Appendices

A The Fourier Transform

A1 Definitions

The one-dimensional *Fourier transform* of the function f(x) is defined as

$$\Im\{f(x)\} = F(u) = \int_{0}^{\infty} f(x) \exp[-i2\pi ux] dx$$
(A1)

The inverse one-dimensional Fourier transformation is defined as

$$\mathfrak{I}^{-1}\left\{F(u)\right\} = f(x) = \int_{-\infty}^{\infty} F(u) \exp[i2\pi ux] du$$
(A2)

The functions f(x) and F(u) are called Fourier transform pair.

The two-dimensional *Fourier transform* of the function f(x, y) is defined as

$$\Im\{f(x,y)\} = F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \exp[-i2\pi(ux+vy)] dxdy$$
(A3)

The corresponding inverse two-dimensional Fourier transformation is defined as

$$\mathfrak{I}^{-1}\left\{F(u,v)\right\} = f(x,y) = \int_{-\infty-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) \exp[i2\pi(ux+vy)]dudv$$
(A4)

The Fourier transformation is a powerful mathematical tool to describe and analyse periodic structures. If x is the time coordinate of a signal (unit: s), then u is the corresponding frequency (unit: 1/s = Hz). In the two-dimensional case (x,y) are often spatial coordinates (units: meter), while (u,v) are the corresponding spatial frequencies (units: 1/meter).

In the following some useful theorems about Fourier transforms are summarized. These formulas are written for the two-dimensional case.

1. Linearity theorem

$$\Im\{af(x,y) + bg(x,y)\} = aF(u,v) + bG(u,v) \tag{A5}$$

where a and b are constants, $F(u,v) = \Im(f(x,y))$ and $G(u,v) = \Im(g(x,y))$.

2. Similarity theorem

$$\Im\{f(ax,by)\} = \frac{1}{|ab|}F\left(\frac{u}{a},\frac{v}{b}\right) \tag{A6}$$

3. Shift theorem

$$\Im\{f(x-a,y-b)\} = F(u,v)\exp[-i2\pi(ua+vb)] \tag{A7}$$

4. Rayleigh's (Parseval's) theorem

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |f(x,y)|^2 dxdy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |F(u,v)|^2 dudv$$
(A8)

5. Convolution theorem

The two-dimensional convolution of two functions f(x, y) and g(x, y) is defined as

$$(f \otimes g)(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x',y')g(x-x',y-y')dx'dy'$$
(A9)

where the \otimes denotes the convolution operation. The convolution theorem states that the Fourier transform of the convolution of two functions is equal to the product of the Fourier transforms of the individual functions:

$$\Im\{f(x,y)\otimes g(x,y)\} = F(u,v)G(u,v) \tag{A10}$$

6. Autocorrelation theorem

$$\Im\left\{\int_{-\infty-\infty}^{\infty} \int_{-\infty}^{\infty} f^*(x',y') f(x+x',y+y') dx' dy'\right\} = \left|F(u,v)\right|^2$$
(A11)

7. Fourier integral theorem

$$\Im \Im^{-1} \{ f(x, y) \} = \Im^{-1} \Im \{ f(x, y) \} = f(x, y)$$
(A12)

8. Differentiation

Differentiation in the spatial domain corresponds to a multiplication with a linear factor in the spatial frequency domain:

$$\Im\left\{\left(\frac{\partial}{\partial x}\right)^{m}\left(\frac{\partial}{\partial y}\right)^{n}f(x,y)\right\} = (i2\pi u)^{m}(i2\pi v)^{n}F(u,v) \tag{A13}$$

A3 The Discrete Fourier Transform

For numerical computations the function to be transformed is given in a discrete form, i. e. f(x) in Eq. (A.1) has to be replaced by the finite series f_k , with integer numbers k = 0,1,...,N-1. The continuous variable x is now described as integer multiple of a sampling interval Δx :

$$x = k\Delta x \tag{A14}$$

The frequency variable u is converted into a discrete variable, too:

$$u = m\Delta u \tag{A15}$$

The discrete representation of Eq. (A.1) is then given by:

$$F_m = \Delta x \sum_{k=1}^{N-1} f_k \exp[-i2\pi k m \Delta x \Delta u]$$
 for $m = 0, 1, ..., N-1$ (A16)

The maximum frequency is determined by the sampling interval in the spatial domain:

$$u_{\text{max}} = N\Delta u = \frac{1}{\Delta x} \tag{A17}$$

The following expression

$$F_m = \frac{1}{N} \sum_{k=0}^{N-1} f_k \exp\left[-i2\pi \frac{km}{N}\right]$$
 (A18)

is therefore defined as one-dimensional *discrete Fourier transform* (DFT). The inverse transformation is given by

$$f_k = \sum_{m=0}^{N-1} F_m \exp\left[i2\pi \frac{km}{N}\right] \tag{A19}$$

Similar considerations lead to the discrete two-dimensional Fourier transform pair:

$$F_{mn} = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} f_{kl} \exp \left[-i2\pi \left(\frac{km + ln}{N} \right) \right]$$
 (A20)

$$f_{kl} = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} F_{mn} \exp\left[i2\pi \left(\frac{km + ln}{N}\right)\right]$$
 (A21)

for m = 0,1,...N-1 and n = 0,1,...N-1

Here a quadratic field of sampling points is used, i. e. the number of points in each row is equal to that in each column.

The computation time for a discrete Fourier transform is mainly determined by the number of complex multiplications. A two-dimensional DFT can be factorised into two one-dimensional DFT's:

$$F_{mn} = \frac{1}{N^2} \sum_{k=0}^{N-1} \left[\sum_{l=0}^{N-1} f_{kl} \exp\left(-i2\pi \frac{nl}{N}\right) \right] \exp\left(-i2\pi \frac{km}{N}\right)$$
(A22)

The one-dimensional DFT can be programmed most effectively using the so called *fast fourier transform* (FFT) algorithms, invented in the 70th of the last century by Cooley and Tookey. These algorithms make use of redundancies and reduce the number of multiplications for a one-dimensional DFT from N^2 to $2N\log_2 N$. The FFT algorithms are not described here, it is referred to the literature [10].

