Reflection Exercise 2

1

I have learned the probability theory 5 years ago when I was an undergraduate student. By reading module 6, I make a good review - there are so many code example which provide me a intuitive image. I can always check my calculation result by doing some programming.

For example, problem #18 in the section of "6.6 Conditional probability":

> 5% of the population is infected.
>
> $$P[S] = 0.05$$
>
> The probability that the test works for an infected person is 0.99.
>
> $$P[T|S] = 0.99$$
>
> The probability of a false positive is 3%.
>
> $$P[T|\overline{S}] = 0.03$$
>
> Probability that if a test is positive, the person is infected: $P[S|T]$.
>
> Probability that if a test is positive, the person is well: $P[\overline{S}|T]$.
>
> $$\begin{aligned} P[TS] &= P[S]P[T|S] = 0.0495 \\ P[\overline{S}] &= 1 - P[S] = 0.95 \\ P[T\overline{S}] &= P[\overline{S}]P[T|\overline{S}] = 0.0285 \\ P[T] &= P[TS] + P[T\overline{S}] = 0.078 \\ P[S|T] &= \frac{P[TS]}{P[T]} = 0.635 \\ P[\overline{S}|T] &= \frac{P[T\overline{S}]}{P[T]} = 0.365 \end{aligned}$$
>
> Code:
>
> ```
> public class LabTestExample {
>
>  public static void main(String[] argv) {
>      double numTrials = 1000000;
>      double positive = 0;
> ```

```
        double infected = 0;
        double well = 0;
        LabTest lab = new LabTest(0.05, 0.99, 0.03);
        for (int n = 0; n < numTrials; n++) {
            lab.nextPatient();
            if (lab.testedPositive()) {
                // INSERT YOUR CODE HERE
                positive++;
                if (lab.isSick()) {
                    infected++;
                } else {
                    well++;
                }
            }
        }
        // AND HERE
        double infectedGivenPositive = infected / positive;
        double wellGivenPositive = well / positive;
        System.out.println("Pr[infected given positive]=" +
infectedGivenPositive + "  theory=" + 0.635);
        System.out.println("Pr[well given positive]=" + wellGivenPositive + "
theory=" + 0.365);
     }

}
```

Result:

```
Pr[infected given positive]=0.6340914353340709  theory=0.635
Pr[well given positive]=0.36590856466592914  theory=0.365
```

It really helps me a lot.

2

At the same time, I review *the Bayes' rule* and *the law of total probability* by traditional math way.

the Bayes' rule:

> This idea
>
> $$Pr[A|B] = \frac{Pr[B|A]Pr[A]}{Pr[B]}$$
>
> is often called *Bayes' rule*.

the law of total probability:

> Thus,
>
> $$\begin{aligned} Pr[A] &= Pr[(A \cap B_1) \cup (A \cap B_2) \cup \ldots \cup (A \cap B_n)] \\ &= Pr[(A \cap B_1)] + Pr[(A \cap B_2)] + \ldots + Pr[A \cap B_n)] \\ &= Pr[A|B_1]Pr[B_1] + Pr[A|B_2]Pr[B_2] + \ldots Pr[A|B_n]Pr[B_n] \end{aligned}$$
>
> This is sometimes called the *law of total probability*

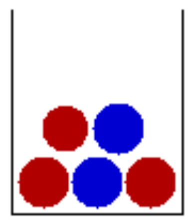
After reading their mathematical proof, I get a deeper understanding of such formula. (I used to just recite them.) Now, I can use them proficiently.

3

At the very begining of module 6, it says that:

> Probability can be somewhat difficult and counterintuitive.

The last part - "6.9 Problem solving" - use traditional math way to show me such "counterintuitive".

> Another example:
>
> 
>
> - Suppose we draw two balls without replacement from an an urn with 3 red balls, and 2 blue balls. What is the probability both have the same color?
>
> - Identify the sample space:
>
> $$\Omega = \{(R, R), (R, B), (B, R), (B, B)\}$$
>
> - Clearly, the event of interest is:
>
> $$A = \{(R, R), (B, B)\}$$
>
> - Note: it is NOT true that
>
> $$Pr[A] = \frac{|A|}{|\Omega|}$$
>
> for this example (because there's conditioning involved - see below)
>
> - That is, the individual outcomes are NOT equally likely.
>
> We have to work these out.

