Applied Physics

(PHC-103/104)

Lecture

Course Instructor: Engr. Abdul Moiz

Department of Biomedical Engineering

Email: abdul.moiz@shu.edu.pk

Definition of Electric Force

- All material is made up of atoms, which contain protons, neutrons, and electrons.
- Protons are positively charged, electrons are negatively charged, and neutrons have no charge.
- Electrons can be transferred from one object to another, causing an imbalance of protons and electrons in an object.
- We call such an object with an imbalance of protons and electrons a charged object.
- A negatively charged object has a greater number of electrons, and a positively charged object has a greater number of protons.
- There is an **electric force** in a system when charged objects interact with other objects.
- Positive charges attract negative charges, so the electric force between them is attractive.

"The electric force is repulsive for two positive charges, or two negative charges."

Or

"Electric force is the attractive or repulsive force between charged objects or point charges."

Properties of Electric Charges

- In a series of simple experiments, it was found that there are two kinds of electric charges, which were given the names positive and negative by Benjamin Franklin (1706–1790).
- **For example**, after rubbing a balloon on your hair on a dry day, you will find that the balloon attracts bits of paper. The attractive force is often strong enough to suspend the paper from the balloon.
- Another important aspect of electricity that arises from experimental observations is that electric charge is always conserved in an isolated system. That is, when one object is rubbed against another, charge is not created in the process. The electrified state is due to a transfer of charge from one object to the other.
- One object gains some amount of negative charge while the other gains an equal amount of positive charge.
- For example, when a glass rod is rubbed on silk, the silk obtains a negative charge equal in magnitude to the positive charge on the glass rod. We now know from our understanding of atomic structure that electrons are transferred in the rubbing process from the glass to the silk.

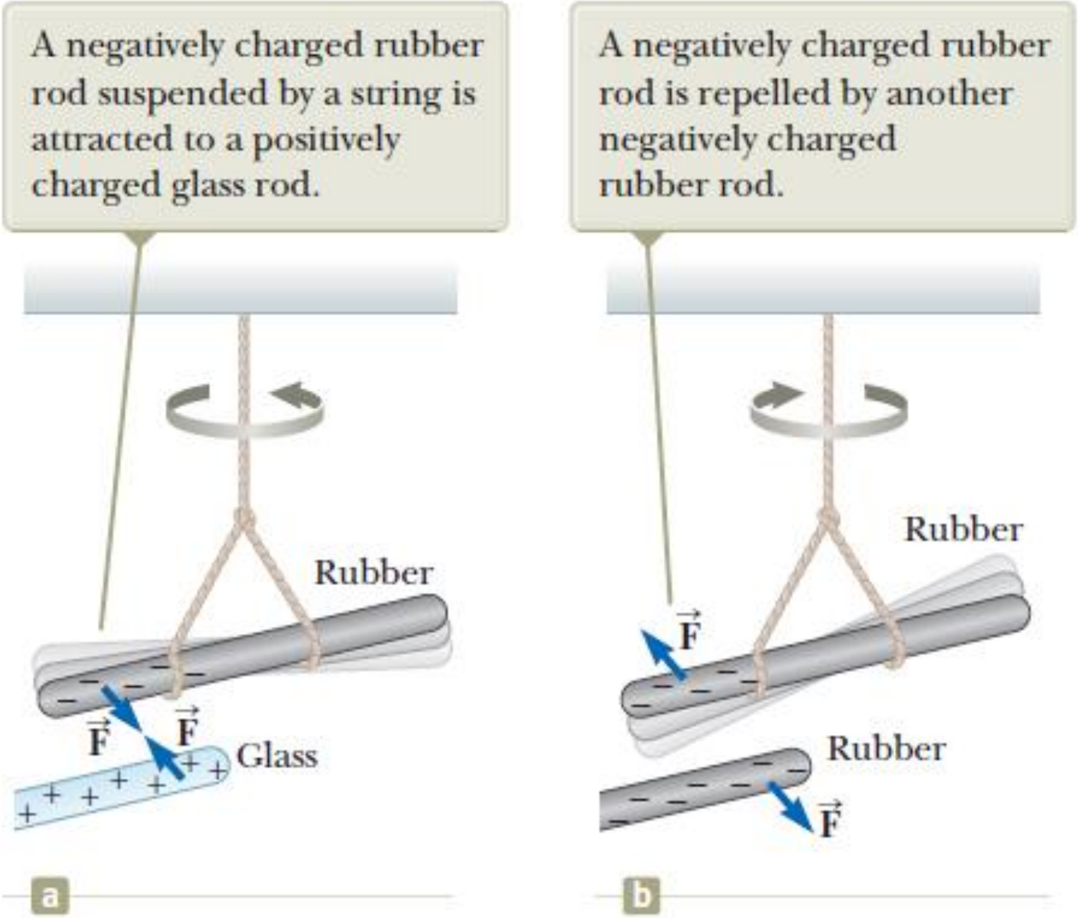


Figure 23.1 The electric force between (a) oppositely charged objects and (b) like-charged objects.

Direction of the Electric Force

- Consider the electric force between two-point charges. Both point charges exert an equal, but opposite electric force on the other, signifying that the forces obey **Newton's third law of motion**. The direction of the electric force between them always lies along the line between the two charges. For two charges of the same sign, the electric force from one charge on the other is repulsive and points away from the other charge.

Charging Objects by Induction

It is convenient to classify materials in terms of the ability of electrons to move through the material:

Electrical **conductors** are materials in which some of the electrons are free electrons[1] that are not bound to atoms and can move relatively freely through the material; electrical **insulators** are materials in which all electrons are bound to atoms and cannot move freely through the material.

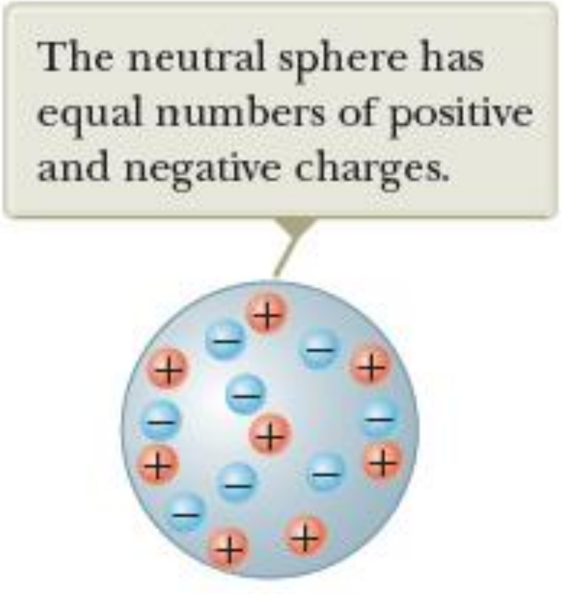
Materials such as glass, rubber, and dry wood fall into the category of electrical insulators. When such materials are charged by rubbing, only the area rubbed becomes charged and the charged particles are unable to move to other regions of the material.

In contrast, materials such as copper, aluminum, and silver are good electrical conductors. When such materials are charged in some small region, the charge readily distributes itself over the entire surface of the material.

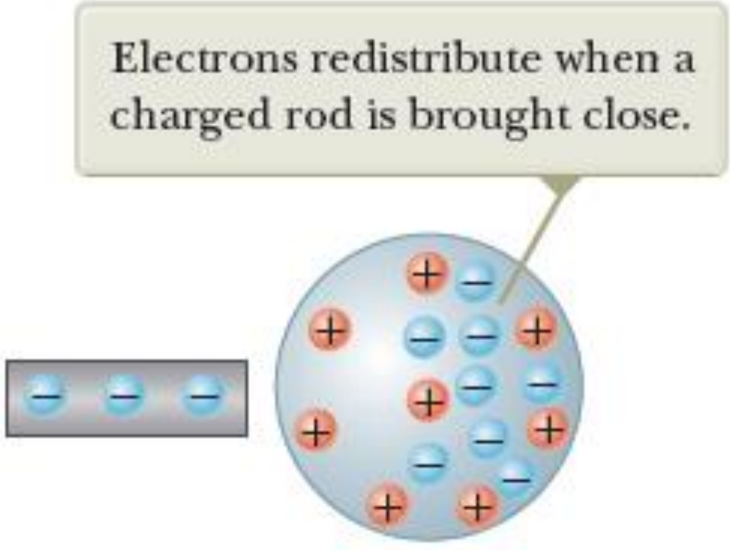
Semiconductors are a third class of materials, and their electrical properties are somewhere between those of insulators and those of conductors. Silicon and germanium are well-known examples of semiconductors commonly used in the fabrication of a variety of electronic chips used in computers, cellular telephones, and home theater systems. The electrical properties of semiconductors can be changed over many orders of magnitude by the addition of controlled amounts of certain atoms to the materials.

- **To understand how to charge a conductor by a process known as induction**, consider a neutral (uncharged) conducting sphere insulated from the ground as shown in Figure a. There are an equal number of electrons and protons in the sphere if the charge on the sphere is exactly zero.