B Phase Transformation of a Spherical Lens

B1 Lens Transmission Function

The effect of an optical component with refractive index n and thickness d on the complex amplitude of a wave is described by a transmission function τ .

$$\tau = \left| \tau \middle| \exp \left[-i \frac{2\pi}{\lambda} (n-1) d \right]$$
 (B1)

This function is calculated in the following for a thin biconvex lens. Such lens consists of two spherical surfaces, see figure B.1. The radius of curvature of the left half lens is R_1 , while that of the right half lens is designated R_2 . Following sign convention is applied: As rays travel from left to right, each convex surface has a positive radius of curvature, while each concave surface has a negative radius of curvature. Due to this convention R_2 has a negative value. Losses due to reflection at the surfaces and due to absorption inside the lens are neglected; i. e. $|\tau|=1$. The refractive index is constant for the entire lens.

The lens thickness is a function of the spatial coordinates x and y:

$$d(x,y) = d_1(x,y) + d_2(x,y)$$

$$= d_{01} - \zeta_1 + (d_{02} - \zeta_2)$$
(B2)

According to figure B.1 it can be written:

$$R_1^2 = r^2 + (R_1 - \zeta_1)^2$$

$$= x^2 + v^2 + R_1^2 - 2R_1\zeta_1 + \zeta_1^2$$
(B3)

and

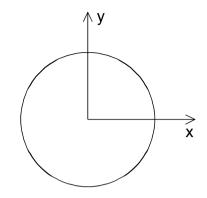
$$R_2^2 = r^2 + (-R_2 - \zeta_2)^2$$

$$= x^2 + y^2 + R_2^2 + 2R_2\zeta_2 + {\zeta_2}^2$$
(B4)

Neglecting the quadratic terms of $\zeta_{1/2}$ leads to:

$$\zeta_1 = \frac{x^2 + y^2}{2R_1} \tag{B5}$$

$$\zeta_2 = -\frac{x^2 + y^2}{2R_2} \tag{B6}$$



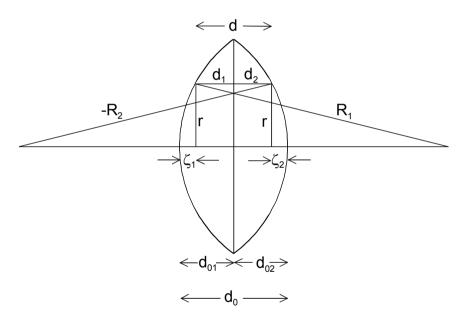


Fig. B.1. Biconvex lens, top view and cross-sectional view

This level of approximation is consistent with the parabolic approximation used in the Fresnel transformation. The thickness is now

$$d(x,y) = d_0 - \frac{x^2 + y^2}{2R_1} + \frac{x^2 + y^2}{2R_2}$$
(B7)

With the lens makers equation

$$\frac{1}{f} = \left(n - 1\right) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \tag{B8}$$

of geometrical optics following lens transmission function is derived:

$$L(x,y) = \exp\left[i\frac{\pi}{\lambda f}(x^2 + y^2)\right]$$
 (B9)

The constant factor $\exp(-i2\pi/\lambda(n-1)d_0)$, which only effects the overall phase, has been neglected.

B2 Correction of Aberrations

In the following the complex amplitude of an object, which is imaged by a lens is calculated. The object is lying in the (ξ, η) coordinate system, the lens is located in the (x,y) system and the image arises in the (ξ', η') system, see figure B.2. The object is described by the complex amplitude $E_o(\xi, \eta)$.

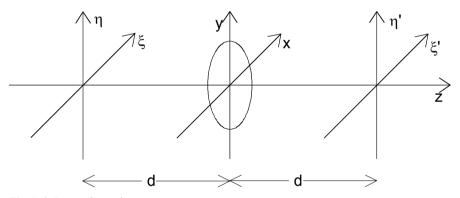


Fig. B.2. Image formation

The complex amplitude in front of the lens is given by

$$E_{O}'(x,y) = \exp\left[-i\frac{\pi}{\lambda d}(x^{2} + y^{2})\right] \int_{-\infty-\infty}^{\infty} E_{O}(\xi,\eta) \exp\left[-i\frac{\pi}{\lambda d}(\xi^{2} + \eta^{2})\right]$$

$$\times \exp\left[i\frac{2\pi}{\lambda d}(x\xi + y\eta)\right] d\xi d\eta$$
(B10)

where the Fresnel approximation is used. The complex amplitude in the image plane in then given by

$$E_{O}^{"}(\xi',\eta')$$

$$= \exp\left[-i\frac{\pi}{\lambda d}(\xi'^{2} + \eta'^{2})\right] \int_{-\infty-\infty}^{\infty} E_{O}^{"}(x,y) L(x,y) \exp\left[-i\frac{\pi}{\lambda d}(x^{2} + y^{2})\right]$$

$$\times \exp\left[i\frac{2\pi}{\lambda d}(x\xi' + y\eta')\right] dxdy$$

$$= \exp\left[-i\frac{\pi}{\lambda d}(\xi'^{2} + \eta'^{2})\right] \int_{-\infty-\infty}^{\infty} \int_{-\infty-\infty}^{\infty} E_{O}(\xi,\eta) \exp\left[-i\frac{\pi}{\lambda d}(x^{2} + y^{2})\right]$$

$$\times \exp\left[-i\frac{\pi}{\lambda d}(\xi^{2} + \eta^{2})\right] \exp\left[i\frac{2\pi}{\lambda d}(x\xi + y\eta)\right] \exp\left[i\frac{2\pi}{\lambda d}(x^{2} + y^{2})\right]$$

$$\times \exp\left[-i\frac{\pi}{\lambda d}(x^{2} + y^{2})\right] \exp\left[i\frac{2\pi}{\lambda d}(x\xi' + y\eta')\right] d\xi d\eta dxdy$$
(B11)

A magnification of 1 and a focal distance of f = d/2 is used for the lens transmission function L(x, y).

The image coordinates can be expressed in terms of the object coordinates:

$$\xi' = -\xi$$
 and $\eta' = -\eta$ (B12)

The minus signs result, because according to the laws of geometrical optics the image is upside down.

The complex amplitude of the image is now

$$E_{O}^{"}(\xi',\eta') = \exp\left[-i\frac{2\pi}{\lambda d}(\xi'^{2} + \eta'^{2})\right]E_{O}(-\xi',-\eta')$$

$$= \exp\left[-i\frac{\pi}{\lambda f}(\xi'^{2} + \eta'^{2})\right]E_{O}(-\xi',-\eta')$$
(B13)

The wavefield in the image plane has to be multiplied therefore by a factor

$$P(\xi', \eta') = \exp\left[i\frac{\pi}{\lambda f} \left(\xi'^2 + {\eta'}^2\right)\right]$$
(B14)

in order to generate the correct phase distribution.

