Assignment 04

Simulation

Since there are some bugs when directly modifing `Queue.java`, I use a different method to simulate (asynchronously). See code and annotation in file `ThreeServerQueue.java`.

Inspection

1

File `ThreeServerQueue.java`:

```
public class ThreeServerQueue {

    public static void main(String[] args) {
        ThreeServerQueue queue = new ThreeServerQueue(1, 0.75, 1);
        if (queue.simulate(1000)) {
            System.out.println(queue.toString());
        }
    }

    ...

}
```

Result:

```
Parameter:
  server number (K): 1
  arrival rate  (λ): 0.75
  service rate  (μ): 1.0
  max client number: 1000000
Results:
  total time:     1333151.2262177207
  arrival time:   1332122.138188411
    - per client: 1.332122138188411  #1
    - rate        0.7506819167196839
  wait time:      2999997.1044096174
    - per client: 2.9999971044096174 #2
  service time:   999175.7426322775
    - per client: 0.9991757426322776 #3
```

```
    - rate:       1.0008249373283933
  system time:    3999172.8470418686
    - per client: 3.999172847041869  #4
  free time:      333975.4835854431
    - per server: 333975.4835854431
  free counter:   249753.0
    - per server: 249753.0
```

2

File `Queue.java`:

```
public class Queue {

    ...

    public static void main(String[] argv) {
        Queue queue = new Queue();
        queue.simulate(1000000);
        System.out.println(queue);
        queue.M.display();
        queue.D.display();
    }

}
```

Result:

```
Simulation results:
  numArrivals:           1000000
  numDepartures:         999999
  avg Wait:              2.9521488083301035 #2
  avg System Time (d)    3.9509479782228114 #4
  avg Interarrival Time: 1.333542668225313  #1
  interarrival Rate (λ): 0.7498822676073846
  avg Service Time:      0.9987995459747511 #3
  service Rate (μ):      1.001201896847157
  Custom number:         3957824.0
  avg Custom Number (m): 3.957824
  probability of Free:   0.25118
```

3

Compare pairs of values which end with `#1`, `#2`, `#3`, `#4` - they are almost equal.

File `ThreeServerQueue.java` is OK.

Question

1

> For the case $K = 2$, plot the system time vs. increasing arrival rate λ.

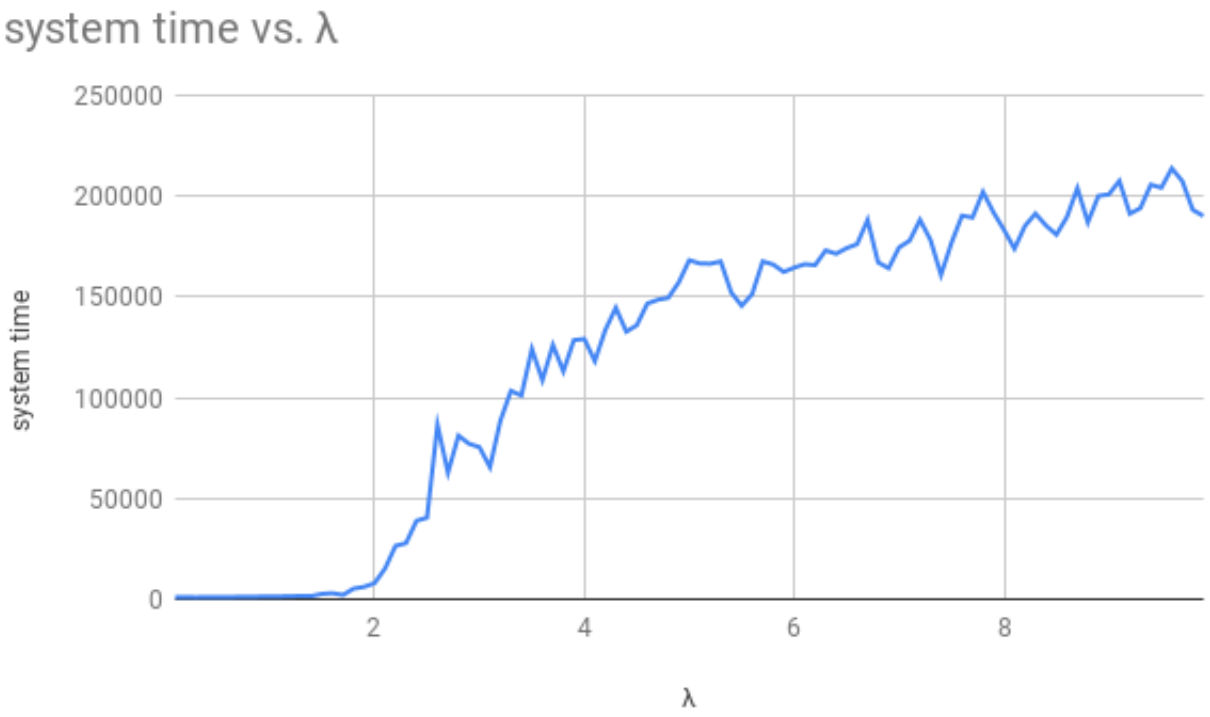
File `ThreeServerQueue.java`:

```
public class ThreeServerQueue {

    public static void main(String[] args) {
        for (double i = 0.1; i < 10; i += 0.1) {
            ThreeServerQueue queue = new ThreeServerQueue(2, i, 1);
            if (queue.simulate(1000)) {
                System.out.println(String.format("%.1f", i)
                        + "\t" + queue.systemTime
                        + "\t" + queue.systemTimePerClient);
            }
        }
    }

    ...

}
```