- When a negatively charged rubber rod is brought near the sphere, electrons in the region nearest the rod experience a repulsive force and migrate to the opposite side of the sphere.
- This migration leaves the side of the sphere near the rod with an effective positive charge because of the diminished number of electrons as in Figure b.



a

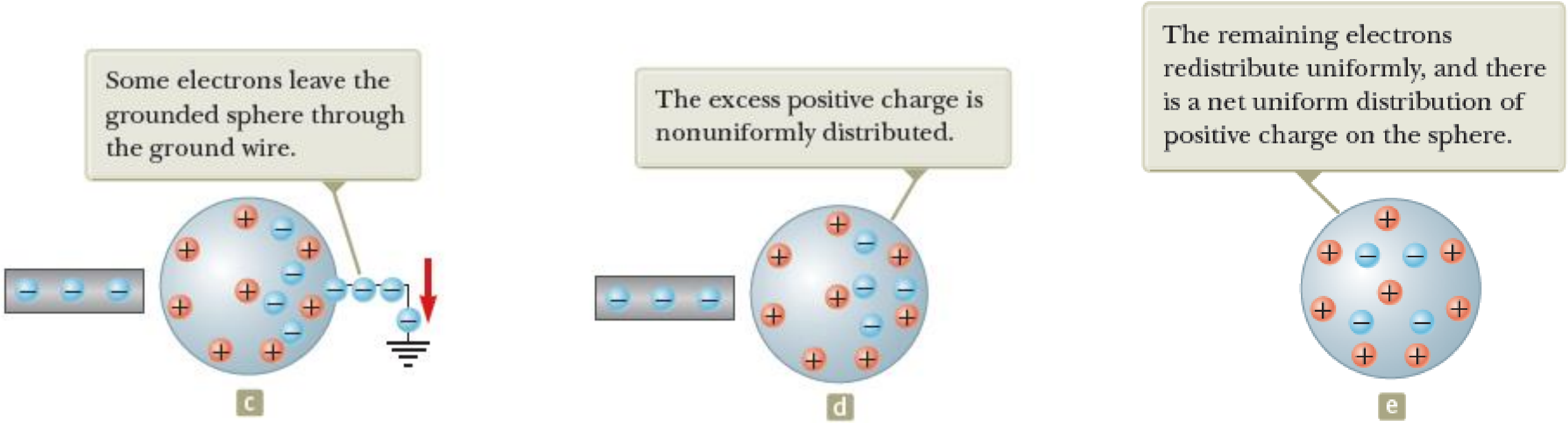


b

- If the same experiment is performed with a conducting wire connected from the sphere to the Earth (Fig. c), some of the electrons in the conductor are so strongly repelled by the presence of the negative charge in the rod that they move out of the sphere through the wire and into the Earth.

- If the wire to ground is then removed (Fig. d), the conducting sphere contains an excess of induced positive charge because it has fewer electrons than it needs to cancel out the positive charge of the protons.

- When the rubber rod is removed from the vicinity of the sphere (Fig. e), this induced positive charge remains on the ungrounded sphere. Notice that the rubber rod loses none of its negative charge during this process.



- A process similar to induction in conductors takes place in insulators. In most neutral molecules, the center of positive charge coincides with the center of negative charge.
- In the presence of a charged object, however, these centers inside each molecule in an insulator may shift slightly, resulting in more positive charge on one side of the molecule than on the other.
- This realignment of charge within individual molecules produces a layer of charge on the surface of the insulator as shown in Figure a.
- The proximity of the positive charges on the surface of the object and the negative charges on the surface of the insulator results in an attractive force between the object and the insulator. Your knowledge of induction in insulators should help you explain why a charged rod attracts bits of electrically neutral paper as shown in Figure b.

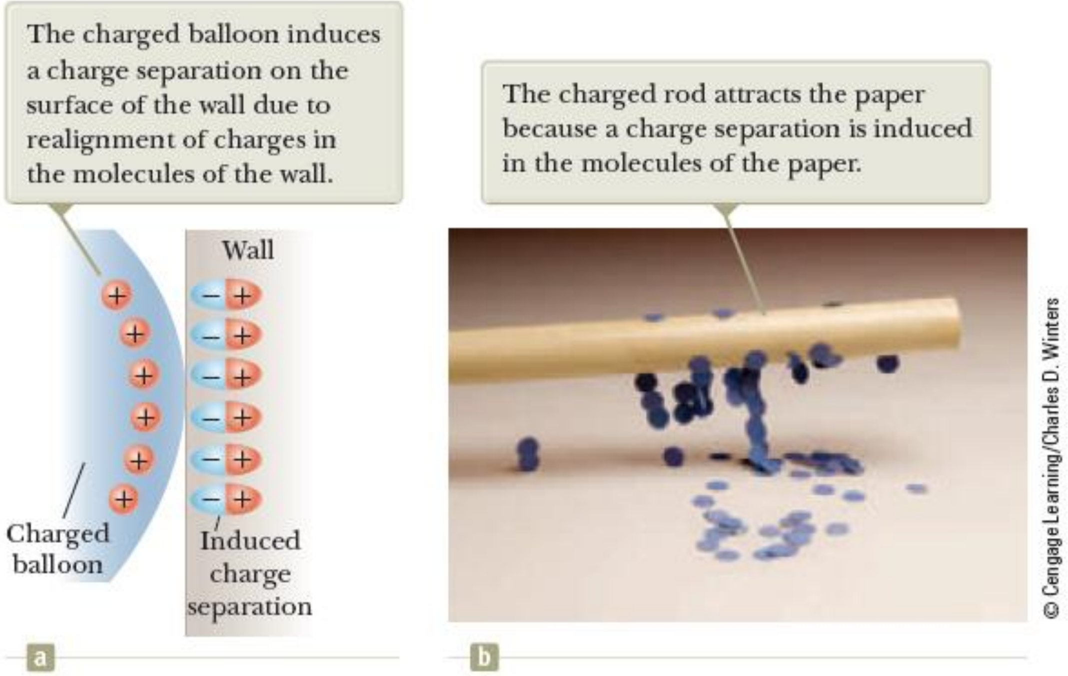


Figure 23.4 (a) A charged balloon is brought near an insulating wall. (b) A charged rod is brought close to bits of paper.

Coulomb's Law

- Charles Coulomb measured the magnitudes of the electric forces between charged objects using the torsion balance, which he invented (Fig.).
- The operating principle of the torsion balance is the same as that of the apparatus used by **Cavendish** to measure the density of the Earth, with the electrically neutral spheres replaced by charged ones. The electric force between charged spheres A and B in Fig. causes the spheres to either attract or repel each other, and the resulting motion causes the suspended fiber to twist.
- Because the restoring torque of the twisted fiber is proportional to the angle through which the fiber rotates, a measurement of this angle provides a quantitative measure of the electric force of attraction or repulsion.
- Once the spheres are charged by rubbing, the electric force between them is very large compared with the gravitational attraction, and so the gravitational force can be neglected.

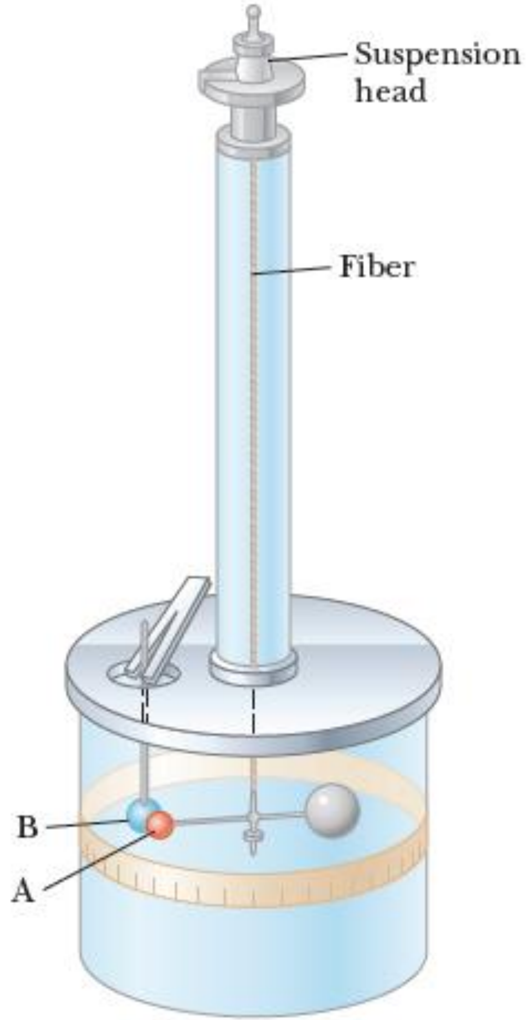


Figure 23.5 Coulomb's balance, used to establish the inverse-square law for the electric force.

- From Coulomb's experiments, we can generalize the properties of the electric force (sometimes called the electrostatic force) between two stationary charged particles.

