Brief review of kinematic formulas for one-dimensional motion

- Average vectors of velocity and acceleration of a point:

$$\langle v \rangle = \frac{\Delta x}{\Delta t} \;, \qquad \langle a \rangle = \frac{\Delta v}{\Delta t} \;,$$

where $\Delta \mathbf{r}$ is the displacement vector (an increment of a radius vector).

- Velocity and acceleration of a point:

$$v = \frac{dx}{dt} \;, \qquad a = \frac{dv}{dt}$$

- Distance covered by a point:

$$s = \int v \, dt,$$

where v is the *modulus* of the velocity vector of a point.

More accurate consideration

$$S(T) = Si + \int_0^T v(t)\,dt$$ The time dependent position (or displacement)

$$v(T) = Vi + \int_0^T a(t)\,dt$$ The time dependent velocity, a(t) is an arbitrary acceleration

$$S(T) = Si + \int_0^T v(t)\,dt = Si + \int_0^T \left(Vi + \int_0^t a(h)\,dh\right) dt = \boxed{Si + \int_0^T Vi\,dt + \int_0^T \left(\int_0^t a(h)\,dh\right) dt}$$

More general expression for S(T), where:

Si – initial position;

Vi – initial velocity;

T – current time;

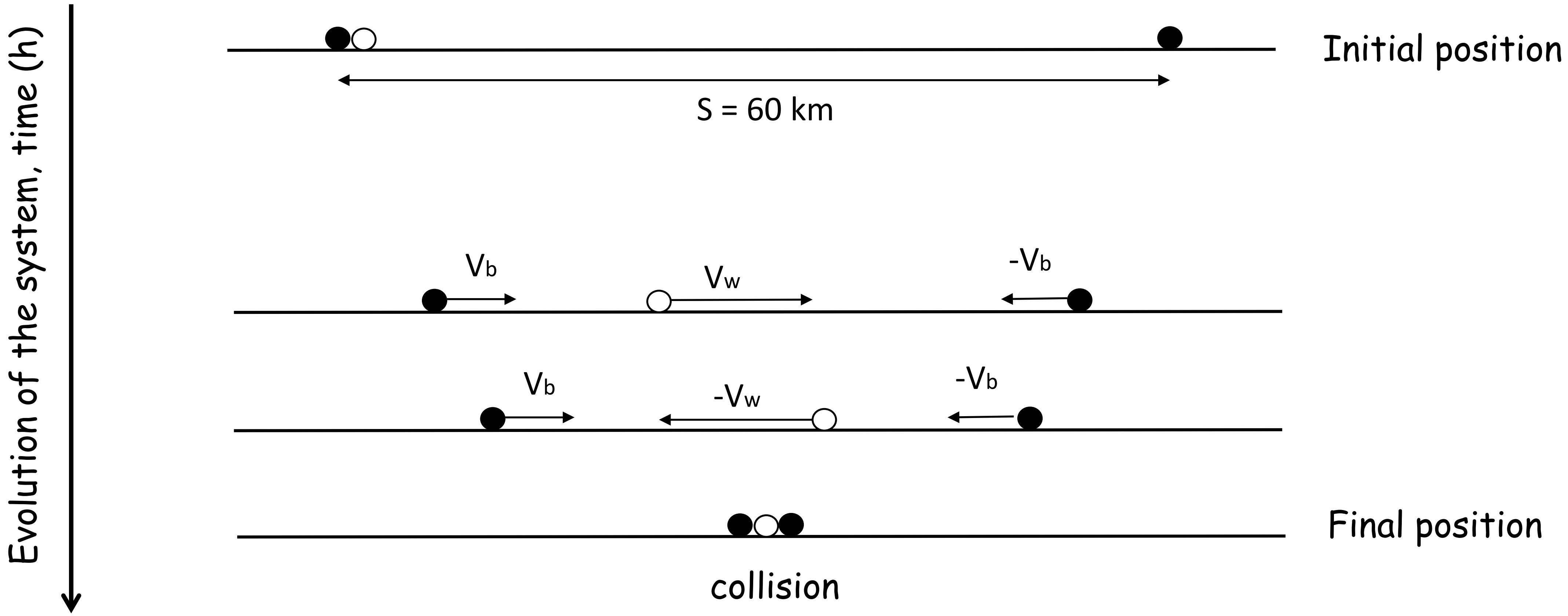
t and h – time variables;

When the acceleration is constant, we get the famous expression for S(T)

