Lecture 14: Discrete -Time Filters

Foundations of Digital Signal Processing

Outline

- Circular Convolution Review
- The DTFT and the DFT: The Relationship
- The Fast Fourier Transform
- Designing the magnitude response
- Designing the phase response

News

- **Homework #6**
  - Due *Thursday*
  - Submit via canvas
- **Coding Problem #4**
  - Due *next week*
  - Submit via canvas

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Building Connections

- **A sliding window**
  - Machine learning project – find red cars

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Deriving Transforms

- **Consider the Inverse Discrete-Time Fourier Transform….**
  - What happens if we sample $\mathrm{X}(\omega)$?

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{+j\omega \mathrm{n}} \, d\omega$$

The Discrete Fourier Transform

- **Circular Convolution**
  - Multiplication property (DFT)

$$x[n]y[n] \leftrightarrow \frac{1}{N} X[k] \circledast Y[k]$$

  - Convolution property (DFT)

$$x[n] \circledast y[n] \leftrightarrow X[k]Y[k]$$

The Discrete Fourier Transform

- **Circular Convolution**
  - Multiplication property (DTFT)

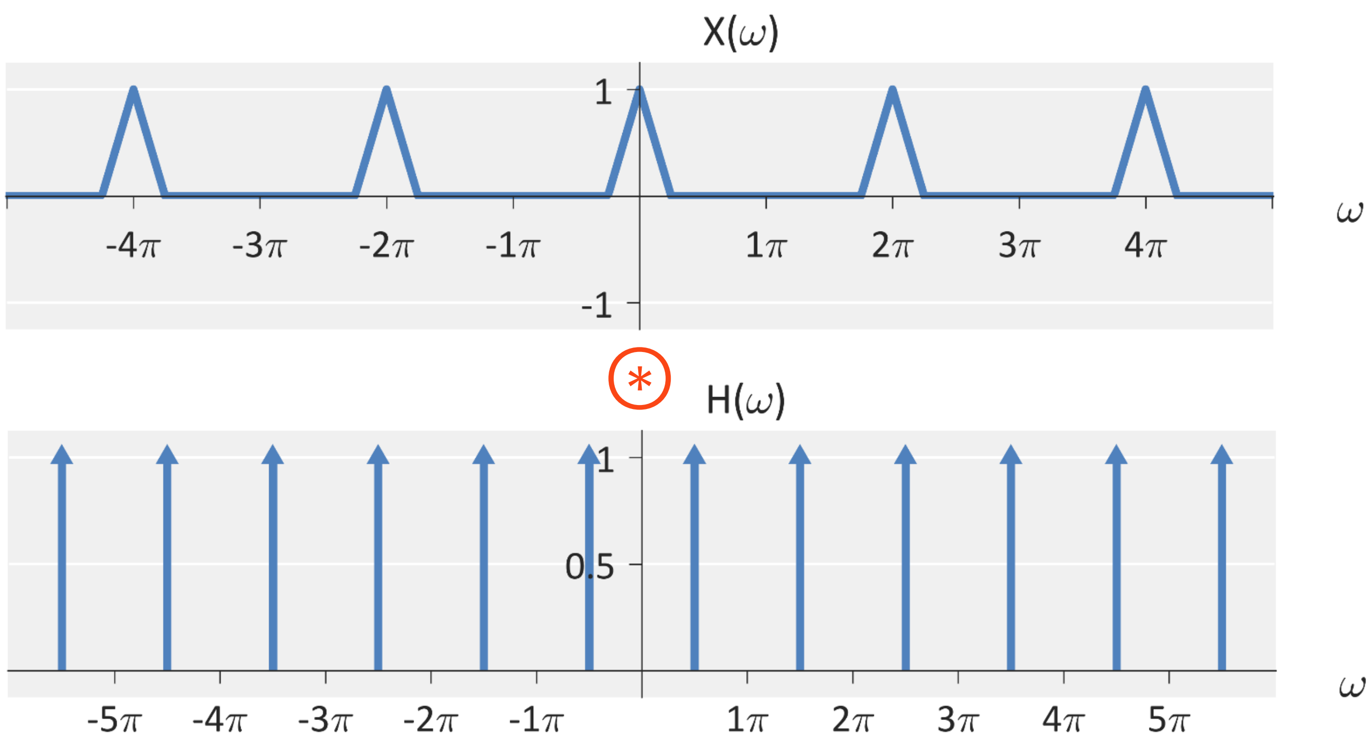
$$x[n]y[n] \leftrightarrow \frac{1}{2\pi} X(\omega) \circledast Y(\omega)$$

  - Convolution property (DTFT)

$$x[n] * y[n] \leftrightarrow X(\omega)Y(\omega)$$

Circular Convolution

- **What is Circular Convolution?**
  - Convolution for periodic signals



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- **What is Circular Convolution?**
  - Convolution for periodic signals
  - Convolve
    - One period of one signal
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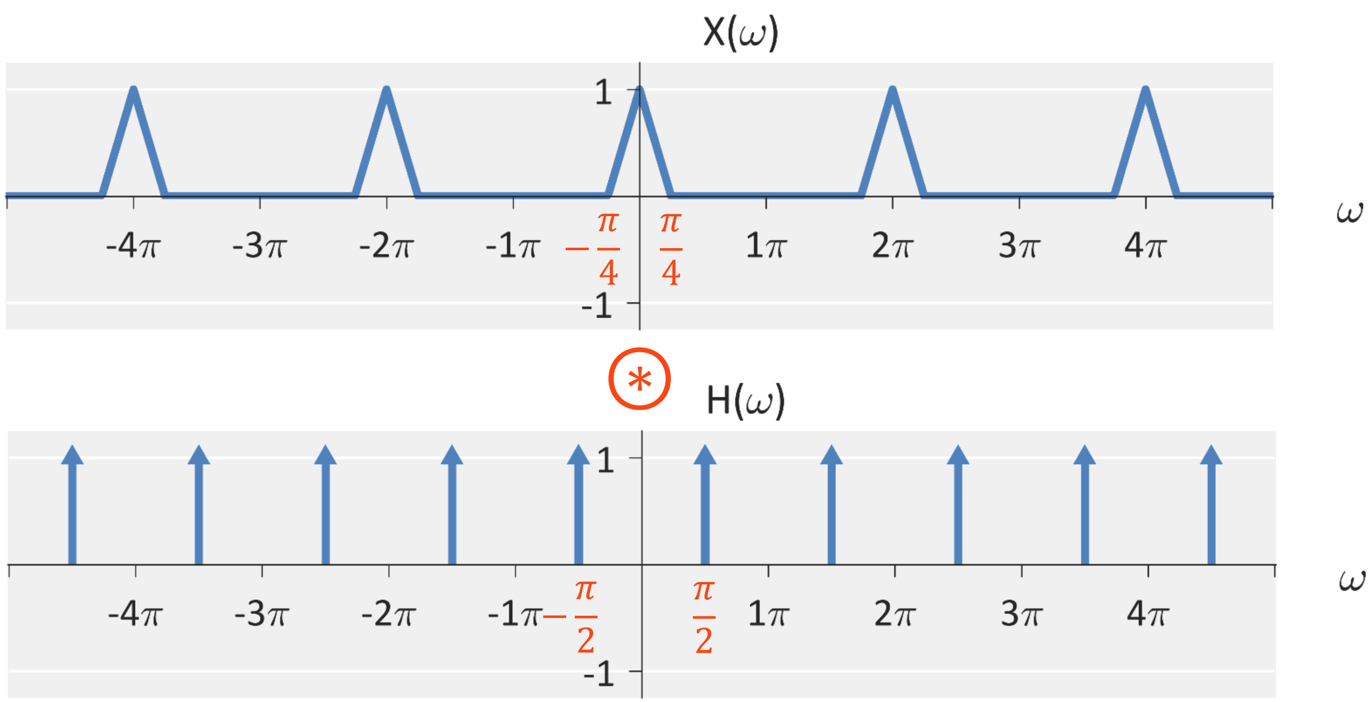