- **We use the term point charge to refer to a charged particle of zero size.**

- The electrical behavior of electrons and protons is very well described by modeling them as point charges.
- From experimental observations, we find that the magnitude of the electric force (sometimes called the Coulomb force) between two-point charges is given by **Coulomb's law**.

$$F_e = k_e \frac{|q_1||q_2|}{r^2} \quad \textbf{(23.1)}$$

where k_e is a constant called the **Coulomb constant.** In his experiments, Coulomb was able to show that the value of the exponent of r was 2 to within an uncertainty of a few percent. Modern experiments have shown that the exponent is 2 to within an uncertainty of a few parts in 10^{16}. Experiments also show that the electric force, like the gravitational force, is conservative.

The value of the Coulomb constant depends on the choice of units. The SI unit of charge is the **coulomb** (C). The Coulomb constant k_e in SI units has the value

$$k_e = 8.987\,6 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \quad \textbf{(23.2)}$$

This constant is also written in the form

$$k_e = \frac{1}{4\pi\epsilon_0} \tag{23.3}$$

where the constant ϵ_0 (Greek letter epsilon) is known as the **permittivity of free space** and has the value

$$\epsilon_0 = 8.854\,2 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2} \tag{23.4}$$

The smallest unit of free charge e known in nature,[2] the charge on an electron ($-e$) or a proton ($+e$), has a magnitude

$$e = 1.602\,18 \times 10^{-19}\ \mathrm{C} \tag{23.5}$$

Table 23.1 **Charge and Mass of the Electron, Proton, and Neutron**

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,176\,5 \times 10^{-19}$	$9.109\,4 \times 10^{-31}$
Proton (p)	$+1.602\,176\,5 \times 10^{-19}$	$1.672\,62 \times 10^{-27}$
Neutron (n)	0	$1.674\,93 \times 10^{-27}$

Example 23.1 The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

SOLUTION

Conceptualize Think about the two particles separated by the very small distance given in the problem statement. In Chapter 13, we mentioned that the gravitational force between an electron and a proton is very small compared to the electric force between them, so we expect this to be the case with the results of this example.

Categorize The electric and gravitational forces will be evaluated from universal force laws, so we categorize this example as a substitution problem.

Use Coulomb's law to find the magnitude of the electric force:

$$F_e = k_e \frac{|e||-e|}{r^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)\frac{(1.60 \times 10^{-19}\text{ C})^2}{(5.3 \times 10^{-11}\text{ m})^2}$$

$$= \boxed{8.2 \times 10^{-8}\text{ N}}$$

Use Newton's law of universal gravitation and Table 23.1 (for the particle masses) to find the magnitude of the gravitational force:

$$F_g = G\frac{m_e m_p}{r^2}$$

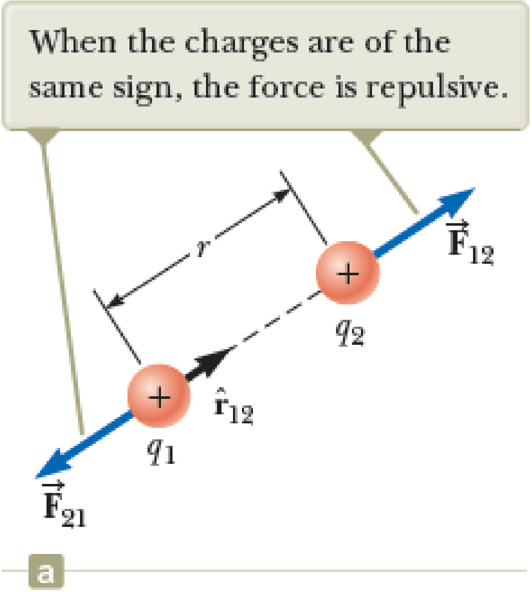
$$= (6.674 \times 10^{-11}\text{ N} \cdot \text{m}^2/\text{kg}^2)\frac{(9.11 \times 10^{-31}\text{ kg})(1.67 \times 10^{-27}\text{ kg})}{(5.3 \times 10^{-11}\text{ m})^2}$$

$$= \boxed{3.6 \times 10^{-47}\text{ N}}$$

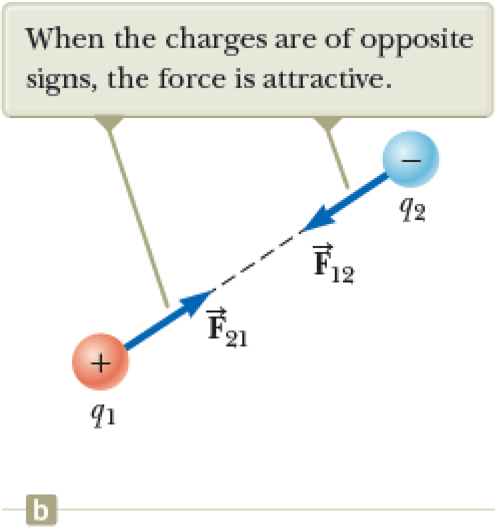
Vector form of Coulomb's law

When dealing with Coulomb's law, remember that force is a vector quantity and must be treated accordingly. Coulomb's law expressed in vector form for the electric force exerted by a charge q_1 on a second charge q_2, written $\vec{\mathbf{F}}_{12}$, is

$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{12} \tag{23.6}$$



where $\hat{\mathbf{r}}_{12}$ is a unit vector directed from q_1 toward q_2 as shown in Figure 23.6a (page 696). Because the electric force obeys Newton's third law, the electric force exerted by q_2 on q_1 is equal in magnitude to the force exerted by q_1 on q_2 and in the opposite direction; that is, $\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$. Finally, Equation 23.6 shows that if q_1 and q_2 have the same sign as in Figure 23.6a, the product q_1q_2 is positive and the electric force on one particle is directed away from the other particle. If q_1 and q_2 are of opposite sign as shown in Figure 23.6b, the product q_1q_2 is negative and the electric force on one particle is directed toward the other particle. These signs describe the *relative* direction of the force but not the *absolute* direction. A negative product indicates an attractive force, and a positive product indicates a repulsive force. The *absolute* direction of the force on a charge depends on the location of the other charge. For example, if an x axis lies along the two charges in Figure 23.6a, the product q_1q_2 is positive, but $\vec{\mathbf{F}}_{12}$ points in the positive x direction and $\vec{\mathbf{F}}_{21}$ points in the negative x direction.



When more than two charges are present, the force between any pair of them is given by Equation 23.6. Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the other individual charges. For example, if four charges are present, the resultant force exerted by particles 2, 3, and 4 on particle 1 is

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{41}$$

Example 23.2 Find the Resultant Force

Consider three point charges located at the corners of a right triangle as shown in Figure 23.7, where $q_1 = q_3 = 5.00\ \mu\text{C}$, $q_2 = -2.00\ \mu\text{C}$, and $a = 0.100$ m. Find the resultant force exerted on q_3.

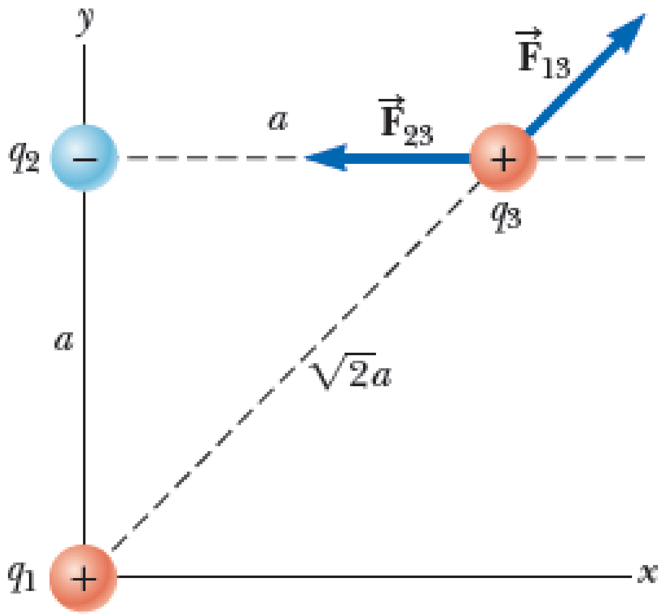


Figure 23.7 (Example 23.2) The force exerted by q_1 on q_3 is $\vec{\mathbf{F}}_{13}$. The force exerted by q_2 on q_3 is $\vec{\mathbf{F}}_{23}$. The resultant force $\vec{\mathbf{F}}_3$ exerted on q_3 is the vector sum $\vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{23}$.

SOLUTION

Conceptualize Think about the net force on q_3. Because charge q_3 is near two other charges, it will experience two electric forces. These forces are exerted in different directions as shown in Figure 23.7. Based on the forces shown in the figure, estimate the direction of the net force vector.

Categorize Because two forces are exerted on charge q_3, we categorize this example as a vector addition problem.

