CS 4442b Assignment 1

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Question 1

A)
P(Water = warm | Play = yes) = 2/3
P(Water = warm | Play = no) = 1
B)
P(Play = yes | Water = warm) = 2/3
P(Play = no | Water = warm) = 1/3
C)
P(Play = yes | Forcast = same) = 1
P(Play = yes | Forcast = change) = 1/2
D) With Laplace smoothing
P(Water = warm | Play = yes) = 3/5
P(Water = warm | Play = no) = 2/3

Question 2

Part A

$$k(x, z) = \beta k_1(x, z) - \alpha k_2(x, z)$$

This kernel is symmetric since both k_1 and k_2 are valid kernels and symmetric themselves. Then

$$\begin{aligned} k(z, x) &= \beta k_1(x, z) - \alpha k_2(x, z) \\ &= \beta k_1(z, x) - \alpha k_2(z, x) \\ &= \beta k_1(x, z) - \alpha k_2(x, z) = k(x, z) \end{aligned}$$

However this violates Mercer's theorem for the matrix K being positive and semidefinite whenever $\alpha \geq \beta$ and $k_2(x,z) < k_1(x,z)$

Part B

$k(x,z) = k_1(x,z)k_2(x,z)$

This kernel is symmetric.

$$\begin{aligned} k(z,x) &= k_1(z,x)k_2(z,x) \\ &= k_1(x,z)k_2(x,z) = k(x,z) \end{aligned}$$

Since both the other kernels are symmetric.

Finally this kernel is positive semidefinite, since both k_1 and k_2 are positive semidefinite, then k will also be positive semidefinite for all values.

Part C

$$k(x,z) = f_1(x)f_1(z) + f_2(x)f_2(z)$$

Let $\phi(x) = [f_1(x), f_2(x)]^T$ then

$$\begin{aligned} k(x,z) &= \phi^T(x)\phi(z) \\ &= [f_1(x), f_2(x)] \begin{bmatrix} f_1(z) \\ f_2(z) \end{bmatrix} \\ &= f_1(x)f_1(z) + f_2(x)f_2(z) \end{aligned}$$

By defining the kernel as above, it is easy to see that it is symmetric

$$\begin{aligned} k(z,x) &= \phi^T(z)\phi(x) \\ &= [f_1(z), f_2(z)] \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} \\ &= f_1(x)f_1(z) + f_2(x)f_2(z) = k(x,z) \end{aligned}$$

For any vector $a = [a_1, \ldots, a_n]^T \epsilon R^n$ and $\phi(x)$ We have

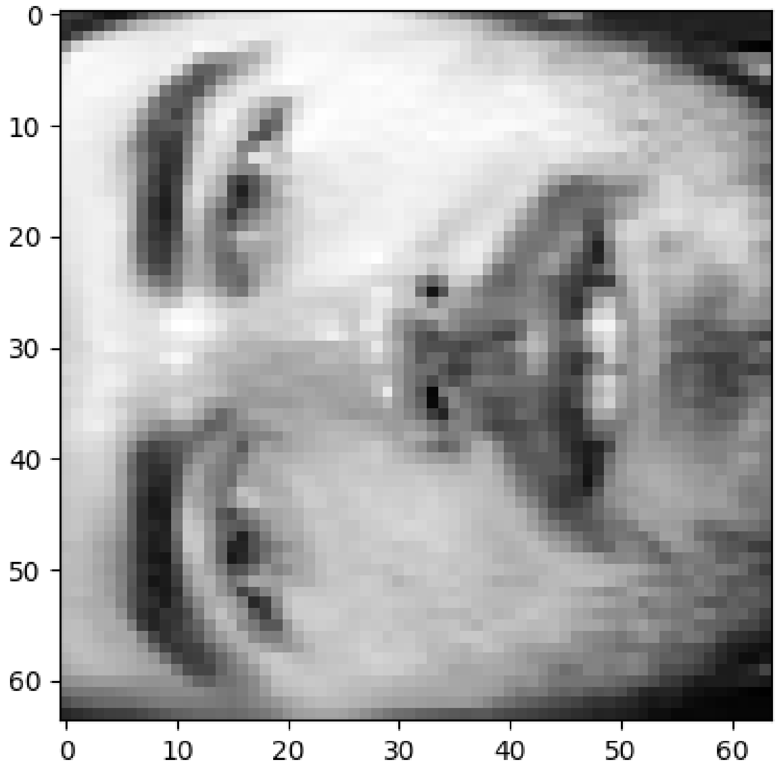
$$\begin{aligned} aKa &= \Sigma_{i=1}^m \Sigma_{j=1}^m a_i K_i j a_j \\ &= \Sigma_{i=1}^m \Sigma_{j=1}^m a_i (\phi(x_i)\phi(x_j)) a_j \\ &= (\Sigma_{i=1}^m a_i \phi(x_i))^2 \geq 0 \end{aligned}$$