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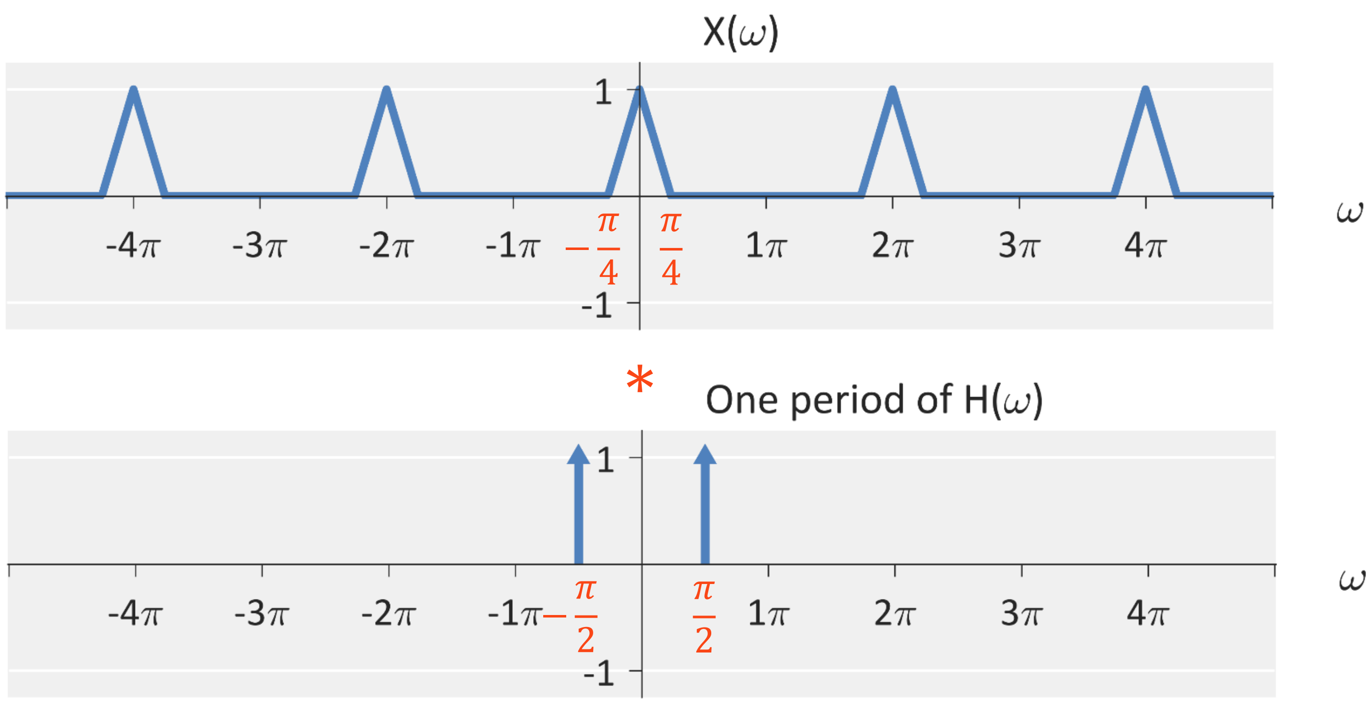

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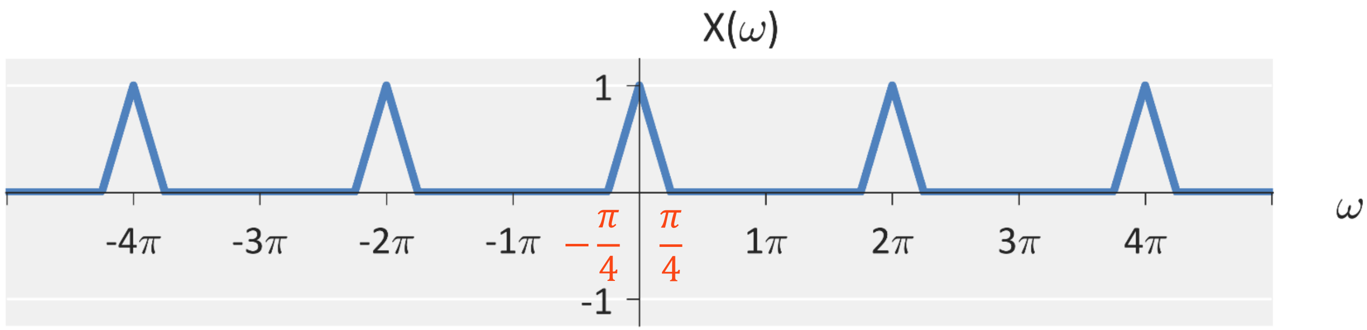

