Module 09

03

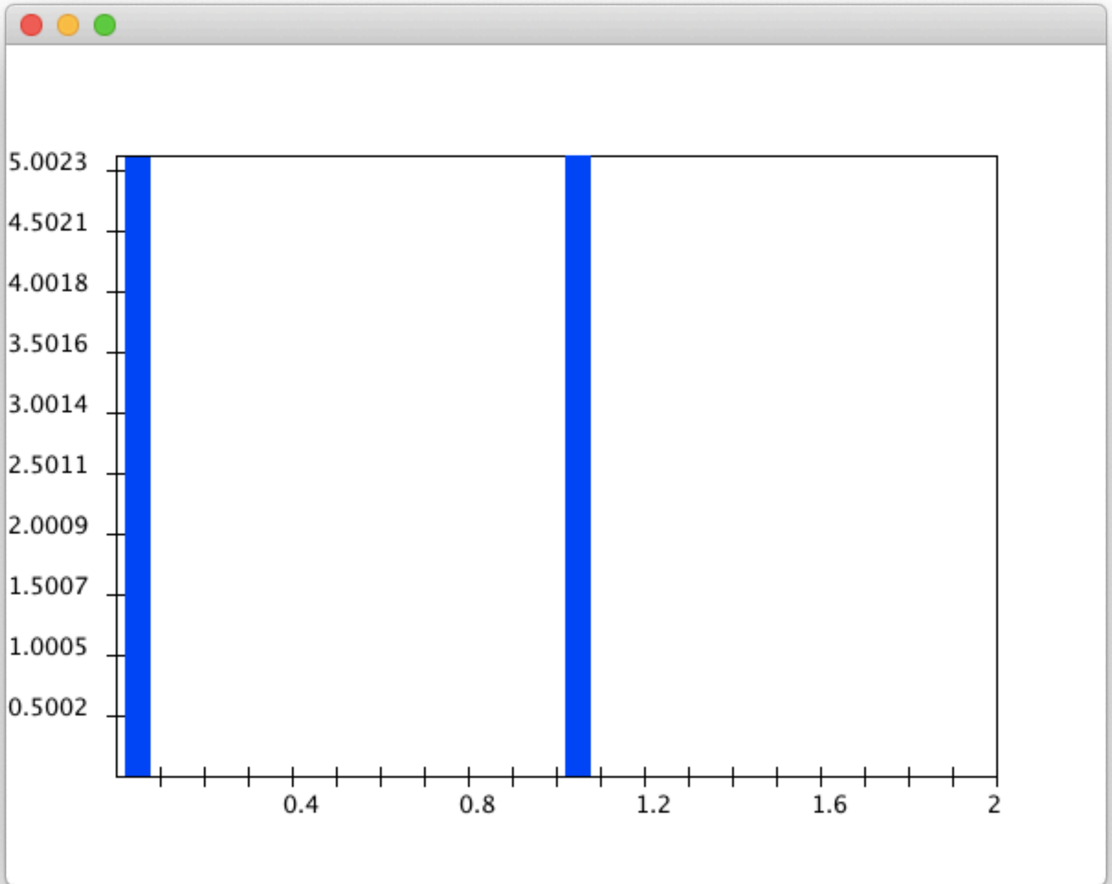
> Suppose that $X \sim Uniform(0,1)$ and $g(X) = X^2$. Modify UniformExample.java to estimate the pdf of $g(X)$ and its average value. You will also need RandTool.java and DensityHistogram.java. What do you notice about the average value of $g(X)$ compared with the average value of X?

See file `UniformExample.java`.