Use Equation 23.1 to find the magnitude of $\vec{\mathbf{F}}_{23}$:

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2.00 \times 10^{-6} \text{ C})(5.00 \times 10^{-6} \text{ C})}{(0.100 \text{ m})^2} = 8.99 \text{ N}$$

Find the magnitude of the force $\vec{\mathbf{F}}_{13}$:

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}\, a)^2}$$

$$= (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(5.00 \times 10^{-6} \text{ C})(5.00 \times 10^{-6} \text{ C})}{2(0.100 \text{ m})^2} = 11.2 \text{ N}$$

Find the x and y components of the force $\vec{\mathbf{F}}_{13}$:

$$F_{13x} = (11.2 \text{ N}) \cos 45.0^\circ = 7.94 \text{ N}$$

$$F_{13y} = (11.2 \text{ N}) \sin 45.0^\circ = 7.94 \text{ N}$$

Find the components of the resultant force acting on q_3:

$$F_{3x} = F_{13x} + F_{23x} = 7.94 \text{ N} + (-8.99 \text{ N}) = -1.04 \text{ N}$$

$$F_{3y} = F_{13y} + F_{23y} = 7.94 \text{ N} + 0 = 7.94 \text{ N}$$

Express the resultant force acting on q_3 in unit-vector form:

$$\vec{\mathbf{F}}_3 = \boxed{(-1.04\hat{\mathbf{i}} + 7.94\hat{\mathbf{j}}) \text{ N}}$$

Electric Field Definition

"An electric charge creates an **electric field**, which is a region of space around an electrically charged particle or object where the charge feels forced."

Electric Field Strength

"The electric field strength at a point in space is defined as the force experienced by a unit positive charge placed at that point. In other words, the magnitude of the strength of the electric field is called **Electric Field Strength** and it can be calculated using the formula:"

$$E = F/q$$

Where,

- *E is the electric field's strength,*
- *F is the electric force, and*
- *q is the test charge.*

Types of Electric Charge

Electric charge is classified into two primary types:

positive and negative

Positive charge is associated with protons, subatomic particles found in an atom’s nucleus, and is indicated by the “+” symbol.

Conversely, negative charge is linked to electrons, which orbit around the nucleus, and is represented by the “-” symbol.

Fields of Force

Newton's universal gravitational law and Coulomb's law enable us to calculate the magnitudes as well as the directions of the gravitational and electric forces, respectively. However one may question, (a) What are the origins of these forces? (b) How are these forces transmitted from one mass to another or from one charge to another?

The answer to (a) is still unknown; the existence of these forces is accepted as it is. That is why they are called basic forces of nature.

To describe the mechanism by which electric force is transmitted, Michael Faraday [1791-1867] introduced the concept of an electric field. According to his theory, it is the intrinsic property of nature that an electric field exists in the space around an electric charge. This electric field is considered to be a force field that exerts a force on other charges placed in that field. For example, a charge q produces an electric field in the space surrounding it.

This field exists whether the other charges are present in space or not. However, the presence of field cannot be tested until another charge q_0 is brought into the field. Thus the field of charge q interacts with q_0 to produce an electrical force. The interaction between q and q_0 is accomplished in two steps: (a) the charge q produces a field and (b) the field interacts with charge q_0 to produce a force **F** on q_0. These two steps are illustrated in Fig. 12.4.

In this figure the density of dots is proportional to the strength of the field at the various points. We may define electric field strength or electric field intensity **E** at any point in the field as

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} \qquad \text{.....} \quad (12.7)$$

where **F** is the force experienced by a positive test charge q_0 placed at the point. The test charge q_0 has to be very small so that it may not distort the field which it has to measure.

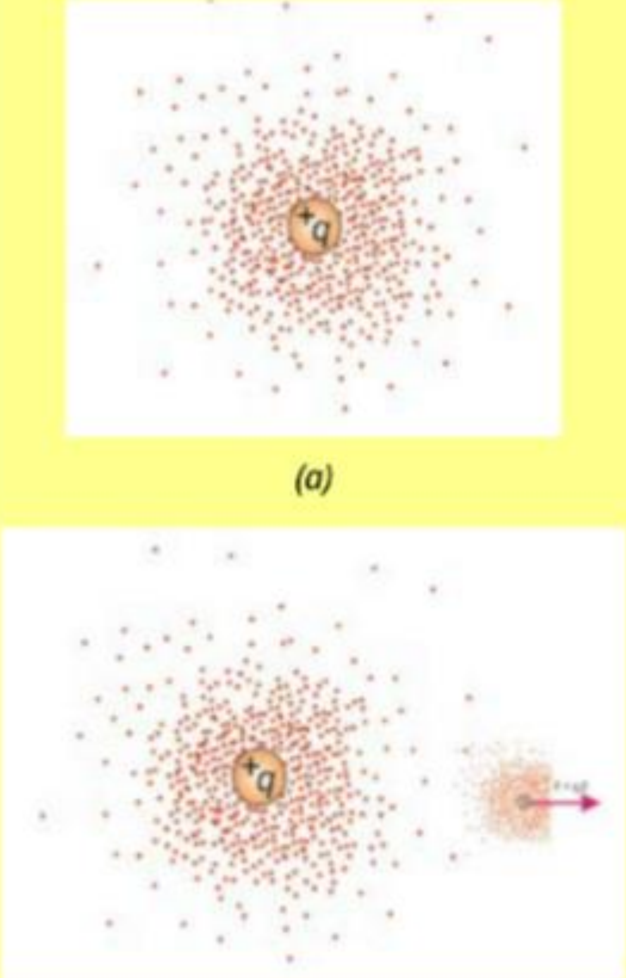


***Fig. 12.4** (a) Dots surrounding the positive charge indicate the presence of the electric field. The density of the dots is proportional to the strength of the electric field at different points, (b) Interaction of the field with the charge q_0*

Electric Field due to Different Charge Distributions

Electric Field Due to Point Electric Charge

Consider a test charge 'q_0' placed at point P in the electric field of a point charge 'q' at a distance 'r' apart.

We want to find out electric field intensity at point 'P' due to a point charge 'q'.

The electrostatic force 'F' between 'q' and 'q_0' can be find out by using expression,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2} \qquad \text{----------------------} \qquad (3)$$

The electric field intensity 'E' due to a point charge 'q' can be obtained by putting the value of electrostatic force in equation

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$

$$E = \frac{\left(\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\right)}{q_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \text{----------------------} \quad (4)$$

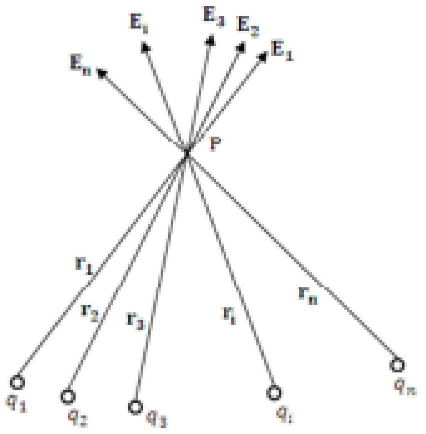
This expression gives the magnitude of electric field intensity due to a point charge 'q'. In vector form, the electric field intensity '**E**' will be:

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{----------------------} \quad (5)$$

where $\hat{r}$ is the unit vector which gives the direction of electric field intensity.

Electric Field due to Many (n) Point Charge

Let $q_1,\ q_2, q_3, \ldots\ldots\ldots, q_n$ are the 'n' point charges as shown in the figure.



The electric field intensity due to assembly of 'n' point charges at a specific field point can be determined by following the procedure mentioned below;

- Calculate the electric field intensity at a field point due to each charge separately by assuming that the other charges are absent.
- Calculate the total electric field intensity by taking the vector sum of intensities of individual point charges.

Now if $\mathbf{E_1},\ \mathbf{E_2},\ \mathbf{E_3}, \ldots\ldots, \mathbf{E_n}$ be the electric field intensities at a field point due to the point charges $q_1,\ q_2, q_3, \ldots\ldots\ldots, q_n$ respectively. Then, the total electric field intensity due to assembly of 'n' point charges will be;

$$\mathbf{E} = \mathbf{E_1} + \mathbf{E_2} + \mathbf{E_3} + \ldots\ldots + \mathbf{E_n} \quad \text{---------------------} \quad (6)$$

where

$\mathbf{E_1}$ = Electric Field Intensity at a Field Point due to Point Charge 'q_1' = $\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^{\,2}}\hat{r_1}$

$\mathbf{E_2}$ = Electric Field Intensity at a Field Point due to Point Charge 'q_2' = $\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^{\,2}}\hat{r_2}$

$\mathbf{E_3}$ = Electric Field Intensity at a Field Point due to Point Charge 'q_3' = $\frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3^{\,2}}\hat{r_3}$

⋮ ⋮ ⋮ ⋮ ⋮ ⋮ ⋮

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$\mathbf{E_n}$ = Electric Field Intensity at a Field Point due to Point Charge 'q_n' = $\frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n^{\,2}}\hat{r_n}$

Putting values in equation (6), we get

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^{\,2}}\hat{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^{\,2}}\hat{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3^{\,2}}\hat{r_3} + \ldots\ldots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n^{\,2}}\hat{r_n}$$

