Physics 1. Mechanics.

Week 5

Forces and Motion 3

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Objectives

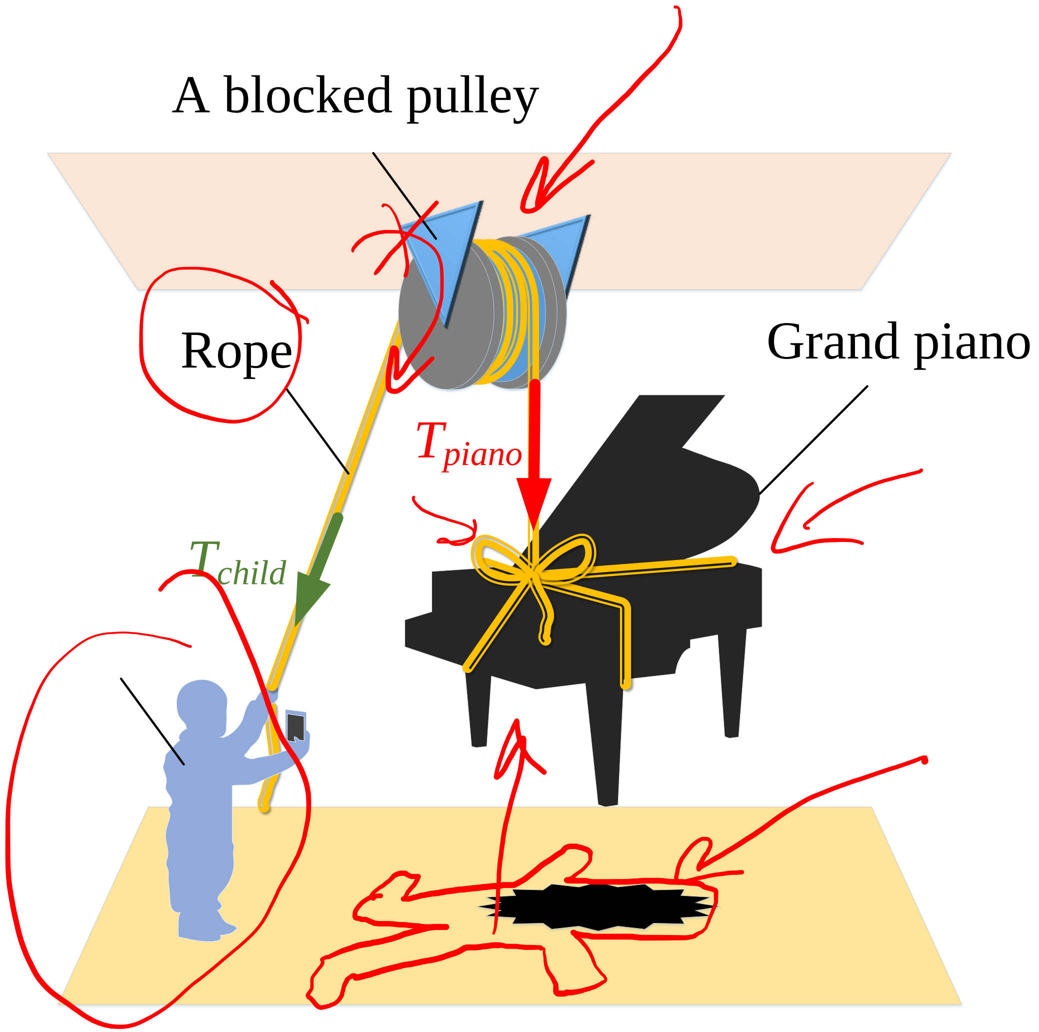
The main objectives of today's lecture are:

- Revisit one last time **Newon's** laws of motion
- Discuss forces occurring in the uniform circular motion
- Become familiar with capstan friction equation

Today’s Question

Assume you need to repair the floor in your apartment under a grand piano whose mass is 500 kg. Quite conveniently, you have a blocked pulley (a disk or a flat cylinder that cannot spin) mounted on the ceiling right above it, and so you throw a rope over the pulley and tie one of its ends around the piano.

Using your inhuman InnoStrength and the power of knowledge, you can easily lift the instrument into the air, but you need someone to hold it while you are working, and so you ask a **neighbor’s kid** to do it for you.