This correction factor depends on the wavelength and on the coordinates of the image plane. It can be neglected, if only the intensity of a wavefield has to be calculated after reconstruction ($I \propto P^*P = 1$). This is also valid, if the phase difference of two wavefields, which are recorded with the *same* wavelength, is computed:

$$\Delta \varphi = \varphi_{1} - \varphi_{2} = i \pi / \lambda f \left(\xi'^{2} + \eta'^{2} \right) + \varphi'_{1} - \left[i \pi / \lambda f \left(\xi'^{2} + \eta'^{2} \right) + \varphi'_{2} \right]$$

$$= \varphi'_{1} - \varphi'_{2}$$
(B15)

This is usually the case in DHI for applications in deformation analysis. However, the correction factor has to be considered, if the phase difference of two wavefields, which are recorded with *different* wavelengths, is computed. This is the case in multi-wavelength DHI for shape measurement.

References

- 1. Abramson N (1983) Light-in-flight recording: High speed holographic motion pictures of ultrafast phenomena. Appl Opt 22:215-232
- 2. Abramson N (1984) Light-in-flight recording 2: Compensation for the limited speed of the light used for observation. Appl Opt 23:1481-1492
- 3. Abramson N (1984) Light-in-flight recording 3: Compensation for optical relativistic effects. Appl Opt 23:4007-4014
- Abramson N (1985) Light-in-flight recording 4: Visualizing optical relativistic phenomena. Appl Opt 24:3323-3329
- 5. Adams M, Kreis T, Jüptner W (1997) Particle size and position measurement with digital holography. In: Proc SPIE vol 3098, pp 234-240
- Adams M, Kreis T, Jüptner W (1999) Particle measurement with digital holography. In: Proc SPIE vol 3823
- 7. Bartels RA, Paul A, Green H, Kapteyn HC, Murnane MM, Backus S, Christov IP, Liu Y, Attwood D, Jacobsen C (2002) Generation of Spatially Coherent Light at Extreme Ultraviolet Wavelengths. Science 297:376-378
- 8. Bisle W (1998) Optische Prüfung an Luftfahrtkomponenten: Weiterentwicklung des Scherografie-Prüfverfahrens für nicht-kooperative Oberflächen von Flugzeugstrukturen. Proc Deutsche Gesellschaft für Luft- und Raumfahrt, annual meeting, Bremen
- 9. Breuckmann B, Thieme W (1985) Computer-aided analysis of holographic interferograms using the phase-shift method. Appl Opt 24:2145-2149
- 10. Brigham EO (1974) The fast fourier transform. Pretince-Hall
- Bryngdahl O, Wyrowski F (1990) Digital Holography computer generated holograms. Progress in Optics 28:1-86
- 12. Butters JN, Leendertz JA (1971) Holographic and Videotechniques applied to engineering measurements. J Meas Control 4:349-354
- 13. Carlsson T, Nilsson B, Gustafsson J (2001) System for acquisition of three-dimensional shape and movement using digital Light-in-Flight holography. Opt Eng 40(01): 67-75
- 14. CCD Primer (2002) product information. Kodak
- Chen DJ, Chiang FP, Tan YS, Don HS (1993) Digital speckle displacement measurement using a complex spectrum method. Appl Opt 32(11): 1839-1848
- Coppola G, De Nicola S, Ferraro P, Finizio A, Grilli S, Iodice M, Magro C, Pierattini G (2003) Evaluation of residual stress in MEMS structures by means of digital holography. In: Proc. SPIE vol 4933, pp 226-31
- 17. Coppola G, Ferraro P, Iodice M, De Nicola S, Finizio A, Grilli S (2004) A digital holographic microscope for complete characterization of microelectromechanical systems. Meas Sci Technol 15: 529-539
- 18. Coquoz O, Conde R, Taleblou F, Depeursinge C (1995) Performances of endoscopic holography with a multicore optical fiber. Appl Opt 34(31):7186-7193
- 19. Creath K (1985) Phase shifting speckle-interferometry. Appl Opt 24(18):3053-3058

- Creath K (1994) Phase-shifting holographic interferometry. In: Holographic Interferometry, Springer Series in Optical Sciences 68, pp 109-150
- 21. Cuche E, Bevilacqua F, Depeursinge C (1999) Digital holography for quantitative phase-contrast imaging. Optics Letters 24(5):291-293
- 22. Cuche E, Marquet P, Depeursinge C (1999) Simultaneous amplitude-contrast and quantitative phase-contrast microscopy by numerical reconstruction of Fresnel off-axis holograms. Appl Opt 38(34):6994-7001
- 23. Cuche E, Marquet P, Depeursinge C (2000) Spatial filtering for zero-order and twinimage elimination in digital off-axis holography. Appl Opt 39(23):4070-4075
- 24. Cuche E, Marquet P, Depeursinge C (2000) Aperture apodization using cubic spline interpolation: application in digital holographic microscopy. Opt Commun 182:59-69
- 25. Demetrakopoulos TH, Mittra R (1974) Digital and optical reconstruction of images from suboptical diffraction patterns. Appl Opt 13(3):665-670
- Demoli N, Mestrovic J, Sovic I (2003) Subtraction digital holography. Appl Opt 42(5):798-804
- 27. De Nicola S, Ferraro P, Finizio A, Pierattini G (2001) Correct-image reconstruction in the presence of severe anamorphism by means of digital holography. Opt Letters 26(13):974-76
- De Nicola S, Ferraro P, Finizio A, Pierattini G (2001) Compensation of Aberrations in Fresnel off-axis Digital Holography. In: Jüptner W, Osten W (eds) Proc 4th International Workshop on Automatic Processing of Fringe Patterns. Akademie, Berlin, pp 407-412
- Doval AF (2000) A systematic approach to TV holography. Meas Sci Technol 11:R1-R36
- Dubois F, Joannes L, Legros J C (1999) Improved three-dimensional imaging with a digital holography microscope with a source of partial spatial coherence. Appl Opt 38(34):7085-7094
- Dubois F, Joannes L, Dupont O, Dewandel JL, Logros JC (1999) An integrated optical set-up for fluid-physics experiments under microgravity conditions. Meas Sci Technol 10:934-945
- 32. Dubois F, Minetti C, Monnom O, Yourassowsky C, Legros JC, Kischel P (2002) Pattern recognition with a digital holographic microscope working in partially coherent illumination. Appl Opt 41(20):4108-4119
- 33. Dubois F, Monnom O, Yourassowsky C, Legros JC (2002) Border processing in digital holography by extension of the digital hologram and reduction of the higher spatial frequencies. Appl Opt 41(14):2621-2626
- 34. Frauel Y, Javidi B (2001) Neural network for three-dimensional object recognition based on digital holography. Optics Letters 26(19):1478-1480
- 35. Frauel Y, Tajahuerce E, Castro MA, Javidi B (2001) Distortion- tolerant three-dimensional object recognition with digital holography. Appl Opt 40(23):3887
- 36. Füzessy Z, Gyimesi F (1984) Difference holographic interferometry: displacement measurement. Opt Eng 23(6):780-783
- 37. Gabor D (1948) A new microscopic principle. Nature 161:777-778
- 38. Gabor D (1949) Microscopy by reconstructed wavefronts. Proc Roy Soc 197:454-487
- Gabor D (1951) Microscopy by reconstructed wavefronts: 2. Proc Phys Soc 64:449-469
- Goodman JW (1975) statistical properties of laser speckle patters. In: Dainty JC (ed) Laser Speckle and Related Phenomena, Topics in Appl Physics vol 9, Springer, Berlin, pp. 9-75
- 41. Goodman JW (1996) Introduction to Fourier Optics 2nd ed. McGraw-Hill

