Applied Physics

(PHC-103/104)

Lecture 11

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What is Magnetism?

"Magnetism is a physical phenomenon resulting from the interaction between magnetic fields and magnetic materials"

Magnetic Fields

"A magnetic field is a region around a magnet or current-carrying wire where magnetic forces can be detected"

Figure 5.1 Compass showing direction of magnetic field

Magnetic Field Lines

"Magnetic field lines are imaginary lines that emerge from the north pole and enter the south pole of a magnet”

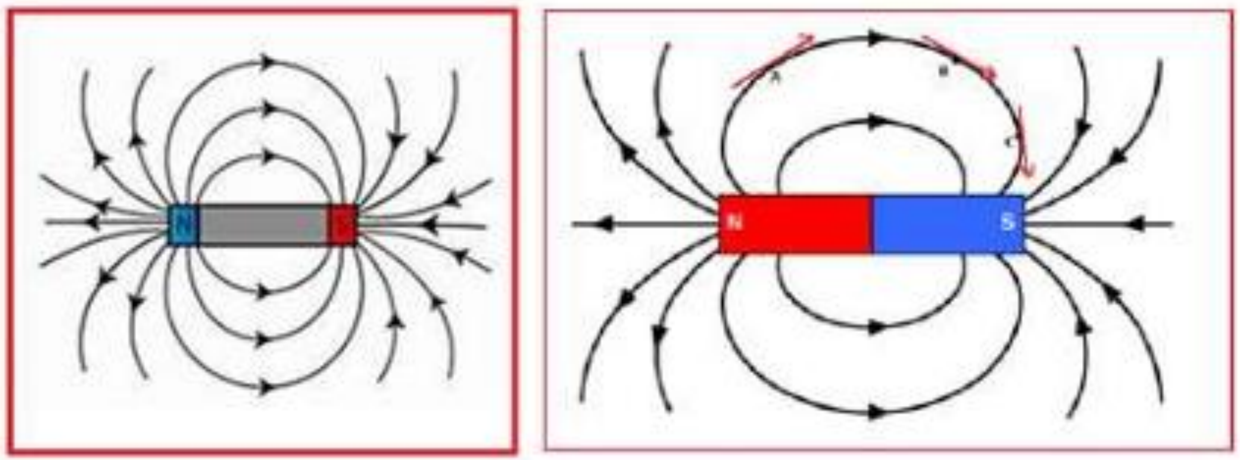


Figure 5.2 Magnetic field lines

Magnetic flux and Magnetic flux density

Magnetic flux is the number of magnetic field lines passing through a given area. It is denoted by ϕ and its unit is weber (Wb).

The number of magnetic field lines crossing unit area kept normal to the direction of field lines is called magnetic flux density. Its unit is Wb/m2

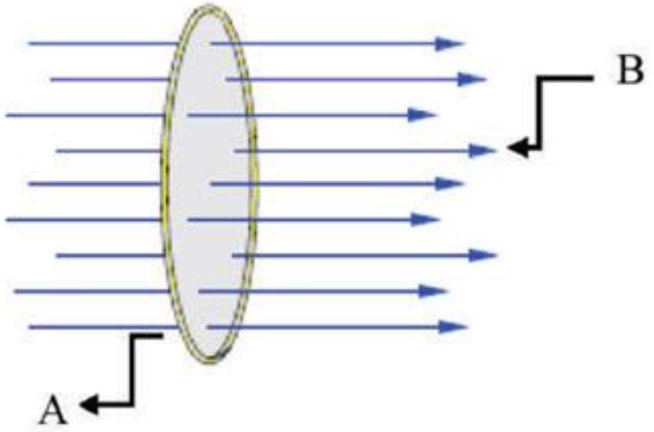


Figure 5.3 Magnetic flux

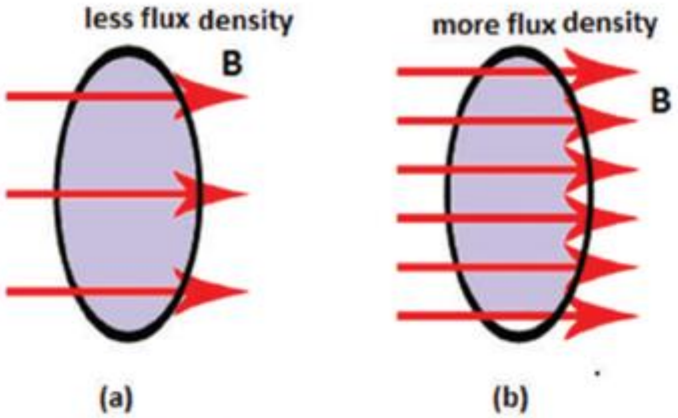


Figure 5.4 Magnetic flux density

Properties of magnetic lines of force

- Magnetic lines of force are closed, continuous curves, extending through the body of the magnet.
- Magnetic lines of force start from the North Pole and end at the South Pole.
- Magnetic lines of force never intersect.
- They will be maximum at the poles than at the equator.
- The tangent drawn at any point on the curved line gives the direction of magnetic field.

