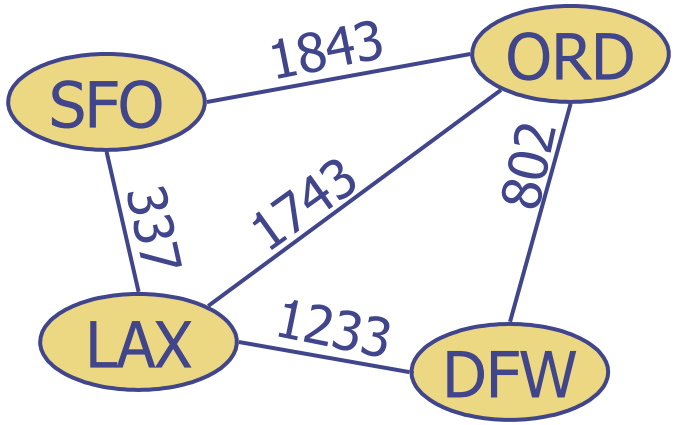
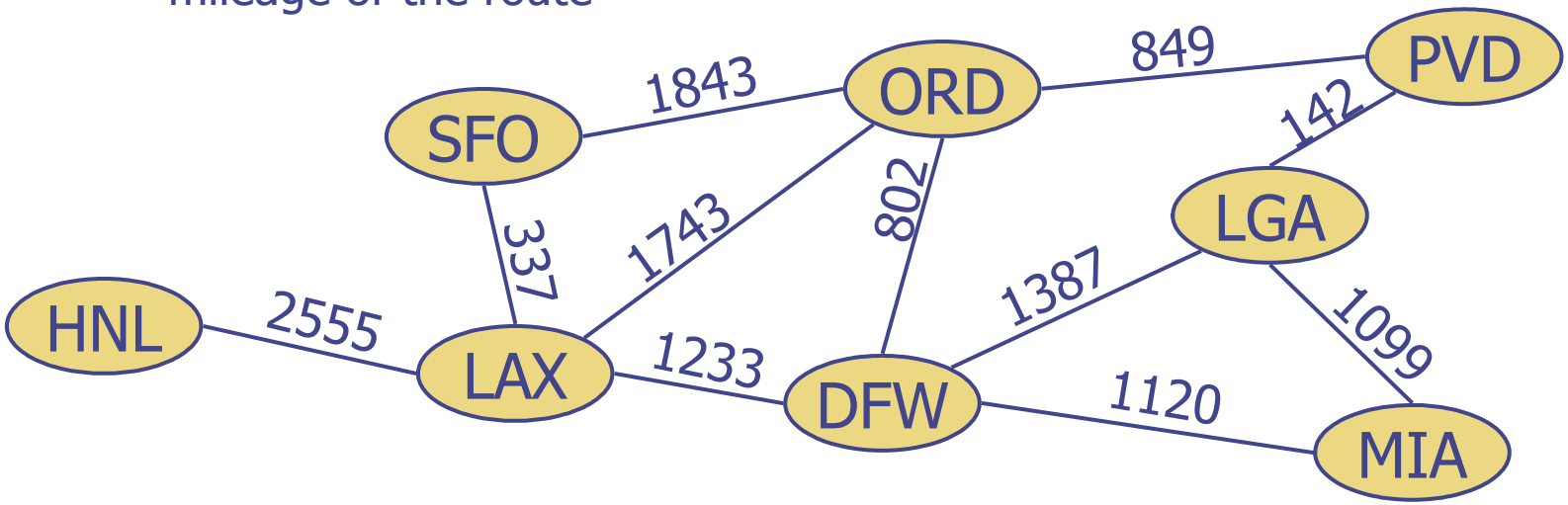
Presentation for use with the textbook Data Structures and Algorithms in Java, 6th edition, by M. T. Goodrich, R. Tamassia, and M. H. Goldwasser, Wiley, 2014

# Graphs



# Graphs

- A graph is a pair $(V, E)$, where
  - $V$ is a set of nodes, called vertices
  - $E$ is a collection of pairs of vertices, called edges
  - Vertices and edges are positions and store elements
- Example:
  - A vertex represents an airport and stores the three-letter airport code
  - An edge represents a flight route between two airports and stores the mileage of the route

