Proposition Let $(M^{2n+1}, \xi = \mathrm{Ker}\,\alpha)$ be a contact mfd and Σ^{2n} a hypersurface, v a contact VF transverse to Σ. Let $\Sigma_\pm := \{x \in \Sigma : \pm\alpha(v_x) > 0\}$. Then $(\pm 1)^n \Sigma_\pm$ are ideal Liouville domain that fill in the contact structure Γ_Σ.

Proof Using the flow of v, we get that a nbhd of Σ is contactomorphic to an open nbhd W of $\Sigma \times \{0\} \subset (\Sigma \times \mathbb{R}, f dt + \beta)$ where $f \in C^\infty \Sigma$, $\beta \in \Omega^1 \Sigma$. The contact condition becomes

$$0 \neq \alpha \wedge (d\alpha)^n = (f dt + \beta) \wedge (df \wedge dt + d\beta)^n = f dt \wedge (d\beta)^n + n \beta \wedge df \wedge dt \wedge (d\beta)^{n-1}$$
$$= (f dt \wedge d\beta + n\beta \wedge df \wedge dt) \wedge (d\beta)^{n-1}$$

and hence $(n\beta \wedge df + f d\beta) \wedge (d\beta)^{n-1} \neq 0$ on Σ.

$\beta' := \frac{1}{f}\beta + dt$. $0 \neq \beta' \wedge (d\beta')^n = (\frac{1}{f}\beta + dt) \wedge (d\beta')^n = dt \wedge (d\beta')^n$.

Hence $d\beta'$ is a symplectic form on $\Sigma_\pm$. Moreover it is an ideal Liouville domain.