APPLIED MATHEMATICS
UNIVERSITY of WASHINGTON

Adaptive Mesh Refinement in Chapel
Part II: A really hard problem, greatly simplified

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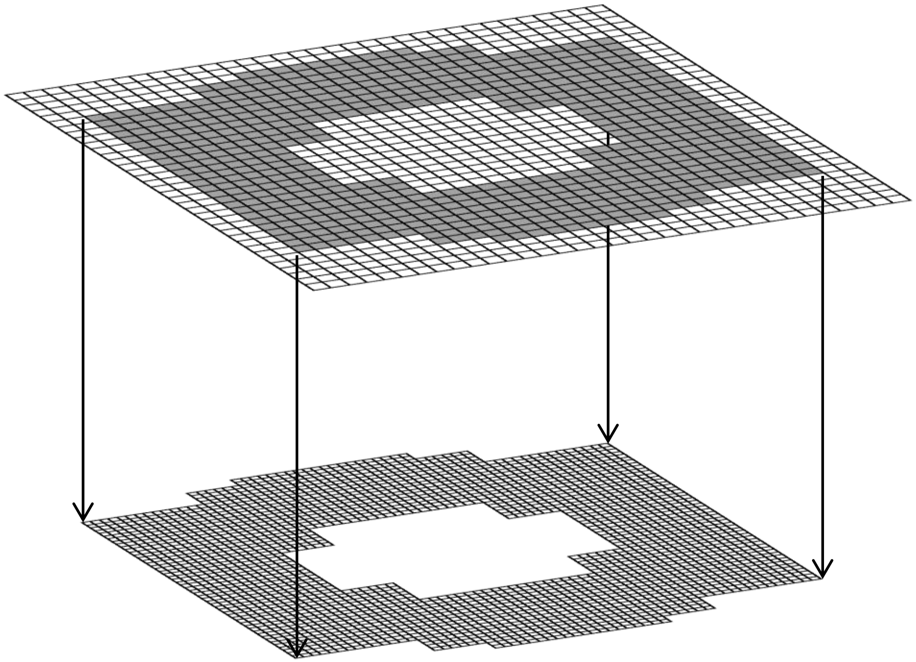
DARPA HPCS CRAY THE SUPERCOMPUTER COMPANY

Overview of two talks

- Previous talk:
  - Several AMR challenges that Chapel makes easy
- This talk:
  - A difficult part of AMR that Chapel sets us up to solve

Data refinement

- Data from a coarse grid provides boundary values to fine grids that it overlaps



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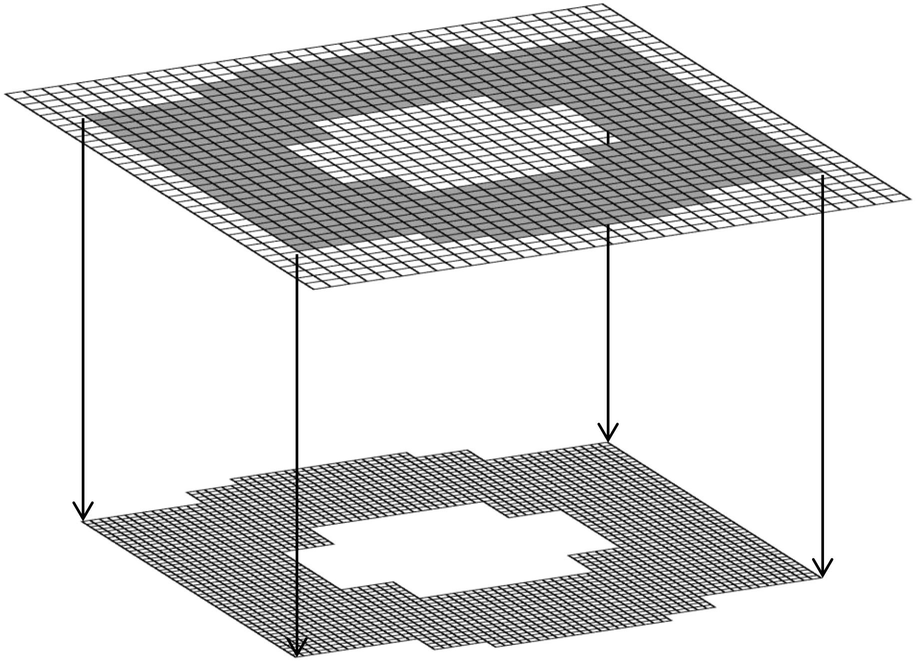
- Only need to fill fine ghost cells that are not overlapped by a fine sibling grid



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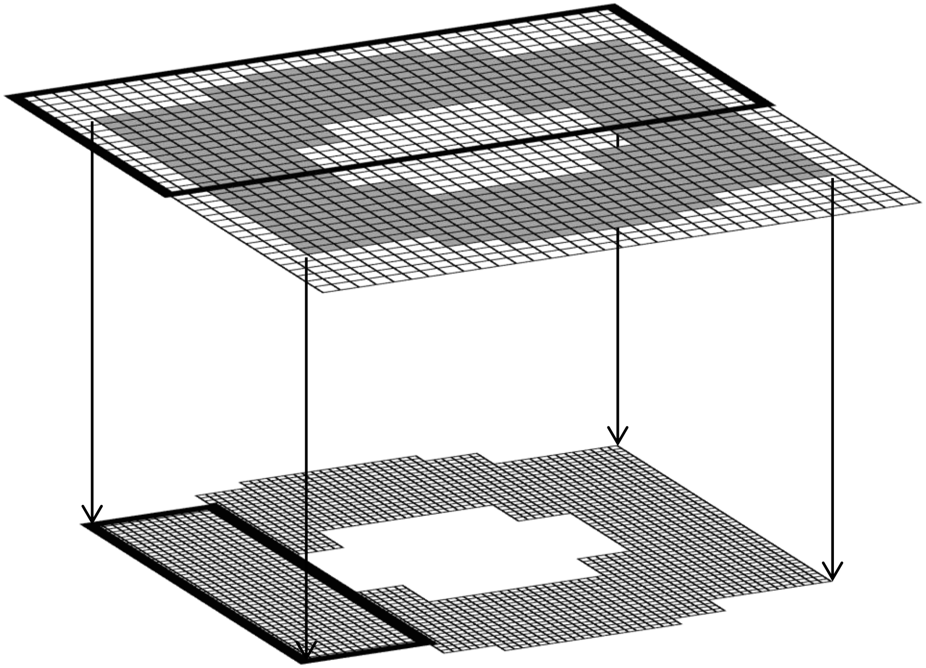