Result:

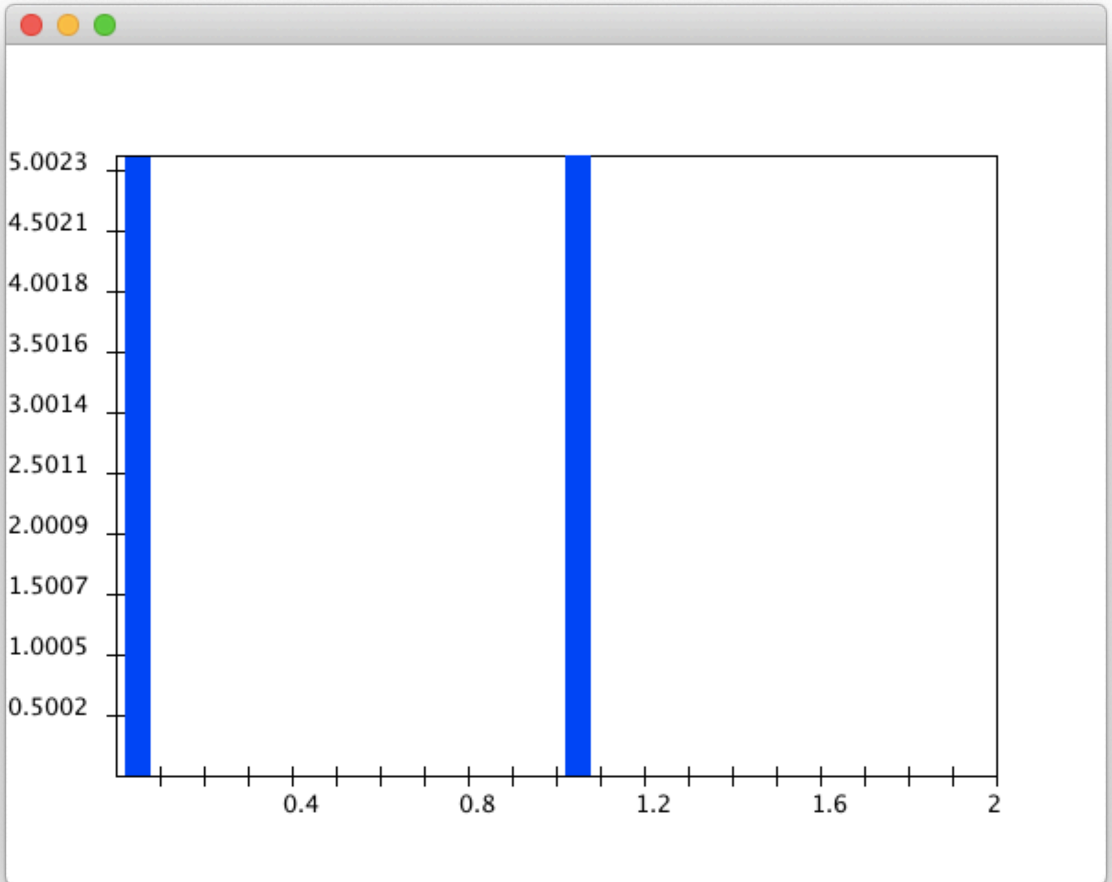
```
E(X)          = 0.50023
E(g(X)=X^2) = 0.50023
```

X



5.0023
4.5021
4.0018
3.5016
3.0014
2.5011
2.0009
1.5007
1.0005
0.5002
0.4
0.8
1.2
1.6
2

