

Lista de Exercícios - Damilo Carvalho / 8ºano

Questão 1-

$$u_{ij} = \begin{cases} i+j, & i < j \\ 0, & i=j \\ j-i, & i > j \end{cases}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 3 & 4 \\ -1 & 2 & 5 \\ -2 & -1 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{10} \begin{pmatrix} -4 & -5 & 2 \\ -11 & -2 & 5 \\ 7 & 5 & -1 \end{pmatrix} \quad x = \frac{1}{10} \begin{pmatrix} -4 & -5 & 2 \\ -11 & -2 & 5 \\ 7 & 5 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$$

$$x = \frac{1}{10} \begin{pmatrix} -20 \\ -30 \\ 20 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$$

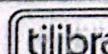
Questão 2 - a)

$$A^{-1} = \frac{1}{-5} \begin{pmatrix} 3 & -4 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -0,6 & 0,8 \\ 0,4 & -0,2 \end{pmatrix}$$

$$x = A^{-1} \cdot B \quad \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad x \Rightarrow \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -0,6 & 0,8 \\ 0,4 & -0,2 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$b) A \begin{pmatrix} 3 & 5 \\ 8 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 10 & 5 \end{pmatrix} \quad AY = B \quad Y = A^{-1} B$$



1 / 1

$$A^{-1} = \frac{1}{-2s} \begin{pmatrix} s & -s \\ -s & 3 \end{pmatrix} = \frac{1}{-2s} \begin{pmatrix} -0,2 & 0,2 \\ 0,32 & -0,12 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0,2 & 0,18 \\ 0,08 & 0,12 \end{pmatrix}$$

c) triangular inferior

$$W = [w_1, w_2, w_3]^T$$

$$w_1 = 5 \quad w_2 = 7 \quad w_3 = 2$$

$$w_1 = 5 \quad 2(5) - w_2 = 7 \Rightarrow 10 - w_2 = 7 \Rightarrow w_2 = 3$$

$$w = \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix}$$

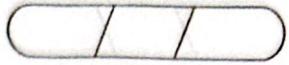
3) a) $AXB = C \Rightarrow AX = CB^{-1}$
 $X = A^{-1}CB^{-1}$

b) $A(B+X) = A$
 $A^{-1}A(B+X) = A^{-1}A = B+X = I \Rightarrow X = I - B$
 $X = I - B$

c) $ACXB = C$

$$ACX = CB^{-1} \Rightarrow X = (AC)^{-1}CB^{-1} \Rightarrow X = C^{-1}A^{-1}(B^{-1})$$

tilibra



d) $(AB)^{-1}(AX) = CC^{-1}$

$$CC^{-1} = I \quad (AB)^{-1}AX = I \quad \Rightarrow AX = AB \Rightarrow X = A^T AB = B$$

$$X = B$$

e) $AB^T X B^{-1} = A^T$

$$AB^T X = A^T B \Rightarrow B^T X = A^{-1} A^T B \Rightarrow X = (B^T)^{-1} A^{-1} A^T B$$

$$X = (B^T)^{-1} A^{-1} A^T B$$

f) $2AX - x = 3B$

$$(2A - I)x = 3B \Rightarrow x = (2A - I)^{-1} 3B$$

Questão 4)

u) $\begin{cases} 3x - 4y = 1 \\ 2x + 6y = 18 \end{cases} \quad S = \{3, 1\}$

$$D = \begin{vmatrix} 3 & -4 \\ 2 & 6 \end{vmatrix} \Rightarrow 18 + 8 = 26$$

$$x = \frac{D_x}{D} \Rightarrow \frac{78 - (3)}{26} = \frac{75}{26} \quad D_x = \begin{vmatrix} 1 & -4 \\ 18 & 6 \end{vmatrix} \Rightarrow 6 + 12 = 18$$

$$y = \frac{D_y}{D} \Rightarrow \frac{26}{26} = 1 \quad D_y = \begin{vmatrix} 3 & -4 \\ 2 & 6 \end{vmatrix} \Rightarrow 18 + 8 = 26$$

/ /

b) $\begin{cases} 5x + 8y = 34 \\ 10x + 16y = 50 \end{cases}$ $D = \begin{vmatrix} 5 & 8 \\ 10 & 16 \end{vmatrix} = 80 - 80 = 0$

mão é penivel $D_x = \begin{vmatrix} 34 & 8 \\ 50 & 16 \end{vmatrix} =$
verdadeiro

$D_y = \begin{vmatrix} 5 & 34 \\ 10 & 50 \end{vmatrix} =$

c) $\begin{cases} x + 2y = 5 \\ 2x - 3y = -4 \end{cases}$ $D = \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = -3 - 4 = -7$