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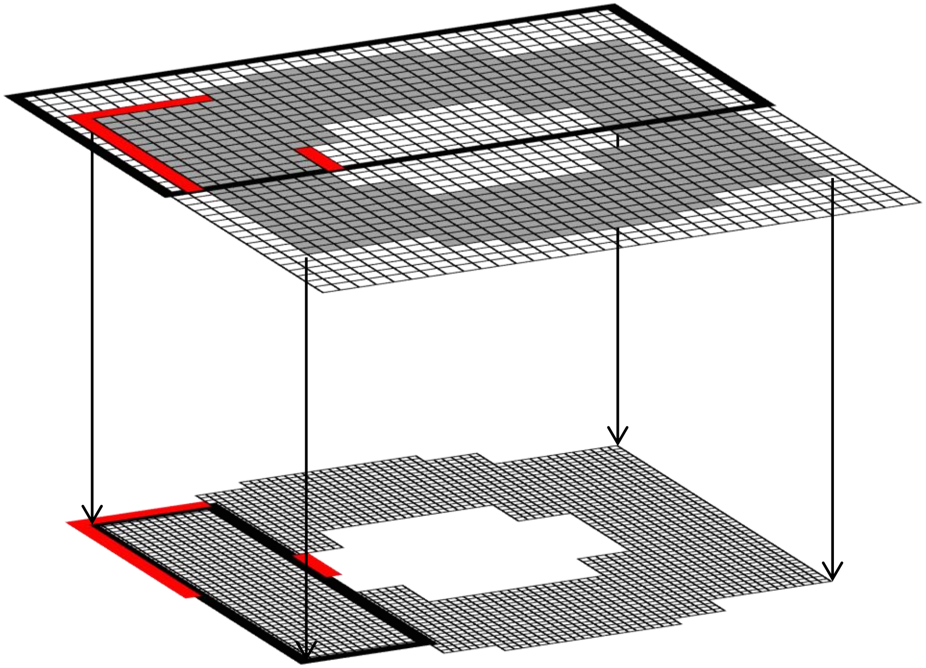

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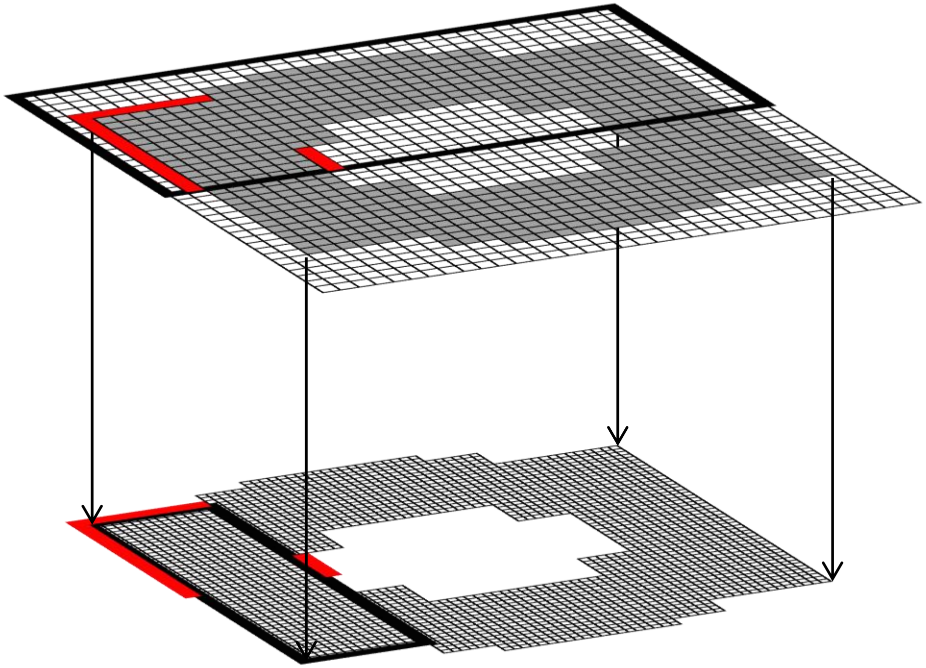