Result:

```
Parameter:
  server number (K): 2
  service rate  (μ): 1.0
  max client number: 1000
```

(Get full data from *table 1* below.)



2

> What is the value of the system time when $\lambda = 1.5$?

Table 1 shows that: the system time is 2665.77 when $\lambda = 1.5$.

3

> What value of λ would cause the queue to become unstable?
>
> What value of λ would cause the queue to become unstable when $K = 3$?
>
> What did you do to assess the accuracy of your estimates?

Define and calculate 4 variables:

- **free time** - the total idle time of all servers.
- **free time per server** - "free time" averaged by each server.
- **free counter** - the counter of the event "a server is idle but the queue is empty".
- **free counter per server** - "free counter" averaged by each server.

When "free counter per server" approaches to its minimum value, the queue become **"stable"**. In other words, event "a server is idle but the queue is empty" rarely happens.

The following results show that:

- $\lambda < 2.2$ cause the queue to become unstable when $K = 2$.
- $\lambda < 3.4$ cause the queue to become unstable when $K = 3$.

K=2

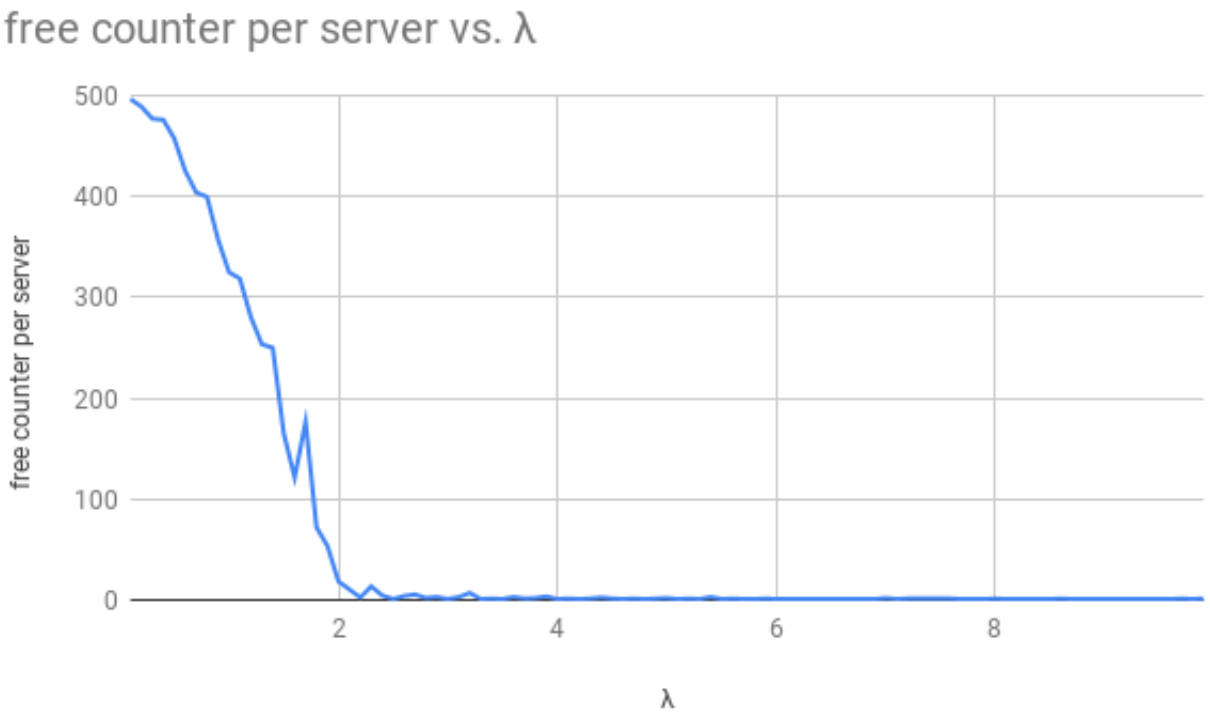
File `ThreeServerQueue.java`:

```
public class ThreeServerQueue {

    public static void main(String[] args) {
