Design Process for a 1.5 kN Liquid Rocket Engine

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Objective and Justification

- The crux of my doctoral research is investigating the effects of vorticity on diffusion limited boundary flames
  - This solves many inherent problems within Hybrid Rocket Engines (HREs) but produces a measurable torque on the engine/vehicle
  - Many analytical papers written in the 60s-80s on swirl nozzle flows. Few experimental investigations performed

- **Objective**: Create a rocket engine to experimentally study the effects if strong vorticity on torque generated and nozzle performance
- Requirements:
  - Produce equivalent angular momentum and thrust to the experimental HRE
  - Control for variations in nozzle geometry and flow rates
- Design Solution: Actively Cooled Liquid Bi-propellant rocket
  - Active cooling controls for nozzle geometry
  - Liquid fuel feed system eliminates variability in flow rates and O/F shifts in HREs
  - K-1 Kerosene produces chemically similar flame to High Density Polyethylene

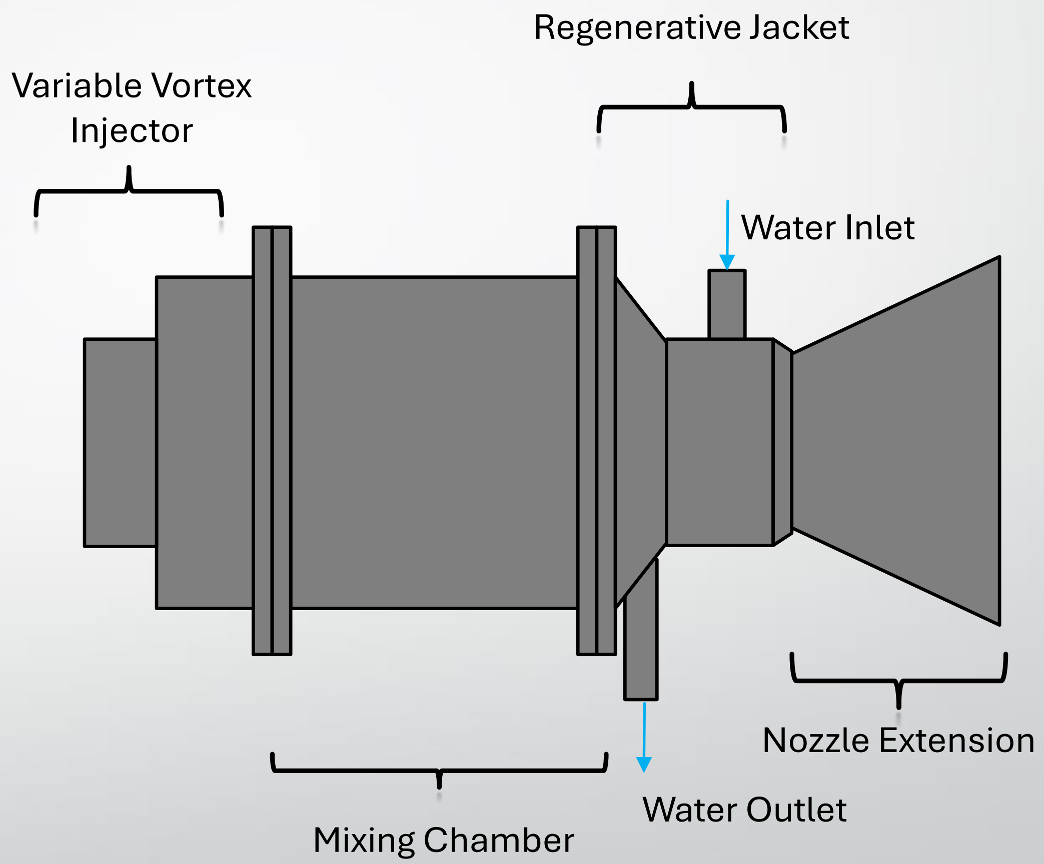
Overview

Major Design Sub-Problems

1. Fluid System Design
2. Injector Design
3. Drives mixing chamber length
4. Method of Characteristics Nozzle
5. Regenerative Cooling Channels

Design Specs

Design Spec	Value
Thrust	1.5 kN
Isp	261 sec
Oxidizer	O2 (l)
Fuel	HDPE/K-1
Burn Duration	15 sec
Chamber Pressure	3.2 MPa (450 psi)
Injector Presure	4.0 MPa (580 psi)



Design Concept

System Design

System is designed from ground up to support both Hybrid & Liquid Bi-propellant swirl studies

Satisfies SMC-S-16 Testing standards and NASA KSC cryogenic system best design practices