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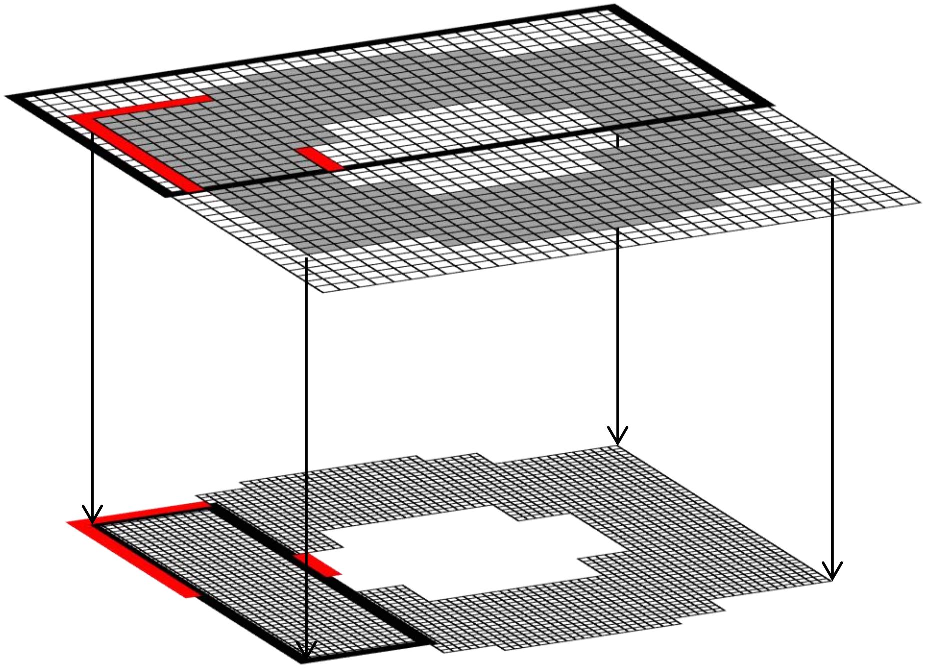
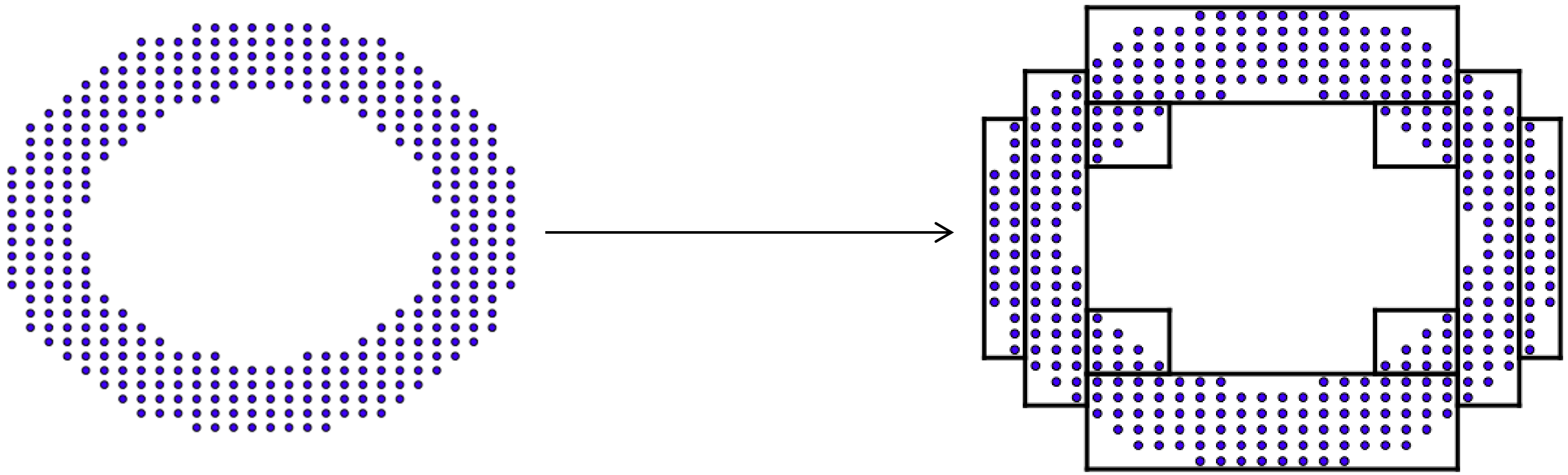
- Chapel: Define an object to store unions of domains, which supports domain subtraction in a set-minus fashion



Regridding



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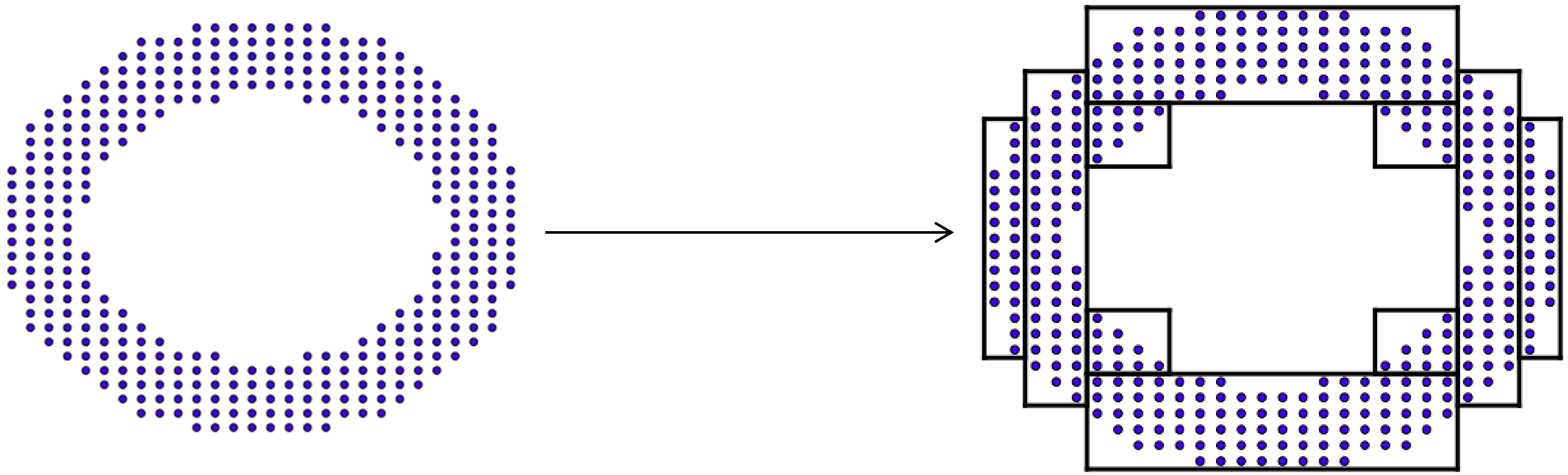

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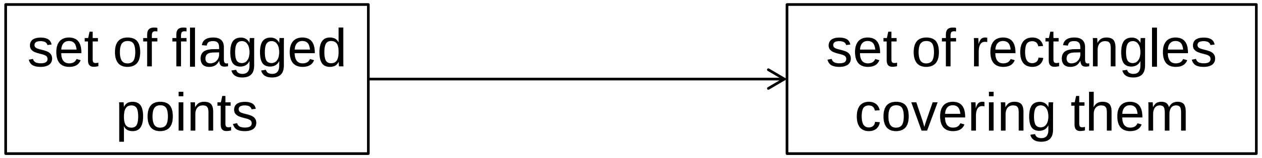

