Sets

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SET BUILDER NOTATION

$$\{x : x \in N \text{ and } x > 10\}$$

first half: what we want to include in our set

Second half: constrains on objects specified in first half for it to be an element of the set.

READING SET BUILDER NOTATION

$\{x : x \times 2 = 5\}$ $\{2.5\}$

$\{x : x = 2k \text{ and } k \in \{1,2,3\}\}$ $\{2,4,6$

$\{x : x \in N \text{ and } \frac{x}{3} \in N\}$

$\{3,6,9,12,15,\ldots\}$

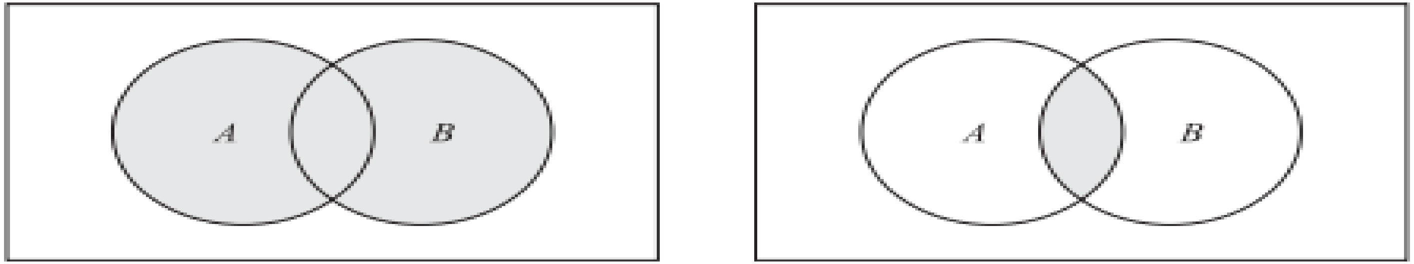
$\{x \mid x = 2y \text{ for some } y \in Z^{+}\}$

$\{2,4,6,8,10,12,\ldots\}$

SET OPERATIONS
Union and Intersection:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$



(a) $A \cup B$ is shaded (b) $A \cap B$ is shaded

Properties of the union operation

- $A \cup \varnothing = A$ Identity law
- $A \cup \boldsymbol{U} = \boldsymbol{U}$ Domination law
- $A \cup A = A$ Idempotent law
- $A \cup B = B \cup A$ Commutative law
- $A \cup (B \cup C) = (A \cup B) \cup C$ Associative law

Properties of the intersection operation

- $A \cap \boldsymbol{U} = A$ Identity law
- $A \cap \varnothing = \varnothing$ Domination law
- $A \cap A = A$ Idempotent law
- $A \cap B = B \cap A$ Commutative law
- $A \cap (B \cap C) = (A \cap B) \cap C$ Associative law

EXAMPLE

$$A = \{1,2,3,4,5\}$$

$$B = \{0,2,4,6,8\}$$

$$C = \{0,5,10,15\}$$

$$D = \{\}$$

Complements, Differences

$$A^{C} = \{x \mid x \in \mathbf{U}, x \notin A\}$$

$$A \setminus B = \{x \mid x \in A, x \notin B\}$$

EXAMPLE

$A = \{1,2,3,4,5\}$

$B = \{0,2,4,6,8\}$

$C = \{0,5,10,15\}$

$D = \{\}$

$A - B$

$B - A$

$C - D$

$(A - C) - (D - B) =$

COMPLEMENT

Properties of complement sets

- $\overline{\overline{A}} = A$ Complementation law
- $A \cup \overline{A} = U$ Complement law
- $A \cap \overline{A} = \varnothing$ Complement law

EXAMPLE

Let $A = \{a, e, i, o, u\}$ (where the universal set is the set of letters of the English alphabet). Then

A^C or $A' = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$.