$g(X) = X^2$



06

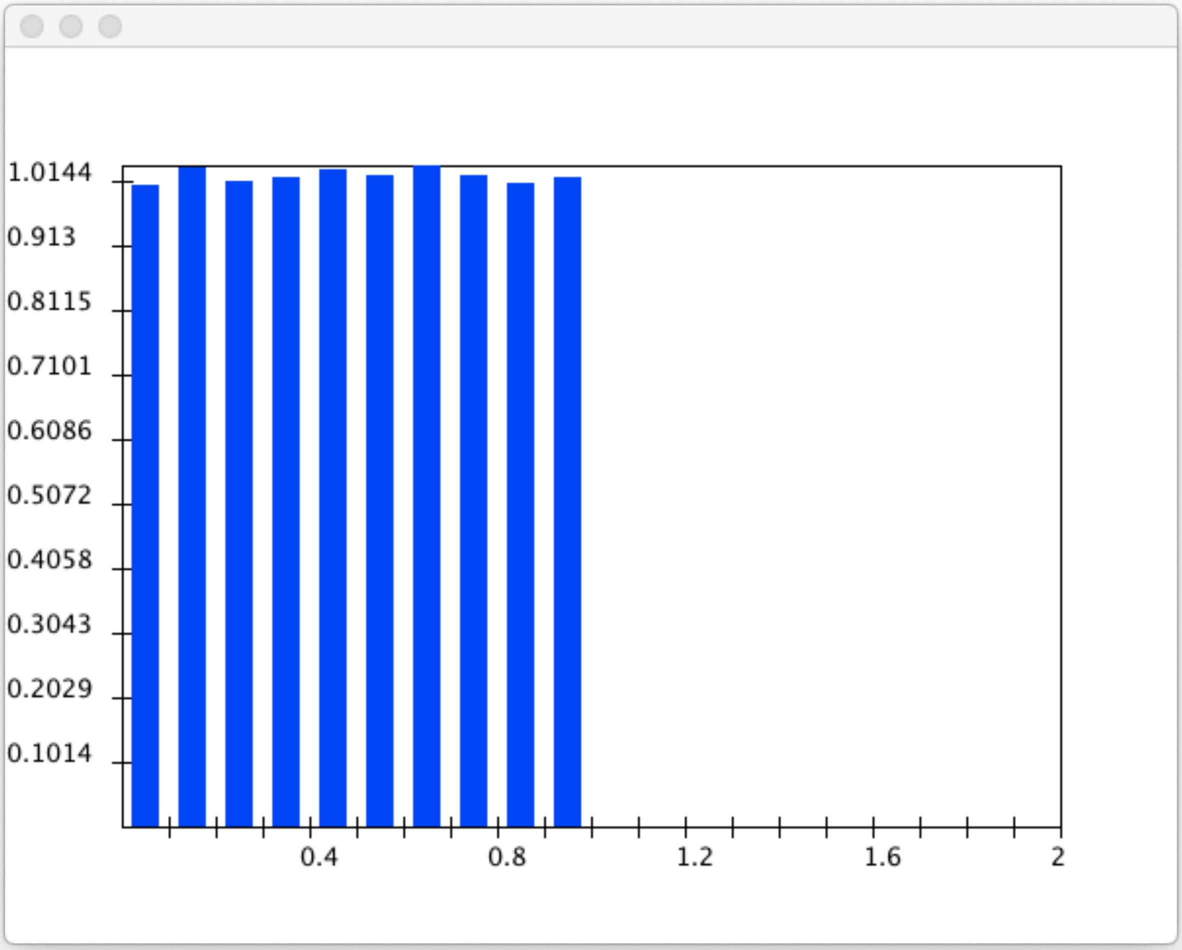
> Suppose $X \sim Uniform(0,1)$ and $g(x)$ is the function $g(x) = (x - 0.5)^2$. Work out $E[g(X)]$ by hand. Then estimate the average value of the rv $g(X)$ by adding code to UniformExample4.java.

See file `UniformExample.java`.