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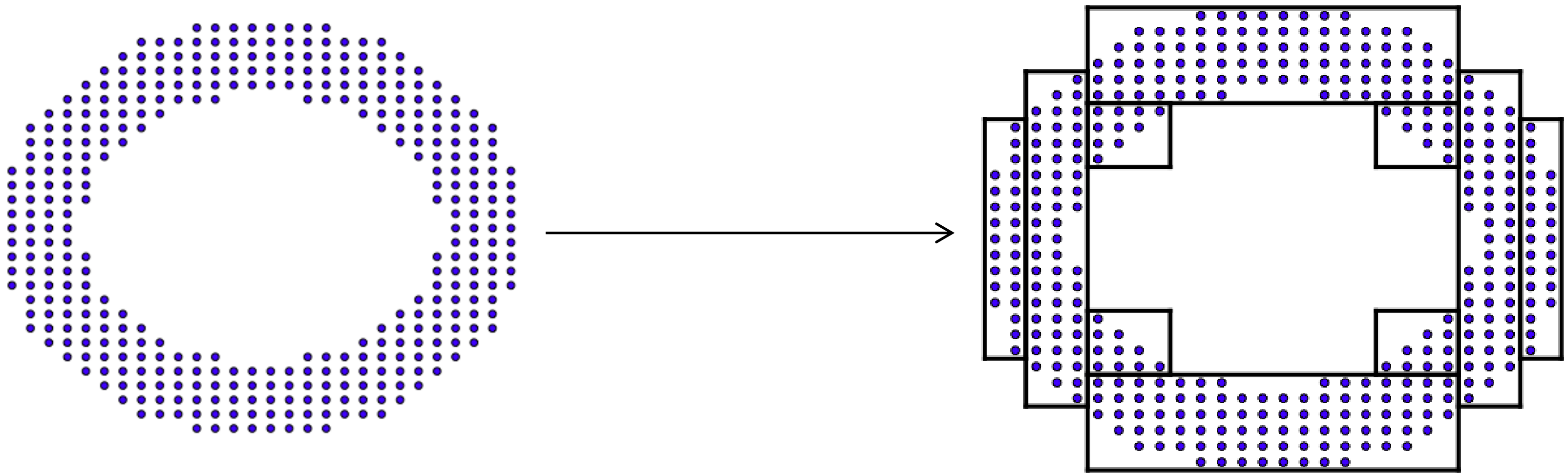
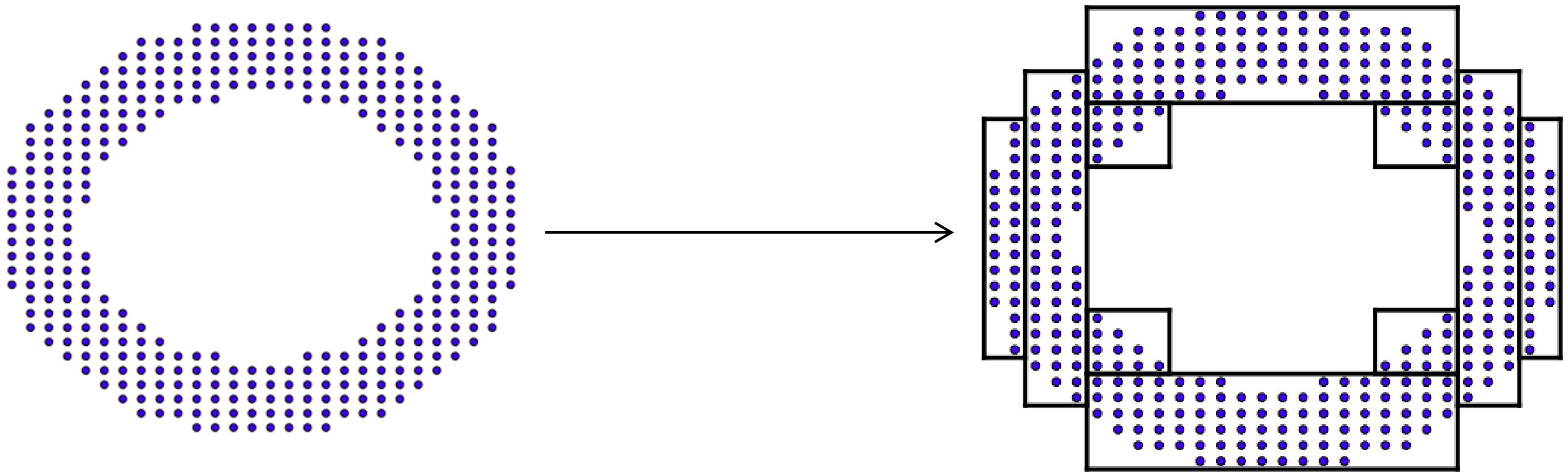


- New grids determined by a **partitioning algorithm** (Berger & Rigoutsos, 1991)

set of flagged points → set of rectangles covering them

Chapel: Boolean array on a large domain → set of subdomains containing all `true` entries

Regridding



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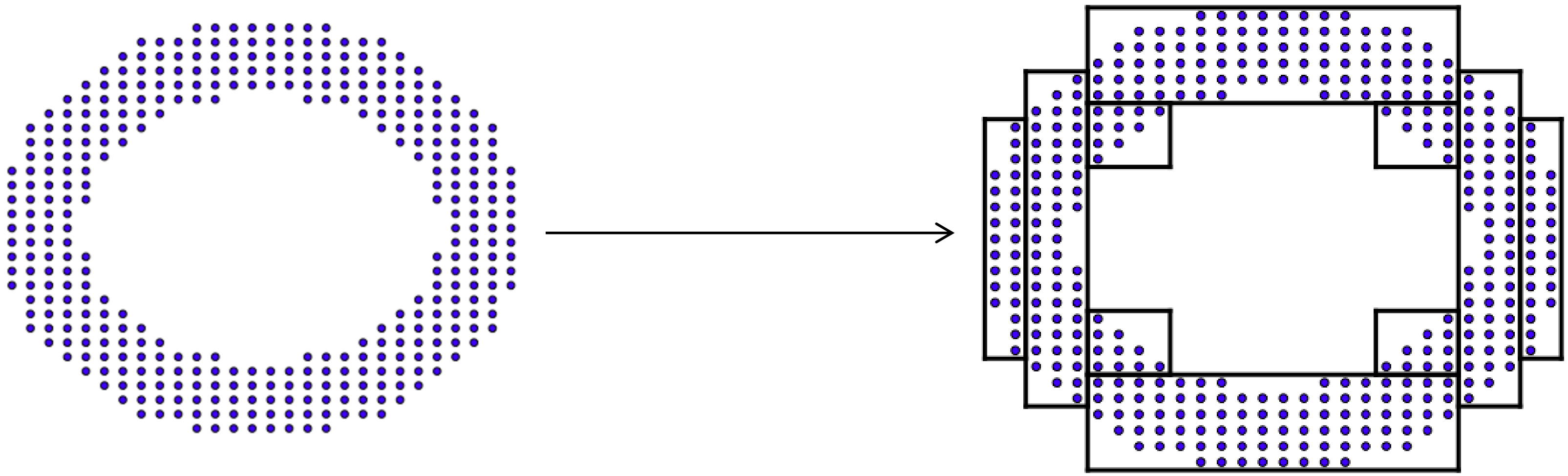

