MAE 3260 Final Group Work:
Exploring a System of Interest

Title:
DUI - Driving Under Inputs

Topic of Interest:
Active Suspension

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Abstract:

We decided to focus on a feedback controller of an active suspension system (quarter-car model). The system includes two masses, one representing the car's chassis and the second representing the entire wheel assembly. The passive spring and shock absorber assembly can be characterized using a stiffness constant k_s and a damping constant b, and the tire's compressibility can be characterized using another spring constant k_t.

Our group decided on this project, as we feel it would be interesting to learn more about a system that we interact with daily (especially given Ithaca's bumpy roads) and can possibly apply in our future careers in engineering. There is abundant documentation on the matter, therefore we aim to prioritize deriving performance requirements and closed loop controllers to meet them. We're interested in seeing what feedback controllers are best for certain applications, such as off-terrain or urban environments.

1. System Description and Modeling Context

The quarter-car model represents one corner (hence "quarter") of a vehicle and consists of:

- Sprung mass
- Unsprung mass Suspension stiffness ksk_sks and damping csc_scs (or passive damper)
- Tire stiffness (modeled as a linear spring; tire damping is usually negligible)
- Actuator force placed in parallel with the suspension (active force input)

The road disturbance acts as a vertical velocity or displacement input zrz_rzr to the tire.

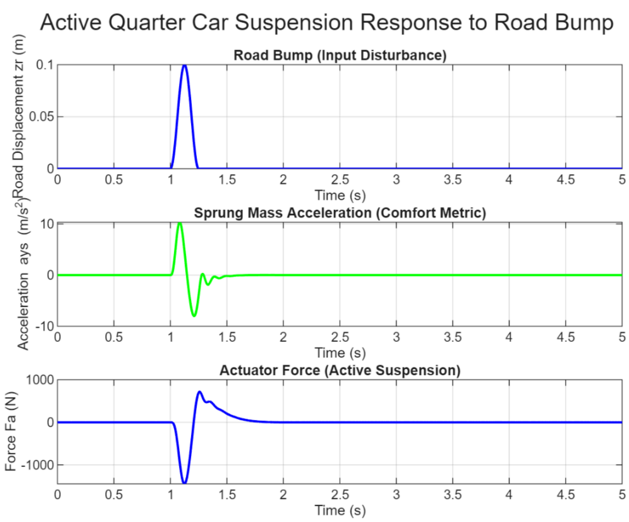
2. Key Assumptions

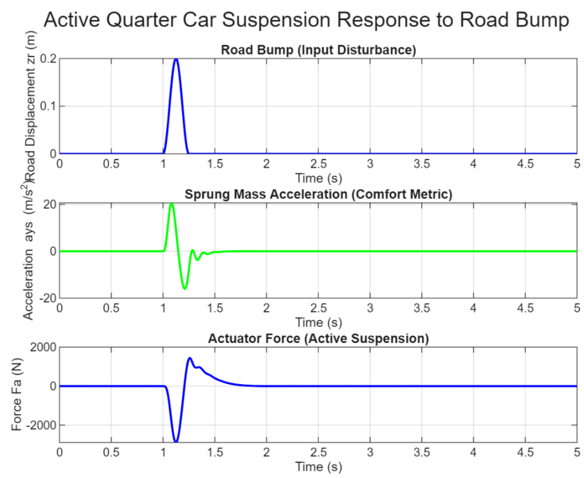
To keep the model linear and tractable for state-space control design, the following standard assumptions are made:

1. The vehicle is perfectly symmetric; only one corner is analyzed (quarter-car approximation).
2. Sprung and unsprung masses are rigid bodies moving only vertically (no pitch or roll).
3. Tire remains in contact with the road at all times
4. Tire stiffness is linear; tire damping is neglected
5. The actuator can generate force in both directions instantaneously
6. Passive damping is present even in the active system
7. Road input is known or can be modeled as isolated bumps

