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SERIAL No. : 18 (CS GROUP)

Q 1) Evaluate the following:

a) $\int \frac{\sin \ln x}{x(3-\cos \ln)^{1/2}} dx$

Use ~~substitution~~ substitution:

Let $u = \ln x$ $\qquad \frac{d}{dx} = \frac{d}{dx} \ln x$

$x = e^x$ and $dx = e^x du$

$I = \int \frac{\sin x}{e\sqrt{3-\cos(x)}} e^x dx \longrightarrow \int \frac{\sin x}{\sqrt{3-\cos x}} dx$

Let $v = 3 - \cos(u)$

$\frac{dv}{dx} = \sin u \longrightarrow \int \frac{1}{\sqrt{v}} dv$

$= 2\sqrt{v} + C$

Put all values:

$\boxed{I = 2\sqrt{3-\cos \ln x} + c}$

b) $\int_0^{\pi/2} \cos 4x \, dx$

$\cos 4x \, dx = \int_0^{\pi/2} \frac{\sin 4x}{4} + c$

Apply limit:

$\frac{\sin(\cancel{4}^2 x . \pi/\cancel{2})}{4} = \frac{\sin 2\pi}{4} = \frac{0}{4} = 0$

If $x = 0$

$\frac{\sin(4.0)}{4} = \frac{\sin 0}{4} = \boxed{0}$

8) $\int_{-6}^{-2/\sqrt{3}} \frac{dx}{x\sqrt{x^2-9}}$

$\therefore \quad x = \quad x^2 - a^2 \quad = a\sec\theta$

$x = 3\sec\theta$

$dx = 3\sec\theta \tan\theta \, d\theta$

$= \sqrt{x^2-9} = \sqrt{9\sec^2\theta - 9} = 3\tan\theta$

$\therefore \quad x = 3\sec\theta$

$I = \int \frac{1}{3} d\theta + \frac{\theta}{3} + C$

$\therefore \quad \sec\theta = \frac{x}{3}$

$I = \frac{1}{3} \sec^{-1}\left(\frac{x}{3}\right) + c$

Apply limit $\quad \sec\theta = \frac{x}{3}$

if $x = -6$

$\sec\theta = \frac{-\cancel{6}^{2}}{\cancel{3}_{1}} = -2$

$\rightarrow \quad \theta = \sec^{-1}(-2) = 1$

$= -2\sqrt{3}$

$\rightarrow \quad \sec\theta = \frac{-2\sqrt{3}}{3}$

$\boxed{I = \frac{1}{3}\sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right) - \sec^{-1}(-2)}$

d) $I = \int \sin\sqrt{2x}\, dx$

Let $u = \sqrt{2x}$

$u^2 = 2x \rightarrow x = \frac{u^2}{2}$

$\frac{du^2}{dx} = dx \rightarrow u.du$

$I = \int \sin u \, . \, u\, du$

Let $v = u$, $dv = du$

$dw = \sin(u)\, dx$, so $w = -\cos(u)$

$\int v\, dw = v.w - \int w\, dv$

$I = -u\cos(x) + \int \cos(u)\, dx$

$I = -u\cos(x) + \sin(x) + C$

$\boxed{I = -\sqrt{2x}\cos(\sqrt{2x}) + \sin\sqrt{2x} + C}$

e) $I = \int \sqrt{1 - \cos x}\, dx$

$\because 1 - \cos x = \frac{2\sin^2 x}{2}$

$I = \int \sqrt{2\sin^2(\frac{x}{2})}\, dx \rightarrow \int \sqrt{2} . \int \sin \frac{x}{2} \, dx$

Let $u = \frac{x}{2} \rightarrow du = \frac{1}{2}\, dx$

$dx = 2\, du$

$\rightarrow I = \int \sqrt{2} \sin x \, dx$

$\rightarrow I = \sqrt{2} \, . -\cos x + c$

$\rightarrow \boxed{I = 2\sqrt{2}\left(-\cos\frac{x}{2}\right) + c}$

Q2 If m and n are positive integers, show that $\int_0^\infty x^m e^{-ax^n} dx$ can be expressed in the form of $\frac{1}{na^{\frac{m+1}{n}}} \Gamma\left(\frac{m+1}{n}\right)$