Symmetric Difference

$$A \oplus B = \{\ x \mid (x \in A \text{ or } x \in B) \text{ and } x \notin A \cap B\}$$

$$A \oplus B = (A \cup B) - (A \cap B)$$

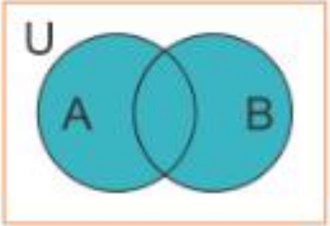
EXAMPLE

1. $\{1, 2, 3\} \oplus \{3, 4, 5\} = \{1, 2, 4, 5\}$
2. {New York, Washington} $\oplus$ $\{3, 4\}$ = {New York, Washington, 3, 4}
3. $\{1, 2\} \oplus \emptyset = \{1, 2\}$

Venn Diagrams: Shows logical relations between a finite collection of sets.

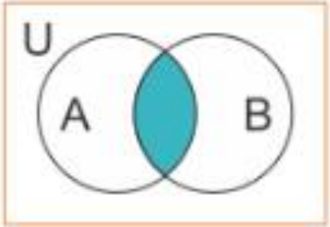
Union of Sets - Consists of all elements in sets A and B.

$A \cup B$



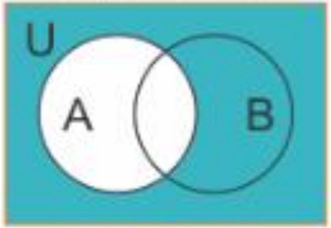
Intersection of Sets - Consists of only the common elements in sets A and B.

$A \cap B$



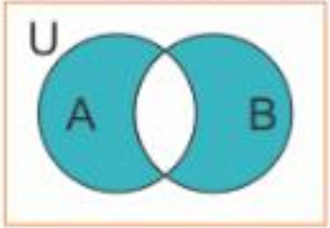
Complement of Set - Consists of elements which do not belong to set A.

$\overline{A}$, A^c or A'



Symmetric Difference of Sets - Consists of elements in sets A and B but not in their intersection.

$A \triangle B$ or $A \ominus B$



SOLVE THIS

$$P_8 = A^C \cap B^C \cap C^C.$$

Applications of Venn diagram

- A number of computer users are surveyed to find out if they have a printer, modem or scanner. Draw separate Venn diagrams and shade the areas, which represent the following configurations.
  a. modem and printer but no scanner
  b. scanner but no printer and no modem
  c. scanner or printer but no modem
  d. no modem and no printer

SOLUTION

- Let

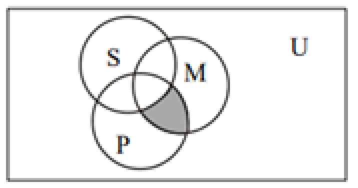
  P represent the set of computer users having printer.

  M represent the set of computer users having modem.

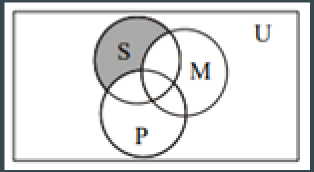
  S represent the set of computer users having scanner

SOLUTION

a. Modem and printer but no Scanner is shaded.

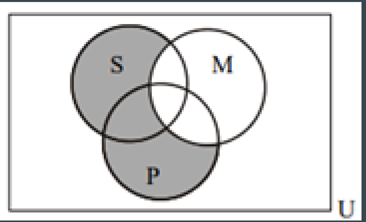


b. Scanner but no printer and no modem is shaded.

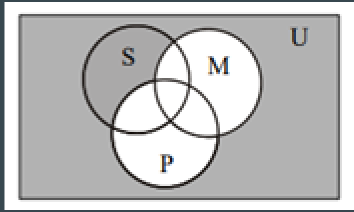


SOLUTION

c. scanner or printer but no modem is shaded.



c. no modem and no printer is shaded.



Task:

Students enrolled in extracurricular activities: Sports, Music, and Drama.

Students in Sports and Music but not Drama.
Students in Drama only, with no Sports and no Music.
Students in Music or Drama but not Sports.
Students who are not in Music and not in Sports.
Students participating in all three activities: Sports, Music, and Drama.

