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Conference Paper · October 2013

DOI: 10.1109/ICAT.2013.6684071

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2013 XXIV International Conference on
Information, Communication and Automation Technologies (ICAT)
October 30 – November 01, 2013, Sarajevo, Bosnia and Herzegovina

Mathematical Modeling of Vehicle Frontal Crash by a Double Spring-Mass-Damper Model

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***Abstract*—This paper presents development of a mathematical model to represent the real vehicle frontal crash scenario.The vehicle is modeled by a double spring-mass-damper system. The front mass m_1 represents the chassi of the vehicle and rear mass m_2 represents the passenger compartment. The physical parameters of the model (Stiffness and dampers) are estimated using Nonlinear least square method (Levenberg-Marquart algorithm) by curve fitting the response of a double spring-mass-damper system to the experimental displacement data from the real vehicle crash. The model is validated by comparing the results from the model with the experimental results from real crash tests available.**

***Index Terms*—Modeling, Vehicle frontal crash, parameters estimation, curve fitting.**

I. Introduction

Car crash test is usually performed in order to ensure safe design standards in crashworthiness (the ability of a vehicle to be plastically deformed and yet maintains a sufficient survival space for its occupants during crash scenario). Nowadays, due to advanced research in computer simulation software, simulated crash tests can be performed beforehand the full-scale crash test. Therefore, cost associated with real crash test can be reduced. Vehicle crashworthiness can be evaluated in four distinct modes: frontal, side, rear and rollover crashes. Several researches have been carried out in this field, which resulted in several novel computational models of vehicle collisions in literature. In [1], a mathematical model is proposed to estimate the maximum occupant deceleration - which is one of the main tasks in the area of crashworthiness study by a Kelvin Model which contains a mass together with spring and damper connected in parallel. An application of physical models composed of springs, dampers and masses joined together in various arrangements for simulating a real car collision with a rigid pole was presented in [2].

In [4], the authors presented an overview of the kinematic and dynamic relationships of a vehicle in a collision, where the work was to identify the parameters of the vehicle crash model using experimental data set. In [6] and [7], a lumped parameter modeling in frontal crash was investigated and analyzed in five degrees of freedom and have been used to analyze the response of occupant during the impact. In [8] and [9], an optimization procedure to assist multibody vehicle model development and validation was proposed. The authors first devised the topological structure of the multibody system representing the structural vehicle components and described the most relevant mechanisms of deformation. In the work of [10], the authors proposed an approach to control the seat belt restraint system force during a frontal crash to reduce thoracic injury.

The main challenge in accident reconstruction is the system identification described as the process of constructing mathematical models of dynamical systems using measured input-output data. In case of vehicle crash, system identification algorithm consists of retrieving the unknown parameters such as the spring stiffness and damping coefficient. A possible approach is to identify these parameters directly from experimental dynamical data.

From literature, System Identification Algorithms (SIA) have been developed for different applications. Among others we can state: subspace identification, genetic algorithm, eigensystem realization algorithm and data-based regressive model approaches. Typical examples where these SIA have been used can be found in [13] and [14], [6] and [17], [19] and [20] respectively.

The main contribution of this paper is the development of a mathematical model for a double spring-mass-damper system which reconstructs a vehicle frontal crash scenario and estimate structural parameters such as natural frequencies,damping factors, spring stiffness and damping coefficients of the system. The model represents the inertia of the vehicle chassis and the passenger compartment. To estimate the physical parameters (stiffness and damping coefficient) of the model, a noninear least square curve fitting estimation method is used. It is noting that the effectiveness and accuracy of simulation modeling results are verified by the real physical experiments. The novelty in this paper as compared to those refered to is that the physical parameters , stiffness and damping coeficients, were first estimated and finally the model was simulated and results compared with experimental results.

