3 Demuestre $A^{i'}_{k}\tilde{A}^{j}_{i'} = \delta^{j}_{k}$

$$A^{i'}_{k}\tilde{A}^{j}_{i'} = \delta^{j}_{k}$$

$$\rightarrow \frac{\partial x^{i'}}{\partial x_{k}} \frac{\partial^{j}}{\partial_{i'}} = \delta^{j}_{k}$$

Por regla de la cadena

$$\rightarrow A^{i'}_{k} \tilde{A}^{j}_{i'} = \delta^{j}_{k} = \frac{\partial x^{j}}{\partial x^{k}}$$

• si $k=j$

$$A^{i'}_{k}\tilde{A}^{j}_{i'} = \frac{\partial x^{j}}{\partial x^{k}} = \frac{\partial x^{j}}{\partial x^{j}} = \frac{\partial x^{k}}{\partial x^{k}} = 1 = \delta^{j}_{k} = \delta^{k}_{k} = \delta^{j}_{j}$$

si $k \neq j$

• $A^{i'}_{k}\tilde{A}^{j}_{i'} = \delta^{j}_{k} = 0$

- establecer la relacion

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

• en un cambio de coordenadas, los elementos de la matriz de transformacion pueden expresarse en terminos de los cosenos de los angulos entre los nuevos ejes y los antiguos

- α, β, γ seran los angulos que forman el nuevo eje con los antiguos

$$A^{1'}_{1} = \cos\alpha,\ A^{1'}_{2} = \cos\beta,\ A^{1'}_{3} = \cos\gamma$$

$$\tilde{A}^{j}_{i'} = A^{i'}_{j}$$

$$A^{i'}_{k} A^{i'}_{j} = \delta^{j}_{k}$$

en este caso $k=j$

$$A^{1'}_{1}A^{1'}_{1} + A^{2'}_{1}A^{2'}_{1} + A^{3'}_{1}A^{3'}_{1} = 1$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

4) Considere el radio vector posicion $r = x^i \hat{i}_i = x\hat{i} + y\hat{j}$ en 2 dimensiones. Demuestre en cuales casos las componentes de r transforman como verdaderas componentes de vectores

a) $\begin{bmatrix} x^1 \\ x^2 \end{bmatrix} \rightarrow \begin{bmatrix} -x^2 \\ x^1 \end{bmatrix}$

$$x^{i'} = A^{i'}_j x^j$$

$$\rightarrow \begin{bmatrix} x^{1'} \\ x^{2'} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x^1 \\ x^2 \end{bmatrix} = \begin{bmatrix} -x^2 \\ x^1 \end{bmatrix}$$ transforma como vector

b) $\begin{bmatrix} x^1 \\ x^2 \end{bmatrix} \rightarrow \begin{bmatrix} x^1 \\ -x^2 \end{bmatrix}$

$$x^{i'} = A^{i'}_j x^j$$

$$\rightarrow \begin{bmatrix} x^{1'} \\ x^{2'} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x^1 \\ x^2 \end{bmatrix} = \begin{bmatrix} x^1 \\ -x^2 \end{bmatrix}$$ transforma como vector

c) $\begin{bmatrix} x^1 \\ x^2 \end{bmatrix} \rightarrow \begin{bmatrix} -x^2 \\ x^1 + x^2 \end{bmatrix}$

$$x^{i'} = A^{i'}_j x^j$$

$$\begin{bmatrix} x^{1'} \\ x^{2'} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x^1 \\ x^2 \end{bmatrix} = \begin{bmatrix} -x^2 \\ x^1 + x^2 \end{bmatrix}$$ transforma como vector

d) $\begin{bmatrix} x^1 \\ x^2 \end{bmatrix} \rightarrow \begin{bmatrix} x^1 + x^2 \\ x^1 - x^2 \end{bmatrix}$

$$x^{i'} = A^{i'}_j x^j$$

$$\begin{bmatrix} x^{1'} \\ x^{2'} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x^1 \\ x^2 \end{bmatrix} = \begin{bmatrix} x^1 + x^2 \\ x^1 - x^2 \end{bmatrix}$$ transforma como vector

2a

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$
$$\partial_i(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$
$$(\partial_i\phi)\psi + (\partial_i\psi)\phi = \phi(\partial_i\psi) + \psi(\partial_i\phi)$$

2d

- $\nabla\cdot(\nabla\times a) = 0$. ¿Que decir de $\nabla\times(\nabla\cdot a)$?