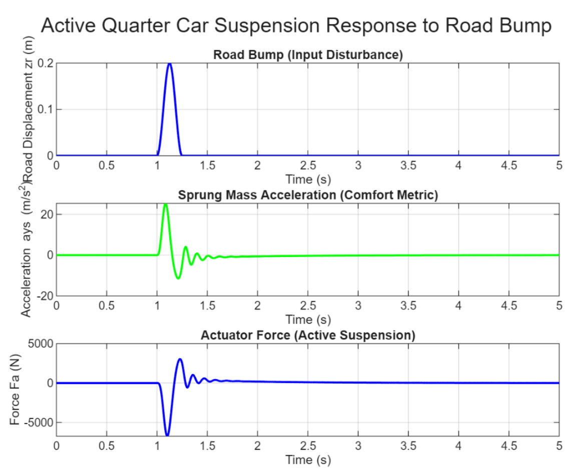
3. Performance Considerations

In general, suspension systems consider road holding (minimizing variations in tire-road contact force to ensure consistent grip, braking, and handling safety), suspension working space (limiting the relative travel between sprung and unsprung masses to prevent the suspension from hitting its mechanical stops and bottoming out), and actuator effort and efficiency. But for the sake of this report, we'll look into acceleration which correlates to ride comfort (the sensation of "bumpiness" is essentially the magnitude of vertical acceleration).





Doubling road bump (0.1 to 0.2m), doubles the sprung-mass acceleration which in real life translates to a rougher bump. We can crank up the C_active from 3000 to 15250 to counteract the acceleration from the bump, thus modeling how active suspension can mitigate the felt effects of bumps.



Zooming out, active suspension systems as a whole use onboard computers and sensors to continuously monitor vehicle motion, road conditions, and body acceleration in real time, then instantly calculate and apply the precise actuator force needed to counteract disturbances and maintain optimal ride comfort and handling.

*script for the graphs above is linked at the bottom

4. Real World Considerations

While the quarter-car model is an extremely useful model for understanding suspension behavior, it is still a highly simplified representation of an actual vehicle. Real vehicles experience numerous effects that are not captured in a two-degree-of-freedom model, and these factors meaningfully influence controller performance, reliability, and safety.

1. Vehicle Dynamics Beyond Two Degrees of Freedom
The quarter-car model assumes only vertical motion of a single corner of the vehicle. In practice, a car simultaneously experiences pitch, roll, yaw, lateral load transfer, and coupling between wheels through the chassis. Road disturbances on one wheel affect the opposite side, especially during cornering or braking. Full-vehicle (7-DOF or higher) models are therefore required when designing production-grade controllers, particularly for stability control, off-road maneuvering, or high-speed handling.

2. Road Profiles Are Highly Variable
The model for a road was also simplified. Actual road excitation is much more variable: potholes, textures, rough gravel, etc. each induce different frequencies and inputs. Controller performance must remain robust across a wide range of inputs, not just a single event.

3. Trade-off Between Ride Comfort and Road Holding
Minimizing chassis acceleration (comfort) often conflicts with maintaining consistent tire load (road handling).

- Too much compliance improves comfort but reduces handling stability.
- Excessive stiffness improves grip but degrades comfort.

Active suspension must balance these competing goals, sometimes prioritizing one based on driving mode.

4. Sensor Noise, Delays, and Integration Challenges
Real implementations rely on accelerometers, wheel-speed sensors, displacement sensors, and state estimators. These can introduce:

- Measurement noise
- Sampling and filtering delays
- Drift or bias in long-term operation

These imperfections can destabilize a controller that performs well in simulation.

5. Durability, Cost, and Maintenance Constraints
Active systems must withstand harsh environments such as temperature variation, vibration, shocks, and corrosion. Hydraulic systems risk leakage. Electromagnetic actuators may require complex cooling. Compared to passive components, active suspension adds weight, cost, and long-term maintenance needs that manufacturers must justify relative to performance benefits.

6. Multi-Objective Design in Real Vehicles
An OEM does not optimize solely for comfort. Real-world suspension design must balance:

- Fuel efficiency (added mass reduces MPG)
- Manufacturing cost
- Reliability
- Compatibility with ADAS and stability control systems
- Customer expectations for different vehicle classes (sedan vs SUV vs off-road)

Our quarter-car study provides insight into vertical dynamics and showcases how an active actuator can reduce acceleration from road inputs, but a real-world system must integrate these broader engineering requirements.