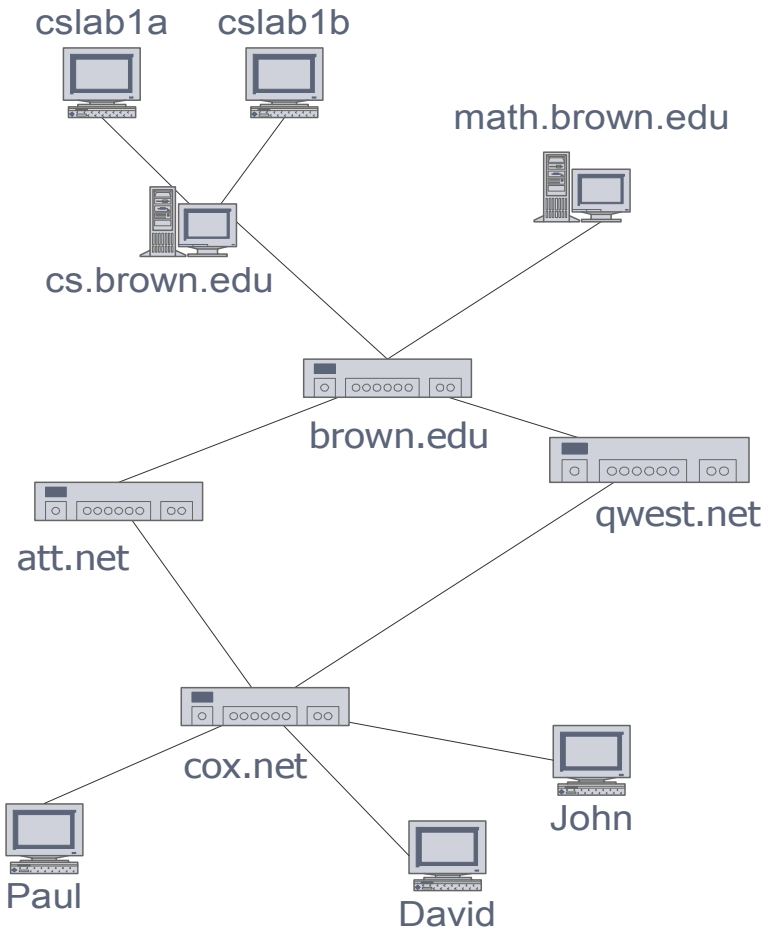


# Edge Types

- Directed edge
  - ordered pair of vertices $(u,v)$
  - first vertex $u$ is the origin
  - second vertex $v$ is the destination
  - e.g., a flight
- Undirected edge
  - unordered pair of vertices $(u,v)$
  - e.g., a flight route
- Directed graph
  - all the edges are directed
  - e.g., route network
- Undirected graph
  - all the edges are undirected
  - e.g., flight network

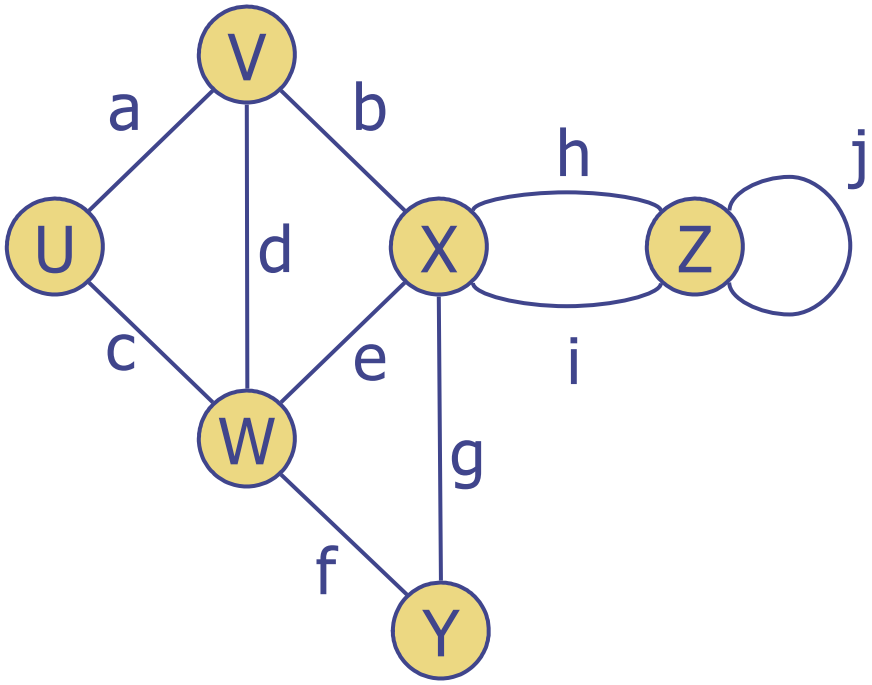

# Applications

- Electronic circuits
  - Printed circuit board
  - Integrated circuit
- Transportation networks
  - Highway network
  - Flight network
- Computer networks
  - Local area network
  - Internet
  - Web
- Databases
  - Entity-relationship diagram