Membership tables of set - Union

A	B	$A \cup B$
1	1	1
1	0	1
0	1	1
0	0	0

Membership tables of Intersection

A	B	$A \cap B$
1	1	1
1	0	0
0	1	0
0	0	0

Membership tables of Difference

A	B	A – B
1	1	0
1	0	1
0	1	0
0	0	0

Membership tables of Complement

A	A^c
1	0
0	1

Computer representation of sets

- Assume that U is finite (and reasonable!)
  - Let U be the alphabet
- Each bit represents whether the element in U is in the set
- The vowels in the alphabet:

  abcdefghijklmnopqrstuvwxyz
  10001000100000100000100000
- The consonants in the alphabet:

  abcdefghijklmnopqrstuvwxyz
  01110111011111011111011111

Computer representation of sets

Consider the union of these two sets:

  1000100010000001000000100000

∨0111011101111110111111011111

____

  11111111111111111111111111

Consider the intersection of these two sets:

  1000100010000001000000100000

∧0111011101111110111111011111

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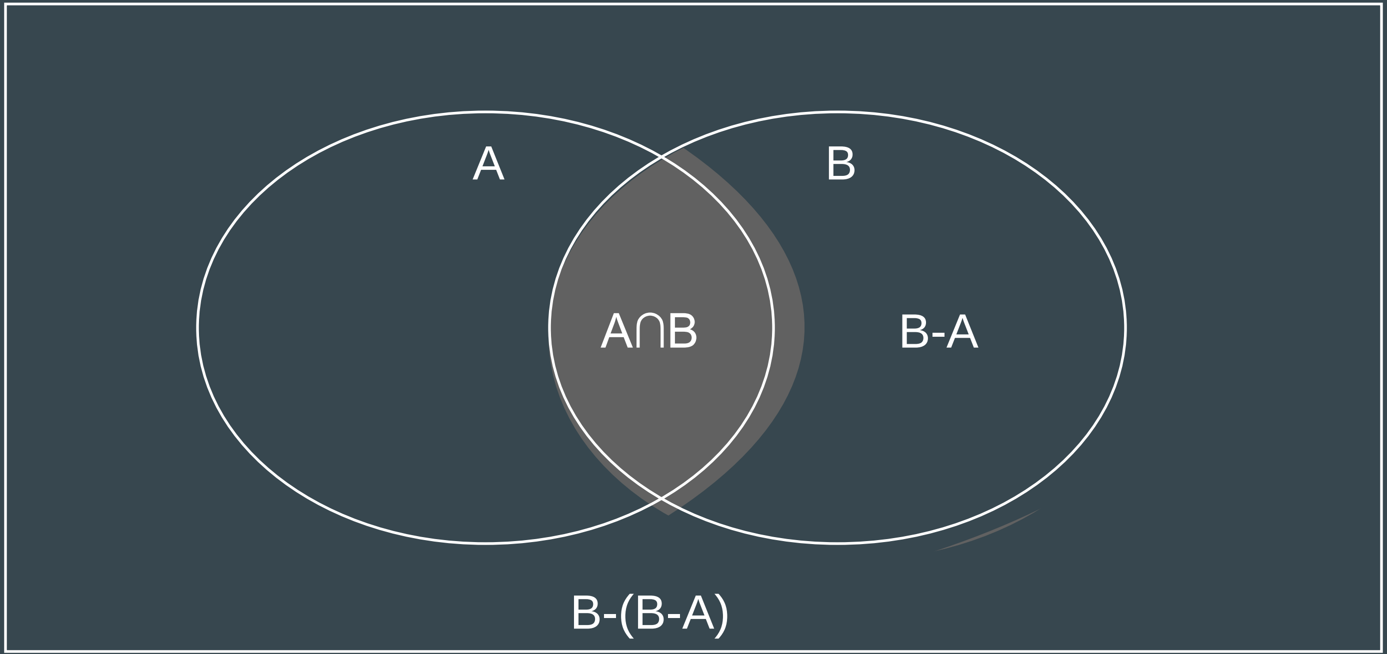
 0000000000000000000000000000

What we are going to prove…

$A \cap B = B - (B - A)$



Proof by membership tables

- The following membership table shows that $A \cap B = B - (B - A)$

A	B	A ∩ B	B-A	B-(B-A)
1	1	1	0	1
1	0	0	0	0
0	1	0	1	0
0	0	0	0	0

- Because the two indicated columns have the same values, the two expressions are identical
- This is similar to Boolean logic!