Solution for Homework Assignment 1 (Kinematics)
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Problem 1 - Fun with Trapezoids.

a) In order to plot the acceleration and displacement with respect to time (Figures 1,2), let us first calculate them at three given time intervals $[0, t_1]$, $[t_1, t_2]$, $[t_2, t_3]$, where $t_1 = 4s$, $t_2 = 10s$, $t_3 = 18s$.

Acceleration:

$t \in [0, 4]$: $v_0 = 0\ m/s$, $v = 4\ m/s$, $a = \frac{v - v_0}{\Delta t} = \frac{4}{4} = 1\ m/s^2$

$t \in [4, 10]$: $v_0 = 4\ m/s$, $v = 4\ m/s$, $a = \frac{v - v_0}{\Delta t} = \frac{0}{6} = 0\ m/s^2$

$t \in [10, 18]$: $v_0 = 4\ m/s$, $v = 0\ m/s$, $a = \frac{v - v_0}{\Delta t} = \frac{-4}{8} = -0.5\ m/s^2$

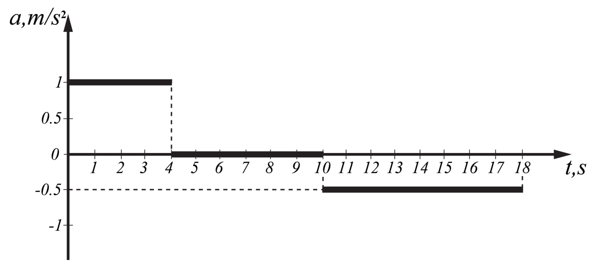

Figure 1: Acceleration with respect to time.

Displacement:

$t \in [0, 4]$: $v_0 = 0\ m/s$, $a = 1\ m/s^2$, $x(\Delta t) = x_0 + v_0 \Delta t + \frac{a \Delta t^2}{2} = 0 + 0 \cdot 4 + \frac{1 \cdot 16}{2} = 8m$ ($a > 0m/s^2 \Rightarrow$ parabola is opening to the top)

$t \in [4, 10]$: $v_0 = 4\ m/s$, $a = 0\ m/s^2$, $x(\Delta t) = x_0 + v_0 \Delta t + \frac{a \Delta t^2}{2} = 8 + 4 \cdot 6 + \frac{0 \cdot 36}{2} = 32m$ ($a = 0m/s^2 \Rightarrow$ line graph)

$t \in [10, 18]$: $v_0 = 4\ m/s$, $a = -0.5\ m/s^2$, $x(\Delta t) = x_{t=18} = x_0 + v_0 \Delta t + \frac{a \Delta t^2}{2} = 32 + 4 \cdot 8 - \frac{0.5 \cdot 64}{2} = 48m$ ($a < 0m/s^2 \Rightarrow$ parabola is opening to the bottom)

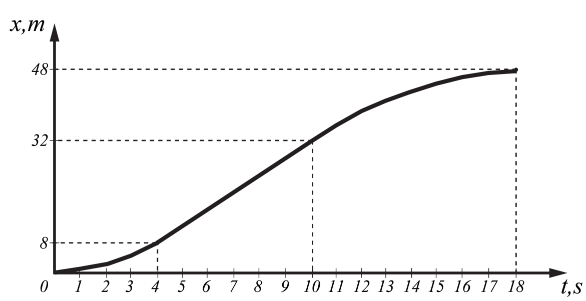

Figure 2: Displacement with respect to time.

b) The displacement over time interval $[0, 18]$ is $\Delta x = x_{t=18} - x_{t=0} = 48 - 0 = 48m$. ($x_{t=18}$ calculated in item a)
The average velocity over time interval $[0, 18]$ can be determined as follows: $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{48}{18} \approx 2.667\ m/s$
<u>Answer</u>: $\Delta x = 48m$, $v_{avg} \approx 2.667\ m/s$.

Problem 2 - Collision Course.
Given an aircraft with velocity $v = 1300\ km/h$ that flies at height $h = 35m$ above initially level ground. Then, at time $t = 0$, the aircraft starts to fly over ground sloping upward at angle $\theta = 4.3°$. Our goal is to determine what time t does the plane strike the ground.

Since movement of aircraft is uniform, the time can be determined as follows: $t = \frac{\Delta x}{v}$, where Δx is the displacement of the aircraft before the collision.
The aircraft will collide with the ground when the ground level will be equal to the altitude of the aircraft h. In fact, we have rectangular triangle (see Figure 3) with angle θ and the opposite leg h. The adjacent leg Δx can be determined using *tangens* definition.