Therefore since the kernel is both symmetric and positive semidefinite, this is a valid kernel.

Question 3

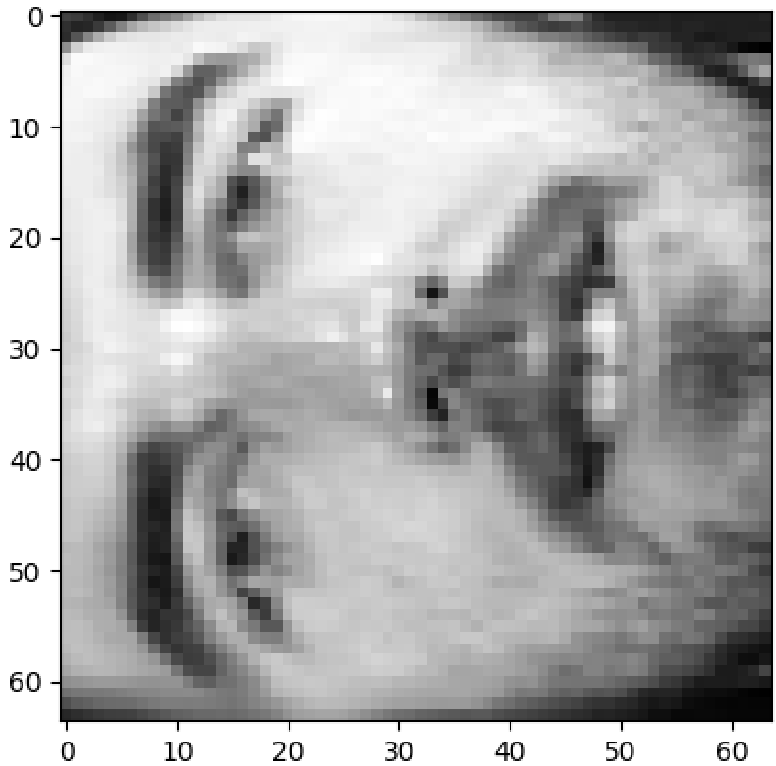
Part A

The 100-th image



Part B