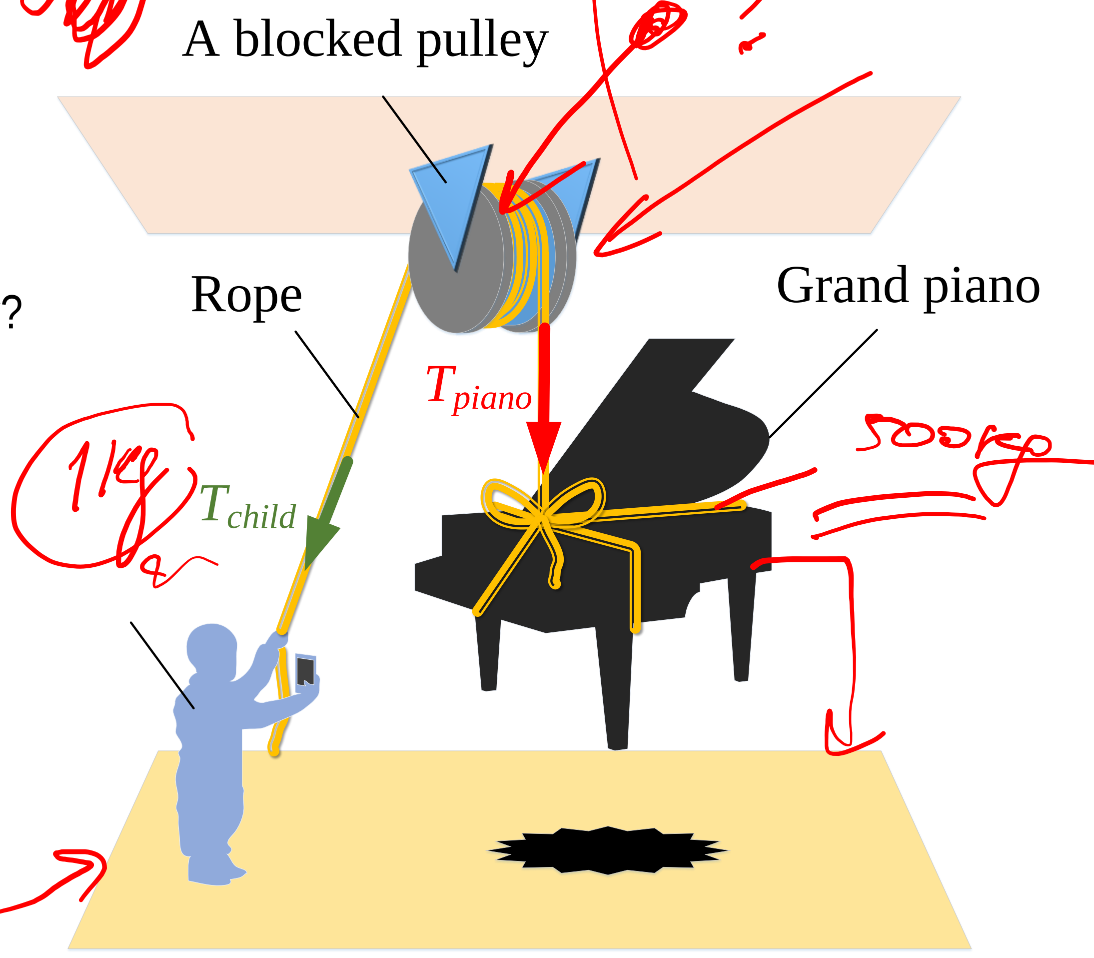
Today's Question

Provided the kid can only lift 1 kg and given the friction coefficient between the rope and pulley to be 0.5,

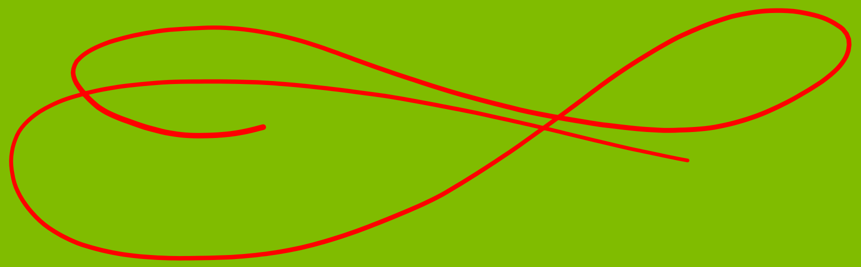
- How many times (in the least) do you need to wrap the line around the pulley so that the child can hold it safely?

Assume both the pulley and the rope can sustain the weight.

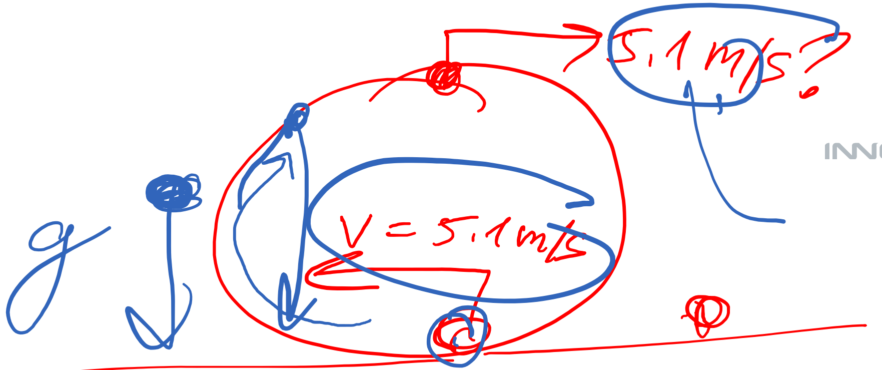
a) 2 times
b) 5 times
c) 7 times
d) Over 10 times



Vertical Circular Loop



Vertical Circular Loop (1)



km/h

Largely because of riding in cars, you are used to horizontal circular motion. Vertical circular motion would be a novelty. In this sample problem, such motion seems to defy the gravitational force.