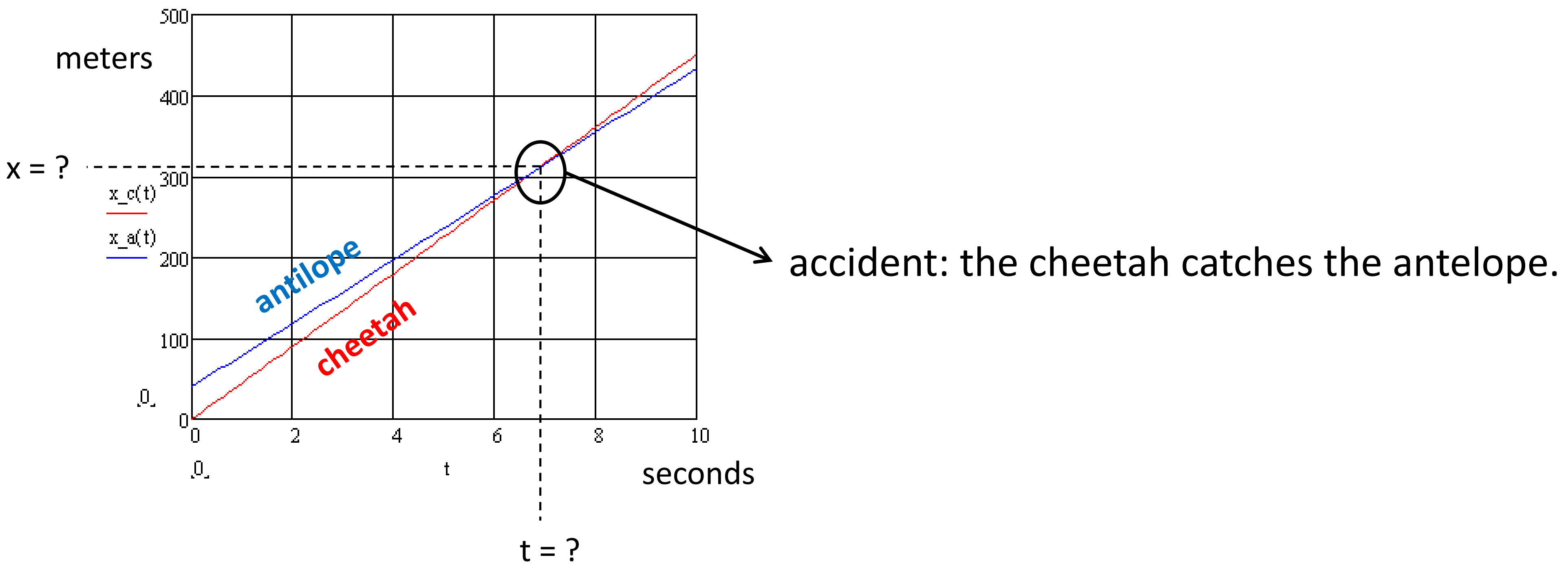
$$\left[S(T) = Si + \int_0^T Vi\,dt + \int_0^T \left(\int_0^t a\,dh\right) dt\right] \rightarrow S(T) = \frac{a \cdot T^2}{2} + Vi \cdot T + Si$$

1.