- 42. Goodman JW, Lawrence RW (1967) Digital image formation from electronically detected holograms. Appl Phys Lett 11:77-79
- 43. Grilli S, Ferraro P, De Nicola S, Finizio A, Pierattini G Meucci R (2001) Whole optical wavefield reconstruction by digital holography. Optics Express 9(6):294-302
- 44. Grunwald R, Griebner U, Elsaesser T, Kebbel V, Hartmann H J, Jüptner W (2001) Femtosecond interference experiments with thin-film micro-optical components. In: Jüptner W, Osten W (eds) Proc 4th International Workshop on Automatic Processing of Fringe Patterns. Akademie, Berlin, pp 33-40
- 45. Guo CS, Zhang L, Wang HT, Liao J, Zhu Y (2002) Phase-shifting error and its elimination in phase-shifting digital holography. Opt Lett 27(19):1687-1689
- 46. Haddad W, Cullen D, Solem J, Longworth J, McPherson A, Boyer K, Rhodes K (1992) Fourier-transform holographic microscope. Appl Opt 31(24): 4973-4978
- 47. Harriharan P (1984) Optical Holography. Cambridge University Press, Cambridge
- 48. Helmers H, Bischoff M, Ehlkes L (2001)ESPI-System with active in-line digital phase stabilization. In: Jüptner W, Osten W (eds) Proc 4th International Workshop on Automatic Processing of Fringe Patterns. Elsevier, pp 673-679
- Hinsch K (2002) Holographic particle image velocimetry. Meas Sci Technol 13:R61-R72
- 50. Holstein D, Hartmann HJ, Jüptner W (1998) Investigation of Laser Welds by Means of Digital Speckle Photography. In: Proc SPIE vol 3478, pp 294-301
- 51. Hung YY (1996) Shearography for non-destructive evaluation of composite structures. Opt Lasers Eng 24:161-182
- 52. Hung YY, Liang CY (1979) Image shearing camera for direct measurement of surface strain. Appl Opt 18:1046-1051
- 53. Huntley JM, Saldner H (1993) Temporal phase-unwrapping algorithm for automated interferogram analysis. Appl Opt 32(17):3047-3052
- 54. Inomato O, Yamaguchi I 2001 Measurements of Benard-Marangoni waves using phase-shifting digital holography. In: Proc SPIE vol 4416:124-127
- 55. Jacquot M, Sandoz P, Tribillon G (2001) High resolution digital holography. Opt Commun 190:87-94
- 56. Javidi B, Nomura T (2000) Securing information by use of digital holography. Optics Letters 25(1):28-30
- 57. Javidi B, Tajahuerce E (2000) Three-dimensional object recognition by use of digital holography. Optics Letters 25(9):610-612
- 58. Jüptner W (1978) Automatisierte Auswertung holografischer Interferogramme mit dem Zeilen-Scanverfahren. In: Kreitlow H, Jüptner W (eds) Proc Frühjahrsschule 78 Holografische Interferometrie in Technik und Medizin.
- Jüptner W (2000) Qualität durch Lasertechnik Zukunft für das 21. Jahrhundert. In: Proc LEF symposium. Erlangen
- 60. Jüptner W, Kreis T, Kreitlow H (1983) Automatic evaluation of holographic interferograms by reference beam phase shifting. In: Proc SPIE vol 398, pp 22-29
- 61. Jüptner W, Pomarico J, Schnars U (1996) Light-in-Flight measurements by Digital Holography. In: Proc. SPIE vol 2860
- 62. Kato J, Yamaguchi I, Matsumura T (2002) Multicolor digital holography with an achromatic phase shifter. Opt Lett 27(16):1403-1405
- 63. Kebbel V, Grubert B, Hartmann HJ, Jüptner W, Schnars U (1998) Application of Digital Holography to Space-Borne Fluid Science Measurements. In: Proc 49th International Astronautical Congress Melbourne paper no. IAF-98-J.5.03
- Kebbel V, Adams M, Hartmann H J, Jüptner W (1999) Digital holography as a versatile optical diagnostic method for microgravity experiments. Meas Sci Technol 10:893-899

