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EDITORIAL

Mixed-integer nonlinear programming 2018

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Abstract
Mixed-Integer Nonlinear Programming (MINLP) is the area of optimization that addresses nonlinear problems with continuous and integer variables. MINLP has proven to be a powerful tool for modeling. At the same time, it combines algorithmic design challenges from combinatorial and nonlinear optimization. The MINLP field has received increased attention over the past two decades with contributions on the theoretical, algorithmic, and computational side originating from a growing community that involves engineers, mathematicians, and operations researchers. This special issue was motivated by a seminar on MINLP that took place in Dagstuhl, Germany in 2018. The purpose of this article is to provide a brief introduction to the field and the articles of the special issue.

Keywords Combinatorial optimization · Nonlinear optimization · MINLP

1 Introduction

Mixed-integer nonlinear programming addresses a very general class of optimization problems with nonlinearities in the objective and/or constraints as well as continuous and integer variables:

$$\left.\begin{aligned} \min\ & f(x,y) \\ & x \in X \subseteq \mathbb{R}^p \\ & y \in Y \subseteq \mathbb{Z}^n \end{aligned}\right\} \tag{1}$$

where $f : (X, Y) \mapsto \mathbb{R}$ and $g : (X, Y) \mapsto \mathbb{R}^m$ are algebraic, often assumed to be recursive compositions of sums and products of univariate functions.

While the origins of mathematical optimization can be traced back to the development of algorithms for linear optimization (Dantzig 1949, 1963), interest was

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expressed very early on in nonlinear optimization (Davidon 1991; Tuy 1964; Falk and Soland 1969; Horst and Tuy 1996; Bazaraa et al. 2006; Boyd and Vandenberghe 2004) and combinatorial optimization (Dantzig et al. 1954; Land and Doig 1960; Balas 1979; Nemhauser and Wolsey 1988). Also early on, attempts were made to extend tools from integer linear programming to the nonlinear case (Beale and Forrest 1976).

Problem (1) offers a very natural way to approach many engineering design problems, especially in chemical engineering where nonlinear functions can capture the complex behavior of chemical processes and integer variables can be used to model discrete decisions, such as choices among different equipment types, and approximate economies of scale through fixed charges. It is therefore not surprising that some of the first systematic approaches to (1) were developed by chemical engineers (Duran and Grossmann 1986). Over the past three decades, the field has experienced a flurry of activity with contributions from engineers, mathematicians, and operations researchers. These contributions have addressed issues of computational complexity, convexification, decomposition, finding feasible solutions, and the development of deterministic and stochastic algorithms for the local and global solution of MINLPs. For additional information, we refer the reader to recent review papers (Belotti et al. 2013; Trespalacios and Grossmann 2014; Kılınç and Sahinidis 2017). As recently reviewed in Kılınç and Sahinidis (2017), applications of MINLP have spanned many areas, including design and control of chemical processes, planning and scheduling, network design, layout design, portfolio optimization, and energy systems. These applications have been facilitated by the recent development of a large number of MINLP codes, including deterministic global optimization codes `BARON` (Sahinidis 1996), `GlobSol` (Kearfott 2009), `LindoGlobal` (Lin and Schrage 2009), `Couenne` (Belotti et al. 2009), `SCIP` (Gleixner et al. 2008), and `ANTIGONE` (Misener and Floudas 2014).

OPTE was launched in 2000 with the specific purpose of addressing optimization problems of interest to mathematicians and engineers. MINLP fits naturally in this context. Sixteen years ago, OPTE published a special issue on mixed-integer optimization (Grossmann and Sahinidis 2003) that included a significant number of MINLP articles. The current special issue was motivated by the 2018 Dagstuhl seminar on MINLP (https://www.dagstuhl.de/en/program/calendar/semhp/?semnr=18081). The seminar was organized to address software development issues, theory at the intersection of integer linear optimization and nonlinear optimization, driving applications, and connections between MINLP and machine learning. The special issue was open to contributors that did not attend the Dagstuhl MINLP seminar.

2 Overview of the special issue articles

Obviously, the presence of integrality conditions makes (1) nonconvex. However, in the MINLP literature, a problem is referred to as *convex* if dropping the integrality conditions results in a convex model; otherwise, it is referred to as *nonconvex*. The manuscripts in this special issue address both types of problems. In the case of nonconvex problems, a recurring theme is bilinearities. Additionally, many of the applications addressed in this issue are in the area of network design and operations.

