Applied Physics

(PHC-103)

Lecture 01 & 02

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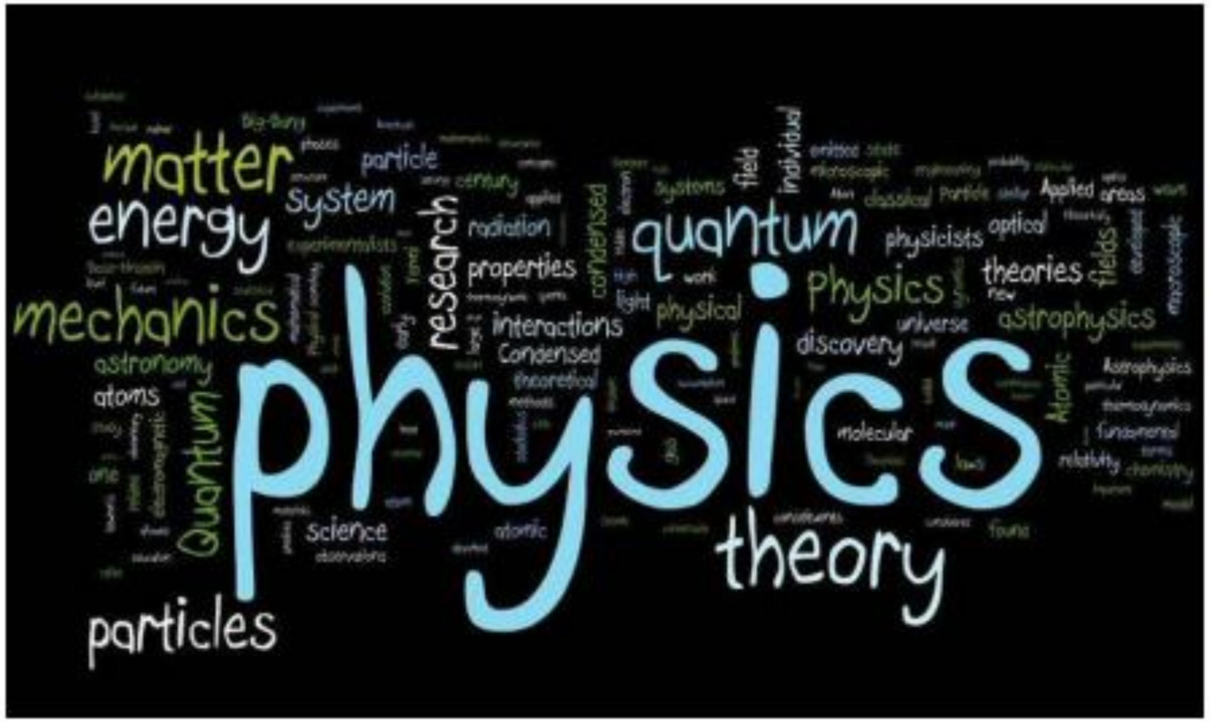
What is Physics?

- A branch of science that involve the study of physical world.
- Energy, Matter and Transaction between them is related.
- It comes from the Greek word "Physica".



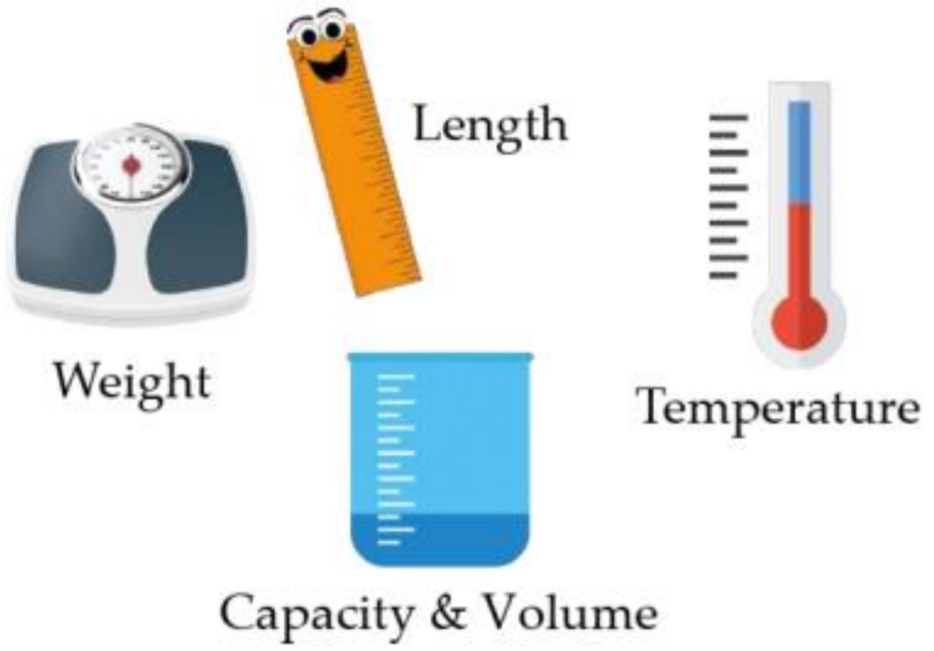
Measurement

- Science and engineering are based on measurements and comparisons. Thus, we need rules about how things are measured and compared, and we need experiments to establish the units for those measurements and comparisons.

- One purpose of physics (and engineering) is to design and conduct those experiments.

- **For example**, physicists strive to develop clocks of extreme accuracy so that any time or time interval can be precisely determined and compared. You may wonder whether such accuracy is actually needed or worth the effort. Here is one example of the worth: Without clocks of extreme accuracy, the Global Positioning System (GPS) that is now vital to worldwide navigation would be useless.

Need of Measurement in Engineering & Science



- We discover physics by learning how to measure the quantities involved in physics. Among these quantities are length, time, mass, temperature, pressure, and electric current.

- We measure each physical quantity in its own units, by comparison with a standard. The unit is a unique name we assign to measures of that quantity.

- **For example**, meter (m) for the quantity length.

Table 1-1 Units for Three SI Base Quantities

Quantity	Unit Name	Unit Symbol
Length	meter	m
Time	second	s
Mass	kilogram	kg

Distance Conversion

X 1000

km | m

/ 1000

X 100

m | cm

/ 100

Mass Conversion

X 1000

kg | gm

/ 1000

X 1000

gm | mg

/ 1000

Time Conversion

X 60

M | S

/ 60

Temperature Conversion

C(9 ⁄ 5) + 32

^{o}C | ^{o}F

(F – 32) × 5 ⁄ 9

Think of Kelvin!!!

Motion

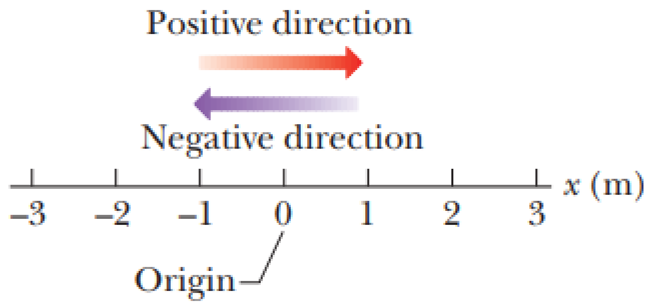
- The world, and everything in it, moves. Even seemingly stationary things, such as a roadway, move with Earth's rotation, Earth's orbit around the Sun, the Sun's orbit around the center of the Milky Way galaxy, and that galaxy's migration relative to other galaxies.

Position and Displacement

- To locate an object means to find its position relative to some reference point, often the origin (or zero point) of an axis.

- A change from position x_1 to position x_2 is called a **displacement** Δx, where

$$\Delta x = x_2 - x_1.$$



- Displacement is an example of a vector quantity,

which is a quantity that has both a direction and a magnitude.

Average Velocity and Average Speed

Actually, several quantities are associated with the phrase "how fast." One of them is the **average velocity** v_{avg}, which is the ratio of the displacement Δx that occurs during a particular time interval Δt to that interval:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \qquad (2\text{-}2)$$

Average speed s_{avg} is a different way of describing "how fast" a particle moves. Whereas the average velocity involves the particle's displacement Δx, the average speed involves the total distance covered (for example, the number of meters moved), independent of direction; that is,

$$s_{avg} = \frac{\text{total distance}}{\Delta t}. \qquad (2\text{-}3)$$

Because average speed does *not* include direction, it lacks any algebraic sign. Sometimes s_{avg} is the same (except for the absence of a sign) as v_{avg}. However, the two can be quite different.