II. Model development

Initially a real vehicle crash experiment was conducted on a typical mid-speed vehicle to pole collision. Its elaboration was the initiative of Robbersmyr (2004). A test vehicle was subjected to impact with a vertical, rigid cylinder. The acceleration field was 100 meter long and had two anchored parallel pipelines. The vehicle was steered using those pipelines that were bolted to the concrete runaway. Setup scheme is shown in Fig.1. During the test, the acceleration was measured in

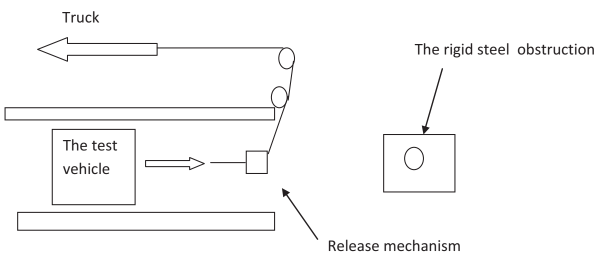

Fig. 1. Vehicle crash Experimental setup [24]

three directions (x - longitudinal, y - lateral, and z - vertical) together with the yaw rate from the center of gravity of the car. Using normal speed and high - speed video cameras, the behavior of the safety barrier and the test vehicle during the collision was recorded. The initial velocity of the car was 35 km/h, and the mass of the vehicle (together with the measuring equipment and dummy) was 873 kg. The obstruction was constructed with two steel components - a pipe filled with concrete and a baseplate mounted with bolts on a foundation. The car undergoing the deformation is shown in Fig.2. The accelerometer is located at the mass center of gravity of the vehicle in the passenger compartment. Since we are interested in the frontal crash, only the measured acceleration in the longitudinal direction is considered in this study. The acclereration data is imported and processed in matlab for analysis. The deformation of the vehicle is obtained by integrating twice the acceleration signal.

Fig. 2. Vehicle undergoing deformation [24]

A. *Mathematical Modeling*

Mathematical models describe the dynamic behavior of a system as a function of time. During frontal crash, the vehicle is subjected to an impulsive force caused by the obstacle. The model for vehicle crash simulates a rigid barrier impact of a vehicle where m_1 and m_2 represent the frame rail (chassis) and passenger compartment masses, respectively.
Parameters to be estimated are springs k_1 and k_2, dampers c_1 and c_2, as shown in Figure 3. When the vehicle impacts on a rigid barrier, the two masses will experience an impulsive force during collision. The method for solving the impact responses of the two masses is adapted from te method used in the free vibration analysis of a two-degrees of freedom damped system [22].

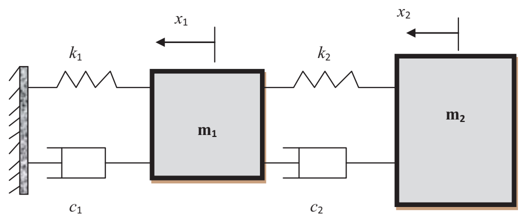

Fig. 3. A double spring-mass-damper model

The dynamic equations of the two mass-spring-damper model are shown in Equation(1).

$$\begin{aligned} &m_1\ddot{x}_1 + (c_1+c_2)\dot{x}_1 + (k_1+k_2)x_1 - c_2\dot{x}_2 - k_2x_2 = 0 \\ &m_2\ddot{x}_2 - c_2\dot{x}_1 + c_2\dot{x}_2 + k_2x_2 - k_2x_1 = 0 \end{aligned} \tag{1}$$

or

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2}$$

The solution for for $x_i, i = 1, 2$, can be respresented as

$$x_i = C_i e^{s_k t} \tag{3}$$

with k= 1,..,4.
where C_i and s_k may be complex numbers. Substituting (3) into (1), we get

$$\begin{aligned} R_{1k}C_1 - R_{2k}C_2 = 0 \\ -S_{1k}C_1 + S_{2k}C_2 = 0 \end{aligned} \tag{4}$$

$$\frac{C_2}{C_1} = \frac{R_{1k}}{R_{2k}} = \frac{S_{1k}}{S_{2k}} \tag{5}$$

$$R_{1k} = R_{2k} * \frac{S_{1k}}{S_{2k}} \tag{6}$$

$$R_{1k} * S_{2k} - R_{2k} * S_{1k} = 0 \tag{7}$$

where
$R_{1k} = m_1 s_k^2 + (c_1+c_2)s_k + (k_1+k_2)$
$R_{2k} = c_2 s_k + k_2$
$S_{1k} = c_2 s_k + k_2$
$S_{2k} = m_2 s_k^2 + c_2 s_k + k_2$

After substituting R_{1k}, R_{2k}, S_{1k} and S_{2k} into (4), we get.

$$\begin{bmatrix} m_1 s_k^2 + (c_1+c_2)s_k + k_1 + k_2 & -c_2 s_k - k_2 \\ -c_2 s_k - k_2 & m_2 s_k^2 + c_2 s_k + k_2 \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{8}$$