        for (double i = 0.1; i < 10; i += 0.1) {
            ThreeServerQueue queue = new ThreeServerQueue(2, i, 1);
            if (queue.simulate(1000)) {
                System.out.println(String.format("%.1f", i)
                        + "\t" + queue.freeTime
                        + "\t" + queue.freeTimePerServer
                        + "\t" + queue.freeCounter
                        + "\t" + queue.freeCounterPerServer);
            }
        }
    }

    ...

}
```

Result:

```
Parameter:
  server number (K): 2
  service rate  (μ): 1.0
  max client number: 1000
```

(Get full data from *table 2* below.)



K=3

File `ThreeServerQueue.java`:

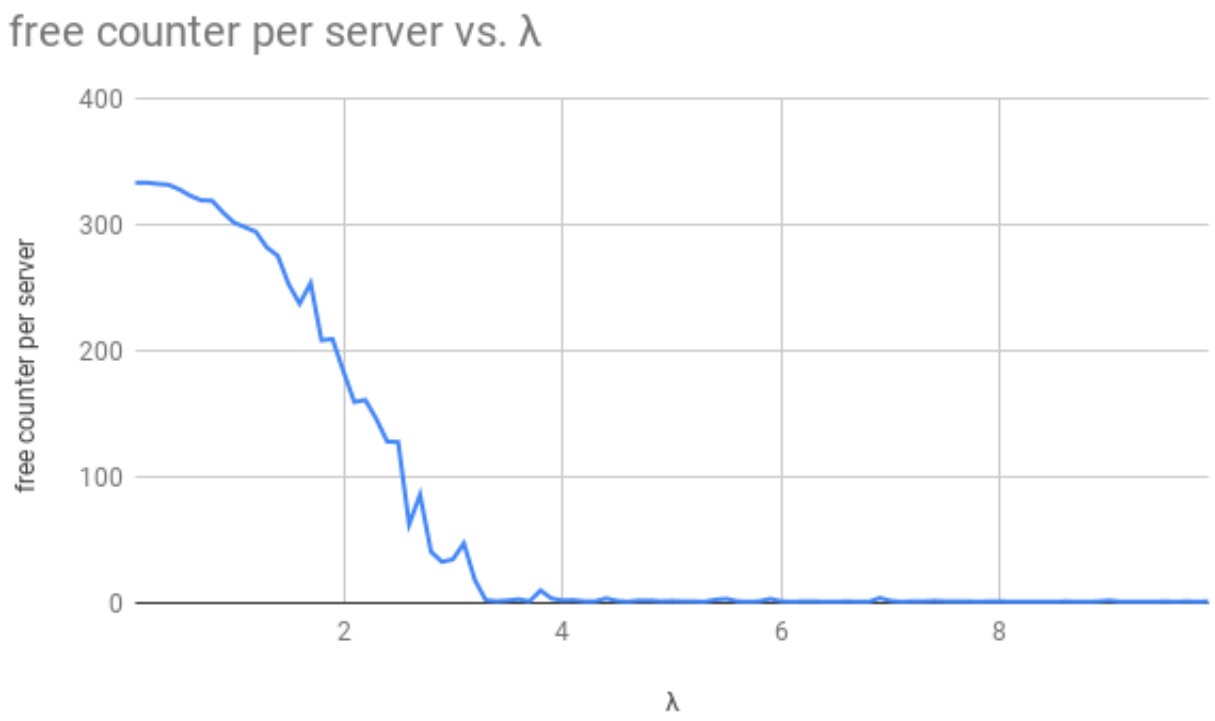
```
public class ThreeServerQueue {

    public static void main(String[] args) {
        for (double i = 0.1; i < 10; i += 0.1) {
            ThreeServerQueue queue = new ThreeServerQueue(3, i, 1);
            if (queue.simulate(1000)) {
                System.out.println(String.format("%.1f", i)
                        + "\t" + queue.freeTime
                        + "\t" + queue.freeTimePerServer
                        + "\t" + queue.freeCounter
                        + "\t" + queue.freeCounterPerServer);
            }
        }
    }
```

```
    ...

}
```