Sample Problem

You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?

(b) What is the time interval Δt from the beginning of your drive to your arrival at the station?

(c) What is your average velocity v_{avg} from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?

While driving a car at 90 km/h, how far do you move while your eyes shut for 0.50 s during a hard sneeze?

A car moves uphill at 40 km/h and then back downhill at 60 km/h. What is the average speed for the round trip?

You are to drive 300 km to an interview. The interview is at 11:15 A.M. You plan to drive at 100 km/h, so you leave at 8:00 A.M. to allow some extra time. You drive at that speed for the first 100 km, but then construction work forces you to slow to 40 km/h for 40 km. What would be the least speed needed for the rest of the trip to arrive in time for the interview?

Acceleration

When a particle's velocity changes, the particle is said to undergo **acceleration** (or to accelerate). For motion along an axis, the **average acceleration** a_{avg} over a time interval Δt is

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}, \tag{2-7}$$

where the particle has velocity v_1 at time t_1 and then velocity v_2 at time t_2. The **instantaneous acceleration** (or simply **acceleration**) is

$$a = \frac{dv}{dt}. \tag{2-8}$$

In words, the acceleration of a particle at any instant is the rate at which its velocity is changing at that instant. Graphically, the acceleration at any point is the slope of the curve of $v(t)$ at that point. We can combine Eq. 2-8 with Eq. 2-4 to write

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}. \tag{2-9}$$

In words, the acceleration of a particle at any instant is the second derivative of its position $x(t)$ with respect to time.

Sample Problem

A particle's position on the x axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with x in meters and t in seconds.

(a) Because position x depends on time t, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

The position of a particle moving along an x axis is given by $x = 12t^2 - 2t^3$, where x is in meters and t is in seconds. Determine (a) the position, (b) the velocity, and (c) the acceleration of the particle at $t = 3.0$ s.

At a certain time a particle had a speed of 18 m/s in the positive x direction, and 2.4 s later its speed was 30 m/s in the opposite direction. What is the average acceleration of the particle during this 2.4 s interval?

Vectors and Scalars

- A particle moving along a straight line can move in only two directions. We can take its motion to be positive in one of these directions and negative in the other. For a particle moving in three dimensions, however, a plus sign or minus sign is no longer enough to indicate a direction. Instead, we must use a vector.

- A **vector** has magnitude as well as direction, and vectors follow certain (vector) rules of combination. Some physical quantities that are vector quantities are displacement, velocity, and acceleration.

- Not all physical quantities involve a direction. Temperature, pressure, energy, mass, We call such quantities **scalars**.

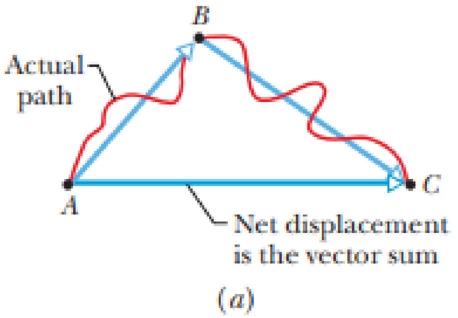
Adding Vectors Geometrically

Suppose that, as in the vector diagram of Fig. 3-2*a*, a particle moves from A to B and then later from B to C. We can represent its overall displacement (no matter what its actual path) with two successive displacement vectors, AB and BC. The *net* displacement of these two displacements is a single displacement from A to C. We call AC the **vector sum** (or **resultant**) of the vectors AB and BC. This sum is not the usual algebraic sum.

In Fig. 3-2*b*, we redraw the vectors of Fig. 3-2*a* and relabel them in the way that we shall use from now on, namely, with an arrow over an italic symbol, as in $\vec{a}$. If we want to indicate only the magnitude of the vector (a quantity that lacks a sign or direction), we shall use the italic symbol, as in a, b, and s. (You can use just a handwritten symbol.) A symbol with an overhead arrow always implies both properties of a vector, magnitude and direction.

We can represent the relation among the three vectors in Fig. 3-2*b* with the *vector equation*

$$\vec{s} = \vec{a} + \vec{b}, \tag{3-1}$$



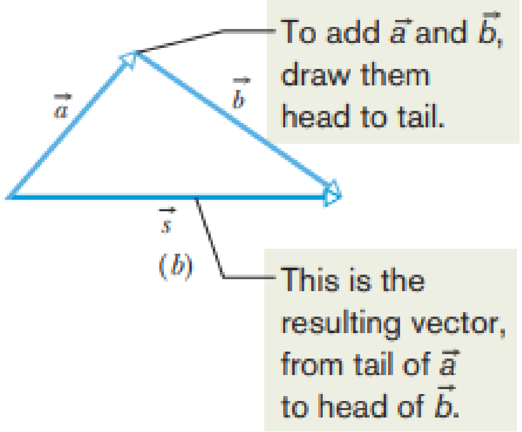


Figure 3-2 (*a*) AC is the vector sum of the vectors AB and BC. (*b*) The same vectors relabeled.

Properties. Vector addition, defined in this way, has two important properties. First, the order of addition does not matter. Adding $\vec{a}$ to $\vec{b}$ gives the same result as adding $\vec{b}$ to $\vec{a}$ (Fig. 3-3); that is,

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \qquad \text{(commutative law).} \tag{3-2}$$

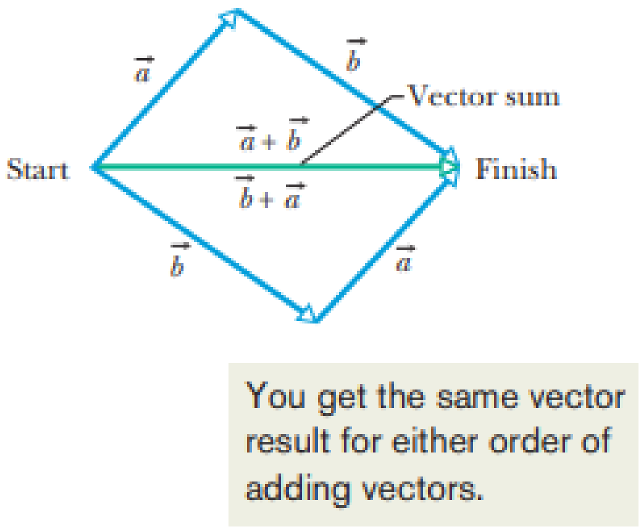


Figure 3-3 The two vectors $\vec{a}$ and $\vec{b}$ can be added in either order; see Eq. 3-2.

Second, when there are more than two vectors, we can group them in any order as we add them. Thus, if we want to add vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, we can add $\vec{a}$ and $\vec{b}$ first and then add their vector sum to $\vec{c}$. We can also add $\vec{b}$ and $\vec{c}$ first and then add *that* sum to $\vec{a}$. We get the same result either way, as shown in Fig. 3-4. That is,

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \qquad \text{(associative law).} \tag{3-3}$$

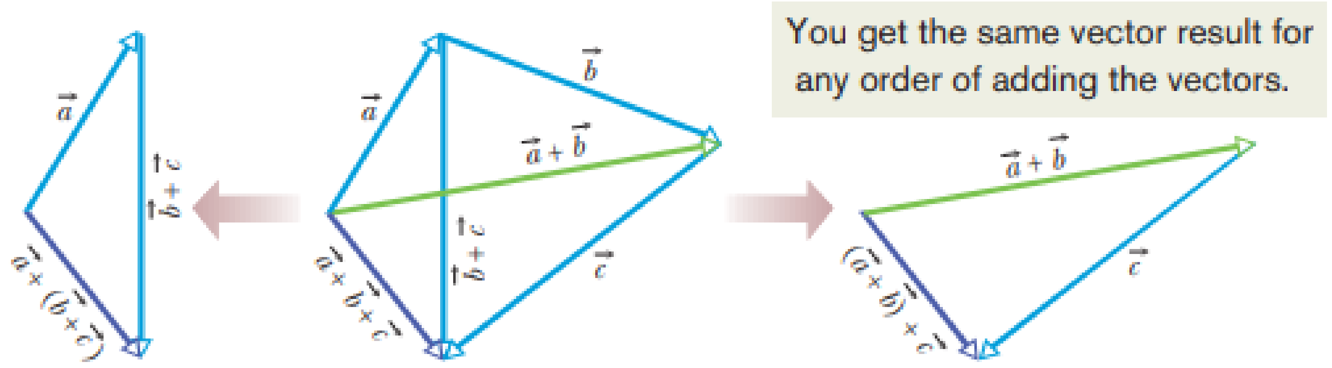


Figure 3-4 The three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ can be grouped in any way as they are added; see Eq. 3-3.