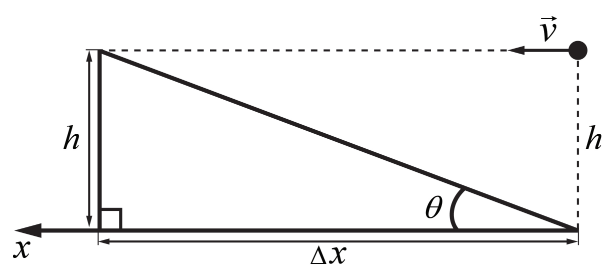

Figure 3: Collision Course.

Apparently, $tan\theta = \frac{h}{\Delta x} \Rightarrow \Delta x = \frac{h}{tan\theta}$.
Furthermore, substitute obtained equation in formula of time: $t = \frac{h}{vtan\theta}$.
Convert speed to m/s: $v = 1300\ km/h \approx 361.111\ m/s$.
Ultimately, $t = \frac{h}{vtan\theta} = \frac{35}{361.111tan4.3°} \approx 1.289s$.
<u>Answer</u>: $t \approx 1.289s$.

Problem 3 - Projectile Range .
Given a motion of a projectile described by a set of equations:
(1) $R = (v_0\ \cos\theta_0)t$
(2) $0 = (v_0\ \sin\theta_0)t - \frac{gt^2}{2}$
Our goal is to show that $R = \frac{v_0^2}{g}\ \sin 2\theta_0$.

Primarily, let us take t out of brackets in (2): $0 = t(v_0 \sin\theta_0 - \frac{gt}{2})$
Indeed, the solution of this equation is either $t = 0$ or $v_0 \sin\theta_0 - \frac{gt}{2} = 0$.
Since we are not interested in the initial moment (the projectile did not move to reach its initial height), we will use the second solution. Let us express t from the obtained equation: $t = \frac{2v_0 sin\theta_0}{g}$.
Substitute the derived formula for time in (1): $R = (v_0 \cos\theta_0)\frac{2v_0 sin\theta_0}{g} = \frac{v_0^2 2sin\theta_0 cos\theta_0}{g}$.
Further, using double angle formula: $sin2\alpha = 2sin\alpha cos\alpha$, we get $\frac{v_0^2 2sin\theta_0 cos\theta_0}{g} = \frac{v_0^2}{g} sin2\theta_0$.
Accordingly, we have shown that $R = \frac{v_0^2}{g} \sin 2\theta_0$ for $t \neq 0$.

Problem 4 - Ball Speed.
By definition a ball is thrown at an angle $\theta = 0°$(horizontally), so the initial velocity projection on the x-axis $v_{ix} = v_i cos\theta = v_i$ and the initial velocity projection on the y-axis $v_{iy} = v_i sin\theta = 0m/s$.
Since the acceleration in the horizontal direction was zero, the initial velocity projection on the x-axis v_{ix} is equal to the final velocity projection on the x-axis v_{fx}. Consequently, $v_i = v_{ix} = v_{fx}$.
Final velocity projection on the y-axis: $v_{fy} = v_{fy0} - gt = v_{iy} - gt = 0 - gt = -gt$.
Time until the ball hits the ground can de determined from the equation $y(t) = y_{fy0} + v_{fy0}t - \frac{gt^2}{2}$. In particular, $v_{fy0} = v_{iy} = 0m/s$ and $y_{fy0} = y_0$. Therefore, $y(t) = y_0 - \frac{gt^2}{2}$ and we can conclude that $t = \sqrt{\frac{2(y_0 - y(t))}{g}}$.
Let us substitute obtained formula for time in v_{fy}: $v_{fy} = -gt = -g\sqrt{\frac{2(y_0-y(t))}{g}} = -\sqrt{2g(y_0 - y(t))}$.
By definition final velocity $v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$.
Based on the fact that $v_f = 3v_i$ we get $3v_i = \sqrt{v_{fx}^2 + v_{fy}^2}$.
Squaring both parts of the equation: $9v_i^2 = v_{fx}^2 + v_{fy}^2$
We have derived $v_{fx} = v_i$ and $v_{fy} = -\sqrt{2g(y_0 - y(t))}$, so substitute them:
$9v_i^2 = v_i^2 + (-\sqrt{2g(y_0 - y(t))})^2$
$8v_i^2 = 2g(y_0 - y(t))$
$v_i = \sqrt{\frac{2g(y_0-y(t))}{8}}$
$v_i = \sqrt{\frac{2\cdot 9.8(20-0)}{8}} = 7m/s$
Answer: $v_i = 7m/s$.