2.3.6

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Lemma 2.3.6 (Contact parallel transport). *Let $\mathbb{R}^2 \times M^{2n-1}$ be a contact manifold with contact form $\lambda + \beta$, where M^{2n-1} is a closed contact manifold with contact form β and λ is a Liouville form on $\mathbb{R}^2$, and let ϕ_t, $t \in \mathbb{R}$, be the flow on M corresponding to the Reeb vector field for β. If $\Lambda \subset M$ is a Legendrian submanifold and $\gamma : [0,1] \to \mathbb{R}^2$ is a path, then there exists a Legendrian embedding*

$$\varphi : [0,1]_s \times \Lambda \to \mathbb{R}^2 \times M, \quad (s,x) \mapsto (\gamma(s), \phi_{-\int_0^s \gamma^*\lambda}(x)).$$

Λ Legendrian means $\beta|_\Lambda = 0$.

We check if ϕ is Legendrian.

$T\varphi(\partial_s) = (\gamma'(s), -\lambda(\gamma'(s))R) \in \ker(\beta+\lambda)$

$T\varphi(v) = (0, T\phi_*(v))$

<u>Claim</u> If $v \in \ker\beta$, then $T\phi_t(v) \in \ker\beta$.

Why? $\left.\frac{d}{dt}\right|_t \phi_t^*\beta = \phi_t^*(\mathcal{L}_R\beta) = 0$

$\phi_t^*\beta = \beta$

$\Rightarrow \beta(T\phi_t(v)) = \beta(v) = 0.$ $\square$