Studied Examples:

As stated, the quarter car model simplifies this suspension system to two degrees of freedom, and does not account for real world conditions such as torsion, the effects of other wheels, and material behavior. However, the quarter car model is useful to get a general idea of the behavior of vehicles on different road conditions.

Under certain requirements, this model is enough to provide a useful system that provides a boost to comfort and utility. These trends also appear in more complicated models, such as in a 3D car model with 8-DOF that can reflect pitch and roll in the car body (*figure 1*), as presented by M. Tian and V. Nguyen [1]. The modeled system had an active suspension controller which provided around a 20% reduction in acceleration responses, and pitch/roll angles of the car, throughout the different tested road profile scenarios [1].

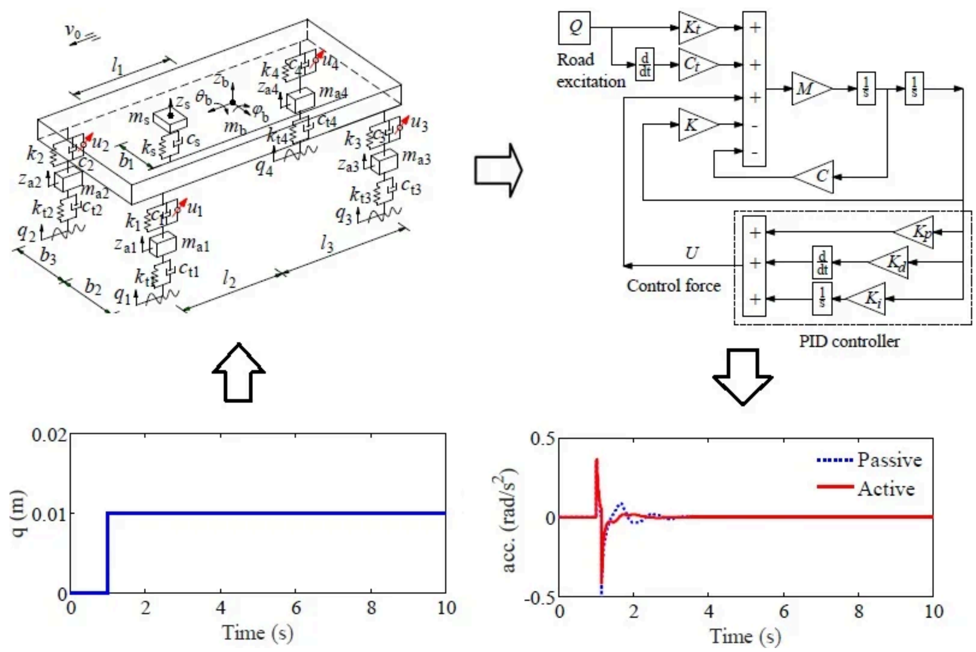


Figure 1 - 3D active suspension model [1]

While we explored a system with. Other papers have explored and discussed the differences between other controller derivations, and the various possible inputs including, the different road profiles. In addition to the PID controller, there are other methods of implementing an active suspension such as a Linear Quadratic Regulator (LQR) controller. [2]

LQR models require simulating various conditions to derive a controller matrix K that would solve the linear feedback control u(t) = -Kx(t). While both systems show an improvement in settling time when compared to the passive system, the conclusion of this paper was that based on the controllers generated, LQR improves settling time at the expense of handling (as displayed through an increase in overshoot in displacement graphs), while PID controllers have a reduced overshoot, but have a slightly longer settling time when compared to the LQR controller [3].