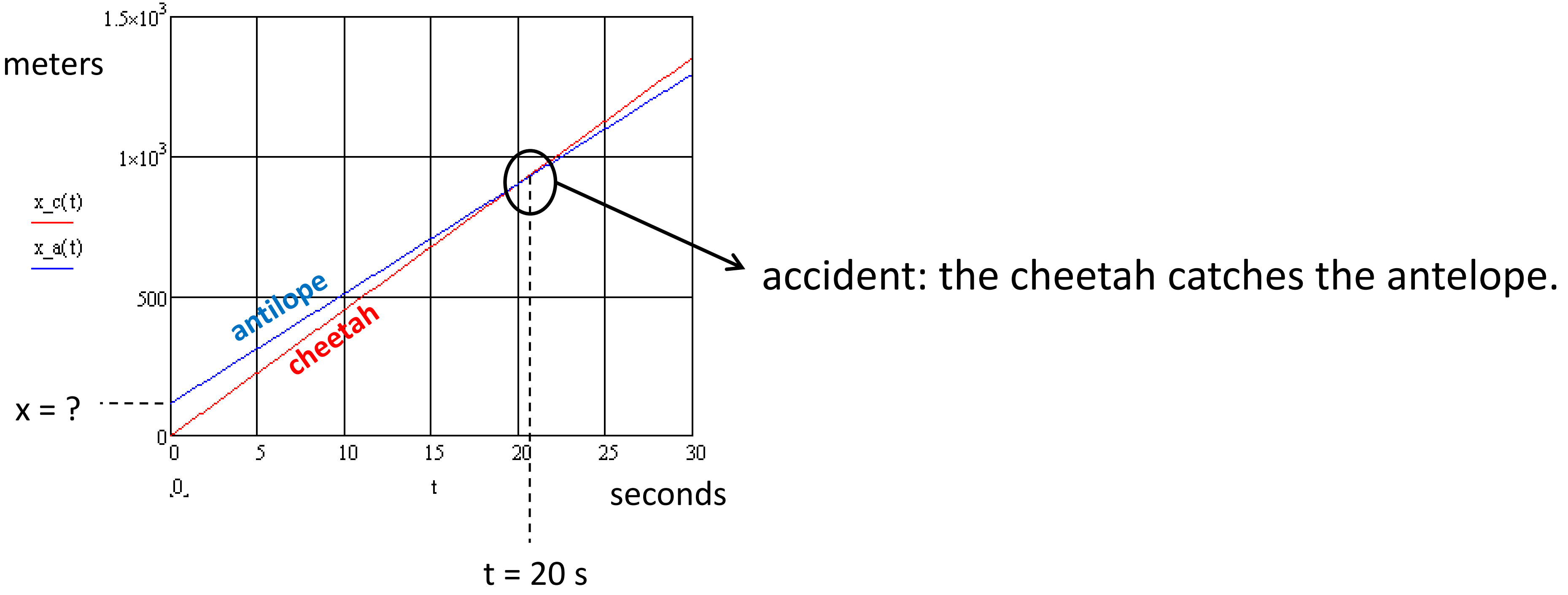
Two ~~trains~~ black material points (MPs), each having a speed of V_b = 30 km/h, are headed at each other on the same straight track. A ~~bird~~ white MP that can fly V_w = 60 km/h flies off the front of one ~~train~~ black MP when they are 60 km apart and heads directly for the other ~~train~~ black MP. On reaching the other ~~train~~ black MP, the ~~(crazy) bird~~ white MP flies directly back to the first ~~train~~ black MP, and so forth. What is the total distance the ~~bird~~ white MP travels before the ~~trains~~ MPs collide?



2. A cheetah is the fastest land mammal, and it can run at speeds of about 101 km/h for a period of perhaps 20 s. The next fastest land animal is an antelope, which can run at about 88 km/h for a much longer time. Suppose a cheetah is chasing an antelope, and both are running at top speed, (a) **If the antelope has a 40-m head start, how long will it take the cheetah to catch him, and how far will the cheetah travel in this time?**