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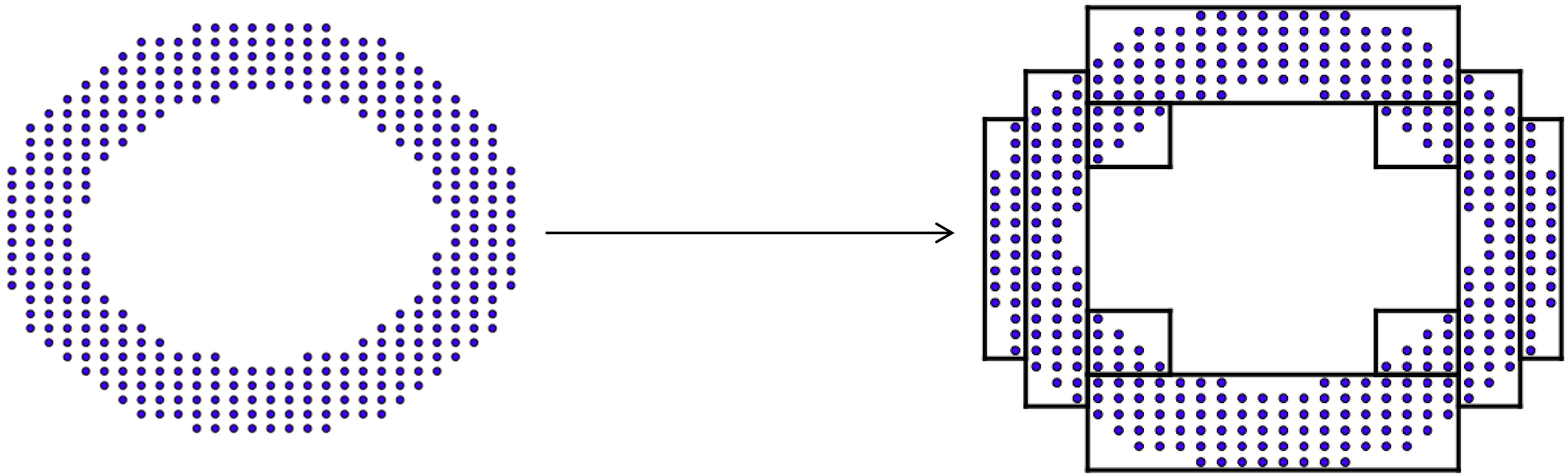

- As with refinement, **unions** of rectangles (domains) are essential
- Subtractions in Berger-Rigoutsos always remove a subset that spans a domain in `rank-1` dimensions; general domain subtraction is convenient, but not necessary
- However, domain subtraction **is** important after partitioning, when refining data onto a newly created level

Unions of domains: `MultiDomain` class

Precedents in AMR libraries:

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Precedents in AMR libraries:

- Chombo *BoxTools* library
  - Class `Box` represents rectangular sets of integer tuples (`IntVects`)
  - Class `IntVectSet` represents irregular sets of integer tuples, supporting full set calculus

Unions of domains: `MultiDomain` class

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  - Class `Box` represents rectangular sets of integer tuples (`IntVects`)
  - Class `IntVectSet` represents irregular sets of integer tuples, supporting full set calculus
- SAMRAI *Hierarchy* library
  - Class `Box` (see above)
  - Classes `BoxArray`, `BoxList`, `BoxTree` represent unions of boxes, supporting various set operations

Unions of domains: `MultiDomain` class

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```
param rank:       int;
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var   subindices:    domain(1);
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Array of child domains

- In principle, `domains` could be an associative domain of rectangular domains
- Tree-based storage of `domains`, with bounding boxes at nodes, will allow better performance for set operations; direction for future improvement

Unions of domains: `MultiDomain` class

- `MultiDomain` operations:

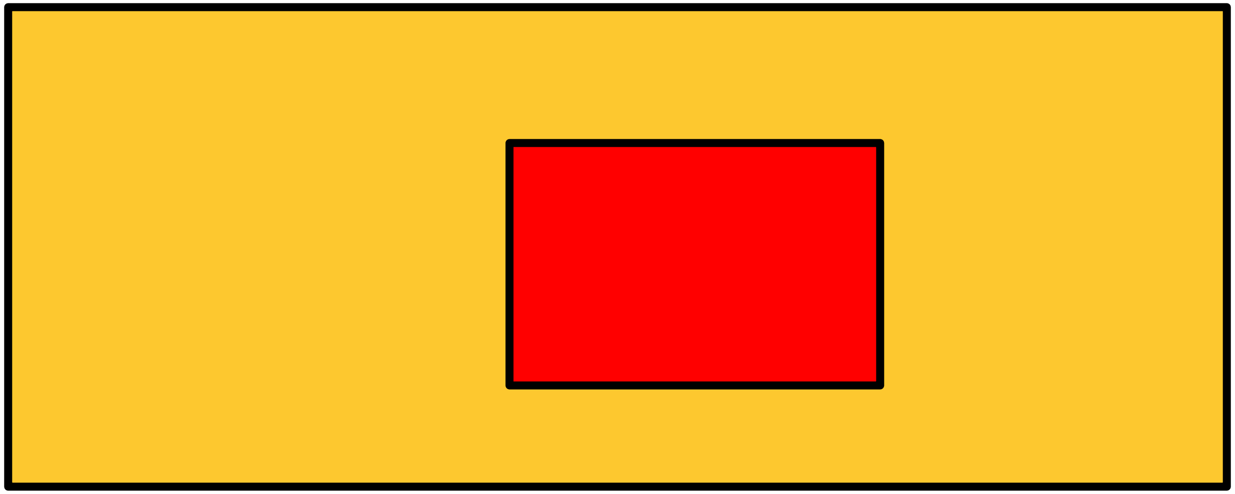
```
MultiDomain = domain;
MultiDomain.add(domain);
MultiDomain = domain – domain;
MultiDomain.subtract(domain);
MultiDomain.intersect(domain);
```