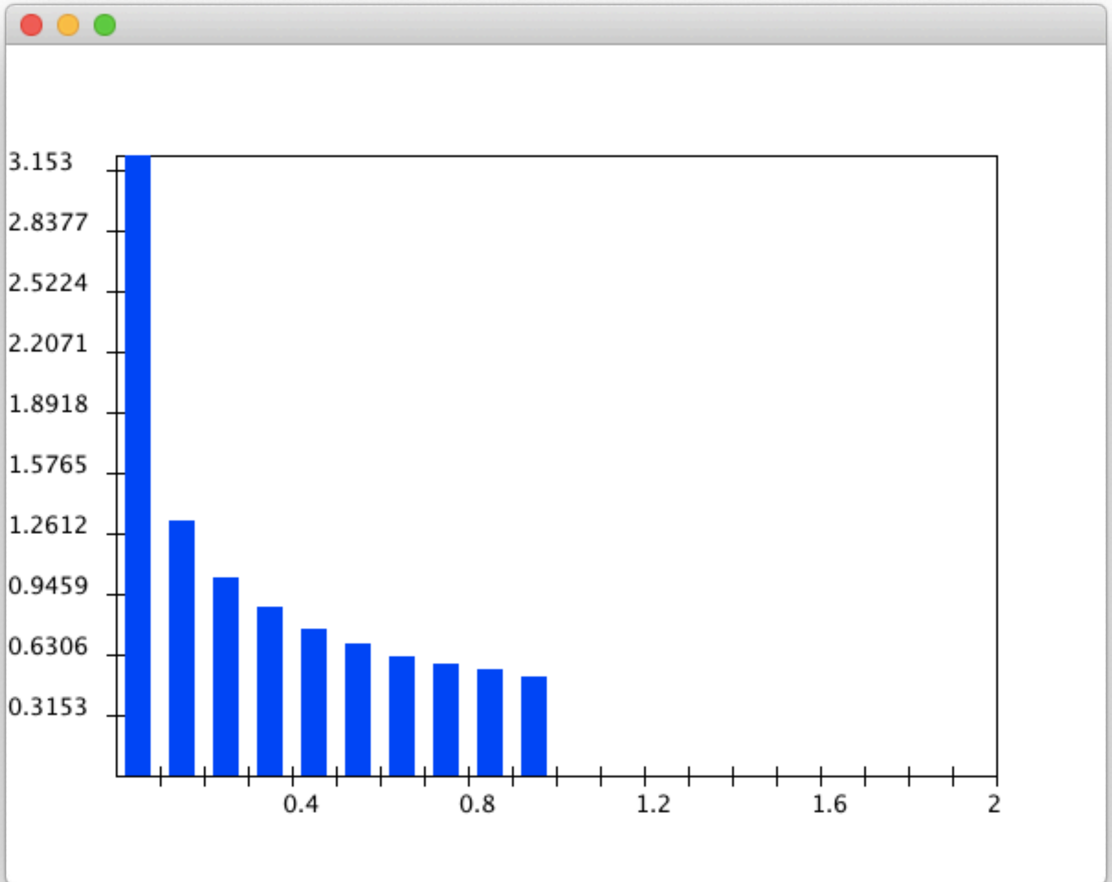
Result:

```
E(x)         = 0.5000630164767703
E(g(x)=x^2) = 0.33308660577988347
```

x


1.0144
0.913
0.8115
0.7101
0.6086
0.5072
0.4058
0.3043
0.2029
0.1014
0.4
0.8
1.2
1.6
2

$g(x) = x^2$



07

> Suppose X is a rv with range $9, 11$ and pmf $Pr[X = 9] = 0.5$ $Pr[X = 11] = 0.5$ and Y is a rv with range $8, 12$ and pmf $Pr[Y = 8] = 0.5$ $Pr[Y = 12] = 0.5$ and Z is a rv with range $8, 10, 12$ and pmf $Pr[Z = 8] = 0.1$ $Pr[Z = 10] = 0.8$ $Pr[Z = 12] = 0.1$ Compute $var[X], var[Y]$ and $var[Z]$.

$$
\begin{aligned}
E(X) &= \sum_k k \cdot P[X = k] = 10 \\
var[X] &= E[(X - E(X))^2] = 1
\end{aligned}
$$