Result:

```
Parameter:
  server number (K): 3
  service rate  (μ): 1.0
  max client number: 1000
```

(Get full data from *table 3* below.)



4

> Write down two applications of the above queueing model?

1. cashier of Whole Foods Market (foggy bottom) - K might be 6 to 16
2. restroom of District House - K might be 2 to 3

Table

1

λ	system time	system time per client

0.1	1010.214058	1.010214058
0.2	1040.67641	1.04067641
0.3	996.765757	0.996765757
0.4	1048.284635	1.048284635
0.5	1046.407754	1.046407754
0.6	1131.969027	1.131969027
0.7	1141.598139	1.141598139
0.8	1194.696502	1.194696502
0.9	1275.277795	1.275277795
1	1300.587145	1.300587145
1.1	1375.855366	1.375855366
1.2	1473.452182	1.473452182
1.3	1607.334601	1.607334601
1.4	1573.059522	1.573059522
1.5	2665.769401	2.665769401
1.6	2986.743366	2.986743366
1.7	2132.933078	2.132933078
1.8	5273.632764	5.273632764
1.9	6128.766925	6.128766925
2	7975.912645	7.975912645
2.1	15225.67974	15.22567974
2.2	26486.41416	26.48641416
2.3	27817.23325	27.81723325
2.4	38934.44848	38.93444848
2.5	40454.15037	40.45415037
2.6	86486.16474	86.48616474
2.7	62818.11874	62.81811874

2.8	81188.92835	81.18892835
2.9	77162.12188	77.16212188
3	75438.29299	75.43829299
3.1	65526.1295	65.5261295
3.2	88634.22068	88.63422068
3.3	103385.4423	103.3854423
3.4	101078.9407	101.0789407
3.5	124152.9791	124.1529791
3.6	108714.7678	108.7147678
3.7	126156.4628	126.1564628
3.8	113086.461	113.086461
3.9	128550.6285	128.5506285
4	129143.9677	129.1439677
4.1	118165.653	118.165653
4.2	133617.0554	133.6170554
4.3	144631.0342	144.6310342
4.4	132754.3855	132.7543855
4.5	135957.971	135.957971
4.6	146686.8374	146.6868374
4.7	148572.4057	148.5724057
4.8	149553.9359	149.5539359
4.9	157210.8069	157.2108069
5	168211.3138	168.2113138
5.1	166604.411	166.604411
5.2	166468.5638	166.4685638
5.3	167594.6878	167.5946878

