FINAL ASSIGNMENT

CALCULUS **20 MARKS** **LAST DATE: 31-01-2025**

Q.1 Evaluate the following:

(a) $\int \frac{\sin \ln x}{x(3-\cos \ln x)^{\frac{1}{2}}} dx$ (b) $\int_0^{\frac{\pi}{2}} \cos^4 x \, dx$ (c) $\int_{-6}^{-2\sqrt{3}} \frac{dx}{x\sqrt{x^2-9}}$

(d) $\int \sin\sqrt{2x} dx$ (f) $\int \sqrt{1-\cos x} \, dx$

Q.2 **If m and n are positive integers , show that $\int_0^{\infty} x^m e^{-ax^n} dx$ can be expressed in the form of $\frac{1}{n a^{\frac{m+1}{n}}} \Gamma\left(\frac{m+1}{n}\right)$.**

Q.3 **Evaluate the following by using an appropriate function:**

a) $\int_0^{\frac{1}{2}} x^4 (1-2x)^3 dx$ b) $\int_0^{\frac{1}{\sqrt{2}}} x^2 \left(1-2x^2\right)^{\frac{1}{2}} dx$

c) $\int_0^{\frac{\pi}{2}} \sin\theta \sqrt{cos^5\theta} d\theta$ d) $\int_0^{\frac{\pi}{4}} sin^3 2\theta \, . cos^6 2\theta \, . d\theta$

Q.4 **Evaluate the double integral :**

(a) $\iint_R (x-3y^2) dA$, $where\ R = \{(x,y) \mid 0 \le x \le 2,\ 1 \le y \le 2\}$

(b) $\int_0^1 \int_0^1 (2-x^2-y^2)\, dy\, dx.$

Q.5 **Evaluate the integral :**

$\iiint_E (xy+z^2) dV, where\ E = \{(x,y,z) \mid 0 \le x \le 2,\ 0 \le y \le 1,\ 0 \le z \le 3\}$

Q.6 **A street vendor sells 'a' hamburgers , 'b' hot dogs and 'c' bottles of water on a given day. He charges $4 for a hamburger, $2.50 for a hot dogs and $1 for a bottle of water. If A=(a , b , c) and P= (4 , 2.5 , 1) , what is the meaning of the dot product A.P?**

Q.7(a) **Find all vectors V such that : (1 , 2 , 1) x V = (3 , 1 , -5)**

(b) **Explain why there is no vector V such that: (1 , 2 , 1) x V = (3 , 1 , 5)**