Exercise 2.1.12

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Exercise 2.1.12 Show that if β is any skew-symmetric bilinear form on the vector space W, there is a basis $u_1, \ldots, u_n, v_1, \ldots, v_n, w_1, \ldots, w_p$ of W such that $\beta(u_j, v_k) = \delta_{jk}$ and all other pairings $\beta(b_1, b_2)$ vanish. A basis with this property is called a **standard basis** for (W, β), and the integer $2n$ is the **rank** of β. $\square$

Let $S := \{w \in W \mid \beta(w, v) = 0 \ \forall v \in W\}$. S is a linear subspace. Let V be a complementary subspace. One then sees $(V, \beta_{|V})$ is a symplectic vectorspace. By Theorem 2.1.3 we get a basis u_i, v_i. Let w_i be arbitrary basis for S. This basis has the properties we wanted. $\square$