- Obviously, the probabilities for the second drawing depend on what happened in the first.

$\Rightarrow$ Conditioning is involved.

- Define key events

R_1 ="first is red"

R_2 ="second is red"_

B_1="first is blue"

B_2 ="second is blue"

- The event of interest is

(R_1 and R_2) OR (B_1 and B_2)

- For additional clarity, we could write this with set notation (so that the combination of events is clear):

$$(R_1 \cap R_2) \cup (B_1 \cap B_2)$$

- Right away, we can see that

$$Pr[R_1] = \frac{3}{5}$$
$$Pr[B_1] = \frac{2}{5}$$

- Also, some conditional probabilities are easy to read off:

$$Pr[R_2|R_1] = \frac{2}{4}$$
$$Pr[B_2|B_1] = \frac{1}{4}$$

- Recall that

$$Pr[R_2 \cap R_1] = Pr[R_2|R_1]Pr[R_1] = \frac{2}{4} \cdot \frac{3}{5} = \frac{6}{20}$$

- Similarly

$$Pr[B_2 \cap B_1] = Pr[B_2|B_1]Pr[B_1] = \frac{1}{4} \cdot \frac{2}{5} = \frac{2}{20}$$

- Now let's return to our event of interest:

$$(R1 \cap R2) \cup (B1 \cap B2)$$

Both events being OR'd (union'ed) are disjoint, so

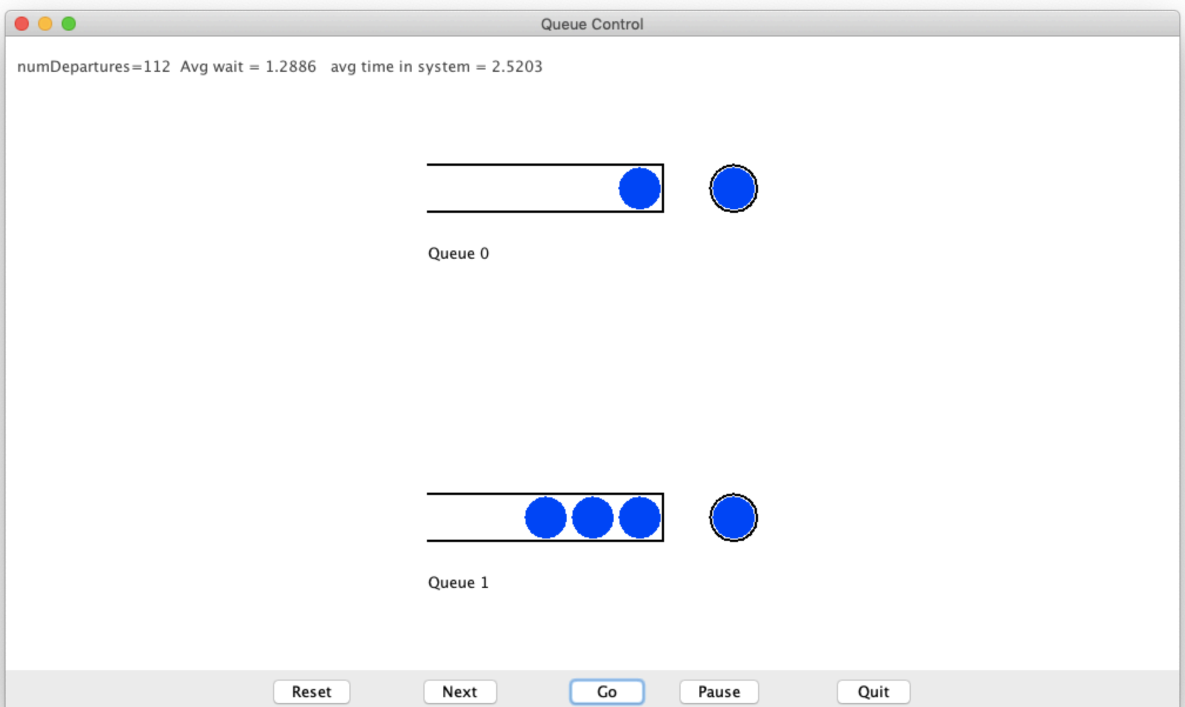
> $$Pr[A] = Pr[R_1 \cap R_2] + Pr[B_1 \cap B_2]$$
>
> - Thus,
>
> $$Pr[A] = \frac{6}{20} + \frac{2}{20} = \frac{8}{20}$$
>
> - So, what did we learn from this example?
>   - In some problems, the conditional probabilities are easy to "read off" from the problem description.
>   - Even though the event of interest seemed complex, it was broken down quite easily.