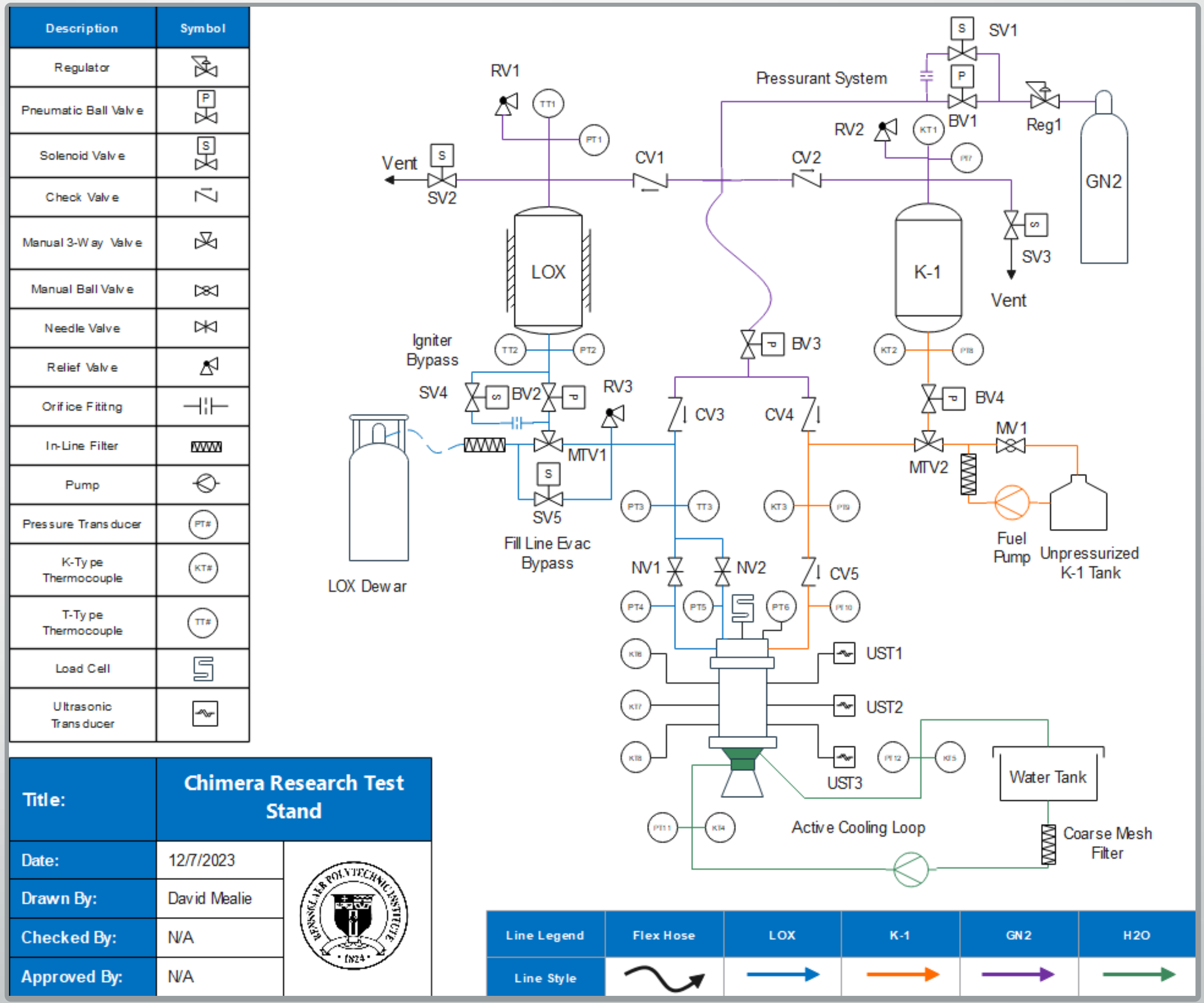
Angular momentum throttled using needle valves feeding into swirl and radial injector volutes

Instrumentation

- Flow Rates measured using pressure drop across micrometer needle valves with characterized Cv
- Rocket performance measured using load cell & chamber pressure
- Hybrid regression measured using 3x ultrasonic transducers w/ additional 3x K-type thermocouples to capture thermal boundary layer

Safety

- Spring loaded relief valves and automated solenoid vent to relieve boiloff in trapped volumes
- Fluid state monitoring along all sections of fluid lines
- Health monitoring and automated abort criteria

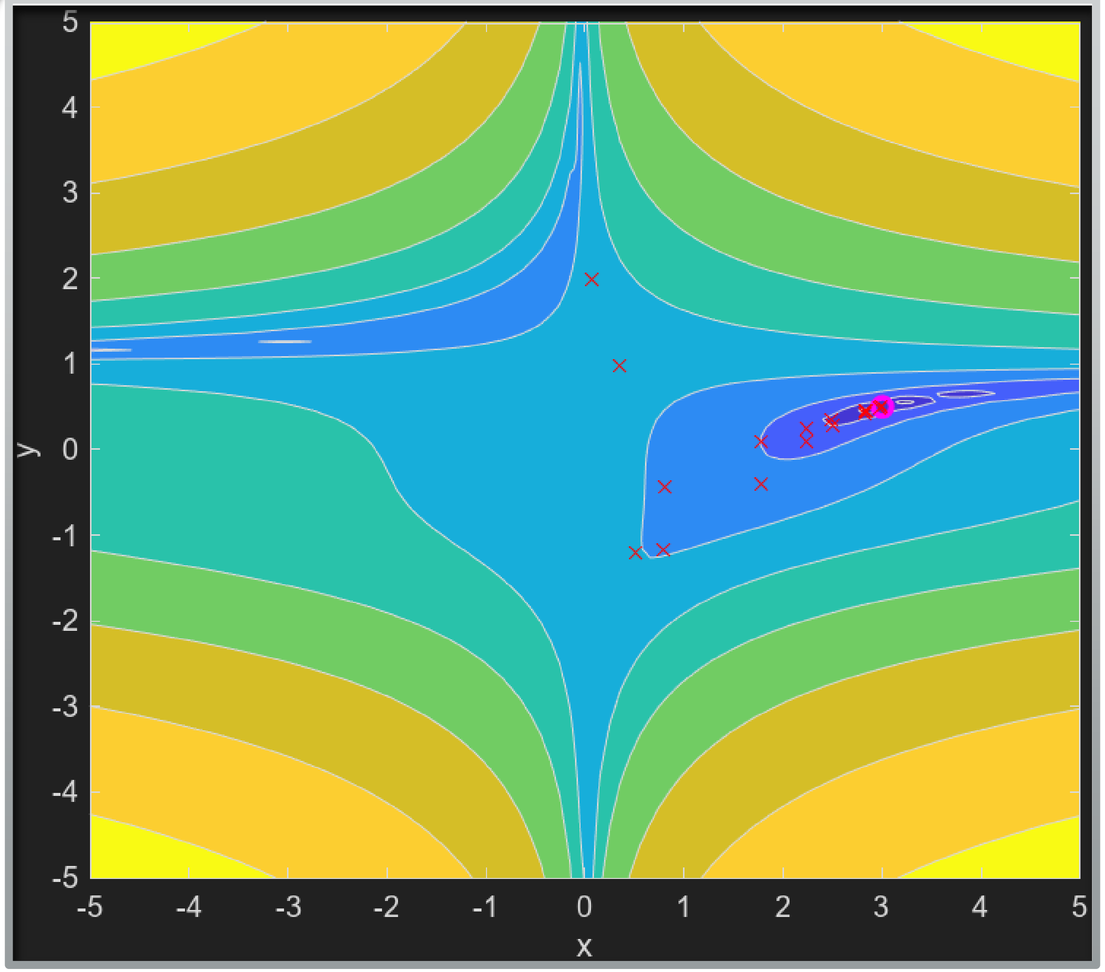


An Aside on Optimization Algorithms

- What does it mean to “Optimize” something?
- Mathematically it describes a problem where :

$$\begin{aligned}&\min f(x)\\&\text{s.t.}\quad c(x) < 0\end{aligned}$$

- Utilizing home-brew optimizer: fminconvict.m
  - based on MATLAB’s paid add on, fmincon.m
  - Quasi-Newton BFGS method
    - Secondary check using steepest-descent method
    - Slower but more robust than fmincon.m
  - Nonlinear constraints applied using Penalty Barrier method



