2.3.7

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Lemma 2.3.7. *Let $\Sigma = \Sigma \times \{0\}$ be a convex hypersurface in $\Sigma \times \mathbb{R}$ equipped with an $\mathbb{R}$-invariant contact structure. Then, for all $a > b > 0$, there exists a contact embedding $i : \Sigma \times (-a, a) \to \Sigma \times (-b, b)$ such that $i|_{\Sigma \times \{0\}} = \mathrm{id}_{\Sigma \times \{0\}}$. In other words, any convex hypersurface has an arbitrarily large invariant neighborhood.*

Let $\alpha = \beta + f\,dt$ with Reeb VF R.
There is correspondence

$$C^\infty M \longleftrightarrow \text{infinitesimal automorphisms of } \alpha$$
$$(X \in \mathfrak{X} M \text{ s.t. } \mathcal{L}_X \alpha = g\alpha)$$
$$\alpha(X) \longleftarrow\!\shortmid X$$
$$H \longmapsto X_H \text{ s.t. } \alpha(X_H) = H$$
$$\iota_{X_H} d\alpha = R(H)\alpha - dH$$

∂_t is an infinitesimal automorphism, let H be the contact Hamiltonian. One sees $f = H$.
H is independent of t. Let ρ be cut-off function
$\operatorname{supp} \rho \subseteq (-b, b)$, $\rho \equiv 1$ on $(-\frac{b}{2}, \frac{b}{2})$.
$H' := \rho H$ generates contact VF supported in $\Sigma \times (-b, b)$, agrees with ∂_t near $\Sigma \times \{0\}$.
The flow gives contract embedding for any a.