

2.1.14

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Exercise 2.1.14 Show that any hyperplane W in a $2n$ -dimensional symplectic vector space (V, ω) is coisotropic. Thus $W^\omega \subset W$ and $\omega|_W$ has rank $2(n-1)$. **Hint:** By Exercise 2.1.12 the 2-form $\omega|_W$ has even rank. Hence there is some nonzero vector $w \in W$ such that $\omega(w, x) = 0$ for all $x \in W$. Show that this vector w spans W^ω . \square

$\dim W = 2n - 1 \Rightarrow \dim W^\omega = 1$. Assume $W^\omega \neq W$. Then $W^\omega \oplus W = V$. But then for a nonzero $v \in W^\omega$ we have $\omega(v, -) \equiv 0$. ∇ .
A fast way to argue about the rank is using Exercise 2.1.13.