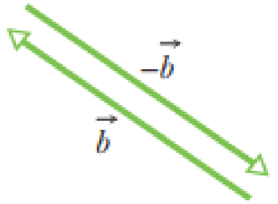


Figure 3-5 The vectors $\vec{b}$ and $-\vec{b}$ have the same magnitude and opposite directions.

The vector $-\vec{b}$ is a vector with the same magnitude as $\vec{b}$ but the opposite direction (see Fig. 3-5). Adding the two vectors in Fig. 3-5 would yield

$$\vec{b} + (-\vec{b}) = 0.$$

Thus, adding $-\vec{b}$ has the effect of subtracting $\vec{b}$. We use this property to define the difference between two vectors: let $\vec{d} = \vec{a} - \vec{b}$. Then

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \qquad \text{(vector subtraction);} \qquad (3\text{-}4)$$

that is, we find the difference vector $\vec{d}$ by adding the vector $-\vec{b}$ to the vector $\vec{a}$. Figure 3-6 shows how this is done geometrically.

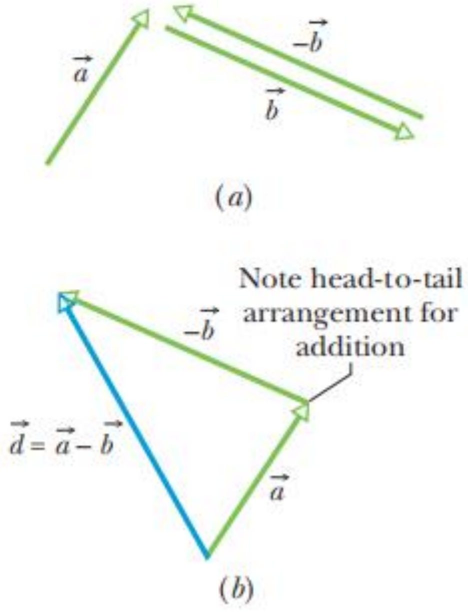
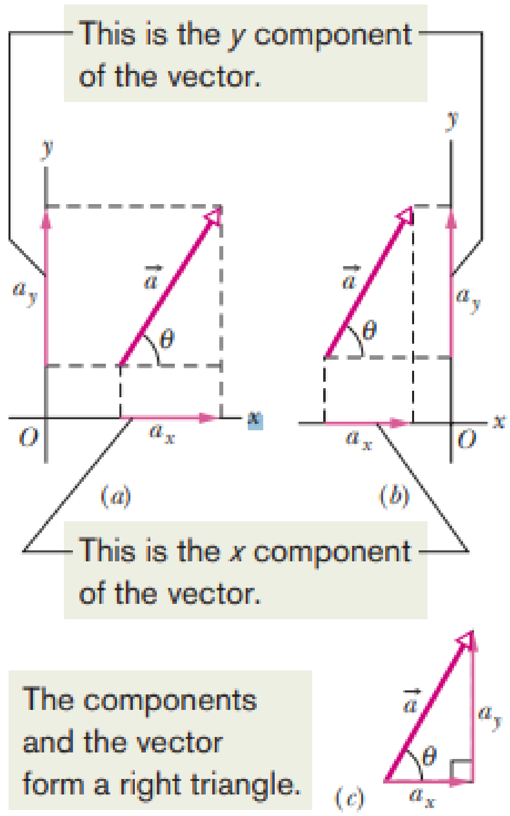


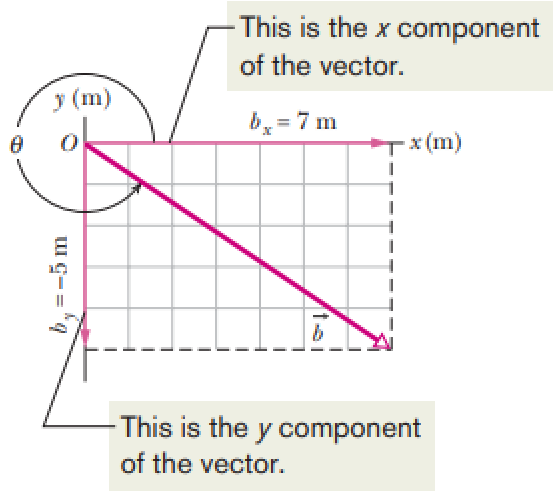
Figure 3-6 (*a*) Vectors $\vec{a}$, $\vec{b}$, and $-\vec{b}$. (*b*) To subtract vector $\vec{b}$ from vector $\vec{a}$, add vector $-\vec{b}$ to vector $\vec{a}$.

Components of Vectors

A **component** of a vector is the projection of the vector on an axis. In Fig. 3-7a, for example, a_x is the component of vector $\vec{a}$ on (or along) the x axis and a_y is the component along the y axis. To find the projection of a vector along an axis, we draw perpendicular lines from the two ends of the vector to the axis, as shown. The projection of a vector on an x axis is its *x component*, and similarly the projection on the y axis is the *y component*. The process of finding the components of a vector is called **resolving the vector.**

A component of a vector has the same direction (along an axis) as the vector. In Fig. 3-7, a_x and a_y are both positive because $\vec{a}$ extends in the positive direction of both axes. (Note the small arrowheads on the components, to indicate their direction.) If we were to reverse vector $\vec{a}$, then both components would be negative and their arrowheads would point toward negative x and y. Resolving vector $\vec{b}$ in Fig. 3-8 yields a positive component b_x and a negative component b_y.





Finding the Components. We can find the components of $\vec{a}$ in Fig. 3-7*a* geometrically from the right triangle there:

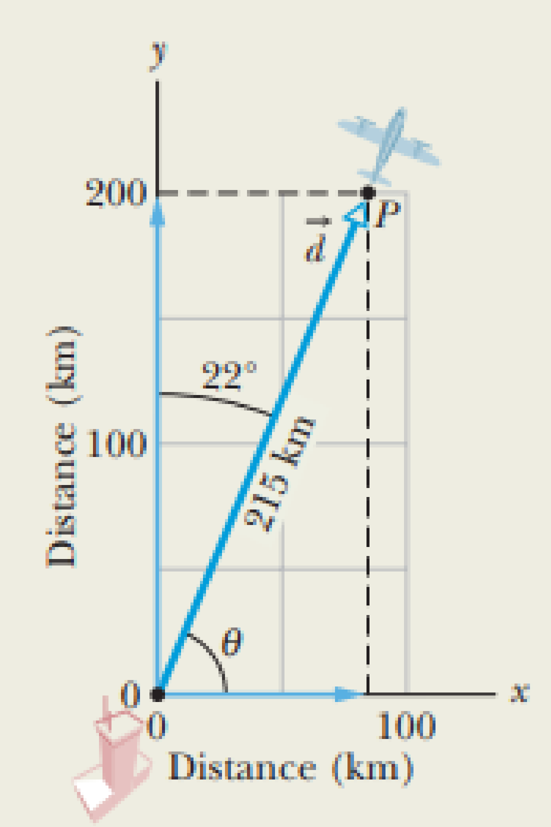
$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta, \tag{3-5}$$

If we know a vector in *component notation* (a_x and a_y) and want it in *magnitude-angle notation* (a and θ), we can use the equations

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x} \tag{3-6}$$

Sample Problem

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. This means that the direction is not due north (directly toward the north) but is rotated 22° toward the east from due north. How far east and north is the airplane from the airport when sighted?



Solution:

We are given the magnitude (215 km) and the angle (22° east of due north) of a vector and need to find the components of the vector.

We draw an *xy* coordinate system with the positive direction of x due east and that of y due north.

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta,$$

$$d_x = d\cos\theta = (215\text{ km})(\cos 68°)$$
$$= 81\text{ km} \qquad \text{(Answer)}$$

$$d_y = d\sin\theta = (215\text{ km})(\sin 68°)$$
$$= 199\text{ km} \approx 2.0 \times 10^2\text{ km}. \qquad \text{(Answer)}$$

Thus, the airplane is 81 km east and 2.0×10^2 km north of the airport.

Practice Problems

•1 What are (a) the x component and (b) the y component of a vector $\vec{a}$ in the xy plane if its direction is 250° counterclockwise from the positive direction of the x axis and its magnitude is 7.3 m?

