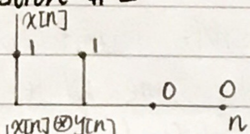
EEE 5502 HW #07 Hudanyun Sheng

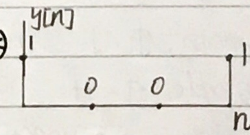
Question #1

I spent 8 hours.

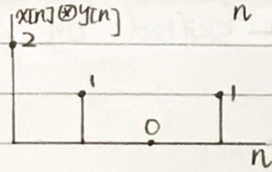
Question #2

(a)




(b)



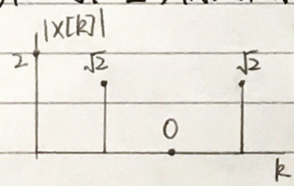

(c) $X[0]=\sum_{n=0}^{3} x[n]e^{-j\frac{2\pi}{4}\cdot 0\cdot n} = \sum_{n=0}^{3} x[n] = 2$

$X[1]=\sum_{n=0}^{3} x[n]e^{-j\frac{2\pi}{4}n\cdot 1} = e^{-j\frac{2\pi}{4}\cdot 0} + e^{-j\frac{2\pi}{4}\cdot 1} = 1+e^{-j\frac{\pi}{2}} = 1-j$

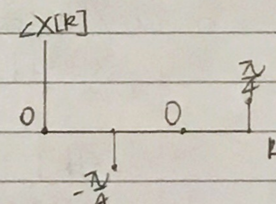
$X[2]=\sum_{n=0}^{3} x[n]e^{-j\frac{2\pi}{4}\cdot 2\cdot n} = e^{0}+e^{-j\frac{2\pi}{4}\cdot 2} = 1+e^{-j\pi} = 0$

$X[3]=\sum_{n=0}^{3} x[n]e^{-j\frac{2\pi}{4}\cdot 3\cdot n} = e^{0}+e^{-j\frac{3}{2}\pi} = 1+e^{-j\frac{3}{2}\pi} = 1+j$

(d) $|X[0]|=2$, $|X[1]|=\sqrt{2}$, $X[2]=0$, $X[3]=\sqrt{2}$




(e) $\angle X[0]=0$. $\angle X[1]=-\frac{\pi}{4}$. $\angle X[2]=0$. $\angle X[3]=\frac{\pi}{4}$



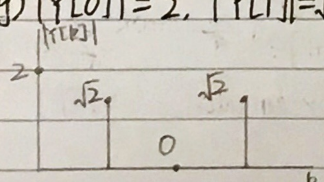

(f) $Y[0]=\sum_{n=0}^{3} y[n]e^{-j\frac{2\pi}{4}\cdot 0\cdot n} = \sum_{n=0}^{3} y[n] = 2$

$Y[1]=\sum_{n=0}^{3} y[n]e^{-j\frac{2\pi}{4}n} = e^{-j\cdot 0}+e^{-j\frac{3}{2}\pi} = 1+j$

$Y[2]=\sum_{n=0}^{3} y[n]e^{-j\frac{2\pi}{4}\cdot 2n} = e^{-j\cdot 0}+e^{-j3\pi} = 1-1=0$

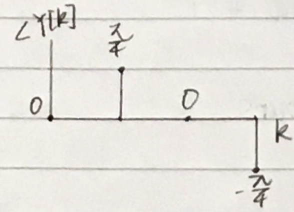
$Y[3]=\sum_{n=0}^{3} y[n]e^{-j\frac{2\pi}{4}\cdot 3n} = e^{-j0}+e^{-j\frac{9}{2}\pi} = 1+e^{-j\frac{\pi}{2}} = 1-j$

(g) $|Y[0]|=2$, $|Y[1]|=\sqrt{2}$, $|Y[2]|=0$, $|Y[3]|=\sqrt{2}$



(h) $\angle Y[0]=0,\ \angle Y[1]=\frac{\pi}{4}$

$\angle Y[2]=0,\ \angle Y[3]=-\frac{\pi}{4}$




(i) In time domain, y[n] corresponds to x[n] shift 1 unit to the left. In this case, the magnitude of the DFT of them are the same; while the phase of the DFT of them negative to each other

Question #3

(a) $X[0]=\sum_{n=0}^{3}x[n]e^{-j\frac{2\pi}{4}\cdot n\cdot 0}=\sum_{n=0}^{3}x[n]=0$

$X[1]=\sum_{n=0}^{3}x[n]e^{-j\frac{2\pi}{4}\cdot n}=e^{0}-e^{-j\frac{\pi}{2}}+e^{-j\pi}-e^{-j\frac{3}{2}\pi}=1+j-1-j=0$