Now for a nontrivial response i.e., for non-zero values of C_1 and C_2, the determinant of their coefficient matrix must vanish. That is:

$$[m_1 s_k^2 + (c_1+c_2)s_k + k_1 + k_2][m_2 s_k^2 + c_2 s_k + k_2] + (c_2 s_k + k_2)^2 = 0 \tag{9}$$

Expansion of (9) leads to a characteristic equation of the system, obtained as shown in (10).

$$s_k^4 + ts_k^3 + us_k^2 + vs_k + w = 0 \tag{10}$$

where
$t = \frac{m_1c_2+m_2(c_1+c_2)}{m_1m_2}$, $u = \frac{m_1k_2+m_2(k_1+k_2)+c_1c_2}{m_1m_2}$,

$v = \frac{k_1c_2+k_2c_1}{m_1m_2}$, $w = \frac{k_1k_2}{m_1m_2}$

Equation (10) is a fourth order polynomial in s and is to be solved to get four roots. All the coefficients of this polynomial are physical parameters of the system shown in Figure 3 and are all positive. For that reason such a polynomial cannot have positive roots. Three allowable configurations of roots are as follows [22]:

1) Two pairs of complex conjugates.
2) One pair of complex conjugates and two real and negative roots.
3) Four real and negative roots.

Case 1: ***Two pairs of complex conjugates***
The system in this case has moderate damping. The rate of decay is defined by p_1, the real part of the root, and the frequency of vibration is specified by q_1, the imaginary part. The two pairs of complex conjugates are:
$(1)s_1 = -p_1 + iq_1$, $s_2 = -p_1 - iq_1$,
$(2)s_3 = -p_2 + iq_2$, $s_4 = -p_2 - iq_2$.
where p_1, p_2, q_1, and q_2 are all positive. s_1 and s_2 are the first pair of complex conjugates, and s_3 and s_4, the second pair.
The two roots s_1 and s_2 in the first pair will yield the solutions X_{11} and X_{21}, where the first subscript refers to the mass index and the second subscript refers to the pair number of the complex conjugate. The displacement components X_{11} and X_{21} due to s_1 and s_2 respectively are given by:

$$\begin{aligned} X_{11} &= A_{11}e^{-p_1t} \times sin(q_1t + \phi_{11}) \\ X_{21} &= A_{21}e^{-p_1t} \times sin(q_1t + \phi_{21}) \end{aligned} \tag{11}$$

where
$A_{11}^2 = 4C_{11}C_{12}$
$A_{21} = 4C_{21}C_{22}$
$C_{11} = m_2s_1^2 + c_2s_1 + k_2$
$C_{21} = c_2s_1 + k_2$
$C_{12} = m_2s_2^2 + c_2s_2 + k_2$
$C_{22} = c_2s_2 + k_2$

The general solution is:

$$\begin{aligned} X_i &= \sum_{j=1}^{2} x_{ij} \\ &= A_{i1}e^{-p_1t}sin(q_1t + \phi_{i1}) + A_{i2}e^{-p_2t}sin(q_2t + \phi_{i2}) \end{aligned} \tag{12}$$

Case 2: ***One pair of complex conjugate and two real and negative roots:***
The general displacement solutions are shown in (13).

$$X_i = A_{i1}e^{-pt}sin(qt + \phi_{i1}) + C_{i3}e^{s_3t} + C_{i4}e^{s_4t} \tag{13}$$

where *i* = 1,2.
Case 3: ***Four real and negative roots:***
The system has a large damping. When it is disturbed, the system will settle to its equilibrium configuration without oscillation. The solutions of the 4^{th} order polynomial yield four real and negative roots. In [2], the authors focused just on the first case. In this paper we will also focus on the third case. The third case is for the system which has a large damping. When it is disturbed, the system will settle to its equilibrium configuration without oscillation. The displacement signal of the real crash is simular to a case of an overdamped vibrating system. Hence the third case would represent the vehicle frontal crash reconstruction. The solution for the case 3 is:

$$X_i(t) = C_{i1}e^{s_1t} + C_{i2}e^{s_2t} + C_{i3}e^{s_3t} + C_{i4}e^{s_4t} \tag{14}$$

with $i = 1, 2$, where s_i are roots of the characteristic equation (10)

