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1. Introduction. Kinematics of point particle

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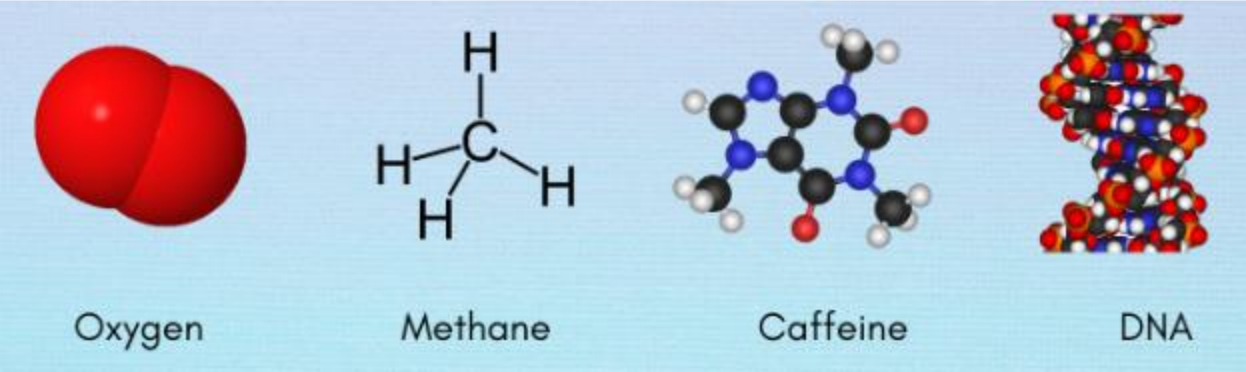
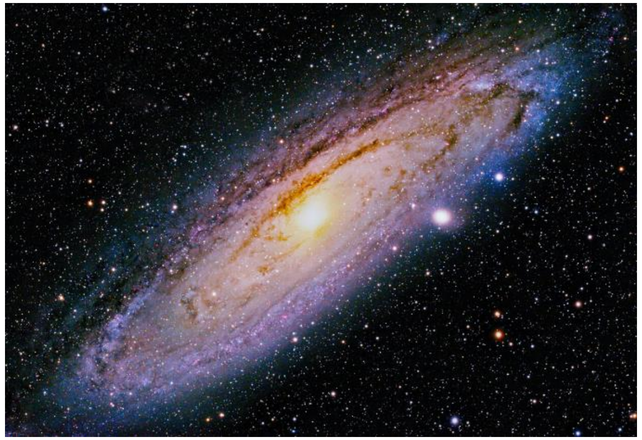
Introduction to Physics

The goal of physics is to know the world around us entirely from the *micro-cosmos (the structure of atoms and molecules)* to the *macro-cosmos (planets, galactics, etc.).*

Physics is defined as a **fundamental science** that studies the structure and properties of matter, the phenomena related to its transformations and the general laws that describe the processes in the universe.

One of the essential aims of learning physics is the correct and complete application in the productive practice of its laws.





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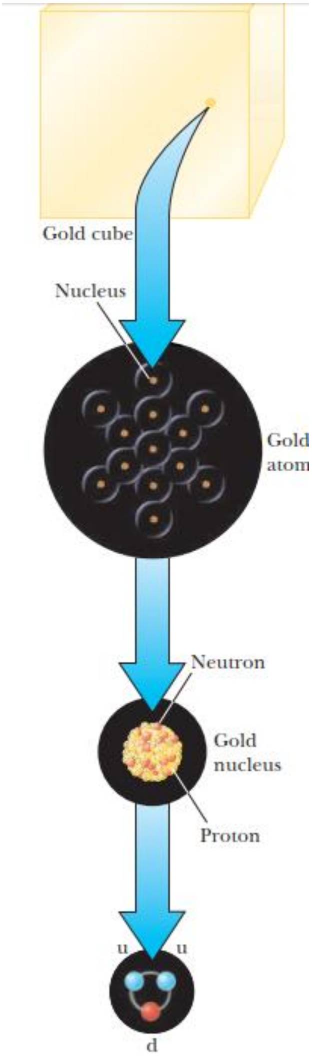
Introduction to Physics

Several forms of matter are known, but they can be grouped into two main forms: **substance** and **physical field**.

Substance is the form of existence of matter that constitutes a certain atomic-molecular structure.

Levels of organization in matter. Ordinary matter consists of atoms, and at the center of each atom is a compact nucleus consisting of protons and neutrons. Protons and neutrons are composed of quarks. The quark composition of a proton is shown.



Quark composition of a proton

The field is the material carrier of interactions and has the property of binding the particles of the substance into systems.

The fundamental property of matter and its mode of existence is **motion**.

Physics describes matter as something that exists in **space** and **time**.

The s***pace is three-dimensional, homogeneous and isotropic, and time is one-dimensional, homogeneous and irreversible.***

The homogeneity property of space means that all points of space are physically equivalent.

The homogeneity property of time is manifested in the physical equivalence of its moments. Different moments of time are equivalent in the sense that any physical process occurs in the same way.