$S = \{1, 2\}$ $x = D_x = -7 = 1$ $D_x = \begin{vmatrix} 5 & 2 \\ -4 & -3 \end{vmatrix} = -15 + 8 = -7$
 $D = -7$

$y = \frac{D_y}{D} = \frac{-14}{-7} = 2$ $D_y = \begin{vmatrix} 1 & 5 \\ 2 & -4 \end{vmatrix} = -4 - 10 = -14$

d) $\begin{cases} 3x + 2y - 5z = 8 \\ 2x - 4y - 2z = -4 \\ x - 2y - 3z = -4 \end{cases}$ $D = \begin{vmatrix} 3 & 2 & -5 \\ 2 & -4 & -2 \\ 1 & -2 & -3 \end{vmatrix} = 3 \cdot \begin{vmatrix} -4 & -2 \\ -2 & -3 \end{vmatrix} + (-2) \cdot \begin{vmatrix} 2 & -5 \\ 1 & -3 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & -5 \\ -4 & -2 \end{vmatrix}$

$$D_x = \begin{vmatrix} 8 & 2 & -5 \\ -4 & -4 & -2 \\ -4 & -2 & -3 \end{vmatrix} = -2 \cdot \begin{vmatrix} 1 & -3 \\ -2 & -3 \end{vmatrix} + (-2) \cdot \begin{vmatrix} 2 & -5 \\ -4 & -2 \end{vmatrix} + (-5) \cdot \begin{vmatrix} 2 & -5 \\ -4 & -2 \end{vmatrix}$$
$$D = 3 \cdot (12 - 4) + (-2) \cdot (-6 + 2) + (-5) \cdot (-4 + 4)$$
$$D = 24 + 8 = 32$$

$$8 \cdot \begin{vmatrix} -4 & -2 \\ -2 & -3 \end{vmatrix} + (-2) \cdot \begin{vmatrix} -4 & -2 \\ -4 & -3 \end{vmatrix} + (-5) \cdot \begin{vmatrix} -4 & -4 \\ -4 & -2 \end{vmatrix} \Rightarrow D = 8 \cdot (12 - 4) + (-2) \cdot (12 - 8) + (-5) \cdot (8 - 16) \Rightarrow 64 - 8 + 40$$

tilibra

$D_x = 96$



$$D_y = \begin{vmatrix} 3 & 8 & -5 \\ 2 & -4 & -2 \\ 1 & -4 & -3 \end{vmatrix} = 3 \cdot \begin{vmatrix} -4 & -2 \\ -4 & -3 \end{vmatrix} + (-8) \cdot \begin{vmatrix} 2 & -2 \\ 1 & -3 \end{vmatrix} + (-5) \cdot \begin{vmatrix} 2 & -4 \\ 1 & -4 \end{vmatrix}$$

$$d = 3 \cdot (12 - 8) + (-8) \cdot (-6 + 2) + (-5) \cdot (-8 + 4)$$

$$d = 12 + 32 + 20 \Rightarrow 64$$

$$D_x = \begin{vmatrix} 3 & 2 & 8 \\ 2 & -4 & -4 \\ 1 & -2 & -4 \end{vmatrix} = 3 \cdot \begin{vmatrix} -4 & -4 \\ -2 & -4 \end{vmatrix} + (-2) \cdot \begin{vmatrix} 2 & -4 \\ 1 & -4 \end{vmatrix} + 8 \cdot \begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix}$$

$$d = 3 \cdot (16 - 8) + (-2) \cdot (-8 + 4) + 8 \cdot (-4 + 4)$$

$$d = 24 + 8 \Rightarrow 32$$

$$\frac{x = D_x}{D} = \frac{96}{32} \Rightarrow 3 \quad y = \frac{D_y}{D} = \frac{64}{32} \Rightarrow 2 \quad z = \frac{32}{32} \Rightarrow 1$$

$$S = \{3, 2, 1\} //$$

$$1) \begin{cases} x + 2y - z = 2 \\ 2x - y + 3z = 9 \\ 3x + 3y - 2z = 3 \end{cases} \quad D = \begin{vmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 3 & -2 \end{vmatrix} = 1 \cdot \begin{vmatrix} 3 & -2 \\ 3 & -2 \end{vmatrix} + (-2) \cdot \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} + (-1) \cdot \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix}$$

$$d = -7 + 2 \cdot (-4 - 9) + 1 \cdot (6 + 3)$$

$$= -7 + 26 - 9 \Rightarrow 10$$

$$D_x = \begin{vmatrix} 2 & 2 & -1 \\ 9 & -1 & 3 \\ 3 & 3 & -2 \end{vmatrix} = 2 \cdot \begin{vmatrix} -1 & 3 \\ 3 & -2 \end{vmatrix} + (-2) \cdot \begin{vmatrix} 9 & 3 \\ 3 & -2 \end{vmatrix} + (-1) \cdot \begin{vmatrix} 9 & -1 \\ 3 & 3 \end{vmatrix}$$