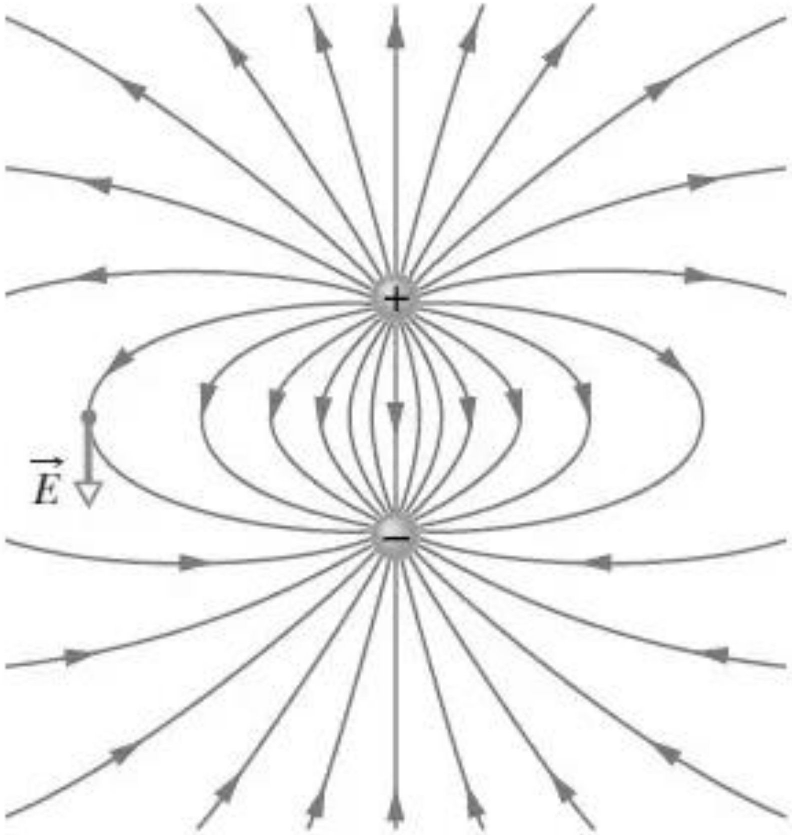
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^{\,2}}\hat{r_1} + \frac{q_2}{r_2^{\,2}}\hat{r_2} + \frac{q_3}{r_3^{\,2}}\hat{r_3} + \ldots\ldots + \frac{q_n}{r_n^{\,2}}\hat{r_n}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^{\,2}}\hat{r_i} \quad \text{---------------------} \quad (7)$$

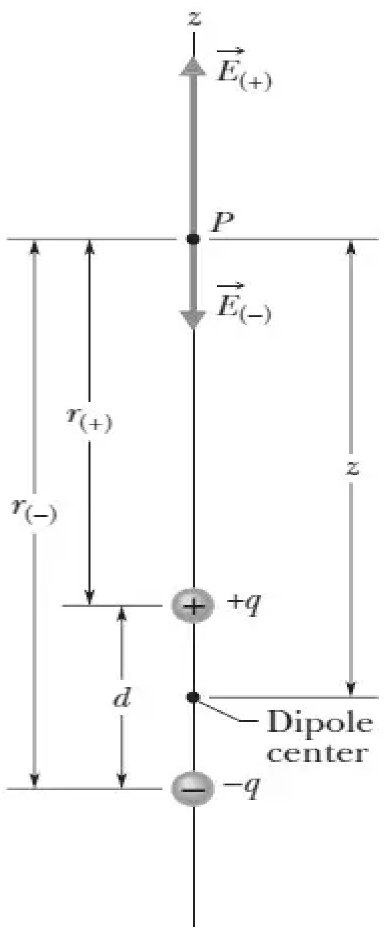
This equation gives the total electric field intensity due to assembly of 'n' point charges at a specific field point.

Electric Field due to a Dipole

The following image shows a setup of two charges that are equal in magnitude and are opposite in sign. This configuration is often called as Electric Dipole.



Let us now find the E due to an electric dipole. The following image shows two charged particles of same magnitude q but of opposite sign. They are separated by a distance d. the axis through the charged particles is called Dipole Axis and we will find the E at point P, which is at a distance z from the mid-point of the dipole axis.



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Using superposition principle for electric fields, the magnitude of E at P is given by the following equation:

$$\begin{aligned} E &= E_{(+)} - E_{(-)} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2} \\ &= \frac{q}{4\pi\varepsilon_0(z-\frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z+\frac{1}{2}d)^2} \end{aligned}$$

Using a little algebra in the above equation and also assuming that distance z is much greater than d (z >> d), we get the following equation.

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}$$

Electric Field Lines

A visual representation of the electric field can be obtained in terms of electric field lines; an idea proposed by Michael Faraday. Electric field lines can be thought of a "map" that provides information about the direction and strength of the electric field at various places. As electric field lines provide information about the electric force exerted on a charge, the lines are commonly called "lines of force".

To introduce electric field lines, we place positive test charges each of magnitude q_0 at different places but at equal distances from a positive charge $+q$ as shown in the figure. Each test charge will experience a repulsive force, as indicated by arrows in Fig. 12.6(a). Therefore, the electric field created by the charge $+q$ is directed radially outward. Fig. 12.6 (b) shows corresponding field lines which show the field direction.

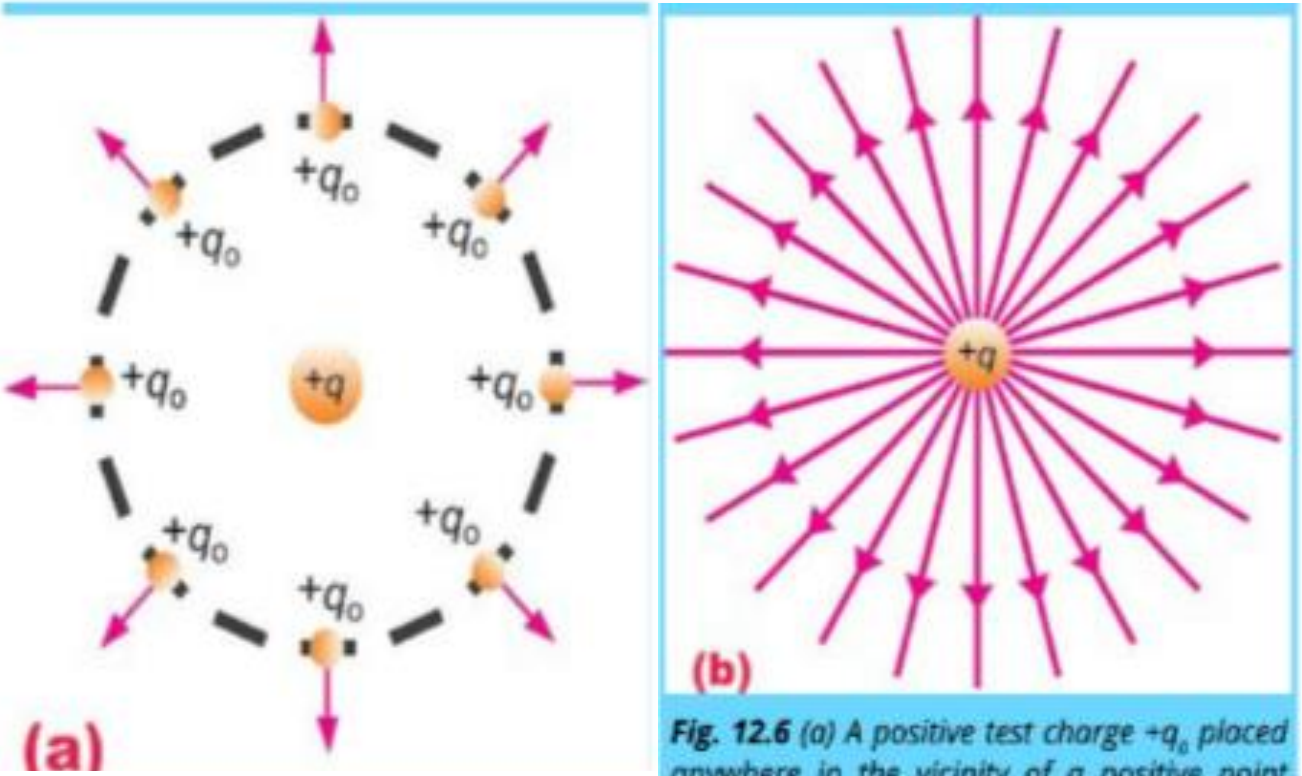


Fig. 12.6 (a) A positive test charge $+q_0$ placed anywhere in the vicinity of a positive point charge $+q$, experiences a repulsive force directed radially outward. (b) the electric field lines are directed radially outward from the positive point charge $+q$.

Fig. 12.7 shows the electric field lines in the vicinity of a negative charge -*q*. In this case the lines are directed radially "inward", because the force on a positive test charge is now of attraction indicating the electric field points inward.