- In a 1901 circus performance, Allo "Dare Devil" Diavolo introduced the stunt of riding a bicycle in a loop-the-loop (shown here).
- Assuming that the loop is a circle with radius R = 2.7 m, what is the least speed v that Diavolo and his bicycle could have at the top of the loop to remain in contact with it there?

$h = 5.4\,m = 2R$

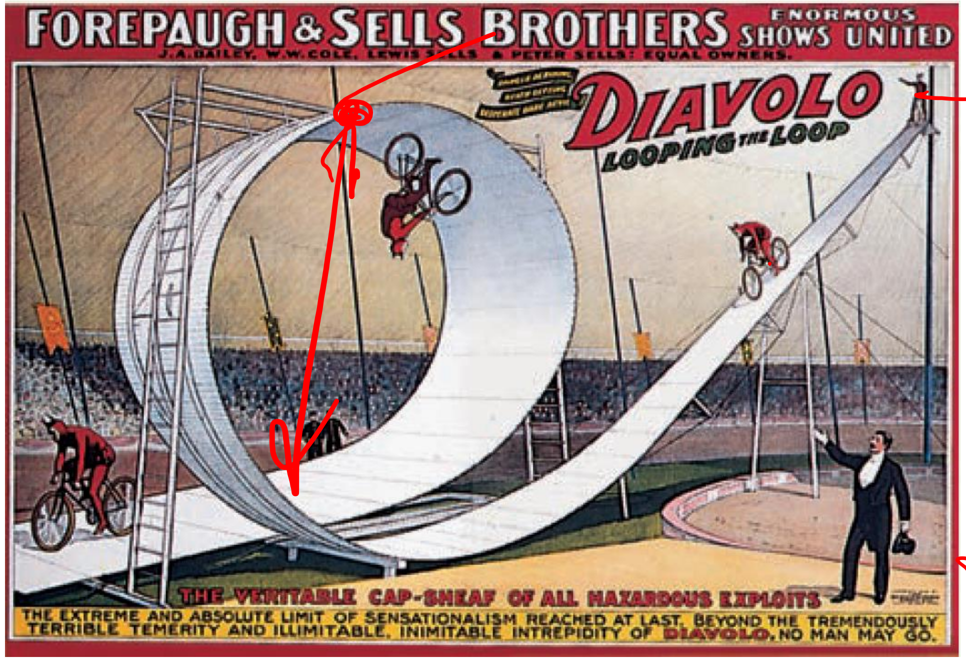
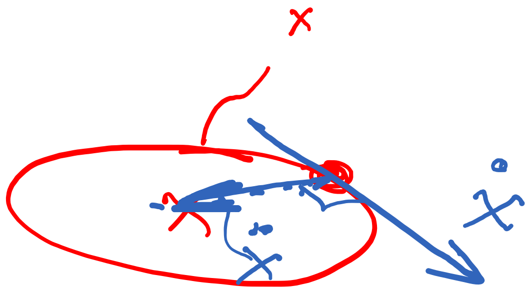


