Module 08

2

> Download and execute [Queue.java](), which is a simulation of a single server queue. Cursorily examine the code.
>
> - What data structures are being used?
> - Where in the code are interarrival times being generated? From what distribution?
> - Where are service times being generated? From what distribution?

The data structure of queue are used.

Interarrival times are generated in function `void scheduleArrival()` called by function `void handleArrival(QueueEvent e)`.

Service times are generated in function `void scheduleDeparture()` called by function `void handleDeparture(QueueEvent e)`.

3

> Add code in method `randomInterarrivalTime()` in [Queue.java]() to estimate the average interarrival time. What does this number have to do with the value of variable `arrivalRate` in the program?

See file `Queue.java`.

It seems that the average interarrival time is the reciprocal of `arrivalRate`.

6

> Examine method `simulate()` in [Queue.java]() and verify that it has this structure. Then examine `init()` to see if the initialization makes sense. Why is there a call to `scheduleArrival()` in `init()`?

The function `init()` must generate the first arrival, or `eventList` will be null and the program ends immediately.

7

> Execute [Queue.java]() to estimate the average time in system.

See exercise #15.

8

> Execute Queue.java to estimate the average waiting time. Subtract this from the estimate of the average system-time. What do you get? Is it what you expect?

See exercise #15..

It seems that the average interarrival time is the sum of *the average wait time* and *the average service time*.

10

See exercise #15.

11

> Fix the service rate at $\mu = 1$ and vary the arrival rate: try $\lambda = 0.5, 0.75, 1.25$. What do you observe when $\lambda = 1.25$?

The larger λ is, the smaller the average waiting time is.

12

> What about it?

$\lambda \to 0$ will cause:

the system time $\to$ *the average waiting time*

14

> For $\lambda = 0.75$ and $\mu = 1$, estimate mm. Then compute $\frac{m}{d}$, where d is the mean system time.

See exercise #15.

It seems that $\frac{m}{d}$ is always 1.

15

> For $\lambda = 0.75$ and $\mu = 1$, estimate the probability that an arriving customer finds the server free. Try this for $\lambda = 0.5, 0.6$ as well. Can you relate this probability to $\lambda = 0.75$ and $\mu = 1$?

See file `Queue.java`.

Result:

```
Simulation results:
  numArrivals:           1000
  numDepartures:         998
  avg Wait:              1.8906618446580803
  avg System Time (d)    2.814388534740915
  avg Interarrival Time: 1.3558307457972822
  interarrival Rate (λ): 0.7375551875481038
  avg Service Time:      0.9253604657777389
  service Rate (μ):      1.0806599557498155
  Custom number:         2944.0
  avg Custom Number (m): 2.944
  probability of Free:   0.312
```

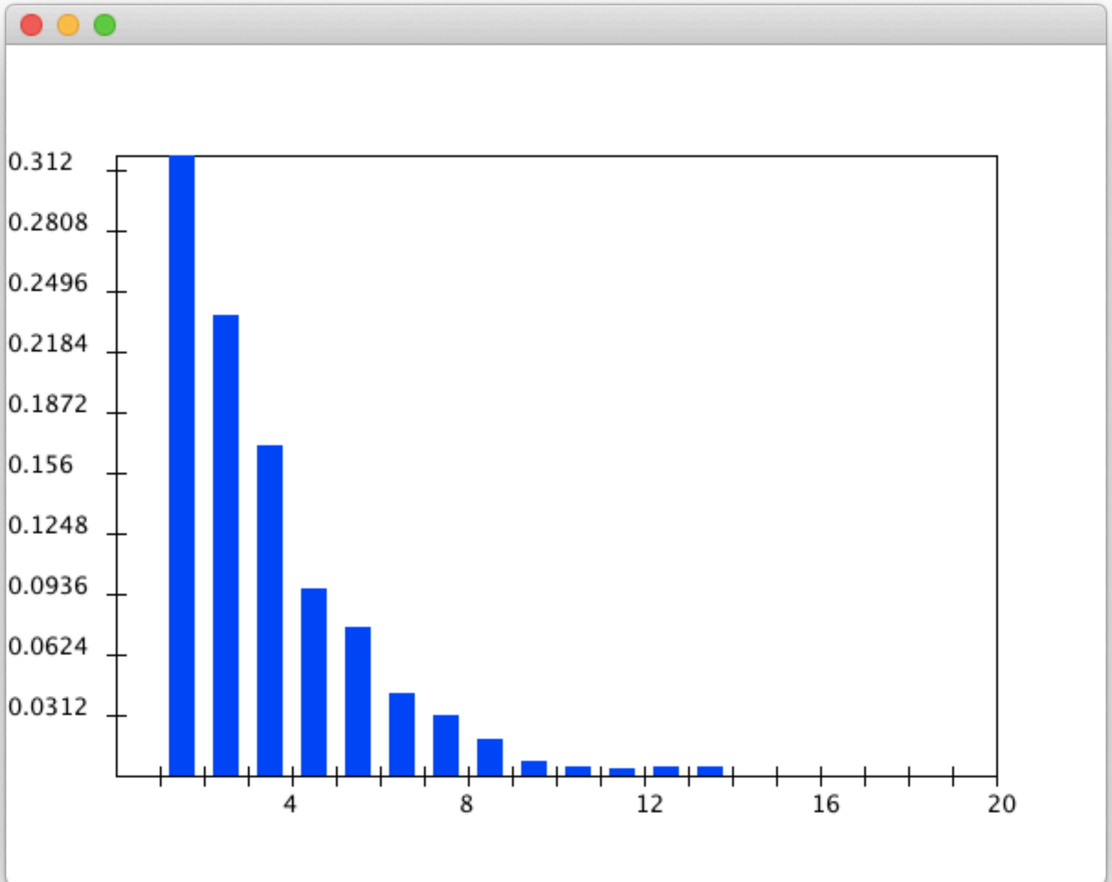
It seems that the probability is equal to $1 - \lambda$.