Magnetic effect of current

- It was on 21st April 1820, Hans Christian Oersted, a Danish Physicist was giving a lecture. He was demonstrating electrical circuits in that class.

- He had to often switch on and off the circuit during the lecture.

- Accidentally, he noticed the needle of the magnetic compass that was on the table. It deflected whenever he switched on and the current was flowing through the wire.

- The compass needle moved only slightly, so that the audience didn’t even notice.

- But it was clear to Oersted that something significant was happening.

- He conducted many experiments to find out a startling effect, the magnetic effect of current.

- Oersted aligned a wire XY such that they were exactly along the North-South direction.
- He kept one magnetic compass above the wire at A and another under the wire at B.
- When the circuit was open and no current was flowing through it, the needle of both the compass was pointing to north.
- Once the circuit was closed and electric current was flowing, the needle at A pointed to east and the needle at B to the west as shown in Figure.
- This showed that current carrying conductor produces magnetic field around it.

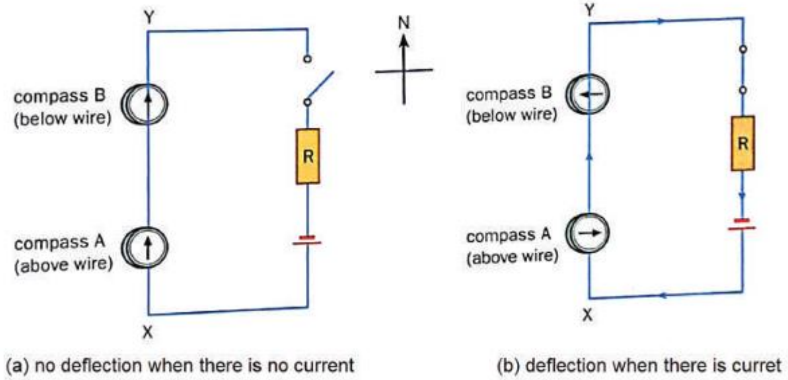
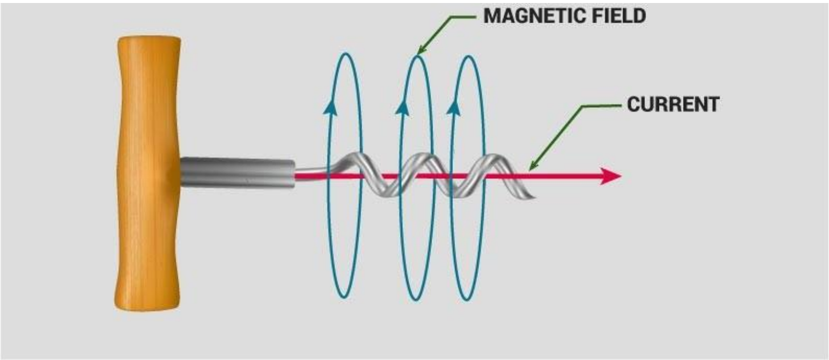


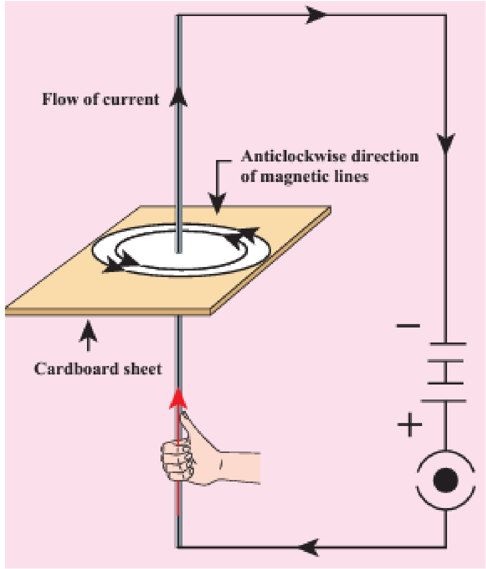
Figure 5.5 Current produces magnetic field