•2 A displacement vector $\vec{r}$ in the xy plane is 15 m long and directed at angle $\theta = 30°$ in Fig. 3-26. Determine (a) the x component and (b) the y component of the vector.

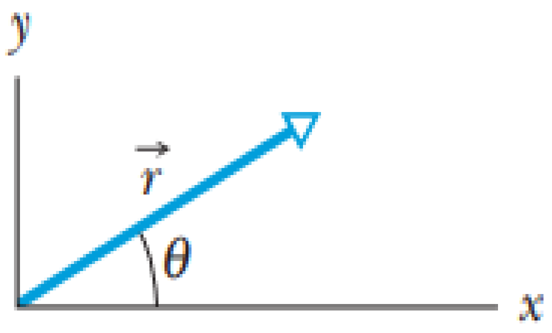


Figure 3-26 Problem 2.

•3 The x component of vector $\vec{A}$ is -25.0 m and the y component is $+40.0$ m. (a) What is the magnitude of $\vec{A}$? (b) What is the angle between the direction of $\vec{A}$ and the positive direction of x?

•4 Express the following angles in radians: (a) 20.0°, (b) 50.0°, (c) 100°. Convert the following angles to degrees: (d) 0.330 rad, (e) 2.10 rad, (f) 7.70 rad.

•6 In Fig. 3-27, a heavy piece of machinery is raised by sliding it a distance $d = 12.5$ m along a plank oriented at angle $\theta = 20.0°$ to the horizontal. How far is it moved (a) vertically and (b) horizontally?

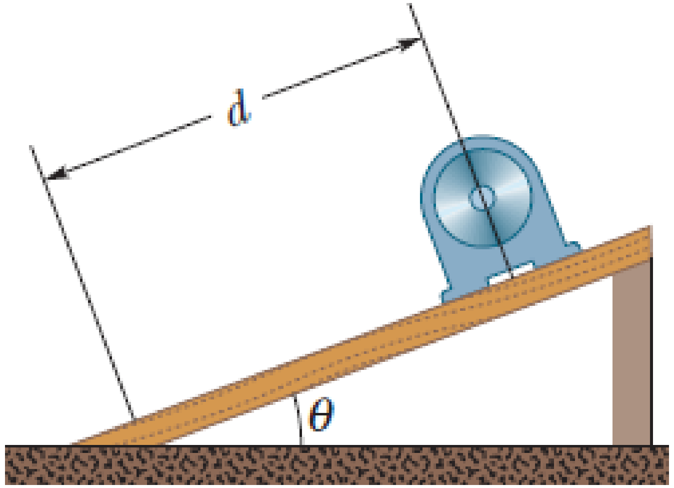


Figure 3-27 Problem 6.

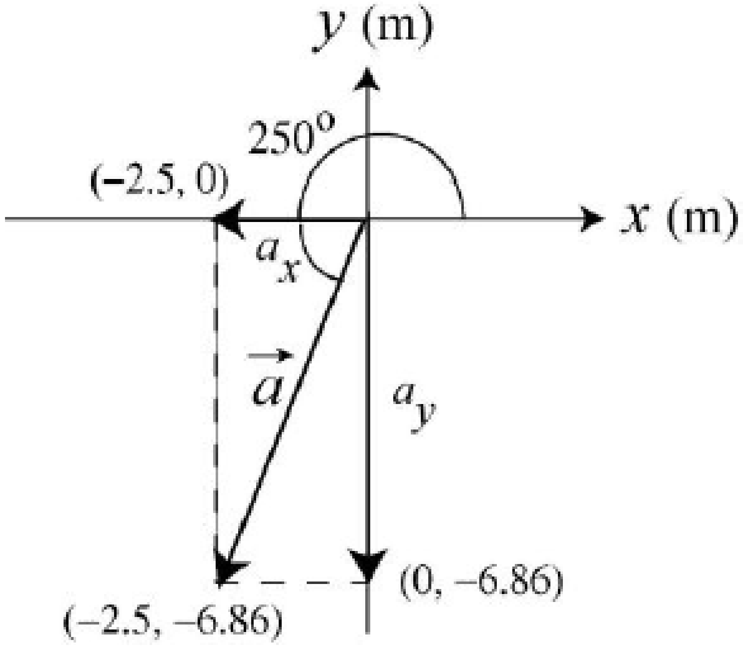
•7 Consider two displacements, one of magnitude 3 m and another of magnitude 4 m. Show how the displacement vectors may be combined to get a resultant displacement of magnitude (a) 7 m, (b) 1 m, and (c) 5 m.

Solutions:

1. **ANALYZE** (a) The x component of $\vec{a}$ is

$$a_x = a\cos\theta = (7.3\text{ m})\cos 250° = -2.50\text{ m},$$

(b) and the y component is $a_y = a\sin\theta = (7.3\text{ m})\sin 250° = -6.86\text{ m} \approx -6.9\text{ m}$. The results are depicted in the figure below:



2. (a) With $r = 15$ m and $\theta = 30°$, the x component of $\vec{r}$ is given by

$$r_x = r\cos\theta = (15 \text{ m}) \cos 30° = 13 \text{ m}.$$

(b) Similarly, the y component is given by $r_y = r \sin\theta = (15 \text{ m}) \sin 30° = 7.5$ m.

3.

ANALYZE (a) The magnitude of the vector $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0 \text{ m})^2 + (40.0 \text{ m})^2} = 47.2 \text{ m}$$

(b) Recalling that $\tan\theta = \tan(\theta + 180°)$,

$\tan^{-1}$ [(40.0 m)/ (– 25.0 m)] = – 58° or 122°.

4. The angle described by a full circle is 360° = 2π rad, which is the basis of our conversion factor.

(a) $20.0° = (20.0°)\dfrac{2\pi \text{ rad}}{360°} = 0.349 \text{ rad}$.

(b) $50.0° = (50.0°)\dfrac{2\pi \text{ rad}}{360°} = 0.873 \text{ rad}$.

(c) $100° = (100°)\dfrac{2\pi \text{ rad}}{360°} = 1.75 \text{ rad}$.

(d) $0.330 \text{ rad} = (0.330 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 18.9°$.

(e) $2.10 \text{ rad} = (2.10 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 120°$.

(f) $7.70 \text{ rad} = (7.70 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 441°$.

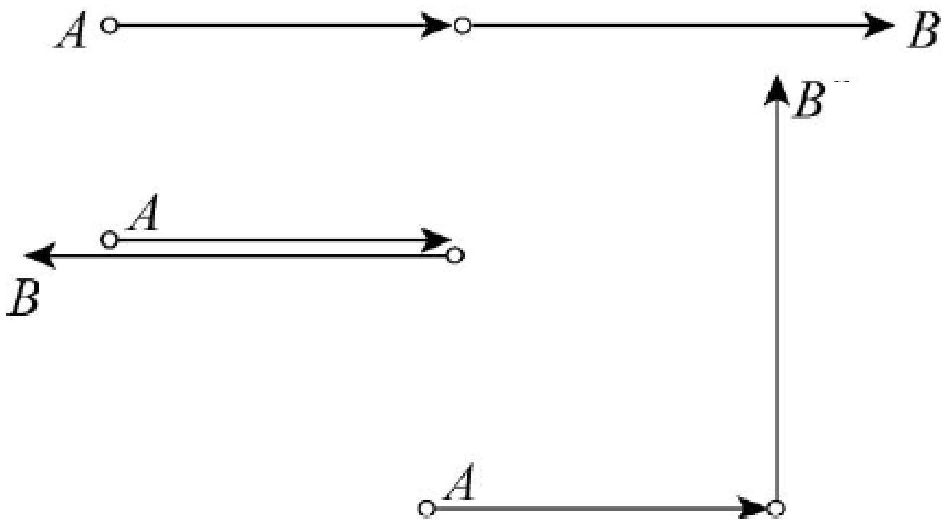
6. (a) The height is $h = d \sin\theta$, where $d = 12.5$ m and $\theta = 20.0°$. Therefore, $h = 4.28$ m.

(b) The horizontal distance is $d \cos\theta = 11.7$ m.

7. (a) The vectors should be parallel to achieve a resultant 7 m long

(b) anti-parallel (in opposite directions) to achieve a resultant 1 m long

(c) and perpendicular to achieve a resultant $\sqrt{3^2+4^2}=5\,\text{m}$ long



Unit Vectors

The unit vectors point along axes.

