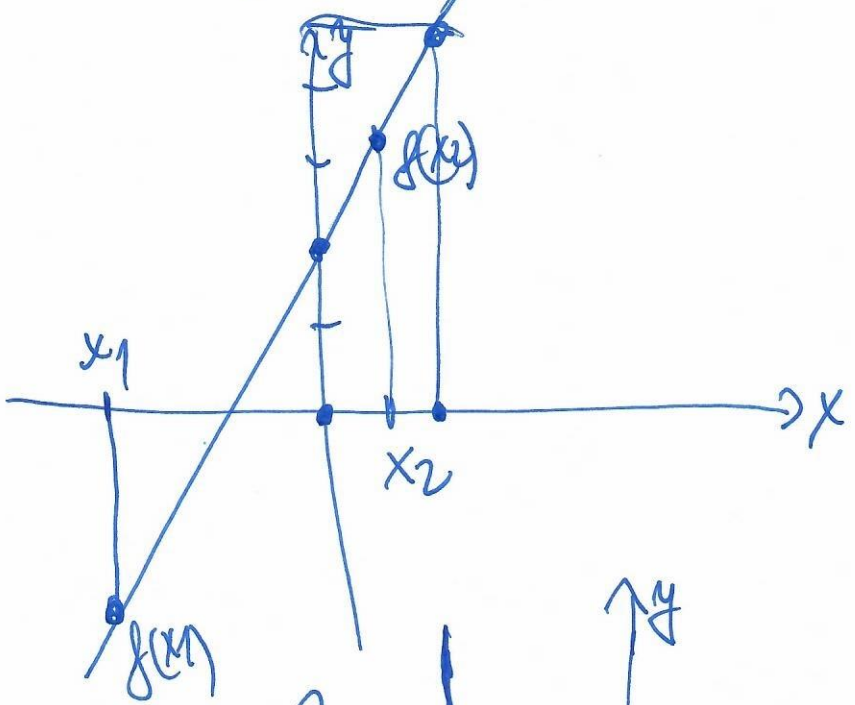
$x_1 \neq x_2$

$3x_1 \neq 3x_2$

$3x_1 + 2 \neq 3x_2 + 2$

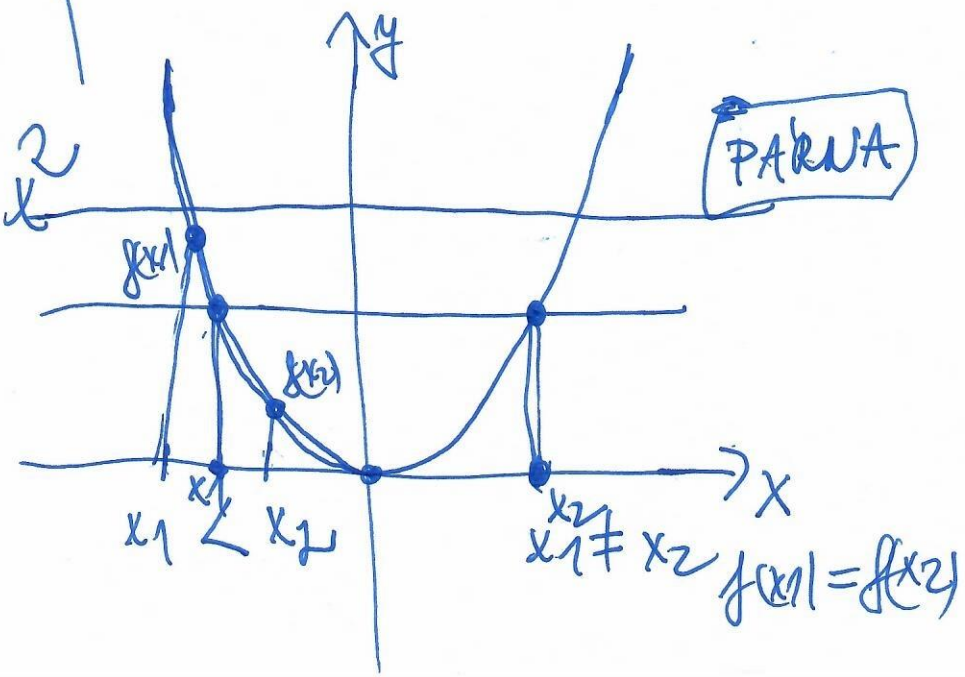
$f(x_1) \neq f(x_2)$

PROSTA'



$y = x^2$

PARNA



KLES.