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Introduction to Physics

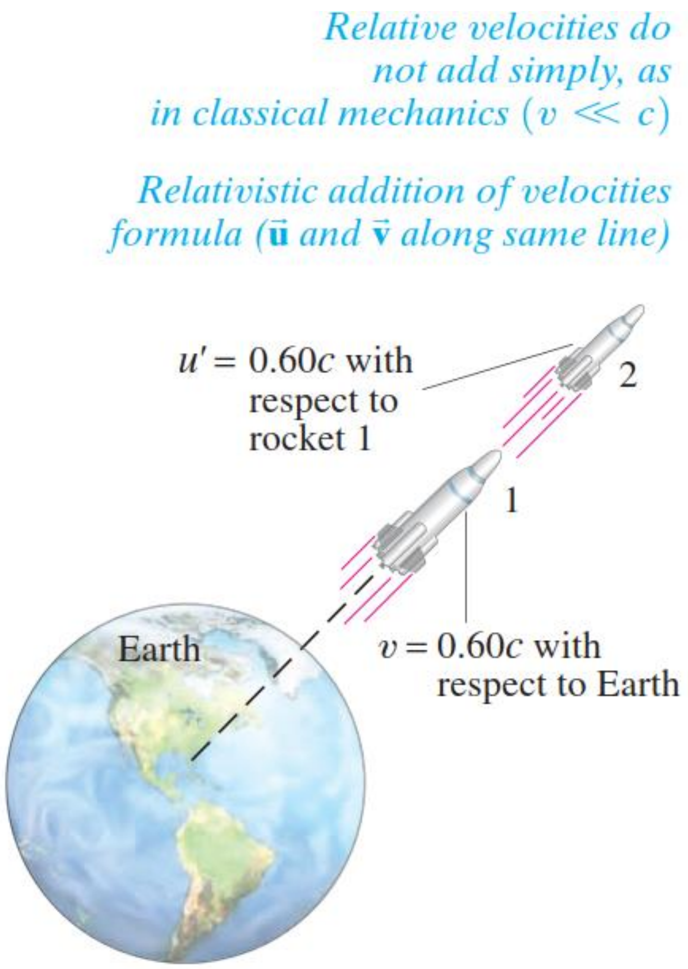
The isotropic property of space is expressed by the physical equivalence of different directions in space. Different directions in space are equivalent in the sense that in a system that has been rotated all processes occur in the same way as before rotation.

Physics studies the simplest and at the same time the most general forms of motion of matter: mechanical, atomic-molecular, electromagnetic, inter-atomic and intranuclear.

These various forms of physical motion are the most common, as they are contained in all the more complex forms of motion of matter, studied by other sciences.

Kinematics of point particle

Mechanics is the part of physics that studies the simplest and most general form of motion of matter that consists in the movement of bodies or their parts relative to each other. This movement is called ***mechanical motion.***



Depending on the value of the speed of bodies, mechanics is classified into ***classical mechanics*** $v \ll c_0$ and ***relativistic mechanics*** $v \sim c_0$.

$$u = \frac{v + u'}{1 + vu'/c^2}$$

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Kinematics of point particle

Classical mechanics studies the movements of bodies at speeds much slower than the speed of light in a vacuum, while relativistic mechanics studies the movements of those bodies that have speeds close to the speed of light in a vacuum.

Classical mechanics is divided into three parts: ***kinematics, dynamics*** and ***statics***.

Kinematics studies the movement of bodies without analyzing the causes that produce it.

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Kinematics of point particle

Dynamics studies the motion of bodies and its causes.

Statics studies the laws of equilibrium of a system of bodies. When the laws that describe the body motion are known, then the laws of equilibrium of the body system can be established. Thus, we conclude that the statics is a particular case of the dynamics.

The fundamental problem of classical mechanics is to ***determine the position of a moving body at any given time***.

The concept of movement or displacement in space only makes sense if another body is indicated, in relation to which the studied body is moving i.e. ***movement is relative***.

Therefore, the description of any movement is only possible if a reference frames is indicated.

The reference frames is the set consisting of the reference body, the coordinate system connected to it and a time measuring instrument (clock), all of which are considered fixed.

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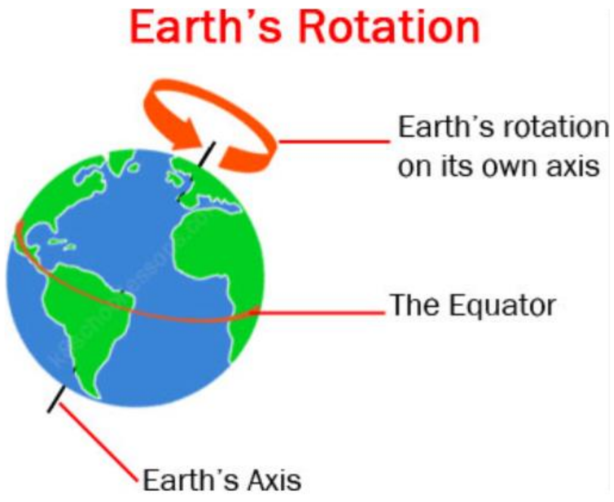
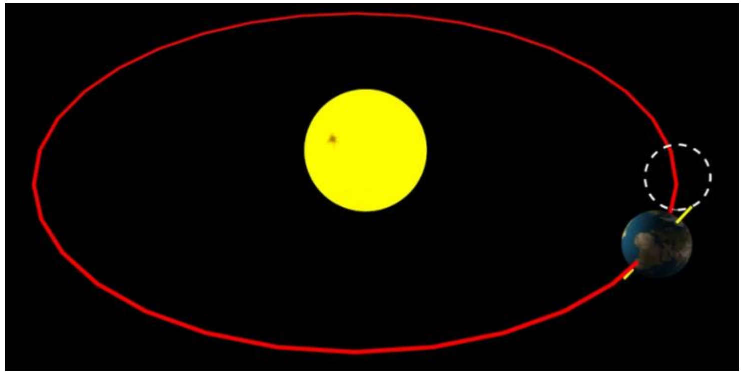
Kinematics of point particle

When describing the movement of bodies in mechanics, depending on the concrete problem, different models are used: ***point particle, absolutely rigid body, absolutely elastic body,*** etc.



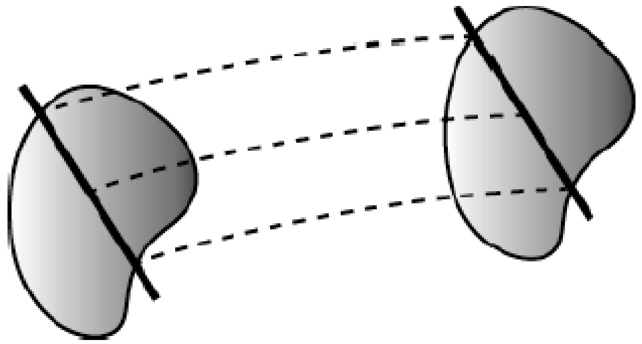
It is called the point particle of the body, the dimensions of which can be neglected compared to the distance traveled or the distances to other bodies.