16

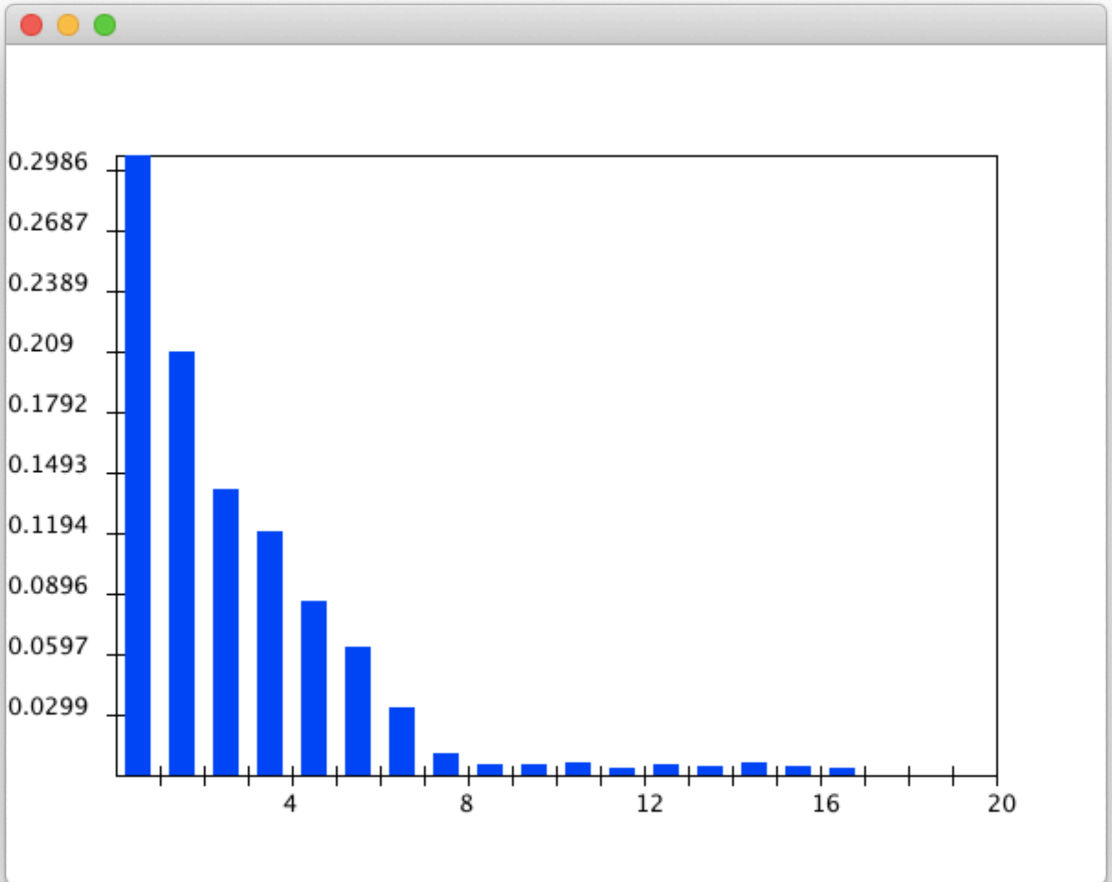
> We will focus on two distributions: the distribution of the number in the system, and the system time. Let rv M denote the number of customers seen by an arriving customer, and let rv D denote the system time experienced by a random customer.
>
> - Is M discrete or continuous? What about D? What is the range of M? Of D?
> - For $\lambda = 0.75$ and $\mu = 1$, obtain the appropriate histogram of M. Which well-known distribution does this look like?
> - For $\lambda = 0.75$ and $\mu = 1$, obtain the appropriate histogram of D. Which well-known distribution does this look like?
>
> Note: use PropHistogram.java or DensityHistogram.java as appropriate.