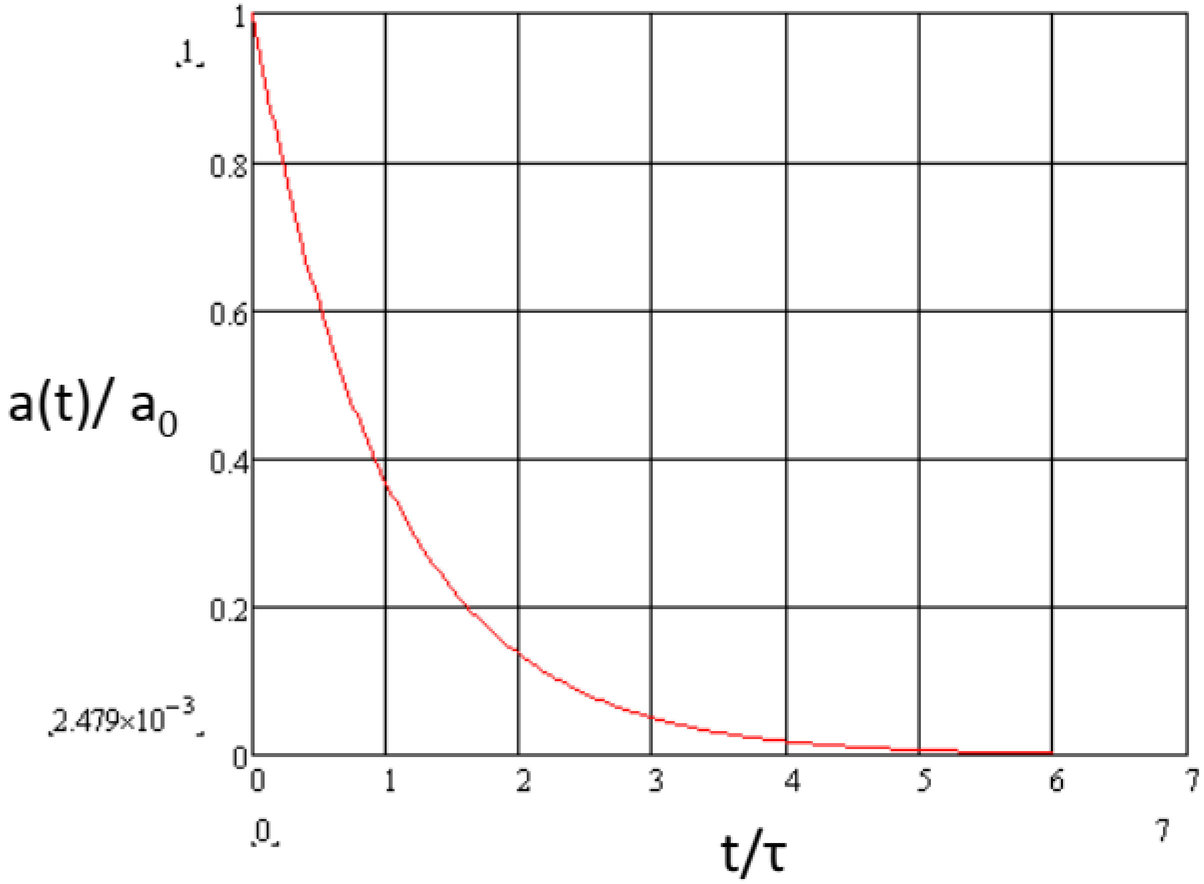
2. A cheetah is the fastest land mammal, and it can run at speeds of about 101 km/h for a period of perhaps 20 s. The next fastest land animal is an antelope, which can run at about 88 km/h for a much longer time. Suppose a cheetah is chasing an antelope, and both are running at top speed. (b) **What is the maximum head start the antelope can have if the cheetah is to catch him within 20 s (at which time the cheetah runs out of breath)?**



3. A motorist traveling 31 m/s passes a stationary motorcycle police officer. 2.5 s after the motorist passes, the police officer starts to move and accelerates in pursuit of the speeding motorist. The motorcycle has constant acceleration of 3.6 m/s^2. (a) How fast will the police officer be traveling when he overtakes the car? Draw curves of x versus t for both the motorcycle and the car, taking t = 0 at the moment the car passes the stationary police officer.

4.

Find the maximum speed v_{max} and the dependence of the distance on time $x(t)$ in the time interval $[0,\infty]$ in the case of the non-constant acceleration $a(t) = a_0 \exp(-t/\tau)$. Assume that the initial velocity is zero.



