Date

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COURSE : CALCULUS AND ANALYTICAL GEOMETRY (MTH - 103)

PICTURE # 1

1. IDENTIFY THE DOMAIN AND RANGE OF FUNCTION THROUGH FOLLOWING GRAPHS

Sol

i) Domain = Real numbers, $x \in (-\infty, \infty)$
Range = $y \in (0, \infty)$

ii) Domain = $x \in (-\infty, \infty)$
Range = $y \in (-\infty, \infty)$

iii) Domain = $x \in (0, \infty)$
Range = $y \in (0, \infty)$

iv) Domain = $x \in (-\infty, \infty)$
Range = $y \in (0, \infty)$

v) Domain = $x \in (-\infty, \infty)$
Range = $y \in (-\infty, \infty)$

Q If $f(x) = 5x+2$ and $g(x) = 2x^2-3$, find (i) fog, (ii) gof, (iii) fof, (iv) gog

(i) fog :

$f(2x^2-3) = 5(2x^2-3)+2$

$fog = 10x^2-15+2$

$\boxed{fog = 10x^2-13}$

(ii) gof :

$g(5x+2) = 2(5x+2)^2-3$

$gof = 2[(5x)^2+2(5x)(2)+(2)^2]-3$

$gof = 2[25x^2+20x+4]-3$

$gof = 50x^2+40x+8-3$

$gof = 50x^2+40x+5$

(iii) fof :

$f(5x+2) = 5(5x+2)+2$

$\boxed{fof = 25x+12}$

(iv) gog:

$g(2x^2-3) = 2(2x^2-3)^2-3$

$gog = 2[(2x^2)^2-2(2x^2)(3)+(3)^2]-3$

$gog = 2[4x^4-12x^2+9]-3$

$gog = 8x^4-24x^2+18-3$

$\boxed{gog = 8x^4-24x^2+15}$

Q If $f(x) = 2x$ and $g(x) = x+1$, find $fog(x)$ for $x=-5$

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Sol.

$$fog = 2(x+1)$$
$$fog = 2x+2$$

for $x = -5$:

$$fog(-5) = 2(-5)+2$$
$$fog(-5) = -10+2$$
$$\boxed{fog(-5) = -8}$$

Q If $f(x) = x+3$ and $g(x) = x^2$, find $gof(x$ for $x = 1$.

$$gof(x) = (x+3)^2$$
$$gof(x) = x^2 + 2(x)(3) + 3^2$$
$$gof(x) = x^2 + 6x + 9$$

for $x = 1$:

$$gof(1) = 1^2 + 6(1) + 9$$
$$gof(1) = 1 + 6 + 9$$
$$\boxed{gof(1) = 16}$$

Q If $c(x) = \cos x$ and $p(x) = x^2+1$, find $poc(x)$.

$$Poc(x) = (\cos x)^3 + 1$$
$$\boxed{Poc(x) = \cos^3 x + 1}$$

PICTURE # 2

Q EVALUATE THE FOLLOWNG:

1) $$\lim_{x \to 0} \frac{3x^3 - 2x^2 + x}{4x^2 + 2x}$$

$$\rightarrow \lim_{x \to 0} \frac{\cancel{x}(3x^2 - 2x + 1)}{\cancel{x}(4x + 2)}$$

$$\rightarrow \lim_{x \to 0} \frac{3x^2 - 2x + 1}{4x + 2}$$

→ Applying limit:

$$\rightarrow \frac{3(0)^2 + 2(0) + 1}{4(0) + 2}$$

$$\rightarrow \boxed{1/2}$$

2) $$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

$$\rightarrow \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$$

→ Apply limit : $3 + 3 = \boxed{6}$

3) $$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 7x + 10}$$

$$\rightarrow \lim_{x \to 2} \frac{x^2 - 2x - 3x + 6}{x^2 - 2x - 7x + 0}$$

$$\rightarrow \lim_{x \to 2} \frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}(x-5)}$$

→ Apply limit : $\frac{2-3}{2-5}$

$$\rightarrow \boxed{1/3}$$

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4) $\lim_{t \to 5} \dfrac{t^2 + 3t - 10}{2t^2 + 13t - 10}$

Apply limit :

$\rightarrow \dfrac{5^2 + 3(5) - 10}{2(5)^2 + 13(5) - 10}$

$\rightarrow \boxed{2/7}$

5) $\lim_{x \to 1} \dfrac{x^n - 1}{x - 1}$

Sol

$= n \cdot 1^{n-1}$

$= \boxed{n}$

6. $\lim_{x \to 0} \dfrac{\sin 2x}{\sin 3x}$

$\because$ $\sin(x) = x$ (for small values of x)

So,

$\rightarrow \lim_{x \to 0} \dfrac{2\cancel{x}}{3\cancel{x}}$

$\rightarrow \boxed{2/3}$

7. $\lim_{\theta \to 0} \frac{\sin\theta}{\tan\theta}$

$\rightarrow \lim_{\theta \to 0} \frac{\cancel{\sin\theta}}{\frac{\cancel{\sin\theta}}{\cos\theta}}$

