Problem 1

a) absolute error: $e = |\pi - \frac{22}{7}| = 0.00126449$

relative error: $\varepsilon = |\frac{\pi - 22/7}{\pi}| = 0.000402499$

b) absolute error: $e = |\pi - 3.1416| = 0.0000073346$

relative error: $\varepsilon = |\frac{\pi - 3.1416}{\pi}| = 0.0000002338$

c) absolute error: $e = |e - 2.718| = 0.000281828$

relative error: $\varepsilon = |\frac{e - 2.718}{e}| = 0.000103679$

d) absolute error: $e = |\sqrt{2} - 1.414| = 0.000213562$

relative error: $\varepsilon = |\frac{\sqrt{2} - 1.414}{\sqrt{2}}| = 0.000151011$

Problem 2

a) (i) $(\frac{4}{5} + \frac{1}{3}) = \frac{17}{15}$

(ii) $fl(\frac{4}{5}) = 0.800$, $fl(\frac{1}{3}) = 0.333$

$(\frac{4}{3} + \frac{1}{3}) = fl(\frac{4}{5}) + fl(\frac{1}{3}) = 1.133 \approx 1.13$

(iii) $fl(\frac{4}{5}) = 0.800$, $fl(\frac{1}{3}) = 0.333$

$(\frac{4}{3} + \frac{1}{3}) = fl(\frac{4}{5}) + fl(\frac{1}{3}) = 1.133 \approx 1.13$

(iv) relative error : $\varepsilon_{ii} = |\frac{\frac{17}{15} - 1.13}{\frac{17}{15}}| \approx 3 \times 10^{-3}$ $\quad \varepsilon_{iii} \approx 3 \times 10^{-3}$

b) (i) $\frac{4}{5} \cdot \frac{1}{3} = \frac{4}{15}$

(ii) $\frac{4}{5} \cdot \frac{1}{3} = fl(\frac{4}{5}) fl(\frac{1}{3}) = 0.800 \cdot 0.333 = 0.266$

(iii) $\frac{4}{5} \cdot \frac{1}{3} = fl(\frac{4}{5}) fl(\frac{1}{3}) = 0.800 \cdot 0.333 = 0.266$

(iv) relative error: $\varepsilon_{ii} = |\frac{\frac{4}{15} - 0.266}{4/15}| \approx 2.5 \times 10^{-3}$ $\quad \varepsilon_{iii} \approx 2.5 \times 10^{-3}$

c) (i) $(\frac{1}{3} - \frac{3}{11}) + \frac{3}{20} = \frac{11-9}{33} + \frac{3}{20} = \frac{139}{660}$

(ii) $fl(\frac{1}{3}) = 0.333$, $fl(\frac{3}{11}) = 0.272$, $fl(\frac{3}{20}) = 0.150$

$(\frac{1}{3} - \frac{3}{11}) + \frac{3}{20} = fl(\frac{1}{3}) - fl(\frac{3}{11}) + fl(\frac{3}{20}) = (0.333 - 0.272) + 0.150 = 0.211$

(iii) $fl(\frac{1}{3}) = 0.333$, $fl(\frac{3}{11}) = 0.273$, $fl(\frac{3}{20}) = 0.150$

$(\frac{1}{3}-\frac{3}{11})+\frac{3}{20} = fl(\frac{1}{3}) - fl(\frac{3}{11}) + fl(\frac{3}{20}) = (0.333-0.273)+0.150 = 0.210$

(iv) relative error: $\varepsilon_{ii} = \left|\frac{139/660 - 0.211}{139/660}\right| \approx 2\times10^{-3}$ $\varepsilon_{iii} = \left|\frac{139/660 - 0.210}{139/660}\right| \approx 3\times10^{-3}$

d) (i) $(\frac{1}{3}+\frac{3}{11})-\frac{3}{20} = \frac{301}{660}$

(ii) $(\frac{1}{3}+\frac{3}{11})-\frac{3}{20} = fl(\frac{1}{3})+fl(\frac{3}{11})-fl(\frac{3}{20}) = 0.333+0.272-0.15 = 0.455$

(iii) $(\frac{1}{3}+\frac{3}{11})-\frac{3}{20} = fl(\frac{1}{3})+fl(\frac{3}{11})-fl(\frac{3}{20}) = 0.333+0.273-0.15 = 0.456$

(iv) relative error: $\varepsilon_{ii} = \left|\frac{301/660 - 0.455}{301/660}\right| \approx 2\times10^{-3}$

$\varepsilon_{iii} = \left|\frac{301/660 - 0.456}{301/660}\right| \approx 1\times10^{-4}$

Problem 3

a). $P = e = 2.718281828$

$p^* = \sum_{n=0}^{5}\frac{1}{n!} = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}$

$= 2.716666667$

$e = |p-p^*| = |2.718281828 - 2.716666671$

$= 0.001615161$

$\varepsilon = \left|\frac{p-p^*}{p}\right| = \left|\frac{2.718281828 - 2.716666671}{2.718281828}\right|$

$= 0.000594185$

b) $P = e = 2.718281828$

$p^* = \sum_{i=0}^{10}\frac{1}{n!} = \frac{1}{0!}+\frac{1}{1!}+\dots\frac{1}{10!}$

$= 2.718281801$

$e = |p-p^*| = 2.718281828 - 2.718281801 = 0.27\times10^{-7}$

$\varepsilon = \left|\frac{p-p^*}{p}\right| = \left|\frac{2.718281828-2.718281801}{2.718281828}\right| = 0.1\times10^{-7}$

Problem 4

$f(x) = \frac{1}{\sqrt{x+2}-\sqrt{x}}$ for every large values $\Rightarrow \sqrt{x+2}\approx\sqrt{x}$, so

the result will not be accurated, since two numbers are too close to each other, which is called loss of significance.

solution: $f(x) = \frac{\sqrt{x+2} + \sqrt{x}}{(\sqrt{x+2} - \sqrt{x})(\sqrt{x+2} + \sqrt{x})} = \frac{\sqrt{x+2} + \sqrt{x}}{x+2-x}$

$= \frac{1}{2}(\sqrt{x+2} + \sqrt{x})$

Problem 5

in attachment