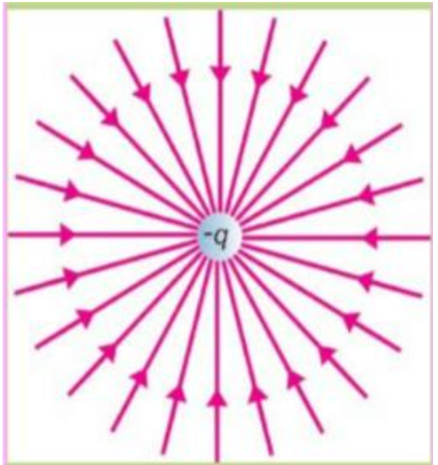


Fig. 12.7 The electric field lines are directed radially inward towards a negative point charge -q.

The electric field lines are curved in case of two identical separated charges. Fig. 12.8 shows the pattern of lines associated with two identical positive point charges of equal magnitude. It reveals that the lines in the region between two like charges seem to repel each other. The behaviour of two identical negatively charges will be exactly the same. The middle region shows the presence of a zero field spot or neutral zone.

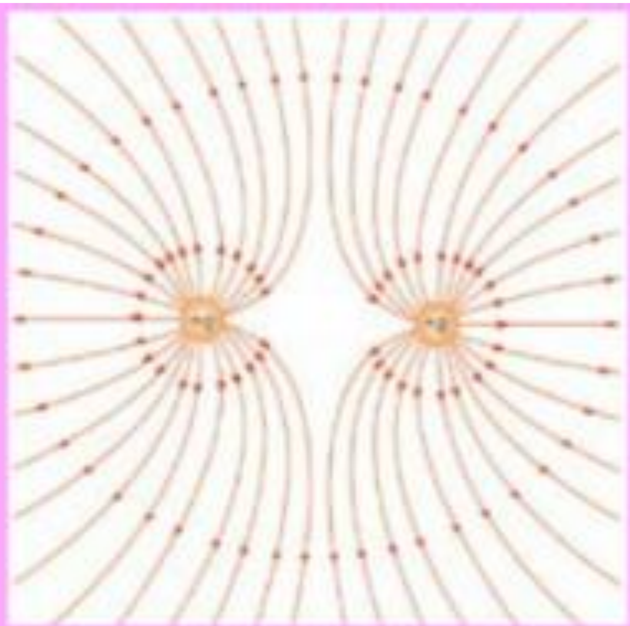


Fig. 12.8 The electric field lines for two identical positive point charges.

The Fig.12.9 shows the electric field pattern of two opposite charges of same magnitudes. The field lines start from positive charge and end on a negative charge. The electric field at points such as 1, 2, 3 is the resultant of fields created by the two charges at these points. The directions of the resultant intensities is given by the tangents drawn to the field lines at these points.

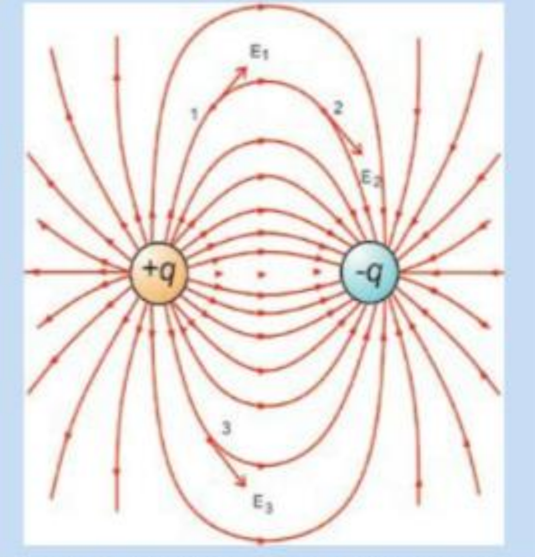


Fig. 12.9 Attractive forces between unlike charges

In the regions where the field lines are parallel and equally spaced, the same number of lines pass per unit area and therefore, field is uniform on all points. Fig.12.10 shows the field lines between the plates of a parallel plate capacitor. The field is uniform in the middle region where field lines are equally spaced.

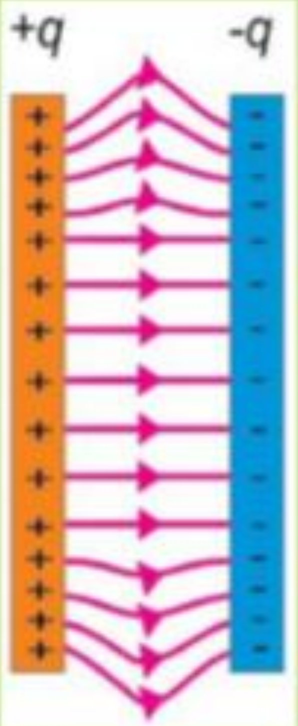


Fig. 12.10 In the central region of a parallel plate capacitor the electric field lines are parallel and evenly spaced, indicating that the electric field there has the same magnitude and direction at all points.

We are now in a position to summarize the properties of electric field lines.

1. Electric field lines originate from positive charges and end on negative charges.
2. The tangent to a field line at any point gives the direction of the electric field at that point.
3. The lines are closer where the field is strong and the lines are farther apart where the field is weak.
4. No two lines cross each other. This is because **E** has only one direction at any given point. If the lines cross, **E** could have more than one direction.

Application of Electrostatics

(i) Xerography (Photocopier)

(ii) Inkjet Printers

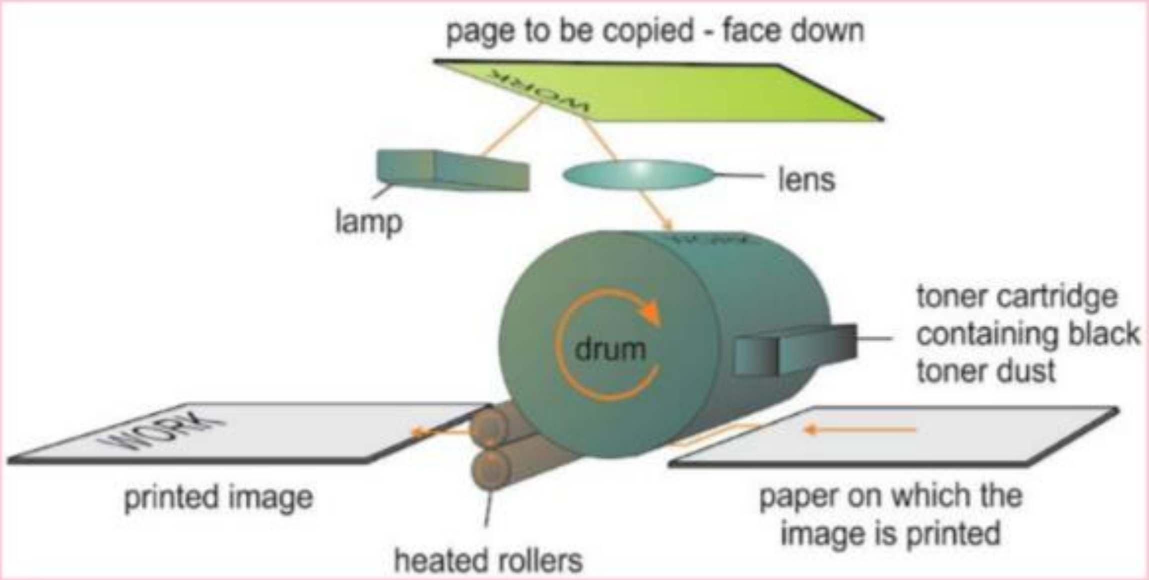


Fig. 12.11 *The basics of photocopying. The lamp transfers an image of the page to the drum, which leaves a static charge. The drum collects toner dust and transfers it to the paper. The toner is melted onto the paper.*

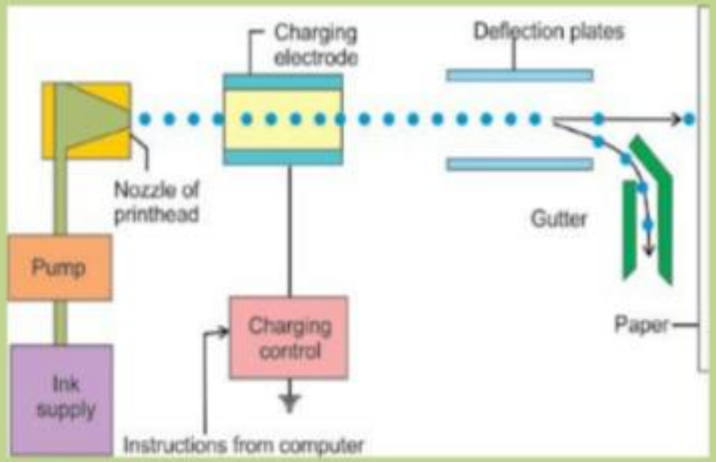


Fig. 12.12 (b) An inkjet printhead ejects a steady flow of ink droplets. The charging electrodes are used to charge the droplets that are not needed on the paper. Charged' droplets are deflected into a gutter by the deflection plates, while uncharged droplets fly straight onto the paper.