Testing results for Beale’s Problem

Injector Design Process

Vortex HRE performance may be characterized by mass-flow-specific angular momentum, $l_{Ang} = vR_{inj}$

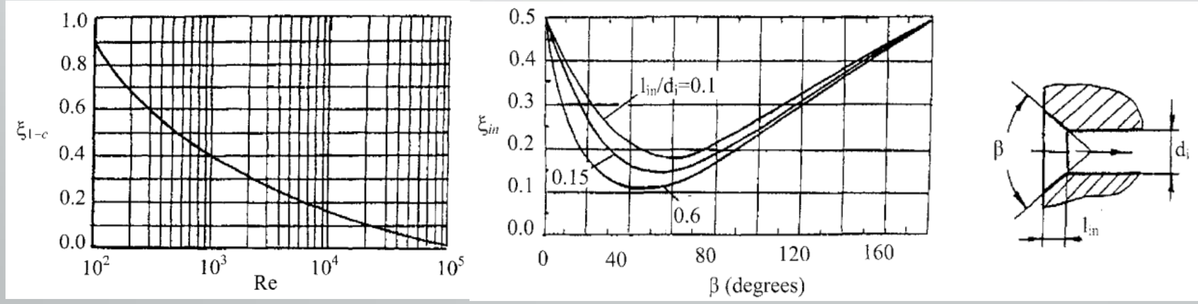
Discharge coefficient of straight cut channel with conical inlet may be parametrized by flow and geometry

Geometric design variables may be used to compute discharge efficiency, injection radius, and angular momentum

$$v = C\sqrt{2\Delta P/\rho}$$

$$C = \frac{C_d}{\underbrace{\sqrt{1-\beta^4}}_{\text{Salvageable dynamic pressure}}}\overbrace{Y}^{\text{Compressible Losses}}$$

$$C_d = \overbrace{\varepsilon}^{\text{Contraction Losses}}\underbrace{\sqrt{\frac{1}{1+\zeta}}}_{\text{Orifice Resistance}}$$



Expansion Factor

$$Y = \begin{cases} \dfrac{\sqrt{\dfrac{\gamma}{\gamma-1}\left(r_c^{\frac{2}{\gamma}} - r_c^{\frac{\gamma+1}{\gamma}}\right)}}{C_d\sqrt{1/\mu^2 - \beta^4 r_c^{2/\gamma}}\sqrt{1-r}} & \text{Unchoked} \\[2ex] \dfrac{\sqrt{\dfrac{\gamma}{\gamma-1}\left(r^{\frac{2}{\gamma}} - r^{\frac{\gamma+1}{\gamma}}\right)}}{C_d\sqrt{1/\mu^2 - \beta^4 r^{2/\gamma}}\sqrt{1-r}} & \text{Choked} \end{cases}$$

where

$$\mu = \begin{cases} \dfrac{\gamma-1}{2\gamma}\left[\dfrac{1-r}{r^{1/\gamma}-r}\right] & \text{Unchoked} \\[2ex] \dfrac{1-r}{r_c(\gamma+1)-r} & \text{Choked} \end{cases} \qquad r = P_2/P_1$$

Velocity Approach Factor

$$\beta = \frac{\mathrm{d_{orf}}}{\mathrm{D_{h_{manifold}}}} \approx \sqrt{\frac{nA_{orf}}{A_{manifold}}}$$

Contraction Coefficient

$$\varepsilon = \mathrm{A_{vc}}/\mathrm{A_{orf}} \quad \varepsilon \to 1 \text{ as } \frac{l}{d} > 1.5$$

Resistance Coefficient

$$\zeta = \frac{\Delta P}{1/2\rho v^2} = \zeta_{inlet} + \zeta_{orf} + \zeta_{fr}$$

$$\zeta_{inlet} = f(\mathrm{Re_d})$$

$$\zeta_{orf} = f(\beta, l_{in})$$

$$\zeta_{fr} = f_{Darcy}\frac{l}{d}$$

Injector Geometry and Optimization

While optimizing Cd may be an obvious choice, doing so in a diameter-limited design may invertedly reduce angular momentum

Objective: Maximize l_{Ang}

Design Variables:

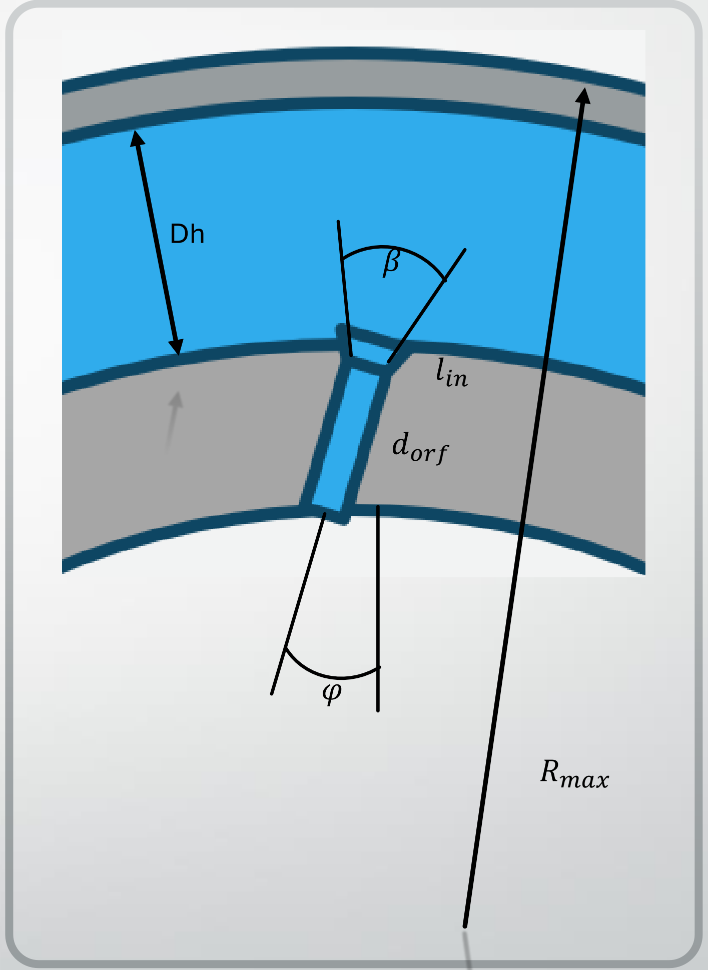
- Orifice count and diameter
- Manifold hydraulic diameter
- Inlet length and geometry