$\rightarrow \lim_{\theta \to 0} \cos\theta$

Apply limit:

$\rightarrow \boxed{\cos 0 = 1}$

8. $\lim_{x \to 0} \frac{\sin 4x}{x}$

$\rightarrow \lim_{x \to 0} \frac{\sin 4x \times 4}{4x}$

Apply limit:

$\rightarrow \boxed{4}$

9. $\lim_{x \to 0} \frac{\sin^2 (x/3)}{x^2}$

Using small angle approximation

$\sin x \sim x$

$\sin\theta = \frac{Perp}{hyp} = \text{angle } \theta$

$\sin^2\left(\frac{x}{3}\right) \sim \left(\frac{x}{3}\right)^2$

$= \frac{\left(\frac{x}{3}\right)^2}{x^2} = \frac{\frac{x^2}{9}}{x^2} = \frac{\cancel{x^2}}{9} \div \cancel{x^2}$

$= \boxed{\frac{1}{9}}$

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10. $\lim_{\phi \to 0} \phi \cot \phi$

$\rightarrow \lim_{\phi \to 0} \frac{\cos \phi}{\sin \phi} \cdot \phi$

$\because \phi / \sin \phi = 1$ and $\because \cos \phi = 1$

$\rightarrow \boxed{1 \times 1 = 1}$

11. $\lim_{x \to \pi/2} (2 \sin x - \cos x + \cot x)$

$\rightarrow$ Apply limit :

$\rightarrow 2 \sin(\pi/2) - \cos(\pi/2) + \cot(\pi/2)$

$\rightarrow 2 - 0 + 0$

$\rightarrow \boxed{2}$

12. $\lim_{x \to 0} \operatorname{cosec} x - \cot x$

$\because \operatorname{cosec} x - \cot x = \frac{1}{\sin x} - \frac{\cos x}{\sin x}$

$\rightarrow \lim_{x \to 0} \frac{1 - \cos x}{\sin x}$

$\rightarrow \lim_{x \to 0} \frac{(1 - \cos x)(1 + \cos x)}{\sin x (1 + \cos x)}$

$\rightarrow \lim_{x \to 0} \frac{\sin^2 x}{\sin (1 + \cos x)} = \frac{\sin x}{1 + \cos x}$

Apply limit :

$\rightarrow \frac{0}{1+1} = \boxed{0}$

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13. $\lim_{\theta \to 0} \dfrac{1 - \cos 5\theta}{1 - \cos 7\theta}$

using approximation:

$\rightarrow$ $1 - \cos x = \dfrac{x^2}{2}$

$\rightarrow$ $\dfrac{(5\theta)^2}{\cancel{2}} \div \dfrac{(7\theta)^2}{\cancel{2}}$

$\rightarrow$ $\boxed{\dfrac{25}{49}}$

14. $\cancel{\lim_{\theta \to 0}}$ $\lim_{x \to a} \dfrac{x^m - a^m}{x^n - a^n}$

$\rightarrow$ $\dfrac{ma^{m-1}}{na^{n-1}}$

$\rightarrow$ $\boxed{\dfrac{m}{n}}$

15. $\lim_{\theta \to 0} \dfrac{1 - \cos \theta}{\sin \theta}$

$\because$ $1 - \cos \theta = \dfrac{\theta^2}{2}$

$\rightarrow$ $\lim_{\theta \to 0} \dfrac{\frac{\theta^2}{2}}{\theta}$

$\rightarrow$ $\boxed{0}$

PICTURE # 3

Q1. Find the derivative of f at any point x in approximate domain, where $f(x)$ is given by :

i) $\sqrt{{}^{1+x}/_{1-x}}$

Let $u = {}^{1+x}/_{1-x}$

$$\frac{du}{dx} = \frac{(1-x).(1) - (1+x).(-1)}{(1-x)^2}$$

$$\frac{du}{dx} = \frac{1-\cancel{x} + 1+\cancel{x}}{(1-x)^2}$$

$$\frac{du}{dx} = \frac{2}{(1-x)^2}$$

$$y = u^{1/2}$$

$$\frac{dy}{du} = \frac{1}{2} U^{1/2 - 1} = \frac{1}{2U^{1/2}}$$

$$\because \frac{dy}{dx} = \frac{du}{dx} \cdot \frac{dy}{du}$$

$$= \frac{2}{(1-x)^2} \cdot \frac{1}{2u^{1/2}}$$

$$= \frac{\cancel{2}}{(1-x)^2 . \cancel{2} \left(\frac{1+x}{1-x}\right)^{1/2}}$$

$$= \frac{1}{(1-x)^{2-\frac{1}{2}} (1+x)^{1/2}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{(1-x)^{3/2} (1+x)^{1/2}}}$$