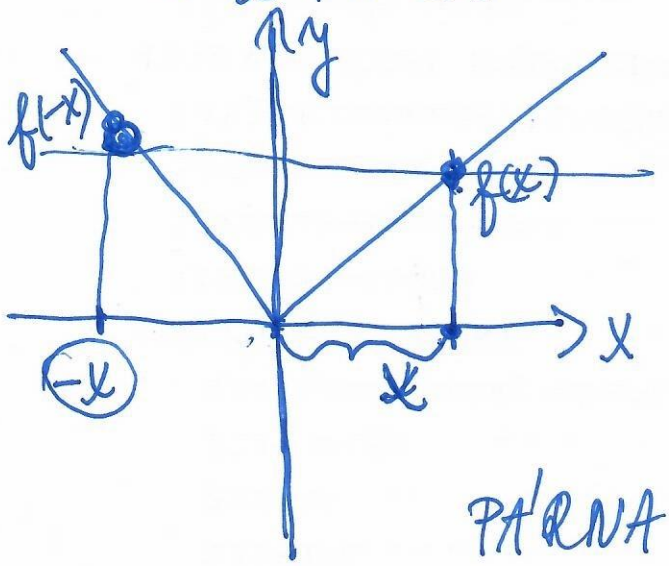
$\langle -\infty, 0 \rangle$

$\langle 0, \infty \rangle$

RASTUCA

$y = |x|$

$$y = \begin{cases} x\,; & x \geq 0 \\ -x\,; & x \leq 0 \end{cases}$$



$f(x) = |x|$

$f(-x) = |-x| = |x|$

$f(x) = f(-x)$

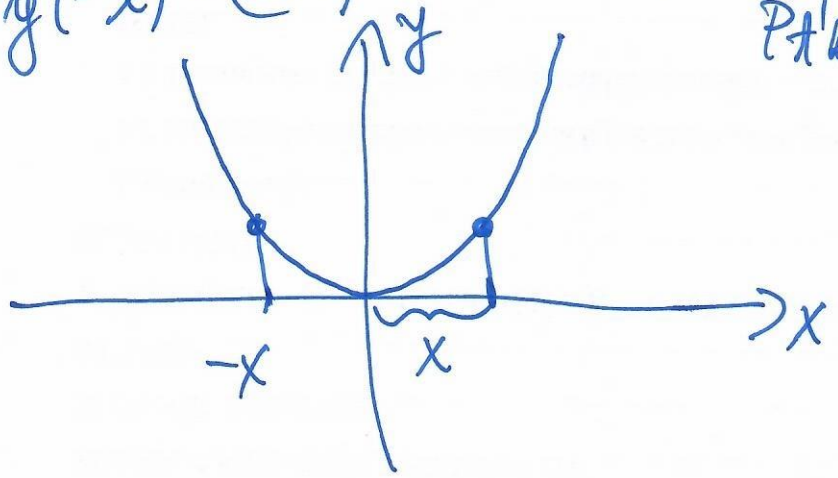
PAR NA

$y = x^2$

$y(-x) = (-x)^2 = x^2$

$f(x) = f(-x)$

PAR NA



$g(f(x))$ $\quad f(x) = \frac{1}{1+x}$

$g(x) = \sqrt{1-x^2}$

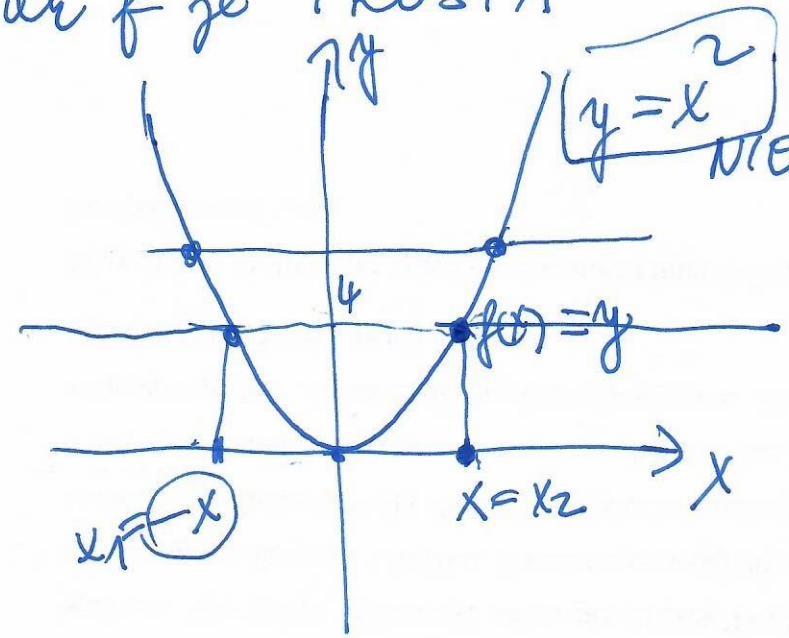
$h_2(x) = \sqrt{1-\left(\frac{1}{1+x}\right)^2}$

INVERZNÁ FUNKCIA

$f: x \to y$

$f^{-1}: y \to x$

ak f je PROSTÁ



NEEXISTUJE INVERZNÁ FUNKCIA

NIE JE PROSTÁ

$y = x^2$

$x = 2$

$f(x) = 4$

$4 \to x$

$4 \to 2$

$4 \to -2$

$y = x^2$

$x = y^2$

$y = \sqrt{x}$

$\langle 0, \infty) = D_f$

INVERZNÁ FUNKCIA

$f: y = \frac{1}{x+1}$ $\quad Df: R - \{-1\}$

$x_1 \neq x_2$

$x_1 + 1 \neq x_2 + 1$

$\frac{1}{x_1+1} \neq \frac{1}{x_2+1}$ PROSTA' $\Rightarrow$ $\exists$ INVERZNA'

$x \leftrightarrow y$

$x = \frac{1}{y+1}$

$x(y+1) = 1$

$x \cdot y + x = 1$

$x \cdot y = 1 - x$

$f^{-1}: \quad y = \frac{1-x}{x}$

$f:$ $x = 3$ $\quad y = \frac{1}{3+1} = \frac{1}{4}$

$x \longrightarrow y$

$3 \longrightarrow \frac{1}{4}$

$f^{-1}:$

$x = \frac{1}{4}$

$y = \frac{1 - \frac{1}{4}}{\frac{1}{4}} = 4 \cdot \left(1 - \frac{1}{4}\right) =$