5.4	152201.3481	152.2013481
5.5	145603.9787	145.6039787
5.6	151438.8591	151.4388591
5.7	167623.1253	167.6231253
5.8	166077.2642	166.0772642
5.9	162422.8189	162.4228189
6	164505.6949	164.5056949
6.1	166236.2224	166.2362224
6.2	165780.9314	165.7809314
6.3	173064.108	173.064108
6.4	171407.8667	171.4078667
6.5	174184.5412	174.1845412
6.6	176183.9038	176.1839038
6.7	188190.8182	188.1908182
6.8	167162.178	167.162178
6.9	164209.5582	164.2095582
7	174556.3878	174.5563878
7.1	177887.9127	177.8879127
7.2	188227.7842	188.2277842
7.3	178202.4481	178.2024481
7.4	160929.6512	160.9296512
7.5	176886.8842	176.8868842
7.6	190316.8968	190.3168968
7.7	189377.4588	189.3774588
7.8	202104.4694	202.1044694
7.9	191932.7595	191.9327595
8	183319.4721	183.3194721

8.1	173906.3111	173.9063111
8.2	185198.5291	185.1985291
8.3	191338.938	191.338938
8.4	185456.1434	185.4561434
8.5	180885.7025	180.8857025
8.6	189621.7756	189.6217756
8.7	203930.516	203.930516
8.8	186990.2784	186.9902784
8.9	200107.0657	200.1070657
9	200856.3754	200.8563754
9.1	207580.8354	207.5808354
9.2	191280.927	191.280927
9.3	194036.3119	194.0363119
9.4	205700.5513	205.7005513
9.5	204164.5564	204.1645564
9.6	213773.6947	213.7736947
9.7	207452.0222	207.4520222
9.8	193177.3409	193.1773409
9.9	190181.357	190.181357

2

λ	free time	free time per server	free counter	free counter per server
0.1	17670.31931	8835.159656	993	496.5
0.2	8843.824447	4421.912224	978	489
0.3	5436.808503	2718.404251	954	477
0.4	4344.180834	2172.090417	952	476
0.5	2800.282254	1400.141127	915	457.5

0.6	2437.007667	1218.503834	851	425.5
0.7	1800.246525	900.1232626	808	404
0.8	1463.81014	731.9050698	799	399.5
0.9	1164.598442	582.299221	715	357.5
1	954.6829458	477.3414729	650	325
1.1	897.6450842	448.8225421	637	318.5
1.2	695.3697989	347.6848994	561	280.5
1.3	600.0913879	300.045694	507	253.5
1.4	540.6654189	270.3327094	500	250
1.5	325.6117131	162.8058565	331	165.5
1.6	222.3736204	111.1868102	243	121.5
1.7	285.0415879	142.5207939	353	176.5
1.8	91.70207213	45.85103607	144	72
1.9	70.85244483	35.42622241	107	53.5
2	25.16875089	12.58437545	37	18.5
2.1	15.47965302	7.73982651	21	10.5
2.2	1.910070821	0.9550354106	5	2.5
2.3	10.27416352	5.137081761	27	13.5
2.4	3.337928115	1.668964058	9	4.5
2.5	0.8378142441	0.418907122	2	1
2.6	5.209892598	2.604946299	8	4
2.7	5.474094657	2.737047328	11	5.5
2.8	2.073283358	1.036641679	4	2
2.9	2.756913163	1.378456581	6	3
3	0.822324221	0.4111621105	2	1
3.1	5.816356614	2.908178307	6	3
3.2	8.509635045	4.254817523	14	7
3.3	0.6305799423	0.3152899711	2	1

3.4	2.07966125	1.039830625	3	1.5
3.5	0.2307336692	0.1153668346	2	1
3.6	2.004342905	1.002171452	6	3
3.7	0.7233894484	0.3616947242	3	1.5
3.8	0.9416313944	0.4708156972	4	2
3.9	3.117528466	1.558764233	7	3.5
4	0.5101150453	0.2550575227	2	1
4.1	0.6726101143	0.3363050572	3	1.5
4.2	0.3252478121	0.1626239061	2	1
4.3	0.514165439	0.2570827195	3	1.5
4.4	1.245198856	0.6225994281	5	2.5
4.5	0.816859605	0.4084298025	3	1.5
4.6	0.4640447226	0.2320223613	2	1
4.7	0.5148497817	0.2574248908	3	1.5
4.8	0.4523561836	0.2261780918	2	1
4.9	1.024884421	0.5124422105	3	1.5
5	0.856575986	0.428287993	4	2
5.1	0.5230869122	0.2615434561	2	1
5.2	0.3347443003	0.1673721501	3	1.5
5.3	0.7806726124	0.3903363062	2	1
5.4	1.797377792	0.8986888959	6	3
5.5	1.437743748	0.7188718741	2	1
5.6	0.4719312399	0.23596562	3	1.5
5.7	1.188579937	0.5942899686	2	1
5.8	1.298008251	0.6490041254	2	1
5.9	1.26588128	0.63294064	3	1.5
6	0.8295474722	0.4147737361	2	1
6.1	0.3786521621	0.1893260811	2	1