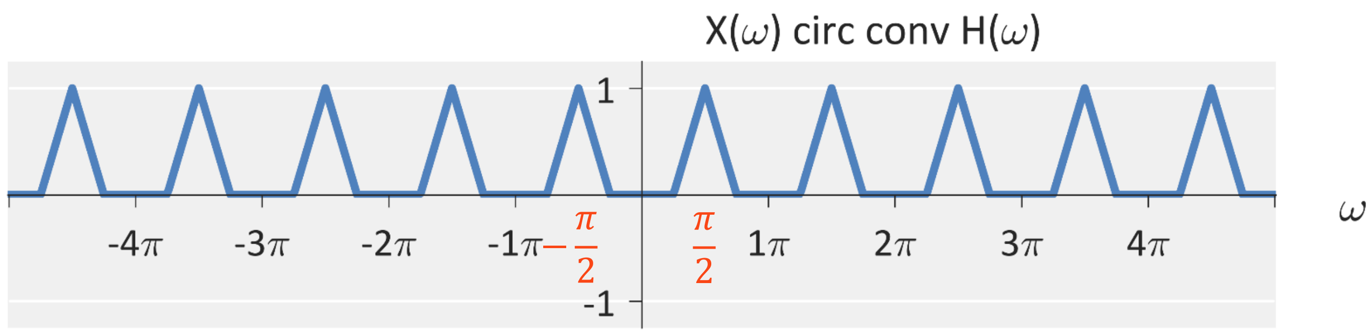

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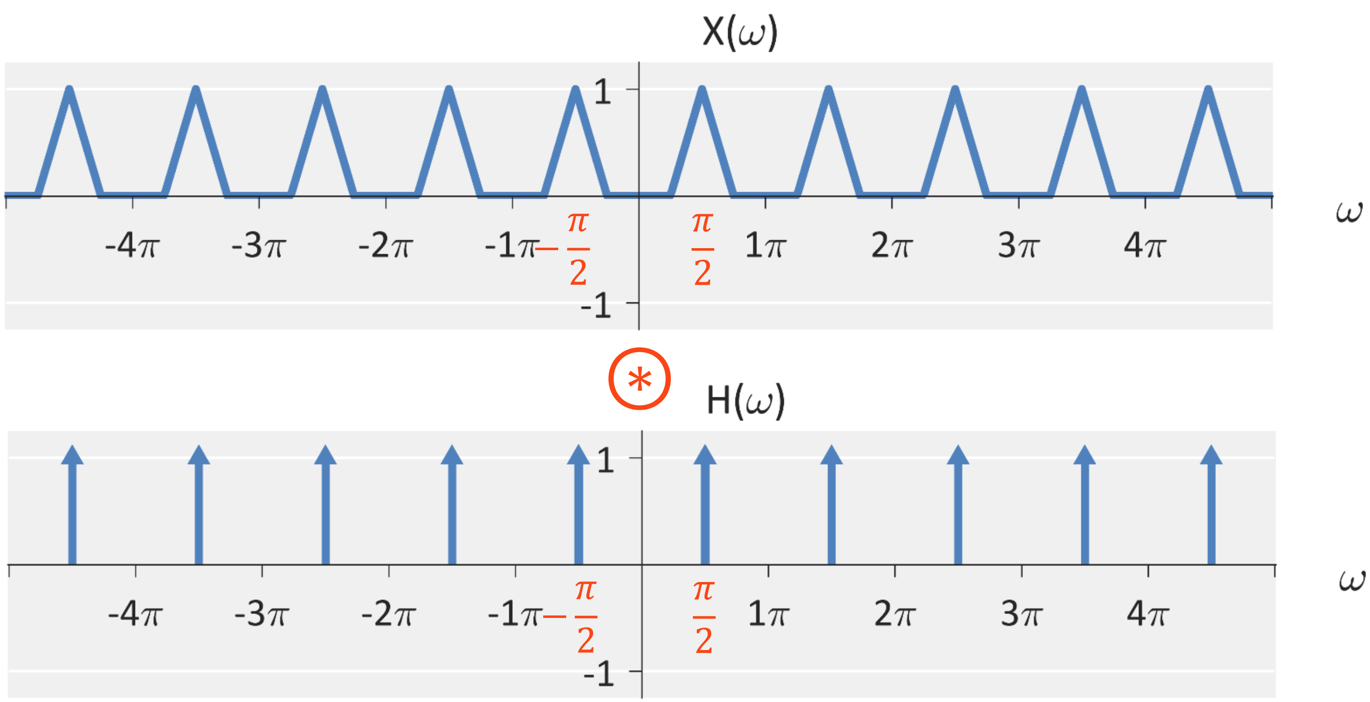

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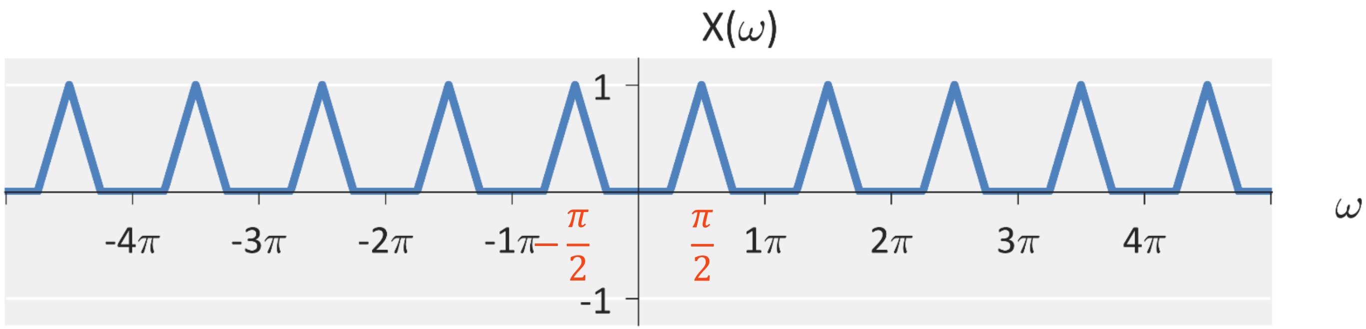

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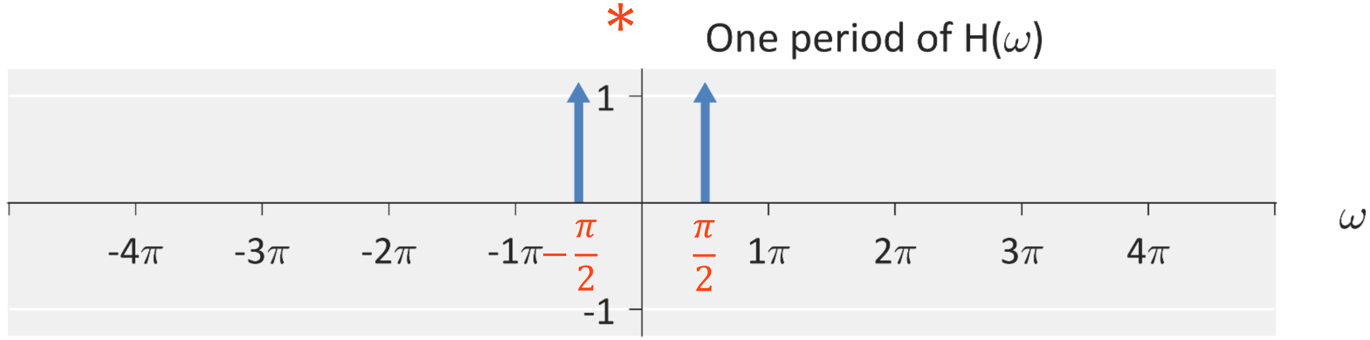

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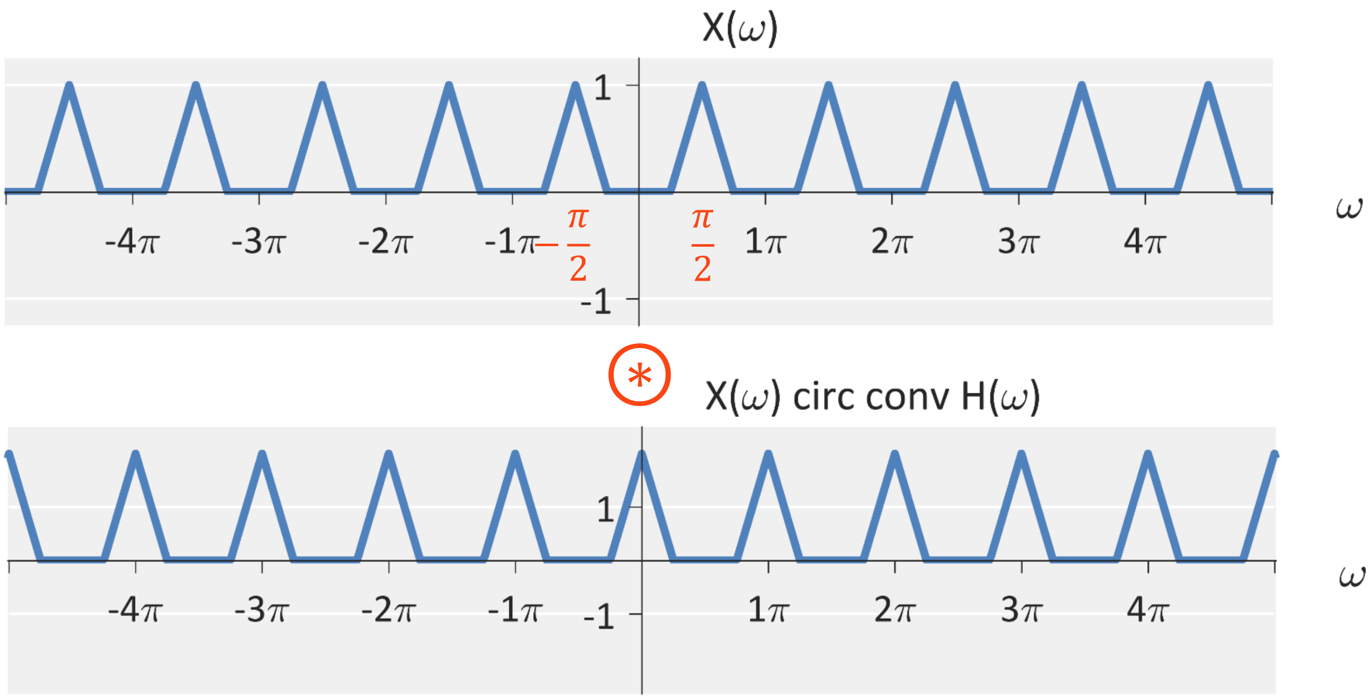

