Predicates and Quantifiers

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Predicates

Understanding

- The word predicate refers to the part of sentence that gives information about subject
- James is a student of Bedford College
- James" is subject
- is the student of Bedford College" is a predicate

Cont...

- Let p stands for "is the students of Bedford college"
- and x is a student at Bedford college
- Symbolized as p(x)

Definition

 A predicate is a sentence that contains a finite number of variables and becomes a proposition when specific value are substituted for the set of all values that may be substituted in place of variable

P(x) denoted the statement "x > 3"

Truth values of p(4) and p(2)

For p(4): 4 > 3 true

p(2): 2 > 3 false

Propositional Functions & Predicates

- •Propositional function (open sentence):
- statement involving one or more variables,
- e.g.: x-3 > 5.
- •Let us call this propositional function P(x), where P is the predicate and x is the variable.

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What is the truth value of P(2)? false
What is the truth value of P(8)? false
What is the truth value of P(9)? true
When a variable is given a value, it is said to be
instantiated
Truth value depends on value of variable
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Propositional Functions

•Let us consider the propositional function Q(x, y, z) defined as:

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•x + y = z.
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•Here, Q is the predicate and x, y, and z are the variables.

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What is the truth value of Q(2, 3, 5) true What is the truth value of Q(0, 1, 2)? false What is the truth value of Q(9, -9, 0)? true
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A propositional function (predicate) becomes a proposition when all its variables are instantiated.

Propositional Function

 By taking a variable subject denoted by symbols such as x, y, z, and applying a predicate one obtains a propositional function (or formula).

Propositional Function contd...

• When an object from the universe is plugged in for *x*, *y*, etc. a truth value results:

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(1) \times x is tall. ...e.g. plug in x = Johnny
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(3) n is a prime number. ...e.g. plug in
$$n = 111$$

Multivariable Predicates

- *Multivariable predicates* generalize predicates to allow descriptions of relationships between subjects. These subjects may or may not even be in the same universe of discourse. For example:
 - Johnny is taller than Debbie.
 - 17 is greater than one of 12, 45.
 - Johnny is at least 5 inches taller than Debbie

Multivariable Propositional Functions

 The multivariable predicates, together with their variables create multivariable propositional functions. In the above examples, we have the following generalizations:

- x is taller than y
- a is greater than one of b, c
- x is at least n inches taller than y

Quantifiers

Quantifiers

• When all the variables in a propositional function are assigned values the resulting statement has a truth value.

Quantifiers contd...

• However, there is another way, called quantification, to create a propositional, to create a proposition from a propositional function.

Types of Quantification

Two types of quantifications

- Universal Quantification
- Existential Quantification

Universal Quantification

 The universal quantification of p(x) is the proposition "p(x) is true for all values of x in the universe of discourse"

The notation

$$\forall x p(x)$$

• Denotes the universal quantification of p(x). Here \forall is called the universal quantifier

Let p(x) be the statement
 "x+1 > x". What is the truth value of the quantification ∀xp(x). Where the universe of discourse is the set of real numbers.

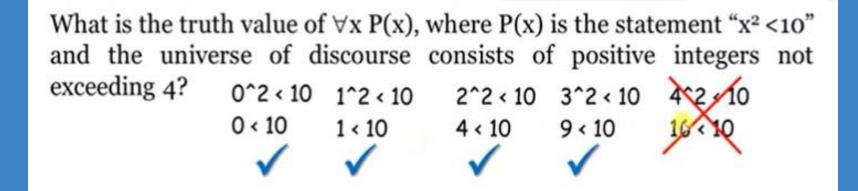
- Solution

• Since p(x) is true for all real number x, the quantification $\forall xp(x)$ is true

• Let Q(x) be the statement "x < 2" what is the truth value of the quantification $\forall x Q(x)$ where the universe of discourse is the real number.

- Solution
- Q(x) is not true for all real numbers x, since, for instance, Q(3) is false.

Thus $\forall xQ(x)$ is false



Important note

• When all of the element the universe of discourse can be listed – say $x_1, x_2, ..., x_n$ it follows that the universal quantification $\forall xp(x)$ is the same as the conjuction

P(x₁)Λp(x₂) Λ p(x₃) Λ ... Λ p(x_n)
 Since this conjuction is true if and only if p(x₁),p(x₂), p(x₃), ..., p(x_n) is true

Existential Quantification

• The existential quantification of p(x) is the proposition

"There exists an element x in the universe of discourse such that p(x) is true"

Use the notation

 $\exists xp(x)$

• For the existential quantification of p(x). Here ∃ is called the existential quantifier

• Let p(x) denoted statement "x > 3" what is the truth value of the quantification $\exists xp(x)$, where the universe of discourse is the set of real number?

• Solution

Since "x>3" is true for all instance, when x = 4 then existential quantification of p(x), which is $\exists xp(x)$, is true

Important Note

When all of the elements in the universe of discourse an be listed say x₁, x₂, ..., x_n
 the existential quantification ∃x p(x)

is same as the disjunction

• $p(x_1) \lor p(x_2) \lor p(x_3) \lor ... \lor p(x_n)$ Since this disjunction is true if and