$= 4 \cdot \frac{3}{4} = 3$

$\frac{1}{4} \longrightarrow 3$

$y = x^3$

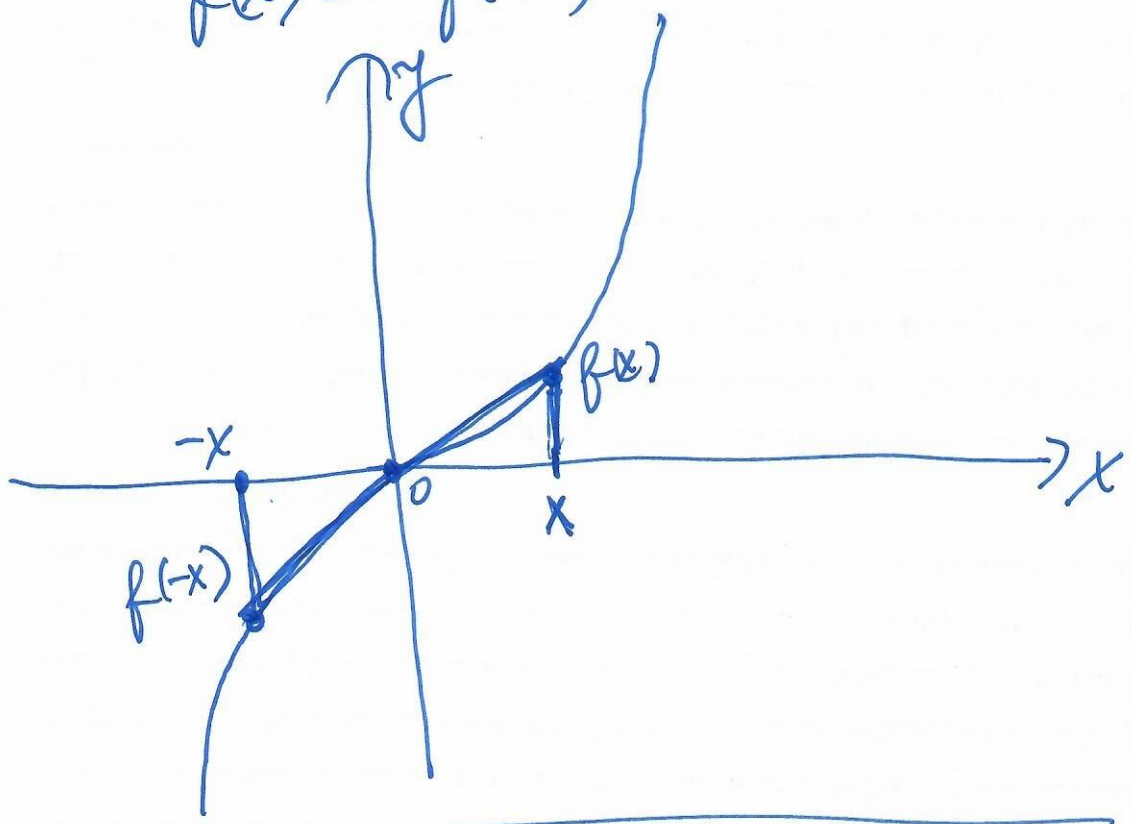
$f(-x) = (-x)^3 = (-x)(-x)^2 = -x \cdot x^2 = -x^3$

$f(x) = x^3$

$f(-x) = -x^3$

NEPÁRNA

$f(x) = -f(-x)$



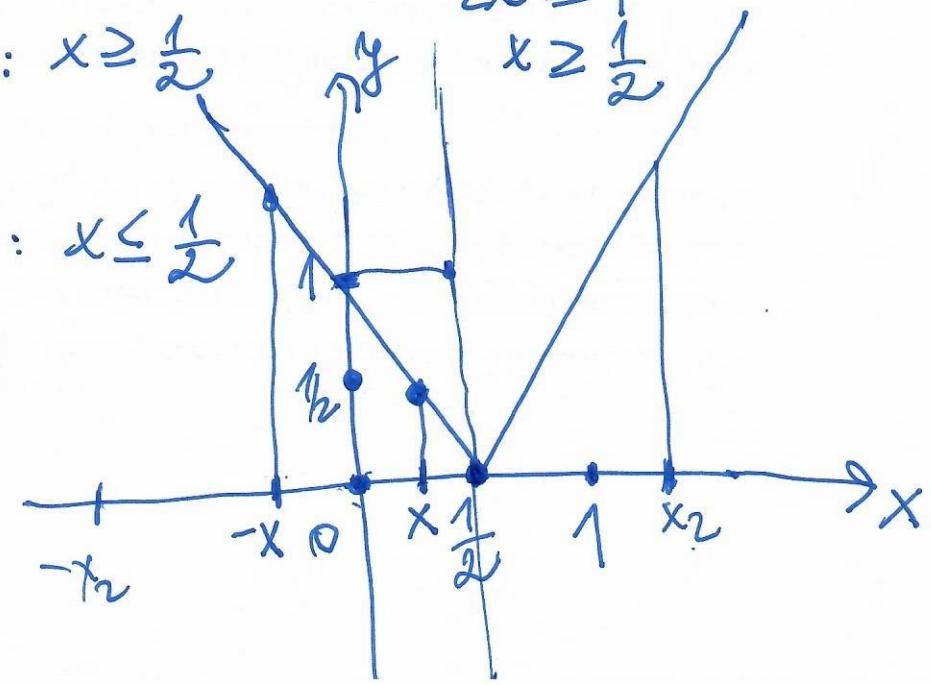
$y = |2x-1|$

$2x-1 \geq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$

$$y = \begin{cases} 2x-1 : x \geq \frac{1}{2} \\ -2x+1 : x \leq \frac{1}{2} \end{cases}$$