B. *Estimation of model parameters by Curve Fitting*

When (14) is curve fitted into displacement experimental data, the constants C_{ik} and s_k (i=1,2 and k=1,...,4) can be easily found and resulting in a system of equations that can be solved for k_2 and c_2.

$$C_{1k} = m_2s_k^2 + c_2s_k + k_2 \tag{15}$$
$$C_{2k} = c_2s_k + k_2 \tag{16}$$

For $k = 1, ..., 4$, equation (15) can be written in a matrix form as

$$\begin{bmatrix} 1 & s_1 \\ 1 & s_2 \\ 1 & s_3 \\ 1 & s_4 \end{bmatrix} \begin{bmatrix} k_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} C_{11} - m_2s_1^2 \\ C_{12} - m_2s_2^2 \\ C_{13} - m_2s_3^2 \\ C_{14} - m_2s_4^2 \end{bmatrix} \tag{17}$$

Let

$$\mathbf{A} = \begin{bmatrix} 1 & s_1 \\ 1 & s_2 \\ 1 & s_3 \\ 1 & s_4 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} C_{11} - m_2s_1^2 \\ C_{12} - m_2s_2^2 \\ C_{13} - m_2s_3^2 \\ C_{14} - m_2s_4^2 \end{bmatrix}, \mathbf{V}_2 = \begin{bmatrix} k_2 \\ c_2 \end{bmatrix}$$

Then, (17) can be represented as

$$\mathbf{A} * \mathbf{V}_2 = \mathbf{B}_2 \tag{18}$$

and, using the pseudo-inverse, we can obtain

$$\mathbf{V}_2 = (\mathbf{A}^T * \mathbf{A})^{-1} * \mathbf{B}_2 \tag{19}$$

The spring stiffness k_1 and damping coefficient c_1 are calculated from (21).

$$\begin{bmatrix} 1 & s_1 \\ 1 & s_2 \\ 1 & s_3 \\ 1 & s_4 \end{bmatrix} \begin{bmatrix} k_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} R_{11} - (m_1s_1^2 + c_2s_1 + k_2) \\ R_{12} - (m_1s_2^2 + c_2s_2 + k_2) \\ R_{13} - (m_1s_3^2 + c_2s_3 + k_2) \\ R_{14} - (m_1s_4^2 + c_2s_4 + k_2) \end{bmatrix} \tag{20}$$

Remark 1: From (6), it was shown that $R_{1k} = R_{2k} * \frac{S_{1k}}{S_{2k}}$. Let

$$\mathbf{V}_1 = \begin{bmatrix} k_1 \\ c_1 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} R_{11} - (m_1s_1^2 + c_2s_1 + k_2) \\ R_{12} - (m_1s_2^2 + c_2s_2 + k_2) \\ R_{13} - (m_1s_3^2 + c_2s_3 + k_2) \\ R_{14} - (m_1s_4^2 + c_2s_4 + k_2) \end{bmatrix}$$

then, we have

$$\mathbf{V}_1 = (\mathbf{A}^T * \mathbf{A})^{-1} * \mathbf{B}_1 \tag{21}$$

Therefore, the estimated parameters are obtained from (19) and (21).

C. *Method for calculating the natural frequencies and damping factor of the model*

The dymamic equation (1) can be rewritten in matix compact form as:

$$\boldsymbol{M} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \mathbf{L} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \boldsymbol{K} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1 \\ 0 \end{bmatrix} \tag{22}$$

with

$$\boldsymbol{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \boldsymbol{L} = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}, \boldsymbol{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

The state space representation of (22) is written as

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_c\mathbf{x} + \mathbf{B}_c\mathbf{u} \\ y &= \mathbf{C}_c\mathbf{x} + \mathbf{D}_c\mathbf{u} \end{aligned} \tag{23}$$

with

$$\boldsymbol{A}_c = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & -\frac{c_1+c_2}{m_1} & \frac{c_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{c_2}{m_2} \end{bmatrix}, \boldsymbol{B}_c = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix},$$

$$\boldsymbol{C}_c = \begin{bmatrix} -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & -\frac{c_1+c_2}{m_1} & \frac{c_2}{m_1} \end{bmatrix}, \boldsymbol{D}_c = \begin{bmatrix} \frac{1}{m_1} \end{bmatrix}$$

Form$\{A_c, B_c, C_c, D_c\}$ the transfer function is obtained as

$$H(s) = C_c(sI - A_c)^{-1} * B_c + D_c \tag{24}$$

A MATLAB function is avalable to convert the state space-realisation to transfer function from which the natural frequencies and damping factor can be obtaine. Therefore, (24) is usefull for extracting the natural frequencies and damping factors of the model.

