# DISCRETE STRUCTURE

Teams code: moimdy3

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# Text Books

Primary:

“Discrete Mathematics and its Applications”, 7th Edition, by Kenneth Rossen.

Reference:

“Fundamentals of Discrete Structures”, by Damian Lyons, Christina Papadakis-Kanaris, Gary Weiss, and Arthur G. Werschulz.

# Grading Scheme

| | |
|---|---|
| Assignments ( 3 or more) | 15% |
| Quizzes (3 or more) | 10% |
| Mid Term | 20% |
| Final Term | 50% |
| | |

# Assignments and Quizzes Policy

1. No assignments will be accepted after due date.
2. Quizzes will not be retaken.
3. Zero tolerance over cheating.
4. If 2 or more students have same assignment , all will bemarked zero

# Logic!

# Logic

- Crucial for mathematical reasoning
- Important for program design
- Used for designing electronic circuitry

- (Propositional )Logic is a system based on propositions.
- A proposition is a (declarative) statement that is either true or false (not both).
- We say that the truth value of a proposition is either true (T) or false (F).
- Corresponds to 1 and 0 in digital circuits

# The Statement/Proposition Game

- "Elephants are bigger than mice."

Is this a statement? yes

Is this a proposition? yes

What is the truth value of the proposition? true

# The Statement/Proposition Game

- "$y > 5$"

Is this a statement? yes

Is this a proposition? no

Its truth value depends on the value of y, but this value is not specified.

We call this type of statement a propositional function or open sentence.

# The Statement/Proposition Game

- "Today is January 27 and 99 < 5."

Is this a statement? yes

Is this a proposition? yes

What is the truth value of the proposition? false

# The Statement/Proposition Game

- "Please do not fall asleep."

Is this a statement? no

It's a request.

Is this a proposition? no

Only statements can be propositions.

# The Statement/Proposition Game

- "If the moon is made of cheese,
- then I will be rich."

Is this a statement? yes

Is this a proposition? yes

What is the truth value of the proposition? probably true

# The Statement/Proposition Game

- "$x < y$ if and only if $y > x$."

Is this a statement? yes

Is this a proposition? yes

… because its truth value
does not depend on
specific values of x and y.

What is the truth value
of the proposition? true

# Combining Propositions

- As we have seen in the previous examples, one or more propositions can be combined to form a single compound proposition.

- We formalize this by denoting propositions with letters such as p, q, r, s, and introducing several logical operators or logical connectives.

# Compound Proposition

- One can connect propositions using “and”, “or”, “not”, “only if” …to form compound proposition:
  - It will not rain tomorrow.
  - Fishes are jumping and the cotton is high.
  - If the ground is wet then it rains last night.
- Truth value of compound proposition depends on truth value of the simple propositions
  - We will formalize above connectives as operations on propositions

**EXAMPLES:** **LOGICAL CONNECTIVES**

# COMPOUND STATEMENT

- Simple statements could be used to build a compound statement.

1. “3 + 2 = 5” **and** “Lahore is a city in Pakistan”
2. “The grass is green” or “ It is hot today”
3. “Discrete Mathematics is **not** difficult to me”

AND, OR, NOT are called LOGICAL CONNECTIVES.

# SYMBOLIC REPRESENTATION

- Statements are symbolically represented by letters such as ***p***, ***q***, ***r***,…

| CONNECTIVE | MEANINGS | SYMBOL | CALLED |
| --- | --- | --- | --- |
| Negation | not | $\sim$ | Tilde |
| Conjunction | and | $\wedge$ | Hat |
| Disjunction | or | $\vee$ | Vel |
| Conditional | if…then… | $\rightarrow$ | Arrow |
| Biconditional | if and only if | $\leftrightarrow$ | Double arrow |

# EXAMPLES:

***p*** **=** “Islamabad is the capital of Pakistan”

***q*** **=** “17 is divisible by 3”

***p*** ∧ ***q*** **=** “**Islamabad is the capital of Pakistan and 17 is divisible by 3”**

***p*** ∨ ***q*** **=** “**Islamabad is the capital of Pakistan or 17 is divisible by 3”**

**~*p*** **=** “It is not the case that **Islamabad is the capital of Pakistan” or simply**

**“Islamabad is not the capital of Pakistan”**

# Logical Operators (Connectives)

- We will examine the following logical operators:
  - Negation (NOT, $\neg$)
  - Conjunction (AND, $\wedge$)
  - Disjunction (OR, $\vee$)
  - Exclusive-or (XOR, $\oplus$)
  - Implication (if – then, $\rightarrow$)
  - Biconditional (if and only if, $\leftrightarrow$)
- Truth tables can be used to show how these operators can combine propositions to compound propositions.

# Negation (NOT)

- Unary Operator, Symbol: ¬

| P | ¬ P |
|---|---|
| true (T) | false (F) |
| false (F) | true (T) |

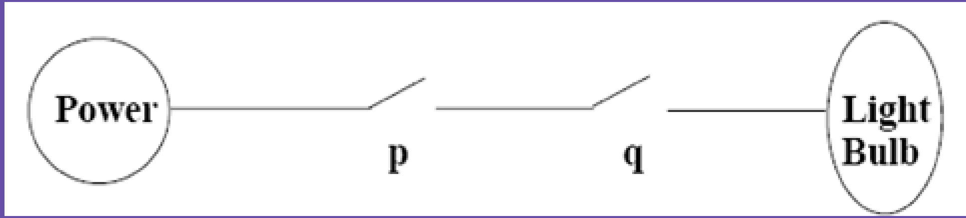
# Conjunction (AND)

