

Bios 6301: Assignment 5

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Due Thursday, 13 October, 1:00 PM

$5^{n=\text{day}}$ points taken off for each day late.

40 points total.

Submit a single knitr file (named `homework5.rmd`), along with a valid PDF output file. Inside the file, clearly indicate which parts of your responses go with which problems (you may use the original homework document as a template). Add your name as `author` to the file's metadata section. Raw R code/output or word processor files are not acceptable.

Failure to name file `homework5.rmd` or include author name may result in 5 points taken off.

Question 1

15 points

A problem with the Newton-Raphson algorithm is that it needs the derivative f' . If the derivative is hard to compute or does not exist, then we can use the *secant method*, which only requires that the function f is continuous.

Like the Newton-Raphson method, the **secant method** is based on a linear approximation to the function f . Suppose that f has a root at a . For this method we assume that we have *two* current guesses, x_0 and x_1 , for the value of a . We will think of x_0 as an older guess and we want to replace the pair x_0, x_1 by the pair x_1, x_2 , where x_2 is a new guess.

To find a good new guess x_2 we first draw the straight line from $(x_0, f(x_0))$ to $(x_1, f(x_1))$, which is called a secant of the curve $y = f(x)$. Like the tangent, the secant is a linear approximation of the behavior of $y = f(x)$, in the region of the points x_0 and x_1 . As the new guess we will use the x -coordinate x_2 of the point at which the secant crosses the x -axis.

The general form of the recurrence equation for the secant method is:

$$x_{i+1} = x_i - f(x_i) \frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}$$

Notice that we no longer need to know f' but in return we have to provide *two* initial points, x_0 and x_1 .

Write a function that implements the secant algorithm. Validate your program by finding the root of the function $f(x) = \cos(x) - x$. Compare its performance with the Newton-Raphson method – which is faster, and by how much? For this example $f'(x) = -\sin(x) - 1$.

The using the values 1 and 0.5 for x_0 and x_1 respectively in the Secant method, and using 0.5 for the value of x in the newton raphson results in the secant method being faster. However when the value of x_0 (for the secant method) and x (for the newton raphson) are the same value and the x_1 (for the secant method) is large then the secant method closer in time to the newton difference. With the values mentioned the secant is faster by 0.0008189678 secs0

```

#Secant Method
x0 <- 1
x1 <- 0.5
f<- function(x) {cos(x) - x}
plot(f, xlim = c(0.5, 1))
secant_method<- function(f, x0, X1, tol=1e-9, max_iter=1000){
  iter<- 0
  while((abs(x1-x0)) >tol) {
    x2<- x1-(f(x1)*(x1-x0))/ (f(x1)-f(x0))
    x0<-x1
    x1<-x2
    iter<- iter+1
    if (iter>max_iter) break
  }
  return (c(x2, iter))
}
secant_method(f,x0,x1)