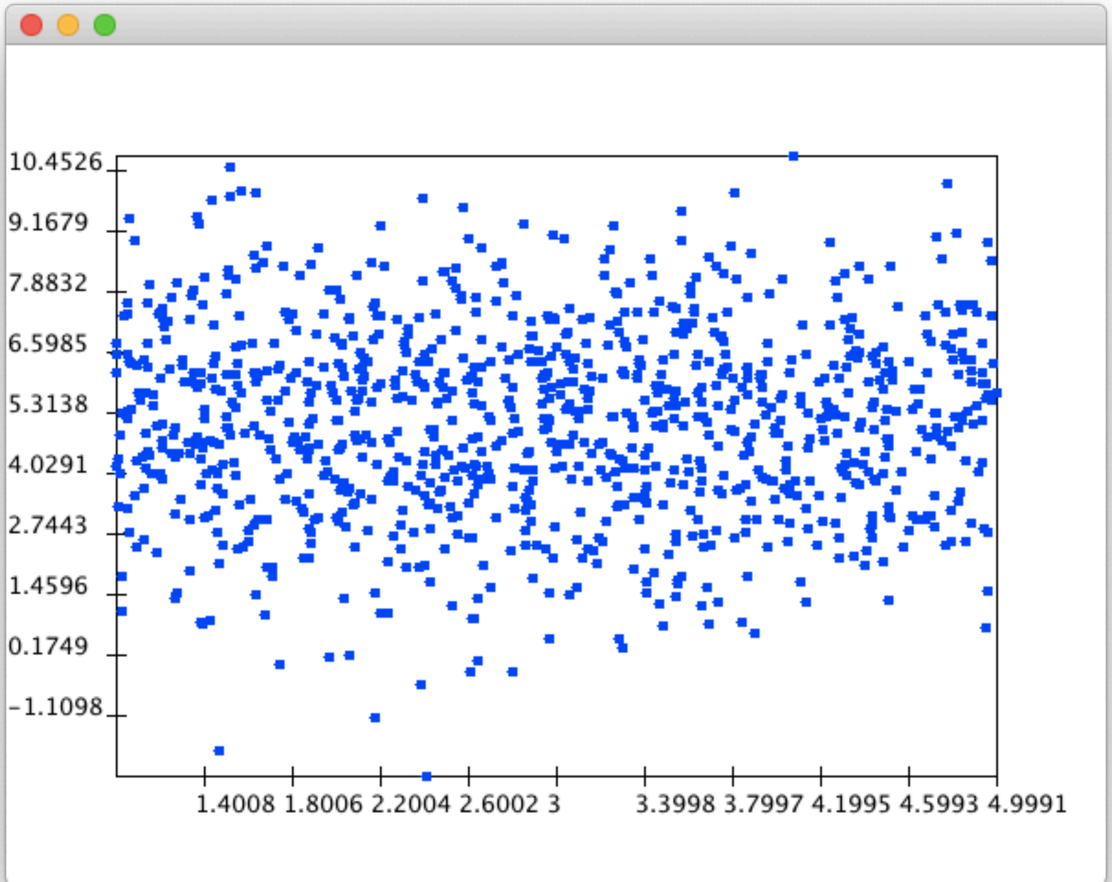
$$
\begin{aligned}
E(Y) &= \sum_k k \cdot P[Y = k] = 10 \\
var[Y] &= E[(X - E(Y))^2] = 4
\end{aligned}
$$

$$
\begin{aligned}
E(Z) &= \sum_k k \cdot P[Z = k] = 10 \\
var[Z] &= E[(X - E(Z))^2] = 2.667
\end{aligned}
$$

08

> Download and execute PointGeneratorExample.java. You will also need PointGenerator.java and PointDisplay.java. Increase the number of points to 1000. Can you guess the distribution of X? Of Y?

See file `PointGeneratorExample.java`.