Electric Flux

When we place an element of area in an electric field, some of the lines of force pass through it (Fig. 12.13 a).

The number of the field lines passing through a certain element of area is known as electric flux through that area. It is usually denoted by Greek letter ϕ. For example the flux ϕ through the area A in Fig. 12.13 (a) is 4 while the flux through B is 2.

In order to give a quantitative meaning to flux, the field lines are drawn such that the number of field lines passing through a unit area held perpendicular to field lines at a point represent the intensity **E** of the field at that point.

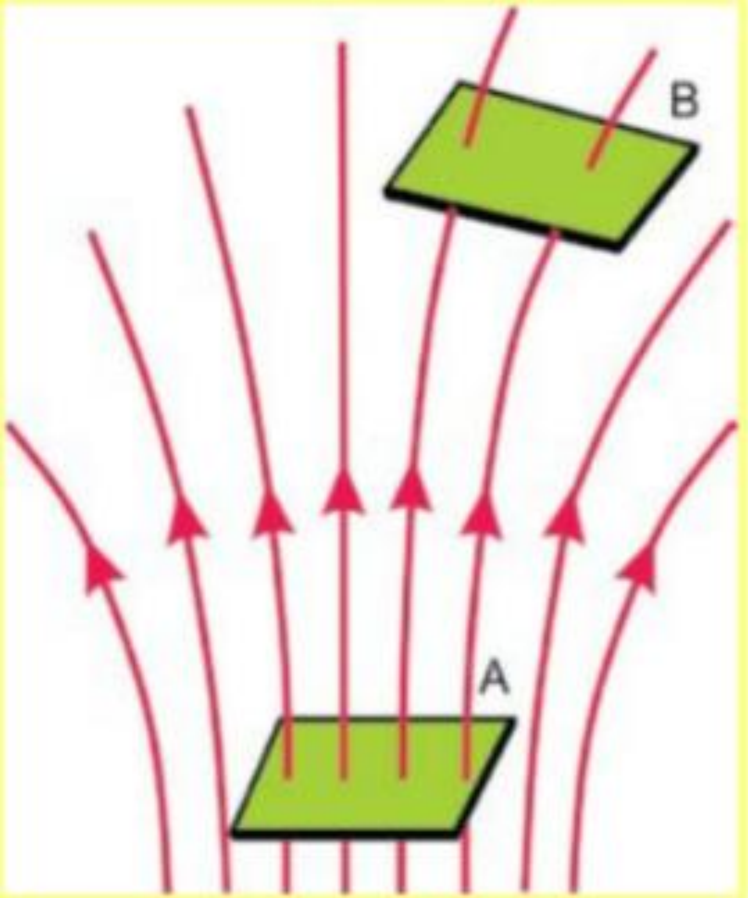


Fig. 12.13 (a) Electric flux through a surface normal to **E**.

In Fig.12.13 (b), area is held perpendicular to the field lines, then $EA_{\perp}$ lines pass through it. The flux ϕ_e in this case is

$$\phi_e = EA_{\perp} \quad \text{.........} (12.10)$$

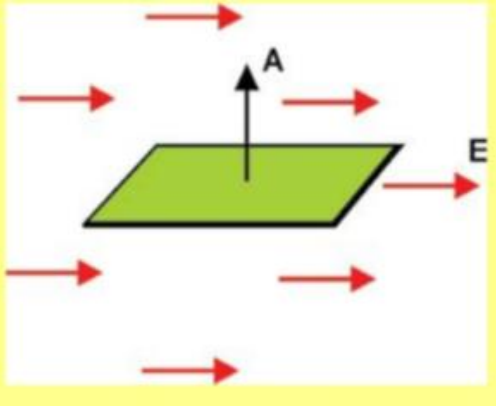


Fig. 12.13 (c) Minimum

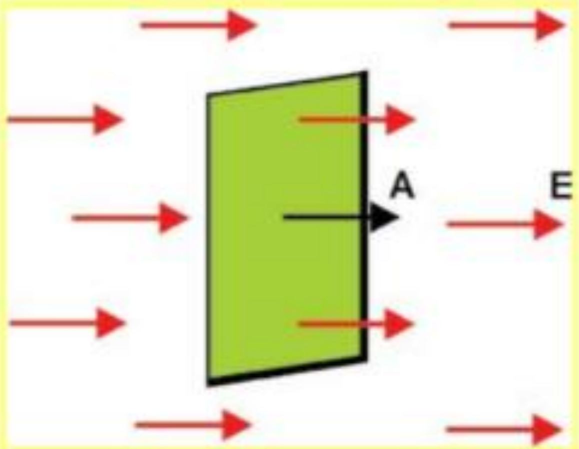


Fig. 12.13 (b) Maximum

where $A_{\perp}$ denotes that the area is held perpendicular to field lines. In Fig. 12.13 (c), area A is held parallel to field lines and, as is obvious no lines cross this area, so that flux ϕ_e in this case is

$$\phi_e = EA_{||} = 0 \quad \text{.........} (12.11)$$

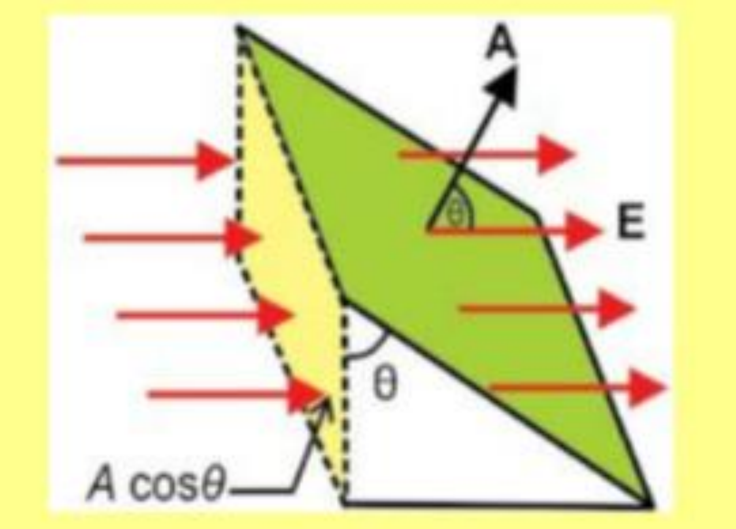


Fig. 12.13 (d)

where $A_{||}$ indicates that A is held parallel to the field lines. Fig. 12.13 (d) shows the case when A is neither perpendicular nor parallel to field lines but is inclined at angle θ with the lines. In this case we have to find the projection of the area which is perpendicular to the field lines. The area of this projection, (Fig. 12.13 d) is $A \cos\theta$. The flux ϕ in this case is

$$\phi_e = EA\cos\theta$$

Usually the element of area is represented by a vector area **A** whose magnitude is equal to the surface area A of the element and whose direction is direction of normal to the area. The electric flux ϕ_e through a patch of flat surface in terms of **E** and **A** is then given by

$$\phi_e = EA\cos\theta = \mathbf{E}.\mathbf{A} \quad \text{.........} (12.12)$$

where θ is the angle between the field lines and the normal to the area.

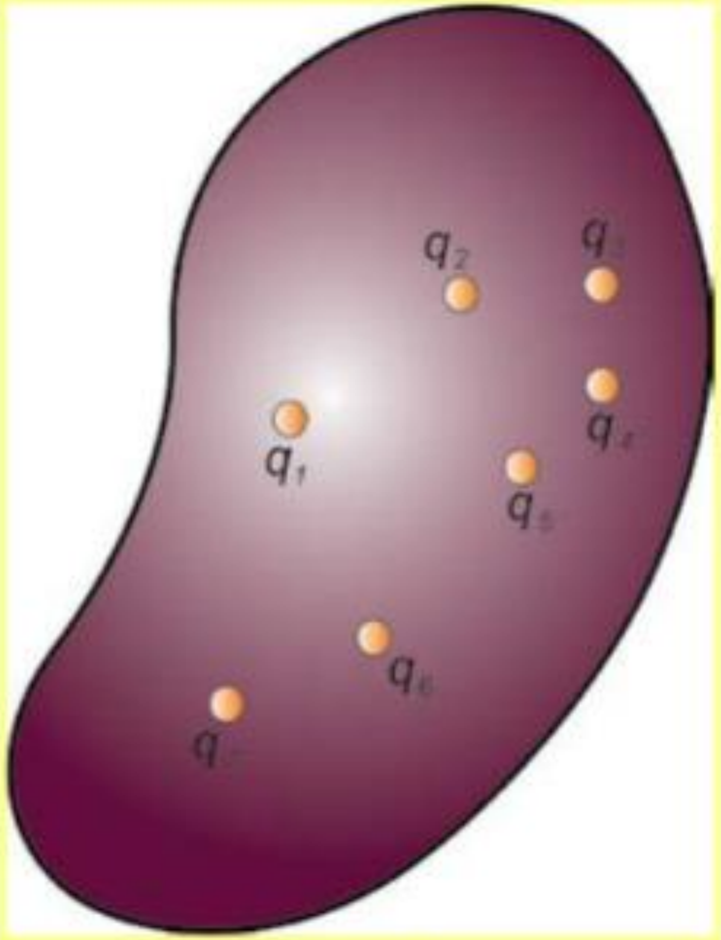


Fig. 12.16

12.7 GAUSS'S LAW

Suppose point charges q_1, q_2, q_3, ..., q_n are arbitrarily distributed in an arbitrary shaped closed surface as shown in Fig. 12.16. Using idea given in previous section, the electric flux passing through the closed surface is