  etc…

- Most operations allow a `MultiDomain` as an argument as well

Domain subtraction

- Recursive procedure, reducing to `rank-1` subtraction at each step



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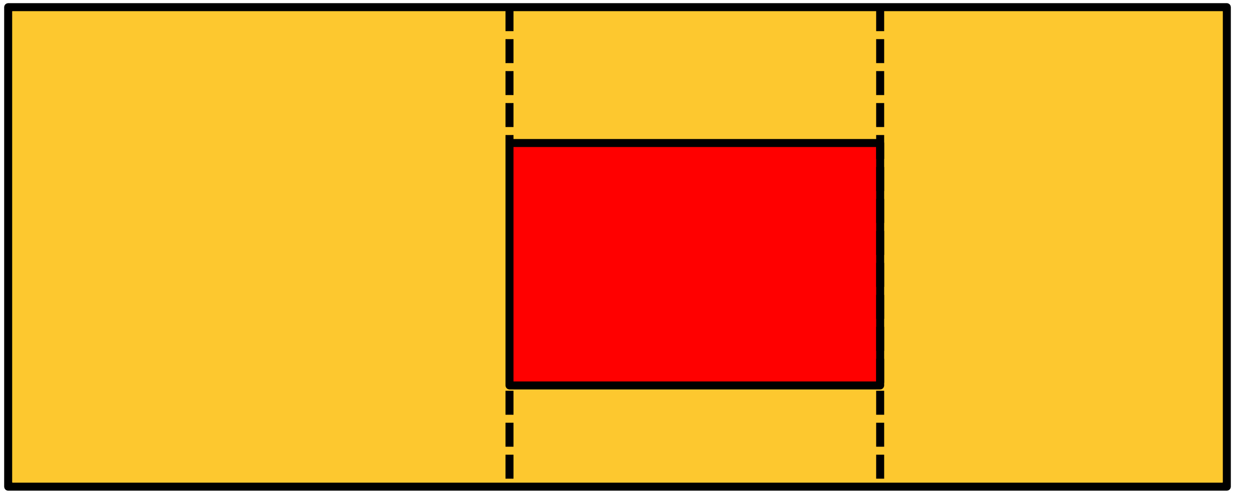

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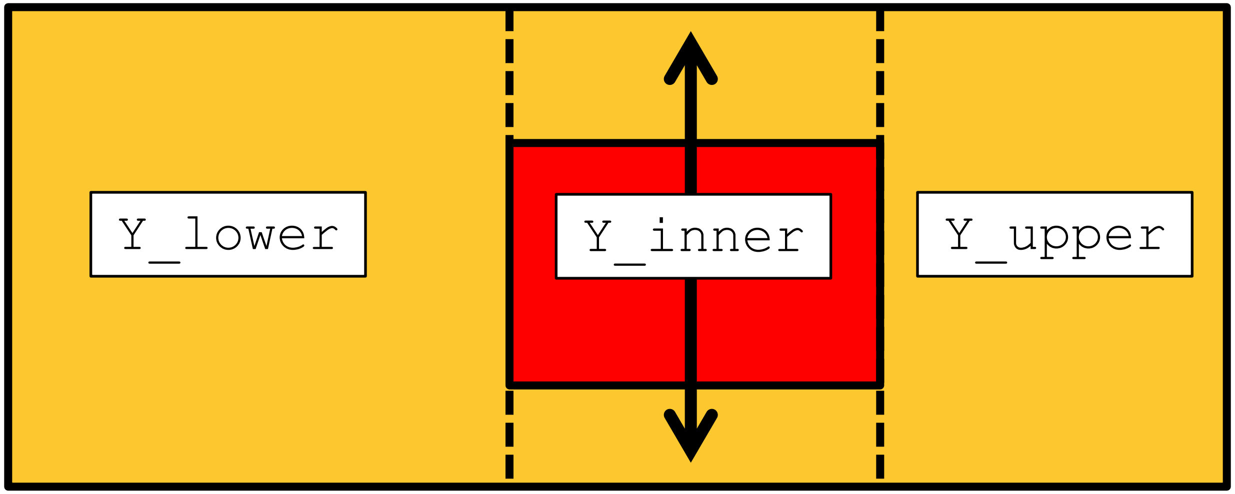

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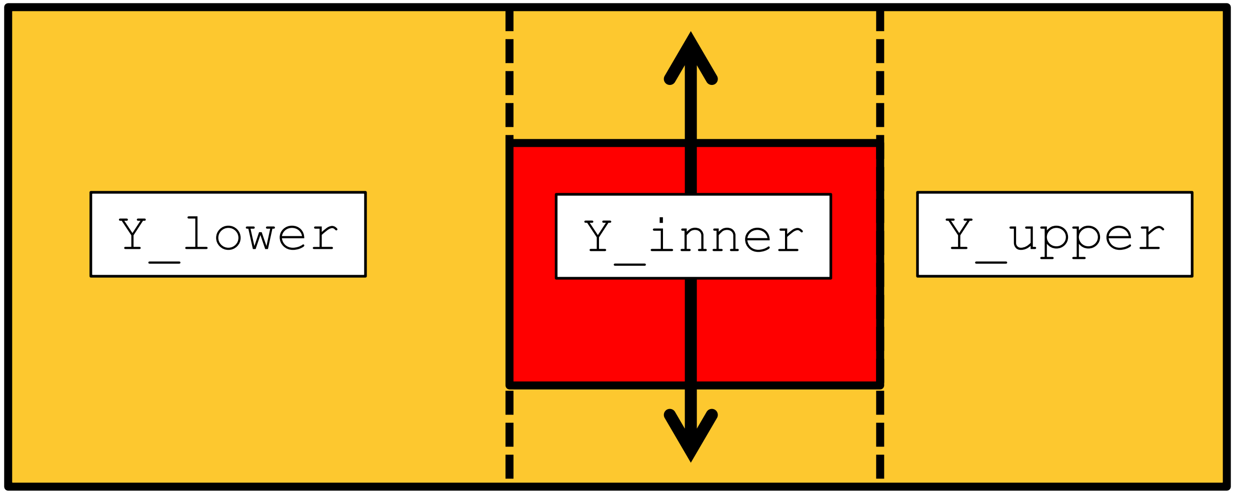