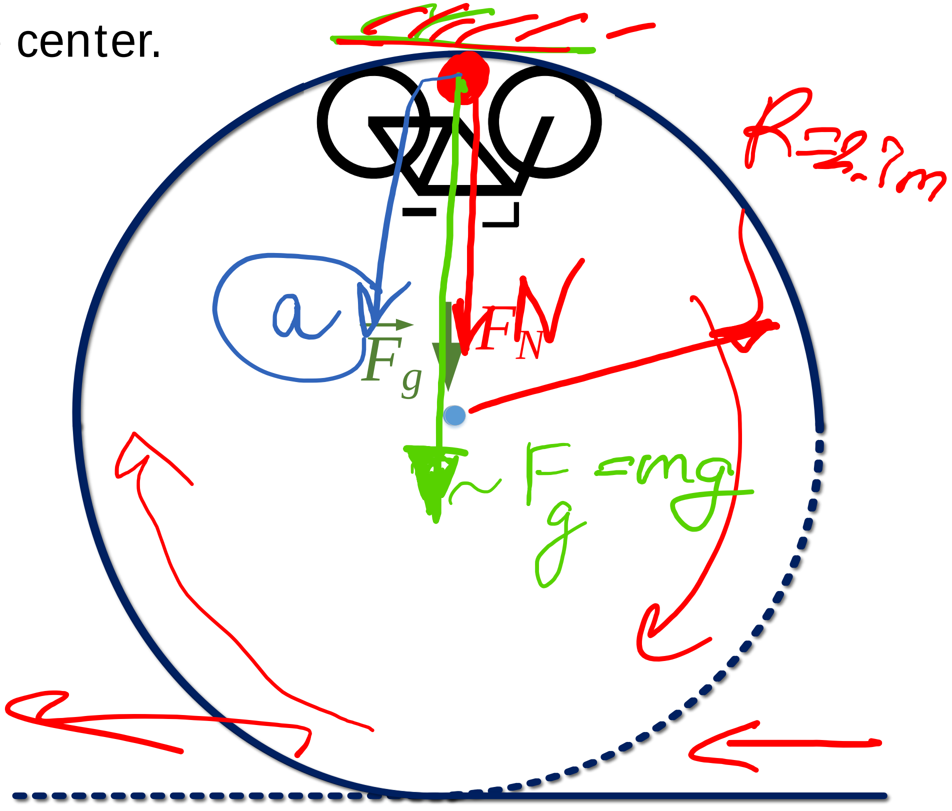
Image credit: Our Textbook

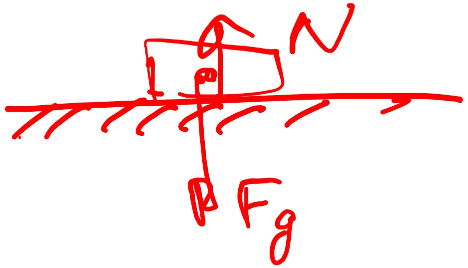
Vertical Circular Loop (2)



Let us assume that Diavolo and his bicycle travel through the top of the loop as a single particle in uniform circular motion. Thus, at the top, the acceleration $\vec{a}$ of this particle must have the magnitude $\boxed{a = \frac{v^2}{r}}$ and is directed downward toward the center.

- Let as analyze the forces on the particle when it is at the top of the loop in the free-body diagram on the right.
- The gravitational force $\vec{F}_g$ is downward along a y axis;
- so is the normal force $\vec{F}_N$ on the particle from the loop (the loop can push down, not pull up);
- so also is the centripetal acceleration of the particle.





$F_{net} = ma$

Vertical Circular Loop (3)

$F_N + mg = m\frac{v^2}{r}$

100N

0N

100N

100N

200N

Thus, Newton's second law for y components gives us

$$-F_N - F_g = m(-a) = -\,m\,\frac{v^2}{r}$$

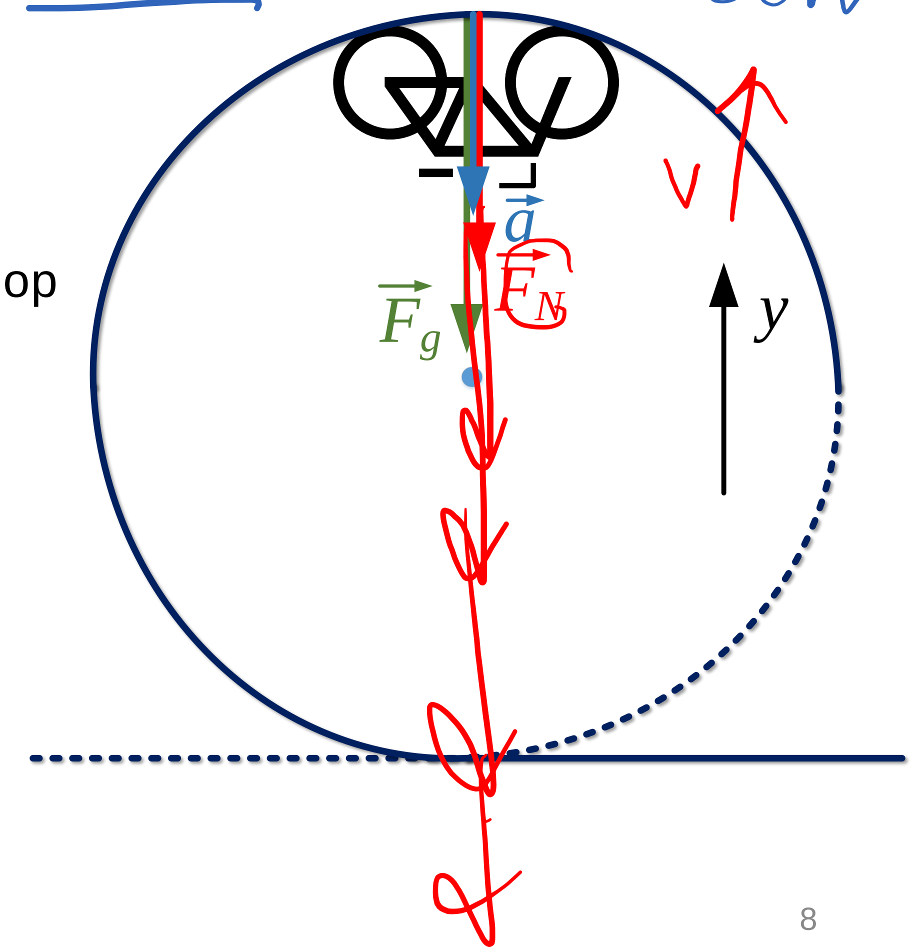
- If the particle has the least speed v needed to remain in contact, then it is on the verge of losing contact with the loop (falling away from the loop), which means

