Predicates and Quantifiers

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Predicates

Understanding

- The word predicate refers to the part of sentence that gives information about subject
- James is a student of Bedford College
- James” is subject
- is the student of Bedford College” is a predicate

Cont…

- Let *p* stands for "is the students of Bedford college"
- and ***x*** is a student at Bedford college
- Symbolized as *p*(x)

Definition

- A predicate is a sentence that contains a finite number of variables and becomes a proposition when specific value are substituted for the set of all values that may be substituted in place of variable

Example

P(x) denoted the statement “x > 3”

Truth values of p(4) and p(2)

For p(4): $4 > 3$ true

p(2): $2 > 3$ false

Propositional Functions & Predicates

- Propositional function (open sentence):
- statement involving one or more variables,
- e.g.: $x-3 > 5$.
- Let us call this propositional function $P(x)$, where P is the predicate and x is the variable.

What is the truth value of $P(2)$? false
What is the truth value of $P(8)$? false
What is the truth value of $P(9)$? true

When a variable is given a value, it is said to be instantiated
Truth value depends on value of variable

Propositional Functions

- Let us consider the propositional function Q(x, y, z) defined as:
- $x + y = z$.
- Here, Q is the predicate and x, y, and z are the variables.

What is the truth value of Q(2, 3, 5) ? true

What is the truth value of Q(0, 1, 2) ? false

What is the truth value of Q(9, -9, 0) ? true

A propositional function (predicate) becomes a proposition when all its variables are instantiated.

Propositional Function

- By taking a variable subject denoted by symbols such as *x*, *y*, *z*, and applying a predicate one obtains a ***propositional function*** (or ***formula***).

Propositional Function contd…

- When an object from the universe is plugged in for *x*, *y*, etc. a truth value results:

(1) x is tall. …e.g. plug in x = Johnny

(2) y is structurally sound. …e.g. plug in
y = GWB

(3) n is a prime number. …e.g. plug in
n = 111

Multivariable Predicates

- ***Multivariable predicates*** generalize predicates to allow descriptions of relationships between subjects. These subjects may or may not even be in the same universe of discourse. For example:
  - Johnny *is taller than* Debbie.
  - 17 *is greater than one of* 12, 45.
  - Johnny *is at least* 5 *inches taller than* Debbie

Multivariable Propositional Functions

- The multivariable predicates, together with their variables create ***multivariable propositional functions***. In the above examples, we have the following generalizations:

  - x is taller than y
  - a is greater than one of b, c
  - x is at least n inches taller than y

Quantifiers

Quantifiers

- When all the variables in a propositional function are assigned values the resulting statement has a truth value.

Quantifiers contd…

- However, there is another way, called quantification, to create a propositional, to create a proposition from a propositional function.

Types of Quantification

- Two types of quantifications
  - Universal Quantification
  - Existential Quantification

Universal Quantification

- The universal quantification of p(x) is the proposition

  "p(x) is true for all values of x in the universe of discourse"

  The notation

  $$\forall x\, p(x)$$

- Denotes the universal quantification of p(x). Here ∀ is called the universal quantifier

Example

- **Let p(x) be the statement "x+1 > x". What is the truth value of the quantification ∀xp(x). Where the universe of discourse is the set of real numbers.**

\- **Solution**

- Since *p(x)* is true for all real number x, the quantification ∀xp(x) is true.

Example

- Let $Q(x)$ be the statement "$x < 2$" what is the truth value of the quantification $\forall x Q(x)$ where the universe of discourse is the real number.

\- Solution

- $Q(x)$ is not true for all real numbers x, since, for instance, $Q(3)$ is false.

  Thus $\forall x Q(x)$ is false

Example

What is the truth value of $\forall x\, P(x)$, where $P(x)$ is the statement "$x^2 < 10$" and the universe of discourse consists of positive integers not exceeding 4?

$0^2 < 10$	$1^2 < 10$	$2^2 < 10$	$3^2 < 10$	$\cancel{4^2 < 10}$
$0 < 10$	$1 < 10$	$4 < 10$	$9 < 10$	$\cancel{16 < 10}$
✓	✓	✓	✓	

Important note

- When all of the element the universe of discourse can be listed – say x_1, x_2, … , x_n it follows that the universal quantification $\forall x p(x)$ is the same as the conjuction

- $P(x_1) \wedge p(x_2) \wedge p(x_3) \wedge \ldots \wedge p(x_n)$

  Since this conjuction is true if and only if $p(x_1)$,$p(x_2)$, $p(x_3)$, … , $p(x_n)$ is true

Existential Quantification

- The existential quantification of p(x) is the proposition

  " There exists an element x in the universe of discourse such that p(x) is true"

  Use the notation

  $$\exists x p(x)$$

- For the existential quantification of p(x). Here $\exists$ is called the existential quantifier

Example

- Let p(x) denoted statement "$x > 3$" what is the truth value of the quantification $\exists x p(x)$, where the universe of discourse is the set of real number?

- <u>Solution</u>

  Since "$x>3$" is true for all instance, when $x = 4$ then existential quantification of p(x), which is $\exists x p(x)$, is true

Important Note

- When all of the elements in the universe of discourse an be listed say x_1, x_2, …, x_n the existential quantification $\exists x\ p(x)$ is same as the disjunction

- $p(x_1) \vee p(x_2) \vee p(x_3) \vee \ldots \vee p(x_n)$ Since this disjunction is true if and