Kinematics of point particle

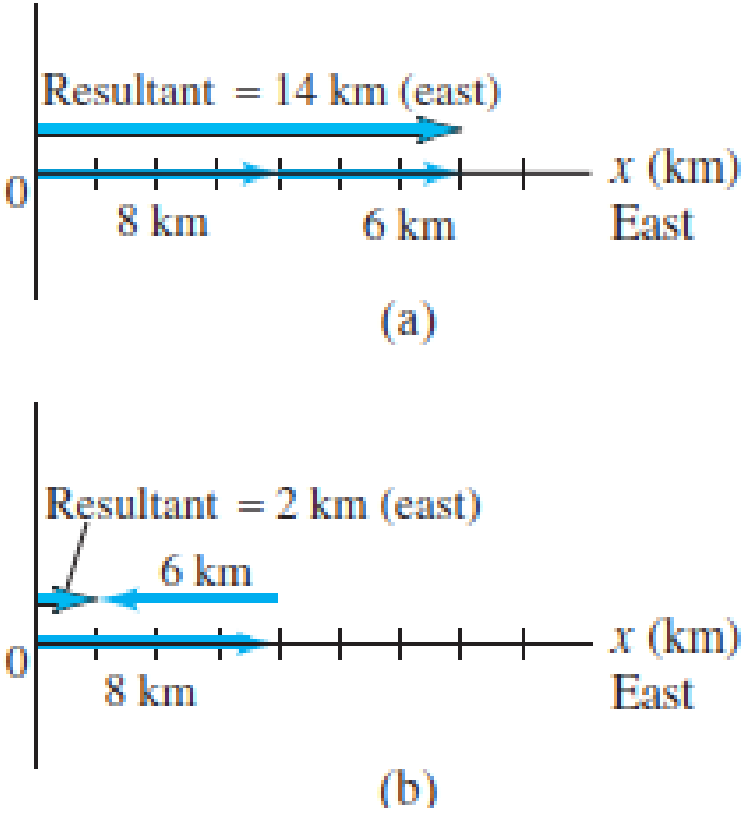
There is a class of problems in which the motion of a body is replaced by the motion of a point. These are the problems in which bodies perform a ***translational movement***.

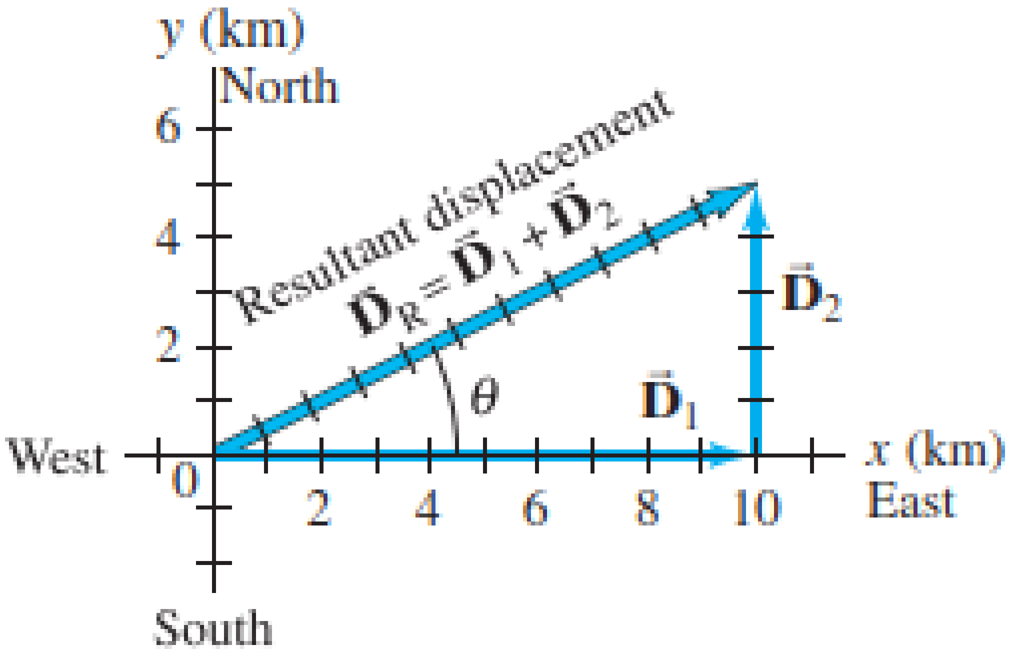
The motion of a body is called translational, if any straight line rigidly associated with moving body remain parallel.