- Kebbel V, Hartmann HJ, Jüptner W (2001) Application of digital holographic microscopy for inspection of micro-optical components. In: Proc SPIE vol 4398, pp 189-98
- Kim MK (1999) Wavelength-scanning digital interference holography for optical section imaging. Optics Letters 24(23):1693-1695
- 67. Kim MK (2000) Tomographic three-dimensional imaging of a biological specimen using wavelength-scanning digital interference holography. (2000) Optics Express 7(9):305-310
- 68. Kim S, Lee B, Kim E (1997) Removal of bias and the conjugate image in incoherent on-axis triangular holography and real-time reconstruction of the complex hologram. Appl Opt 36(20):4784-4791
- 69. Klein MV, Furtak TE (1986) Optics. 2nd ed Wiley, New York
- Kolenovic E, Lai S, Osten W, Jüptner W (2001) Endoscopic shape and deformation measurement by means of Digital Holography. In: Jüptner W, Osten W (eds) Proc 4th International Workshop on Automatic Processing of Fringe Patterns. Akademie, Berlin, pp 686-691
- 71. Kreis T (1996) Holographic Interferometry. Akademie, Berlin
- 72. Kreis T (2002) Frequency analysis of digital holography. Opt Eng 41(4):771-778
- Kreis T (2002) Frequency analysis of digital holography with reconstruction by convolution. Opt Eng 41(8):1829-1839
- 74. Kreis T, Jüptner W (1997) Principles of digital holography. In: Jüptner W, Osten W (eds) Proc 3rd International Workshop on Automatic Processing of Fringe Patterns. Akademie, Berlin, pp 353-363
- 75. Kreis T, Jüptner W (1997) Suppression of the dc term in digital holography. Opt Eng 36(8):2357-2360
- Kreis T, Jüptner W, Geldmacher J (1998) Digital Holography: Methods and Applications. In: Proc SPIE vol 3407, pp 169-177
- 77. Kreis T, Adams M, Jüptner W 1999 Digital in-line holography in particle measurement. In: Proc SPIE vol 3744
- 78. Kreis T, Aswendt P, Höfling R (2001) Hologram reconstruction using a digital micromirror device. Opt Eng 40(6):926-933
- 79. Kreis T, Adams M, Jüptner W (2002) Aperture synthesis in digital holography. In: Proc SPIE vol 4777, pp 69-76
- Kreuzer HJ, Pawlitzek RA (1997) Numerical Reconstruction for in-line Holography in Reflection and under glancing Incidence. In: Jüptner W, Osten W (eds) Proc 3rd International Workshop on Automatic Processing of Fringe Patterns. Akademie, Berlin, pp 364-367
- 81. Kronrod MA, Merzlyakov NS, Yaroslavski LP (1972) Reconstruction of holograms with a computer. Sov Phys-Tech Phys USA 17 (2):333-334
- 82. Krupka R, Walz T, Ettemeyer A (2002) Industrial Applications of Shearography for Inspection of Aircraft Components. Proc 8th ECNDT, Barcelona
- 83. Kulak M, Pisarek J (2001) Speckle photography in the examination of composites. In: Jüptner W, Osten W (eds) Proc 4th International Workshop on Automatic Processing of Fringe Patterns. Elsevier, pp 528-530
- 84. Lai S, Neifeld M (2000) Digital wavefront reconstruction and its application to image encryption. Opt Commun 178:283-289
- 85. Lai S, Kemper B, von Bally G (1999) Off-axis reconstruction of in-line holograms for twin-image elimination. Optics Communications 169:37-43
- 86. Lai S, King B, Neifeld M A (2000) Wave front reconstruction by means of phase-shifting digital in-line holography. Optics Communications 173:155-160

- 87. Lai S, Kolenovic E, Osten W, Jüptner W (2002) A deformation and 3D-shape measurement system based on phase-shifting digital holography. In: Proc SPIE vol 4537, pp 273-276
- 88. Le Clerc F, Gross M (2001) Synthetic-aperture experiment in the visible with on-axis digital heterodyne holography. Opt Lett 26(20):1550-1552
- Le Clerc F, Collot L, Gross M (2000) Numerical heterodyne holography with twodimensional photodetector arrays. Optics Letters 25(10):716-718
- Lee WH (1978) Computer-generated Holograms: Techniques and Applications. Progress in Optics 16: 120-232
- 91. Leith EN, Upatnieks J (1962) Reconstructed wavefronts and communication theory. Jour Opt Soc Amer 52:1123-1130
- 92. Leith EN, Upatnieks J (1964) Wavefront reconstruction with diffused illumination and threedimensional objects. Journ Opt Soc Amer 54:1295-1301
- 93. Liu G, Scott PD (1987) Phase retrieval and twin-image elimination for in-line Fresnel holograms. J Opt Soc Am A 4(1): 159-165
- 94. Lokberg O (1980) Electronic Speckle Pattern Interferometry. Phys Techn 11:16-22
- 95. Lokberg O, Slettemoen GA (1987) Basic Electronic Speckle Pattern Interferometry. Applied Optics and Optical Engineering 10:455-505
- Macovski A, Ramsey D, Schaefer LF (1971) Time Lapse Interferometry and Contouring using Television Systems. Appl. Opt. 10(12):2722-2727
- 97. Matoba O, Naughton TJ, Frauel Y, Bertaux N, Javidi B (2002) Real-time three-dimensional object reconstruction by use of a phase-encoded digital hologram. Appl Opt 41(29):6187-6192
- 98. Milgram JH, Weichang Li (2002) Computational reconstruction of images from holograms. Appl Opt 41(5): 853-864
- 99. Nadeborn W, Andrä P, Osten W (1995) A robust procedure for absolute phase measurement. Optics and Lasers in Engineering 22
- 100. Neumann DB (1980) Comparative Holography. In: Tech Digest Topical Meeting on Hologram Interferometry and Speckle Metrology, paper MB2-1. Opt Soc Am
- 101. Nilsson B, Carlsson T (1998) Direct three-dimensional shape measurement by digital light-in-flight holography. Appl Opt 37(34):7954-7959
- 102. Nilsson B, Carlsson T (1999) Digital light-in-flight holography for simultaneous shape and deformation measurement. In: Proc. SPIE vol 3835, pp 127-134
- 103. Nilsson B, Carlsson T 2000 Simultaneous measurement of shape and deformation using digital light-in-flight recording by holography. Opt Eng 39(1):244-253
- 104. Onural L (2000) Sampling of the diffraction field. Appl Opt 39(32):5929-5935
- 105. Onural L, Özgen MT (1992) Extraction of three-dimensional object-location information directly from in-line holograms using Wigner analysis. J Opt Soc Amer A 9(2):252-260
- 106. Onural L, Scott PD (1987) Digital decoding of in-line holograms. Opt Eng 26(11):1124-1132
- 107. Osten W, Nadeborn W, Andrä P (1996) General hierarchical approach in absolute phase measurement. In: Proc SPIE vol 2860
- 108. Osten W, Kalms M, Jüptner, Tober G, Bisle W, Scherling D (2000) Shearography system for the testing of large scale aircraft components taking into account noncooperative surfaces. In: Proc SPIE vol 4101B
- 109. Osten W, Seebacher S, Jüptner W (2001) Application of digital holography for the inspection of microcomponents. In: Proc SPIE vol 4400, pp 1-15
- 110. Osten W, Seebacher S, Baumbach T, Jüptner W (2001) Absolute shape control of microcomponents using digital holography and multiwavelength contouring. In: Proc SPIE vol 4275, pp 71-84

