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Proposition Let (W, J, ϱ) be a Stein mfd. Then (W, ω) with $\omega = -d(d\varrho \circ J)$ is a Weinstein mfd and for every regular value c of ϱ the symplectic mfd $(\varrho^{-1}((-\infty, c]), \omega)$ is a Weinstein filling of it's boundary.

Proof ω non degenerate: $g(-,-) = \omega(-, J-)$ is Riemannian metric, so $\omega(v, Jv) > 0$ $\forall v$. ✓ Hence ω is a symplectic form.

ω has Liouville form $-d\varrho \circ J$, so there is a Liouville VF Y given by $i_Y\omega = -d\varrho \circ J$.

ϱ is an exhausting function. Is Y gradient like for ϱ? $\mathcal{L}_Y \varrho = d\varrho(Y) = i_Y\omega \circ J(Y) = \omega(Y, JY) > 0$. ✓

Is ϱ Morse? Writing things out in coordinates, one checks $\mathrm{Hess}\,\varrho$ invertable in critical points.

$Y \pitchfork \varrho^{-1}c$ since $d\varrho \neq 0$ on $\varrho^{-1}c$ and $Y(\varrho) > 0$. $\Rightarrow$ Y outward pointing on $\varrho^{-1}((-\infty, c])$. $\square$