5. A point moves rectilinearly in one direction. Fig. 1.1 shows

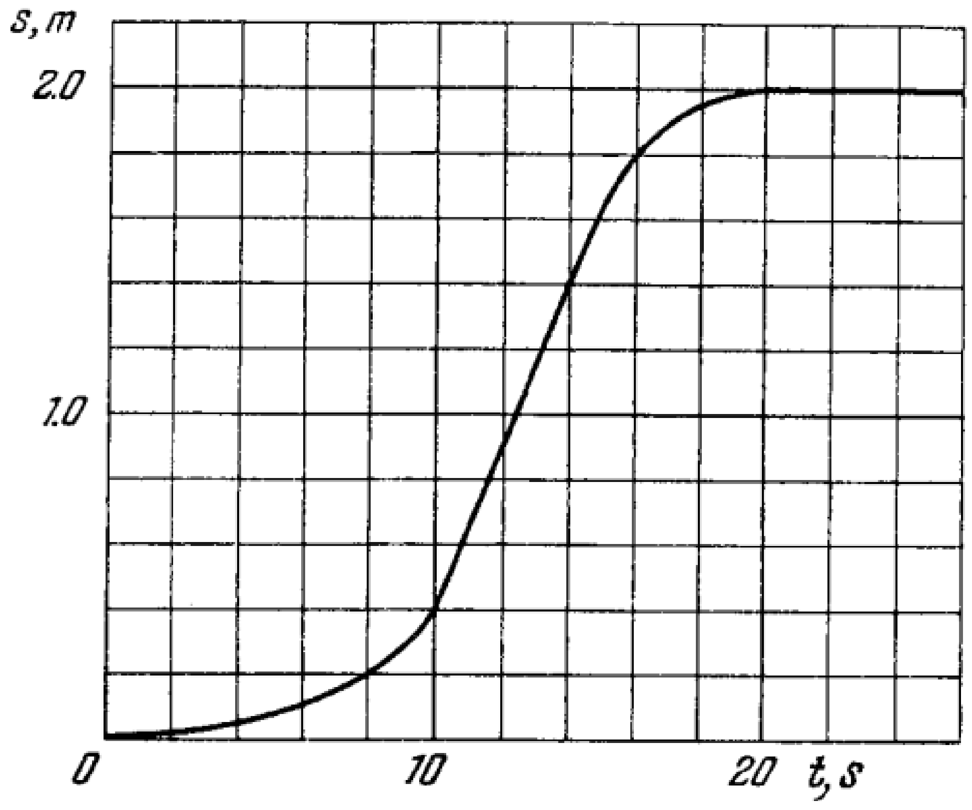


Fig. 1.1.

the distance s traversed by the point as a function of the time t. Using the plot find:

(a) the average velocity of the point during the time of motion;

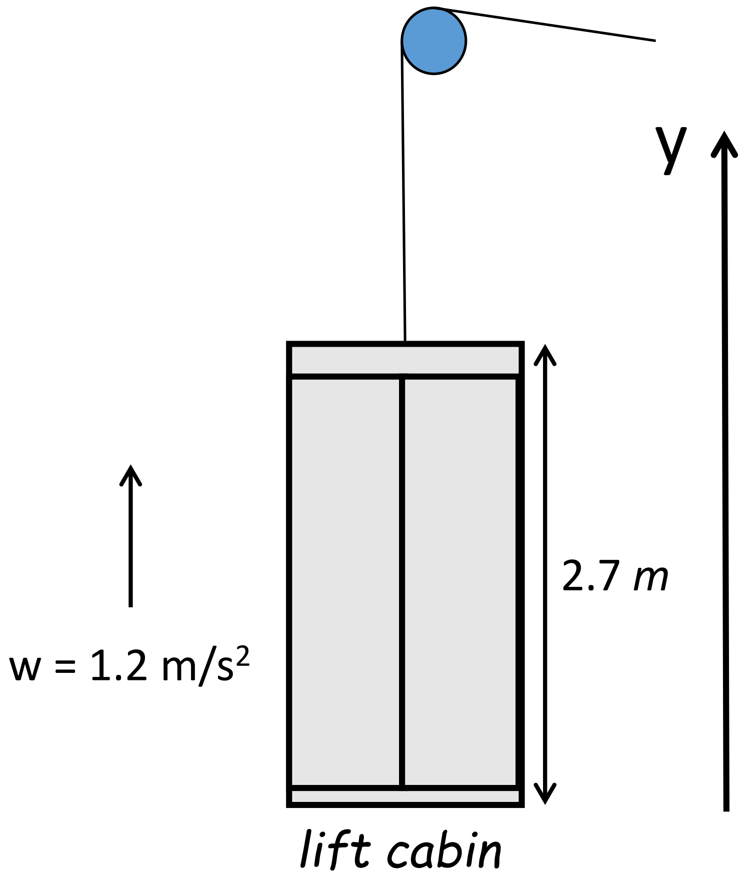
(b) the maximum velocity;

(c) the time moment t_0 at which the instantaneous velocity is equal to the mean velocity averaged over the first t_0 seconds.

6. An elevator car whose floor-to-ceiling distance is equal to 2.7 m starts ascending with constant acceleration 1.2 m/s^2; 2.0 s after the start a bolt begins falling from the ceiling of the car. Find:

(a) the bolt's free fall time;

(b) the displacement and the distance covered by the bolt during the free fall in the reference frame fixed to the elevator shaft.



7.

The position of a particle moving along an x axis is given by $x(t) = 12t^2 - 2t^3$, where x is in meters and t is in seconds. Determine (a) the position, (b) the velocity, and (c) the acceleration of the particle at $t = 3.0$ s. (d) What is the maximum positive coordinate reached by the particle and (e) at what time is it reached? (f) What is the maximum positive velocity reached by the particle and (g) at what time is it reached? (h) What is the acceleration of the particle at the instant the particle is not moving (other than at $t = 0$)? (i) Determine the average velocity of the particle between $t = 0$ and $t = 3$ s.