Constraints:

- Mass flow through orifice must match design point

Optimization Problem

$$\max l_{Ang}\left(d_{orf}, n_{orf}, D_{h_{man}}, l_{in}, \beta\right)$$
$$\text{s.t.} \quad \dot{m} = nCA_{orf}\sqrt{2\rho\Delta P}$$



Note: φ is not to scale

Chamber Length

Chamber length is largely governed by the residence time required to mix and vaporize atomized droplets

Chamber length may be calculated from characteristic length, $L^* = V_c/A_t$

Procedure:

Breakup Length:

$$L_b = 5.45 d_{orf} \left(\frac{\rho_l}{\rho_g}\right)^{2/3} \left[We \frac{(1-\cos(\theta))^2}{\sin^3(\theta)}\right]^{-1/3}$$

Droplet Diameter

$$d_D = \frac{2\pi\sigma}{\rho_g U_{jet}^2} = \frac{2\pi\rho_l}{\rho_g We} \qquad We = \frac{\rho_l U^2 h}{\sigma}$$

Characteristic Length:

$$L^* = \xi \frac{d_D^2 \left(\left(\frac{2}{\gamma+1}\right)\left\{1+\left(\frac{\gamma-1}{2}\right)M^2\right\}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \sqrt{\gamma R T_t}}{4\left(\frac{\gamma}{C_p \rho_l}\right)\ln(1+B)}$$

$$\xi = \frac{\frac{U_l}{U_g} + \frac{3}{10}}{2+S} \qquad S = \frac{Pr}{2B} \qquad B = \frac{Cp(T_{fl} - T_{vap})}{h_{vap}}$$

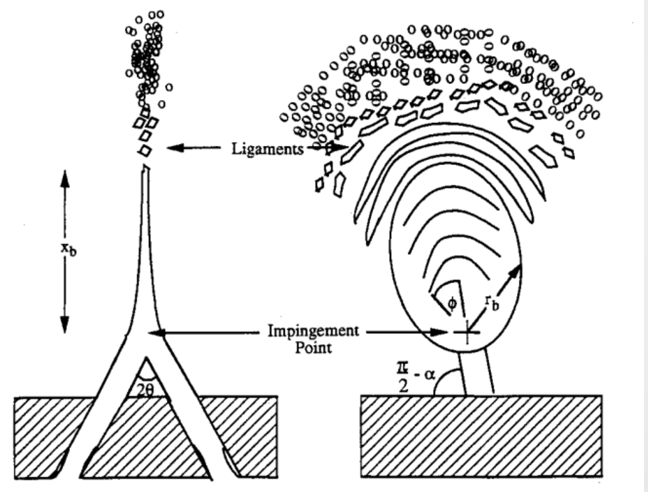
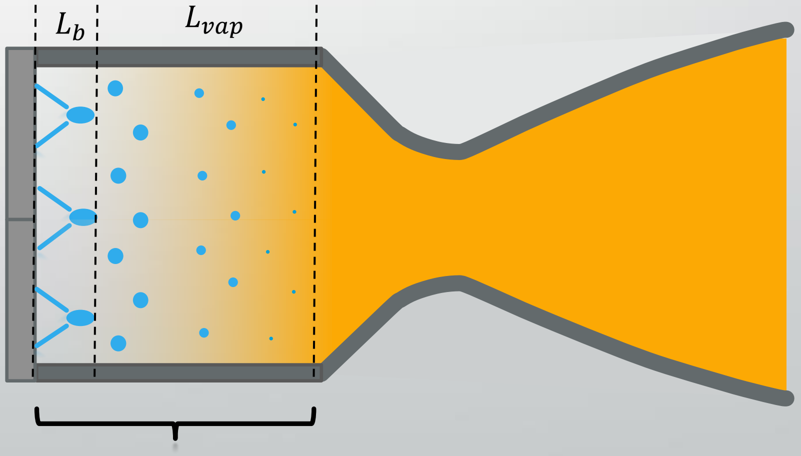


Fig. 1 Schematic diagram of a typical impinging jet spray.



$$L_c = L_b + L^* \left(\frac{A_c}{A_t}\right)$$

Method of Characteristics Nozzle

Iterative method for solving hyperbolic field equations

Reduces the PDEs into set of ODEs

May be applied to isentropic, supersonic flows

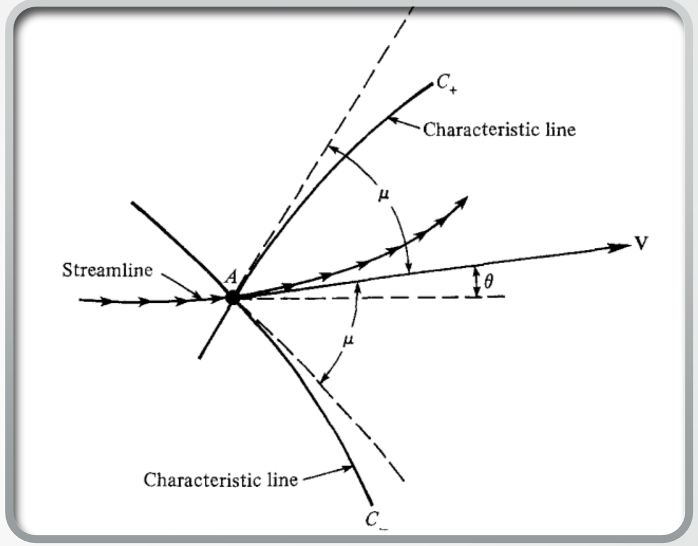
Characteristic lines extend beyond domain of flow field, allowing the nozzle contour to be traced from a streamline at the nozzle throat