$$\nabla(\nabla\times a) = \partial_i(\nabla\times a)^i = \partial_i(\varepsilon^{ijk}\partial_j a_k) = \overbrace{\varepsilon^{ijk}}^{\text{asimetrico}}\underbrace{\partial_i\partial_j}_{\text{simetrico}} a_k = 0$$

- $\nabla\times(\nabla\cdot a) = ?$

- $\nabla\cdot a =$ es un escalar (divergencia de a)
- El rotacional $\nabla\times$ solo esta definido para campos vectoriales, asi que $\nabla\times(\nabla\cdot a)$ no esta definido

2F

$$\nabla\times(\nabla\times a) = \nabla(\nabla\cdot a) - \nabla^2 a$$

$$\nabla\times(\nabla\times a) = \varepsilon^{ijk}\partial_j((\nabla\times a)_k) = \varepsilon^{ijk}\partial_j(\varepsilon_{klm}\partial_l a_m)$$

$$\varepsilon^{ijk}\varepsilon_{klm} = \delta^i_l\delta^j_m - \delta^i_m\delta^j_l$$

$$\nabla\times(\nabla\times a) = (\delta^i_l\delta^j_m - \delta^i_m\delta^j_l)\partial_j\partial_l a_m = \partial_m\partial_i a_m - \partial_l\partial_l a^i$$

$$= \nabla(\nabla\cdot a) - \nabla^2 a$$

2 Demuestre

- $\cos(3\alpha) = \cos^3\alpha - 3\cos\alpha\,\text{sen}^2\alpha$
- $\text{Sen}(3\alpha) = 3\cos^2\alpha\,\text{sen}\,\alpha - \text{sen}^3\alpha$

$$z = \cos\alpha + i\,\text{sen}\,\alpha$$
$$z^3 = (\cos\alpha + i\,\text{sen}\,\alpha)^3$$
$$\cos 3\alpha + i\,\text{sen}\,3\alpha = \cos^3\alpha + 3\cos^2\alpha(i\,\text{sen}\,\alpha) + 3\cos\alpha(i\,\text{sen}\,\alpha)^2 + (i\,\text{sen}\,\alpha)^3$$
$$\cos 3\alpha + i\,\text{sen}\,3\alpha = (\cos^3\alpha - 3\cos\alpha\,\text{sen}^2\alpha) + i(3\cos^2\alpha\,\text{sen}\,\alpha - \text{sen}^3\alpha)$$

$$\cos 3\alpha = \cos^3\alpha - 3\cos\alpha\,\text{sen}^2\alpha$$
$$i\,\text{sen}\,3\alpha = i(3\cos^2\alpha\,\text{sen}\,\alpha - \text{sen}^3\alpha)$$

5 Encuentre las raíces

a) $\sqrt{2i}$

$$z^{\frac{1}{n}} = |z|^{1/n}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\,\text{sen}\left(\frac{\theta + 2\pi k}{n}\right)\right] \; ; \tan\theta = \frac{y}{x} \; ; z = x + yi$$

$k = 0, \dots, n-1$; $n = 2$; $\theta = \frac{\pi}{2} \rightarrow$ ya que $x = 0$ y $y > 0$

$z = 2i$; $|z| = \sqrt{2^2} = 2$

- $k = 0 \rightarrow z_0 = \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + i\,\text{sen}\left(\frac{\pi}{4}\right)\right] = 1 + i$
- $k = 1 \rightarrow z_1 = \sqrt{2}\left[\cos\left(\frac{3\pi}{4}\right) + i\,\text{sen}\left(\frac{3\pi}{4}\right)\right] = -1 - i$

b) $\sqrt{1 - \sqrt{3}i}$

$z = 1 - \sqrt{3}i$; $|z| = \sqrt{1^2 + \sqrt{3}^2} = 2$; $\tan\theta = \frac{-\sqrt{3}}{1} \rightarrow \theta = \frac{-\pi}{3}$

- $k = 0$

$$z_0 = \sqrt{2}\left[\cos\left(-\frac{\pi}{6}\right) + i\,\text{sen}\left(-\frac{\pi}{6}\right)\right] = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$$

- $k = 1$

$$z_1 = \sqrt{2}\left[\cos\left(\frac{5\pi}{6}\right) + i\,\text{sen}\left(\frac{5\pi}{6}\right)\right] = -\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$$

c) $(-1)^{1/3}$

$z = -1$; $|z| = 1$ $x < 0$ $\rightarrow \theta = \pi$

- $k = 0$

$$z_0 = 1\left[\cos\left(\frac{\pi}{3}\right) + i\,\text{sen}\left(\frac{\pi}{3}\right)\right] = \frac{1}{2} + \frac{\sqrt{3}}{2}i \; ;$$

