Multi-agent safety invariant set.

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Background

There are many smart agents $\{\mathrm{A}_1, \mathrm{A}_2, \mathrm{A}_3 \dots \mathrm{A}_\mathrm{n}\}$ living in a shared space. Each agent is capable of controlling its own behavior by:

$$\dot{x}_i = f_i(x_i, u_i) \tag{1.1}$$

For easier representation, also define its integration:

$$F_i(x, u, t) \triangleq \ state\ after\ controling\ x\ using\ u\ for\ time\ t \tag{1.2}$$

For each two of them, certain pair of state is dangerous, known as crush. We use the following Boolean function to denote that:

$$Crush_i^j(x_i, x_j) \tag{2}$$

Agents are not capable of communicating with each other. However, they can observe the state of any agent in the system.

Given such a system, this article is proposing a strategy for those agents to peacefully live in the space, by applying control supervisor based on safe invariant set.

For each agent, its control u_i is limited by supervision set R_i:

$$u_i \in R_i(x_1, x_2, \dots x_n) \tag{3}$$

In the following pages, the word "regulation" will be used to represent such supervision set.

Principle 1. Non-correlated regulation (NCR)

The author thinks that one agent is never capable of considering all the other agents in a correlated manner at real time. The computational power of a single agent is limited, such that it can only regulate its behavior using following form:

$$R_i(x_1, x_2, \dots x_n) = \cap_{j=0, j\neq i}^{n} R_i^j(x_i, x_j) \tag{4}$$

First, for each other agent A_j, A_i works out a two-body regulation R_i^j considering only the states of A_j and A_i. Then A_i chooses its final regulation to be the intersection of all such two body regulations.

Principle 2. Basic agent right.

There can be many different results of R_i^j considering the abundance of the agents. No matter what they are, the intersection of them in eq (4) cannot be empty, or otherwise there would be no choice for $\mathrm{A_i}$ to make.

In fact, each other agent may cast certain regulation upon $\mathrm{A_i}$, but they are not clever enough to think in $\mathrm{A_i}$'s perspective. To achieve the non-emptiness of $\mathrm{A_i}$'s final regulation, one solution is to make the contract of "basic agent right":

For each agent $\mathrm{A_i}$, there exist a control known as the "basic agent right", denoted as $\mathrm{b_i}$. $\mathrm{A_i}$ shall be always allowed to take $\mathrm{b_i}$ as its next control. On the other hand, $\mathrm{A_i}$'s control needs to be regulated such that it won't harm the basic right of any other agents. ($\mathrm{b_i}$ is known by all agents.)

In the later pages, we will show that the peaceful space can be achieved if every NCR agent observes the principle of basic agent right.

Notion of backup safety.

For some two agents $\mathrm{A_i}$, $\mathrm{A_j}$ in the n-agent space, one necessary condition for "eventually no crush between $\mathrm{A_i}$ and $\mathrm{A_j}$ for all possible trajectories starting from x_i, x_j" is "backup safety":

$$BS_j^i(x_j, x_i) = BS_i^j(x_i, x_j) \triangleq \forall t \geq 0,\ !Crush_i^j\ (F_i(x_i, b_i, t), F_j(x_j, b_j, t)) \tag{5}$$

Proof of necessity: since there are many other agents in the space, it is possible that $\mathrm{A_i}$'s final regulation set get maximumly squeezed by other agents, such that $\forall t > 0$, $\mathrm{A_i}$ has nothing to do but to take the backup choice b_i. The same goes for $\mathrm{A_j}$. If such situation happens (such situation can always happen no matter how the agents are regulated), eq (5) is equivalent to the definition of "eventually no crush between $\mathrm{A_i}$ and $\mathrm{A_j}$, starting from x_i, x_j".

Eq (5) must hold true for all selection of $\mathrm{i, j}$ to avoid the risk of collision. It is also easy to show that always $BS_i^j(x_i, x_j)$ indicates no collision between $\mathrm{A_i}$, $\mathrm{A_j}$.

Notion of moral bottom line

To avoid a complicated story, we further limit the dynamic to discrete time axis.

$$x_i(k+1) = F_i(x_i(k), u_i(k), dt) \qquad (6)$$

As mention before, $\mathrm{A_i}$, $\mathrm{A_j}$ must try to keep eq (5) all the time since from their perspective doing so is necessary for collision prevention.

Suppose $BS_i^j(x_i(k), x_j(k)) = true$ at moment k. To keep such property at $k+1$, A_j must satisfy:

$$R_j^i(x_j, x_i) \subseteq M_j^i(x_j, x_i) \triangleq \{u_j | BS_i^j\left(F_i(x_i, b_i, dt),\ F_j(x_j, u_j, dt)\right)\} \qquad (7)$$

From the principle 2 basic agent right, A_j must respect the basic right of A_i. It is possible for A_i to choose b_i at k, thus A_j must constraint its action in $M_j(x_j, x_i)$ so as to avoid potential backup safe regression. The constraint of M_j^i, known as A_j's moral bottom line over A_i, must also be enforced to everyone in the space to rule out the risk of collision.

$$BS_i^j(x_i, x_j) \rightarrow M_j^i(x_j, x_i) \neq \emptyset \qquad (8)$$

Proof of eq (8): by showing b_j is an element of $M_j^i(x_j, x_i)$.

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Knowing that each agent is at least regulated by moral bottom line, there is one design of regulation set that can be sufficient for the purpose of peaceful living:

$$R_i^j(x_i, x_j) = \{u_i |\ \forall u_j \in\ M_j^i(x_j, x_i),\ BS_i^j\left(F_i(x_i, u_i, dt),\ F_j(x_j, u_j, dt)\right)\} \qquad (8)$$

It is not hard to prove that:

$$b_i \in R_i^j(x_i, x_j) \subseteq M_i^j(x_i, x_j) \qquad (9)$$

So here such regulation R_i^j observes moral bottom line, while also protects A_i's own basic right.

When A_i uses (8) and A_j observes (7), it can be proved that for any step k:

$$BS_i^j(x_i(k), x_j(k)) \rightarrow\ BS_i^j(x_i(k+1), x_j(k+1)) \qquad (10)$$

The backup safe invariance between A_i and A_j is then achieved.

$R_i(x_1, x_2, \dots x_n)$, as the intersection of R_i^j, is always respecting other's basic right, while protecting A_i's own basic right, using (9), and can be proved to be not empty if $BS_i^j(x_i, x_j)$. Together with (10), the design is done.

Summary

I am so good.