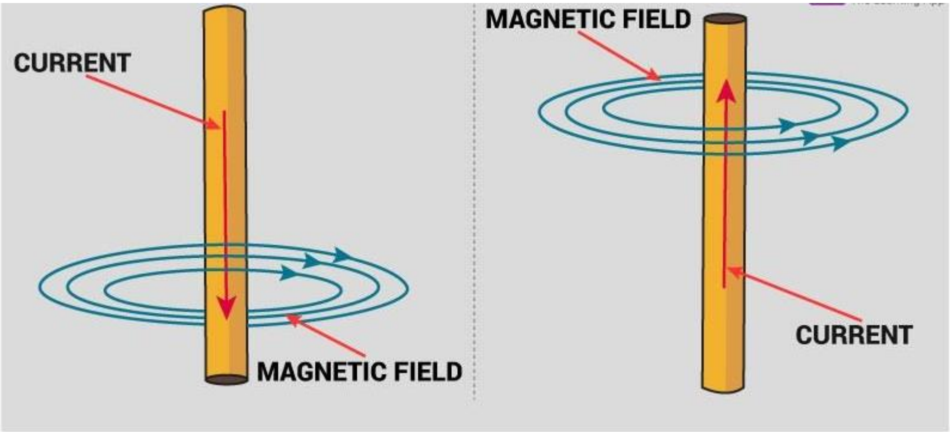
Direction of the magnetic lines around a current carrying conductor

The direction of the magnetic lines around a current carrying conductor can be easily understood using the right-hand thumb rule.

Hold the wire with four fingers of your right hand with thumbs-up position. If the direction of the current is towards the thumb, then the magnetic lines curl in the same direction as your other four fingers as shown in Figure. This shows that the magnetic field is always perpendicular to the direction of current.







Force on a charge moving in a uniform magnetic field

H.A.Lorentz found that a charge moving in a magnetic field, in a direction other than the direction of magnetic field, experiences a force. It is called the magnetic Lorentz force.

Consider a point charge of magnitude "q" moving in a uniform magnetic field of induction having strength B with a velocity v. The angle between velocity and magnetic intensity is "θ". Force experienced by the charge depends directly upon the following factors:

(i) The magnitude q of the charge:

$$F \propto q \quad \ldots\ldots\ldots\ldots\ldots\ldots (1$$

(ii) The velocity v of the charge:

$$F \propto v \quad \ldots\ldots\ldots\ldots\ldots\ldots (2$$

(iii) The strength of magnetic field

$$F \propto B \quad \ldots\ldots\ldots\ldots\ldots\ldots (3$$

(iv) The sine of the angle between force and velocity field:

$$F \propto \sin\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

By combining above four factors:

$$F \propto qvB \sin\theta$$

$$F = \text{Constant} \times qvB \sin\theta$$

This constant of proportionality depends upon the system of units used. If 1 coulomb charge enters perpendicularly in a magnetic field of 1 tesla with a velocity of 1m/s and experiences a force of 1N then in S.I. System of units value of this constant will be unity. Therefore,

$$\boxed{F = qvB \sin\theta}$$

CONDITION OF MAXIMA:

Force on a charged particle in a magnetic field will be maximum if it enters perpendicularly in the magnetic field, i.e. the angle between field $\overline{B}$ and velocity $\overline{V}$ is $\theta = 90^\circ$

$$F = qvB\ \mathrm{Sin}90^\circ$$

$$\boxed{F = qvB} \qquad (\text{as Sin}90^\circ = 1)$$

CONDITION OF MINIMA:

Force on a charged particle in a magnetic field will be minimum if it enters along the direction of magnetic field, i.e. the angle between field $\overline{B}$ and velocity $\overline{V}$ is $\theta = 0^\circ$