Sol

$$\int_0^\infty x^m e^{-ax^n} dx$$

$$\rightarrow \int_0^\infty x^m e^{-ax^n} = \frac{1}{na(m+1)n} \cdot \gamma\left(\frac{m+1}{n}\right)$$

Let $v = ax^n$, $x^n = \frac{u}{a}$

and $dx = \frac{1}{n} \cdot \frac{1-n}{n} dx$

$$\rightarrow \int_0^\infty x^m e^{-ax^n} dx = \int_0^\infty \left(\frac{u}{a}\right)^{\frac{m}{n}} \cdot e^{x} \cdot \frac{1}{n} \cdot \frac{1-n}{n} dx$$

Simplify :

$$\frac{1}{na(m+1)/n} \int_0^\infty \frac{u(m+1)}{(n-1)} \cdot e^{-u} dx$$

$$\rightarrow \int_0^\infty \frac{u(m+1)}{n-1} e^{-u} dx = \frac{\gamma(m+1)}{n}$$

$$\rightarrow \boxed{\int_0^\infty x^m e^{-ax^n} dx = \frac{1}{na(m-1)/n} \gamma\left(\frac{m+1}{n}\right)}$$

Ans

Q 3) Evaluate:

a) $\int_0^{\frac{1}{2}} x^4 (1-2x)^3 \, dx$

Let $u = 1-2x$, $du = -2dx$

$x = \frac{1-u}{2}$

$x = 0 \qquad u = 1$

$x = \frac{1}{2} \qquad u = 0$

$$\int_0^{1/2} x^4 (1-2x)^3 \, dx = \int^0 \left(\frac{1-u}{2}\right)^4 u^3 \left(\frac{1}{2}\right) du$$

Simplify: $\frac{1}{32} \int_0^1 (1-u)^4 \, u^3 \, du$

$$\rightarrow \frac{1}{32} \int_0^1 (1 - 4u + 6u^2 - 4u^3 + u^4)$$

$$\rightarrow \frac{1}{32} \int_0^1 (u^3 + 4u^4 + 6u^5 - 4u^6 + u^7) \, du$$

$$\rightarrow \frac{1}{32} \left(\frac{u^2}{4} - \frac{4u^5}{5} + \frac{6u^6}{6} - \frac{4u^7}{7} + \frac{4u^8}{8} \right]_0^1$$

For Beta function: $B(4,5) \quad \frac{\Gamma(4)\Gamma(5)}{\Gamma(9)}$

$$= \frac{3! \cdot 4!}{8!}$$

$$= \frac{6 \times 24}{40320}$$

$$= \frac{144}{40320} = \frac{1}{280}$$

$$= \frac{1}{32}\left(\frac{1}{280}\right) = \boxed{\frac{1}{8960}}$$

b) $\int_0^{1/\sqrt{2}} x^2(1-2x^2)^{1/2}\,dx$

Let $u = 1-2x^2$ $\quad x^3 = \frac{1-4}{2}$

$du = -4x\,dx$

when $x = 1/\sqrt{2}$ $\quad u = 0$

$\rightarrow \int_0^{1/\sqrt{2}} x^2(1-2x^2)\,dx = \int_1^0 \frac{1-4}{2} \cdot u\left(\frac{-1}{4}\right) du$

$\rightarrow \frac{1}{8}\int_0^1 u(u-1)\,du = \frac{1}{8}\int_0^1 (u^2-u)\,du$

$\rightarrow \frac{1}{8}\left[\frac{u^3}{3} - \frac{u^2}{2}\right]_0^1$

$\rightarrow \frac{1}{8}\left(\frac{1}{3} - \frac{1}{2}\right) = \frac{1}{8} \cdot \frac{1}{6} = \boxed{-\frac{1}{48} \text{ Ans}}$

c) $\int_0^{\pi/2} \sin\theta \sqrt{\cos^5\theta}\, d\theta$

$\sqrt{\cos 5\theta}$ as $\cos\theta^{5/2}$

$\int_0^{\pi/2} \sin\theta\,[\cos\theta]^{5/2}\,d\theta$

Let $u = \cos\theta$, $du = -\sin\theta\,d\theta$

when $\theta = 0$, $u = 1$ when $\theta = \pi/2$, $u = 0$

$\int_0^{\pi/2} \sin\theta\,(\cos\theta)^{5/2}\,d\theta = \int (u)^{5/2}(-1)\,du$