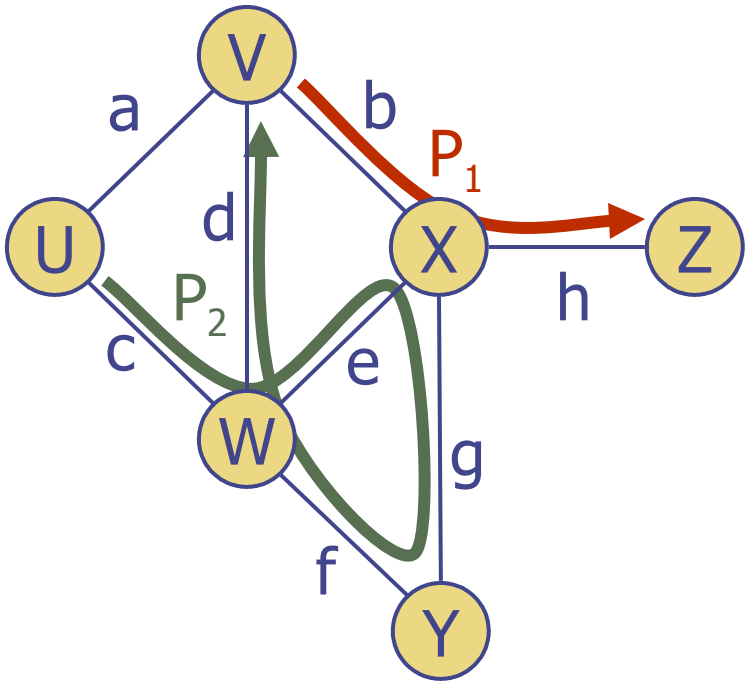
# Terminology

- End vertices (or endpoints) of an edge
  - U and V are the endpoints of a
- Edges incident on a vertex
  - a, d, and b are incident on V
- Adjacent vertices
  - U and V are adjacent
- Degree of a vertex
  - X has degree 5
- Parallel edges
  - h and i are parallel edges
- Self-loop
  - j is a self-loop



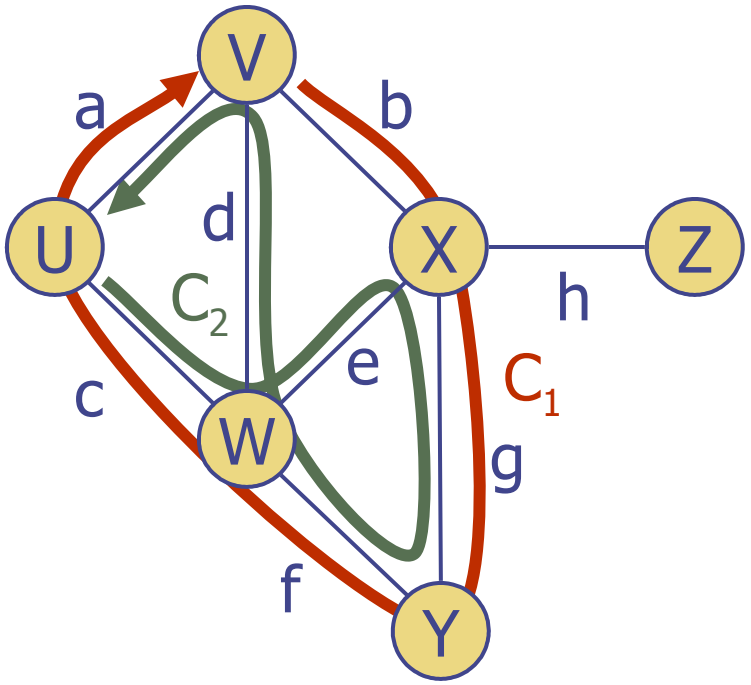
# Terminology (cont.)