$$F = qvB\ \mathrm{Sin}0^\circ$$

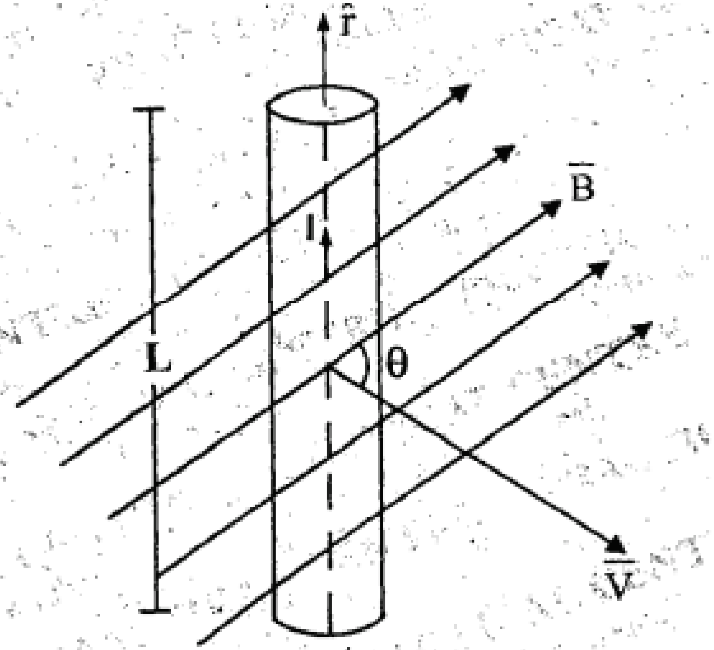
$$\boxed{F = 0} \qquad (\text{as sin}0^\circ = 0)$$

Force on a current carrying conductor in a magnetic field

A wire placed in magnetic field experiences a force when electric current is passed through it. It is due to the free electrons present in the conductor.

Consider a conductor wire of length "L" placed in magnetic field B. Let "A" be the cross-sectional area of the wire and "n" be the number of free electrons in unit volume of wire. The charge moving through the unit volume of elements is

$$q = nALe \quad \text{———————} \quad (a)$$



As the force on a charged particle moving in a magnetic field is given by:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

OR $$\vec{F} = nALe(\vec{v} \times \vec{B}) \qquad (1)$$

If the direction of drift velocity is specified by unit vector $\hat{r}$ then,

$$\vec{v} = v\hat{r}$$

Equation (1) => $F = nA\, Le\,(v\, r \times B)$

$$\vec{F} = nAve\,(L\, r \times \vec{B}) \quad \text{——(2)}$$

∴ But $L\, r = L$

Equation (2) => $\vec{F} = nAve\,(\vec{L} \times \vec{B})$ —— (3)

Where $\vec{L}$ is the vector length. The magnitude of velocity of charge passing through length L of conductor in time t is given by,

$$v = \frac{L}{t}$$

Equation (3) => $\vec{F} = nA\,\frac{L}{t}\,e\,(\vec{L} \times \vec{B})$

$$\vec{F} = \frac{nALe}{t}\,(\vec{L} \times \vec{B})$$

From equation "a" $\vec{F} = \frac{q}{t}\,(\vec{L} \times \vec{B})$

But $\frac{q}{t} = I$

∴ $\vec{F} = I\,(\vec{L} \times \vec{B})$

OR $F = BIL\, Sin\theta$

CONDITION OF MAXIMA:

Force on a current carrying conductor will be maximum when conductor is placed perpendicularly on to the field, that is the angle between field $\vec{B}$ and vector length $\vec{L}$ is $\theta = 90^\circ$

$$F = BIL \sin 90^\circ$$

$$F = BIL \qquad (\text{as } \sin 90^\circ = 1)$$

CONDITION OF MINIMA:

Force on a current carrying conductor will be minimum when conductor is placed parallel to the field, that is the angle between field $\vec{B}$ and vector length $\vec{L}$ is $\theta = 0^\circ$

$$F = BIL \sin 0^\circ$$

$$F = 0 \qquad (\text{as } \sin 0^\circ = 0)$$

DIRECTION OF FORCE:

The force is always a vector quantity. A vector quantity has both magnitude and direction. It means we should know the direction in which the force would act. The direction is often found using what is known as Fleming’s Left-hand Rule (formulated by the scientist John Ambrose Fleming).

The law states that while stretching the three fingers of left hand in perpendicular manner with each other, if the direction of the current is denoted by the middle finger of the left hand and the second finger is for direction of the magnetic field, then the thumb of the left hand denotes the direction of the force or movement of the conductor.