NIEJE ANI PÁRNA ANI NEPÁRNA

SYMETR. $X = \frac{1}{2}$

OHRANIČENÁ FUNKCIA

$f(x)$

ak $\exists K : \forall x \in D_f \quad f(x) > K$

OHRANIČENÁ ZDOLA

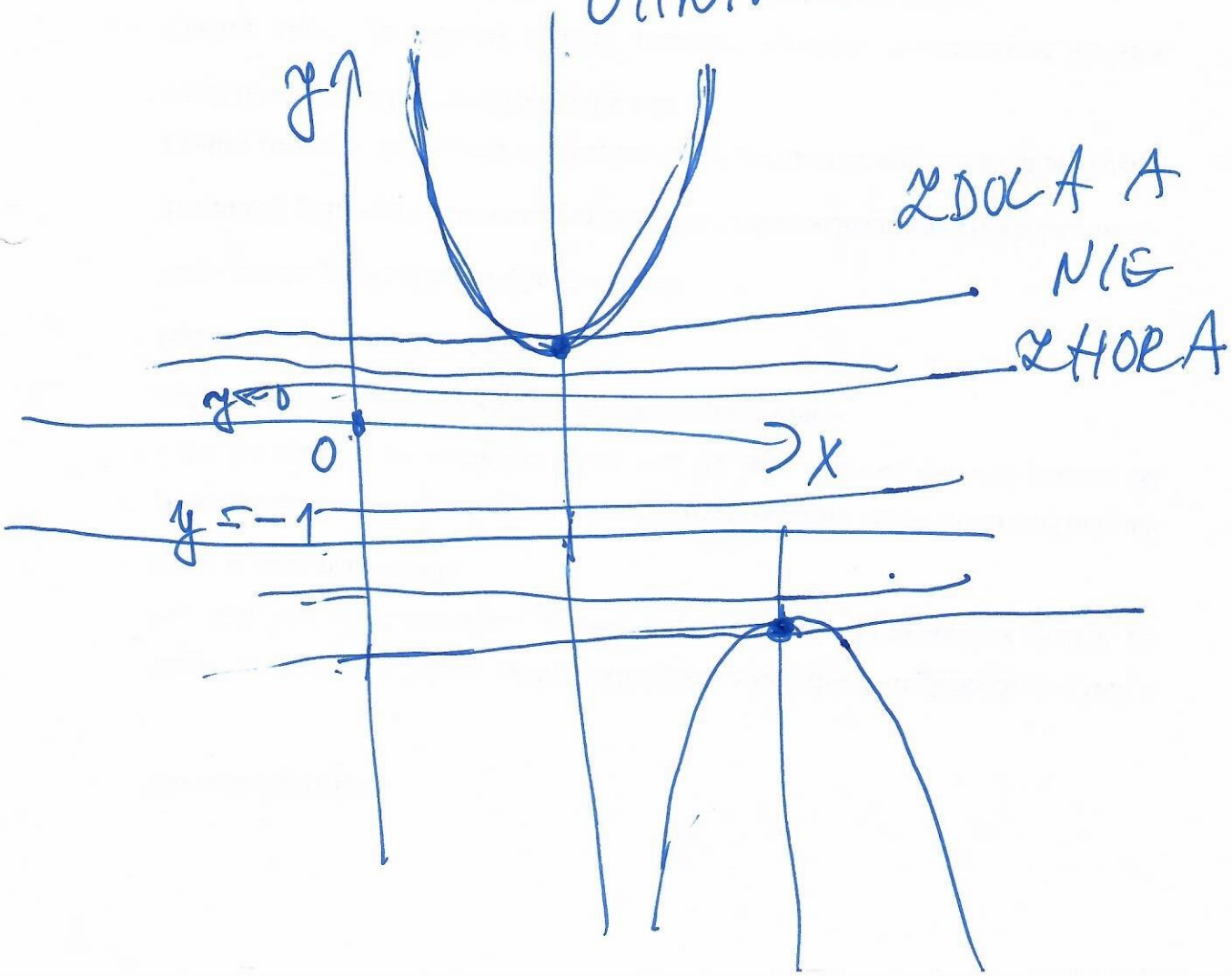
$\exists L : \forall x \in D_f \quad f(x) < L$

