Before we get to those two properties, we need to clarify a few terms first :

- A **stationary distribution** of a Markov chain is a probability distribution that remains unchanged in the Markov chain as time progresses. Typically, it is represented as a row vector **π** whose entries are probabilities summing to 1, and given transition matrix **P**, it satisfies $\pi = \pi \mathbf{P}$

- The **limiting distribution** of a Markov chain seeks to describe how the process behaves a long time after . For it to exist, the following limit must exist for any states i and j : $L_{i,j} = \lim_{n\to\infty} \mathbb{P}(X_n = j \mid X_0 = i).$

- The first property we want to understand is described as follows :

  - Given a transition matrix **P** in which no element is equal to 0, no matter how we change the initial state **π0** as long as **π0** is not consist of all zero elements (such as [0, 0, 0]), the **limiting distribution** will not be affected.

  - Exercise 1:

    1. Question 1 : For the given initial state and the given transition matrix **P** in Exercise.java, after how many times will the distribution reach the limiting distribution? Please use the Exercise1.java to plot times vs each of the element in each step of distribution using Function class.

2. Question 2 : Is it true that the **limiting distribution** will not be affected by the initial state? Please use the Exercise1.java to plot out the result of each limiting distribution of each initial states vs the number of that initial state.

Pseudocode :

```
public class MarkovChain {

    public static void main (String[] argv)
    {
        double[][] transition = {
                       {0.9, 0.075, 0.025},
                       {0.15, 0.8, 0.05},
                       {0.25, 0.25, 0.5}};

        double[] prev = {0.1, 0.2, 0.7};
        double[] next;
        int num;    // Number of times when it reaches limiting
distribution.
        Function f1 = new Function("f1");
        Function f2 = new Function("f2");
        Function f3 = new Function("f3");


        for (int cnt = 0; cnt < 5; cnt++) {
          //  Set your initial state of the distribution

          for (int k = 0; k < num; k++) {
               // Calculate the next state using previous state
and transition matrix
          }

        }


        // Display result.
        f.show ();
```

```
}
```

- The second property we want to test is that :
    - For a Markov process which possesses a stationary distribution, it satisfies the equation that $\pi_i P_{ij} = \pi_j P_{ji}$, this equation is called *detailed balance equation*.
    - Exercise 2 :
        - Does the detailed balance equation still stand given the initial state and the transition matrix we've had tried before? Please use the Exercise2.java to test this equation for each initial state and print out those results.