III. Results and discussion

A. Parameters estimation from curve fitting approach

From the curve fitting, the values for s_i and C_{1j} (j=1,...,4) are found to be: $C_{11} = -253.6$, $C_{12} = 83.75$, $C_{13} = 333$, $C_{14} = -163.2$, $s_1 = -7.7540$, $s_2 = -6.238$,$s_3 = -9.603$, $s_4 = -10.87$.
The result from the curve fitting is shown in Figure 4.

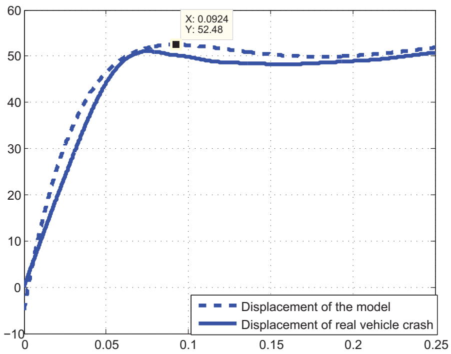

Fig. 4. Result from curve fitting

B. Vehicle crash experimental data analysis

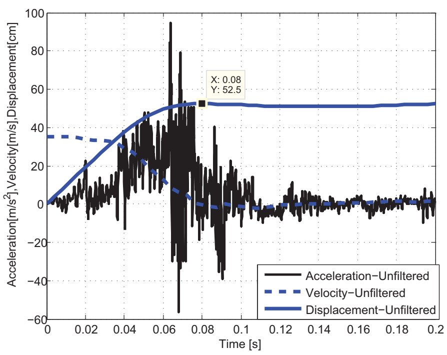

Fig. 5. Unfiltered data plot from the real vehicle crash test

It is observed from Figure 5, that the dynamic crash from the real vehicle crash test is 53.17 cm and occurs at time t_c= 0.078 s, when the unfiltered data are used in the analysis. The filtered data result in a dynamic crash of 51.11 cm at time t_c= 0.0745 s as shown in Figure 6.
The initial velociy for both filtered and unfiltered data is closer to 35 km/h (i.e. 34.99 km/h for the unfiltered data and 35.28 km/h for the filtered data).

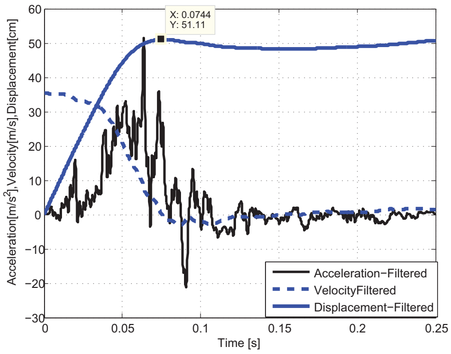

Fig. 6. Filtered data plot from the real vehicle crash test

C. Results from the model

Four different cases are considered in this section as a sample of results. Let m_1 be the mass of the chassis, m_2 the mass of passenger compartment and $m_t = 873kg$ the total mass of the vehicle.

Case 1: $(m_1 < m_2)$: $m_1 = \frac{1}{3}m_t$, $m_2 = \frac{2}{3}m_t$
From Figure 7, the dynamic crash of m_2 is 80 cm which is the displacement of the passenger compartment. Therefore this model cannot represent the vehicle crash scenarion. It is observed that, the time for dynamic crash is longer than that for the real crash (i.e. 0.17 s instead of 0.078 s).The dynamic crash of m_1 is 42.5 cm and occurs after 0.17s.