$X[2]=\sum_{n=0}^{3}x[n]e^{-j\frac{2\pi}{4}\cdot n\cdot 2}=e^{0}-e^{-j\pi}+e^{-2j\pi}-e^{-3j\pi}=\cancel{2}\,4$

$X[3]=\sum_{n=0}^{3}x[n]e^{-j\frac{3\pi}{2}n}=e^{0}-e^{-j\frac{3}{2}\pi}+e^{-j3\pi}-e^{-j\frac{9}{2}\pi}=0$

(b) $X_e[0]=\sum_{n=2n}x[n]e^{-j\frac{2\pi}{2}0\cdot n}=x[0]+x[2]=2$

$X_e[1]=\sum_{n=2n}x[n]e^{-j\frac{2\pi}{2}\cdot 1\cdot n}=x[0]e^{0}+x[2]e^{-j\pi}=\cancel{4}\,0$

(c) $X_o[1]=\sum_{n=2n+1}x[n]e^{-j\frac{2\pi}{2}\cdot n}=x[1]e^{0}+x[3]\cancel{e^{j\pi}}=\cancel{0}\ -2$

$X_o[3]=\sum_{n=2n+1}x[n]e^{-j\frac{2\pi}{2}\cdot 1n}=x[1]e^{0}+x[3]e^{-j\pi}=\cancel{2}\ 0$

(d) $X[k]=\sum_{n=0}^{3}x[n]e^{-j\frac{2\pi}{4}nk}=\sum_{n=0}^{3}x[n]e^{-j\frac{\pi}{2}nk}$

$X[k+4]=\sum_{n=0}^{3}x[n]e^{-j\frac{\pi}{2}n(k+4)}=\sum_{n=0}^{3}x[n]e^{-j\frac{\pi}{2}nk}\cdot e^{-j\frac{\pi}{2}n\cdot 4}$

$=\sum_{n=0}^{3}x[n]e^{-j\frac{\pi}{2}nk}\ e^{-j2n\pi}=\sum_{n=0}^{3}x[n]e^{-j\frac{\pi}{2}nk}=X[k]$

$\therefore X[k]$ is periodic with a fundamental periods of 4.

(e) $X_e[k]=\sum_{n=0}^{1}x[2n]e^{-j\frac{2\pi}{2}nk}=\sum_{n=0}^{1}x[2n]e^{-j\pi nk}$. $X_e[k+2]=\sum_{n=0}^{1}x[2n]e^{-j\pi n(k+2)}=\sum_{n=0}^{1}x[2n]e^{-j\pi nk}=X_e[k]$

$X_o[k]=\sum_{n=0}^{1}x[2n+1]e^{-j\pi nk}$. $X_o[k+4]=\sum_{n=0}^{1}x[2n+1]e^{-j\pi n(k+2)}=\sum_{n=0}^{1}x[2n+1]e^{-j\pi nk}=X_o[k]$

$\therefore X_e[k]$ and $X_o[k]$ are periodic with a fundamental periods of 2.

(f) $X_e[k]+e^{-j(2\pi/4)k}X_o[k]=\sum_{n=0}^{1}x[2n]e^{-j\pi nk}+\sum_{n=0}^{1}x[2n+1]e^{-j\pi nk-j(\frac{\pi}{2})k}$

$=\sum_{n=0}^{1}x[2n]e^{-j\pi nk}+\sum_{n=0}^{1}x[2n+1]e^{-j(\pi n+\frac{\pi}{2})k}$

$=\sum_{n=0}^{1}x[2n]e^{-j\frac{4\pi}{4}nk}+\sum_{n=0}^{1}x[2n+1]e^{-j(\frac{4\pi n+2\pi}{4})k}$

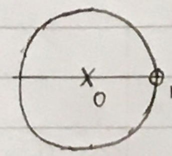
$=\sum_{n=0}^{1}x[2n]e^{-j\frac{2\pi}{4}2nk}+\sum_{n=0}^{1}x[2n+1]e^{-j\frac{2\pi}{4}(2n+1)k}=\sum_{n=0}^{3}x[n]e^{-j\frac{2\pi}{4}nk}$

$=X[k]$

And base on part d, we know it has a fundamental period of 4.