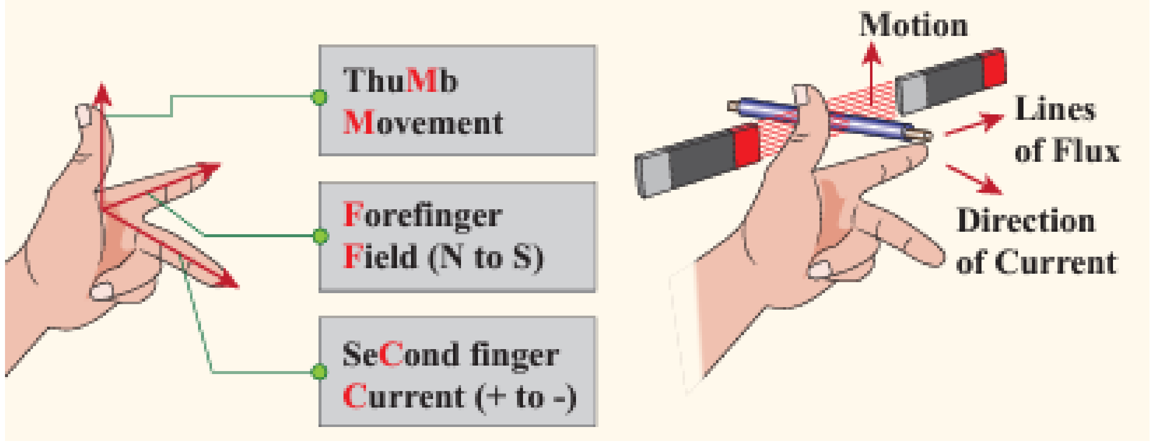
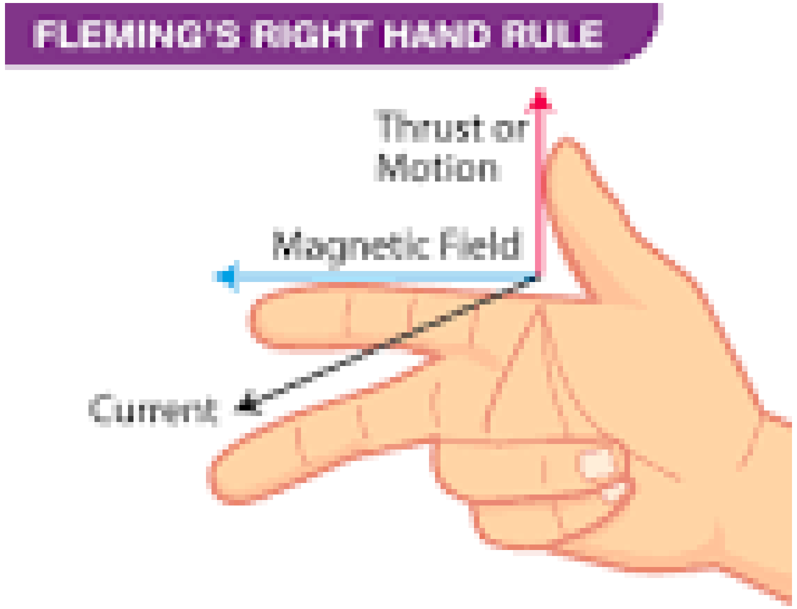


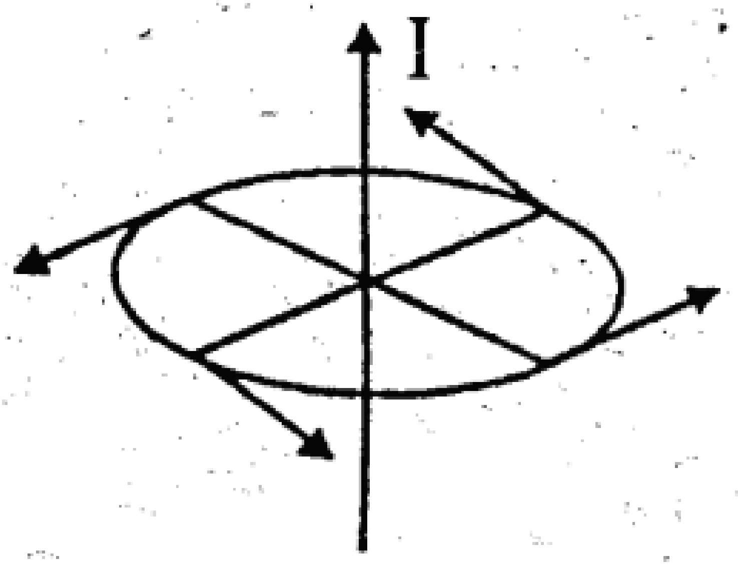
Figure 5.9 Fleming's left hand rule



The Biot-Savart law

The relation between the magnetic field of a current carrying conductor and current passing through a conductor was studied by Biot and Savat.

Consider a long straight wire carrying a current "I" and a closed curve consisting of a circle of radius 'r' with the wire at the centre as shown in the figure.