$\int_0^1 u^{5/2}\,du = \frac{u^{7/2}}{7/2} = \frac{2}{7}u^{7/2}$

$\boxed{\frac{2}{7}\left[u^{7/2}\right]_0^1 = \frac{2}{7}[1-0] = \frac{2}{7}}$ Ans

d) $\int_0^{\pi/2} \sin^3 2\theta \cdot \cos^6 2\theta$

Let $u = \sin\theta$, $du = \cos\theta\, d\theta$, $\cos^2\theta = 1-u^2$

when $\theta = 0$, $u = 1$

when $\theta = \frac{\pi}{4}$, $v = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$= \int_0^{\pi/2} (\sin\theta)^{3/2} (\cos\theta)^3 \, d\theta$

$= \int_0^{\sqrt{2}/2} u^{3/2} (1-u^2) \, du$

Expanding :

$\int_0^{\pi/2} u^{3/2}(1-u^2), \, du = \int^{\sqrt{2}/2} (u^{3/2} - u^{7/2})$

$\int_0^{\pi/2} u^{3/2} \, du - \int^{\sqrt{2}/2} u^{7/2} \, du$

for $u^{3/2} \, du = \frac{u^{5/2}}{5/2} = \frac{2}{5} u^{5/2}$

for $u^{7/2} \, du = \frac{u^{9/2}}{9/2} = \frac{2}{9} u^{9/2}$

$\rightarrow \frac{2}{5}\left(\frac{\sqrt{2}}{2}\right)^{5/2} - \frac{2}{9}\left(\frac{\sqrt{2}}{2}\right)^{9/2}$

$\rightarrow \boxed{\frac{\sqrt{2}}{20} - \frac{\sqrt{2}}{72}}$ Ans

Q4) Evaluate the double integral:

a) $\iint_R (x - 3y^2)\, dA$, where $R = \{(x,y) \mid 0 \le x \le 2,\ 1 \le y \le 2\}$

$$\int_0^2 \int_1^2 (x - 3y^2)\, dy\, dx$$

inner : $\int_1^2 (x - 3y^2)\, dy = \int_1^2 x\, dy - \int_1^2 3y^2\, dy$

for $\int_1^2 x\, dy$:

$$\int_1^2 x\, dy = x \int_1^2 1.\, dy = x(2-1) = x$$

for $\int_1^2 3y^2\, dy$:

$$\int_1^2 3y^2\, dy = 3 \int y^2\, dy = 3\left[\frac{y^3}{3}\right]_1^2 = (y^3)^2 = (2)^3 - (1)^3$$

$$\boxed{\int_1^2 3y^2\, dy = 7}$$

Outer : $\int_0^2 x - 7\, dx = \int_0^2 x\, dx - \int_0^2 7\, dx$

for $\int_0^2 2dx$ =

$$\int_0^2 x\, dx = \left[\frac{x^2}{2}\right]_0^2 = \frac{2^2}{2} - \frac{0^2}{2} = 2$$

for $\int_0^2 7\, dx = 7 \int_0^2 1.\, dx = 14$

$$\int_0^2 (x - 7)\, dx = 2 - 14 = \boxed{-12} \text{ Ans}$$

b) $\int_0^1 \int_0^1 (2-x^2-y^2)\, dy\, dx$

inner:

$\int_0^1 (2-x^2-y^2)\, dy = \int_0^1 2\,dy - \int_0^1 x^2 dy - \int_0^1 y^2\, dy$

for $\int_0^1 2\,dy = 2\int_0^1 1\,dy = 2.(1-0) = 2$

for $\int_0^1 x^2\, dy = x^2 \int_0^1 1\,dy = x^2 (1-0)$

for $\int_0^1 y^2\, dy = \frac{y^3}{3} = \frac{1}{3}$

combining: $\int_0^1 (2-x^2-y^2)\, dy = 2-x^2-\frac{1}{3}$

outer: $\int_0^1 (2x^2-\frac{1}{3})\, dx = \int_0^1 2dx - \int_0^1 x^2 dx - \int_0^1 \frac{1}{3} dx$

for $\int_0^1 2dx = 2\int_0^1 1.\, dx = 2.\,(1-0) = 2$

for $\int_0^1 x^2\, dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - (1-0) = \frac{1}{3}$

Combining:

$\int_0^1 (2x^2 - \frac{1}{3})\, dx = 2 - \frac{1}{3} - \frac{1}{3} = \frac{2-2}{3}$