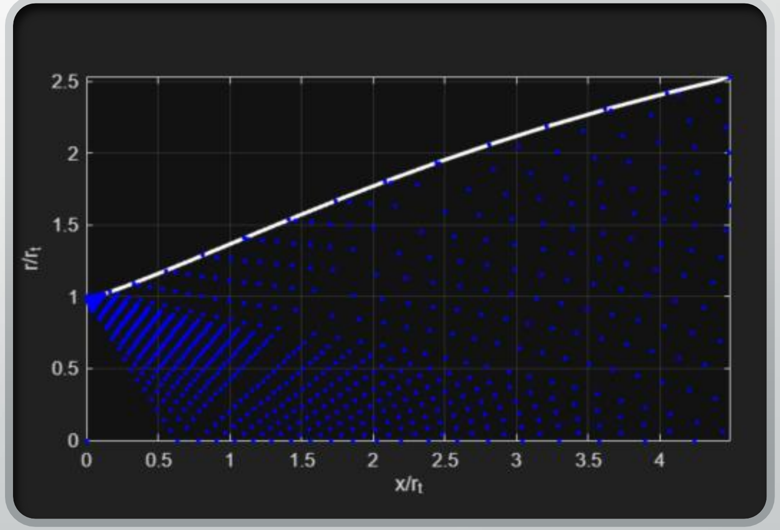
Points within the field may be solved via compatibility eqs

$$\frac{dr}{dx}_{car} = \tan(\theta \mp \mu)$$

$$d(\theta - \nu) = -\frac{1}{\sqrt{M^2 - 1} + \cot(\theta)} \frac{dr}{r} \quad \text{along } C_+$$

$$d(\theta + \nu) = \frac{1}{\sqrt{M^2 - 1} - \cot(\theta)} \frac{dr}{r} \quad \text{along } C_-$$





Improving Method of Characteristics

Traditional implementation of MoC codes generally rely uniform throat conditions conditions (axial flow at M=1)

Boundary conditions may be made more accurate by referencing transonic throat solutions

1. Correction 1: Throat Efficiency Factor η^*
   - Real throat flows are 2 dimensional and vary in mach number
   - This creates choking effect – reducing nozzle flow
   - Efficiency factor used to correct for 2D throat flows
2. Sauer's Solution
   - Analytic solution for mach number and velocity components within proximity to sonic line

Information from transonic flow may be used to inform flow at wall using a predictor-corrector approach

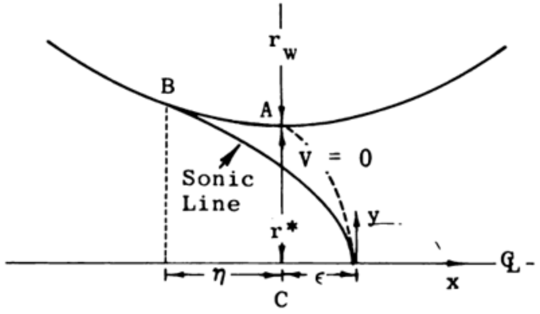
Methodology nets 1-2 s Isp gains

$$\eta^* = \frac{1}{\gamma-3}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left(4\frac{r_w}{r^*}-1\right)\left\{\left[1+\frac{\gamma-1}{2}\left(\frac{4\frac{r_w}{r^*}}{4\frac{r_w}{r^*}-1}\right)^2\right]^{\frac{\gamma-3}{2(\gamma-1)}} - \left[1+\left(\frac{\gamma-1}{2}\right)\left(\frac{4\frac{r_w}{r^*}-2}{4\frac{r_w}{r^*}-1}\right)^2\right]^{\frac{\gamma-3}{2(\gamma-1)}}\right\} \tag{6-38}$$

$$\alpha = \sqrt{\frac{2}{(k+1)\rho_S y_S}}$$

$$\rho_K = \sqrt{\frac{2\rho_S y_S}{k+1}} \tag{2C(b)}$$

$$\eta = \epsilon = \frac{y_S}{8}\sqrt{2(k+1)\frac{y_S}{\rho_S}}$$



Truncated Ideal Contour Optimization

Truncated Ideal Contour (TIC) nozzles select a greater mach number than what is needed, and truncate once exit conditions are met

- Specifying a length fraction relative to a 15 deg cone nozzle sets constant mass/Surface area constraint
- Losses from over-expansion create upper bound on length fraction
- For any given length fraction there is a mach number that maximizes thrust coefficient

Objective: Maximize C_F

Design Variables:

- Mach number of infinitely expanded nozzle
- Length Faction

Optimization Problem:

$$\max C_F(M_\infty, f)$$

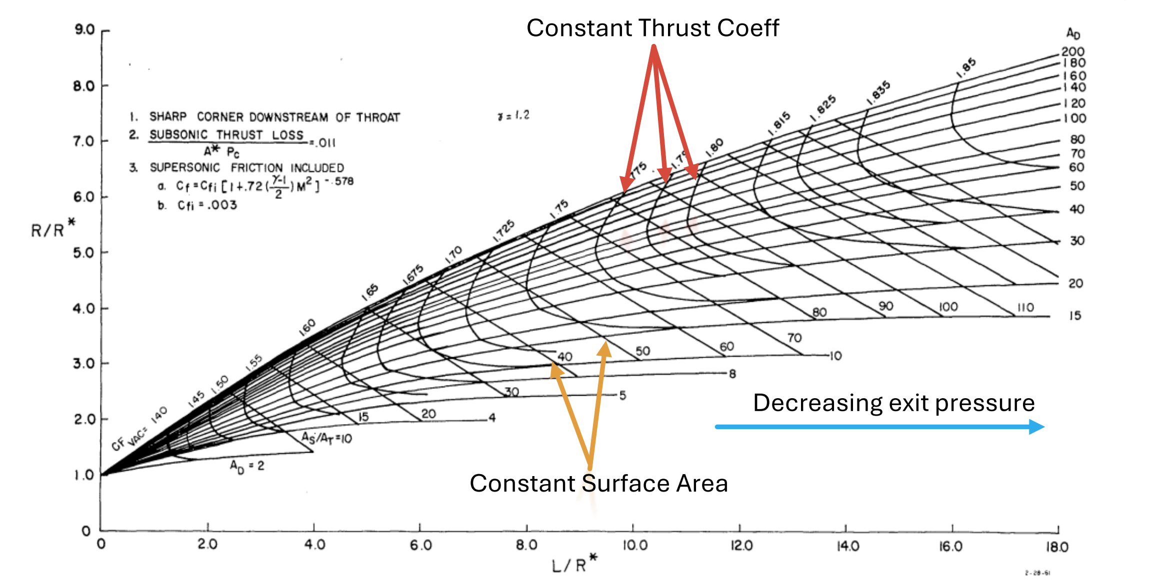


Fig. 1 Perfect nozzle shapes

Regenerative Cooling Analysis

Method of Characteristics may be used to define M at all points along the wall

This defines Pressure, temperature, and all related heat transfer parameters into nozzle wall

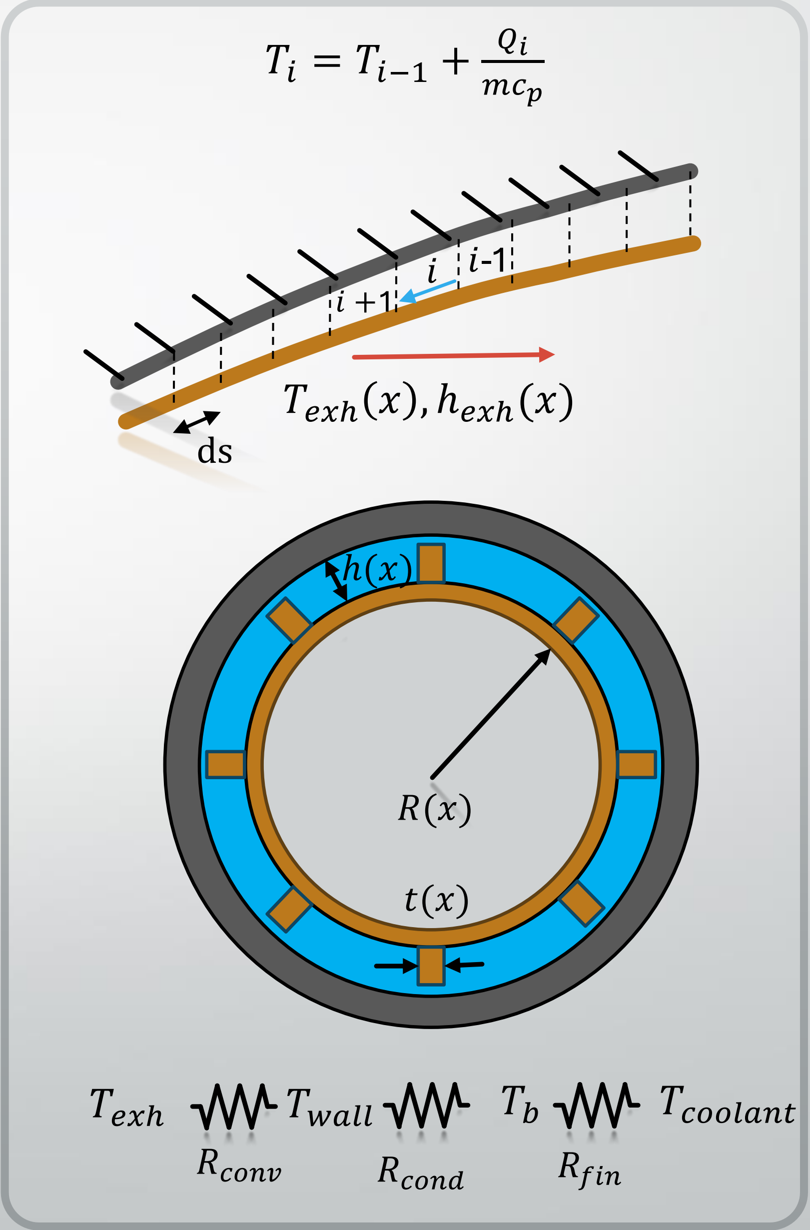
- Heat Transfer calculated using series resistance model with fins approximating channel walls
- Since $h(x) \ll R(x)$ a Leveque approximation holds, and the fins are treated as rectilinear
- Jacket is discretized with heat transfer, flow rate, and pressure drop calculated at all points

$$m = \sqrt{2h_{conv}/kt}$$

$$L_c = h + t/2$$

$$\eta_{fin} = \frac{\tanh(mL_c)}{mL_c}$$

$$R_{fin} = \frac{1}{h_{conv}(A_{base} + \eta_{fin}A_{fin})}$$



Regenerative Cooling Optimization

Water Cooling with 1.5 kg/s

Objective: Minimize Pressure losses in cooling jacket

Design Variables:

- Channel Height
- Channel Wall (fin) Thickness

Design variables specified at nozzle inlet, throat, and exit, and interpolated via natural spline