Question #4

(a) $Y(z) = X(z) - X(z)\cdot z^{-1} \Rightarrow H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-1}$

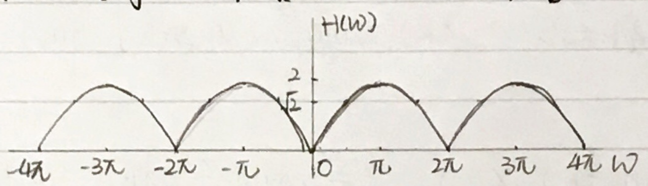



zero: $z=1$. Pole: $z=0$

(b) $H(z) = 1 - z^{-1}$

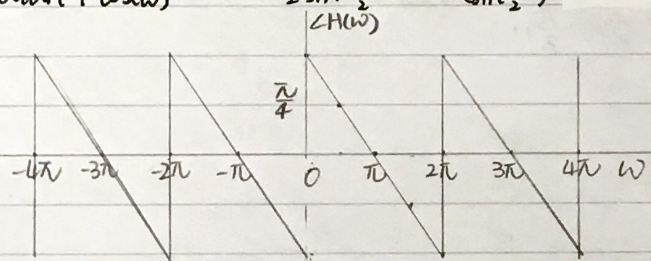
$H(\omega) = 1 - e^{-j\omega} = 1 - \cos(\omega) + j\sin(\omega) = e^{-\frac{j}{2}\omega}(e^{\frac{j}{2}\omega} - e^{-\frac{j}{2}\omega}) = e^{-\frac{j}{2}\omega}\cdot 2j\sin(\frac{\omega}{2})$

(c) $|H(\omega)| = |1-\cos(\omega) + j\sin(\omega)| = \sqrt{(1-\cos\omega)^2 + \sin^2\omega} = \sqrt{2(1-\cos\omega)} = 2|\sin\frac{\omega}{2}|$




(d) $\angle H(\omega) = \operatorname{atan}\frac{\sin(\omega)}{1-\cos(\omega)} = \operatorname{atan}\frac{2\sin(\frac{\omega}{2})\cos(\frac{\omega}{2})}{2\sin^2\frac{\omega}{2}} = \operatorname{atan}\left(\frac{\cos\frac{\omega}{2}}{\sin\frac{\omega}{2}}\right) = \operatorname{atan}(\cot\frac{\omega}{2}) = \frac{\pi}{2} - \frac{\omega}{2}$ when $\omega \neq \pm 2k\pi$, $k = 0, 1, \ldots$

(e)




(f) High pass filter

(g) The inverse is not still stable.

(h) This is not a minimum phase system.

(i) $y[n] = 0$

(j) $x[n] = 10 + 5\cos(\pi/3n) - 13 = 5\cos(\pi/3n) - 3$ $X(\omega) = 5\pi\sum_k\{\delta(\omega - \frac{\pi}{3} - 2\pi k) + \delta(\omega + \frac{\pi}{3} - 2\pi k)\}$

$H(\frac{\pi}{3}) = \frac{1}{2} + \frac{\sqrt{3}}{2}j$, $H(-\frac{\pi}{3}) = \frac{1}{2} - \frac{\sqrt{3}}{2}j$

$Y(\omega) = X(\omega)\cdot H(\omega) = 5\pi\sum_k\{\delta(\omega - \frac{\pi}{3} - 2\pi k)\cdot\frac{1}{2} + \delta(\omega - \frac{\pi}{3} - 2\pi k)\frac{\sqrt{3}}{2}j + \frac{1}{2}\delta(\omega + \frac{\pi}{3} - 2\pi k) - \frac{\sqrt{3}}{2}j\,\delta(\omega + \frac{\pi}{3} - 2\pi k)\}$

$\therefore y[n] = 2.5\cos(\frac{\pi}{3}n) - 2.5\sqrt{3}\sin(\frac{\pi}{3}n) = 5\sin(\frac{\pi}{6} - \frac{\pi}{3}n)$

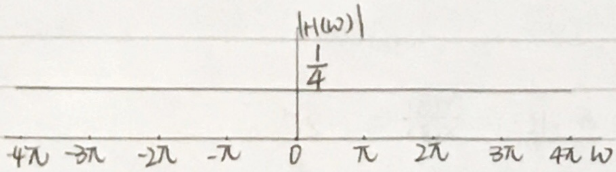
Question #5

(a) Zeros: $z = \pm 0.5j$. Poles: $z = \pm 2j$

$H(z) = \frac{(z-0.5j)(z+0.5j)}{(z-2j)(z+2j)} = (4 + z^{-2})/(4 + 16z^{-2})$

$H(\omega) = (4 + e^{-2j\omega})/(4 + 16e^{-2j\omega}) = \frac{1}{4}$

(b) $|H(\omega)| = \frac{|4 + e^{-2j\omega}|}{|4 + 16e^{-2j\omega}|} = \frac{|4 + \cos(2\omega) - j\sin(2\omega)|}{|4 + 16\cos(2\omega) - 16j\sin(2\omega)|} = \frac{1}{4}\frac{|5\cos\omega + 3j\sin\omega|}{|5\cos\omega - 3j\sin\omega|} = \frac{1}{4}$




(c) This is a all pass filter.