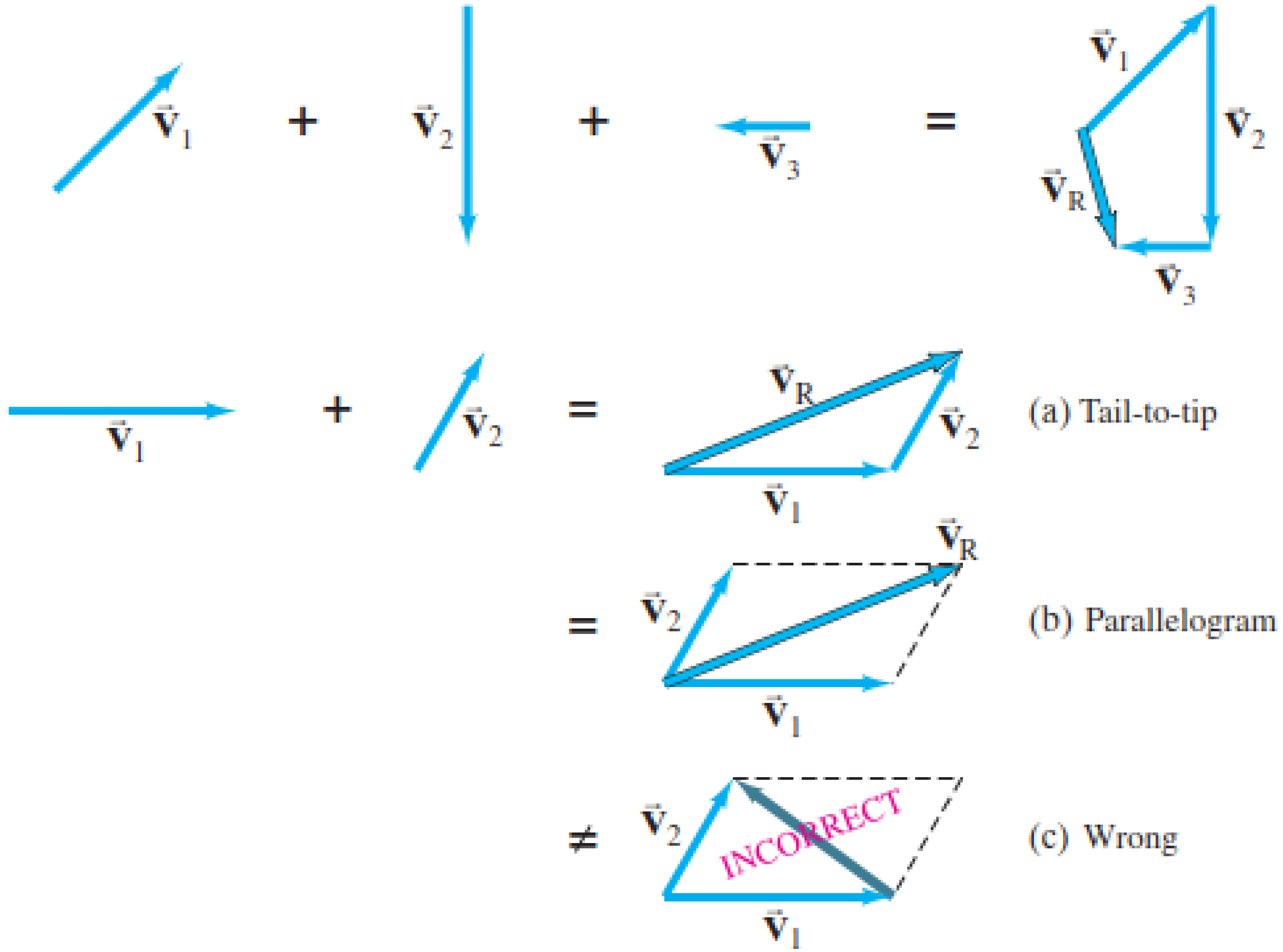
Vectors

Combining vectors in





Vectors


$\vec{\mathbf{v}}_1$
+
$\vec{\mathbf{v}}_2$
+
$\vec{\mathbf{v}}_3$
=
$\vec{\mathbf{v}}_1$
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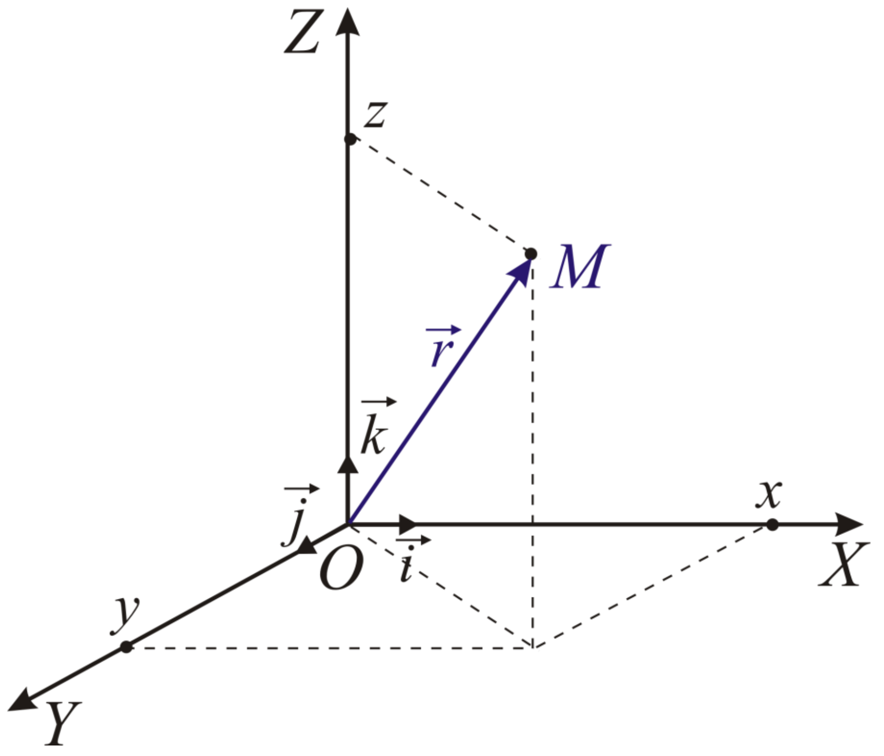
$\vec{\mathbf{v}}_R$
$\vec{\mathbf{v}}_3$
$\vec{\mathbf{v}}_1$
+
$\vec{\mathbf{v}}_2$
=
$\vec{\mathbf{v}}_R$
$\vec{\mathbf{v}}_2$
$\vec{\mathbf{v}}_1$
(a) Tail-to-tip
$\vec{\mathbf{v}}_R$
=
$\vec{\mathbf{v}}_2$
$\vec{\mathbf{v}}_1$
(b) Parallelogram
≠
$\vec{\mathbf{v}}_2$
INCORRECT
$\vec{\mathbf{v}}_1$
(c) Wrong

Kinematics of point particle

Let us analyze the general case of the motion of the material point on a curve in space with respect to a reference with a three-dimensional Cartesian coordinate system.

At some point in time the position of the body is determined by ***the position vector*** $\vec{r}$ which ***joins the origin O of the coordinate frame with the point M where the body is located.***



Kinematics of point particle

The motion of a point particle is characterized by the ***trajectory*** and the ***law of motion***.