After removing the mean ($m \approx 132.384...$)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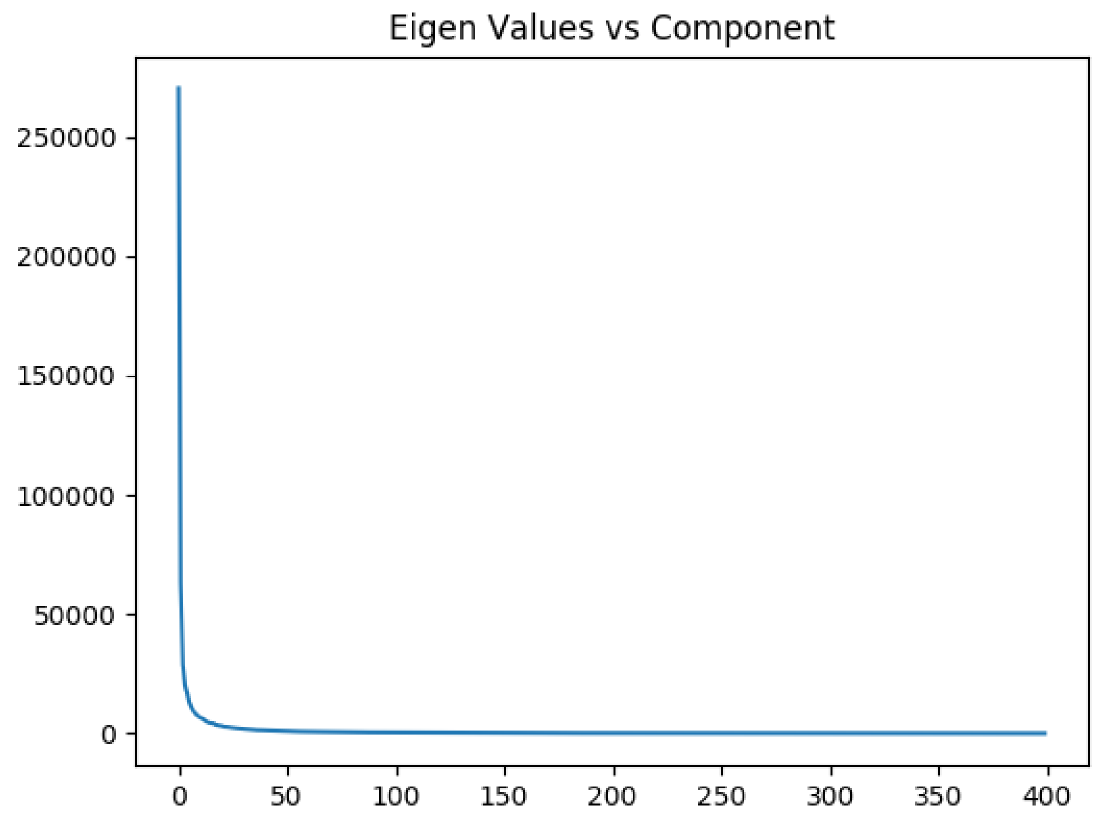
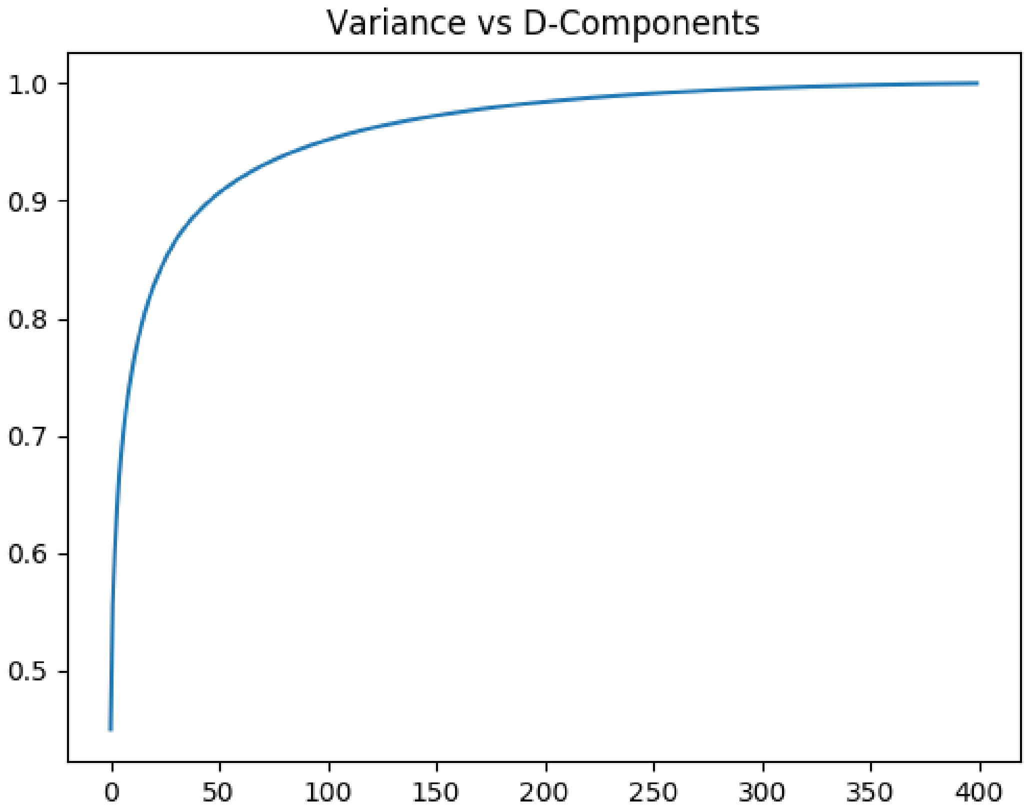
Part C



