ECE 182

Homework 1

1) The electric field of a combination of two monochromatic plane waves is given by

$$\mathbf{E} = (e^{-j\beta z} + \Gamma e^{j\beta z})\hat{\mathbf{x}}$$

where $\beta = \omega_0\sqrt{\varepsilon_0\mu_0}$ and Γ is a known constant.

   a) Find the magnetic field $\mathbf{H}$

   b) Find the optical intensity (time average power flow density) using the monochromatic (frequency domain) Poynting vector $\mathbf{S}$

2)

Dielectric Media. Identify the media described by the following equations, regarding linearity, dispersiveness, spatial dispersiveness, and homogeneity.
(a) $\mathscr{P} = \epsilon_0\chi\mathscr{E} - a\nabla \times \mathscr{E}$,
(b) $\mathscr{P} + a\mathscr{P}^2 = \epsilon_0\mathscr{E}$,
(c) $a_1\partial^2\mathscr{P}/\partial t^2 + a_2\partial\mathscr{P}/\partial t + \mathscr{P} = \epsilon_0\chi\mathscr{E}$,
(d) $\mathscr{P} = \epsilon_0[a_1 + a_2\exp[-(x^2 + y^2)]]\mathscr{E}$,
where χ, a, a_1, and a_2 are constants.

3) Starting with the paraxial (parabolic) approximation for a spherical wave (valid for $\sqrt{x^2 + y^2} \ll z$), derive an expression for Gaussian beam by replacing the real coordinate z with a complex coordinate $z + jz_0$. Identify (give an expression for) the beam width as a function of z and Fresnel distance.

4) A nonabsorbing medium of refractive index n_0 contains impurities characterized by susceptibility $\chi = \chi' + j\chi''$, where $\chi', \chi'' \ll 1$. Show that the refractive index and absorption coefficient are given approximately by $n \approx n_0 + \chi'/2n_0$, $\alpha \approx -k_0\,\chi''/n_0$

5) **Group Velocity in a Resonant Medium.** Determine as expression for the group velocity v_g of a resonant medium with refractive index given by

$$n(\nu) \approx n_0 + \frac{\chi'(\nu)}{2n_0}$$

$$\chi''(\nu) = -\chi_0 \frac{\nu_0 \Delta\nu}{4} \frac{1}{(\nu_0 - \nu)^2 + (\Delta\nu/2)^2}$$

$$\chi'(\nu) = 2\frac{\nu - \nu_0}{\Delta\nu}\chi''(\nu).$$

Plot the group velocity v_g as a function of the frequency ν

6) Pulse broadening in an Optical Fiber. A Gaussian pulse of width $\tau_0 = 100\,ps$ travels a distance of 1 km through an optical fiber made of fused silica with the characteristics shown in the figure below. Estimate the time delay τ_d and the width of the received pulse if the wavelength is (a) $0.8\,\mu m$; (b) $1.55\,\mu m$.