OHRANIČENÁ ZHORA

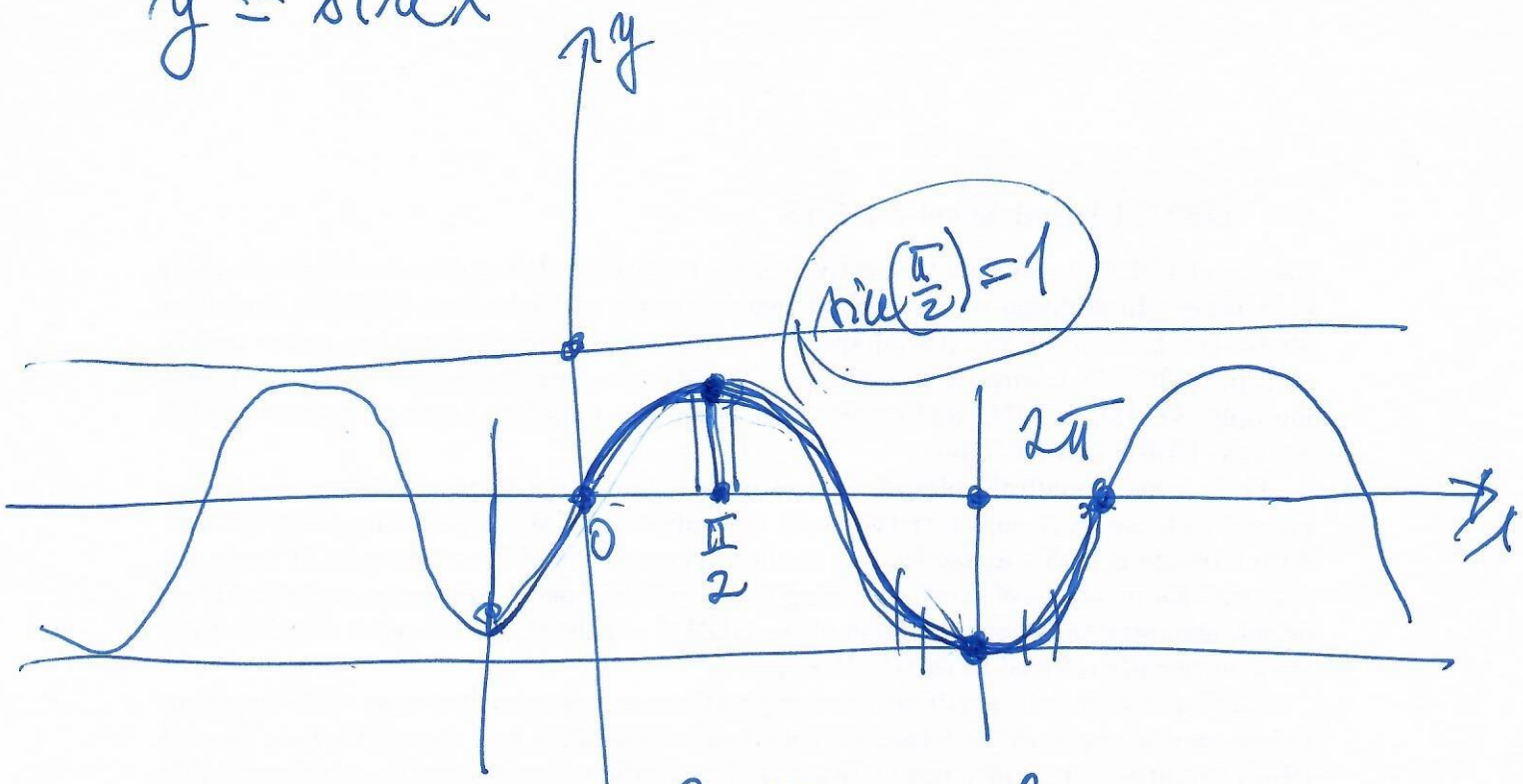
$\exists K : \forall x \in D_f$

$-K \leq f(x) < K$

OHRANIČENÁ



$y = \sin x$



$-3 < \sin x < 3$

$-100 < \sin x < +100$

$-1 \leq \sin x \leq 1$

OHRANIČENÁ FUNKCIA

PERIÓDA FUNKCIE

$AR\ \exists p: f(x+p) = f(x)\ \forall x \in Df$

PERIODICKÁ

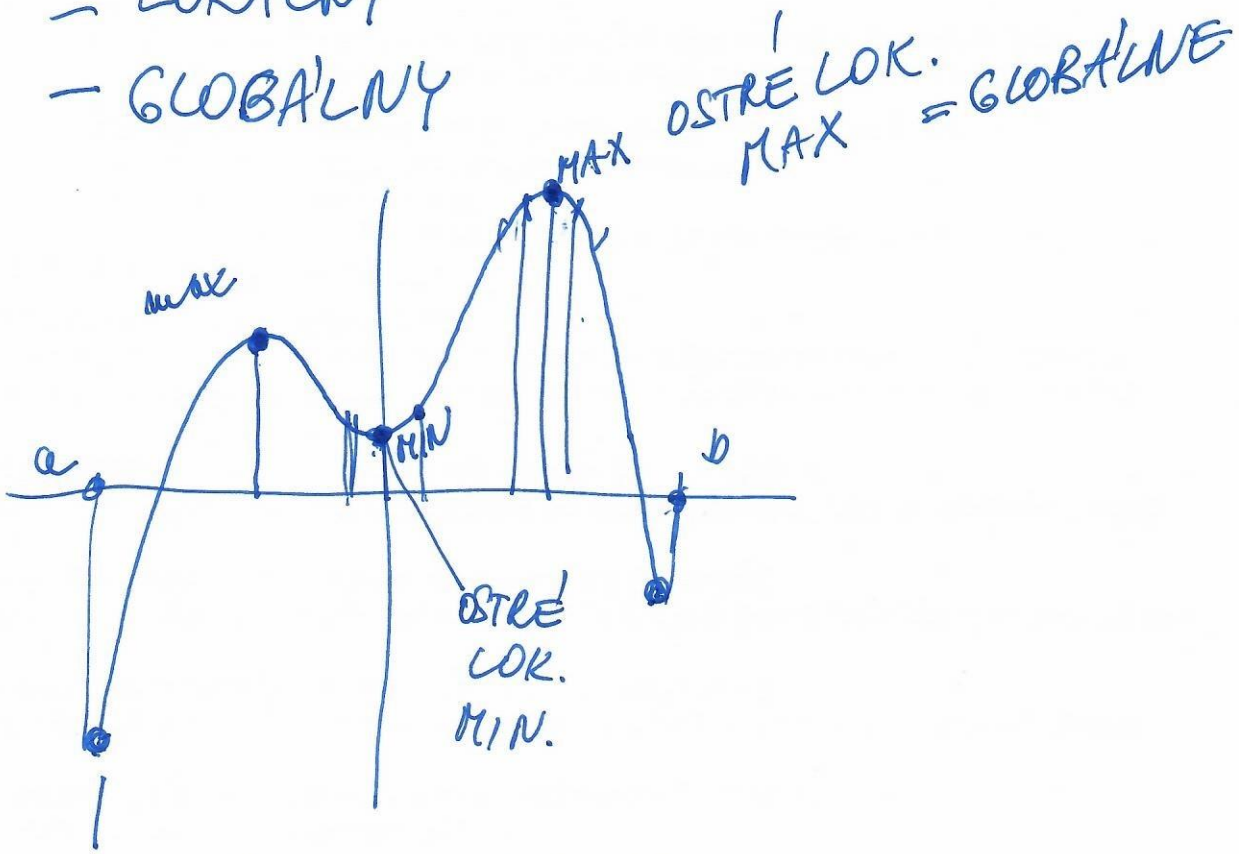
$MIN\{p : p > 0\} = P$ PERIÓDA FUNKCIE

$\sin(x + 4\pi) = \sin x$; $\sin(x + 8\pi) = \sin x$