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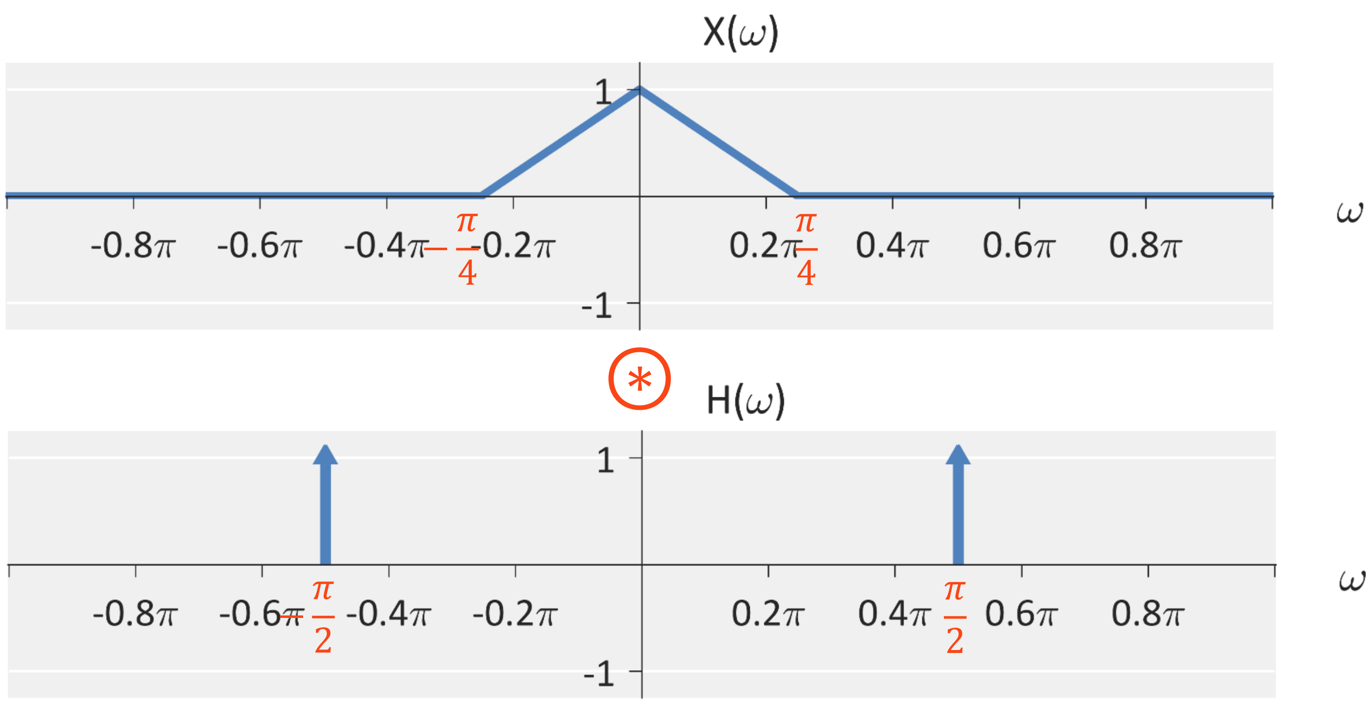

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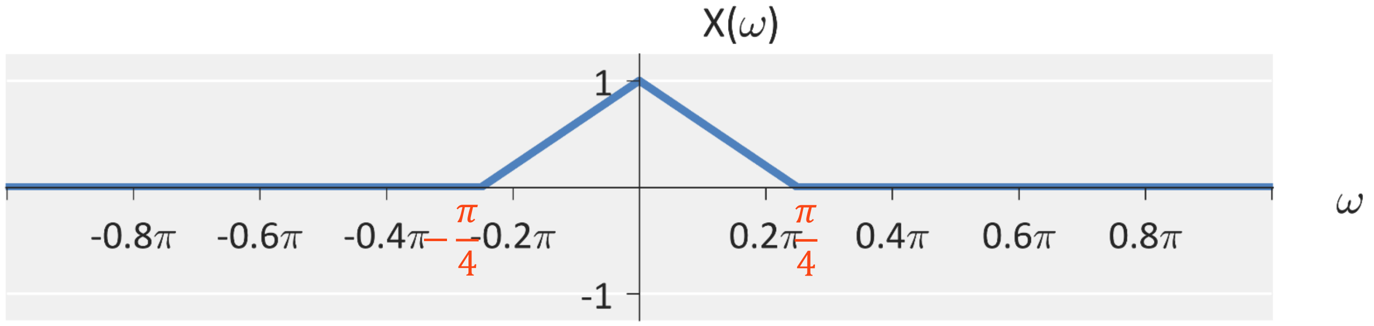