6.2	0.1971613466	0.09858067332	2	1
6.3	0.2765663701	0.1382831851	2	1
6.4	0.5054060003	0.2527030002	2	1
6.5	0.5426509767	0.2713254884	2	1
6.6	0.3765596615	0.1882798307	2	1
6.7	0.2630756091	0.1315378046	2	1
6.8	0.364101484	0.182050742	2	1
6.9	0.1506630642	0.07533153208	2	1
7	1.245291725	0.6226458625	4	2
7.1	0.2725029209	0.1362514605	2	1
7.2	0.7627242273	0.3813621136	3	1.5
7.3	0.3077152542	0.1538576271	3	1.5
7.4	0.453521672	0.226760836	3	1.5
7.5	0.6229602317	0.3114801158	3	1.5
7.6	0.2490231732	0.1245115866	3	1.5
7.7	0.2154954514	0.1077477257	2	1
7.8	0.2382549428	0.1191274714	2	1
7.9	0.1556470091	0.07782350457	2	1
8	0.4813357394	0.2406678697	3	1.5
8.1	0.8341818876	0.4170909438	2	1
8.2	0.2007625646	0.1003812823	2	1
8.3	0.6172179144	0.3086089572	2	1
8.4	0.1925873884	0.09629369418	2	1
8.5	0.596923849	0.2984619245	2	1
8.6	0.7952882805	0.3976441402	3	1.5
8.7	0.7069173388	0.3534586694	2	1
8.8	0.1646923661	0.08234618304	2	1
8.9	0.5349981916	0.2674990958	2	1

9	0.2642837578	0.1321418789	2	1
9.1	0.3550504014	0.1775252007	2	1
9.2	0.1229946254	0.06149731269	2	1
9.3	0.04144409998	0.02072204999	2	1
9.4	0.01541217526	0.007706087629	2	1
9.5	0.2747564556	0.1373782278	2	1
9.6	0.6668590206	0.3334295103	2	1
9.7	0.4565450459	0.2282725229	3	1.5
9.8	0.2227082586	0.1113541293	2	1
9.9	0.2728998223	0.1364499112	3	1.5

3

λ	free time	free time per server	free counter	free counter per server
0.1	26978.33779	8992.779263	1000	333.3333333
0.2	13769.71498	4589.904992	1000	333.3333333
0.3	8638.400289	2879.466763	997	332.3333333
0.4	7020.947013	2340.315671	995	331.6666667
0.5	4694.092377	1564.697459	984	328
0.6	4161.720923	1387.240308	969	323
0.7	3204.865498	1068.288499	958	319.3333333
0.8	2703.502689	901.167563	957	319
0.9	2245.712236	748.5707454	929	309.6666667
1	1903.24289	634.4142968	905	301.6666667
1.1	1852.643078	617.5476925	894	298
1.2	1531.906601	510.6355338	883	294.3333333
1.3	1389.775952	463.2586506	846	282
1.4	1291.304807	430.4349358	826	275.3333333
1.5	975.3321134	325.1107045	758	252.6666667
1.6	822.9246793	274.3082264	712	237.3333333
1.7	898.2759868	299.4253289	761	253.6666667