```

```
## [1] 0.7390851 5.0000000
```

```

# Newton Raphson Method
f<- function(x) {cos(x) - x}
d<- function(x) {-sin(x) - 1}
x <- 0.5
newton_raphson <-function(f, x, d, tol=1e-9, n=1000) {
  i <- 0
  while(abs(f(x)) > tol){
    x<- x - (f(x)/d(x))
    i<- i+1
    if (i>n) break
  }
  return (c(x, i))
}
newton_raphson(f,x,d)

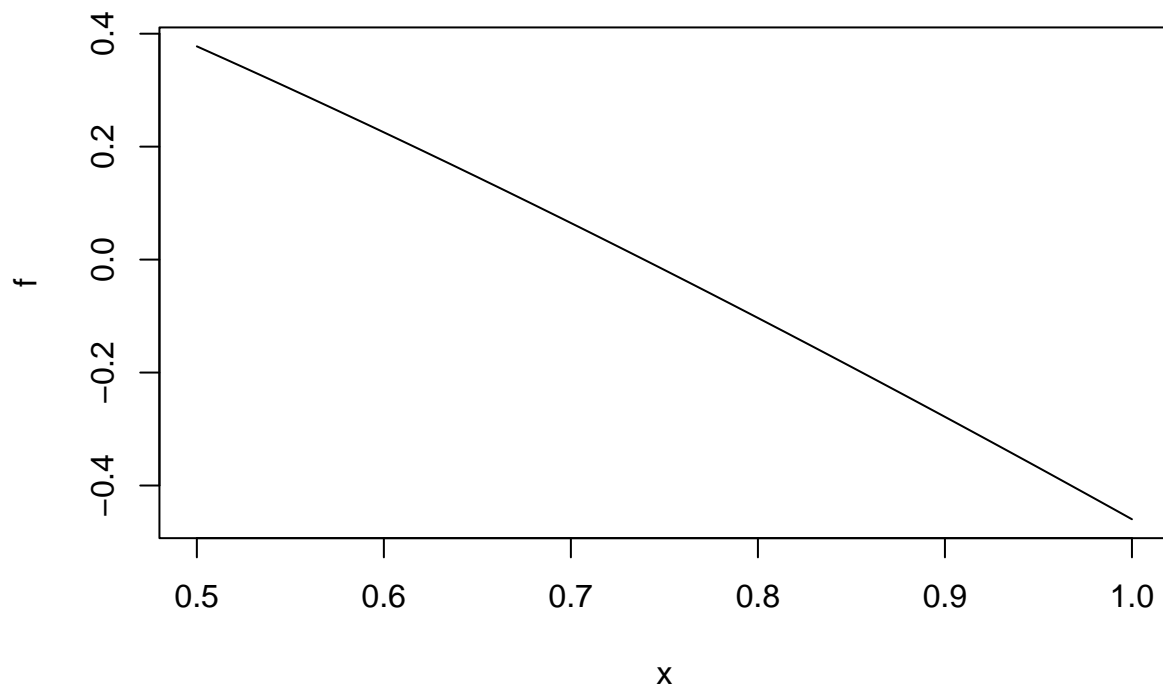
```

```
## [1] 0.7390851 4.0000000
```

```

#Testing Time
#Secant Time
start.time <- Sys.time()
x0 <- 1
x1 <- 0.5
f<- function(x) {cos(x) - x}
plot(f, xlim = c(0.5, 1))

```



```
secant_method<- function(f, x0, X1, tol=1e-9, max_iter=1000){
  iter<- 0
  while((abs(x1-x0)) >tol) {
    x2<- x1-(f(x1)*(x1-x0))/ (f(x1)-f(x0))
    x0<-x1
    x1<-x2
    iter<- iter+1
    if (iter>max_iter) break
  }
  x2
}
secant_method(f,x0,x1)
```

```
## [1] 0.7390851
```

```
end.time <- Sys.time()
time.taken <- end.time - start.time
time.taken
```

```
## Time difference of 0.04193306 secs
```

```
#Newton Raphson Time
start.time <- Sys.time()
f<- function(x) {cos(x) - x}
```

```

d<- function(x) {-sin(x) - 1}
x <- 0.5
newton_raphson <-function(f, x, d, tol=1e-9, n=1000) {
  i <- 0
  while(abs(f(x)) > tol){
    x<- x - (f(x)/d(x))
    i<- i+1
    if (i>n) break
  }
  x
}
newton_raphson(f,x,d)

```

```
## [1] 0.7390851
```

```

end.time <- Sys.time()
time.taken <- end.time - start.time
time.taken

```

```
## Time difference of 0.008968115 secs
```

```

#Secant Time
start.time <- Sys.time()
secant_method(f,x0,x1)

```

```
## [1] 0.7390851
```

```

end.time <- Sys.time()
time.taken.secant <- end.time - start.time
time.taken.secant

```

```
## Time difference of 0.00188303 secs
```

```

#Newton Raphson Time
start.time <- Sys.time()
newton_raphson(f,x,d)

```

```
## [1] 0.7390851
```

```

end.time <- Sys.time()
time.taken.nr <- end.time - start.time
time.taken.nr

```

```
## Time difference of 0.001691103 secs
```

```
abs(time.taken.secant-time.taken.nr)
```

```
## Time difference of 0.000191927 secs
```

Question 2

20 points

The game of craps is played as follows (this is simplified). First, you roll two six-sided dice; let x be the sum of the dice on the first roll. If $x = 7$ or 11 you win, otherwise you keep rolling until either you get x again, in which case you also win, or until you get a 7 or 11, in which case you lose.