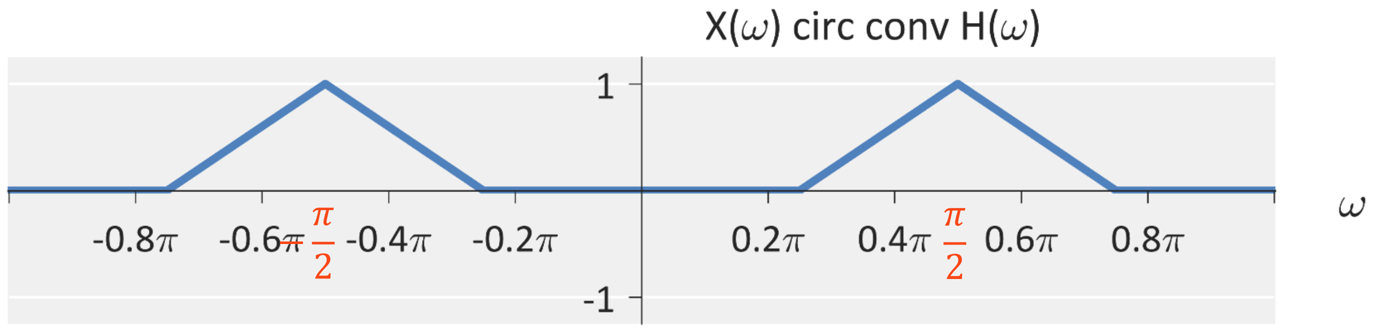

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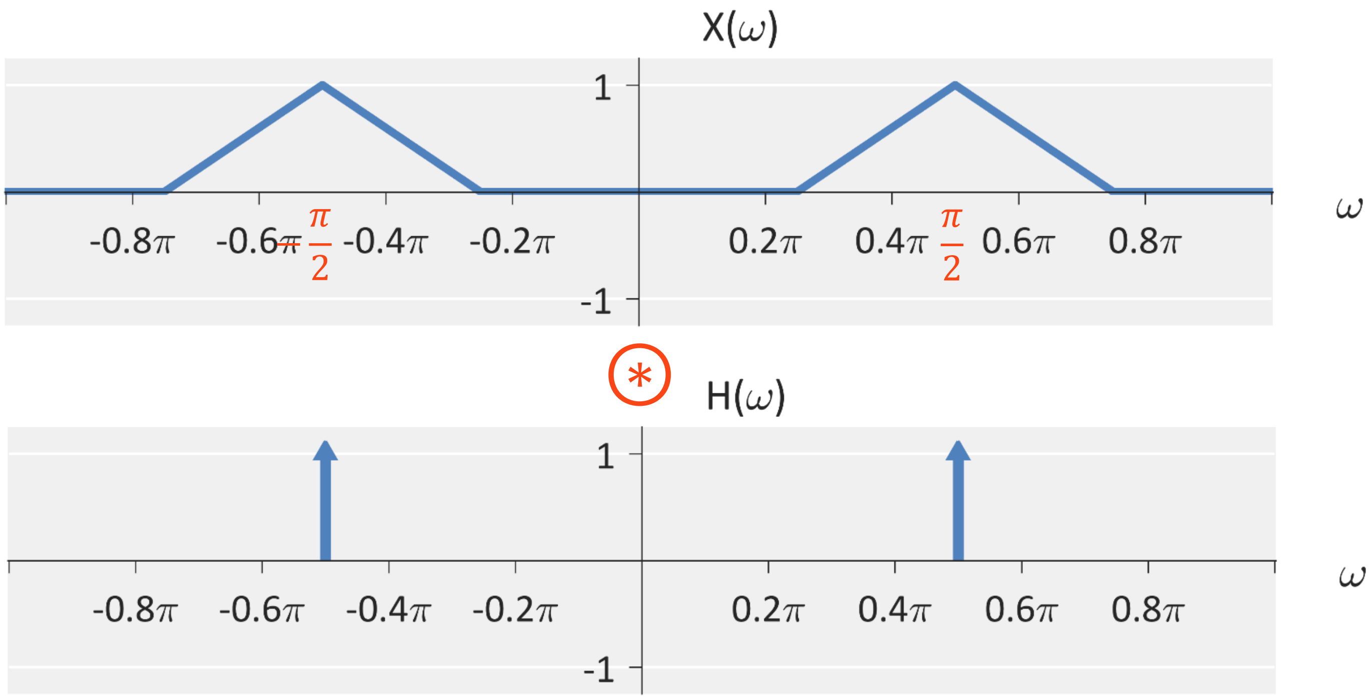

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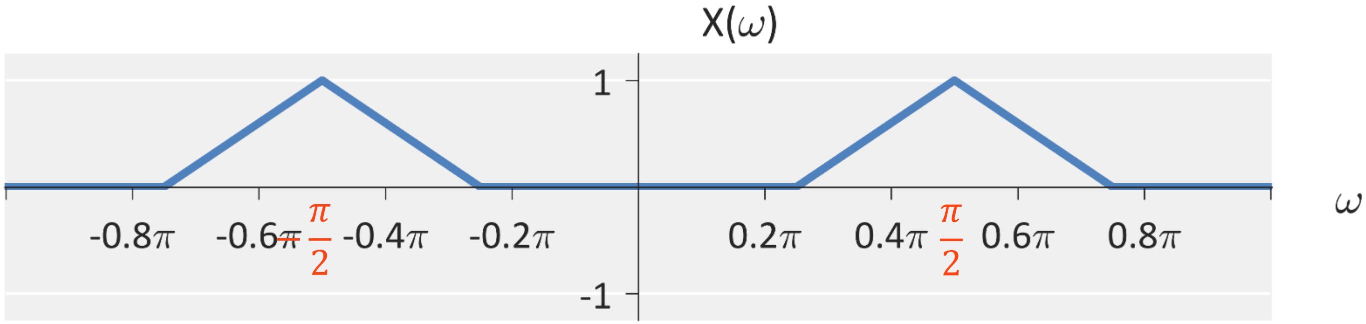

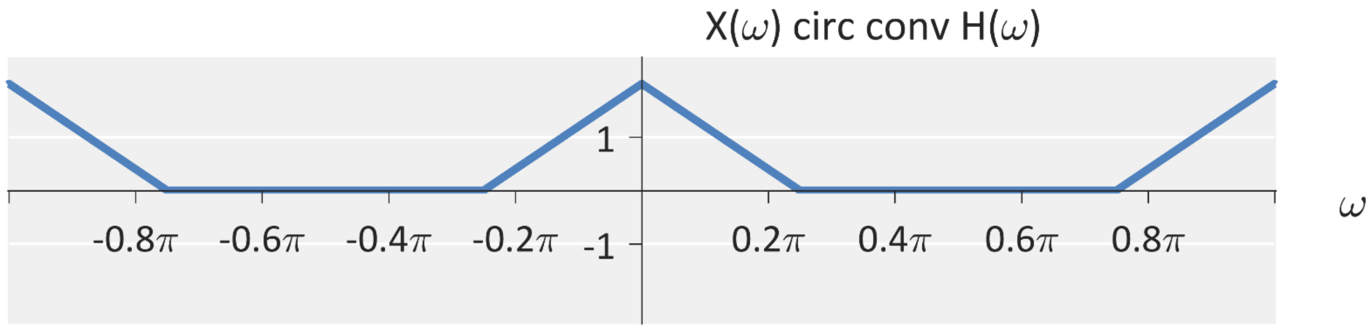

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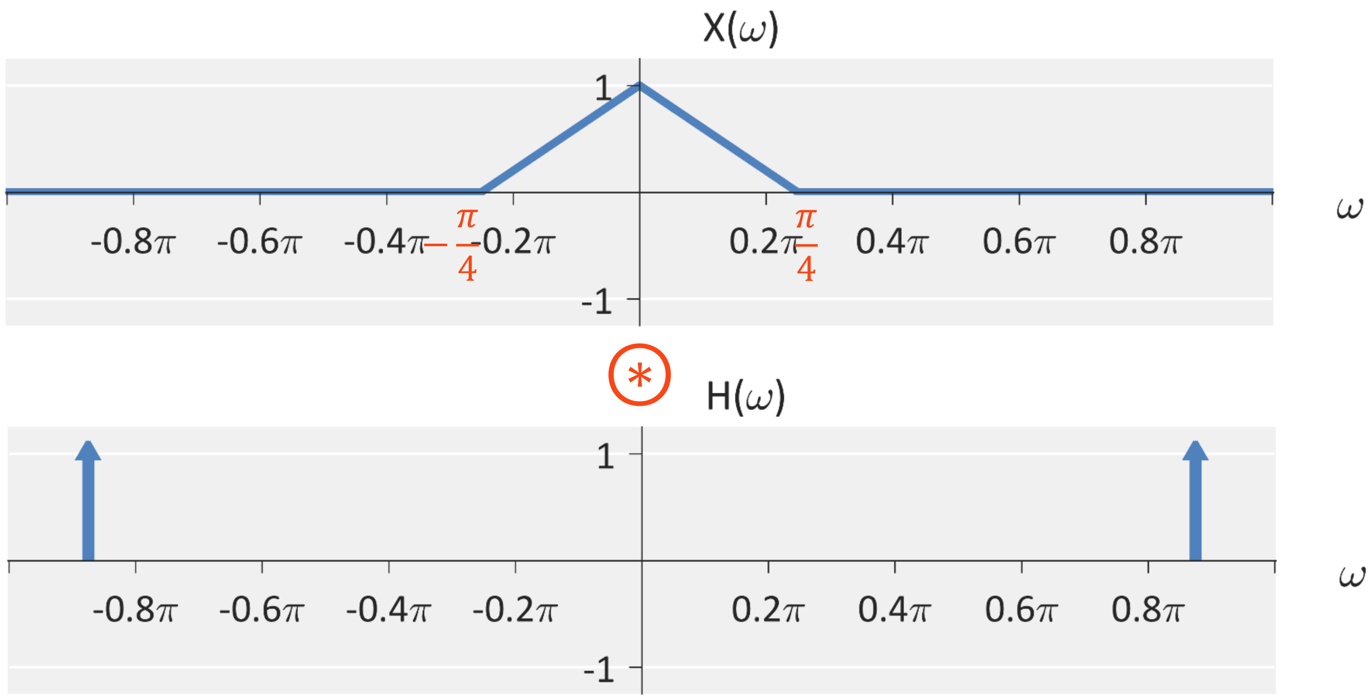

