Logistic Regression

November 5, 2020

1 Logistic Regression and Gradient Descent

Implementation of Logistic Regression using NumPy. And classification of IRIS data set using One vs All approach.

[3]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import load_iris
```

1.1 Task 1

- Implement the fixed learning rate stochastic gradient descent algorithm.

[4]:
```
# Calculate the error
def calculate_error(X, W, Y):
    error = 0
    for i, x in enumerate(X):
        error = np.log(1 + np.exp(-Y[i]*np.dot(2*W, X[i])))
    return error/len(X)

# Logistic Regression
def log_regression(X_in, Y_in, X_out, Y_out, T=2000, learning_rate=0.001):
    W = np.zeros((X_in.shape[1],))
    errors_in = []
    errors_out = []

    for i in range(T):
        n = np.random.randint(0,len(X_in))
        x_n = X_in[n]
        E_dev = (-Y_in[n] * x_n)/(1 + np.exp((Y_in[n] * np.dot(W, x_n))))
        W = W - learning_rate * E_dev
        # Calculate insample error
        errors_in.append(calculate_error(X_in, W, Y_in))
        # Calculate out of sample error
        errors_out.append(calculate_error(X_out, W, Y_out))
    return W, errors_in, errors_out
```

1.2 Task 2

- Prepare the data

1.2.1 IRIS Data Set

- Load IRIS data set of size 150
- Split the data into training (80% = 120 examples) and test data (20% = 30 examples) sets

[61]:
```
# Load the iris data
iris = load_iris()
X_data = iris.data
Y_data = iris.target

# Shuffle the data
rng = np.random.RandomState(0)
permutation = rng.permutation(len(X_data))
X_data, Y_data = X_data[permutation], Y_data[permutation]

# Split the data to train and test
X_train, X_test, Y_train, Y_test = X_data[:120], X_data[120:], Y_data[:120],
 ↪Y_data[120:]
```

1.2.2 One vs All

- Generate Dk for each class where Dk = {(xn, y'n = 2[yn = k]-1)} for n in N

[6]:
```
# Generate labels for OVA decomposition
def generate_D_Y(Y, k):
    Y_copy = np.copy(Y)
    for i, y in enumerate(Y_copy):
        Y_copy[i] = 2 * (int(y) == k) - 1
    return Y_copy
```

[62]:
```
D0_Y_train = generate_D_Y(Y_train, 0)
D1_Y_train = generate_D_Y(Y_train, 1)
D2_Y_train = generate_D_Y(Y_train, 2)

D0_Y_test = generate_D_Y(Y_test, 0)
D1_Y_test = generate_D_Y(Y_test, 1)
D2_Y_test = generate_D_Y(Y_test, 2)
```

1.3 Task 3

- Plot E_in() and E_out() as a function of t, and briefly state your findings.

Calculate the errors and weights generated by running the 'log_regression()' function on each class data set

[63]:
```
W0, errors_in0, errors_out0 = log_regression(X_train, D0_Y_train, X_test,␣
 ↪D0_Y_test)
W1, errors_in1, errors_out1 = log_regression(X_train, D1_Y_train, X_test,␣
 ↪D1_Y_test)
W2, errors_in2, errors_out2 = log_regression(X_train, D2_Y_train, X_test,␣
 ↪D2_Y_test)
```

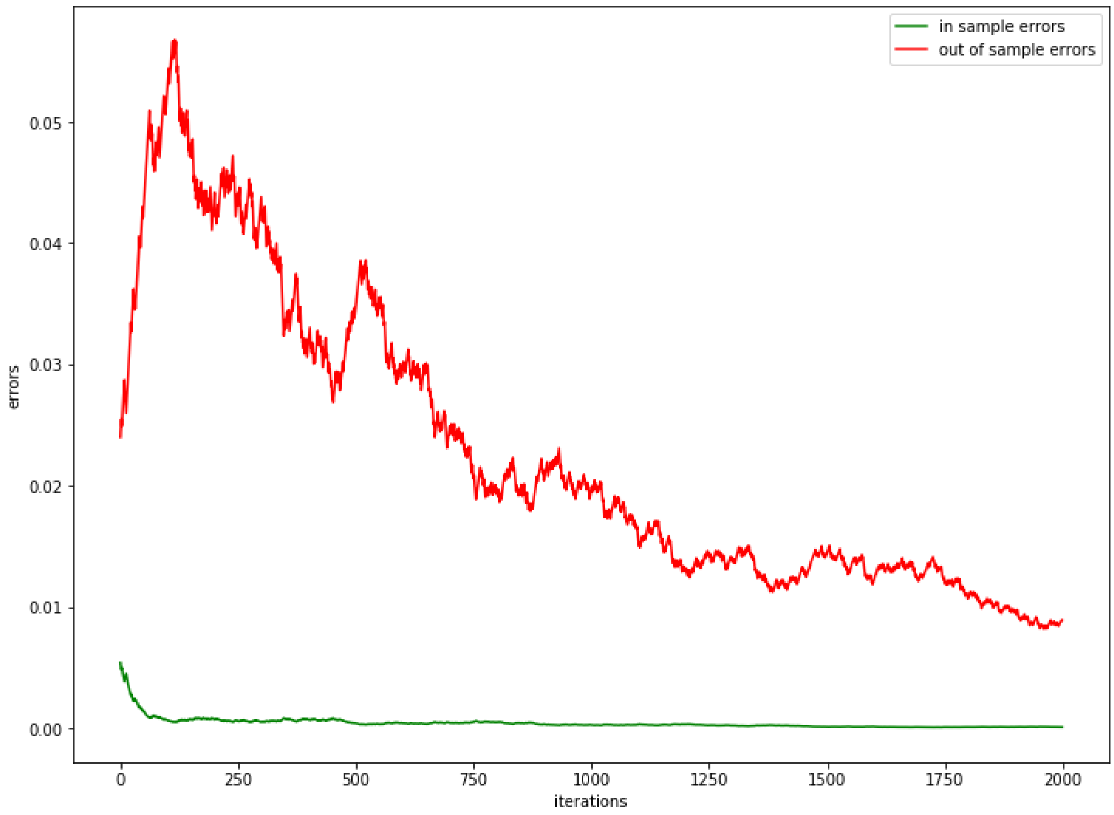
1.3.1 Errors graphs of each class Dk

[69]:
```
# Plot the errors
def plot_errors(errors_in, errors_out):
    plt.rcParams["figure.figsize"] = (12,9)