M is discrete and its range $M \geq 1$. D is continuous and its range $D \geq 0$.

It seems that $M \sim Exponential(0.334)$.



It seems that $D \sim Exponential(0.355)$.



19

> Download and execute [AsynchBoids.java](). What do you notice? Does it work?

It works.

20

> Examine the code in the molecular simulation from [Module 4](). Is this synchronous or asynchronous?

It is synchronous.

21

> Find a simulation of the Game-of-Life. Is this synchronous or asynchronous?

It is asynchronous.

22

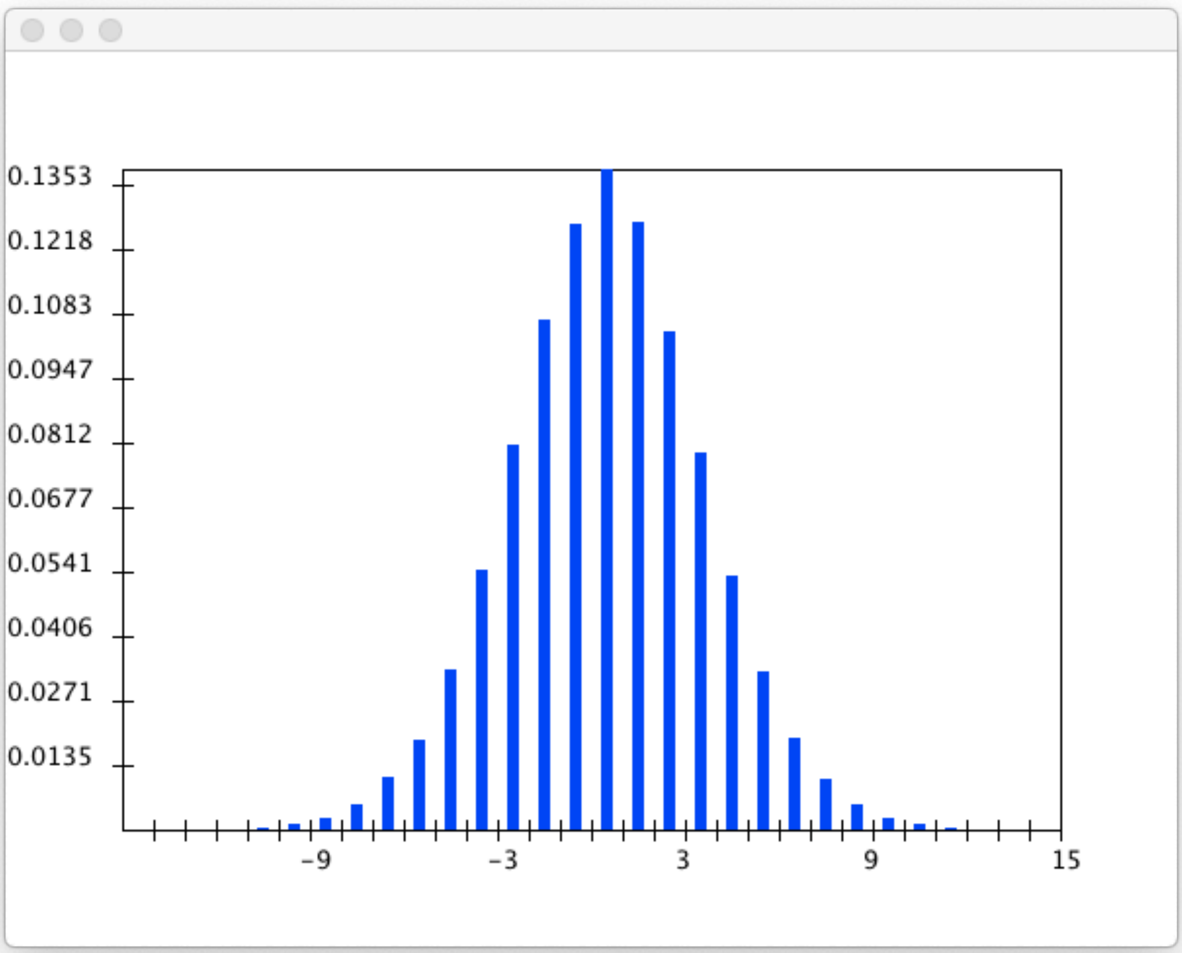
> Fill in code in [Raindrop.java]() to obtain histograms of X and T respectively. Use $s = 1, h = 10, p = 0.5$.
>
> - What is the likely distribution of X?
> - Vary h: try $h = 20, 30, 40, 50$. What is the relationship between $E[T]$ and h?
>
> Note: use [PropHistogram.java]() or [DensityHistogram.java]() as appropriate.