Part D

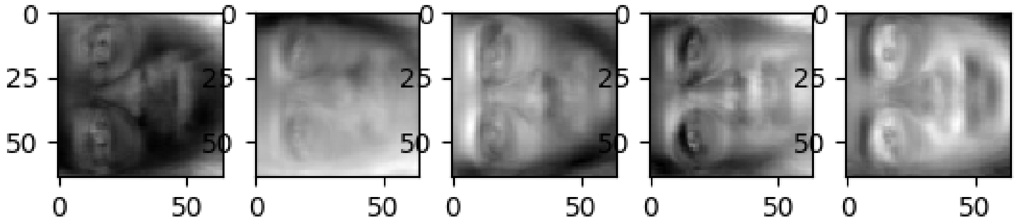
The eigen value computed for the 400-th component is $1.8307756442497e-23 \approx 0$. Note the reason it is not zero exactly is due to either numpy using n-1 in its eigen value decomposition or due to floating point arithmetic percision. However, ignoring that fact, the reason that the n-th eigen value is 0, is due to the nature of the covariance matrix used for the components. When the k-th component is equal to the size of the set, the only possible eigen value is then 0, since there are no more dimensions that could be seperated from the n dimension data set. This is the point at which all previous variance has been accounted for.

Part E

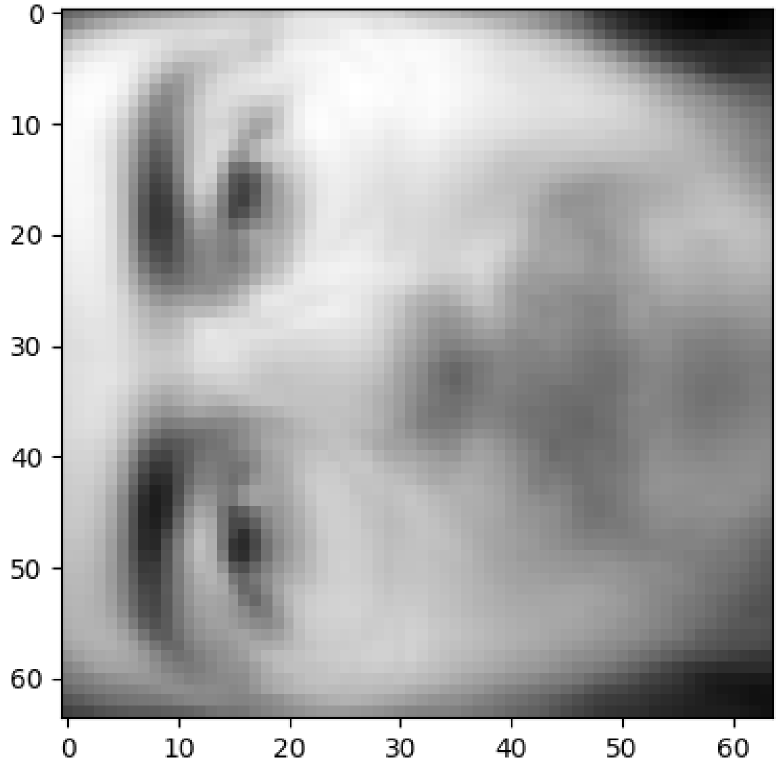


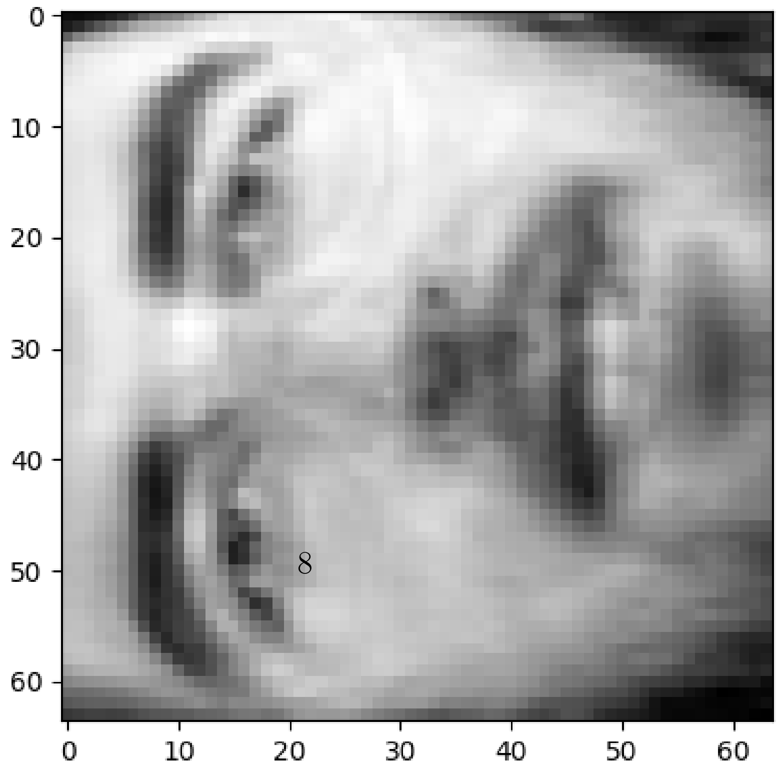
To select the dimensionality of the data you would select a principle component which selects for most of the variance. From above any x value greater than 200 would suffice.

