Exercise 2.1.10

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Exercise 2.1.10 Let (V, ω) be a symplectic vector space and $W \subset V$ be any subspace. Prove that the quotient $V' = W/W \cap W^\omega$ carries a natural symplectic structure. $\square$

$\omega|_{W \cap W^\omega} \equiv 0$ since if $v, w \in W \cap W^\omega$ then $\omega(v, w) = 0$ since $v \in W$, $w \in W^\omega$.
Hence we can define the skew-symmetric bilinear form $\tilde{\omega} : V' \times V' \to \mathbb{R}$ given by $\tilde{\omega}([v_1], [v_2]) := \omega(v_1, v_2)$.
We show $\tilde{\omega}$ is non degenerate.
Let $v \in W$ such that $\tilde{\omega}([v], [w]) = 0$ $\forall w \in W$. Then $\omega(v, w) = 0$ $\forall w \Rightarrow$ $v \in W \cap W^\omega \Rightarrow [v] = 0$. $\square$