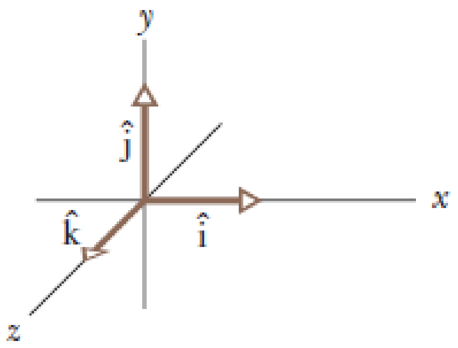


Figure 3.13 Unit vectors î, ĵ, and k̂ define the directions of a right-handed coordinate system.

Unit Vectors

A **unit vector** is a vector that has a magnitude of exactly 1 and points in a particular direction. It lacks both dimension and unit. Its sole purpose is to point—that is, to specify a direction. The unit vectors in the positive directions of the x, y, and z axes are labeled $\hat{i}$, $\hat{j}$, and $\hat{k}$, where the hat ˆ is used instead of an overhead arrow as for other vectors (Fig. 3-13). The arrangement of axes in Fig. 3-13 is said to be a **right-handed coordinate system.** The system remains right-handed if it is rotated rigidly. We use such coordinate systems exclusively in this book.

Unit vectors are very useful for expressing other vectors; for example, we can express $\vec{a}$ and $\vec{b}$ of Figs. 3-7 and 3-8 as

$$\vec{a} = a_x\hat{i} + a_y\hat{j} \tag{3-7}$$

and

$$\vec{b} = b_x\hat{i} + b_y\hat{j}. \tag{3-8}$$

These two equations are illustrated in Fig. 3-14. The quantities $a_x\hat{i}$ and $a_y\hat{j}$ are vectors, called the **vector components** of $\vec{a}$. The quantities a_x and a_y are scalars, called the **scalar components** of $\vec{a}$ (or, as before, simply its **components**).

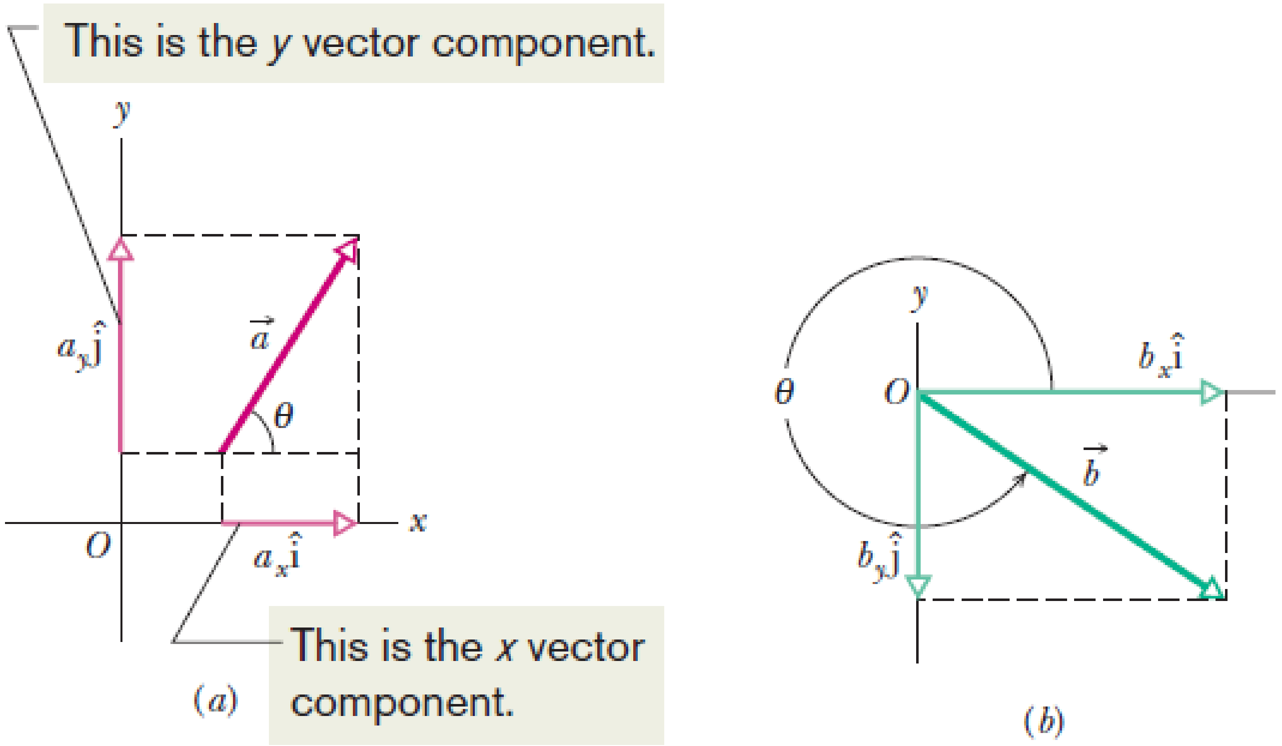


Figure 3-14 (*a*) The vector components of vector $\vec{a}$. (*b*) The vector components of vector $\vec{b}$.

Adding Vectors by Components

To start, consider the statement

$$\vec{r} = \vec{a} + \vec{b}, \tag{3-9}$$

which says that the vector $\vec{r}$ is the same as the vector $(\vec{a} + \vec{b})$. Thus, each component of $\vec{r}$ must be the same as the corresponding component of $(\vec{a} + \vec{b})$:

$$r_x = a_x + b_x \tag{3-10}$$

$$r_y = a_y + b_y \tag{3-11}$$

$$r_z = a_z + b_z. \tag{3-12}$$

In other words, two vectors must be equal if their corresponding components are equal.

This procedure for adding vectors by components also applies to vector subtractions. Recall that a subtraction such as $\vec{d} = \vec{a} - \vec{b}$ can be rewritten as an addition $\vec{d} = \vec{a} + (-\vec{b})$. To subtract, we add $\vec{a}$ and $-\vec{b}$ by components, to get

$$d_x = a_x - b_x, \quad d_y = a_y - b_y, \quad \text{and} \quad d_z = a_z - b_z,$$

where

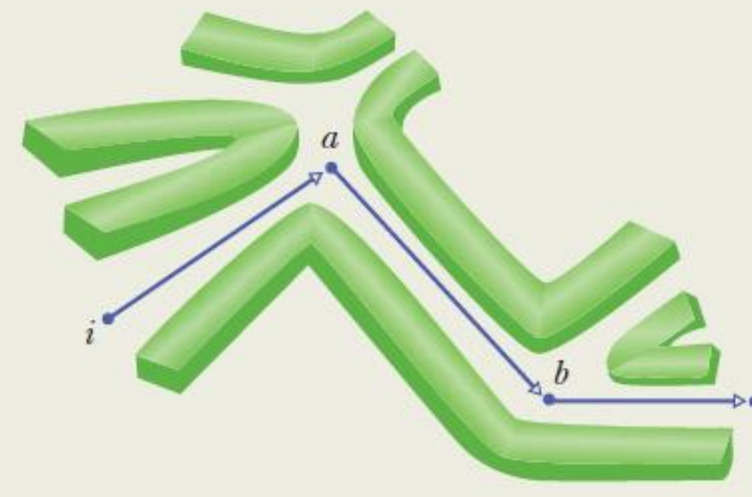
$$\vec{d} = d_x\hat{\mathrm{i}} + d_y\hat{\mathrm{j}} + d_z\hat{\mathrm{k}}. \tag{3-13}$$

Sample Problem

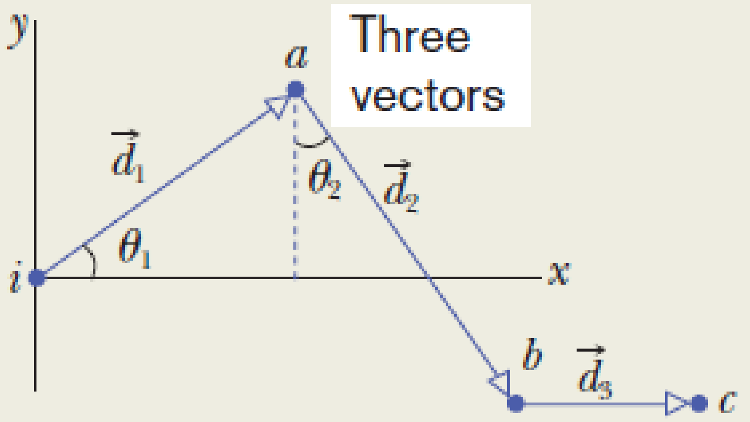
A hedge maze is a maze formed by tall rows of hedge. After entering, you search for the center point and then for the exit. Figure 3-16*a* shows the entrance to such a maze and the first two choices we make at the junctions we encounter in moving from point *i* to point *c*. We undergo three displacements as indicated in the overhead view of Fig. 3-16*b*:

$$d_1 = 6.00 \text{ m} \qquad \theta_1 = 40°$$
$$d_2 = 8.00 \text{ m} \qquad \theta_2 = -60°$$
$$d_3 = 5.00 \text{ m} \qquad \theta_3 = 0°,$$

where the last segment is parallel to the superimposed *x* axis. When we reach point *c*, what are the magnitude and angle of our net displacement $\vec{d}_{\text{net}}$ from point *i*?