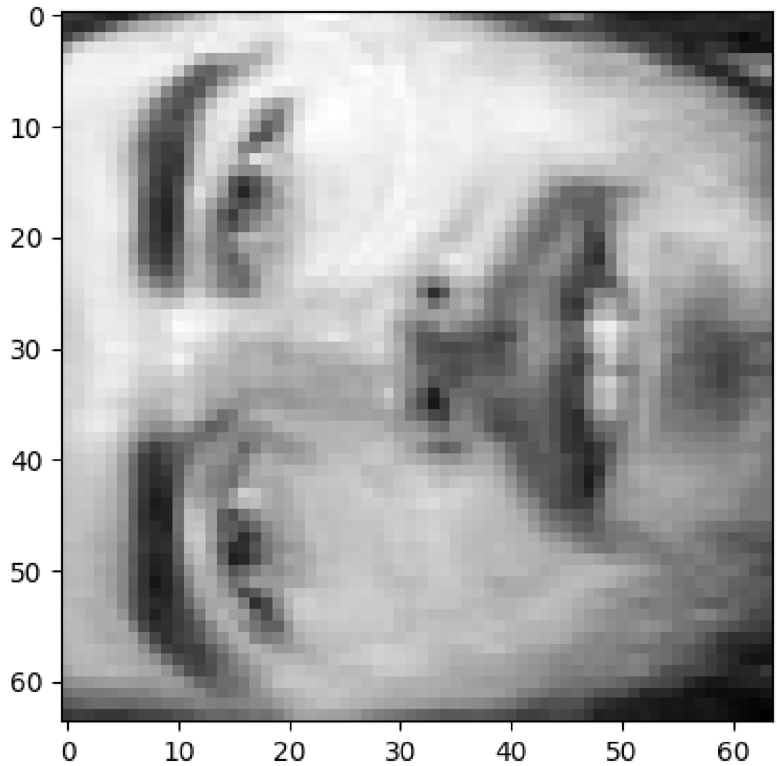
Part F



Part G







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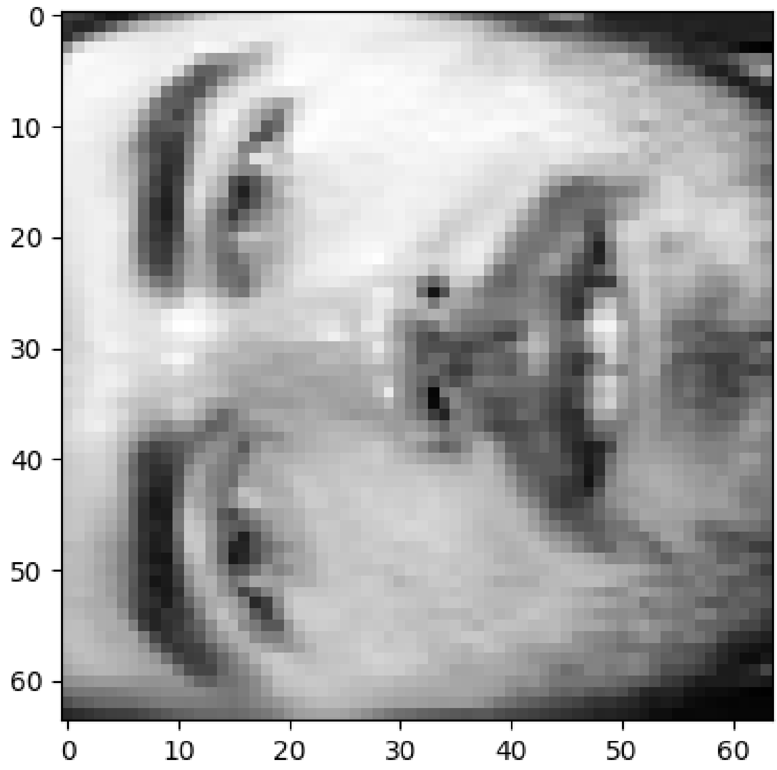