This example tells me that: sometime we may "read off" some key point when try to solving a problem, and we should use strict mathematical tools rather than intuition to deal with a problem. I believe that it is really important - not only on this cource but everywhere.

4

There is a very interesting visual demo at the beginning of module 7.

By playing with this demo and thinking about the principle, I realize that there are still some important concepts in the probability theory I have to learn.



5

There is an interesting question - "how computers generate a random variable?". The answer is in the section of "7.10 Generating random values from distributions".

By reading the instruction, I do exercise #48.

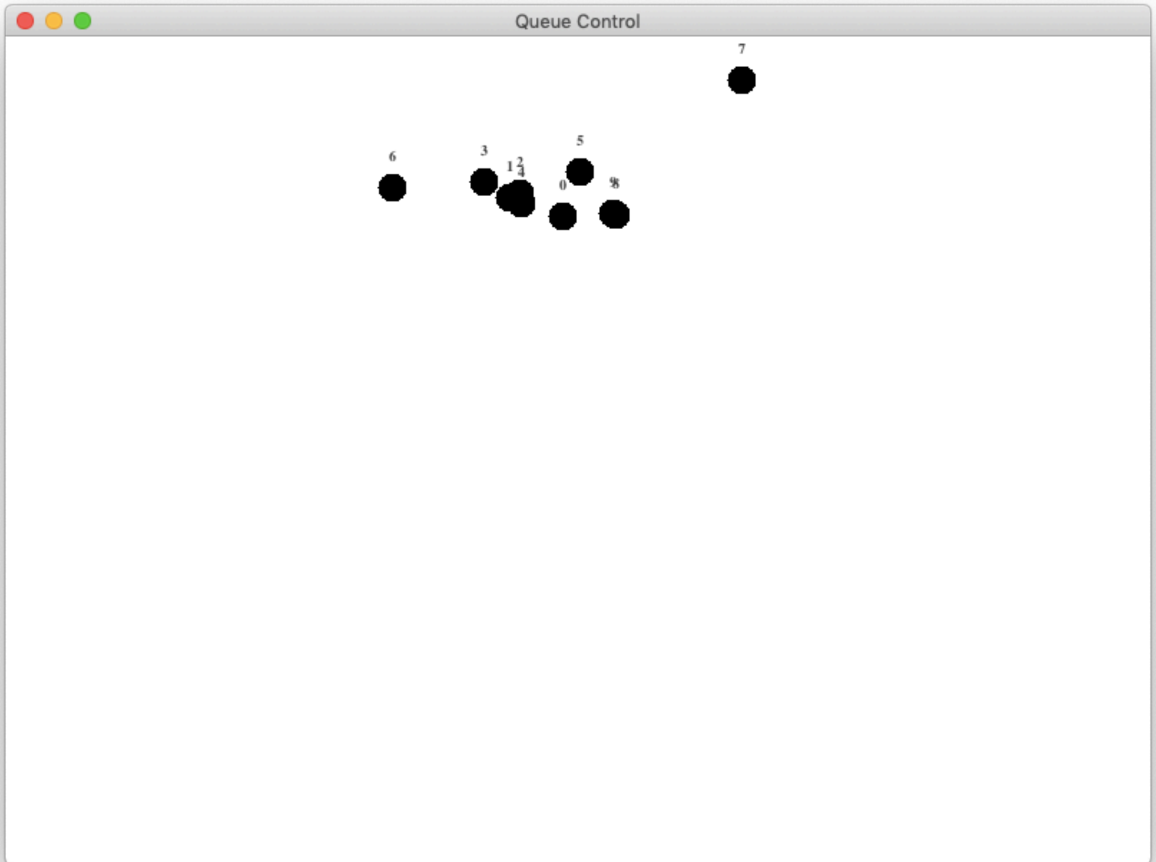
```
public class ExponentialGenerator {

    static double x = 1;

    public static void main(String[] argv) {
        int numTrials = 100000;

        // Exponential parameter.
        double gamma = 4.0;

        // Make a density histogram. NOTE: change code in PropHistogram
accordingly.
        PropHistogram hist = new PropHistogram(0, 5, 20);
        for (int n = 0; n < numTrials; n++) {
            double x = generateNext(gamma);
            hist.add(x);
        }

        hist.display();
    }


    static double generateNext(double gamma) {
        // INSERT YOUR CODE HERE.
        return -Math.log((1 - uniform()) / gamma);
    }

    static double uniform() {
        int M = 1 << 16 - 1;
        int a = 48271;
        x = (a * x) % M;
        return x / (double) M;
    }

}
```