(*a*)



(*b*)

Solution

(1) To find the net displacement $\vec{d}_{net}$, we need to sum the three individual displacement vectors:

$$\vec{d}_{net} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3.$$

(2) To do this, we first evaluate this sum for the x components alone,

$$d_{net,x} = d_{1x} + d_{2x} + d_{3x}, \qquad (3\text{-}16)$$

and then the y components alone,

$$d_{net,y} = d_{1y} + d_{2y} + d_{3y}. \qquad (3\text{-}17)$$

(3) Finally, we construct $\vec{d}_{net}$ from its x and y components.

$$d_{1x} = (6.00\text{ m})\cos 40° = 4.60\text{ m}$$
$$d_{2x} = (8.00\text{ m})\cos(-60°) = 4.00\text{ m}$$
$$d_{3x} = (5.00\text{ m})\cos 0° = 5.00\text{ m}.$$

Equation 3-16 then gives us

$$d_{net,x} = +4.60\text{ m} + 4.00\text{ m} + 5.00\text{ m}$$
$$= 13.60\text{ m}.$$

Similarly, to evaluate Eq. 3-17, we apply the y part of Eq. 3-5 to each displacement:

$$d_{1y} = (6.00\text{ m})\sin 40° = 3.86\text{ m}$$
$$d_{2y} = (8.00\text{ m})\sin(-60°) = -6.93\text{ m}$$
$$d_{3y} = (5.00\text{ m})\sin 0° = 0\text{ m}.$$

Equation 3-17 then gives us

$$d_{net,y} = +3.86\text{ m} - 6.93\text{ m} + 0\text{ m}$$
$$= -3.07\text{ m}.$$

$$d_{net} = \sqrt{d_{net,x}^2 + d_{net,y}^2} \qquad (3\text{-}18)$$
$$= \sqrt{(13.60\text{ m})^2 + (-3.07\text{ m})^2} = 13.9\text{ m}. \qquad \text{(Answer)}$$

$$\theta = \tan^{-1}\left(\frac{d_{net,y}}{d_{net,x}}\right) \qquad (3\text{-}19)$$
$$= \tan^{-1}\left(\frac{-3.07\text{ m}}{13.60\text{ m}}\right) = -12.7°. \qquad \text{(Answer)}$$

Practice Problems

•8 A person walks in the following pattern: 3.1 km north, then 2.4 km west, and finally 5.2 km south. (a) Sketch the vector diagram that represents this motion.

•9 Two vectors are given by

$$\vec{a} = (4.0\ \text{m})\hat{\text{i}} - (3.0\ \text{m})\hat{\text{j}} + (1.0\ \text{m})\hat{\text{k}}$$

and

$$\vec{b} = (-1.0\ \text{m})\hat{\text{i}} + (1.0\ \text{m})\hat{\text{j}} + (4.0\ \text{m})\hat{\text{k}}.$$

In unit-vector notation, find (a) $\vec{a} + \vec{b}$, (b) $\vec{a} - \vec{b}$, and (c) a third vector $\vec{c}$ such that $\vec{a} - \vec{b} + \vec{c} = 0$.

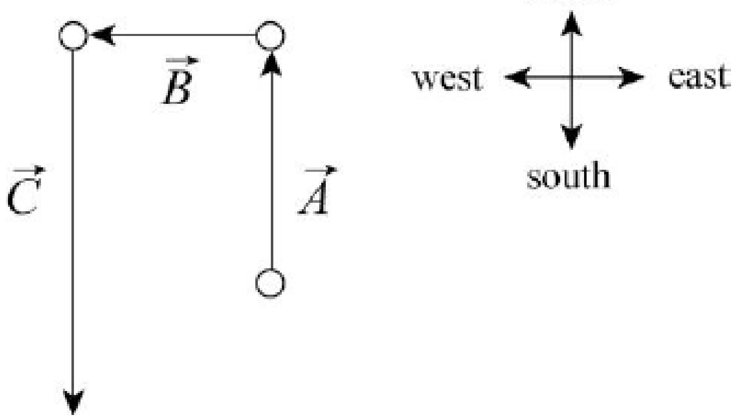
•10 Find the (a) x, (b) y, and (c) z components of the sum $\vec{r}$ of the displacements $\vec{c}$ and $\vec{d}$ whose components in meters are $c_x = 7.4, c_y = -3.8, c_z = -6.1; d_x = 4.4, d_y = -2.0, d_z = 3.3$.

•11 SSM (a) In unit-vector notation, what is the sum $\vec{a} + \vec{b}$ if $\vec{a} = (4.0\ \text{m})\hat{\text{i}} + (3.0\ \text{m})\hat{\text{j}}$ and $\vec{b} = (-13.0\ \text{m})\hat{\text{i}} + (7.0\ \text{m})\hat{\text{j}}$? What are the (b) magnitude and (c) direction of $\vec{a} + \vec{b}$?

Solutions:

8.

The vector diagram representing the motion is shown next:



10. The x, y, and z components of $\vec{r} = \vec{c} + \vec{d}$ are, respectively,

(a) $r_x = c_x + d_x = 7.4 \text{ m} + 4.4 \text{ m} = 12 \text{ m}$,

(b) $r_y = c_y + d_y = -3.8 \text{ m} - 2.0 \text{ m} = -5.8 \text{ m}$, and

(c) $r_z = c_z + d_z = -6.1 \text{ m} + 3.3 \text{ m} = -2.8 \text{ m}$.

9. All distances in this solution are understood to be in meters.

(a) $\vec{a} + \vec{b} = [4.0 + (-1.0)]\,\hat{\text{i}} + [(-3.0) + 1.0]\,\hat{\text{j}} + (1.0 + 4.0)\hat{\text{k}} = (3.0\hat{\text{i}} - 2.0\hat{\text{j}} + 5.0\,\hat{\text{k}})$ m.

(b) $\vec{a} - \vec{b} = [4.0 - (-1.0)]\hat{\text{i}} + [(-3.0) - 1.0]\hat{\text{j}} + (1.0 - 4.0)\hat{\text{k}} = (5.0\,\hat{\text{i}} - 4.0\hat{\text{j}} - 3.0\,\hat{\text{k}})$ m.

(c) The requirement $\vec{a} - \vec{b} + \vec{c} = 0$ leads to $\vec{c} = \vec{b} - \vec{a}$, which we note is the opposite of what we found in part (b). Thus, $\vec{c} = (-5.0\,\hat{\text{i}} + 4.0\,\hat{\text{j}} + 3.0\,\hat{\text{k}})$ m.

11.

$$r_x = a_x + b_x = (4.0\text{ m}) + (-13\text{ m}) = -9.0\text{ m}$$
$$r_y = a_y + b_y = (3.0\text{ m}) + (7.0\text{ m}) = 10.0\text{ m}.$$

Thus $\vec{r} = (-9.0\text{m})\hat{\text{i}} + (10\text{m})\hat{\text{j}}$.

(b) The magnitude of $\vec{r}$ is $r = |\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0\text{ m})^2 + (10\text{ m})^2} = 13\text{ m}$.

(c) The angle between the resultant and the $+x$ axis is given by

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{10.0\text{ m}}{-9.0\text{ m}}\right) = -48° \text{ or } 132°.$$

Since the x component of the resultant is negative and the y component is positive, characteristic of the second quadrant, we find the angle is 132° (measured counterclockwise from $+x$ axis).

Multiplying Vectors

There are three ways in which vectors can be multiplied, but none is exactly like the usual algebraic multiplication. As you read this material, keep in mind that a vector-capable calculator will help you multiply vectors only if you understand the basic rules of that multiplication.