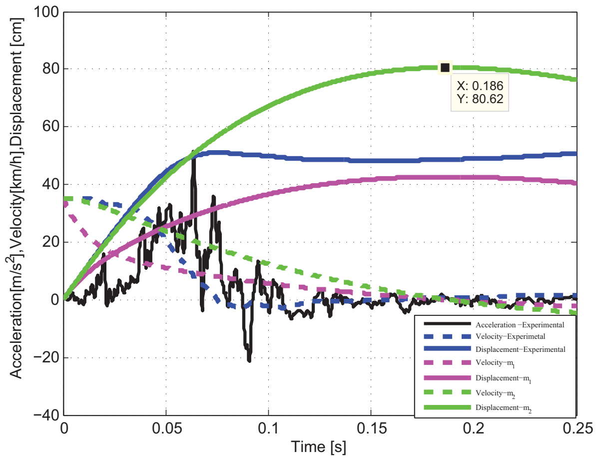

Fig. 7. Comparative analysis between Vehicle crash test and model results for $m_1 = \frac{1}{3}m_t$

Case 2: $(m_1 > m_2)$: $m_1 = \frac{2}{3}m_t$, $m_2 = \frac{1}{3}m_t$
From Figure 8, the dynamic crash of the passenger compartment m_2 is 66.2 cm . The time for dynamic crash increases futher up to 0.15 s. The dynamic crash of m_1 is 45.5 cm and occurs after 0.13s.Therefore for this case, the model cannot represent the vehicle crash scenarion.

Case 3: $(m_1 < m_2)$: $m_1 = \frac{1}{4}m_t$, $m_2 = \frac{3}{4}m_t$
From Figure 9, the dynamic crash of the passenger compartment m_2 is 69 cm . The time for dynamic crash is 0.15s. The dynamic crash of m_1 is 30.9 cm and occurs after 0.14s. This also cannot represent the real vehicle crash test.

Case 4: $(m_1 > m_2)$: $m_1 = \frac{3}{4}m_t$, $m_2 = \frac{1}{4}m_t$
From Figure 10, the dynamic crash of the passenger compartment m_2 is 49.8 cm and the time for dynamic crash is 0.11s. The dynamic crash of m_1 is 35.5 cm and occurs after 0.1s. Therefore, this case can represent the vehicle crash senarion because the dynamic crash is much closer to that from the real vehicle crash and the time is relatively small as compared to other cases.

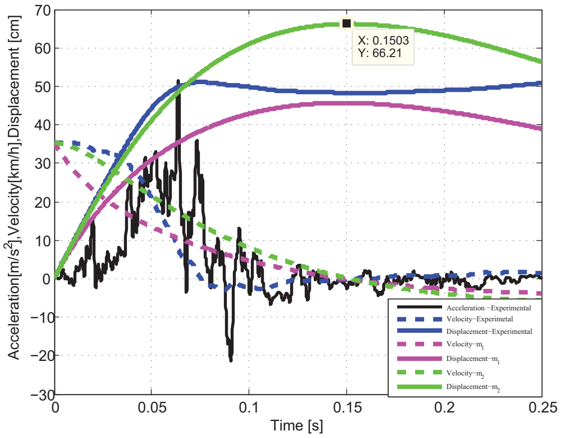

Fig. 8. Comparative analysis between Vehicle crash test and model results for $m_2 = \frac{1}{3} m_t$

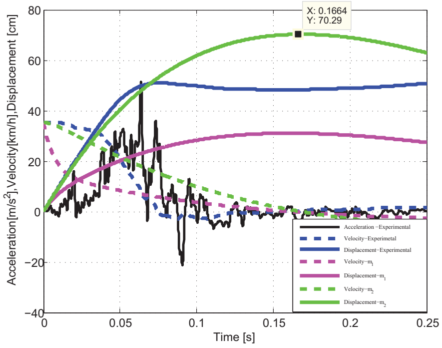

Fig. 9. Comparative analysis between Vehicle crash test and model results for $m_1 = \frac{1}{4} m_t$

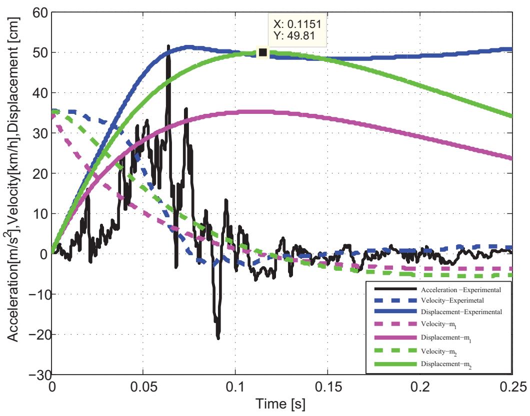

Fig. 10. Comparative analysis between Vehicle crash test and model results for $m_2 = \frac{1}{4} m_t$