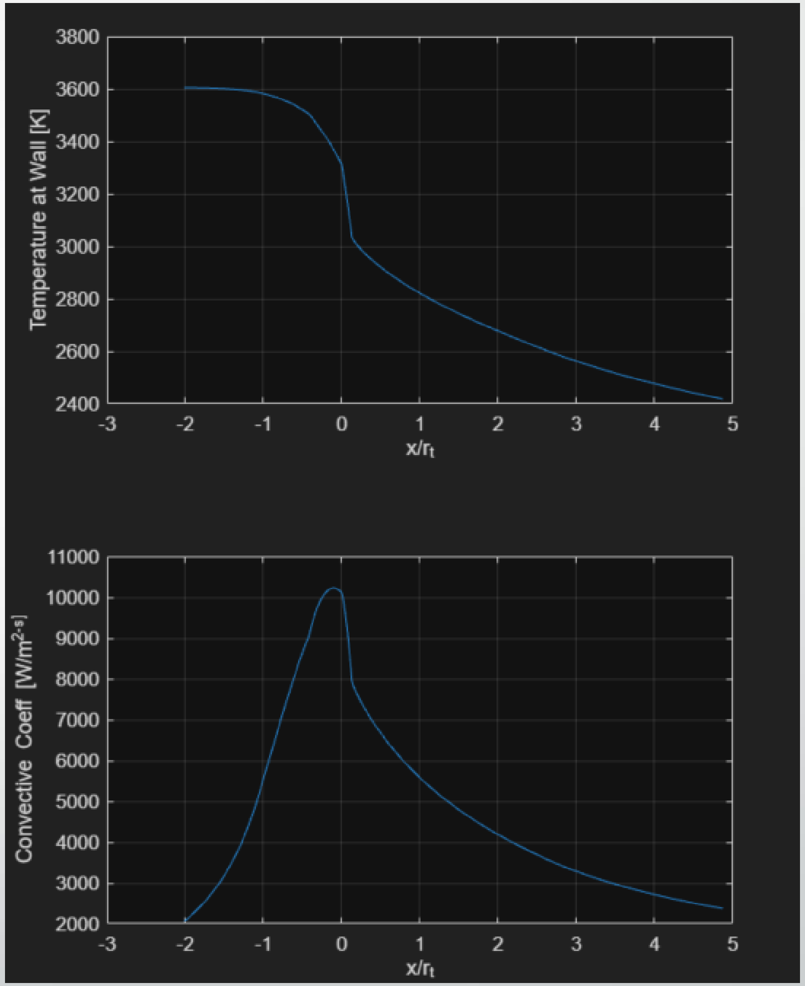
Constraints:

- Minimum channel size governed by end mill diameter
- All points on the interior nozzle wall must be below 550 K

Optimization Problem

$$\min \Delta P\big(h(x), t(x)\big)$$

$$\text{s.t.} \quad T_{wall} < 550\ K\ \forall x, \qquad \frac{\pi R(x) - nt(x)}{n} > d_{tool}$$