$$F_N = 0$$

- Thus, solving the equation on top for v yields

$$\boxed{v = \sqrt{gR} = 5.1\text{ m/s}}$$

$$\boxed{-F_g = +\cancel{m}g = +\cancel{m}\frac{v^2}{r}} \rightarrow v =$$



Capstan (Friction) Equation

Capstan Friction (1)

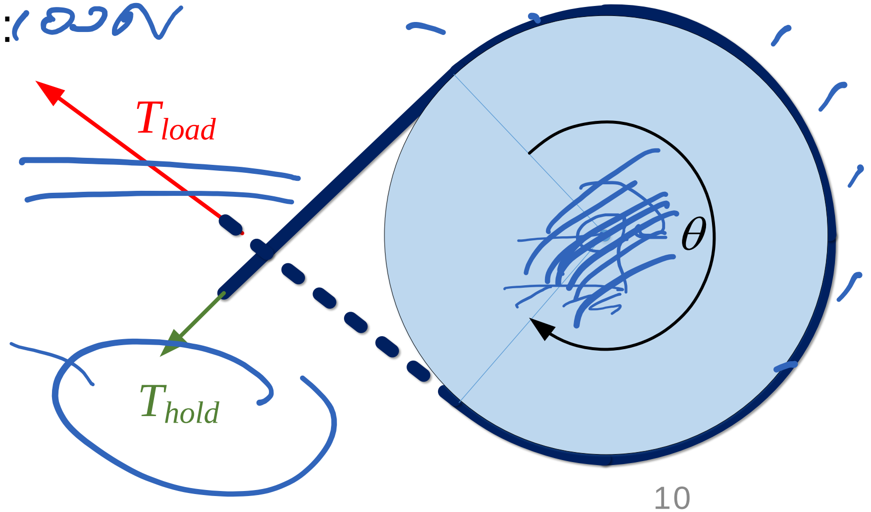
A very curious case of friction occurs when a rope or a flexible line is wound around a cylinder (a capstan).

- Because of the interaction of frictional forces and tension, the tension on a line wrapped around a capstan may be different on either side of the capstan.

Image credit: fotosearch

The principle by which a capstan-type device operates is as follows: 1000N

- A small holding force exerted on one side can carry a much larger loading force on the other side.



m_1g 1kg 1000kg m_2g

Capstan Friction (2)

Imagine that we have dissected the rope into n infinitely small segments or parts.