$\sin(x + 2\pi) = \sin x$

EXTRÉMY FUNKCIE

- LOKÁLNY
- GLOBÁLNY

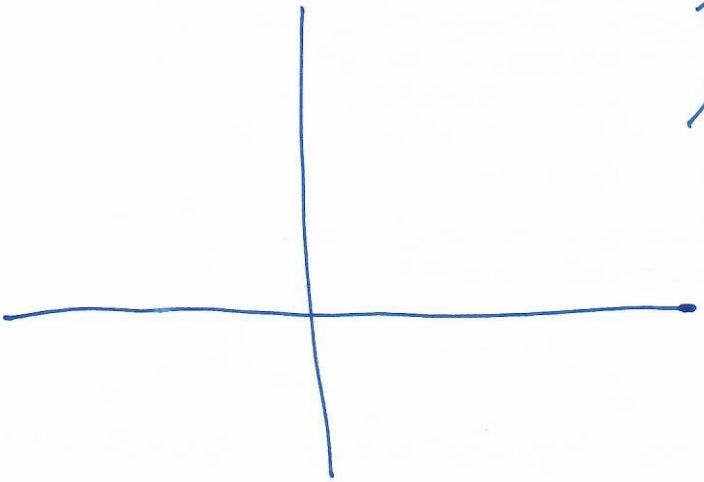


$y = kx + q$ (lin. funkcia)

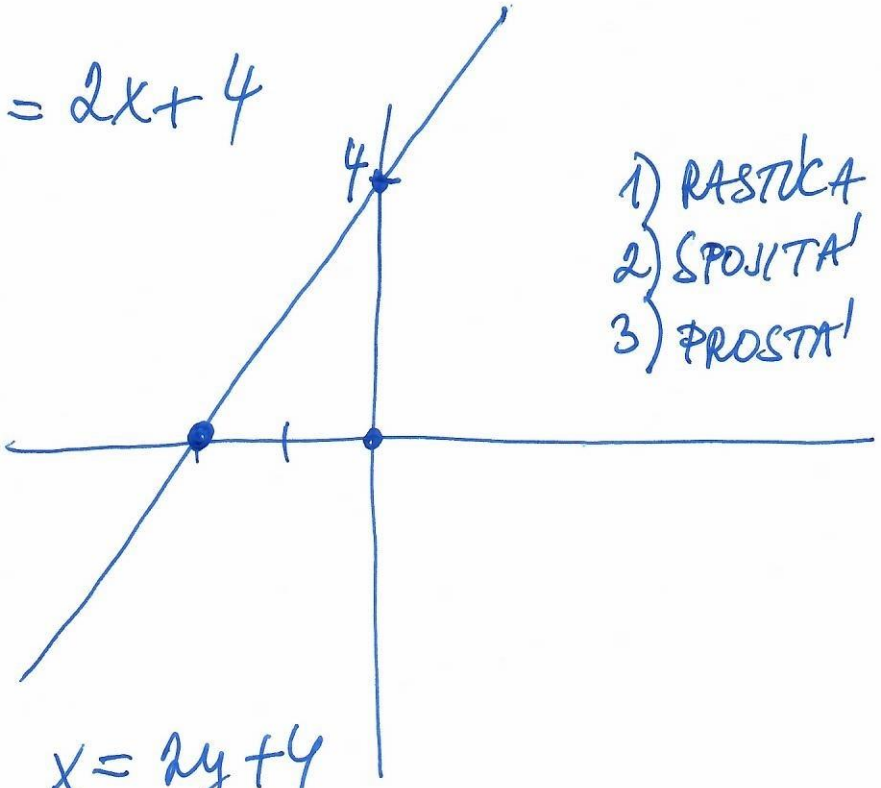
$k > 0$ RASTÚCA

$k < 0$ KLESAJÚCA

$k = 0$ KONŠT.
NERASTÚCA
NEKLESAJÚCA



$f: y = 2x + 4$



1) RASTÚCA
2) SPOJITÁ
3) PROSTÁ

$x = 2y + 4$

$x - 4 = 2y$

$f^{-1}: y = \frac{x-4}{2}$

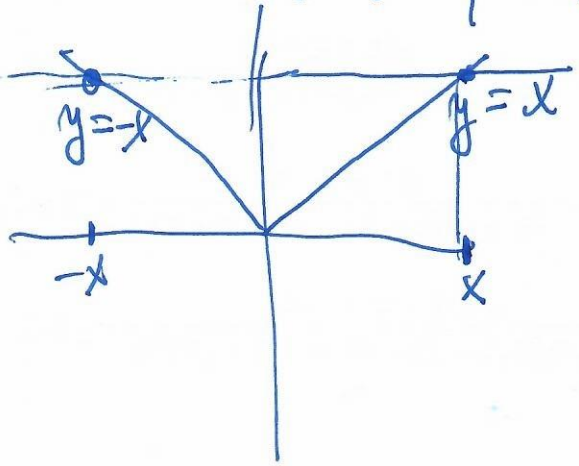
AK f JE RASTÚCA, TAK AJ f^{-1} JE RASTÚCA

f^{-1} JE KLES.