$$d = 2 \cdot (2 - 9) + (-2) \cdot (-18 - 9) + (-1) \cdot (27 + 3)$$

$$d = -14 + 54 + -30 \Rightarrow 10$$

1 1 1

$$d_1 = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 4 & 3 \\ 3 & 3 & -2 \end{vmatrix} = 1 \begin{vmatrix} 4 & 3 \\ 3 & -2 \end{vmatrix} + (-2) \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} + (-1) \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix}$$

$$d = (-18 - 9) + (-2) \cdot (-4 - 9) + (-1) \cdot (6 - 27)$$

$$d = -27 + 26 + 21 \Rightarrow 20$$

$$d_2 = \begin{vmatrix} 1 & 2 & 2 \\ 2 & -1 & 9 \\ 3 & 3 & 3 \end{vmatrix} = 1 \cdot \begin{vmatrix} -1 & 9 \\ 3 & 3 \end{vmatrix} + (-2) \cdot \begin{vmatrix} 2 & 9 \\ 3 & 3 \end{vmatrix} + (-2) \cdot \begin{vmatrix} 2 & -1 \\ 3 & 3 \end{vmatrix}$$

$$d = (-3 - 27) + (-2) \cdot (6 - 27) + (-2) \cdot (6 + 3)$$

$$d = -30 + 42 - 18 \Rightarrow -6$$

$$x = \frac{D_x}{D} = \frac{10}{10} \Rightarrow 1 \quad y = \frac{D_y}{D} = \frac{20}{10} \Rightarrow 2 \quad z = \frac{D_z}{D} = \frac{-6}{10} \Rightarrow -0,6$$

$$S = \{1, 2, -0,6\}$$

$$b) \begin{cases} x + 3z + 6 = -2 \\ 2x - 4y = -4 \\ 3x - 2y - 5z = 26 \end{cases} \Rightarrow \begin{cases} x + 0y + 3z = -8 \\ 2x - 4y + 0z = -4 \\ 3x - 2y - 5z = 26 \end{cases}$$

$$D = \begin{vmatrix} 1 & 0 & 3 \\ 2 & -4 & 0 \\ 3 & -2 & -5 \end{vmatrix} = 1 \cdot \begin{vmatrix} -4 & 0 \\ -2 & -5 \end{vmatrix} + 0 \cdot \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix}$$

$$d = 20 + 3 \cdot (-4 + 12) \Rightarrow 44$$



$$D_x = \begin{vmatrix} -2 & 0 & 1 \\ -4 & -4 & 0 \\ 26 & -2 & -5 \end{vmatrix} = -2 \cdot \begin{vmatrix} -4 & 0 \\ -2 & -5 \end{vmatrix} + 0 + 1 \cdot \begin{vmatrix} -4 & -4 \\ 26 & -2 \end{vmatrix}$$

$$d = -2 \cdot (20) + (8 + 104)$$

$$d = -40 + 112 \Rightarrow 72$$

$$D_y = \begin{vmatrix} 1 & -2 & 1 \\ 2 & -4 & 0 \\ 3 & 26 & -5 \end{vmatrix} = 1 \cdot \begin{vmatrix} -4 & 0 \\ 26 & -5 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & 1 \\ 3 & -5 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & -4 \\ 3 & 26 \end{vmatrix}$$

$$d = 20 + 20 + 64 \Rightarrow 104$$

$$D_z = \begin{vmatrix} 1 & 0 & -2 \\ 2 & -4 & -4 \\ 3 & -2 & 26 \end{vmatrix} = 1 \cdot \begin{vmatrix} -4 & -4 \\ -2 & 26 \end{vmatrix} + 0 + (-2) \cdot \begin{vmatrix} 2 & -4 \\ 3 & -2 \end{vmatrix}$$

$$d = (-104 - 8) + (-2) \cdot (-4 + 12)$$

$$d = -112 - 16 \Rightarrow -128$$

$$x = \frac{176}{44} \Rightarrow 4 \quad y = \frac{132}{44} \Rightarrow 3 \quad z = \frac{-48}{44} \Rightarrow -\frac{12}{11}$$

$$S = \left\{ 4, 3, -\frac{12}{11} \right\}$$

5) a) SPI, por uma equação é múltiplo da outra

b) SPI, a segunda equação é o dobro da primeira

c) SPD

6) a) $\begin{cases} 3x + my = 2 \\ x - y = 1 \end{cases} \Rightarrow D = \begin{vmatrix} 3 & m \\ 1 & -1 \end{vmatrix} \Rightarrow -3 - m \neq 0 \\ m \neq -3$