The trajectory *represents the geometric place of all the successive points through which the body passes during the movement.*

The law of motion *is the law of variation as a function of time of the position vector of the moving point particle.*

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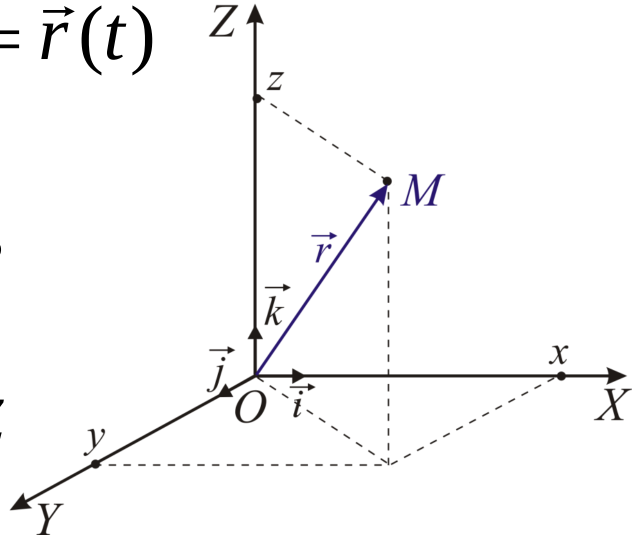
Kinematics of point particle

The motion is therefore determined when the function is known $\vec{r} = \vec{r}(t)$ which represents the vector form of the ***equation of motion***.



The position vector can also be written according to its coordinates:

$$\vec{r} = x \cdot \vec{i} + y \cdot \vec{j} + z \cdot \vec{k},$$

where $\vec{i}, \vec{j}, \vec{k}$ – are the ***verses*** (*unit vectors*) of the *OX*, *OY* and *OZ* axes of the chosen reference frame.

Therefore, the motion is also known if we know how the coordinates of the point particle change over time, i.e. the laws of motion in Cartesian coordinates:

$$\begin{cases} x = x(t) \\ y = y(t). \\ z = z(t) \end{cases}$$

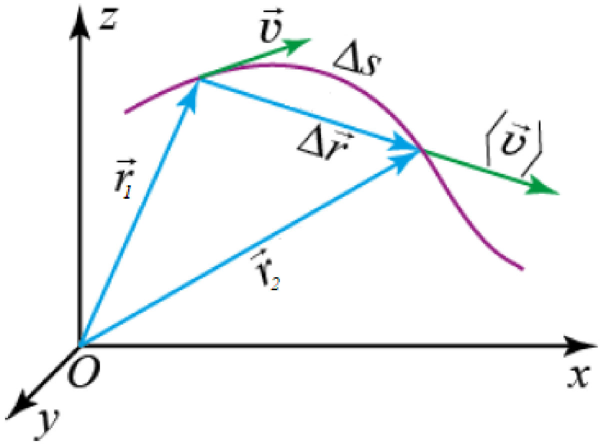
Eliminating time from these relations results in the equation of the trajectory of the material point.

Kinematics of point particle

Changing the position of the material point determines the ***displacement vector***,

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

corresponding to the time interval $\Delta t = t_2 - t_1$.



The speed of variation of the position of the point particle can be characterized by the ***average velocity vector***

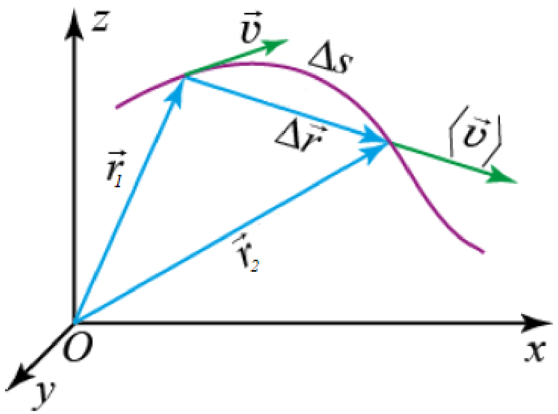
$$\vec{v}_{med} = \langle\vec{v}\rangle = \frac{\Delta\vec{r}}{\Delta t}, \qquad [v]_{SI} = \frac{\text{m}}{\text{s}}.$$

We notice that for different time intervals Δt the average speed can have not only different meanings, but also different values. It does not contain enough information about the movement of the point particle at each point of the trajectory.

Kinematics of point particle

THUS we introduce ***instantaneous speed:***

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$



When $\Delta t \to 0$ the secant $\Delta \vec{r}$ tends to the tangent to the trajectory. Therefore the instantaneous velocity vector $\vec{v}$ is oriented along the tangent to the trajectory in the direction of the motion of the point particle.

As the time when Δt decreases Δs gets closer to $|\Delta \vec{r}|$.

Because of this

$$v = |\vec{v}| = \left| \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \right| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}.$$

Thus, the numerical value of the instantaneous speed is equal to the first derivative of traveled s in relation to time.

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Kinematics of point particle

Considering $\vec{r} = x \cdot \vec{i} + y \cdot \vec{j} + z \cdot \vec{k},$ we get $\vec{v} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k},$

or $\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k},$ where $v_x = \frac{dx}{dt},\ v_y = \frac{dy}{dt},\ v_z = \frac{dz}{dt}.$

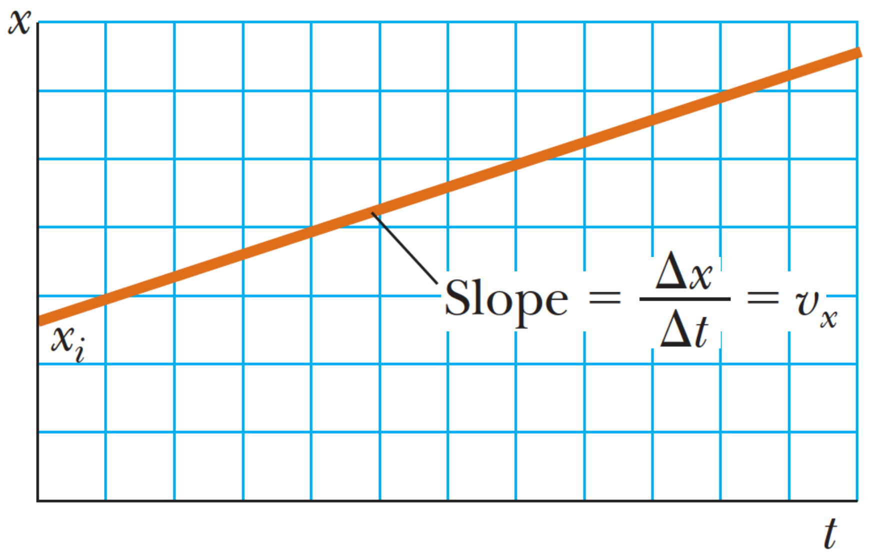
The absolute value of the velocity vector is calculated by the relation:

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}.$$

Kinematics of point particle

When we know the components of velocity υ_x, υ_y, υ_z, ie velocity $\vec{\upsilon}$, the coordinates of the point particle *x*, *y*, *z* can be determined at any time:

$$\upsilon_x = \frac{dx}{dt} \Rightarrow dx = \upsilon_x \cdot dt \Rightarrow \int_{x_0}^{x} dx = \int_{t_0}^{t} \upsilon_x dt \Rightarrow \qquad x - x_0 = \int_{t_0}^{t} \upsilon_x dt,$$



Position–time graph for a particle under constant velocity. The value of the constant velocity is the slope of the line.

Kinematics of point particle

For $$v_y = \frac{dy}{dt} \Rightarrow dy = v_y \cdot dt \Rightarrow \int_{y_0}^{y} dy = \int_{t_0}^{t} v_y dt \Rightarrow \quad y - y_0 = \int_{t_0}^{t} v_y dt,$$

$$v_z = \frac{dz}{dt} \Rightarrow dz = v_z \cdot dt \Rightarrow \int_{z_0}^{z} dz = \int_{t_0}^{t} v_z dt \Rightarrow \quad z - z_0 = \int_{t_0}^{t} v_z dt.$$

When $\left(|\vec{v}| = const\right)$ ***- the motion is called uniform,*** and the law of motion is:

$$x = x_0 + v_x(t - t_0), \quad y = y_0 + v_y(t - t_0), \quad z = z_0 + v_z(t - t_0).$$

When the velocity of body does not remain constant in size, the motion is called ***non-uniform***.

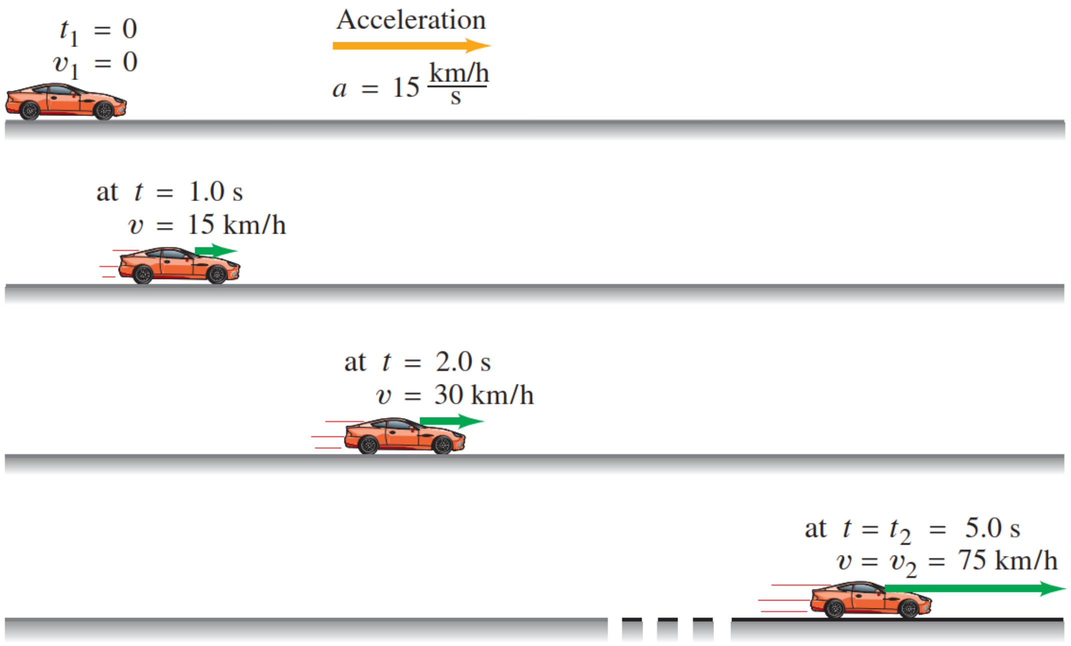
Kinematics of point particle

***Acceleration** characterizes the speed of variation of the velocity vector.*

We call the ***average acceleration*** the vector quantity equal to the ratio of the variation of the velocity $\Delta\vec{v}$ to the time interval *Δt:* $\vec{a}_{med} = \langle\vec{a}\rangle = \dfrac{\Delta\vec{v}}{\Delta t}$, $[a]_{SI} = \dfrac{\text{m}}{\text{s}^2}$.

A more detailed motion feature is possible with the use of ***instantaneous acceleration***

$$\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}.$$

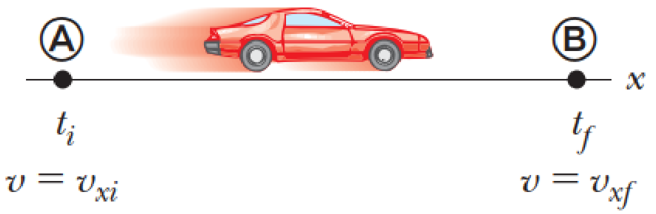


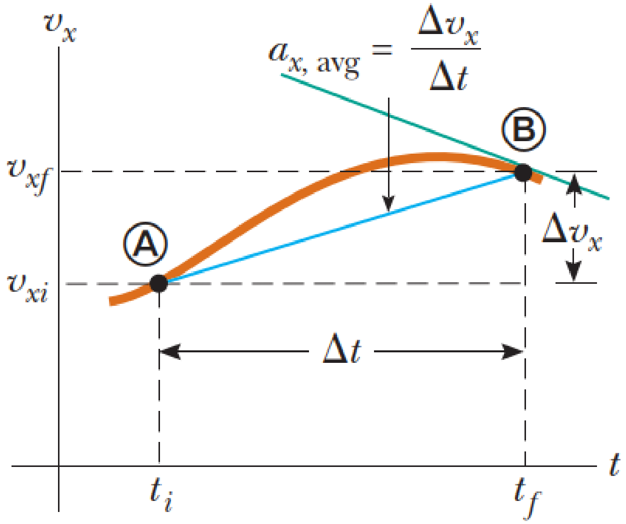
Kinematics of point particle

Given that $\vec{v} = v_x \vec{i} + v_y \vec{j} + v_z \vec{k}$

We obtain $\vec{a} = \frac{dv_x}{dt} \cdot \vec{i} + \frac{dv_y}{dt} \cdot \vec{j} + \frac{dv_z}{dt} \cdot \vec{k}$ or $\vec{a} = a_x \cdot \vec{i} + a_y \cdot \vec{j} + a_z \cdot \vec{k}$.