- 111. Osten W, Baumbach T, Seebacher S, Jüptner W (2001) Remote shape control by comparative digital holography. In: Jüptner W, Osten W (eds) Proc 4th International Workshop on Automatic Processing of Fringe Patterns. Akademie, Berlin, pp 373-382
- 112. Osten W, Baumbach T, Jüptner W (2002) Comparative digital holography. Opt Lett 27(20):1764-1766
- 113. Ostrovsky YI, Butosov MM, Ostrovskaja GV (1980) Interferometry by Holography. Springer, New York
- 114. Owen R B, Zozulya A (2000) In-line digital holographic sensor for monitoring and characterizing marine particulates. Opt Eng 39(8):2187-2197
- 115. Owen RB, Zozulya A, Benoit MR, Klaus DM (2002) Microgravity materials and life sciences research applications of digital holography. Appl Opt 41(19): 3927-3935
- 116. Papp Z, Janos K (2001) Digital holography by two reference beams. In: Proc SPIE vol 4416, pp 112-115
- 117. Pedrini G, Tiziani H (2002) Short-coherence digital microscopy by use of a holographic imaging system. Appl Opt 41(22):4489-4496
- 118. Pedrini G, Zou YL, Tiziani H (1995) Digital double-pulsed holographic interferometry for vibration analysis. J Mod Opt 42(2):367-374
- 119. Pedrini G, Zou Y, Tiziani H (1997) Simultaneous quantitative evaluation of in-plane and out-of-plane deformations by use of a multidirectional spatial carrier. Appl Opt 36(4):786
- 120. Pedrini G, Fröning P, Fessler H, Tiziani HJ (1998) In-line digital holographic interferometry. Appl Opt 37(26):6262-6269
- 121. Pedrini G, Schedin S, Tiziani H (1999) Lensless digital holographic interferometry for the measurement of large objects. Optics Communications 171:29-36
- 122. Pedrini G, Fröning P, Tiziani H, Santoyo F (1999) Shape measurement of microscopic structures using digital holograms. Opt Commun 164:257-268
- 123. Pedrini G, Schedin S, Tiziani H (2000) Spatial filtering in digital holographic microscopy. J Mod Opt 47(8):1447-1454
- 124. Pedrini G, Titiani H J, Alexeenko I (2002) Digital-holographic interferometry with an image-intensifier system. Appl Opt 41(4):648
- 125. Pettersson S-G, Bergstrom H, Abramson N (1989) Light-in-flight recording 6: Experiment with view-time expansion using a skew reference wave. Appl Opt 28:766-770
- 126. Pomarico J, Schnars U, Hartmann HJ, Jüptner W (1996) Digital recording and numerical reconstruction of holograms: A new method for displaying Light-in-flight. Applied Optics 34(35):8095-8099
- 127. Powell RL, Stetson KA (1965) Interferometric Vibration Analysis by Wavefront reconstructions. J Opt Soc Amer 55:1593-1598
- 128. Schnars U (1994) Direct phase determination in hologram interferometry with use of digitally recorded holograms. Journ Opt Soc Am A 11(7):2011-2015, reprinted (1997) In: Hinsch K, Sirohi R (eds). SPIE Milestone Series MS 144, pp 661 665
- 129. Schnars U (1994) Digitale Aufzeichnung und mathematische Rekonstruktion von Hologrammen in der Interferometrie VDI-Fortschritt-Berichte series 8 no 378 VDI, Düsseldorf
- 130. Schnars U, Jüptner W (1993) Principles of direct holography for interferometry. In: Jüptner W, Osten W (eds) FRINGE 93 Proc. 2nd International Workshop on Automatic Processing of Fringe Patterns. Akademie, Berlin, pp 115-120
- 131. Schnars U, Jüptner W (1994) Direct recording of holograms by a CCD-target and numerical reconstruction. Applied Optics 33(2):179-181

- 132. Schnars U, Jüptner W (1994) Digital reconstruction of holograms in hologram interferometry and shearography. Applied Optics 33(20):4373-4377, reprinted (1997) In: Hinsch K, Sirohi R (eds). SPIE Milestone Series MS 144, pp 656 660
- 133. Schnars U, Jüptner W (1995) Digitale Holografie. In: annual conference of the Deutsche Gesellschaft für angewandte Optik. Handout, Binz
- 134. Schnars U, Geldmacher J, Hartmann HJ, Jüptner W (1995) Mit digitaler Holografie den Stoßwellen auf der Spur. F&M 103(6):338-341
- 135. Schnars U, Kreis T, Jüptner W (1995) CCD recording and numerical reconstruction of holograms and holographic interferograms. In: Proc SPIE vol 2544, pp 57-63
- 136. Schnars U, Kreis T, Jüptner W (1995) Numerische Rekonstruktion von Hologrammen in der interferometrischen Messtechnik, In: Waidelich W (ed) Proc LASER 95. Springer, Heidelberg
- 137. Schnars U, Osten W, Jüptner W, Sommer K (1995) Advances of Digital Holography for Experiment Diagnostics in Space. In: Proc 46th International Astronautical Congress Oslo, paper no. IAF-95-J.5.01
- 138. Schnars U, Hartmann HJ, Jüptner W (1995) Digital recording and numerical reconstruction of holograms for nondestructive testing. In: Proc SPIE vol 2545, pp 250-253
- 139. Schnars U, Kreis T, Jüptner W (1996) Digital recording and numerical reconstruction of holograms: Reduction of the spatial frequency spectrum. Optical Engineering 35(4):977-982
- 140. Schreier D (1984) Synthetische Holografie. VCH, Weinheim
- 141. Schwomma O (1972) austrian patent 298,830
- 142. Seebacher S (2001) Anwendung der digitalen Holografie bei der 3D-Form- und Verformungsmessung an Komponenten der Mikrosystemtechnik. University Bremen publishing house, Bremen
- 143. Seebacher S, Osten W, Jüptner W (1998) Measuring shape and deformation of small objects using digital holography. In: Proc SPIE vol 3479, pp 104-115
- 144. Seebacher S, Baumbach T, Osten W, Jüptner W (2000) Combined 3D-shape and deformation analysis of small objects using coherent optical techniques on the basis of digital holography. In: Proc SPIE vol 4101B, pp 520-531
- 145. Sjoedahl M, Benckert L R (1993) Electronic speckle photography: analysis of an algorithm giving the displacement with subpixel accuracy. Appl Opt 32(13):2278-2284
- 146. Skarman B, Becker J, Wozniak K (1996) Simultaneous 3D-PIV and temperature measurements using a new CCD-based holographic interferometer. Flow Meas Instrum 7(1):1-6
- 147. Sollid J E (1969) Holographic interferometry applied to measurements of small static displacements of diffusely reflecting surfaces. Appl Opt 8:1587-1595
- 148. Stadelmaier A, Massig JH (2000) Compensation of lens aberrations in digital holography. Optics Letters 25(22):1630-1632
- 149. Steinbichler H (2004) Shearography NDT. Product information, Steinbichler, Neubeuern
- 150. Steinchen W, Yang L (2003) Digital Shearography. SPIE press
- 151. Stetson KA, Powell RL (1965) Interferometric hologram evaluation and real-time vibration analysis of diffuse objects. J Opt Soc Amer 55:1694-1695
- 152. Stetson KA, Brohinsky R (1985) Electrooptic holography and its application to hologram interferometry. Appl Opt 24(21):3631-3637
- 153. Stetson KA, Brohinsky R (1987) Electrooptic holography system for vibration analysis and nondestructive testing. Opt Eng 26(12):1234-1239
- 154. Synnergren P, Sjödahl M (2000) Mechanical testing using digital speckle photography. Proc SPIE vol 4101B