    # Plot the target function
    plt.plot(range(2000), errors_in, 'g', label = 'in sample errors')
    plt.plot(range(2000), errors_out, 'r', label = 'out of sample errors')

    plt.xlabel('iterations')
    plt.ylabel('errors')
    plt.legend()
    plt.show()
```

1.3.2 Class 0 (D0)

- E_in and E_out as a function of t, while training on the D0

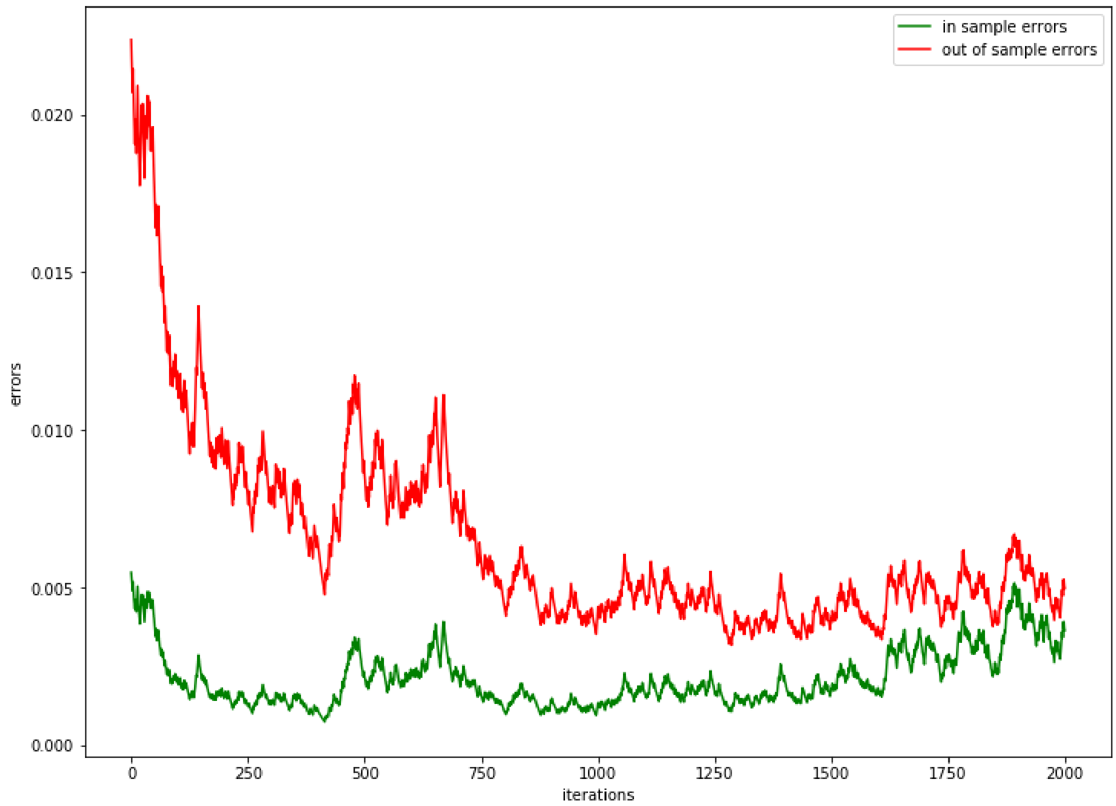
[70]:
```
# Plot the error function for D0
plot_errors(errors_in0, errors_out0)
```



1.3.3 Class 1 (D1)

- E_in and E_out as a function of t, while training on the D1

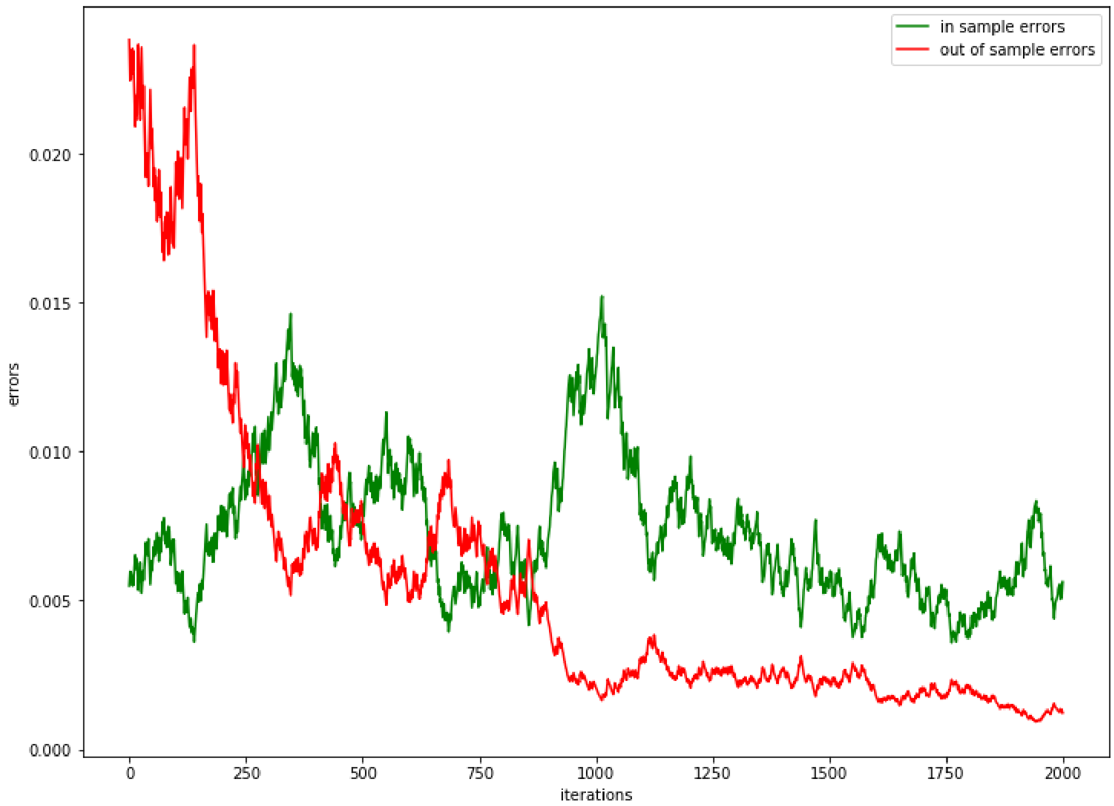
[71]:
```
# Plot the error function for D1
plot_errors(errors_in1, errors_out1)
```



1.3.4 Class 2 (D2)

- E_in and E_out as a function of t, while training on the D2

[72]:
```python
# Plot the error function for D2
plot_errors(errors_in2, errors_out2)
```



1.3.5 Accuracy of the Model

- Claculate the Y_pred using g(x) = argmax(sigmoid(dot(Wk.T,x)))
- Calculate the accuracy of the model on the original test data set by comparison of Y_pred and actual labels

[58]:
```
# Calculate the probability that x belongs to the class
def sigmoid(x):
    return 1/(1+np.exp(-x))