Acceleration magnitude (absolute value): $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$.





A car, modeled as a particle, moving along the x axis from A toB

Velocity–time graph (brown) for the particle moving in a straight line. The slope of the blue straight line connecting A and B is the average acceleration of the car during the time interval Dt = tf- ti. The slope of the green line is the instantaneous acceleration of the car at point B.

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Kinematics of point particle

When we know the acceleration projections $a_x, a_y, a_z,$ the respective projections of the velocity of the material point can be determined :

$$a_x = \frac{dv_x}{dt} \Rightarrow dv_x = a_x \cdot dt \Rightarrow \int_{v_{x0}}^{v_x} dv_x = \int_{t_0}^{t} a_x \cdot dt \Rightarrow v_x = v_{x0} + a_x(t - t_0).$$

The equation of uniformly varied along to a line motion is obtained by considering the constant acceleration and substituting the velocity obtained from expression $dx = v_x dt,$

$$dx = (v_{x0} + a_x t)dt \Rightarrow \int_{x_0}^{x} dx = \int_{0}^{t} v_{x0} \cdot dt + \int_{0}^{t} a_x \cdot t \cdot dt \Rightarrow$$

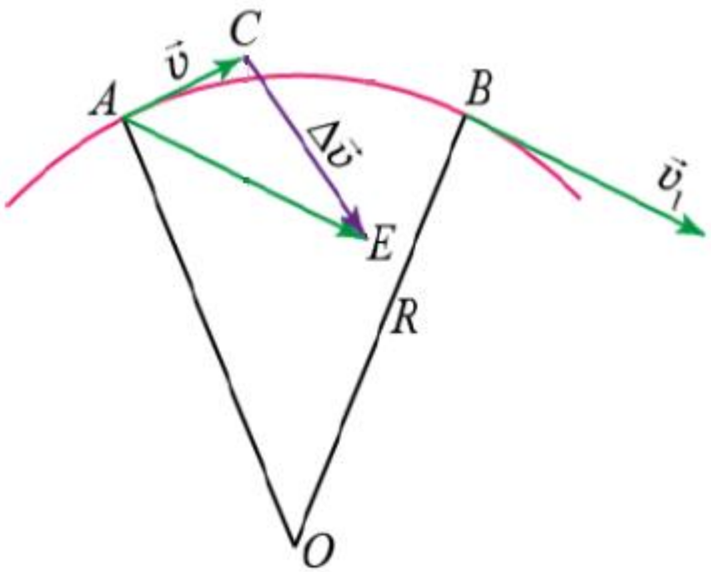
$$x = x_0 + v_{x0}t + \frac{a_x t^2}{2}$$ – ***the law of uniformly varied linear motion.***

If $a_x = const > 0 \ (v_{x1} > v_{x0})$ – ***uniformly accelerated movement,***

and $a_x = const < 0 \ (v_{x1} < v_{x0})$ – ***evenly slow motion.***

Kinematics of point particle

In what follows we consider a body that moves on a curved trajectory and we admit that in point A it has speed $\vec{v}$, and in point B – speed $\vec{v}_1 = \vec{v} + \Delta\vec{v}$.



We notice that at the curvature motion of the point particle the velocity vector can vary not only in the mode, as in the case of the rectilinear motion, but also by the direction.

Moving the vector through a parallel translational $\vec{v}_1$ with origin at point A, we find the difference vector $\Delta\vec{v} = \vec{v}_1 - \vec{v}$.

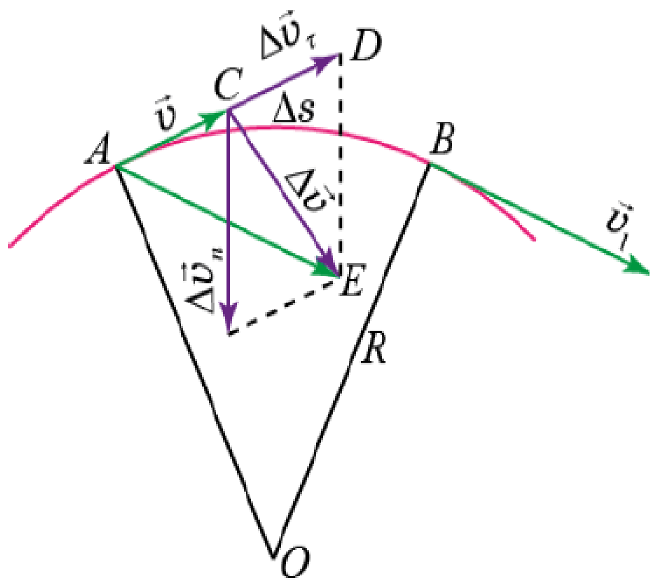
From the geometry of the figure we realized that the vector $\Delta\vec{v}$ can be decomposed into two components.

Kinematics of point particle

From point A in the direction of velocity $\vec{\upsilon}$ we draw the vector $\overrightarrow{AD}$ equal in the mode with the vector $\vec{\upsilon}_1$.

