Module 10

02

> Why is this true? What is the minimum (unconstrained) value of $f(x, y) = 3x + 4y$?

For a "linear objective, unconstrained variables" problem, the minimum value must be $+\infty$ and the maximum valus must be $-\infty$.

The minimum (unconstrained) value of $f(x, y) = 3x + 4y$ is $-\infty$ (not existent in general).

03

> Go back to your calc book and find an example of a "hard to differentiate" function.

$$f(x) = \ln(x) + \cos(x)$$

04

> What's an example of a function that's continuous but not differentiable? Consider the (weird) function $f(x)$ where $f(x) = 1$ if x is rational and $f(x) = 0$ otherwise. Is this continuous? Differentiable?

One example:

$$g(x) = |x|$$

It is continuous but not differentiable when $x = 0$.

Function $f(x)$ (Dirichlet function) is netither continuous nor differentiable.

05

> Consider the function:
>
> $$f(x) = \frac{x}{\mu_1 - \lambda x} + \frac{1 - x}{\mu_2 - \lambda(1 - x)}$$
>
> Compute the derivative $f'(x)$. Can you solve $f'(x) = 0$?

Get derivative $f'(x)$:

$$f(x) = \frac{x}{\mu_1 - \lambda x} + \frac{1-x}{\mu_2 - \lambda(1-x)}$$
$$f'(x) = \frac{\mu_1}{x^2} - \frac{\mu_2}{(1-x)^2}$$

Calculate $f'(x) = 0$:

$$\begin{aligned} f'(x) = 0 \Longrightarrow \frac{\mu_1}{x^2} &= \frac{\mu_2}{(1-x)^2} \\ \Longrightarrow (1-x)^2 \mu_1 &= \mu_2 x^2 && (x \neq 0\ ,\ x \neq 1) \\ \Longrightarrow (\mu_2 - \mu_1)x^2 + 2\mu_1 x - \mu_1 &= 0 && (x \neq 0\ ,\ x \neq 1) \end{aligned}$$

So:

$$x = \begin{cases} \frac{1}{2} & (\mu_1 = \mu_2) \\ \frac{-\mu_1 \pm \sqrt{\mu_1^2 - \mu_1(\mu_2 - \mu_1)}}{\mu_2 - \mu_1} & (\mu_1 \neq \mu_2\ ,\ \mu_1^2 - \mu_1(\mu_2 - \mu_1) \geq 0\ ,\ x \neq 0\ ,\ x \neq 1) \end{cases}$$

06

> Download and execute BracketSearch.java.
>
> - What is the running time in terms of M and N?
> - If we keep MN constant (e.g., $MN = 24$), what values of M and N produce best results?

See file `BracketSearch.java`.

Result:

```
a=4.560280445054108 b=4.950464868160342 bestf=2.502058677967597
```

The running times is MN.

$M = 6\ ,\ N = 4$ will get the best result:

```
a=4.629629629629631 b=4.753086419753088 bestf=2.500347523243408
```

07

> Draw an example of a function for which bracket-search fails miserably, that is, the true minimum is much lower than what's found by bracket search even for large M and N.

$$f(x) = x \sin(100x);$$

08

> What is the number of function evaluations in terms of M and N for the bracket-search algorithm?

The time complexity is $O(MN)$.

09

> What is the number of function evaluations in terms of M and N for the 2D bracket-search algorithm? How does this generalize to n dimensions?

The time complexity of 2D bracket-search is $O(M^2N)$.

The time complexity of n-D bracket-search is $O(M^nN)$.

10

> Add code to [MultiBracketSearch.java]() to find the minimum of $f(x_1, x_2) = (x_1 - 4.71)^2 + (x_2 - 3.2)^2 + 2(x_1 - 4.71)^2(x_2 - 3.2)^2$.

See file `MultiBracketSearch.java`.

Result:

```
Bracketing search: x1=4.691358024691359 x2=3.2098765432098757 numFuncEvals=138
```

11

> Modify [BracketSearch2.java]() to use the proportional-difference stopping condition.

See file `BracketSearch2.java`.

16

> Download and execute [GradientDemo.java]().
>
> - How many iterations does it take to get close to the optimum?
> - What is the effect of using a small α (e.g, $\alpha = 0.001$)?
> - In the method `nextStep()`, print out the current value of x, and the value of $xf'(x)$ before the update.
> - Set $\alpha = 1$. Explain what you observe.
> - What happens when $\alpha = 10$?

1

about 200 times

2

more iteration times needed

3

See file `GradientDemo.java`.

4

It doesn't work.

5

It doesn't work.

17

> Download GradientDemo2.java and examine the function being optimized.
>
> - Fill in the code for computing the derivative.
> - Try an initial value of x at 1.8. Does it converge?
> - Next, try an initial value of x at 5.8. What is the gradient at the point of convergence?

See file `GradientDemo2.java`.

Both $x = 1.8$ and $x = 5.8$ converge and their gradient are 0.