In Dey et al. (2019), Dey, Santana and Wang address quadratically constrained quadratic programs where the variables can be partitioned into two sets such that fixing the variables in any one set renders the program linear. This is a class of problems with many applications, including pooling problems and network design problems. The authors develop an SOCP-representable relaxation, a new branching rule, and a branch-and-bound algorithm for this problem. They also present computational experiments from a structural engineering problem that involves updating finite elements. The results demonstrate that the proposed approach outperforms BARON and a piecewise discretization approach.

The short communication by D'Ambrosio et al. (2019) address a very challenging problem in the design of smooth direction fields on surfaces. The primary goal of this work is expose a disciplined MINLP modeling approach to a problem that henceforth has been addressed by greedy heuristics. The problem represents a challenge for current MINLP solvers since it involves tens of thousands of constraints and variables.

Motivated by a network transmission problem, González Rueda et al. (2019) present a modification to the classical successive linear programming algorithm for solving nonlinear and mixed-integer nonlinear optimization problems. In addition to a theoretical analysis of the proposed algorithm, the authors demonstrate that their developments are useful in practice by optimizing part of the Spanish gas transmission network.

A very thorough review and comparison of solvers for convex MINLPs is presented by Kronqvist et al. (2019). The authors first review solvers based on convexification, decomposition, and branch-and-bound for this class of problems. An impressive number of 16 MINLP solvers are then compared on 335 problems and the results are analyzed based on the degree of nonlinearity and discreteness of problems. The results demonstrate that, for the convex MINLPs solved, the authors' recently developed SHOT solver (Kronqvist et al. 2016) has an edge, followed very closely by the global solver BARON (Khajavirad and Sahinidis 2018; Kılınç and Sahinidis 2019).

The article by Pecci et al. (2019) investigates the problem of optimal placement of control valves in water supply networks. A nonconvex MINLP model is developed, along with a tailored branch-and-bound algorithm. The proposed approach is evaluated in benchmark problems and an operational water supply network from the UK, for which the authors approach is demonstrated to outperform BARON and SCIP.

Koster and Kuhnke (2019) address the design of water usage and treatment networks. The presence of contaminants leads to bilinearities in mass balances. The authors develop an adaptive discretization-based approach that alternates between the solution of mixed-integer linear programs and quadratically constrained programs. Computational results demonstrate that this tailored approach outperforms BARON and SCIP by finding high-quality solutions for many difficult problems.

Yet another network optimization problem is addressed in the manuscript of Burlacu et al. (2019). The starting point of the authors is a system of nonlinear parabolic partial differential equations for modeling nonlinear gas physics. Integer variables are used to model the switching of valves and compressors. The authors tackle this problem by discretization and iterative application of mixed-integer linear programming to solve the nonconvex MINLP to global optimality. The MILP-based approach outperforms BARON and SCIP.

The University of Utah's north chiller plant is optimized by Blackburn et al. (2019). The authors model the problem as a mixed-integer quadratic program, for which they develop a heuristic. The authors' approach finds near-optimal solutions in a fraction of the time of their branch-and-bound implementation, and facilitates online implementation of their model. The solutions obtained by the authors' approach improve operator solutions by reducing chiller energy costs up to 34%.

The design and operation of water distribution networks for high-rise buildings is addressed by Altherr et al. (2019). The authors show that even simple versions of this problem are strongly NP-hard. Then, they develop a tailored branch-and-bound algorithm for solving the problem. Enforcing resiliency constraints leads to tight relaxations and an algorithm that outperforms SCIP and BARON.

Industrial gas networks under uncertainty are optimized by Cay et al. (2019). Ramping constraints on production levels over multiple time periods make the problem challenging. The authors develop a heuristic to solve the underlying MINLP and use a convex relaxation to demonstrate that their heuristic provides near-optimal solutions.

3 Conclusions

A majority of the papers in the special issue address optimization problems over networks and/or optimization problems over bilinear constraints. Many of the papers develop tailored algorithms that outperform general-purpose MINLP solvers. Interestingly, over 20 general-purpose MINLP solvers are reported in this special issue. This is a rather remarkable development in an area for which only a handful of solvers existed a decade ago.

Fueled by a variety of applications, MINLP will continue to advance and provide powerful tools for analytics and decision making. In addition to many applications, we foresee significant algorithmic developments in this area over the next decade. These will come about from advances in convexification, decomposition, and parallel implementations to address large-scale problems, including those that arise from the modeling of uncertainty and machine learning.

The author wishes to thank the organizers of the Dagstuhl seminar on MINLP that motivated this special issue. Thanks are also due to the referees who offered high-quality reviews in a timely manner, thus making possible the publication of this issue in about a year's time after papers were submitted. As OPTE increases its emphasis on core optimization theory and algorithms, we anticipate that MINLP will be addressed by many papers that we publish. Another special issue on MINLP is already slated to accept submissions at the end of this year.

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