Let us assume that

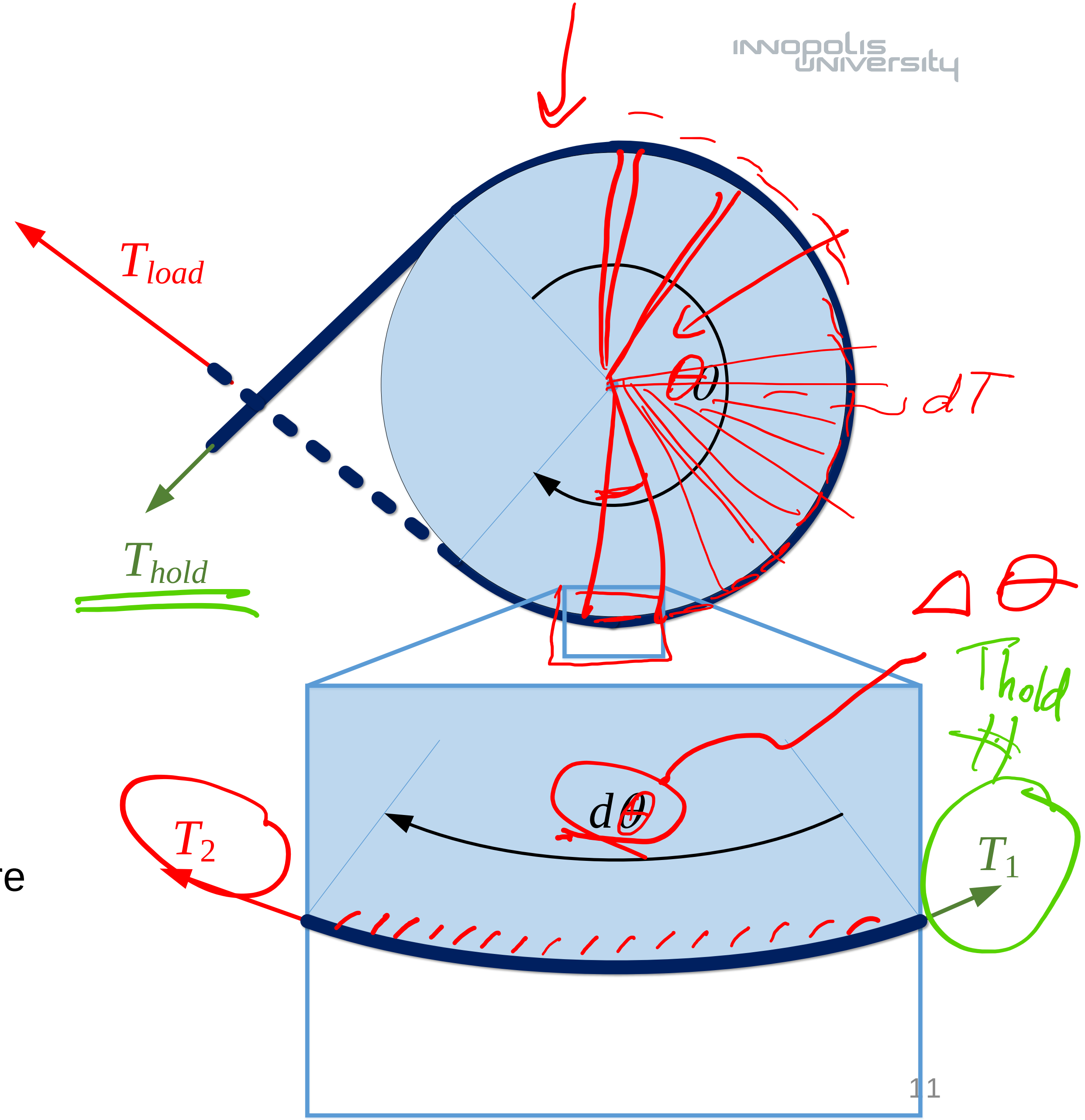
- Each section of the rope is in equilibrium, and
- The contact between the rope and capstan is uniform along the whole length.

Therefore, we can extrapolate the findings of our future investigation of the rope's section onto the whole line.