- 155. Tajahuerce E, Javidi B (2000) Encrypting three-dimensional information with digital holography. Appl Opt 39(35):6595-6601
- 156. Tajahuerce E, Matoba O, Verral S, Javidi B (2000) Optoelectronic information encryption with phase-shifting interferometry. Appl Opt 39(14):2313-2320
- 157. Tajahuerce E, Matoba O, Javidi B (2001) Shift-invariant three-dimensional object recognition by means of digital holography. Appl Opt 40(23):3877-3886
- 158. Takaki Y, Ohzu H (1999) Fast numerical reconstruction technique for high-resolution hybrid holographic microscopy. Appl Opt 38(11):2204-2211
- 159. Takaki Y, Ohzu H (2000) Hybrid holographic microscopy: visualization of threedimensional object information by use of viewing angles. Appl Opt 39(29):5302-5308
- 160. Takaki Y, Kawai H, Ohzu H (1999) Hybrid holographic microscopy free of conjugate and zero-order images. Appl Opt 38(23):4990-4996
- 161. Trolinger JD (1991) Particle and Flow Field Holography Combustion Measurements. Chigier N (ed). Hemisphere Publishing Corporation, pp 51-89
- 162. Wagner C, Seebacher S, Osten W, Jüptner W (1999) Digital recording and numerical reconstruction of lensless Fourier holograms in optical metrology. Appl Opt 38(22):4812-4820
- 163. Wagner C, Osten W, Seebacher S (2000) Direct shape measurement by digital wavefront reconstruction and multiwavelength contouring. Opt Eng 39(1):79-85
- 164. Winnacker A (1984) Physik von Laser und Maser. BI-Verlag, Mannheim
- 165. Wozniak K, Skarman B (1994) Digital Holography in Flow Visualization. Final Report for ESA/ ESTEC purchase order 142722, Noordwijk
- 166. Xiao X, Puri I (2002) Digital recording and numerical reconstruction of holograms: an optical diagnostic for combustion. Appl Opt 41(19):3890-3898
- 167. Xu L, Miao J, Asundi A (2000) Properties of digital holography based on in-line configuration. Opt Eng 39(12):3214-3219
- 168. Xu L, Peng X, Miao J, Asundi A K (2001) Studies of digital microscopic holography with applications to microstructure testing. Appl Opt 40(28):5046-5051
- 169. Xu L, Peng X, Asundi A, Miao J, (2001) Hybrid holographic microscope for interferometric measurement of microstructures. Opt Eng 40(11):2533-2539
- 170. Yamaguchi I, Saito H (1969) Application of holographic interferometry to the measurement of poisson's ratio. Jap Journal of Appl Phys 8:768-771
- 171. Yamaguchi I , Zhang T (1997) Phase-shifting digital holography. Optics Letters 22(16):1268-1270
- 172. Yamaguchi I, Kato J, Ohta S, Mizuno J (2001) Image formation in phase-shifting digital holography and applications to microscopy. Appl Opt 40(34):6177-6186
- 173. Yamaguchi I, Inomoto O, Kato J (2001) Surface shape measurement by phase shifting digital holography. In: Jüptner W, Osten W (eds) Proc 4th International Workshop on Automatic Processing of Fringe Patterns. Akademie, Berlin, pp 365-372
- 174. Yamaguchi I, Matsumura T, Kato J (2002) Phase-shifting color digital holography. Opt Lett 27(13):1108-1110
- 175. Yang S, Xie X, Thuo Y, Jia C (1999) Reconstruction of near-field in-line holograms. Optics Communications 159:29-31
- 176. Yaroslavskii LP, Merzlyakov NS (1980) Methods of digital holography. Consultants Bureau, New York
- 177. Yu L, Cai L (2001) Iterative algorithm with a constraint condition for numerical reconstruction of a three-dimensional object from its hologram. J Opt Soc Am A 18(5):1033-1045
- 178. Zhang T, Yamaguchi I (1998) Three-dimensional microscopy with phase-shifting digital holography. Optics Letters 23(15):1221-1223

- 179. Zhang T, Yamaguchi I (1998) 3D microscopy with phase-shifting digital holography. In: Proc SPIE vol 3479:152-159
- 180. Zou Y, Pedrini G, Tiziani H (1996) Surface contouring in a video frame by changing the wavelength of a diode laser. Opt Eng 35(4):1074-1079

Index

aberrations 136, 147 aircraft industry 73, 85 amplitude 6, 21 amplitude transmission 21 amplitude transmittance 22 angular frequency 6 aperture 48, 125 aperture size 123 Ar-Ion laser 101 autocorrelation function 14, 104 Autocorrelation theorem 142

backprojection approach 109

Charged Coupled Device 41, 61 Coherence 10 coherence distance 13, 14 coherence length 11, 104, 101 coherence time 11 Comparative Digital Holography 3, 116, 117 Comparative interferometry 117 complex amplitude 7, 16, 21, 41, 59 complex degree of coherence 15 Computer Generated Holography 1 conjugate object wave 23 conjugate reference 43 constructive interference 8 Contouring 31 contrast 18, 65, 69 convolution approach 53, 52, 95 convolution theorem 52, 142 correction factor 44, 53, 148 cross correlation 15 cross correlation function 133