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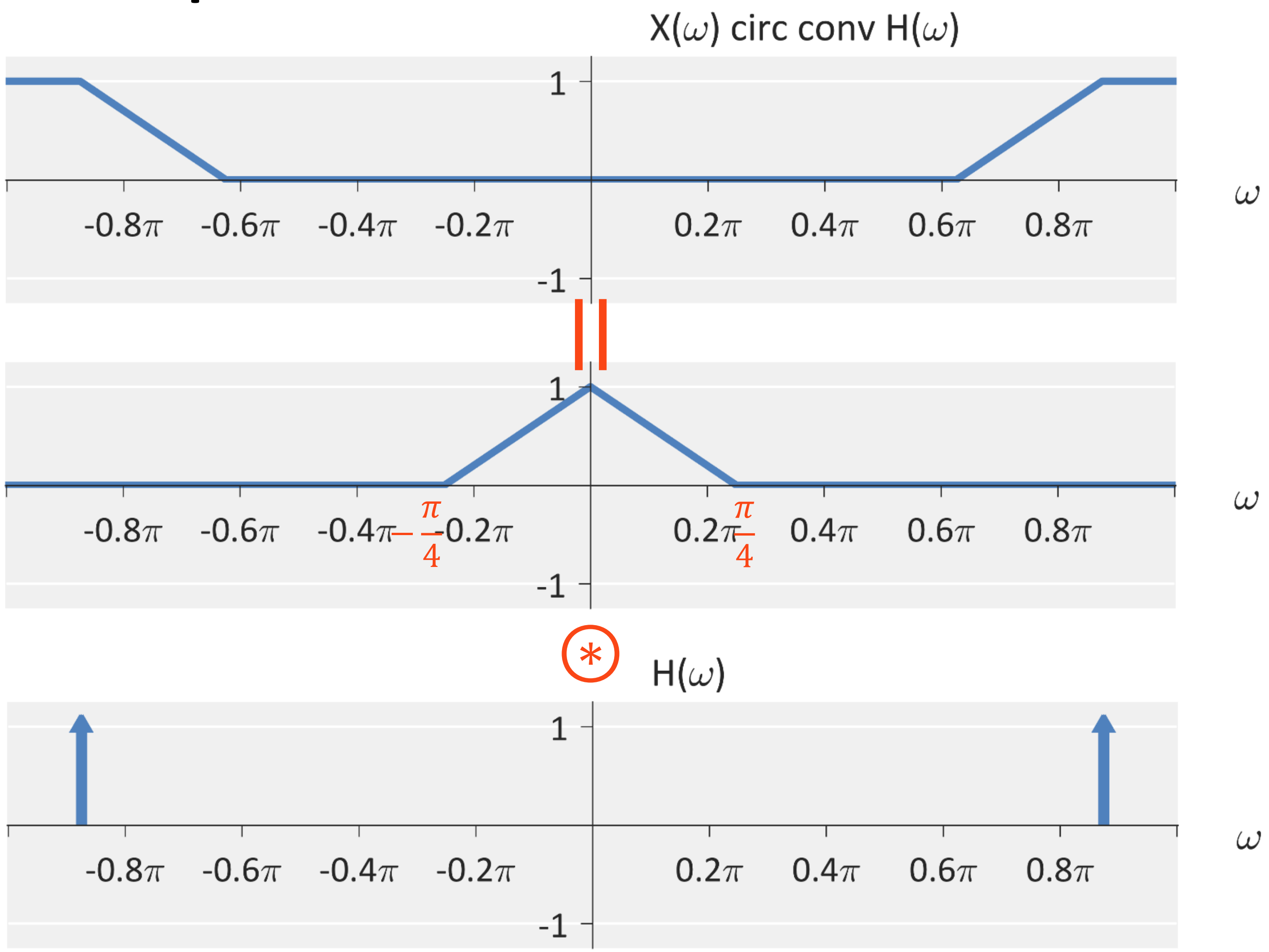

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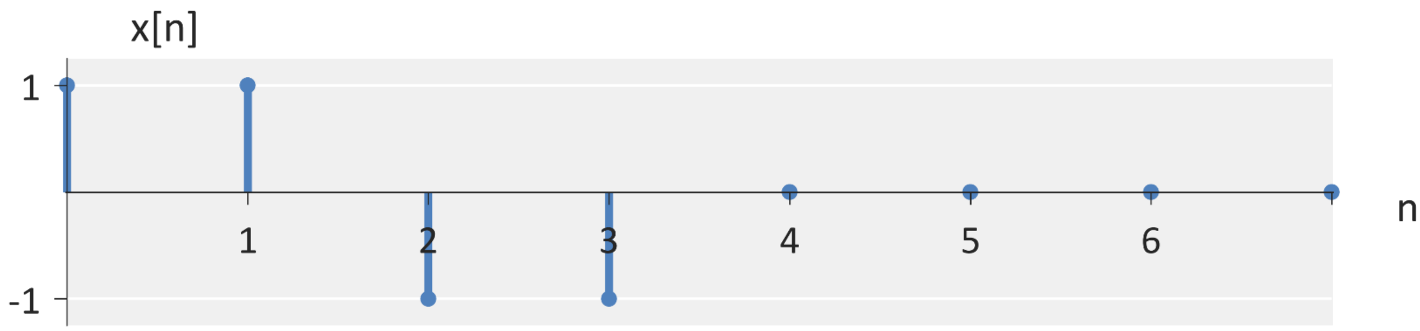

$\circledast$

$$h[n] = \delta[n-6]$$

Assume these are *finite* length-8 signals

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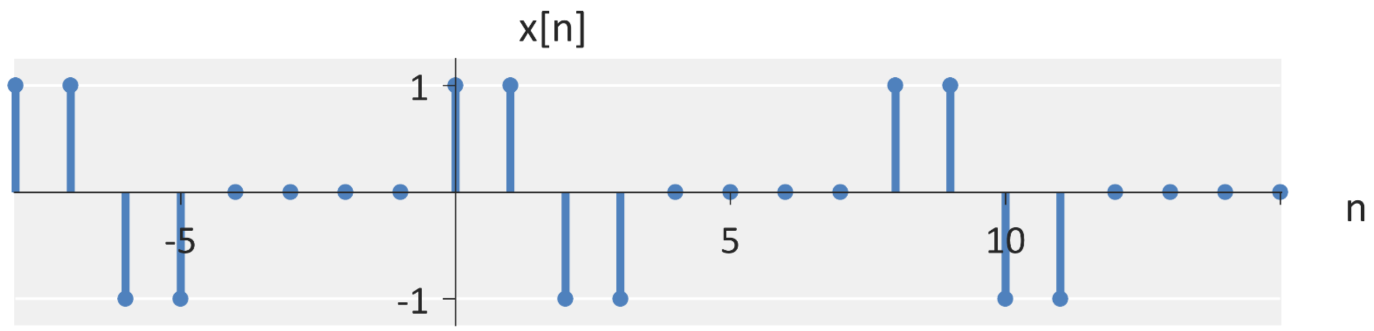