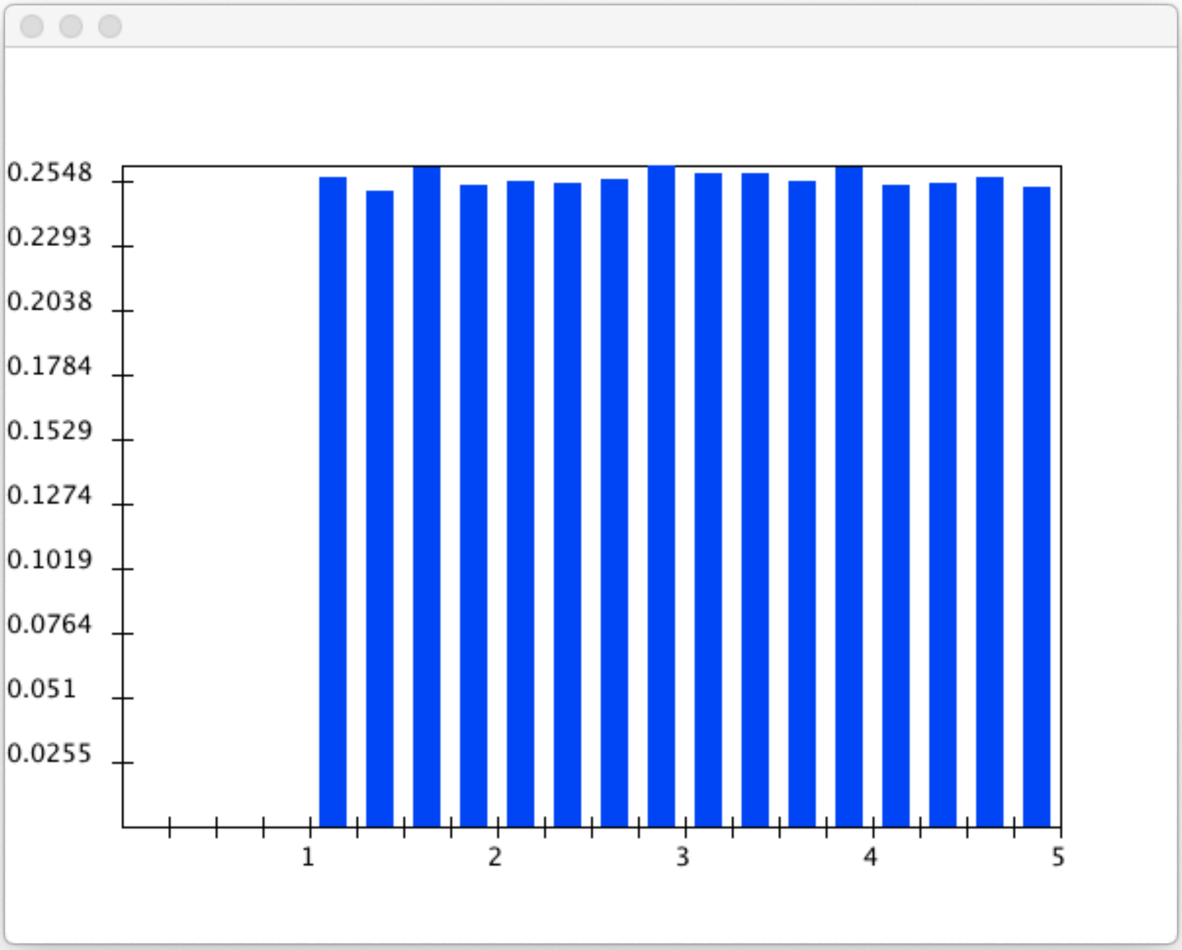
It seems that $X \sim Uniform(0, 5)$ and $Y \sim N(5, \sigma^2)$.