Write a program to simulate a game of craps. You can use the following snippet of code to simulate the roll of two (fair) dice:

1. The instructor should be able to easily import and run your program (function), and obtain output that clearly shows how the game progressed. Set the RNG seed with `set.seed(100)` and show the output of three games. (lucky 13 points)

```
set.seed(100)
iterations=3
craps_game1 <- function(results=c(TRUE, FALSE)) {
  roll_1 <- sum(ceiling(6*runif(2)))
  if (results==TRUE) {print(roll_1)}
  if (roll_1 == 7 | roll_1 == 11) {
    if (results==TRUE) {print("You Win")}
    else {return(1) }
  }
  else{
    roll_again <- sum(ceiling(6*runif(2)))
    if (results==TRUE) {print(roll_again)}
    while (roll_again != roll_1 & roll_again != 7 & roll_again != 11){
      roll_again <- sum(ceiling(6*runif(2)))
    }
    if (results==TRUE) {print(roll_again)}
  }
  if (roll_again == roll_1) {
    #print(c(roll_again, ("You rolled the same thing twice, YOU WIN!!!")))
    if (results==TRUE) {print("You Win")}
    else {return(1)}
  }
  else {
    #print(c(roll_again, ("Sorry you lost you rolled")))
    if (results==TRUE) {print("You Lose")}
    else {return(0)}
  }
}

for (i in 1:iterations) {
  craps_game1(results=TRUE)
}
```

```
## [1] 4
## [1] 5
## [1] 6
## [1] 8
## [1] 6
## [1] 10
```

```
## [1] 5
## [1] 10
## [1] 5
## [1] 8
## [1] 9
## [1] 9
## [1] 5
## [1] 11
## [1] "You Lose"
## [1] 6
## [1] 9
## [1] 9
## [1] 11
## [1] "You Lose"
## [1] 6
## [1] 7
## [1] "You Lose"
```

1. Find a seed that will win ten straight games. Consider adding an argument to your function that disables output. Show the output of the ten games. (7 points)

```
seed <- 1
i = 0
while(i!=10){
  seed <- seed + 1
  set.seed(seed)
  i = 0
  for (j in 1:10) {
    output<- craps_game1(results=FALSE)
    i = i + output
  }
}
```

Question 3

5 points

This code makes a list of all functions in the base package:

```
objs <- mget(ls("package:base"), inherits = TRUE)
funs <- Filter(is.function, objs)
formals(funs)
```

```
## Warning in formals(fun): argument is not a function
```

```
## NULL
```

Using this list, write code to answer these questions.

1. Which function has the most arguments? (3 points) The Scan function has the most arguments.

```

most <- 0
mostfuns<- NULL

for (i in 1:length(funs)) {
  n <- length(formals(funs[[i]]))
  if (n > most) {
    most <- n
    mostfuns <- funs[i]
  }
}
mostfuns

## $scan
## function (file = "", what = double(), nmax = -1L, n = -1L, sep = "",
##   quote = if (identical(sep, "\n")) "" else "'", dec = ".",
##   skip = 0L, nlines = 0L, na.strings = "NA", flush = FALSE,
##   fill = FALSE, strip.white = FALSE, quiet = FALSE, blank.lines.skip = TRUE,
##   multi.line = TRUE, comment.char = "", allowEscapes = FALSE,
##   fileEncoding = "", encoding = "unknown", text, skipNul = FALSE)
## {
##   na.strings <- as.character(na.strings)
##   if (!missing(n)) {
##     if (missing(nmax))
##       nmax <- n/pmax(length(what), 1L)
##     else stop("either specify 'nmax' or 'n', but not both.")
##   }
##   if (missing(file) && !missing(text)) {
##     file <- textConnection(text, encoding = "UTF-8")
##     encoding <- "UTF-8"
##     on.exit(close(file))
##   }
##   if (is.character(file))
##     if (file == "")
##       file <- stdin()
##     else {
##       file <- if (nzchar(fileEncoding))
##         file(file, "r", encoding = fileEncoding)
##       else file(file, "r")
##       on.exit(close(file))
##     }
##   if (!inherits(file, "connection"))
##     stop("'file' must be a character string or connection")
##   .Internal(scan(file, what, nmax, sep, dec, quote, skip, nlines,
##     na.strings, flush, fill, strip.white, quiet, blank.lines.skip,
##     multi.line, comment.char, allowEscapes, encoding, skipNul))
## }
## <bytecode: 0x0000026df30c9df8>
## <environment: namespace:base>

```

1. How many functions have no arguments? (2 points) 227 functions have no arguments

```
empty <- c()
for (i in 1:length(funs)) {
  n <- length(formals(funs[[i]]))
  if (n == 0) {
    empty<- append (empty, funs[[i]])
  }
}
length(empty)
```

```
## [1] 227
```

Hint: find a function that returns the arguments for a given function.