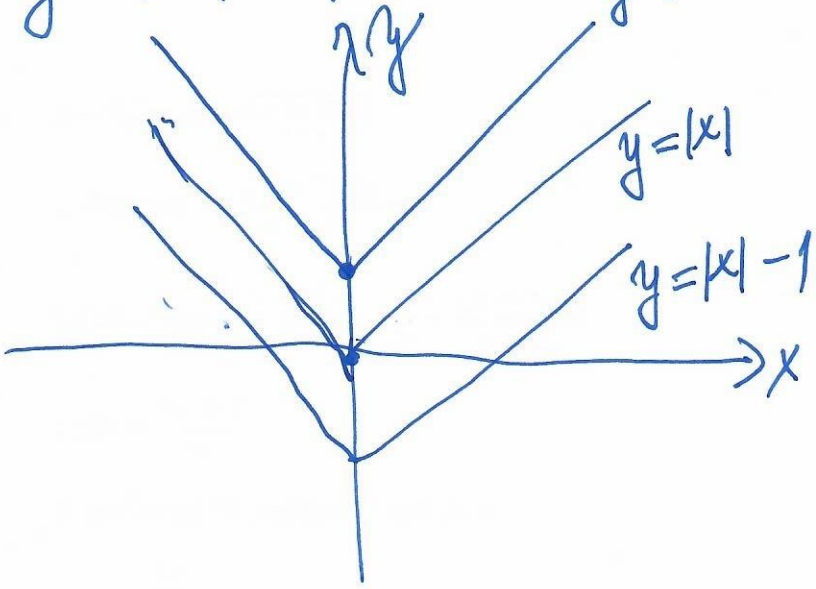
KLES.

$$y = |x| = \begin{cases} x & ; x \geq 0 \\ -x & ; x \leq 0 \end{cases}$$

- PÁRNA
- NEEK. INVERZNÁ!



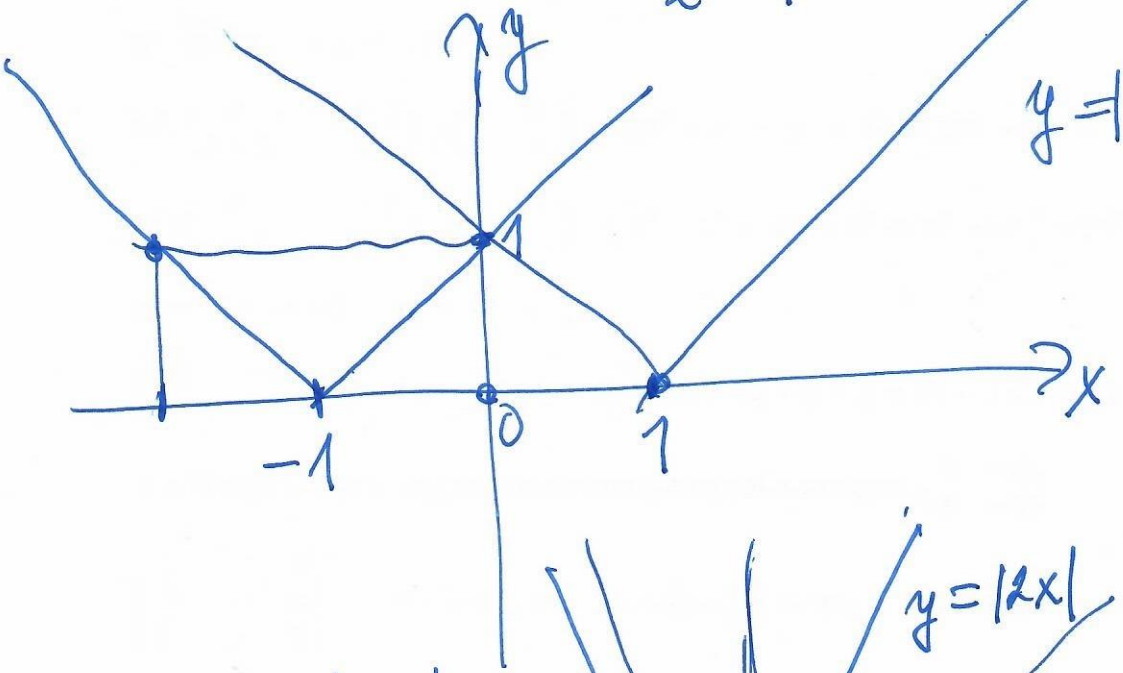
$y = |x| + 1$



$y = |x + 1|$ $\quad x+1=0$ $\quad x=-1$

NIE JEST PARZYSTA (ANI NIEP.)