Biot-Savat deduced from the experience that the magnetic field 'B' around a long straight current carrying conductor is directly Proportional to the twice of current I passing through the conductor and is inversely Proportional to the distance 'r' from the conductor, mathematically.

$$B \propto \frac{2I}{r}$$

$$B = \frac{\mu_o}{{}^{2}\cancel{4}\pi} \times \frac{\cancel{2}I}{r}$$

$$\boxed{B = \frac{\mu_o I}{2\pi r}}$$

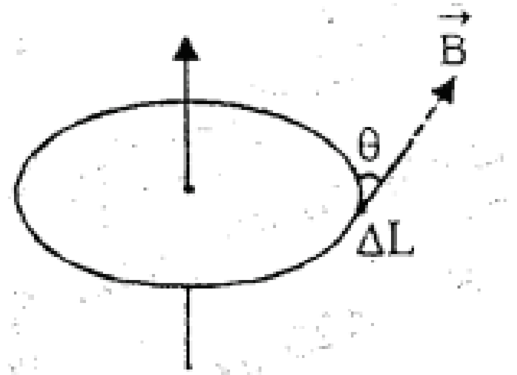
Where μ_o is the constant called the permeability of force space and its value is $4\pi \times 10^{-7}$ weber /A.m.

Amperes's Law

A current carrying conductor has a magnetic field around it. Ampere's Law provides the relation between the magnetic flux density and the current enclosed.

Consider a long straight.

Wire carrying a current 'I' with a magnetic field in the form of a closed curve consisting of a circle of radius 'r' around it. Let us divide the magnetic field into a large number of small length of elements ΔL, ΔL_2, ΔL_3, ………. The direction of each length of element is along the direction of magnetic field which is tangent on the circle at each length of element. Taking dot product of tangential component of magnetic field with all length of elements.



$$\vec{B}.\Delta\vec{L}_1 = B\Delta L_1 \text{Cos}\theta$$

$$\vec{B}.\Delta\vec{L}_1 = B\Delta L_1 \text{Cos}0^\circ$$

$$\vec{B}.\Delta\vec{L}_1 = B\Delta L_1$$

Similarly $\vec{B}.\Delta\vec{L}_2 = B\Delta L_2$

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$$\vec{B}.\Delta\vec{L}_N = B\Delta L_N$$

The sum of all dot Products is

$$\Sigma \vec{B} . \vec{\Delta L} = \Sigma B \Delta L$$

As tangial component of magnetic field 'B' remains constant at all length of elements, therefore,

$$\Sigma \vec{B} . \vec{\Delta L} = B \Sigma \Delta L$$

Where $\Sigma \Delta L$ is the circumference of the circle which is $2\pi r$ and 'B' is given by the Biot – Savat's Las as

$$B = \frac{\mu_o I}{2\pi r}$$

$$\therefore \quad \Sigma B . \Delta L = \frac{\mu_o I}{2\pi r} \times 2\pi r$$

$$\Sigma B . \Delta L = \mu_o I$$

$$\therefore \quad \Sigma \vec{B} . \vec{\Delta L} = \mu_o \times \text{Current}$$

This relation is called Ampere's Law and is stated as.

> ***"The sum of the dot Product of the tangential component of magnetic field of induction and the length of an element of a closed curve taken in the magnetic field is μ_o times the current passing through the conductor."***

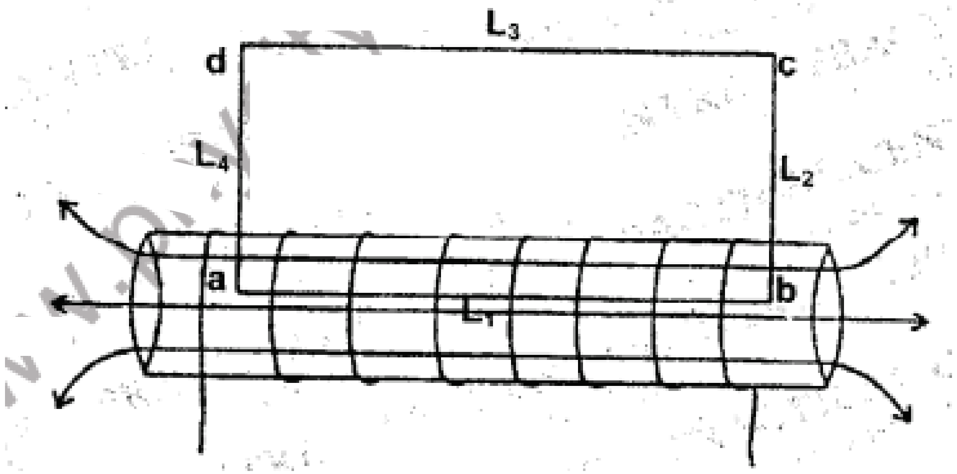
Application of Amperes's Law

I) FIELD DUE TO A SOLENOID:

