2.3.8

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Corollary 2.3.8. *Let $\Sigma \subset (M,\xi)$ be a convex hypersurface with dividing set Γ. Fix a contact form β on Γ such that $\xi|_\Gamma = \ker\beta$ and consider $\Gamma \times \mathbb{R}^2_{x,y}$ equipped with the contact form $\alpha = \beta - y\,dx$. Let $D(R) \subset \mathbb{R}^2$ be the open disk of radius R. Then for any $R > 0$ there exists a contact embedding*

$$j : (\Gamma \times D(R), \ker\alpha) \to (M,\xi)$$

such that $j|_{\Gamma\times\{0\}} = \mathrm{id}_\Gamma$.

Let $\beta + f dt$ contact form for $(\Sigma \times \mathbb{R}, \xi)$. Then one can take $\beta_{|\Gamma}$ as contact form on Γ since $d(\beta + f dt)(\partial_t, Y) = \pm df(Y) \neq 0$ and $\langle \partial_t, Y_p \rangle \oplus T_p\Gamma = T_p M$.

We construct a nbhd differ.

$$\begin{aligned} \Gamma \times (-\varepsilon,\varepsilon)_u \times \mathbb{R}_s &\xrightarrow{\varphi} M \\ p, u, s &\longmapsto Fl_u^{-uY} Fl_s^{\partial_t} p \end{aligned}$$

(Y and ∂_t commute).

$T\varphi(\partial_u) = -uY$

$T\varphi(\partial_s) = \partial_t$

$(\beta + f dt)(-uY, \partial_t) = 1$

We also know $Y \in \ker d\beta, \ker\beta$

From this one can follow

$$\varphi^*(\beta + f dt) = \beta_{|\Gamma} + u\,ds$$

So for some $D(\varepsilon)$ true.

We can embed $\Gamma \times D(R) \to \Gamma \times (-\varepsilon,\varepsilon) \times \mathbb{R}_s$ for any R by $p, x, y \mapsto p, \frac{\varepsilon}{R}x, \frac{R}{\varepsilon}y$