See file `Raindrop.java`.

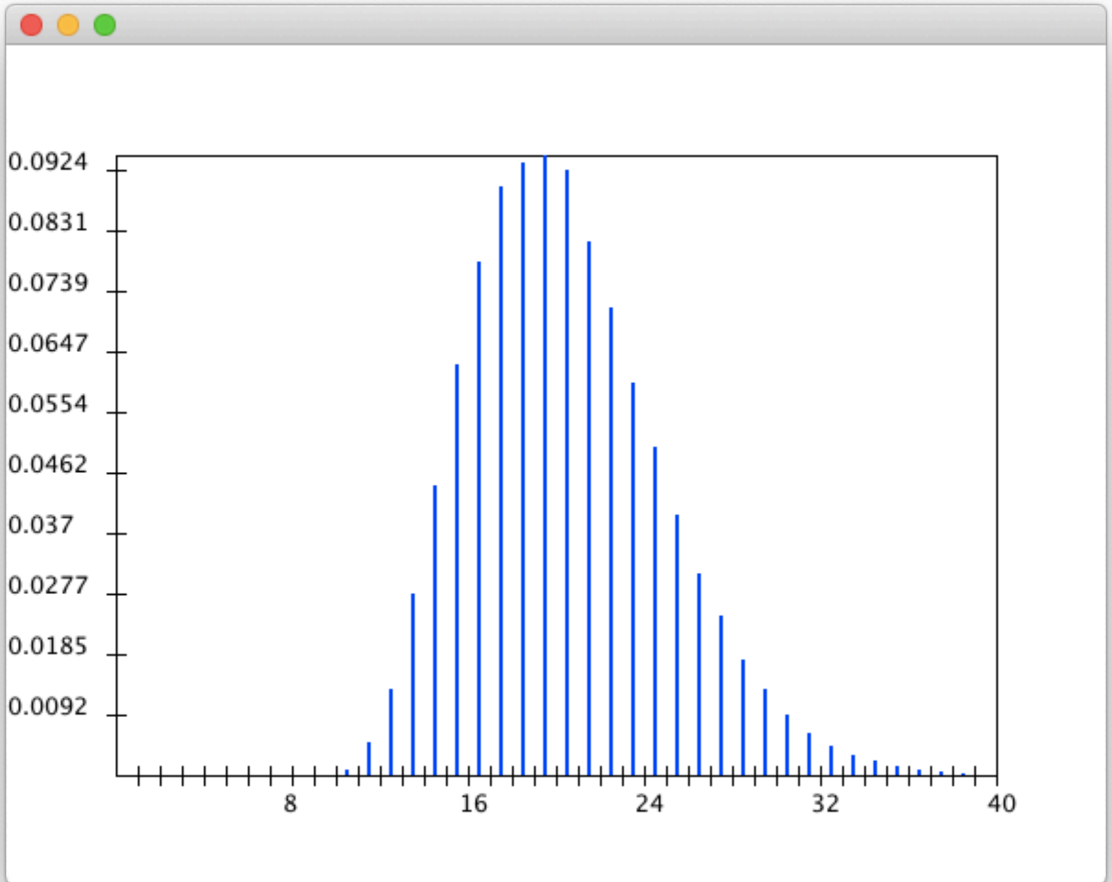
Result:

```
E[T] = 19.99368
```

It seems that $X \sim N(0, \sigma^2)$



T



Also,

```
h = 20 ===> E[T] = 40.0269
h = 30 ===> E[T] = 60.03735
h = 40 ===> E[T] = 80.03158
h = 50 ===> E[T] = 100.01916
```

It seems that $E[T] = \frac{h}{p}$.

23

> Do you know the historical significance of the distributions of X and T?

(I don't know.)

24

> What is the size of the eventlist for the single-server queue? For the three-queue example?

Each item of single-server queue has 2 variable. Each item of three-queue has 3 variable.

25

> If the eventlist has n events, how long does it take for each operation (in order-notation)?

The time compliexty of enqueue is $O(\log n)$.

The time complexity of enqueue is $O(1)$.