- $k = 1$

$$z_1 = 1\left[\cos\pi + i\,\text{sen}\,\pi\right] = -1$$

- $k = 2$

$$z_2 = 1\left[\cos\left(\frac{5\pi}{3}\right) + i\,\text{sen}\left(\frac{5\pi}{3}\right)\right] = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

d) $8^{1/6}$

$Z=8$; $|Z|=8$; $x>0 \rightarrow \theta=0$

• $K=0$

$Z_0=\sqrt{2}\left[\cos(0)+i\,sen(0)\right]=\sqrt{2}$

• $K=1$

$Z_1=\sqrt{2}\left[\cos\left(\frac{\pi}{3}\right)+i\,sen\left(\frac{\pi}{3}\right)\right]=\frac{\sqrt{2}}{2}+i\,\frac{\sqrt{6}}{2}$

• $K=2$

$Z_2=\sqrt{2}\left[\cos\left(\frac{2\pi}{3}\right)+i\,sen\left(\frac{2\pi}{3}\right)\right]=-\frac{\sqrt{2}}{2}+i\,\frac{\sqrt{6}}{2}$

• $K=3$

$Z_3=\sqrt{2}\left[\cos(\pi)+i\,sen(\pi)\right]=-\sqrt{2}$

• $K=4$

$Z_4=\sqrt{2}\left[\cos\left(\frac{4\pi}{2}\right)+i\,sen\left(\frac{\pi 4}{3}\right)\right]=-\frac{\sqrt{2}}{2}-i\frac{\sqrt{6}}{2}$

• $K=5$

$Z_5=\sqrt{2}\left[\cos\left(\frac{5\pi}{3}\right)+i\,sen\left(\frac{5\pi}{3}\right)\right]=\frac{\sqrt{2}}{2}-i\,\frac{\sqrt{6}}{2}$

e) $\sqrt[4]{-8-8\sqrt{3}i}$

$z=-8-8\sqrt{3}i$; $|z|=16$; $\tan\theta=\left(\frac{-8\sqrt{3}}{-8}\right)\rightarrow\theta=\frac{\pi}{3}$ (como esta en el tercer cuadrante $\theta=\frac{4\pi}{3}$)

• $K=0$

$z_0=2\left[\cos\left(\frac{\pi}{3}\right)+i\,sen\left(\frac{\pi}{3}\right)\right]=1+\sqrt{3}i$

$K=1$

$z_1=2\left[\cos\left(\frac{5\pi}{6}\right)+i\,sen\left(\frac{5\pi}{6}\right)\right]=-\sqrt{3}+i$

• $K=2$

$z_2=2\left[\cos\left(\frac{4\pi}{3}\right)+i\,sen\left(\frac{4\pi}{3}\right)\right]=-1-\sqrt{3}i$

$K=3$

$z_3=2\left[\cos\left(\frac{11\pi}{6}\right)+i\,sen\left(\frac{\pi 11}{6}\right)\right]=\sqrt{3}-i$

6) Demuestre

a) $\log(-ie) = 1 - \frac{\pi}{2} i$

$\text{Log}(z) = \ln|z| + i(\theta + 2\pi n)$

$z = -ie \; ; |z| = \sqrt{e^2} = e \; ; \theta = -\frac{\pi}{2} \; (x=0; y<0)$

con $n=0$

$\log(-ie) = \ln(e) + i\left(-\frac{\pi}{2} + 2\pi n\right) = 1 - \frac{\pi}{2} i$

b) $\log(1-i) = \frac{1}{2}\ln(2) - \frac{\pi}{4} i$

$z = 1-i \; ; |z| = \sqrt{2} \; ; \theta = -\frac{\pi}{4}$

con $n=0$

$\log(1-i) = \ln\sqrt{2} + i\left(-\frac{\pi}{4} + 2\pi n\right) = \frac{1}{2}\ln(2) - \frac{\pi}{4} i$

c) $\log(e) = 1 + 2n\pi i$

$z = e \; ; |z| = e \; ; \theta = 0$

$\log(e) = \ln e + i(0 + 2\pi n)$

$= 1 + 2\pi n i$

d) $\log(i) = \left(2n + \frac{1}{2}\right)\pi i$

$z = i \; ; |z| = i \; ; \theta = \pi/2$

$\log(i) = \ln(i) + i\left(\frac{\pi}{2} + 2\pi n\right) = \left(2n + \frac{1}{2}\right)\pi i$