b) $\begin{cases} 3x + 2(m-1)y = 1 \\ mx - 4y = 0 \end{cases} \Rightarrow D = \begin{vmatrix} 3 & 2(m-1) \\ m & -4 \end{vmatrix} \Rightarrow 3(-4) - 2m(m-1) \\ \Rightarrow -12 - 2m^2 + 2m \neq 0 \\ \Rightarrow \Delta = (-2)^2 - 4(2)(12) \\ = 4 - 96 = -92 < 0$

c) $\begin{cases} x - y = 2 \\ x + my = -3 \\ -x + y - z = 4 \end{cases} \Rightarrow \begin{cases} x - y + 0z = 2 \\ x + my + z = 0 \\ -x + y - z = 4 \end{cases} \Rightarrow \begin{vmatrix} 1 & -1 & 0 \\ 1 & m & 1 \\ -1 & 1 & -1 \end{vmatrix}$

$D = 1 \cdot \begin{vmatrix} m & 1 \\ 1 & -1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix} + 0 \Rightarrow 1 \cdot (-m-1) + 1 \cdot (-1+1) \neq 0 \\ -m-1+0 \neq 0 \\ m \neq -1$

d) $\begin{cases} mx + y - z = 4 \\ x + my + z = 0 \\ x - y = 2 \end{cases} \Rightarrow D = \begin{vmatrix} m & 1 & -1 \\ 1 & m & 1 \\ 1 & -1 & 0 \end{vmatrix} = m \cdot \begin{vmatrix} m & 1 \\ -1 & 0 \end{vmatrix} + (-1) \cdot \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$

$$m \neq -1 \quad -1 \cdot 1 \cdot m = m \cdot (1) + 1 + (-1) \cdot (-1-m)$$

$$1 \cdot 1 \cdot -1 = m + 1 + 1 + m \neq 0 \\ 2m \neq -2 \Rightarrow m \neq \frac{-2}{2} \Rightarrow$$

tilibra

$$7) \begin{cases} 6x + 2y = 750 \\ x + y = 225 \end{cases} \rightarrow x = 225 - y \Rightarrow x = 225 - 150$$

x = peças corretas

$$6(225 - y) 2y = 750$$

y = peças incorretas

$$1350 - 6y + 2y = 750$$

$$-4y = 750 - 1350$$

$$x = 75$$

$$+y = \frac{-600}{-4} \Rightarrow y = 150$$

$$y = 150$$

$$8) \begin{cases} 0,6x + 0,2y = 300 \\ x + y = 540 \end{cases}$$

$$x = 540 - y \quad x = 540 - 60 \Rightarrow x = 480$$

$$x = -210$$

$$x = carro \quad x = 480$$

$$0,6(540 - y) + 0,2y = 300$$

$$y = moto \quad y = 60$$

$$324 - 0,6y + 0,2y = 300$$

$$m = 60 \text{ cm}$$

$$-0,4y = 300 - 324 \Rightarrow -24$$

$$m = 30 \text{ cm}$$

$$y = \frac{-24}{-0,4} \Rightarrow 60 \quad y = 60$$

$$9) \begin{cases} 2x + 5y + 10z = 500 \\ x + y + z = 42 \end{cases}$$

$$2x + 5y + 10x = 500$$

$$x + y + z = 42$$

$$12x + 5y = 500$$

$$x = 3y + 42 - z$$

$$x + y + z = 42$$

$$5(2x + y) = 5 \cdot 42 \Rightarrow 10x + 5y = 210 \quad 2x + y = 42$$

$$= 460$$

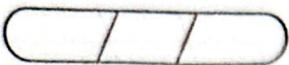
$$x = 20$$

$$z = 20$$

$$y = 52$$

$$(12x + 5y) - (10x + 5y) = 500 - 460 \Rightarrow 2x = 40 \Rightarrow x = 20$$

$$2x + y = 42 \Rightarrow 2(20) + y = 42 \Rightarrow y = 52$$



10)

11

$$x = \text{kiba} = 77$$

$$x + y = 109$$

$$y = \text{Akamaru} = 32$$

$$z = \text{tamaku} = 65$$

$$x + z = 142$$

$$y + z = 97$$

$$348$$

$$2x + 2y + 2z = 348 \Rightarrow x + y + z = 174$$

$$x + y = 109$$

$$z = 174 - 109 \Rightarrow 65$$

$$x + z = 142 \Rightarrow x + 65 = 142 \Rightarrow x = 77$$

$$x + y = 109 \Rightarrow 77 + y = 109 \Rightarrow y = 32$$