1.8	620.0452161	206.6817387	625	208.3333333
1.9	597.3470175	199.1156725	628	209.3333333
2	493.4075165	164.4691722	551	183.6666667
2.1	431.491456	143.8304853	478	159.3333333
2.2	378.5000213	126.1666738	483	161
2.3	293.2184561	97.73948535	438	146
2.4	282.2439685	94.08132284	384	128
2.5	259.5105232	86.50350774	383	127.6666667
2.6	136.8462431	45.61541438	187	62.33333333
2.7	132.140127	44.046709	258	86
2.8	67.24642309	22.41547436	122	40.66666667
2.9	66.29733594	22.09911198	98	32.66666667
3	63.5315203	21.17717343	104	34.66666667
3.1	86.76894612	28.92298204	142	47.33333333
3.2	34.23286393	11.41095464	55	18.33333333
3.3	2.746869711	0.9156232369	7	2.333333333
3.4	3.551176867	1.183725622	4	1.333333333
3.5	0.7246583608	0.2415527869	6	2
3.6	4.771031417	1.590343806	9	3
3.7	1.315082282	0.4383607607	4	1.333333333
3.8	12.63510102	4.211700341	30	10
3.9	7.458916164	2.486305388	11	3.666666667
4	1.992971951	0.6643239837	6	2
4.1	2.026267838	0.6754226128	7	2.333333333
4.2	0.9474993265	0.3158331088	4	1.333333333
4.3	1.241743872	0.4139146238	4	1.333333333
4.4	3.475826785	1.158608928	11	3.666666667
4.5	2.317991954	0.7726639846	5	1.666666667
4.6	0.7710838173	0.2570279391	3	1
4.7	1.034785174	0.3449283913	6	2
4.8	1.60417449	0.5347248299	6	2

4.9	1.681188574	0.5603961912	4	1.333333333
5	1.606684018	0.5355613393	5	1.666666667
5.1	1.184850018	0.3949500059	4	1.333333333
5.2	1.136429086	0.3788096952	4	1.333333333
5.3	1.353056219	0.4510187398	3	1
5.4	3.697708134	1.232569378	8	2.666666667
5.5	3.217929606	1.072643202	10	3.333333333
5.6	1.43473079	0.4782435966	4	1.333333333
5.7	2.000986318	0.6669954392	3	1
5.8	2.18968337	0.7298944567	4	1.333333333
5.9	3.040884999	1.013628333	10	3.333333333
6	1.425422126	0.4751407087	4	1.333333333
6.1	0.6811195234	0.2270398411	3	1
6.2	0.4691933252	0.1563977751	4	1.333333333
6.3	0.5481217645	0.1827072548	4	1.333333333
6.4	0.80948004	0.26982668	3	1
6.5	1.271044306	0.4236814353	3	1
6.6	1.368714412	0.4562381373	4	1.333333333
6.7	0.5750927519	0.191697584	3	1
6.8	0.591945587	0.1973151957	3	1
6.9	2.223765697	0.7412552322	12	4
7	2.398200314	0.7994001046	5	1.666666667
7.1	0.6047323935	0.2015774645	3	1
7.2	1.361678011	0.4538926703	4	1.333333333
7.3	0.7224523807	0.2408174602	4	1.333333333
7.4	1.294870659	0.431623553	5	1.666666667
7.5	1.123047109	0.3743490364	4	1.333333333
7.6	0.676293005	0.2254310017	4	1.333333333
7.7	0.5124948073	0.1708316024	4	1.333333333
7.8	0.4081434781	0.136047826	3	1
7.9	0.6850353944	0.2283451315	4	1.333333333

8	0.9481288321	0.316042944	4	1.333333333
8.1	1.438563796	0.4795212653	3	1
8.2	0.3621185673	0.1207061891	3	1
8.3	1.241361177	0.413787059	3	1
8.4	0.3255725525	0.1085241842	3	1
8.5	1.029680405	0.3432268018	3	1
8.6	1.359141398	0.4530471326	4	1.333333333
8.7	1.204961301	0.4016537672	3	1
8.8	0.3215539609	0.1071846536	3	1
8.9	1.148211997	0.3827373324	4	1.333333333
9	1.04183539	0.3472784632	6	2
9.1	0.7239007351	0.241300245	3	1
9.2	0.2092157604	0.06973858679	3	1
9.3	0.337550502	0.112516834	3	1
9.4	0.06896764607	0.02298921536	3	1
9.5	0.5198979413	0.1732993138	4	1.333333333
9.6	1.293968569	0.4313228565	3	1
9.7	1.142816755	0.3809389182	4	1.333333333
9.8	0.4518060807	0.1506020269	3	1
9.9	0.5846564142	0.1948854714	4	1.333333333