$T_2 > T_1$

The resulting tension forces on each of the ends are

$$T_2 = T_1 + dT$$



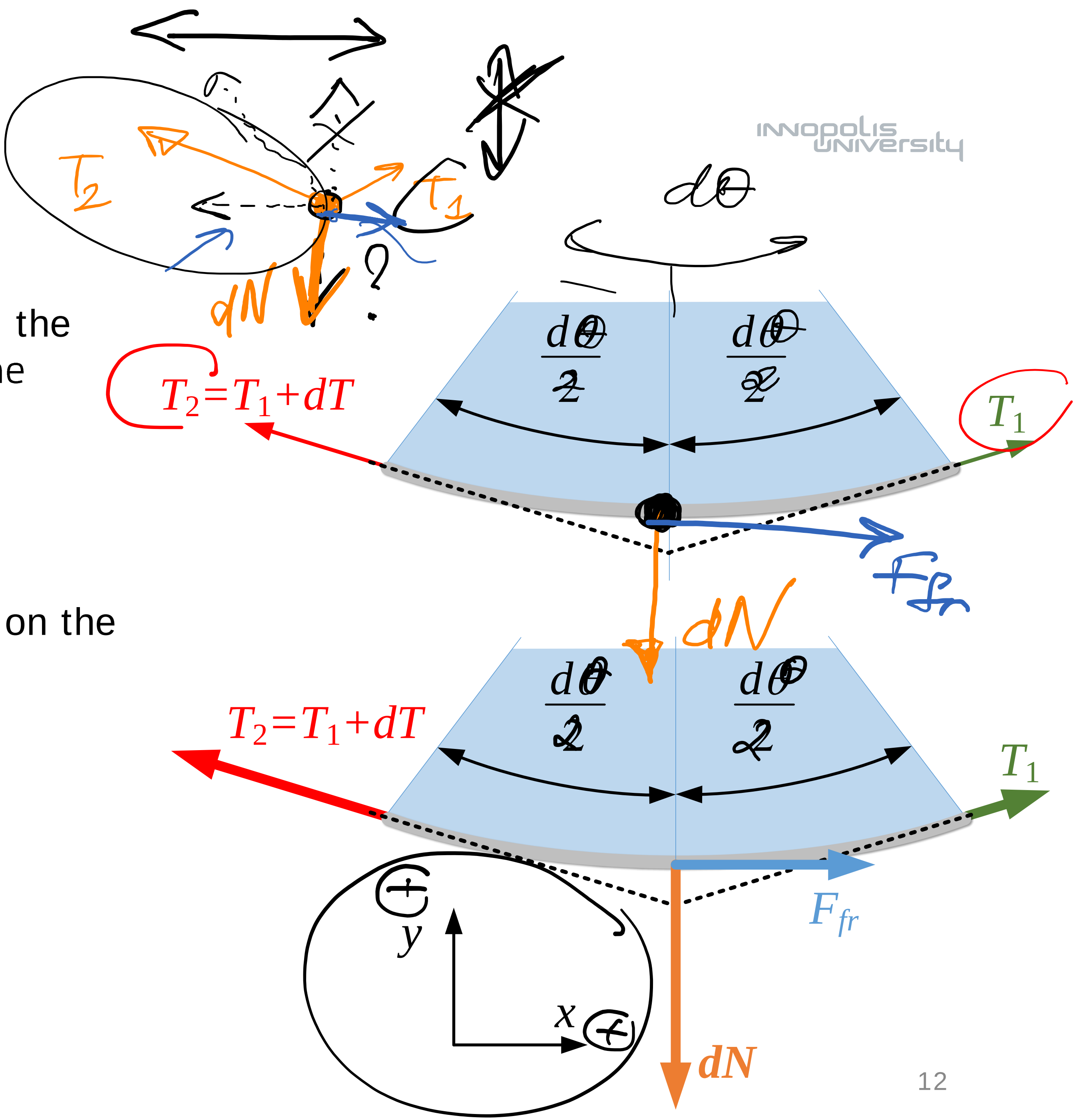
$\theta = \sum_{0}^{n} d\theta$

Capstan Friction (3)

Now, let us discuss what forces act between the rope's and capstan's segments in order for the system to remain in equilibrium.

$$\vec{T}_1 + \vec{T}_2 + d\vec{N} + \vec{F}_{fr} = 0$$

- Now, let us find projections of these forces on the horizontal and vertical axis.



Capstan Friction (4)

$$\int \frac{dT}{T_1} = \int_{T_{hold}}^{T_{load}} \frac{1}{T_1}\,dt = \ln\left|\frac{T_{load}}{T_{hold}}\right| \Leftrightarrow \ln\frac{T_{load}}{T_{hold}}$$

T_{load}, T_{hold}

$$\boxed{\int \frac{dT}{T_1} = \int \mu \cdot d\theta}$$

$$\cos(d\theta) \approx 1,\quad \sin(d\theta) \approx d\theta$$

- x axis: $\sum F = 0$

$$T_{2x} = F_{fr} + T_{1x}$$

$$(T_1 + dT)\cdot\cos\left(\frac{d\theta}{2}\right) = T_1 \cdot \cos\left(\frac{d\theta}{2}\right)$$

$$\cancel{T_1} + dT = \cancel{T_1} + \mu \cdot dN$$

- y axis:

$$T_{2y} + T_{1y} = dN$$

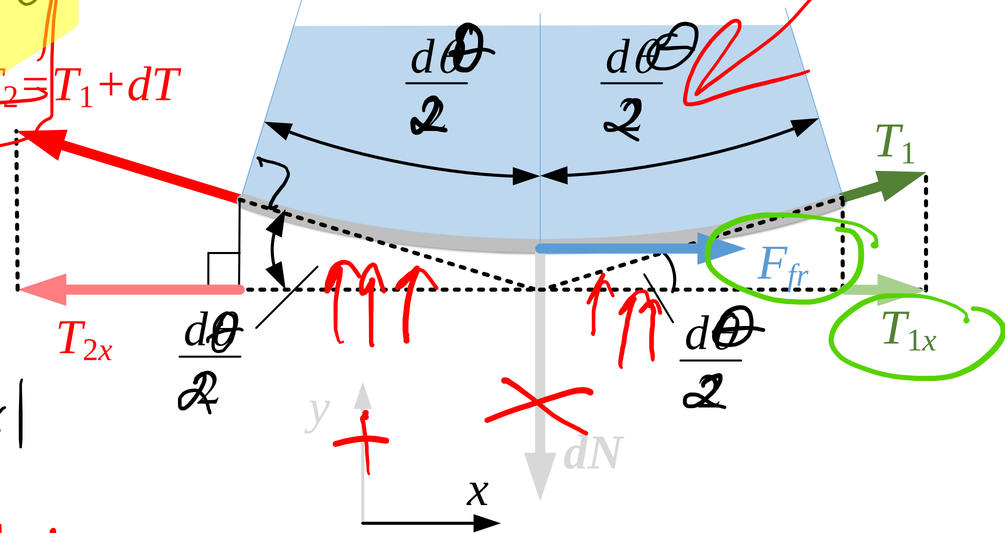
$$(T_1 + dT)\sin\left(\frac{d\theta}{2}\right) + T_1 \cdot \sin\frac{d\theta}{2} =$$