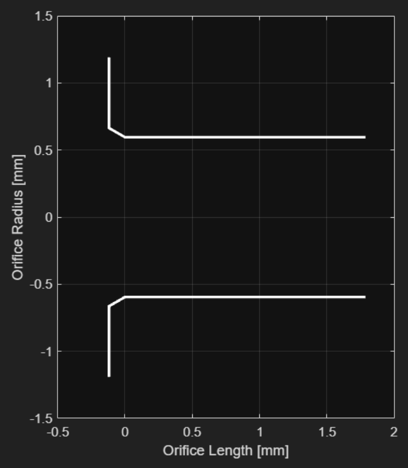


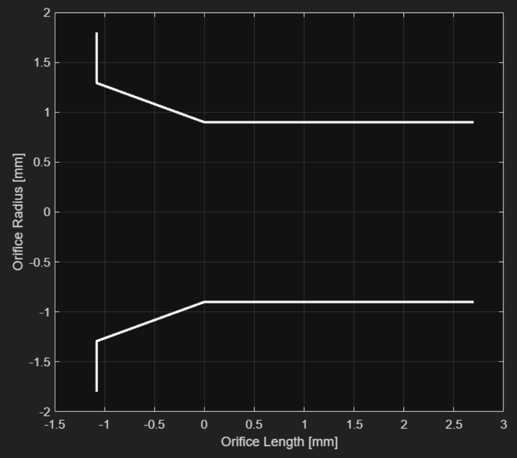
Solution Discussion

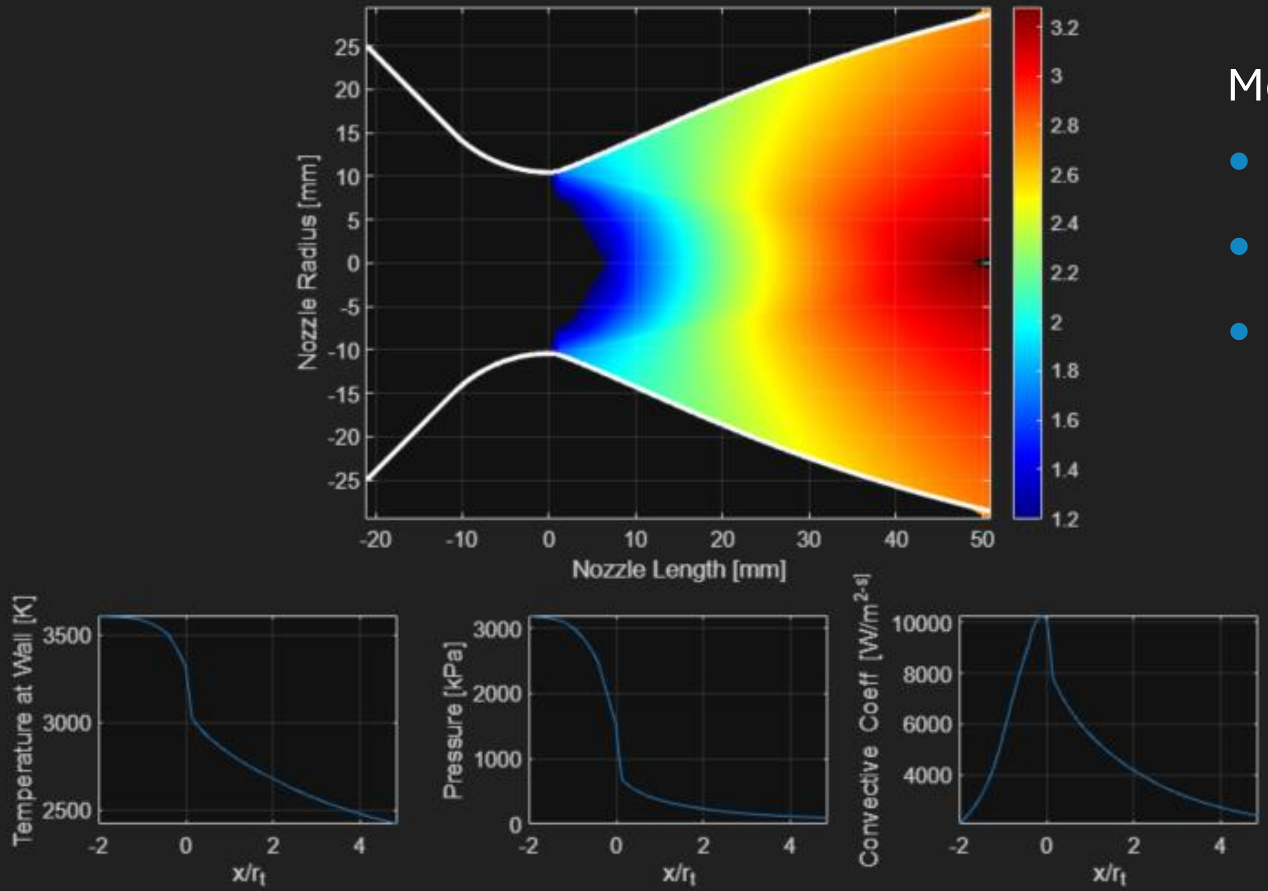
Injector Optimization

- Optimizer favored smaller, more numerous orifice in smaller liquid bi
- As OD was kicked out (for HRE variant) fewer, larger orifice were preferable
- Inlet Angle likely converged to exact angles
  - Due to linear interpolation scheme
  - Done to ensure manufacturable solution

Design Spec	2” OD Liquid Bi-Prop	4” OD HRE
Orifice Diameter	1.19 mm (3/64”)	1.80 mm
Number of orifici	12	5
Inlet Angle	60 deg	40 deg
Inlet length	.12 mm	1.08 mm
Orifice Length	1.79 mm	2.70 mm
Discharge Coeff	.916	.943
Injection Radius	1.42 cm	3.27 cm
Injection Velocity	30.41 m/s	31.3 m/s
Mean Droplet Diam	59 µm	192 µm
Chamber Length	10.4 cm	N/A







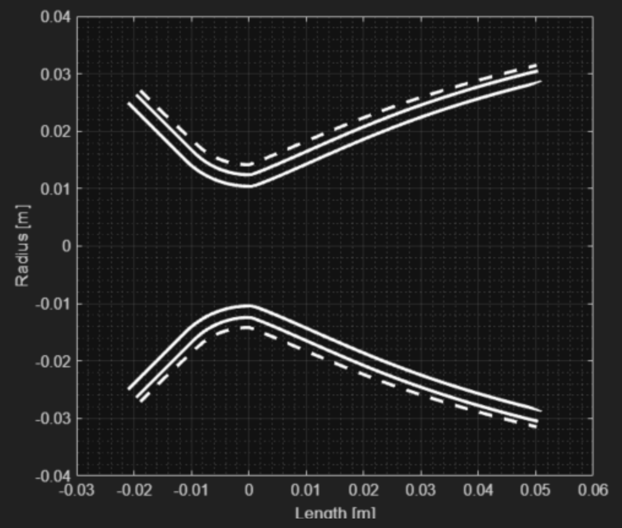
Solution Discussion

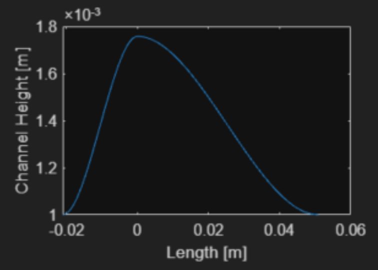
Method of Characteristics

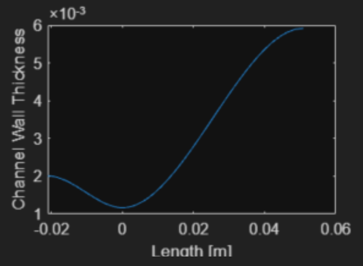
- Thrust Coefficient: 1.58
- Area Ratio: 7.51
- Peak heat transfer coeff: 10 kW/m^2-K

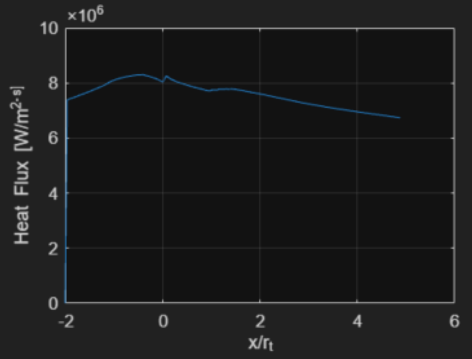
Solution Discussion

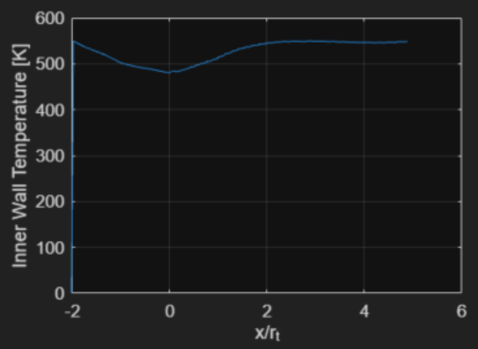
Regenerative Jacket Optimization

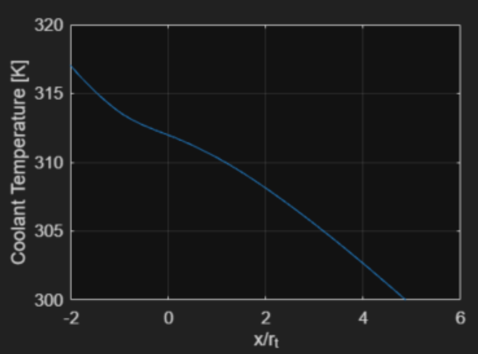


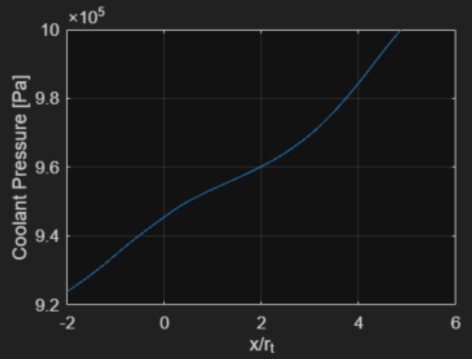












- Pressure drop: 11.8 psi
- Design converged to 18 channels
- Peak velocity: 15 m/s
- Reverse-Expander-cycle problem
  - High surface area relative to coolant flow rates

Questions?

References:

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