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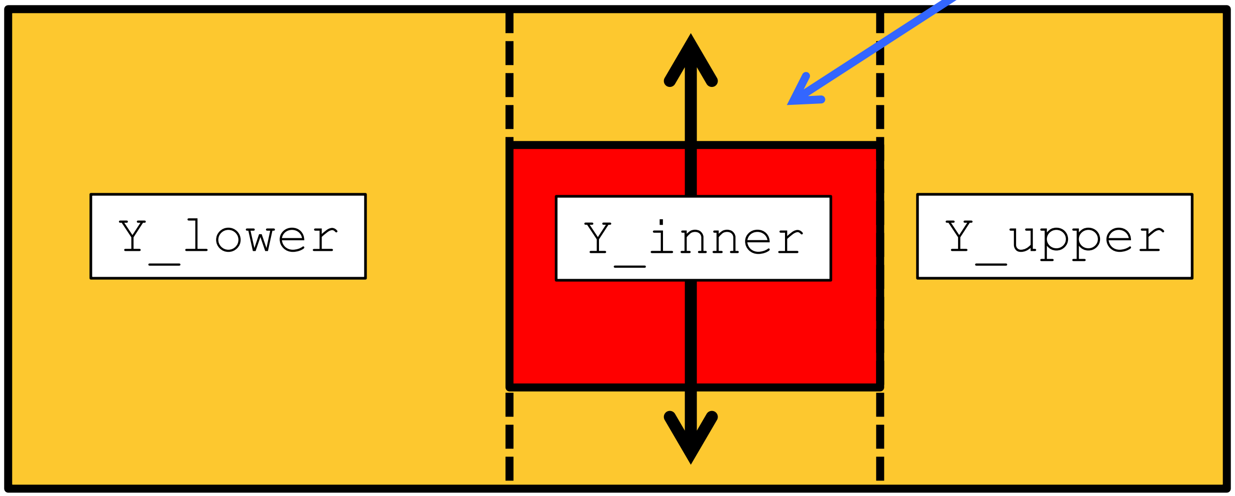
Domain subtraction

`Y_inner-Red` is much more complicated than this in higher dimensions

- Recursive procedure, reducing to



- Calculate `Yellow - Red`, working along the horizontal:
  - `Yellow` splits into 3 pieces: `Y_lower`, `Y_inner`, and `Y_upper`, any of which may be empty
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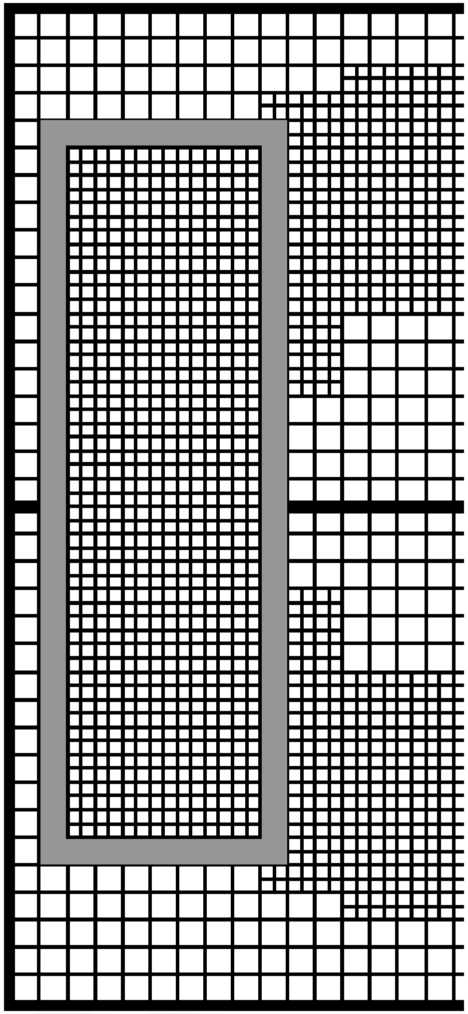
  - Now calculate `Y_inner-Red`, but project onto remaining dimensions since `Y_inner.dim(1) == Red.dim(1)`

Class `GridCFGhostRegion`

- Represents ghost cells of a fine grid that will receive data from "coarse neighbor" grids



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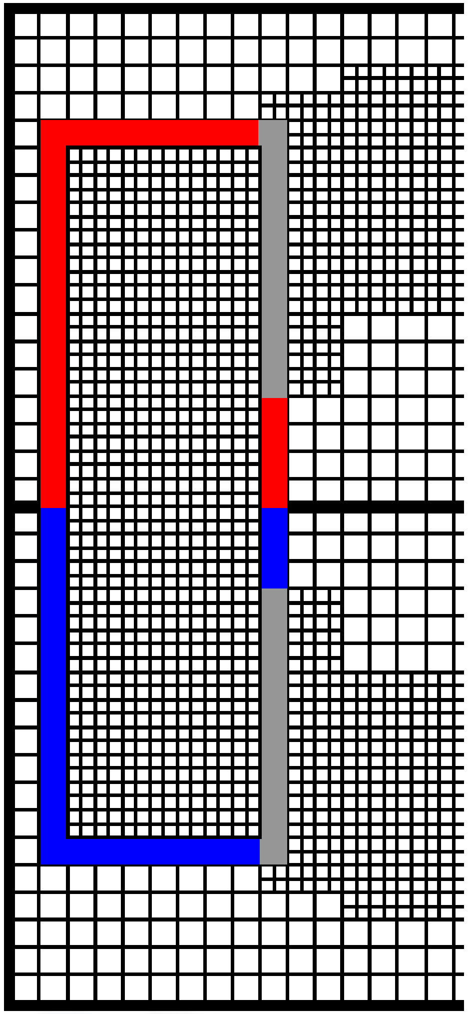