A Solenoid is a long hollow pipe on which wire is wounded. The turns of the winding are closely spaced and may consist of one or more layers. The magnetic field produced by the solenoid in the middle of the solenoid is stronger and uniform but it is weaker and negligible outside the solenoid because magnetic lines are crowded and runs parallel with the axis of solenoid inside but they diverge outside the solenoid.

In order to determine the magnetic field B, consider a rectangular path abcda as shown in the figure. Let this path be divided into four elements of lengths as,

$ab = L_1$, $bc = L_2$, $cd = L_3$ and $da = L_4$. Such that the sum of dot product of magnetic field and the length of element is,



$$\sum \vec{B}.\Delta\vec{L} = BL_1Cos\theta_1 + BL_2Cos\theta_2 + BL_3Cos\theta_3 + BL_4Cos\theta_4$$

as L_1 is parallel to the magnetic field and lies inside the solenoid therefore,

$$\theta = 0^o$$

$$BL_1\ Cos\theta_1 = BL_1$$

L_2 and L_4 are perpendicular to the mangnetic field i.e. $\theta_2 = 90^o$ and $\theta_4 = 90^o$

$$BL_2Cos\theta_2 = 0$$

and $$BL_4Cos\theta_4 = 0$$

and L_3 lies outside the solenoid where the field is negligibly weaker,

i.e. $$B = 0$$

$\therefore$ $$BL_3Cos\theta_3 = 0$$

Putting all the values in eq (1).

$$\sum \vec{B}.\Delta\vec{L} = BL_1 + 0 + 0 + 0$$

$$\sum \vec{B}.\Delta\vec{L} = BL_1 \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

According to the ampere's law

$$\sum \vec{B}.\Delta\vec{L} = \mu_o \text{ x current} \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

If N is the number of turns of coil then

$$\text{Current} = NI$$

And if 'n' is the number of turns per unit length, then

$$n = \frac{N}{L_1}$$

OR $$N = nL_1$$

$\therefore$ $$\text{Current} = nL_1 I$$

$\therefore$ eq. (3) => $$\sum \vec{B}.\Delta\vec{L} = \mu_o nL_1 I \qquad \ldots\ldots\ldots\ldots\ldots (4)$$

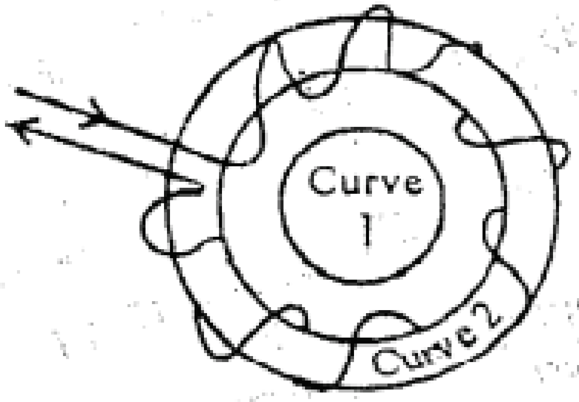
By comparing equation (2) and (4), we get

$$\mu_o n\cancel{L}_1 I = B\cancel{L}_1$$

OR $$\boxed{\mathbf{B = \mu_o n I}}$$

II) FIELD DUE TO A TOROID:

A Toroid or a toroidal coil is a circular solenoid. When a current pass from a toroid circular magnetic lines of induction form inside the toroid. The field outside the toroid is almost zero, as magnetic lines are confined inside of the toroid only.



For determining the field B, let us divide the loop into a large number of very small elements, so that each element is considered as straight. The direction of $\vec{B}$ at any point is tangent to the circle passing through that point and having center at the center of the toroid. *By symmetry* $\vec{B}$ *is constant everywhere along the loop and its direction is parallel to each of the* elementary length $\Delta\vec{L}$, such that the sum of all dot products of $\vec{B}$ and $\Delta\vec{L}$ is given by

$$\sum \vec{B}.\Delta\vec{L} = B\Delta L_1 \text{Cos}0^\circ + B\Delta L_2 \text{Cos}0^\circ + \text{-------}$$

$$\sum \vec{B}.\Delta\vec{L} = \sum B\Delta L$$

OR $$\sum \vec{B}.\Delta\vec{L} = B\sum \Delta L$$

Where $\sum \Delta L$ is the circumference of the toroidal coil. If "r" is radius of toroid then:

$$\sum \Delta L = 2\pi r$$

$$\therefore \qquad \sum \vec{B} . \Delta \vec{L} = B \times 2\pi r \text{ ----------- (1)}$$

But according to Ampere's Law

$$\sum B . \Delta L = \mu_o \times \text{current}$$

If I is the current flowing through each turn of the toroid "N" then the current enclosed will be NI.