# Choose the class with the most probability
def predict(p):
    Y_pred = []
    for i in range(len(p[0])):
        Y_pred.append(np.argmax(p[:,i]))
    return Y_pred

# Calculate the accuracy of Y_pred
def calculate_accuracy(Y_pred, Y):
    correct = 0
    for i in range(len(Y)):
        if Y_pred[i] == Y[i]:
```

```
            correct += 1
    return correct/len(Y)
```

[67]:
```
prob0 = sigmoid(np.dot(X_test, W0))
prob1 = sigmoid(np.dot(X_test, W1))
prob2 = sigmoid(np.dot(X_test, W2))
probs = np.array([prob0, prob1, prob2])

Y_pred = predict(probs)
print('accuracy of the model:', calculate_accuracy(Y_pred, Y_test))
```

```
accuracy of the model: 0.7
```

[68]:
```
print('predictions:', np.array(Y_pred))
print('labels:     ', Y_test)
```

```
predictions: [0 2 0 0 2 0 2 2 2 1 2 2 2 2 0 1 2 2 0 2 2 2 2 0 0 0 2 2 2 0]
labels:      [0 2 0 0 2 0 2 1 1 1 2 2 1 1 0 1 2 2 0 1 1 1 1 0 0 0 2 1 2 0]
```

1.3.6 Summary

The error graphs show that the error is fluctuating a lot but generally decreases as t increases. That means that the weights are updated correctly. The accuracy of the model is 0.7. It can be seen that class 1 is highly misclassified in class 2. That means that the model calculates the probability that the actual example from class 1 belongs to class 1 is less than it belongs to class 2. The reason for this might be that these two classes have similar features and that the number of examples is very limited.