2.3.4

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<u>Lemma</u> (Flexibility) Let $(\Sigma \times \mathbb{R}_t, \beta_i + f dt)$ ctct mfds with $\beta_i \in \Omega^1 \Sigma$, $f \in C^\infty \Sigma$ s.t. $\beta_0 \equiv \beta_1$ near Γ and β_1, β_0 are exact deformation equivalent ($d\beta_t$ $t \in [0,1]$ symplectic).

Then there is 1-parameter family of diffeos $\phi_s : \Sigma \times \mathbb{R} \to \Sigma \times \mathbb{R}$ s.t.

(1) $\phi_0 = id$

(2) $T\phi_1 (\ker(f dt + \beta_0)) = \ker(f dt + \beta_1)$

or equivalently $\phi_1^*(f dt + \beta_1) = (f dt + \beta_0) g$

for nonzero $g \in C^\infty \Sigma \times \mathbb{R}$.

(3) $\phi_s(\Sigma \times \{0\}) \pitchfork$ to ∂_t

(4) $\phi_s \equiv id$ near $\Gamma \times \mathbb{R}$.

<u>Proof</u> Let X_s be time-dependent VF with flow ϕ_s. Then we want

$\phi_s^*(\alpha_s) = g_s \alpha_0$ $\forall s$ where $\alpha_s = f dt + \beta_s$ are contact forms

$$\phi_s^*(\mathcal{L}_{X_s}(\alpha_s) + \dot{\alpha}_s) = \dot{g}_s \alpha_0 = \phi_s^*(\mu_s \alpha_s)$$

$$d i_{X_s} \alpha_s + i_{X_s} d\alpha_s + \dot{\alpha}_s = \mu_s \alpha_s .$$

Let $X_s \in \ker \alpha_s$ $\forall s$ and $\mu_s = \dot{\alpha}_s(R_{\alpha_s})$

Then solve for $i_{X_s} d\alpha_s = \mu_s \alpha_s - \dot{\alpha}_s$

(1), (2) are satisfied by construction.

Near Γ we have $\dot{\alpha}_s \equiv 0$, $\mu_s = 0 \Rightarrow X_s = 0$.

$\Rightarrow$ (4) as well.

The X_s is t-invariant since everything else is

Hence $\mathcal{L}_{\partial_t} X_s = 0$. It remains to show $\phi_s \Sigma \pitchfork \partial_t$.

If $\partial_t \in T\phi_s \Sigma$ then $\phi_s^* \partial_t \in T\Sigma$.

But $\phi_0^* \partial_t = \partial_t \notin T\Sigma$. So $\phi_s^* \partial_t$ not constant,

$\Rightarrow \mathcal{L}_{X_s} \partial_t \neq 0 \Rightarrow \mathcal{L}_{\partial_t} X_s \neq 0$ ↯ ∎

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$\Rightarrow \mathcal{L}_{X_s} \partial_t \neq 0 \Rightarrow \mathcal{L}_{\partial_t} X_s \neq 0$ ↯. $\square$