DC term 56, 67 decorrelation 116 demodulation 38 depth of field 95 depth of focus 1, 21, 109 destructive interference 9 Differentiation 143 diffraction 5, 15, 41, 107, 109 diffraction efficiency 115 diffuser 93 Digital Fourier Holography 55 digital hologram 48, 96, 117 Digital Holographic Interferometry 42, 71. 134 Digital Holographic Microscopy 3, 95 Digital Holography 2, 41, 71, 104, 107, Digital Mirror Device 3, 115 Digital Speckle Pattern Interferometry Digital Speckle Photography 133 diode laser 12 discrete Fourier transform 143 displacement vector 29, 30, 73, 127 displacement vector field 76 double-exposure holography 26 dve laser 101 dynamic evaluation 86 dynamic range 66

eddy current 85 electrical field 5 electromagnetic wave 5 Electronic Speckle Pattern Interferometry 2, 125, 134 Electro-Optic Holography 129 encrypting of information 3 Endoscopic Digital Holography 111

fast fourier transform 144 flaws 85 focal distance 43 Fourier hologram 1 Fourier holography 68 Fourier integral theorem 143 Fourier transform 52, 55 Fourier Transform Method 38 Fourier transformation 46, 141 frame grabber 112 Frame-transfer architecture 63 Frequency 6 Fresnel approximation 45, 95 Fresnel Approximation 44 Fresnel hologram 2 Fresnel transform 48, 71 Fresnel transformation 45 Fresnel-Kirchhoff integral 16, 17, 41, 52, 60, 98 fringe 9 Full-frame architecture 64

glass fibres 112 grating 106

heterodyne 38 heterodyne techniques 3 hierarchical phase unwrapping 90 high pass filter 56 hologram 1, 21, 41 holographic interferogram 26, 71, 114, Holographic Interferometry 1, 26, 36, 134 holography 1, 21, 48 Huygens' principle 15, 17 Huygens' principle 16

illumination direction 76 image plane holograms 125, 135 **Imaging Equations 23** impact loading 73 impulse response function 52 inclination factor 17, 53 incoherent light 13 information encryption 120 in-line 96, 107, 109 in-plane 127, 133 intensity 6, 18, 21, 45 interference 8 interference pattern 1 interference phase 28, 30, 38, 71, 72, 73, 85, 128, 135, 136 interferogram 71 interferometer 102 Interline-transfer architecture 63

inversion 60

Laplace operator 5 laser 12, 21, 101 laser diode 112 laser-doppler-anemometry 107 lateral magnification 25 LED 98 lens transmission factor 53 Lens Transmission Function 145 light-in-flight holography 3, 102 linear polarized light 5 Linearity theorem 142 longitudinal magnification 25

Mach-Zehnder interferometer 92, 98 magnetic field 5 magnification 25, 54, 95 Maxwell equations 5, 7 Michelson interferometer 10, 106, 132 microscopy 95 monochromatic 8, 12 multiwavelength contouring 89

Non-Destructive Testing 85 numerical focussing 136 Numerical hologram reconstruction 1 numerical reconstruction 104, 44, 107

object wave 1, 23 objective speckle pattern 19 observation direction 76, 87 off-axis 48 Optical fibres 87 optical path difference 101 Optical Reconstruction 114 orthoscopic image 26 out-of plane 127 out-of-plane deformation 30

Partially coherent light 13 Particle Distribution 107 particle image velocimetry 107 particle tracking 3 penetrant testing 85 perspective 1, 21 phase 6, 21, 45 phase aberrations 44 phase object 35 phase shift 122

phase shift angle 128 phase shifting 98, 128, 135 phase shifting digital holography 3, 59 Phase shifting Holographic Interferometry 37 phase unwrapping 38, 73 phase-doppler-anemometry 107 phase-shift 113 phase-shifting Digital Holography 120 photo effect 61 photographic emulsions 64, 65 photographic plate 21 photographic plates 135 photons 5 photorefractive crystals 135 piezo electric transducer 98 piezoelectric transducer 59 piezoelectric translator 37 pixel 61 pixel distance 48, 53 plane reference wave 43 plane wave 6 Poisson ratio 76, 78 printer 116 pseudoscopic image 26

quantum optics 5

real image 23, 41, 114 real time technique 26 reconstruction 41 reference wave 1, 21, 41, 105 refocusing 98 refractive index 35, 92 resolution 48, 53, 64, 95, 135 rigid body motions 82, 132 ruby laser 74, 109

sensitivity 65
sensitivity vector 30, 76
Shape Measurement 86
shearogram 130
shearography 2, 85, 129
Shift theorem 142
shutter 62
Similarity theorem 142
skeletonizing 38
Spatial coherence 13
spatial frequency 9, 19, 64, 68, 73
spatial light modulator 114
Spatial Separation 57

speckle 125 speckle interferogram 125, 130 speckle pattern 133 speckle photography 2, 133 speckle size 19, 48, 125, 127 speckles 18 spectral width 12 speed of light 5, 105 spherical reference wave 55, 96 stability 69 strains 133 subjective speckle pattern 19 subpixel evaluation 134 superposition 8 Suppression 56 Synthetic Apertures 122 synthetic wavelength 33, 88, 90

telecentric imaging system 34 Temporal Coherence 10 temporal phase unwrapping 113 thermal expansion coefficient 76, 82 thermoplastic films 135 tilted reference wave 58 tomography 106, 109 torsions 78 transient deformations 73 transparent media 92 TV-holography 125 twin image 1, 93 two-illumination-point method 31 Two-Illumination-Point method 86 two-wavelength contouring 119 two-wavelength method 31,88

Ultrasonic testing 85 unwrapped phase 75

vacuum chamber 82 vacuum permittivity 7 vibration isolation 129, 133 vibrations 69, 113 virtual image 21, 23, 41, 57, 114 virtual lens 44 visibility 12

wave equation 5 wave number 6 wave vector 6 wavefront 9, 15, 103, 105 wavelength 6 x-ray 85

Young's interferometer 13, 14, 15

Young's modulus 76, 80

zero order 48, 56, 113 zero padding 66