EEE 5502 Code #1

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Question #1

I spent 2 hours.

Question #2

(a) $S=\sum_{n=0}^{N-1} x[n]y[n] = \sum_{n=0}^{N-1} x[n]\cdot x[n] = \sum_{n=0}^{N-1} |x[n]|^2 = E_x$

(b) $S=\sum_{n=0}^{N-1} x[n]y[n]$. Based on Cauchy-Schwarz inequality, we have:

$$S=\sum_{n=0}^{N-1} x[n]y[n] \le \sum_{n=0}^{N-1} \frac{|x[n]|^2}{2} + \frac{|y[n]|^2}{2} = \sum_{n=0}^{N-1} \frac{|x[n]|^2}{2} + \sum_{n=0}^{N-1} \frac{|y[n]|^2}{2} = 1.$$

The equality holds if and only if $x[n]=y[n]$.

$\therefore$ S is maximized when $x[n]=y[n]$. The maximum value of S is 1.

(c)

$$C=\frac{\sum_{n=0}^{N-1} x[n]y[n]}{\sqrt{\sum_{n=0}^{N-1}|x[n]|^2}\sqrt{\sum_{n=0}^{N-1}|y[n]|^2}}$$

$$\because -\sqrt{\sum_{n=0}^{N-1}|x[n]|^2\sum_{n=0}^{N-1}|y[n]|^2} \le \sum_{n=0}^{N-1} x[n]y[n] \le \sqrt{\sum_{n=0}^{N-1}|x[n]|^2\sum_{n=0}^{N-1}|y[n]|^2}$$

$\therefore -1 \le C \le 1$

$\therefore$ The maximum value of C is 1. The minimum value of C is -1.

(d) $x[n]=ay[n]$, $C=\dfrac{\sum_{n=0}^{N-1} a|y[n]|^2}{\sqrt{a^2\sum_{n=0}^{N-1}|y[n]|^2}\sqrt{\sum_{n=0}^{N-1}|y[n]|}} = 1$

$y[n]=-ax[n]$, $C=\dfrac{\sum_{n=0}^{N-1} -a|x[n]|^2}{\sqrt{\sum_{n=0}^{N-1}|x[n]|^2}\sqrt{\sum_{n=0}^{N-1}a^2|x[n]|^2}} = -1$

(e) C is positive when two vectors are positively related, C is negative when they are negatively related.

The numerator of C gets large when the values at same positions are large, it gets small when the peak values for two vectors are totally different.

The denominator of C is the ~~L2~~ L2 norm of 2 vectors. It is a kind of normalization, which exclude the case when the magnitude of the vectors are big, C gets big even if they are less similar.