$= \frac{6}{3} - \frac{2}{3} = \boxed{\frac{4}{3}}$ Ans

Q5 Evaluate the integral:

$\iiint_E (xy+z^2)\,dv$, where $E = \{(x,y,z) \mid 0 \le x \le 2,$
$0 \le y \le 1,\ 0 \le z \le 3\}$

Solving with respect to z:

$\int_0^3 (xy + z^2)\,dz$

$xy \int_0^3 dz + \int_0^3$

1) $xy\left(z \int_0^3\right) = xy(3-0) = 3xy$

2) $\int_0^3 z^2\,dz = \frac{z^3}{3}\Big]_0^3 = \frac{27}{3} - 0 = 9$

$\boxed{\int_0^3 = 3xy + 9}$

Solving with respect to y:

$\int_0^1 3xy\,dy \int_0^1 9\,dy \rightarrow 3x \int_0^1 y\,dy = 3x \left|\frac{y^2}{2}\right|_0^1 = \frac{3x}{2}$

2) $\int_0^1 9\,dy = 9y\Big|_0^1 = 9(1-0) = 9$

$\boxed{\int_0^1 y = \frac{3x}{2} + 9}$

Solving with respect to x:

$\rightarrow \int_0^2 \left(\frac{3x}{2} + 9\right) \rightarrow \int_0^2 \frac{3x}{2}\,dx \int_0^2 9\,dx$

$\rightarrow \frac{3}{2}\int_0^2 x\,dx = \frac{3}{2}\cdot\frac{x^2}{2}\Big|_0^2 = \frac{3}{2}\cdot\frac{4}{2} = 3$

$\int_0^2 9\,dx = 9(x)_0^2 = 9(2-0) = 18$

$\boxed{3 + 18 = 21}$

Q6) A street vendor sells 'a' hamburgers 'b' hotdogs and 'c' bottles of water on a given day. He charges \$ 4 for a hamburger, \$ 2.50 for a hot dog and \$1 for a bottle of watter. If $A = (a, b, c)$ and $P = (4, 2.5, 1)$, what is the meaning of the dot product $A.P$?

Since their product is scalar,

$$x.y = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

total from hamburgers:

$R_1 = 4a$

from hot dogs $= R_2 = 2.5b$

from soft drinks $= R_3 = c$

total is $R = 4a + 2.5b + c$

let $A = \{a, b, c\}$ and $P = \{4, 2.5, 1\}$

and these products as:

$$\boxed{A.P = 4a + 2.5b + c}$$

Ans

Q7) (a) Find all vectors V such that:

$$(1, 2, 1) \times V = (3, 1, -5)$$

(a) $(1, 2, 1) \times V = (3, 1, -5)$

let

$V = (v_1, v_2, v_3)$ are unknown.

$(1, 2, 1) \times (v_1, v_2, v_3) = (3, 1, -5)$

Using cross product A and B is :

$A = a_1, a_2, a_3$ and b_1, b_2, b_3.

The cross product becomes :

$(1, 2, 1) \times (v_1, v_2, v_3) = 2v_3 - 1v_2, 1v_1 - 1v_3, 1v_2 - 2v_1)$

1) $2z - y = \boxed{3}$

2) $x - z = \boxed{1}$

3 $y - 2x = \boxed{-5}$

For v_1, v_2 and v_3 :

$v_1 = v_3 + 1$, $v_2 = 2v_3 - 3$

Let $v_3 = t$ (real no.)

$v_1 = t + 1$ $v_2 = 2t - 3$

So, $\boxed{V = (t+1, 2t-3, t)}$

Ans

(b) Explain why there is no vector V such that $(1, 2, 1) \times V = (3, 1, 5)$

$(1, 2, 1) \times V = (3, 1, 5)$

cross product of two vectors is perpendicular to both vectors

$A = (1, 2, 1)$ and $B = V$ result of $A \times B$ perpendicular to A

Date

Check (3, 1, 5) is perpendicular to (1, 2, 1) the dot product of two vectors

$A = a_1, a_2, a_3$

$B = b_1, b_2, b_3$

$A.B = a_1 b_1 + a_2 b_2 + a_3 b_3$

for $(1, 2, 1)$ & $(3, 1, 5) = 1.3 + 2.1 + 1.5$

$= \boxed{10}$

The dot product is not zero, because vectors are not perpendicular.