- Path
  - sequence of alternating vertices and edges
  - begins with a vertex
  - ends with a vertex
  - each edge is preceded and followed by its endpoints
- Simple path
  - path such that all its vertices and edges are distinct
- Examples
  - $P_1$=(V,b,X,h,Z) is a simple path
  - $P_2$=(U,c,W,e,X,g,Y,f,W,d,V) is a path that is not simple



# Terminology (cont.)

- Cycle
  - circular sequence of alternating vertices and edges
  - each edge is preceded and followed by its endpoints
- Simple cycle
  - cycle such that all its vertices and edges are distinct
- Examples
  - $C_1$=(V,b,X,g,Y,f,W,c,U,a,↵) is a simple cycle
  - $C_2$=(U,c,W,e,X,g,Y,f,W,d,V,a,↵) is a cycle that is not simple



# Properties

## Property 1

$\sum_v \deg(v) = 2m$

Proof: each edge is counted twice

## Property 2

In an undirected graph with no self-loops and no multiple edges

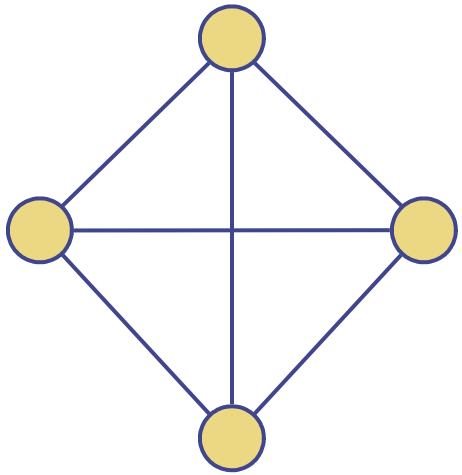
$m \leq n\,(n-1)/2$

Proof: each vertex has degree at most $(n-1)$

## What is the bound for a directed graph?

## Notation

| | |
|---|---|
| $n$ | number of vertices |
| $m$ | number of edges |
| $\deg(v)$ | degree of vertex $v$ |



## Example

- $n = 4$
- $m = 6$
- $\deg(v) = 3$

# Vertices and Edges

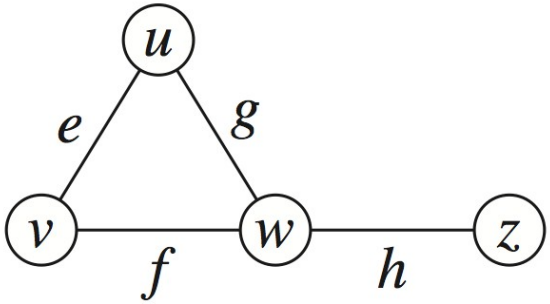
- A **graph** is a collection of **vertices** and **edges**.
- We model the abstraction as a combination of three data types: Vertex, Edge, and Graph.
- A **Vertex** is a lightweight object that stores an arbitrary element provided by the user (e.g., an airport code)
  - We assume it supports a method, element(), to retrieve the stored element.
- An **Edge** stores an associated object (e.g., a flight number, travel distance, cost), retrieved with the element( ) method.

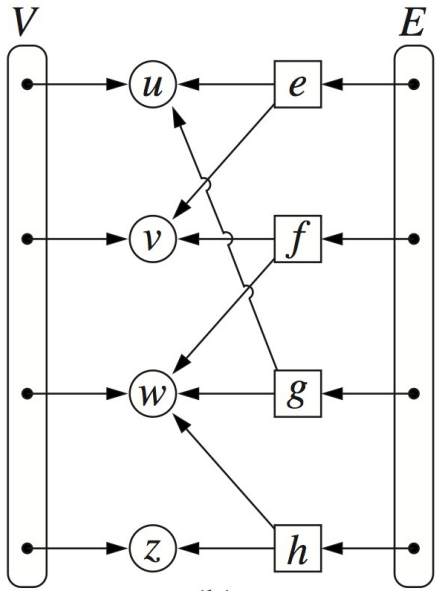
# Graph ADT

| | |
|---|---|
| numVertices( ): | Returns the number of vertices of the graph. |
| vertices( ): | Returns an iteration of all the vertices of the graph. |
| numEdges( ): | Returns the number of edges of the graph. |
| edges( ): | Returns an iteration of all the edges of the graph. |
| getEdge($u$, $v$): | Returns the edge from vertex $u$ to vertex $v$, if one exists; otherwise return null. For an undirected graph, there is no difference between getEdge($u$, $v$) and getEdge($v$, $u$). |
| endVertices($e$): | Returns an array containing the two endpoint vertices of edge $e$. If the graph is directed, the first vertex is the origin and the second is the destination. |
| opposite($v$, $e$): | For edge $e$ incident to vertex $v$, returns the other vertex of the edge; an error occurs if $e$ is not incident to $v$. |
| outDegree($v$): | Returns the number of outgoing edges from vertex $v$. |
| inDegree($v$): | Returns the number of incoming edges to vertex $v$. For an undirected graph, this returns the same value as does outDegree($v$). |
| outgoingEdges($v$): | Returns an iteration of all outgoing edges from vertex $v$. |
| incomingEdges($v$): | Returns an iteration of all incoming edges to vertex $v$. For an undirected graph, this returns the same collection as does outgoingEdges($v$). |
| insertVertex($x$): | Creates and returns a new Vertex storing element $x$. |
| insertEdge($u$, $v$, $x$): | Creates and returns a new Edge from vertex $u$ to vertex $v$, storing element $x$; an error occurs if there already exists an edge from $u$ to $v$. |
| removeVertex($v$): | Removes vertex $v$ and all its incident edges from the graph. |
| removeEdge($e$): | Removes edge $e$ from the graph. |