In a separate paper, where researchers J. Bharali and M. Buragohain modeled a three degree of freedom system, PID and LQR based fuzzy control schemes were designed and compared to a standard LQR controller. Fuzzy control systems involve adjusting parameters along intervals, in this studied example, the PID controller's Kp, Ki, Kd values are adjusted dynamically based on the input parameters to the controller which include relative displacement and relative velocity [3]. An graphical visualization of how the Kd values change along intervals is provided in their paper and also seen below (*figure 2)*

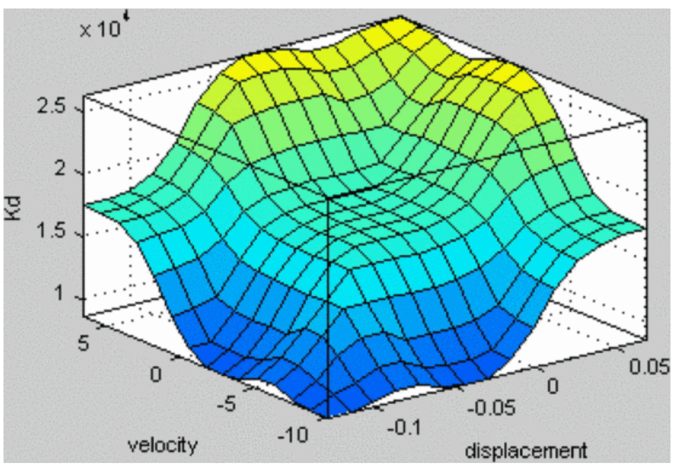


Figure 2: Fuzzy surface of Kd [3]

The fuzzy LQR system works on a similar principle, however with a larger matrix. After presenting various graphs showing outputs such as passenger acceleration, suspension deflection and sprung mass displacement, the researchers displayed the effectiveness of the dynamic controllers to settle the systems when compared to the passive suspension system, as well as the greatest effectiveness in the fuzzy LQR system in drastically reducing overshoot across the different system responses. One example can be seen below, a graph of the mass displacement vs time was provided in the paper and displays the mentioned trends (*figure 3)*.

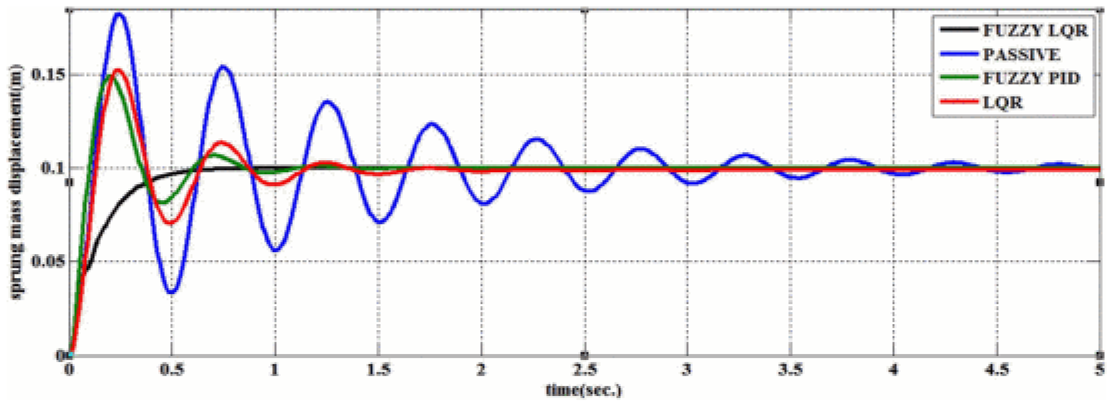
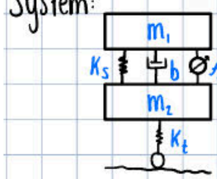


Figure 3: Sprung mass displacement vs time [3]

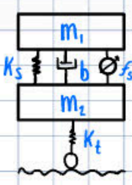
5. State Space Model Derivation

System:



m_1: car chassis mass
m_2: tire mass
K_s: suspension/passive spring constant
K_t: tire compressibility / spring constant
b: suspension spring constant
f_s: active component from feedback control
↳ control input