$$\phi_e = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \frac{q_3}{\varepsilon_0} + \ldots\ldots + \frac{q_n}{\varepsilon_0}$$

$$\phi_e = \frac{1}{\varepsilon_0} \times (q_1 + q_2 + q_3 \ldots\ldots + q_n)$$

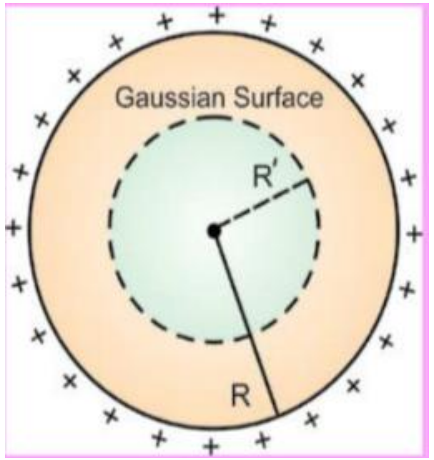
$$\phi_e = \frac{1}{\varepsilon_0} \times (\text{total charge enclosed by closed surface})$$

$$\phi_e = \frac{1}{\varepsilon_0} \times Q \quad \ldots\ldots(12.16)$$

where $Q = q_1 + q_2 + q_3 + \ldots\ldots + q_n$, is the total charge enclosed by closed surface. Eq.12.16 is mathematical expression of Gauss's law which can be stated as,

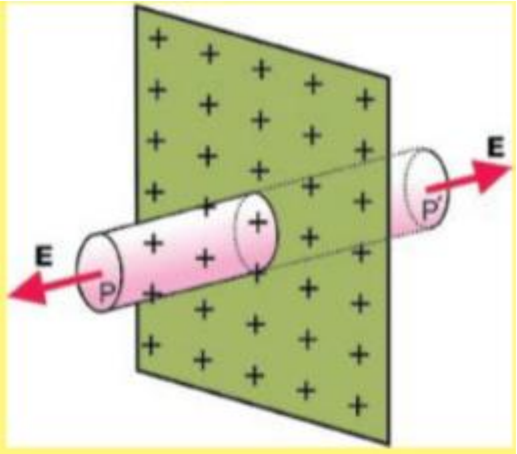
"The flux through any closed surface is $1/\varepsilon_0$ times the total charge enclosed in it".

Application of Gauss's Law
(complete derivation H.W)

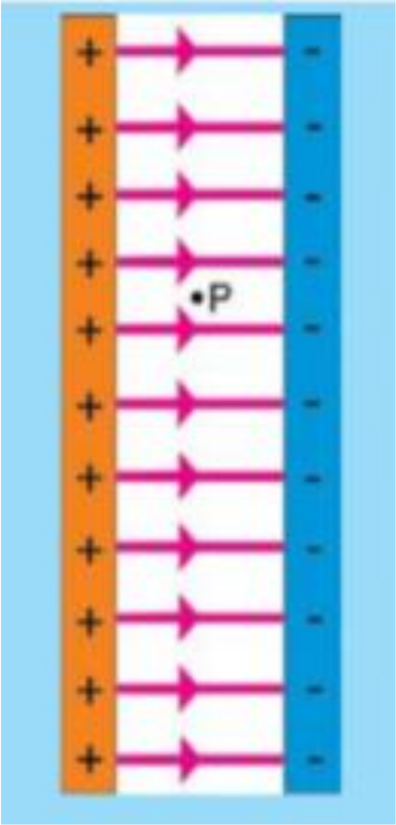


(a) Intensity of Field Inside a Hollow Charged Sphere

(b) Electric Intensity Due to an Infinite Sheet of Charge



(c) Electric Intensity Between Two Oppositely Charged Parallel Plates



Practice Problems

1. Two point charges of $+2\ \mu C$ and $-3\ \mu C$ are placed 0.5 m apart. Calculate the force between them.

2. A charge of $5\ \mu C$ experiences a force of $0.2\ N$ in an electric field. Find the magnitude of the electric field.

3. Calculate the force between two charges $q_1 = +3\ \mu C$ and $q_2 = +5\ \mu C$ separated by 1 m in air.

4. **Example 12.1 : Charges q_1 = 100μC and q_2 = 50μC are located in xy-plane at position $r_1 = 3.0\hat{j}$ and $r_2 = 4.0\hat{i}$ respectively, where the distances are measured in metres. Calculate the force on q_2 (Fig. 12.3).**

Solution:

The electric force is given by Coulomb's law:

$$F = k\frac{|q_1 q_2|}{r^2}$$

where:

- $k = 9 \times 10^9\,\mathrm{Nm^2/C^2}$,
- $q_1 = +2\,\mu C = 2 \times 10^{-6}\,\mathrm{C}$,
- $q_2 = -3\,\mu C = -3 \times 10^{-6}\,\mathrm{C}$,
- $r = 0.5\,\mathrm{m}$.

Solution:

The relationship between electric force and electric field is:

$$F = qE$$

where:

- $F = 0.2\,\mathrm{N}$,
- $q = 5\,\mu C = 5 \times 10^{-6}\,\mathrm{C}$.

Rearrange to find E:

$$E = \frac{F}{q}$$

Solution:

Use Coulomb's law:

$$F = k\frac{|q_1 q_2|}{r^2}$$

Substitute values:

4.

$q_1 = 100\mu C \qquad q_2 = 50\mu C$

Position vector of q_2 relative to q_1

$= \mathbf{r}_{21} = \mathbf{r}_2 - \mathbf{r}_1 = 4\hat{i} - 3\hat{j}$

r= magnitude of $\mathbf{r}_{21} = \sqrt{(4m)^2 + (-3m)^2} = 5$ m

$$\hat{\mathbf{r}}_{21} = \frac{4\hat{i} - 3\hat{j}}{5}$$

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{21}$$

$$= \frac{9x10^9 Nm^2C^{-2}x100x10^{-6}Cx50x10^{-6}C}{(5m)^2} x \frac{4\hat{i} - 3\hat{j}}{5}$$

$$= 1.44\hat{i} - 1.08\hat{j}$$

Magnitude of $\mathbf{F}_{21} = F = \sqrt{(1.44)^2 + (-1.08)^2} = 1.8N$

Direction of $\mathbf{F}_{21} = \tan^{-1}\left(\frac{-1.08}{1.44}\right) = -37^0$ with x-axis

5. Find the electric field at a point 0.2 m away from a charge of $+5\ \mu C$.

6. Calculate the electric field due to a charge of $-2\ \mu C$ at a distance of 1 m.

7. A charge of $+10\ \mu C$ creates an electric field of $1000\ N/C$ at a certain point. Find the distance of the point from the charge.

8. At what distance from a $+1\ \mu C$ charge is the electric field $450\ N/C$?

The electric field E due to a point charge is given by:

$$E = \frac{k \cdot |Q|}{r^2}$$

9. Find the flux through a cube of side 2 m if a charge of $1\,\mu C$ is placed at its center.

Solution:

Using Gauss's Law:

$$\Phi = \frac{q}{\varepsilon_0}$$

Where:

- $q = 1\,\mu C = 1 \times 10^{-6}\,\mathrm{C}$
- $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$ (permittivity of free space)

10. The electric flux through a spherical surface of radius 0.2 m is $2 \times 10^4\,\mathrm{N \cdot m^2/C}$. Find the charge enclosed.

Solution:

Using Gauss's Law:

$$\Phi = \frac{q}{\varepsilon_0} \implies q = \Phi \varepsilon_0$$

Where:

- $\Phi = 2 \times 10^4\,\mathrm{N \cdot m^2/C}$
- $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$

11. **Problem:**

A uniform electric field $E = 500\,\mathrm{N/C}$ is directed perpendicular to a flat surface of area $A = 2\,\mathrm{m}^2$.

Calculate the electric flux through the surface.

Solution:

The electric flux Φ_E is given by:

$$\Phi_E = E \cdot A \cdot \cos\theta$$

Since the field is perpendicular to the surface, $\theta = 0^\circ$, and $\cos 0^\circ = 1$.

Reason: θ = angle between the electric field and the normal to the surface

Since the electric field is directed perpendicular to the surface, it is parallel to the normal to the surface.- When two vectors are parallel, the angle between them is 0°.