Class `GridCFGhostRegion`

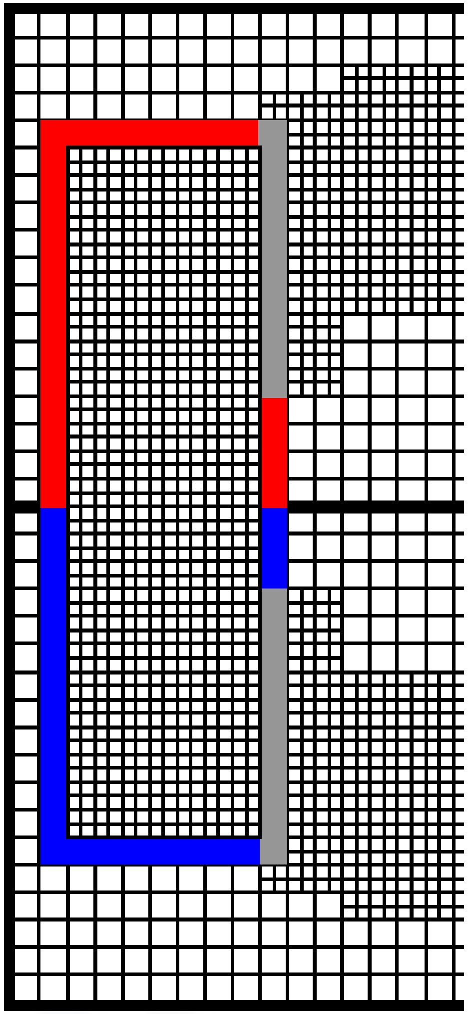
- Represents ghost cells of a fine grid that will receive data from "coarse neighbor" grids
- Fields are:

```
const grid: Grid;
```

The fine grid in question

```
const coarse_neighbors: domain(Grid);

const multidomains: [coarse_neighbors]
    MultiDomain(dimension, stridable=true);
```



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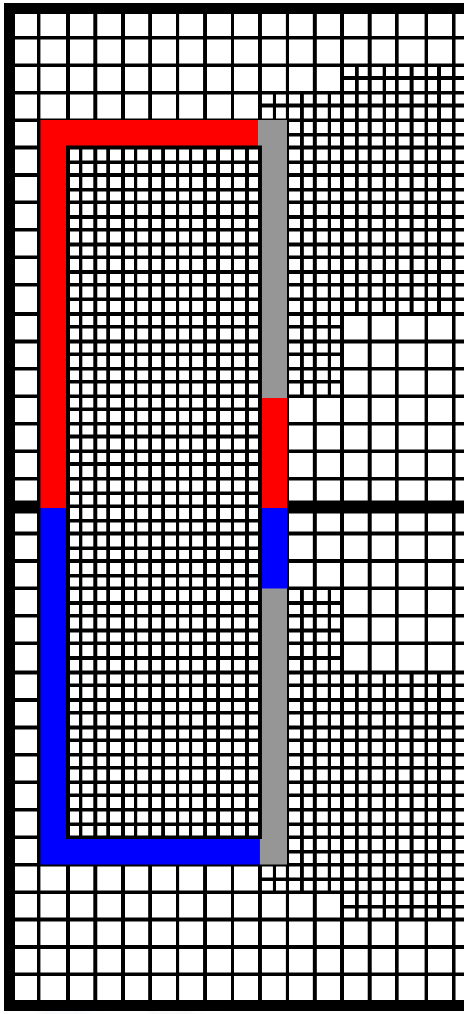
- Constructor also needs to know:
  - `parent_level` of `grid`
  - `coarse_level`
  - `ref_ratio`, the refinement ratio between `coarse_level` and `parent_level`



Class GridCFGhostRegion

```
for coarse_grid in coarse_level.grids {
  var fine_intersection =
        grid.extended_cells( refine(coarse_grid.cells, ref_ratio) );

  if fine_intersection.numIndices > 0 {
    var boundary_multidomain = fine_intersection - grid.cells;

    for (neighbor, region) in parent_level.sibling_ghost_regions(grid)
    {
      if fine_intersection(region).numIndices > 0 then
            boundary_multidomain.subtract(region);
    }

    if boundary_multidomain.length > 0 {
      coarse_neighbors.add(coarse_grid);
      multidomains(coarse_grid) = boundary_multidomain;
    }
    else delete boundary_multidomain;
  }
}
```

Iterate over coarse grids; all are potentially coarse neighbors

Class GridCFGhostRegion

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  }
}
```

Intersect the coarse grid (interior only) – in fine index space – with the fine grid (ghost cells included)

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    }
    else delete boundary_multidomain;
  }
}
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If `fine_intersection` is empty, there's no reason to continue

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boundary_multidomain will still contain the ghost region that overlaps sibling grids

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Iterate over the grid's SiblingGhostRegion; a `these()` method has been defined to make the object iterable

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Subtract the region of overlap, pre-scanning for obviously disjoint cases

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If boundary_multidomain is nonempty, update the `GridCFGhostRegion` fields

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```

Otherwise, get rid of it

Class `GridCFGhostRegion`

```
for coarse_grid in coarse_level.grids {
  va
                                                                  );
  if
```

- MultiDomains greatly simplify the hard part
  - Internally, MultiDomains heavily rely on Chapel infrastructure for domains
  - Simple ≠ cheap; misuse of MultiDomains can be expensive

```
    }
    else delete boundary_multidomain;
  }
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- MultiDomains greatly simplify the hard part
  - Internally, MultiDomains heavily rely on Chapel infrastructure for domains
  - Simple ≠ cheap; misuse of MultiDomains can be expensive
- Development of full AMR framework also required:
  - Assemble Grid data structures into Level data structures
  - Define spatial variables on GridCFGhostRegions
  - Space-time interpolation from coarse to fine variables

Conclusions

Final recap of code size:

Language	Parallelism	SLOC[1]	Tokens	Relative size (tokens)
C++ (D≤6) [3]	Dist. mem.	40200	261427	100%
Fortran (2D+3D) [2]	Serial	16562	151992	58%
2D		8297	71639	27%
3D		8265	80353	31%
Chapel (any D)	Shared mem.	1988	13783	**5%**

[1] source lines of code, [2] AMRClaw, [3] Chombo BoxTools+AMRTools

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Suggestion for future language evaluation

- Use rectangular set operations (“box calculus”) as a problem representative of, and more tractable than, AMR

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- Dimension-independence
  - After defining MultiDomains, spatial dimension only appears in variable declarations
- Clean, clear iteration syntax
  - Ability to define any object as an iterator with `these()` method

Recall Chapel’s main goal:

- **Improve programmer productivity**

Where can I learn more?

Chapel:

http://chapel.cray.com

Today's presentations, and many more:

http://chapel.cray.com/presentations.html

Chapel source:

https://sourceforge.net/projects/chapel

Application studies:

http://chapel.svn.sourceforge.net/viewvc/chapel/trunk/…

AMR:	…test/studies/amr
SSCA2:	…test/users/jglewis/ssca2_version2
PTRANS:	…test/studies/hpcc/PTRANS/jglewis