(d) The system is invertible.

(e) This is not a minimum phase system.

(f) $X(\omega) = 10 \cdot 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k) = 20\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$. $Y(\omega) = X(\omega) \cdot H(\omega) = 5\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$

$\therefore y[n] = 2.5$

$H(\frac{\pi}{3}) =$

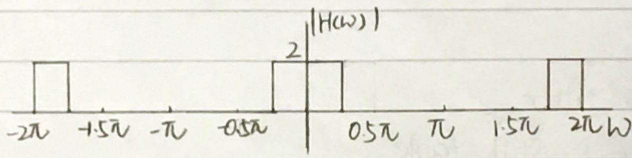
(g) $x[n] = -5\sin(\frac{\pi}{3} n)$. $X(\omega) = -\frac{5\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\omega - \frac{\pi}{3} - 2\pi k) - \delta(\omega + \frac{\pi}{3} - 2\pi k)\}$

$Y(\omega) = X(\omega) H(\omega) = -\frac{5}{4} \frac{\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\omega - \frac{\pi}{3} - 2\pi k) - \delta(\omega + \frac{\pi}{3} - 2\pi k)\}$

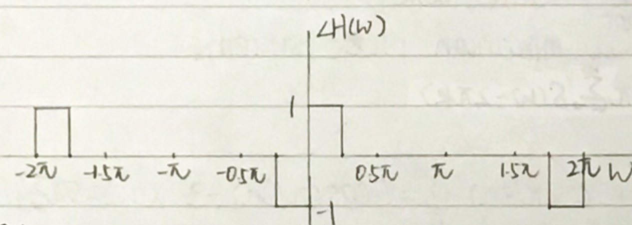
$\therefore y[n] = -\frac{5}{4} \sin(\frac{\pi}{3} n)$.

Question #6

(a)




(b)




(c) Low pass filter.

(d) $x[n] = -5\sin(\frac{\pi}{3} n)$. $X(\omega) = -\frac{5\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\omega - \frac{\pi}{3} - 2\pi k) - \delta(\omega + \frac{\pi}{3} - 2\pi k)\}$

$\therefore y[n] = 0$

(e) $x[n] = 5\cos(\frac{\pi}{3} n + \frac{\pi}{2}) - 3 = -5\sin(\frac{\pi}{3} n) - 3$. $X(\omega) = -\frac{5\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\omega - \frac{\pi}{3} - 2\pi k) - \delta(\omega + \frac{\pi}{3} - 2\pi k)\} - 6\pi \sum_k \delta(\omega - 2\pi k)$

$Y(\omega) = H(\omega) X(\omega)$

$= -12\pi \sum_k \delta(\omega - 2\pi k)$

$\therefore y[n] = -12$

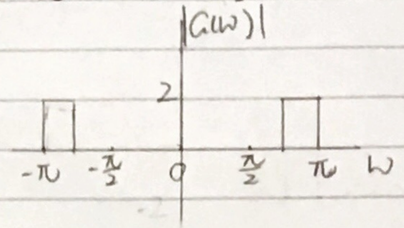
(f) The system is not invertible.

(g) $G(\omega) = \frac{1}{2\pi} H(\omega) \circledast \mathcal{F}\{\cos(\pi n)\}$

$$= \frac{1}{2\pi} H(\omega) \circledast \pi \sum_{k=-\infty}^{\infty} \{\delta(\omega - \pi - 2\pi k) + \delta(\omega + \pi - 2\pi k)\}$$

$$= \frac{1}{2} H(\omega) \circledast \sum_{k=-\infty}^{\infty} \{\delta(\omega - \pi - 2\pi k) + \delta(\omega + \pi - 2\pi k)\}$$

High pass filter.




(h) $G(\omega) = \frac{1}{2\pi} H(\omega) \circledast \pi \sum_{k=-\infty}^{\infty} \{\delta(\omega - \frac{\pi}{2} - 2\pi k) + \delta(\omega + \frac{\pi}{2} - 2\pi k)\}$

$$= \frac{1}{2} H(\omega) \circledast \sum_{k=-\infty}^{\infty} \{\delta(\omega - \frac{\pi}{2} - 2\pi k) + \delta(\omega + \frac{\pi}{2} - 2\pi k)\}$$

Band pass filter.