09

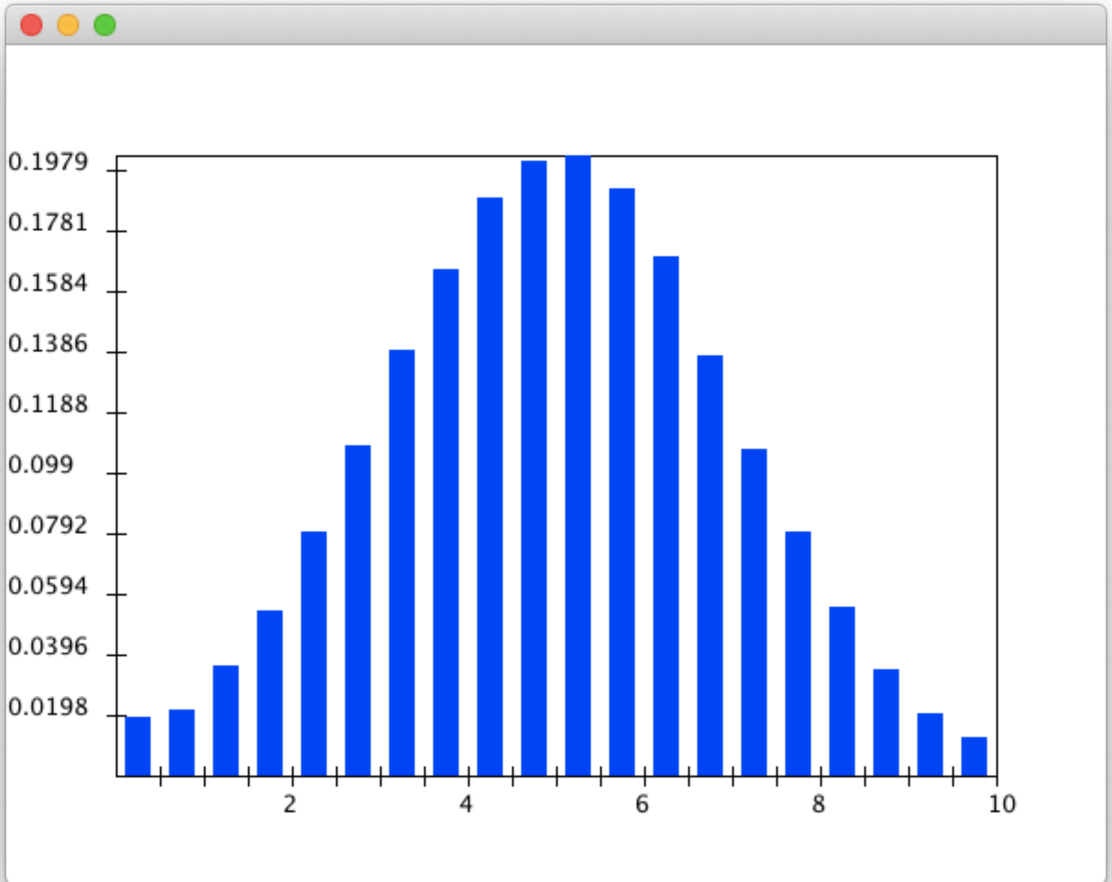
> Download and modify [PointGeneratorExample2.java]() to display density histograms for X and Y
> .

See file `PointGeneratorExample2.java`.

X


0.2548
0.2293
0.2038
0.1784
0.1529
0.1274
0.1019
0.0764
0.051
0.0255
1
2
3
4
5

Y



10

> Download and examine PointGeneratorExample3.java. The first part of the code estimates $Pr[Y \in [5,7]]$. Add code below to estimate $Pr[Y \in [5,7] | X \in [3,4]]$. Are these events independent?

See file `PointGeneratorExample3.java`.

Result:

```
Pr[Y in [5,7]]               = 0.34286
Pr[Y in [5,7] | X in [3,4]] = 0.341532114095291
```

X, Y are independent., since $Pr[Y \in [5,7]] = Pr[Y \in [5,7] | X \in [3,4]]$.

24

> Execute UniformVarianceExample.java and obtain the two estimates.

See file `UniformVarianceExample.java`.

Result:

```
Mean estimate: 0.5216167255358847
Std-dev estimate: 0.28058637197873465
```

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> If $\mu' = 0.48$ and $\sigma' = 0.27$ and obtain the number of samples needed.

$$f = 0.049$$

27

> If $\mu' = 0.48$ and $\sigma' = 0.27$, obtain f when $n = 500$ samples are used.

$$f = 0.049$$

28

> Express δ as a function of σ and n.

$$\delta = \frac{1.96\sigma'}{\sqrt{n}}$$

29

> Use Stats.java to collect statistics about the single-server queue in Queue.java.
>
> - Estimate the mean interarrival time. How many samples are needed for a 5% ($f = 0.05$) confidence interval?
> - Estimate the mean time in system. Again, how many samples are needed for a 5% ($f = 0.05$) confidence interval? Consider that the true mean system time is $E[S] = 4.0$ when the arrival rate is 0.75 and the service rate is 1.0.
>
> Explain why it may be inappropriate to use our usual procedure (the standard-normal approximation) to compute confidence intervals when estimating the mean system time.

See file `Queue.java`.

Result:

```
Simulation results:
  numArrivals:     1000
  numDepartures:   998
  avg Wait:        1.8906618446580803
  avg System Time: 2.814388534740915
```