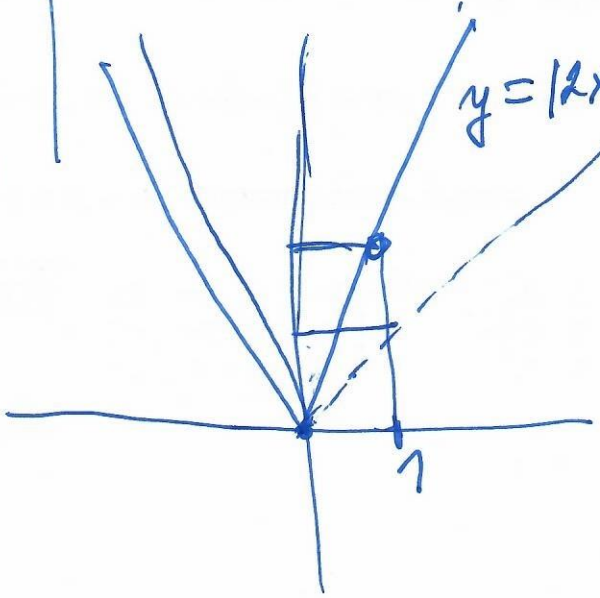
$y = |x - 1|$ $\quad x=1$



$y = |2x|$

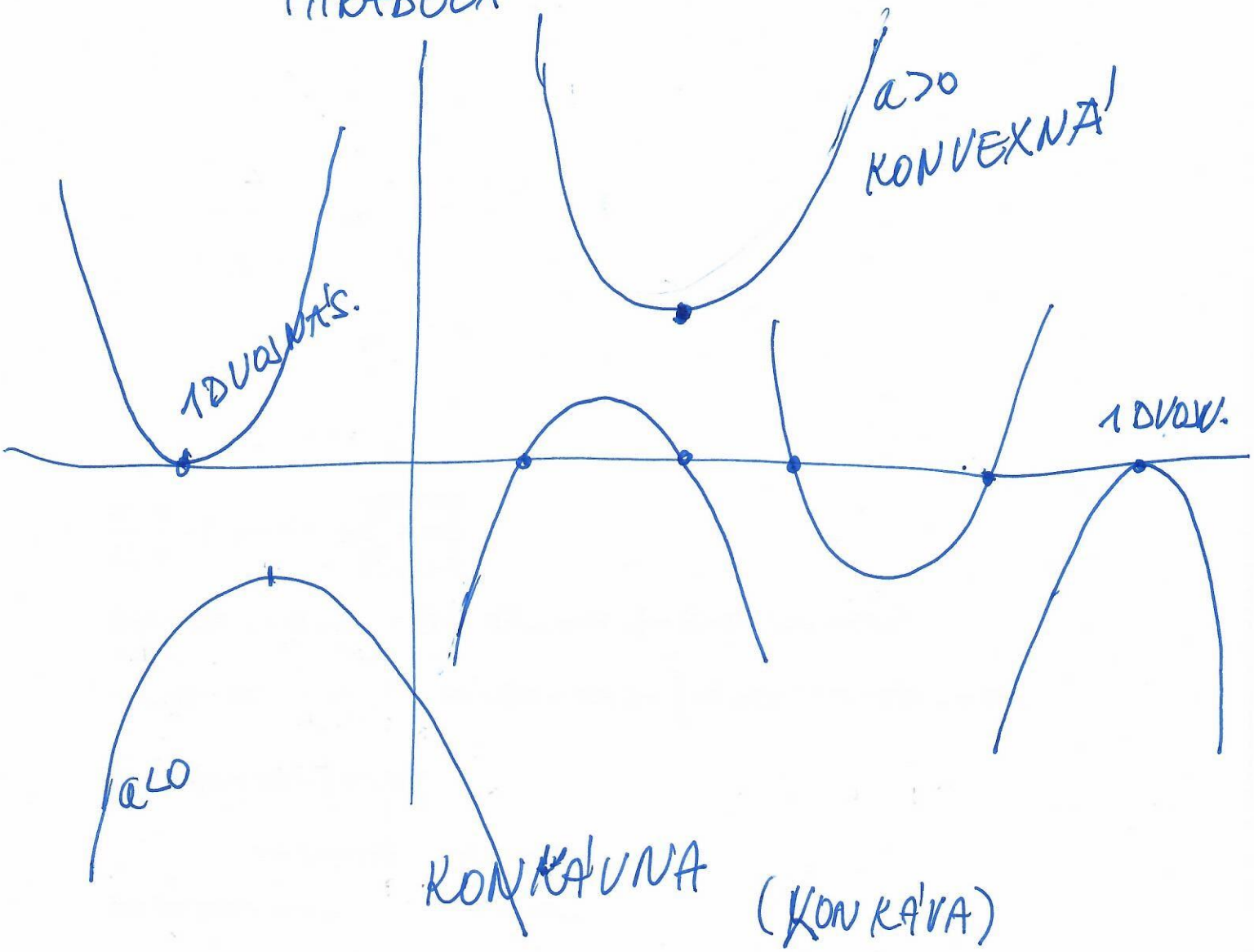
$k > 1$

$k < -1$



$y = ax^2 + bx + c$

PARABOLA



$ax^2 + bx + c = 0$ ● NEMÁ REÁLNY KOREŇ

$y = 2x^2 + x - 4$

$y = 2\left(x^2 + \frac{x}{2} - 2\right) = 2\left[\left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 2\right] =$

$x^2 + \frac{x}{2} + \frac{1}{16}$

$= 2\left[\left(x + \frac{1}{4}\right)^2 - \frac{33}{16}\right]$ KONVEXNÁ

≥ 0 $V\left[-\frac{1}{4}; -\frac{33}{8}\right]$