→



$x_1, \dot{x}$: chassis position & velocity

$x_2, \dot{x}_2$: tire position & velocity

d: road displacement

* force from spring: $F = -Kx$
* force from damper: $F = -b\dot{x}$

inputs: f_s, d outputs: $\dot{x}_1$, $\ddot{x}_1$

for m_1:

$$\sum F = m_1\ddot{x}_1 = b(\dot{x}_2 - \dot{x}_1) + K_s(x_2 - x_1) + f_s$$

for m_2:

$$\sum F = m_2\ddot{x}_2 = -b(\dot{x}_2 - \dot{x}_1) - K_s(x_2 - x_1) + K_t(d - x_2)$$

$$m_1\ddot{x}_1 + b(\dot{x}_1 - \dot{x}_2) + K_s(x_1 - x_2) = f_s$$
$$m_2\ddot{x}_2 + b(\dot{x}_2 - \dot{x}_1) + K_s(x_2 - x_1) + K_t x_2 = K_t d$$

$$\dot{x} = A\underline{x} + B\underline{u} \qquad \underline{\dot{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \Rightarrow \underline{\dot{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} x_5 \\ x_6 \\ x_1 \\ x_2 \end{bmatrix}$$
$$y = C\underline{x} + D\underline{u}$$

$$\ddot{x}_1 = b/m_1(\dot{x}_2 - \dot{x}_1) + K_s/m_1(x_2 - x_1) + f_s/m_1$$
$$\ddot{x}_2 = -b/m_2(\dot{x}_2 - \dot{x}_1) - K_s/m_2(x_2 - x_1) + K_t/m_2(d - x_2) - f_s/m_2$$

$$x_3 = b/m_1(x_2 - x_1) + K_s/m_1(x_6 - x_5) + f_s/m_1$$
$$x_4 = b/m_2(x_1 - x_2) + K_s/m_2(x_5 - x_6) + K_t/m_2(d - x_6) - f_s/m_2$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K_s/m_1 & K_s/m_1 & -b/m_1 & b/m_1 \\ K_s/m_2 & -(K_s+K_t)/m_2 & b/m_2 & -b/m_2 \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m_1 & 0 \\ -1/m_2 & K_t/m_2 \end{bmatrix} \begin{bmatrix} f_s \\ d \end{bmatrix}$$

$$\boxed{\dot{x} = A\underline{x} + B\underline{u} \rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K_s/m_1 & K_s/m_1 & -b/m_1 & b/m_1 \\ K_s/m_2 & -(K_s+K_t)/m_2 & b/m_2 & -b/m_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m_1 & 0 \\ 0 & K_t/m_2 \end{bmatrix} \begin{bmatrix} f_s \\ d \end{bmatrix}}$$

Since we're focusing on comfort will focus on velocity and acceleration of Chassis:

$$y = \begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \end{bmatrix}$$

$$\boxed{y = C\underline{x} + D\underline{u} \rightarrow \begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -K_s/m_1 & K_s/m_1 & -b/m_1 & b/m_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \end{bmatrix} \begin{bmatrix} f_s \\ d \end{bmatrix}}$$