- Binary Operator, Symbol: ∧

| P | Q | P∧ Q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

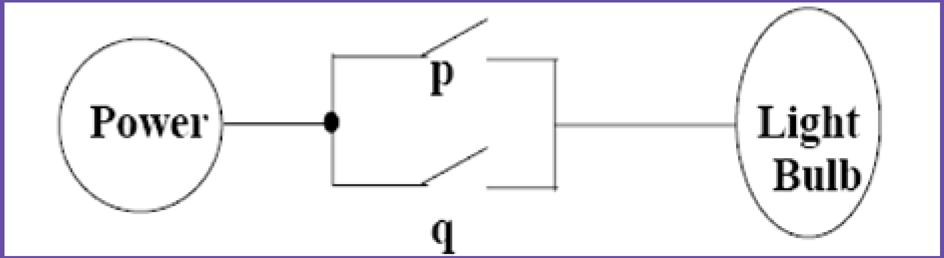
# Example (1/2)

- **Example** (Circuit interpretation of AND and OR)
- The following circuit can be represented using p ^ q.



# Example (2/2)

The following circuit can be represented using p v q.



# Disjunction (OR)

- Binary Operator, Symbol: ∨

| P | Q | P ∨ Q |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

# Exclusive Or (XOR)

- Binary Operator, Symbol: $\oplus$

| P | Q | $P \oplus Q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

# Implication (if - then)

- Binary Operator, Symbol: $\rightarrow$

| P | Q | P→Q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

# Bi-conditional (iff)

- Binary Operator, Symbol: $\leftrightarrow$

| P | Q | $P \leftrightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

# Truth Tables of Compound Propositions

**TABLE 7** The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

| $p$ | $q$ | $\neg q$ | $p \vee \neg q$ | $p \wedge q$ | $(p \vee \neg q) \rightarrow (p \wedge q)$ |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | T | T | F | F |
| F | T | F | F | F | T |
| F | F | T | T | F | F |

# Examples

Use truth tables to verify the associative laws

a) $(p \lor q) \lor r \equiv p \lor (q \lor r)$.

b) $(p \land q) \land r \equiv p \land (q \land r)$.

# Exercise:

Let $p$ and $q$ be the propositions

$p$ : You drive over 65 miles per hour.

$q$ : You get a speeding ticket.

Write these propositions using $p$ and $q$ and logical connectives (including negations).

a) You do not drive over 65 miles per hour.
b) You drive over 65 miles per hour, but you do not get a speeding ticket.
c) You will get a speeding ticket if you drive over 65 miles per hour.
d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.
f) You get a speeding ticket, but you do not drive over 65 miles per hour.
g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

# CONVERSE, CONTRAPOSITIVE, AND INVERSE

- We can form some new conditional statements starting with a conditional statement

  $p \rightarrow q$.

- There are three related conditional statements that occur so often that they have special names.

# CONVERSE, CONTRAPOSITIVE, AND INVERSE

| | |
|---|---|
| $p \rightarrow q$ | **The proposition $q \rightarrow p$ is called the converse** |
| | The proposition $\neg q \rightarrow \neg p$ is called the **contrapositive** |
| | The proposition $\neg p \rightarrow \neg q$ is called the **inverse** |

# Truth Table

| P | Q | Cond | Converse | Inverse | Contrapositive |
|---|---|---|---|---|---|
| P | Q | $p \to q$ | q$\to p$ | $\neg p \to \neg q$ | $\neg$q$\to \neg p$ |
| T | T | T | T | T | T |
| T | F | F | T | T | F |
| F | T | T | F | F | T |
| F | F | T | T | T | T |

# CONVERSE, CONTRAPOSITIVE, AND INVERSE

- We will see that of these three conditional statements formed from $p \rightarrow q$, only the contrapositive always has the same truth value as $p \rightarrow q$.

# Analysis

- The contrapositive, $\neg q \rightarrow \neg p$, of a conditional statement $p \rightarrow q$ always has the same truth value as $p \rightarrow q$.
- The contrapositive is false only when $\neg p$ is false and $\neg q$ is true, that is, only when $p$ is true and $q$ is false.
- Neither the converse, $q \rightarrow p$, nor the inverse, $\neg p \rightarrow \neg q$, has the same truth value as $p \rightarrow q$ for all possible truth values of $p$ and $q$.

Note that when $p$ is true and $q$ is false, the original conditional statement is false, but the converse and the inverse are both true.

## EXAMPLE

- What are the contrapositive, the converse, and the inverse of the conditional statement

"The home team wins whenever it is raining."

The Inverse ?

The Converse?

The contrapositive?

# *Solution:*

Because “ q whenever p” is one of the ways to express the conditional statement $p \rightarrow q$, the original statement can be rewritten as
“If it is raining, then the home team wins.”

The inverse is
“If it is not raining, then the home team does not win.”

The converse is
“If the home team wins, then it is raining.”

The contrapositive of this conditional statement is
“If the home team does not win, then it is not raining.”

Only the contrapositive is equivalent to the original statement.

# Note-

Performing any two actions always result in the third one.

For example-

- Inverse of converse is contrapositive.
- Inverse of contrapositive is converse.
- Converse of inverse is contrapositive.
- Converse of contrapositive is inverse.
- Contrapositive of inverse is converse.
- Contrapositive of converse is inverse.

**a)** If it snows today, I will ski tomorrow.

- Inverse

  "If it does not snow today, then I will not ski tomorrow."

- Contrapositive

  ''If I don't ski tomorrow, then it will not have snowed today."

- Converse

  "I will ski tomorrow only if it snows today."

# Exercise:

- State the converse, contrapositive, and inverse of each of these conditional statements.
  a) If it snows today, I will ski tomorrow.
  b) I come to class whenever there is going to be a quiz.
  c) A positive integer is a prime only if it has no divisors other than 1 and itself