A summary of main results is shown in Table I. The values for k_2 and c_2 are the first and second entry of vector $\mathbf{V}_2$ in (19). The values for k_2 and c_2 depend on the value of mass m_2 taken into consideration. Values for k_1 and c_1 are obtained from vector $\mathbf{V}_1$ in (21). The stiffness coefficients which result in a closer vehicle crash reconstruction are found to be $k_1 = 74681 N/m$, $k_2 = 45821 N/m$ and the damping coefficients are: $c_1 = 18176 Ns/m$, $c_2 = 11196 Ns/m$ when the mass of the chassis is $\frac{3}{4}$ the total mass of the vehicle (m_t), where the dynamic crash of the passenger compartment is equal to 49.8 cm and occurs after 0.11s (see sub section **4.3 case 4**, Figure 10). It is observed that the passenger compartment m_2 is the one that reconstructs the vehicle crash. When the mass of the chassis is greater than that of the passenger compartment, the results from the model are closer to the expected values. For example, when m_1 =582 kg (i.e.$\frac{3}{4}$ of the total mass of the vehicle) and m_2 =291 kg (i.e. $\frac{1}{4}$ of the mass of the vehicle), the dynamic crash of the passenger compartment is 49.8 cm which is closer to 51.11 cm (the dynamic crash from the real vehicle crash).

TABLE I
PARAMETERS ESTIMATION

Parameters	Case 1	Case 2	Case 3	Case4
	m_1=1/3 m_t	m_1=2/3 m_t	m_1=1/4 m_t	m_1=3/ 4 m_t
	Dynamic crash[cm]=80	Dynamic crash[cm]= 66	Dynamic crash[cm]=69	Dynamic crash[cm]=49.8
	Time of crash[s]=0.17	Time of crash[s]= 0.15	Time of crash[s]=0.14	Time of crash[s]=0.11
k_1 [N/m]	45929		74681	
k_2 [N/m]	40731		45821	
c_1 [Ns/m]	13687		18176	
c_2 [Ns/m]	9952		11196	

From the estimated spring stiffness and damping coeficients, the system matrices are

$$\boldsymbol{A}_c = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -184.0428 & 69.9824 & -44.8599 & 17.0997 \\ 209.9473 & -209.9473 & 51.2990 & -51.2990 \end{bmatrix}, \boldsymbol{B}_c = \begin{bmatrix} 0 \\ 0 \\ 0.0015 \\ 0 \end{bmatrix},$$

$$\boldsymbol{C}_c = \begin{bmatrix} -184.0428 & 69.9824 & -44.8599 & 17.0997 \end{bmatrix}, \boldsymbol{D}_c = \begin{bmatrix} 0.0015 \end{bmatrix}$$

$$H(s) = C_c(sI - A_c)^{-1} * B_c + D_c$$

$$H(s) = \frac{0.001527s^4 + 0.07835s^3 + 0.3207s^2 + 2.495e - 15s + 9.113 * 10^{-5}}{s^4 + 96.16s^3 + 1818s^2 + 1.168e04s + 2.395e04}$$

The natural frequencies expressed in rad/seconds are ω_n={4.3367, 6.2256, 12.0619, 73.5347}

The system has only one damping factor $\zeta = 1$, therefore the system is critically damped.

IV. CONCLUSION AND FITURE WORK

In this paper, it is presented a method to estimate the parameters of a double spring-mass-damper model of a vehicle frontal crash. It was observed that the model results were much closer to the experimental test results. The overall behavior of the model matches the real vehicle's crush.Two of the main parameters characterizing the collision are the maximum dynamic crush - which describes the highest car's deformation and the time at which it occurs- t_m. They are pertinent to the occupant crashworthiness since they help to assess the maximum intrusion to the passenger's compartment.

It can be concluded from this study, that a double spring-mass-damper model can represent the real vehicle crash scenarion when the mass m_1 representing the chassis of the vehicle is less than m_2, the mass representing the passenger compartment. In all cases, the front part of the vehicle undergoes smaller deformation than the passenger compartment.The time at which the maximum chassis displacement occurs is slightly shorter than the time for the passenger compartment because of its additional compression by the rest of the car. In our future work, We would like to extend our study to a three-mass-spring-damper model taking into consideration the nonlinearity of the spring and damper. The three masses will be representing the engine, the suspension and the passenger compartment mass respectively, interconnected by springs and dampers. Also injury mechanisms of the occupant such as Head Injury Criterion (HIC) would be considered in future work.

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