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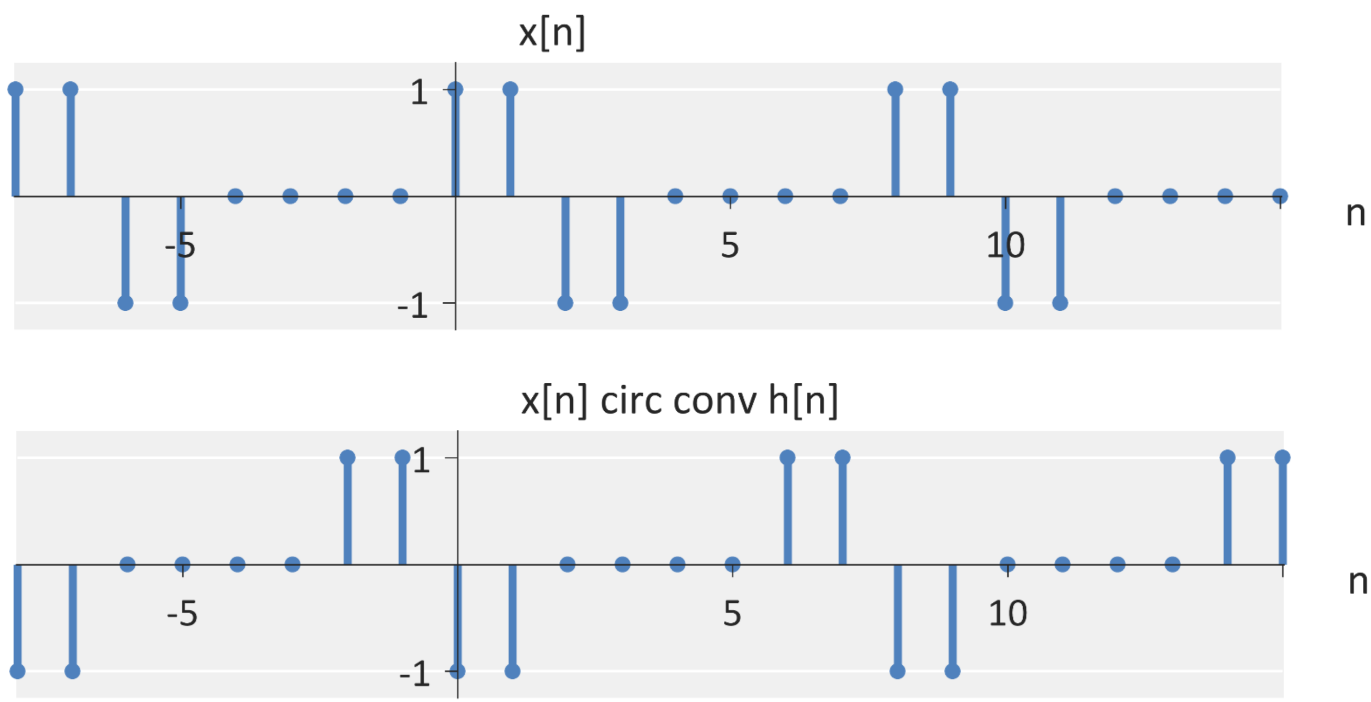

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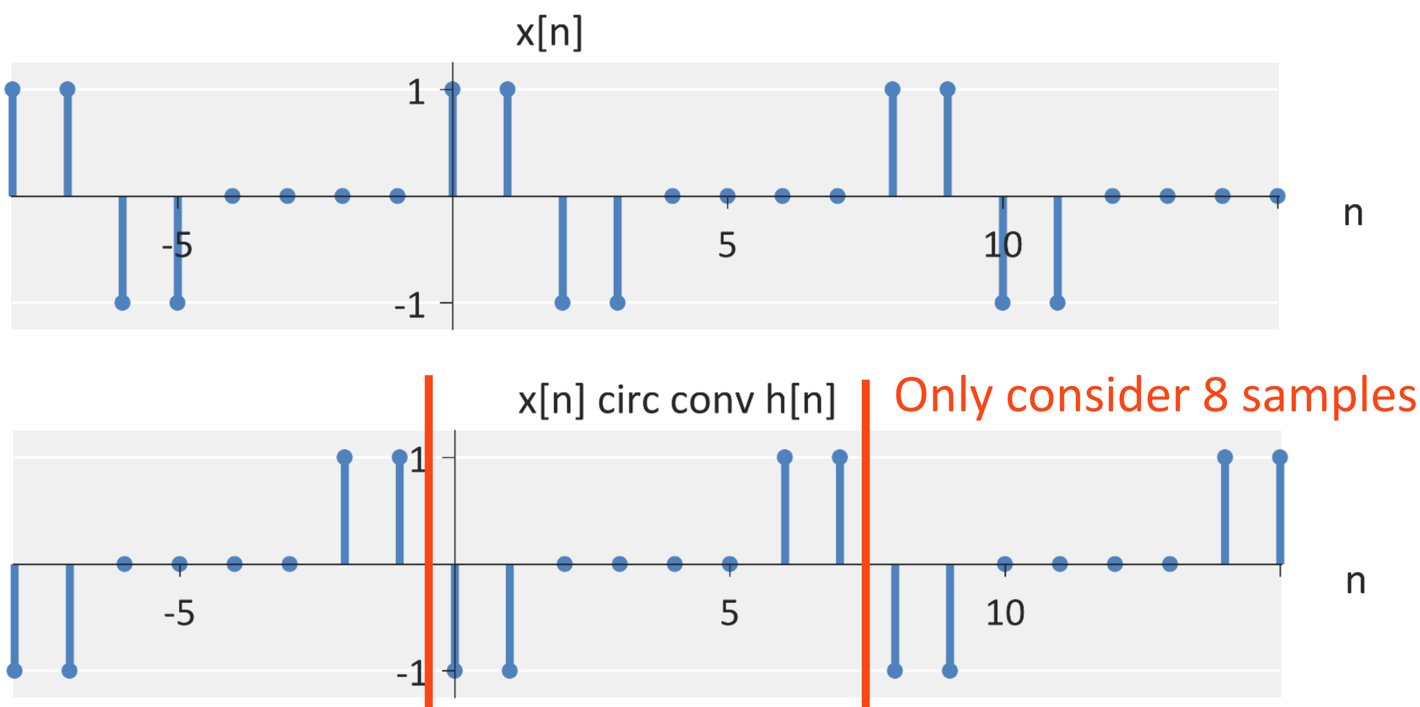