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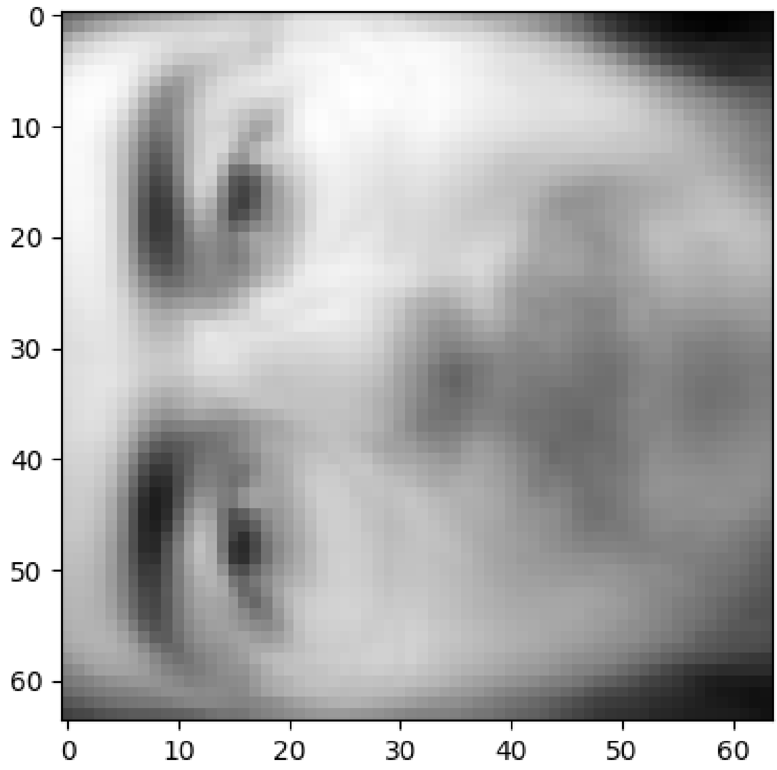
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