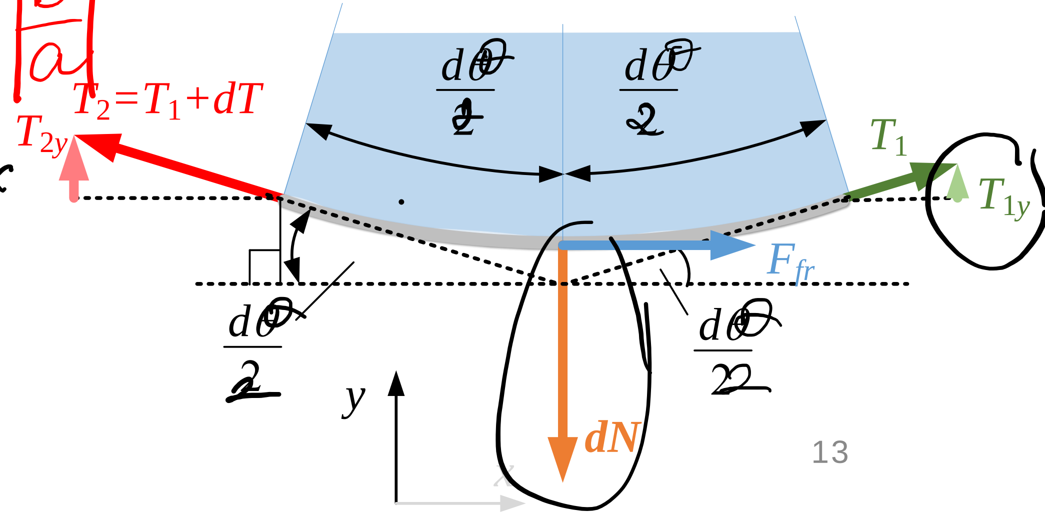
$$(T_1 + dT)\cdot\frac{d\theta}{2} + T_1 \cdot \frac{d\theta}{2} = dN$$

$$\int_a^b \frac{1}{x}\,dx = \ln|x| \rightarrow \ln\left|\frac{b}{a}\right|$$

$$\int a \cdot dx = a\int 1 \cdot dx$$

$$0 \rightarrow \theta$$





Capstan Friction (5)

1 kg —— 500 kg
×500

Finally,

$$\boxed{\ln\frac{T_2}{T_1} = \mu\theta} \Rightarrow \boxed{\frac{T_2}{T_1} = e^{\mu\theta}}$$

- Expanding this solution to the original holding and loading forces yields

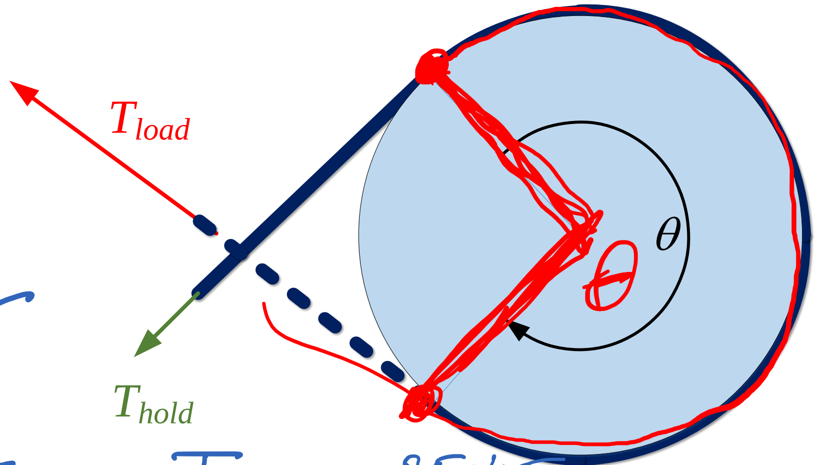
$$\boxed{T_{load} = T_{hold} \cdot e^{\mu\theta}}$$

$\mu \cdot \theta = 1$

$T_{load} \approx 2.71 \cdot T_{hold}$

Number of turns	Coefficient of friction μ				
	0.1	0.2	0.3	0.4	0.5
1	1.9	3.5	6.6	12	23
2	3.5	12	43	152	535
3	6.6	43	286	1,881	12,392
4	12	152	1,881	23,228	286,751
5	23	535	12,392	286,751	6,635,624

Image credit: fotosearch



$T_{load} = T_{hold} \cdot e^{0.5 \cdot 4\pi}$

Today’s Question

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a) 2 times
b) 5 times
c) 7 times
d) Over 10 times

Thank you for your attention!

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