

PR1 VYPOČÍTAJTE DERIVACIU FUNKCIE

a) $y = 7x^4 - 12x^3 + 2\sqrt{x}$ $y' = 28x^3 - 36x^2 + \frac{1}{\sqrt{x}}$

b) $y = (x^2 - 2x + 5)(3x - 2)$ $y' = 9x^2 - 16x + 19$

c) $y = \frac{1-x}{1+x}$ $y' = \frac{-2}{(1+x)^2}$

d) $y = \operatorname{tg} x - 3x \log_4 x$ $y' = \frac{1}{\cos^2 x} - 3 \log_4 x - \frac{3}{\ln 4}$

e) $y = 4 \cdot 3^x \cdot 2^{-x}$ $y' = 4 \cdot \left(\frac{3}{2}\right)^x \ln\left(\frac{3}{2}\right)$

f) $y = e^{\sin^2 x^3}$ $y' = 6x^2 e^{\sin^2 x^3} \sin x^3 \cos x^3$

g) $y = \sin(\cos 2x)$ $y' = -2 \cos(\cos 2x) \sin 2x$

h) $y = \ln|\cot x|$ $y' = \frac{-\sin x}{2 \cot x}$

i) $y = 2^{\operatorname{tg} x}$ $y' = \frac{\ln 2 \cdot 2^{\operatorname{tg} x}}{\cos^2 x}$

j) $y = \operatorname{arccot} \frac{3}{\sqrt{x}}$ $y' = \frac{-3 \operatorname{arccot}^2 \sqrt{x}}{2\sqrt{x}(1+x)}$

k) $y = \arcsin\left(\frac{\ln x}{2}\right)$ $y' = \frac{1}{x\sqrt{4 - \ln^2 x}}$

l) $y = \operatorname{arccot} \frac{x}{1+x^2}$ $y' = \frac{x^2 - 1}{(1+x^2)^2 + x^2}$

m) $y = 10^{\sqrt{x}} \cdot x$ $y' = 10^{\sqrt{x}} \left(\frac{\ln 10 \sqrt{x}}{2} + 1 \right)$

n) $y = 3^{\cot x} \cdot \arcsin x$ $y' = 3^{\cot x} \left(-\frac{\ln 3 \arcsin x}{\sin^2 x} + \frac{1}{1-x^2} \right)$

o) $y = \log_5 (\operatorname{tg} x^3)$ $y' = \frac{3x^2}{\ln 5 \operatorname{tg}^3 x \cos^2 x^3}$

p) $y = \ln(\operatorname{arctg} \sqrt{5x})$ $y' = \frac{5}{2 \operatorname{arctg} \sqrt{5x} \cdot (1+5x) \cdot \sqrt{5x}}$

r) $y = (\ln x)^x$ $y' = (\ln x)^x \left(\ln \ln x + \frac{1}{\ln x} \right)$

s) $y = (3x)^{\sin x}$ $y' = (3x)^{\sin x} \cdot (\cos x \ln 3x + \sin x)$

t) $y = (\cot x)^{\operatorname{arccot} x}$ $y' = \cot x^{\operatorname{arccot} x} \left(\frac{-\ln^x(\cot x)}{1-x^2} + \frac{\operatorname{arccot} x}{\cot x (-\sin^2 x)} \right)$

PR2) POMOCOU DEFINÍCIE VÝPOČÍTATE DERIVATÍV
FUNKCIE $f(x)$ V BODE a , AK:

a) $a=2$ $f(x) = \frac{1}{x}$ $f'(a) = -\frac{1}{4}$

b) $a=4$ $f(x) = \sqrt[3]{x+4}$ $f'(a) = \frac{1}{12}$

c) $a=0$ $f(x) = \begin{cases} x \sin x & x \leq 0 \\ x^2 & x > 0 \end{cases}$ $f'(a) = 0$

d) $a=2$ $f(x) = |3x-6|$ $f'(a) = \text{?}$

e) $a=2$ $f(x) = \sqrt{4x+1}$ $f'(a) = \frac{2}{3}$

f) $a=3$ $f(x) = |x-3|$ $f'(a) = \text{?}$

PR3) 1) NÁJDITE ROVNICU DOTYČNICE A NORMÁLY KU
GRAFU FUNKCIE $f(x) = e^{-x} \cos 2x$ V BODE $A[0, 2]$

$[t: y = -x + 1; n: y = x + 1]$

2) NÁJDITE t A n KU GRAFU $f(x) = \frac{3x-4}{2x-3}$ V BODE $A[2, \frac{2}{5}]$
 $[t: y = -x + 4; n: y = x]$

3) NÁJDITE t A n KU GRAFU $f(x) = e^{1-x^2}$, KT. PRŮCHÁDZA
PRISEČNÍKOM GRAFU FUNKCIE S $n: y = 1$

$\left[\begin{array}{ll} t_1: y = -2x + 3 & n_1: y = \frac{1}{2}x + \frac{1}{2} \\ t_2: y = -2x + 3 & n_2: y = -\frac{1}{2}x + \frac{1}{2} \end{array} \right]$

4) NÁJDITE t A n KU $f(x) = x^2 - 2x + 3$, AK $t \parallel n: 3x - y + 5 = 0$
 $[t: 4y = 12x - 13; n: 12y = -4x + 61]$

5) NÁJDITE t A n KU $f(x) = \ln x$, AK $t \perp n: x + 2y - 2 = 0$

$[t: y = 2x - \ln 2 - 1; n: y = \frac{1}{2}x - \ln 2 + \frac{1}{4}]$