By doing such programming exercise, I also know "how computer generate the random number" and the concept of "seed".

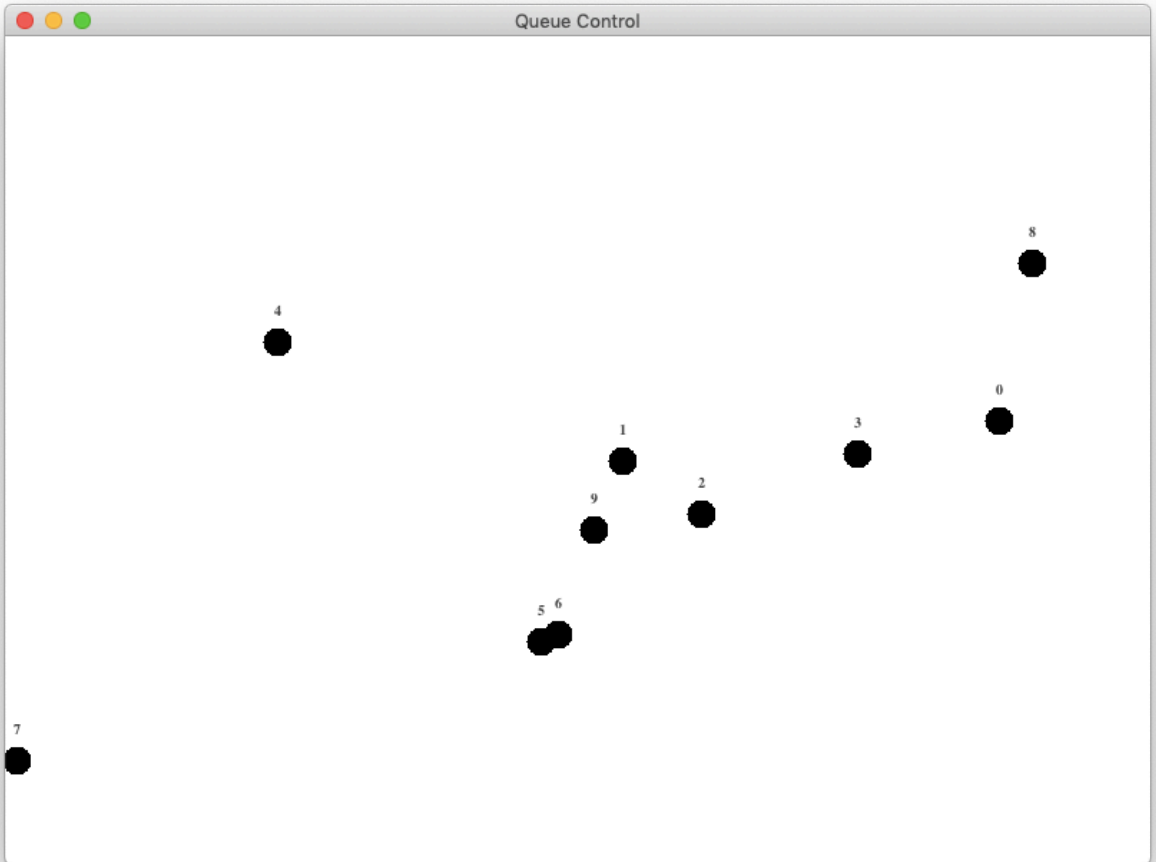
6

Section "8.5 Synchronous simulations" in module 8 mentions asynchronous and synchronous simulation. It provide us two visual demos.

Boid - synchronous simulation:



Boid - asynchronous simulation:



It helps me to understand the difference between them:

- synchronous simulation: fixed time steps.
- asynchronous simulation: based on special events.