Matlab script

```
% MATLAB Script for Modeling and Visualizing an Active Suspension System (Quarter Car Model)

% This script models a quarter car active suspension system.
% Key variables:
% - Road bump (zr): Modeled as a smooth half-sine wave bump.
% - Up-down acceleration (ays): Sprung mass acceleration, correlating to bumpiness/comfort.
% - Suspension actuator (Fa): Uses a skyhook damping control to mitigate acceleration.

% Parameters are typical for a passenger car.
% Simulation uses ode45 to solve the differential equations.

clear; clc; close all;

% Define parameters
Ms = 300;    % Sprung mass (kg)
Mu = 50;     % Unsprung mass (kg)
Ks = 20000;  % Suspension stiffness (N/m)
Cs = 1500;   % Suspension damping (Ns/m)
Kt = 200000; % Tire stiffness (N/m)
C_active = 15250; % Active skyhook damping coefficient (Ns/m) - tunes the actuator to mitigate acceleration

% Road bump function: Smooth half-sine wave bump (height 0.1 m, duration 0.25 s, starting at t=1 s)
bump_height = 0.2;   % Bump height (m)
bump_start = 1;      % Start time (s)
bump_duration = 0.25; % Duration (s)
zr_func = @(t) bump_height / 2 * (1 - cos(2*pi*(t - bump_start)/bump_duration)) .* ...
          (t >= bump_start & t < bump_start + bump_duration);

% Simulation time
tspan = [0 5]; % From 0 to 5 seconds
initial_conditions = [0; 0; 0; 0]; % [ys; yu; vys; vyu] at equilibrium

% ODE function for quarter car model
% States: x = [ys; yu; vys; vyu]
quarter_car_ode = @(t, x, Fa_func, params) [
  x(3);  % dys/dt = vys
  x(4);  % dyu/dt = vyu
  (1/params.Ms) * (-params.Ks*(x(1)-x(2)) - params.Cs*(x(3)-x(4)) + Fa_func(t, x));  % ays
  (1/params.Mu) * (params.Ks*(x(1)-x(2)) + params.Cs*(x(3)-x(4)) - params.Kt*(x(2) - zr_func(t)) - Fa_func(t, x))  %
ayu
];

% Parameters struct
params.Ms = Ms;
params.Mu = Mu;
params.Ks = Ks;
params.Cs = Cs;
params.Kt = Kt;

% --- Active Suspension (Skyhook: Fa = -C_active * vys to mitigate sprung acceleration) ---
Fa_active = @(t, x) -C_active * x(3);
[t_active, x_active] = ode45(@(t,x) quarter_car_ode(t, x, Fa_active, params), tspan, initial_conditions);

% Compute accelerations, zr, and Fa for active
zr_active = arrayfun(zr_func, t_active);
ays_active = (1/Ms) * (-Ks*(x_active(:,1)-x_active(:,2)) - Cs*(x_active(:,3)-x_active(:,4)) + Fa_active(0, x_active(1,:)'));
Fa_active_vals = arrayfun(@(i) Fa_active(t_active(i), x_active(i,:)), (1:length(t_active))');

% Visualization
```

```
figure('Name', 'Active Quarter Car Suspension Simulation', 'NumberTitle', 'off');

% Subplot 1: Road Bump (zr)
subplot(3,1,1);
plot(t_active, zr_active, 'b-', 'LineWidth', 1.5);
title('Road Bump (Input Disturbance)');
xlabel('Time (s)');
ylabel('Road Displacement zr (m)');
grid on;

% Subplot 2: Sprung Mass Acceleration (ays) - Measures bumpiness/comfort
subplot(3,1,2);
plot(t_active, ays_active, 'g-', 'LineWidth', 1.5);
title('Sprung Mass Acceleration (Comfort Metric)');
xlabel('Time (s)');
ylabel('Acceleration ays (m/s^2)');
grid on;

% Subplot 3: Actuator Force (Fa)
subplot(3,1,3);
plot(t_active, Fa_active_vals, 'b-', 'LineWidth', 1.5);
title('Actuator Force (Active Suspension)');
xlabel('Time (s)');
ylabel('Force Fa (N)');
grid on;

% Adjust figure
sgtitle('Active Quarter Car Suspension Response to Road Bump');
set(gcf, 'Position', [100 100 800 600]);
```

References

- *[] D. Karnopp, M. J. Crosby, and R. A. Harwood, “Vibration control using semi-active force generators,” Journal of Engineering for Industry, vol. 96, no. 2, pp. 619–626, 1974.*
- *[] T. D. Gillespie, Fundamentals of Vehicle Dynamics. SAE International, 1992.*
- *[] “Robust Control of an Active Suspension,” Mathworks.com, 2019. https://www.mathworks.com/help/robust/gs/active-suspension-control-design.html*
- *[1] V. Nguyen and M. Tian, “Control performance of suspension system of cars with PID control based on 3D dynamic model,” Journal of Mechanical Engineering, Automation and Control Systems, vol. 1, no. 1, pp. 1–10, Jun. 2020, doi: 10.21595/jmeacs.2020.21363.*
- *[2] R. Darus and N. I. Enzai, "Modeling and control active suspension system for a quarter car model," 2010 International Conference on Science and Social Research (CSSR 2010), Kuala Lumpur, Malaysia, 2010, pp. 1203-1206, doi: 10.1109/CSSR.2010.5773718.*
- *[3] J. Bharali and M. Buragohain, "Design and performance analysis of Fuzzy LQR; Fuzzy PID and LQR controller for active suspension system using 3 Degree of Freedom quarter car model," 2016 IEEE 1st International Conference on Power Electronics, Intelligent Control and Energy Systems (ICPEICES), Delhi, India, 2016, pp. 1-6, doi: 10.1109/ICPEICES.2016.7853369.*