# Edge List Structure

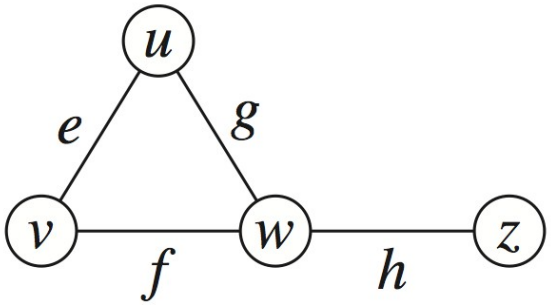
- Vertex object
  - element
  - reference to position in vertex sequence
- Edge object
  - element
  - origin vertex object
  - destination vertex object
  - reference to position in edge sequence
- Vertex sequence
  - sequence of vertex objects
- Edge sequence
  - sequence of edge objects

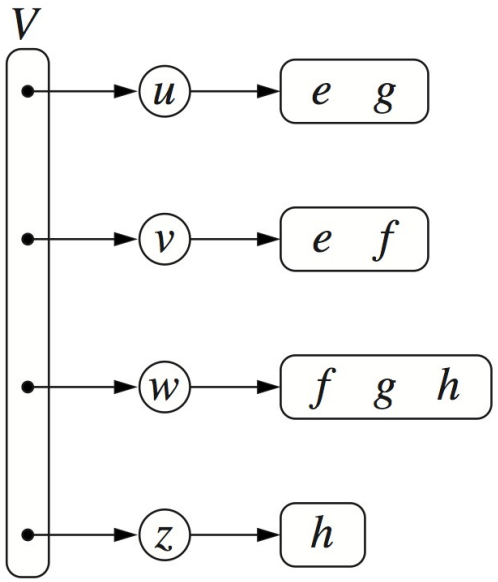


# Adjacency List Structure

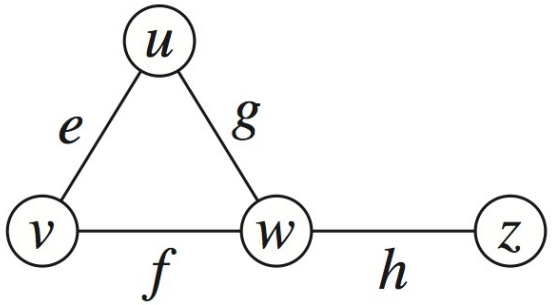
- Incidence sequence for each vertex
  - sequence of references to edge objects of incident edges
- Augmented edge objects
  - references to associated positions in incidence sequences of end vertices



# Adjacency Matrix Structure

- Edge list structure
- Augmented vertex objects
  - Integer key (index) associated with vertex
- 2D-array adjacency array
  - Reference to edge object for adjacent vertices
  - Null for non nonadjacent vertices
- The "old fashioned" version just has 0 for no edge and 1 for edge




| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| u → 0 | | e | g | |
| v → 1 | e | | f | |
| w → 2 | g | f | | h |
| z → 3 | | | h | |

# Performance

| ▪ $n$ vertices, $m$ edges<br>▪ no parallel edges<br>▪ no self-loops | Edge List | Adjacency List | Adjacency Matrix |
|---|---|---|---|
| Space | $n + m$ | $n + m$ | $n^2$ |
| incidentEdges($v$) | $m$ | $\deg(v)$ | $n$ |
| areAdjacent ($v$, $w$) | $m$ | $\min(\deg(v), \deg(w))$ | 1 |
| insertVertex($o$) | 1 | 1 | $n^2$ |
| insertEdge($v$, $w$, $o$) | 1 | 1 | 1 |
| removeVertex($v$) | $m$ | $\deg(v)$ | $n^2$ |
| removeEdge($e$) | 1 | 1 | 1 |