Multiplying a Vector by a Scalar

If we multiply a vector $\vec{a}$ by a scalar s, we get a new vector. Its magnitude is the product of the magnitude of $\vec{a}$ and the absolute value of s. Its direction is the direction of $\vec{a}$ if s is positive but the opposite direction if s is negative. To divide $\vec{a}$ by s, we multiply $\vec{a}$ by $1/s$.

Multiplying a Vector by a Vector

There are two ways to multiply a vector by a vector: one way produces a scalar (called the *scalar product*), and the other produces a new vector (called the *vector product*).

The Scalar Product

The **scalar product** of the vectors $\vec{a}$ and $\vec{b}$ in Fig. 3-18*a* is written as $\vec{a} \cdot \vec{b}$ and defined to be

$$\vec{a} \cdot \vec{b} = ab \cos \phi, \tag{3-20}$$

The commutative law applies to a scalar product, so we can write

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}.$$

When two vectors are in unit-vector notation, we write their dot product as

$$\vec{a} \cdot \vec{b} = (a_x\hat{\text{i}} + a_y\hat{\text{j}} + a_z\hat{\text{k}}) \cdot (b_x\hat{\text{i}} + b_y\hat{\text{j}} + b_z\hat{\text{k}}), \tag{3-22}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z. \tag{3-23}$$

Dot product of unit vectors:

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{i}} = 0$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$$

$$\hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{j}} = 0$$

The Vector Product

The **vector product** of $\vec{a}$ and $\vec{b}$, written $\vec{a} \times \vec{b}$, produces a third vector $\vec{c}$ whose magnitude is

$$c = ab \sin \phi, \tag{3-24}$$

The direction of $\vec{c}$ is perpendicular to the plane that contains $\vec{a}$ and $\vec{b}$. Figure 3-19*a* shows how to determine the direction of $\vec{c} = \vec{a} \times \vec{b}$ with what is known as a **right-hand rule.** Place the vectors $\vec{a}$ and $\vec{b}$ tail to tail without altering their orientations, and imagine a line that is perpendicular to their plane where they meet. Pretend to place your *right* hand around that line in such a way that your fingers would sweep $\vec{a}$ into $\vec{b}$ through the smaller angle between them. Your outstretched thumb points in the direction of $\vec{c}$.

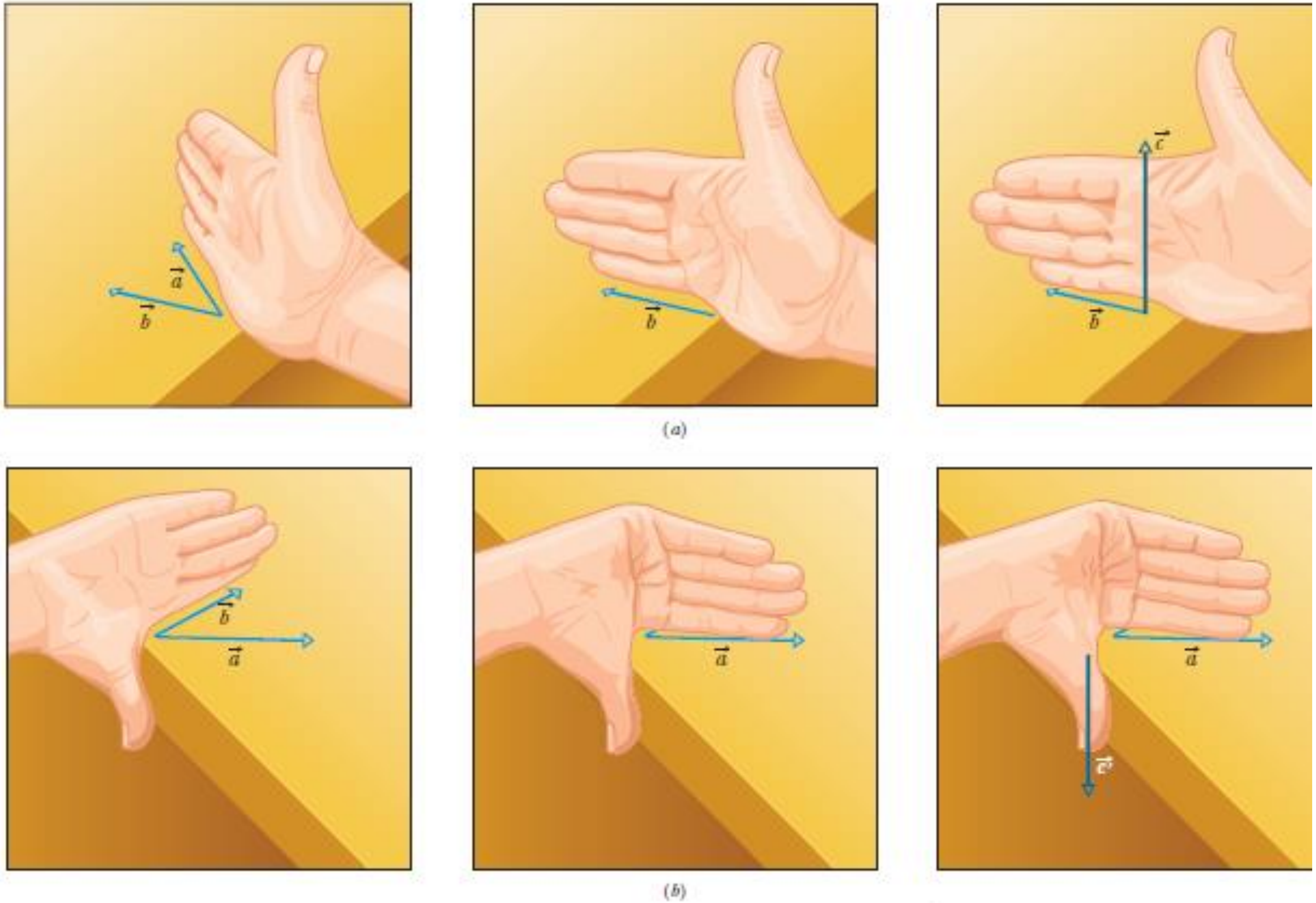


Figure 3-19 Illustration of the right-hand rule for vector products. (*a*) Sweep vector $\vec{a}$ into vector $\vec{b}$ with the fingers of your right hand. Your outstretched thumb shows the direction of vector $\vec{c} = \vec{a} \times \vec{b}$. (*b*) Showing that $\vec{b} \times \vec{a}$ is the reverse of $\vec{a} \times \vec{b}$.

$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b}). \qquad (3\text{-}25)$$

In other words, the commutative law does not apply to a vector product.

In unit-vector notation, we write

$$\vec{a} \times \vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}) \times (b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}}), \qquad (3\text{-}26)$$

$$a_x\hat{\mathrm{i}} \times b_x\hat{\mathrm{i}} = a_x b_x(\hat{\mathrm{i}} \times \hat{\mathrm{i}}) = 0,$$

because the two unit vectors $\hat{\mathrm{i}}$ and $\hat{\mathrm{i}}$ are parallel and thus have a zero cross product. Similarly, we have

$$a_x\hat{\mathrm{i}} \times b_y\hat{\mathrm{j}} = a_x b_y(\hat{\mathrm{i}} \times \hat{\mathrm{j}}) = a_x b_y\hat{\mathrm{k}}.$$

Cross product of unit vector

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$

$$\vec{i} \times \vec{j} = \vec{k} \qquad \vec{j} \times \vec{k} = \vec{i} \qquad \vec{k} \times \vec{i} = \vec{j}$$

$$\vec{j} \times \vec{i} = -\vec{k} \qquad \vec{k} \times \vec{j} = -\vec{i} \qquad \vec{i} \times \vec{k} = -\vec{j}$$

Sample Problem

What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$? (*Caution:* Although many of the following steps can be bypassed with a vector-capable calculator, you will learn more about scalar products if, at least here, you use these steps.)