Thus its first component $\Delta\vec{\upsilon}$ is the vector $\overrightarrow{CD}$ noted with $\Delta\vec{\upsilon}_\tau$ represents the variation of the velocity modulus over time Δt:

$$\Delta\vec{\upsilon}_\tau = \vec{\upsilon}_1 - \Delta\vec{\upsilon}.$$

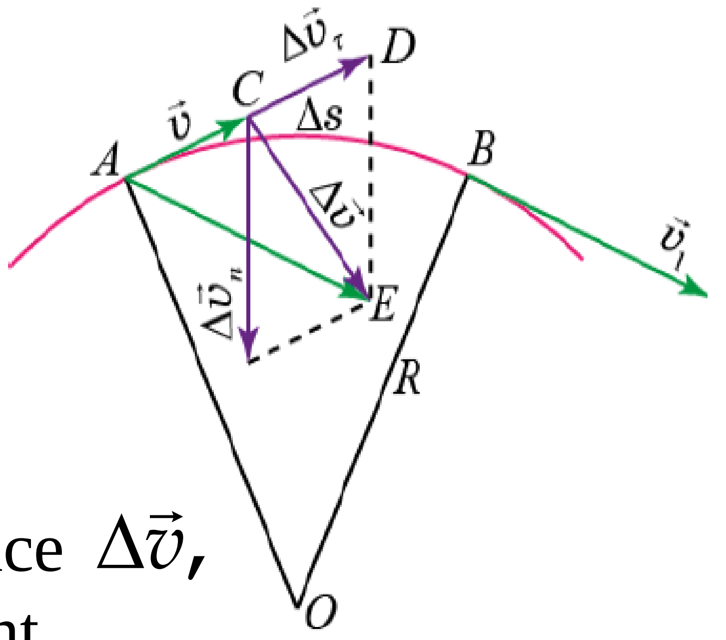


The second component of the vector $\Delta\vec{\upsilon}$, namely $\Delta\vec{\upsilon}_n$, characterizes the variation of the direction velosity in the same time interval Δt.

Le us determine how fast the speed mode changes over time.

Kinematics of point particle

For the first component of the difference vector $\Delta\vec{v}$, tangentially oriented to the trajectory of the material point we will have:



$$a_\tau = \lim_{\Delta t \to 0} \frac{\Delta v_\tau}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}.$$

This is called ***tangential acceleration.***

Le us to determine the second component of the difference $\Delta\vec{v}$, vector oriented perpendicular to the trajectory of the material point.

For this we assume that point *B* is quite close to point *A*. In this case we can consider that Δs is an arc of circle with *radius R* that tends towards the chord *AB*

$$\left(AB \approx \Delta s \approx v \cdot \Delta t\right).$$

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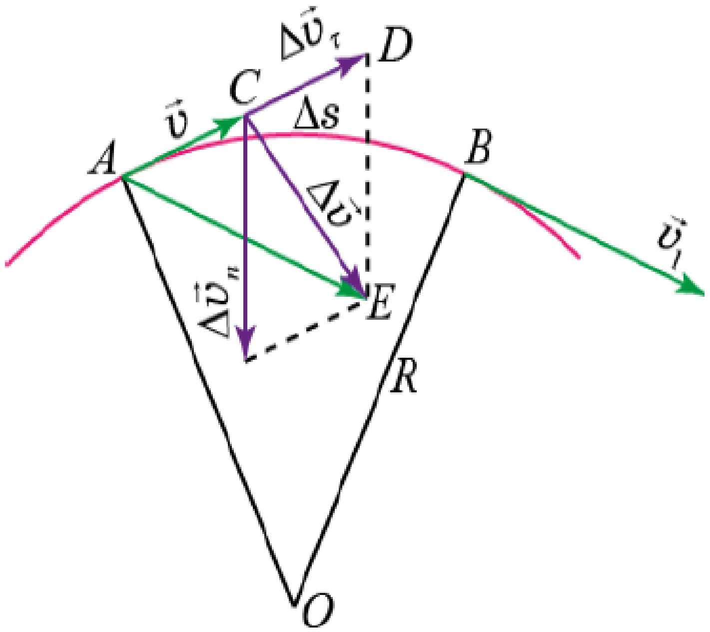
Kinematics of point particle

From the similarity of the triangles *ADE* and *OAB* it can be written:

$$\frac{DE}{AE}=\frac{AB}{BO}\Rightarrow\frac{|\Delta v_n|}{v_1}=\frac{\Delta s}{R}\Rightarrow$$



or $$\frac{|\Delta v_n|}{v_1}=\frac{v\cdot\Delta t}{R}\Rightarrow\frac{|\Delta v_n|}{\Delta t}=\frac{v\cdot v_1}{R}.$$

When $\Delta t\to 0$ $v_1\to v$, vector $\Delta\vec{v}_n$ it is oriented towards the center of curvature of the trajectory at the given point.

$$a_R=a_n=\lim_{\Delta\to 0}\frac{|\Delta\vec{v}_n|}{\Delta t}=\frac{v^2}{R}$$ – called **centripetal** (***normal) acceleration***.

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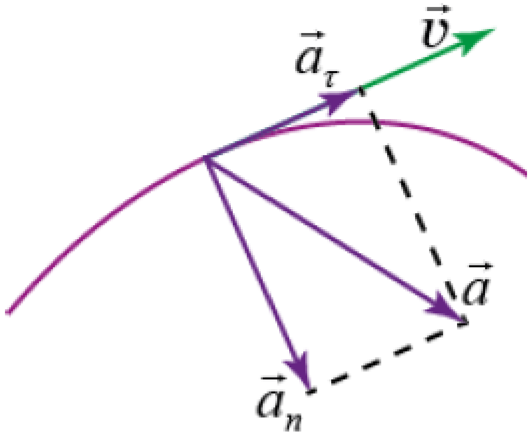
Kinematics of point particle

The total acceleration is equal to the vector sum of the tangential and centripetal acceleration components



$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{a}_\tau + \vec{a}_n.$$

Total acceleration magnitude $a = \sqrt{a_\tau^2 + a_n^2}.$

Thus ***the tangential component of the acceleration characterizes the speed of variation of the speed module, being oriented along the tangent to the trajectory, and the centripetal one - the speed of variation of the speed direction, being oriented towards the center of curvature of the trajectory at the considered point.***

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Kinematics of point particle