(ii) $\dfrac{3x^2 - x}{x\sqrt{1+x^2}}$

let $u = 3x^2 - x$ and $v = x\sqrt{1+x^2}$

$u' = 6x - 1$

$v' = x.(1+x^2)^{1/2}$

$v' = \sqrt{1+x^2} + x.\frac{1}{2}(1+x^2)^{-1/2}.2x$

$v' = \sqrt{1+x^2} + \dfrac{x^2}{\sqrt{1+x^2}}$

$$\frac{dy}{dx} = \frac{(6x-1)(x\sqrt{1+x^2}) - (3x^2-x)\left(\sqrt{1+x^2} + \frac{x^2}{\sqrt{1+x^2}}\right)}{(x\sqrt{1+x^2})^2}$$

$$\boxed{\frac{dy}{dx} = \frac{3+x}{(1+x^2)(\sqrt{1+x^2})}}$$

iii) $\sqrt[5]{x^2 + 2x + 3}$

let $u = x^2 + 2x + 3$

$= u^{1/5}$

$\cancel{\frac{du}{dx}} = \frac{d}{du}(u^{1/5}) = \frac{1}{5}u^{-4/5}.\frac{du}{dx}$

$\frac{du}{dx} = 2x + 2$

$\frac{dy}{dx} = \frac{1}{5}(x^2+2x+3)^{-4/5}.(2x+2)$

$$\boxed{\frac{dy}{dx} = \frac{2(x+1)}{5(x^2+2x+3)^{4/5}}}$$

iv) $\sqrt{x^2 + a^2}$ $(a \in R)$

$(x^2+a^2)^{1/2}$

let $u = x^2 + a^2$

$\frac{du}{dx} = 2x$

$y = u^{1/2}$

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$$\frac{dy}{du} = \cancel{2x} \quad \frac{1}{2u^{1/2}}$$

$$\frac{dy}{dx} = \cancel{2}x \cdot \frac{1}{\cancel{2}(x^2+a^2)^{1/2}}$$

$$\boxed{\frac{dy}{dx} = \frac{x}{\sqrt{x^2+a^2}}}$$

Q2. Defining a function f by $y = f(x)$, determine dy/dx in the following cases:

i) $y = \sin^3 x$

$$\frac{d}{dx} y = \frac{d}{dx}(\sin^3 x) \cdot \frac{d}{dx}(\sin x)$$

$$\boxed{dy/dx = 3\sin^2 x \cdot \cos x}$$

ii) $y = 2\cos 3x + 3\sin 2x$

$$\frac{dy}{dx} = 2\frac{d}{dx}\cos 3x + 3\frac{d}{dx}\sin 2x$$

$$\frac{dy}{dx} = 2(-\sin 3x).3 + 3(\cos 2x).2$$

$$\boxed{\frac{dy}{dx} = -6\sin 3x + 6\cos 2x}$$

iii) $y = \tan(2x-3)$

$$\frac{dy}{dx} = \frac{d}{dx}\tan(2x-3)$$

$$= \sec^2(2x-3).\ 2$$

$$\boxed{\frac{dy}{dx} = 2\sec^2(2x-3)}$$

iv) $y = \sin 4x \cdot \cos 5x$

$$\frac{dy}{dx} = \cos 5x (\cos 4x).4 + (\sin 4x).(-\sin 5x).5$$

$$\boxed{\frac{dy}{dx} = 4\cos 4x \cdot \cos 5x - 5\sin 5x \cdot \sin 4x}$$

v) $y = \sin^2 x \cdot \cos^3 x$

$$\frac{dy}{dx} = (\cos^3 x).(2\sin x)(\cos x) + \sin^2 x(3\cos x - \sin x)$$

$$\frac{dy}{dx} = (\cos^3 x)(2\sin x \cos x) + \sin^2 x.(-3\cos x \sin x)$$

$$\boxed{\frac{dy}{dx} = 2\sin x \cos^4 x - 3\sin^3 x \cos^2 x}$$

vi) $y = x \cos x$

$$dy/dx = \cos x (1) + (x)(-\sin x)$$

$$\boxed{\frac{dy}{dx} = \cos x - x\sin x}$$

vii) $y = \sqrt{\sin 3x}$

$$y = (\sin 3x)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(\sin 3x)^{\frac{1}{2}-1}.(\cos 3x).3$$

$$= \frac{1}{2}(\sin 3x)^{-1/2}.(3\cos 3x)$$

$$\boxed{\frac{dy}{dx} = \frac{3\cos 3x}{2(\sin 3x)^{1/2}}}$$

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viii) $y = \dfrac{\sin 2x}{\cos^2 3x}$

$$\frac{dy}{dx} = \frac{\cos^2 3x\,(\cos 2x).2 - \sin 2x\,(2\cos 3x)(-\sin 3x).3}{(\cos^2 3x)^2}$$

$$\frac{dy}{dx} = \frac{(\cos^2 3x)(2\cos 2x) + \sin 2x\,(2\cos 3x)(\sin 3x).3}{(\cos^2 3x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{(\cos^2 3x)(2\cos 2x) + 6\sin 2x\,(2\cos 3x)(\sin 3x)}{(\cos^2 3x)^2}}$$