The angle between the directions of two vectors is included in the definition of their scalar product (Eq. 3-20):

$$\vec{a} \cdot \vec{b} = ab \cos \phi. \qquad (3\text{-}28)$$

Calculations: In Eq. 3-28, a is the magnitude of $\vec{a}$, or

$$a = \sqrt{3.0^2 + (-4.0)^2} = 5.00, \qquad (3\text{-}29)$$

and b is the magnitude of $\vec{b}$, or

$$b = \sqrt{(-2.0)^2 + 3.0^2} = 3.61. \qquad (3\text{-}30)$$

$$\vec{a} \cdot \vec{b} = (3.0\hat{i} - 4.0\hat{j}) \cdot (-2.0\hat{i} + 3.0\hat{k})$$

$$= -6.0.$$

Substituting this result and the results of Eqs. 3-29 and 3-30 into Eq. 3-28 yields

$$-6.0 = (5.00)(3.61) \cos \phi,$$

so $$\phi = \cos^{-1} \frac{-6.0}{(5.00)(3.61)} = 109° \approx 110°. \qquad \text{(Answer)}$$

In Fig. 3-20, vector $\vec{a}$ lies in the xy plane, has a magnitude of 18 units, and points in a direction 250° from the positive direction of the x axis. Also, vector $\vec{b}$ has a magnitude of 12 units and points in the positive direction of the z axis. What is the vector product $\vec{c} = \vec{a} \times \vec{b}$?

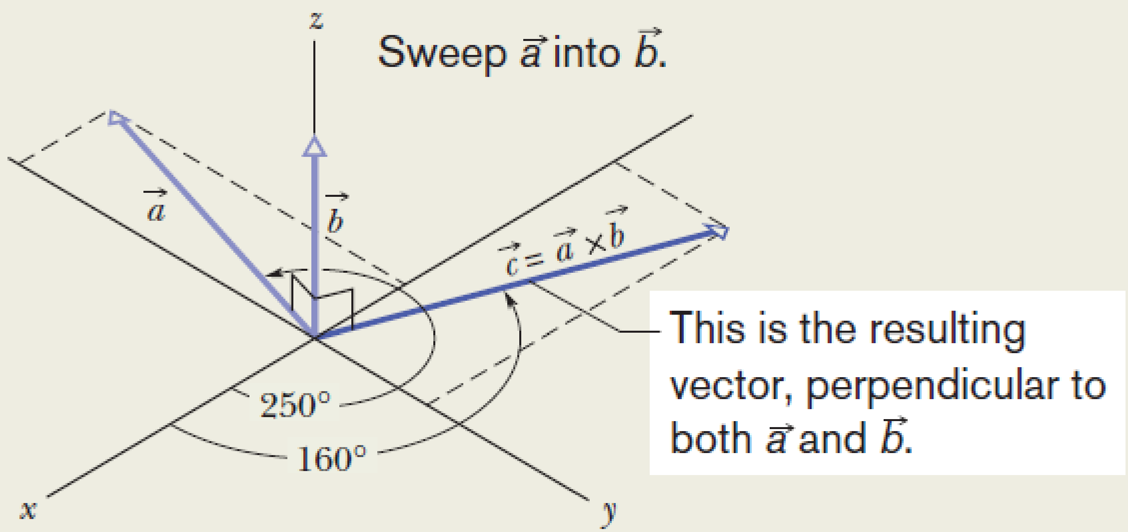


Figure 3-20 Vector $\vec{c}$ (in the xy plane) is the vector (or cross) product of vectors $\vec{a}$ and $\vec{b}$.

Calculations: For the magnitude we write

$$c = ab \sin \phi = (18)(12)(\sin 90°) = 216. \qquad \text{(Answer)}$$

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

$$= -12\hat{i} - 9\hat{j} - 8\hat{k}. \qquad \text{(Answer)}$$

Practice Problems

•34 Two vectors are presented as $\vec{a} = 3.0\hat{i} + 5.0\hat{j}$ and $\vec{b} = 2.0\hat{i} + 4.0\hat{j}$. Find (a) $\vec{a} \times \vec{b}$, (b) $\vec{a} \cdot \vec{b}$, (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$

•36 If $\vec{d}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{d}_2 = -5\hat{i} + 2\hat{j} - \hat{k}$, then what is $(\vec{d}_1 + \vec{d}_2) \cdot (\vec{d}_1 \times 4\vec{d}_2)$?

•37 Three vectors are given by $\vec{a} = 3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k}$, $\vec{b} = -1.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$, and $\vec{c} = 2.0\hat{i} + 2.0\hat{j} + 1.0\hat{k}$. Find (a) $\vec{a} \cdot (\vec{b} \times \vec{c})$, (b) $\vec{a} \cdot (\vec{b} + \vec{c})$, and (c) $\vec{a} \times (\vec{b} + \vec{c})$.

••38 GO For the following three vectors, what is $3\vec{C} \cdot (2\vec{A} \times \vec{B})$?

$$\vec{A} = 2.00\hat{i} + 3.00\hat{j} - 4.00\hat{k}$$

$$\vec{B} = -3.00\hat{i} + 4.00\hat{j} + 2.00\hat{k} \qquad \vec{C} = 7.00\hat{i} - 8.00\hat{j}$$

••39 Vector $\vec{A}$ has a magnitude of 6.00 units, vector $\vec{B}$ has a magnitude of 7.00 units, and $\vec{A} \cdot \vec{B}$ has a value of 14.0. What is the angle between the directions of $\vec{A}$ and $\vec{B}$?

Solutions:

34.

(a) $\vec{a}\times\vec{b} = 2.0\hat{k}$.

(b) $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y$ yields $(3.0)(2.0) + (5.0)(4.0) = 26.$

(c) $\vec{a}+\vec{b} = (3.0 + 2.0)\,\hat{i} + (5.0 + 4.0)\,\hat{j} \Rightarrow (\vec{a}+\vec{b})\cdot\vec{b} = (5.0)\,(2.0) + (9.0)\,(4.0) = 46$.

37.

(a) We note that $\vec{b}\times\vec{c} = -8.0\hat{i} + 5.0\hat{j} + 6.0\hat{k}$. Thus,

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (3.0)\,(-8.0) + (3.0)(5.0) + (-2.0)\,(6.0) = -21.$$

(b) We note that $\vec{b}+\vec{c} = 1.0\hat{i} - 2.0\hat{j} + 3.0\hat{k}$. Thus,

$$\vec{a}\cdot(\vec{b}+\vec{c}) = (3.0)\,(1.0) + (3.0)\,(-2.0) + (-2.0)\,(3.0) = -9.0.$$

(c) Finally,

$$\begin{aligned}\vec{a}\times(\vec{b}+\vec{c}) &= [(3.0)(3.0) - (-2.0)(-2.0)]\,\hat{i} + [(-2.0)(1.0) - (3.0)(3.0)]\,\hat{j} \\ &\quad + [(3.0)(-2.0) - (3.0)(1.0)]\,\hat{k} \\ &= 5\hat{i} - 11\hat{j} - 9\hat{k}\end{aligned}$$

38.

$$2\vec{A} \times \vec{B} = 2\left(2.00\hat{\mathrm{i}} + 3.00\hat{\mathrm{j}} - 4.00\hat{\mathrm{k}}\right) \times \left(-3.00\hat{\mathrm{i}} + 4.00\hat{\mathrm{j}} + 2.00\hat{\mathrm{k}}\right)$$
$$= 44.0\hat{\mathrm{i}} + 16.0\hat{\mathrm{j}} + 34.0\hat{\mathrm{k}}.$$

Next, making use of

$$\hat{\mathrm{i}} \cdot \hat{\mathrm{i}} = \hat{\mathrm{j}} \cdot \hat{\mathrm{j}} = \hat{\mathrm{k}} \cdot \hat{\mathrm{k}} = 1$$
$$\hat{\mathrm{i}} \cdot \hat{\mathrm{j}} = \hat{\mathrm{j}} \cdot \hat{\mathrm{k}} = \hat{\mathrm{k}} \cdot \hat{\mathrm{i}} = 0$$

we have

$$3\vec{C} \cdot \left(2\vec{A} \times \vec{B}\right) = 3\left(7.00\hat{\mathrm{i}} - 8.00\hat{\mathrm{j}}\right) \cdot \left(44.0\hat{\mathrm{i}} + 16.0\hat{\mathrm{j}} + 34.0\hat{\mathrm{k}}\right)$$
$$= 3[(7.00)(44.0) + (-8.00)(16.0) + (0)(34.0)] = 540.$$

39. From the definition of the dot product between $\vec{A}$ and $\vec{B}$, $\vec{A} \cdot \vec{B} = AB\cos\theta$, we have

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

With $A = 6.00$, $B = 7.00$ and $\vec{A} \cdot \vec{B} = 14.0$, $\cos\theta = 0.333$, or $\theta = 70.5°$.