OR $$\sum \vec{B} . \Delta \vec{L} = \mu_0 \text{NI. ----------- (2)}$$

By comparing eq. (1) and (2), we get $\Rightarrow \mu_0 NI = B \times 2\pi r$

OR $$\boxed{B = \frac{\mu_o NI}{2\pi r}}$$

Problems

- 1. A charge of 2 μC is moving with a velocity of 3 m/s in a magnetic field of 0.5 T. If the angle between the velocity and magnetic field is 30°, what is the force experienced by the charge?

- 2. A proton with a charge of 1.6 x 10^-19 C is moving with a velocity of ? m/s in a magnetic field of 0.2 T. If the angle between the velocity and magnetic field is 45°, The force experienced by the proton is 4.525 x 10^-14 N.

- 3. An electron with a charge of -1.6 x 10^-19 C is moving with a velocity of 5 x 10^6 m/s in a magnetic field of 0.1 T. If the angle between the velocity and magnetic field is 60°, what is the force experienced by the electron?

- 4. A current of 2 A is flowing through a wire of length 0.5 m in a magnetic field of 0.3 T. If the angle between the wire and magnetic field is 30°, what is the force experienced by the wire?

- 5. A charge of 5 μC is moving with a velocity of 4 m/s in a magnetic field of 0.2 T. If the angle between the velocity and magnetic field is 45°, what is the force experienced by the charge? If the charge is then accelerated to a velocity of 6 m/s, what is the new force experienced by the charge?

- 6. A proton with a charge of 1.6 x 10^-19 C is moving with a velocity of 3 x 10^6 m/s in a magnetic field of 0.1 T. If the angle between the velocity and magnetic field is 60°, what is the force experienced by the proton? If the magnetic field is then increased to 0.2 T, what is the new force experienced by the proton?

- 7. A charge of 2 μC is moving with a velocity of 5 m/s in a magnetic field of 0.5 T. If the angle between the velocity and magnetic field is 30°, what is the force experienced by the charge? If the charge is then moved to a region with a magnetic field of 0.8 T and an angle of 45°, what is the new force experienced by the charge?

- 8. A proton with a charge of 1.6 x 10^-19 C is moving with a velocity of 2 x 10^6 m/s in a magnetic field of 0.2 T. If the angle between the velocity and magnetic field is 60°, what is the force experienced by the proton? If the proton is then accelerated to a velocity of 3 x 10^6 m/s and the magnetic field is increased to 0.3 T, what is the new force experienced by the proton?

- 9. A charge of 5 μC is moving with a velocity of 4 m/s in a magnetic field of 0.3 T. If the angle between the velocity and magnetic field is 45°, what is the force experienced by the charge? If the charge is then moved to a region with a magnetic field of 0.5 T and an angle of 30°, what is the new force experienced by the charge?

- 10. An electron with a charge of -1.6 x 10^-19 C is moving with a velocity of 6 x 10^6 m/s in a magnetic field of 0.1 T. If the angle between the velocity and magnetic field is 60°, what is the force experienced by the electron? If the electron is then accelerated to a velocity of 8 x 10^6 m/s and the magnetic field is increased to 0.2 T, what is the new force experienced by the electron?

- 11. A wire of length 0.5 m carries a current of 10A in a uniform magnetic field of 0.2 T. The wire makes an angle of 30∘ with the magnetic field. Find the force on the wire.

- 12. A solenoid has 1000 turns/m and carries a current of 3 A. Find the magnetic field inside the solenoid.

- 13. A toroid has 500 turns, a mean radius of 0.1 m, and carries a current of 2 A. Calculate the magnetic field inside the toroid.

- 14. Find the magnetic field at the center of a circular loop of radius 0.2 m carrying 10 A.

- 15. A charge of 2 μC2 moves at a velocity of 500 m/s perpendicular to a magnetic field of ? T. The magnetic force is 2×10 −4 N .