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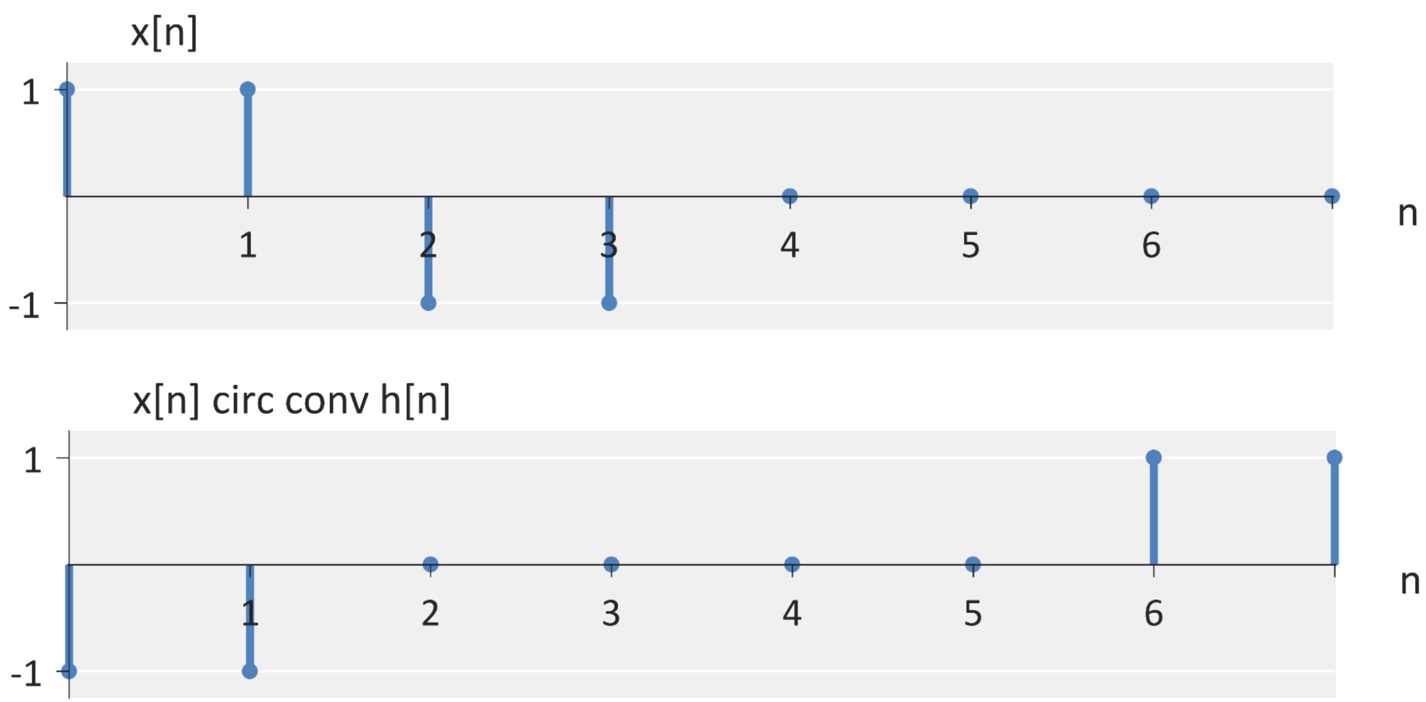

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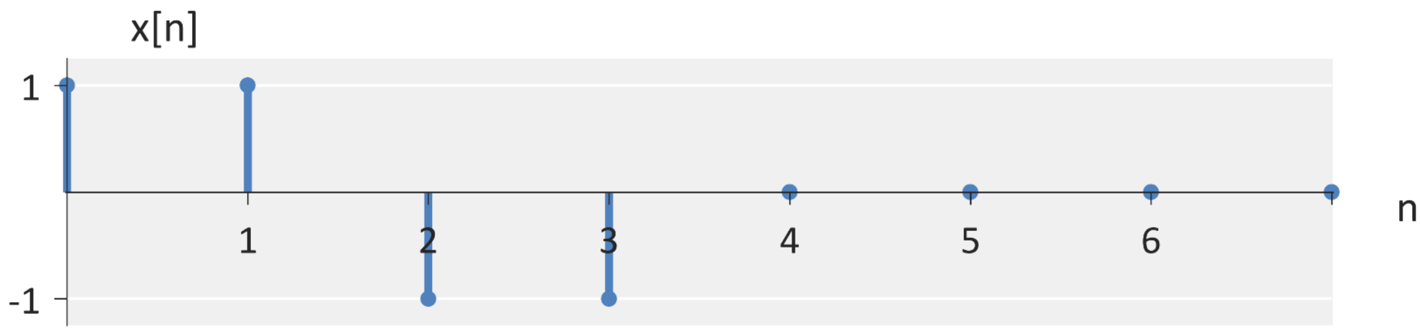

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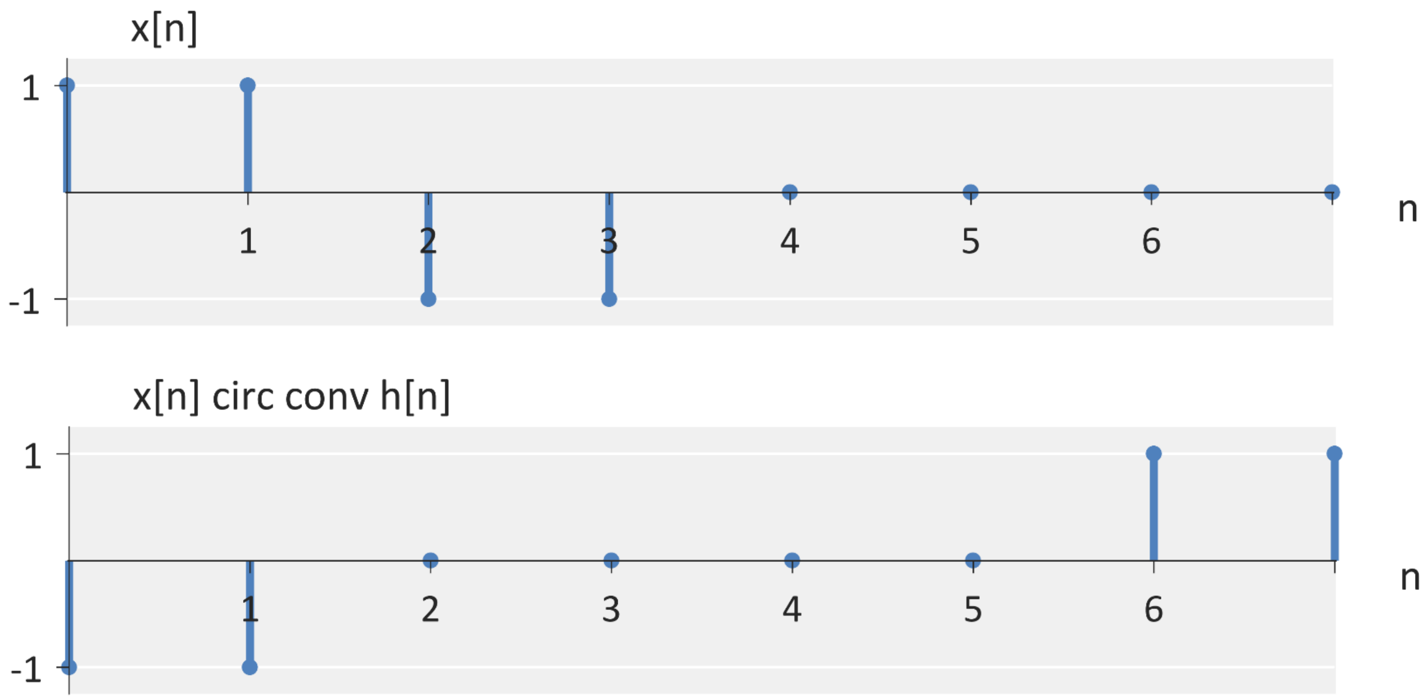

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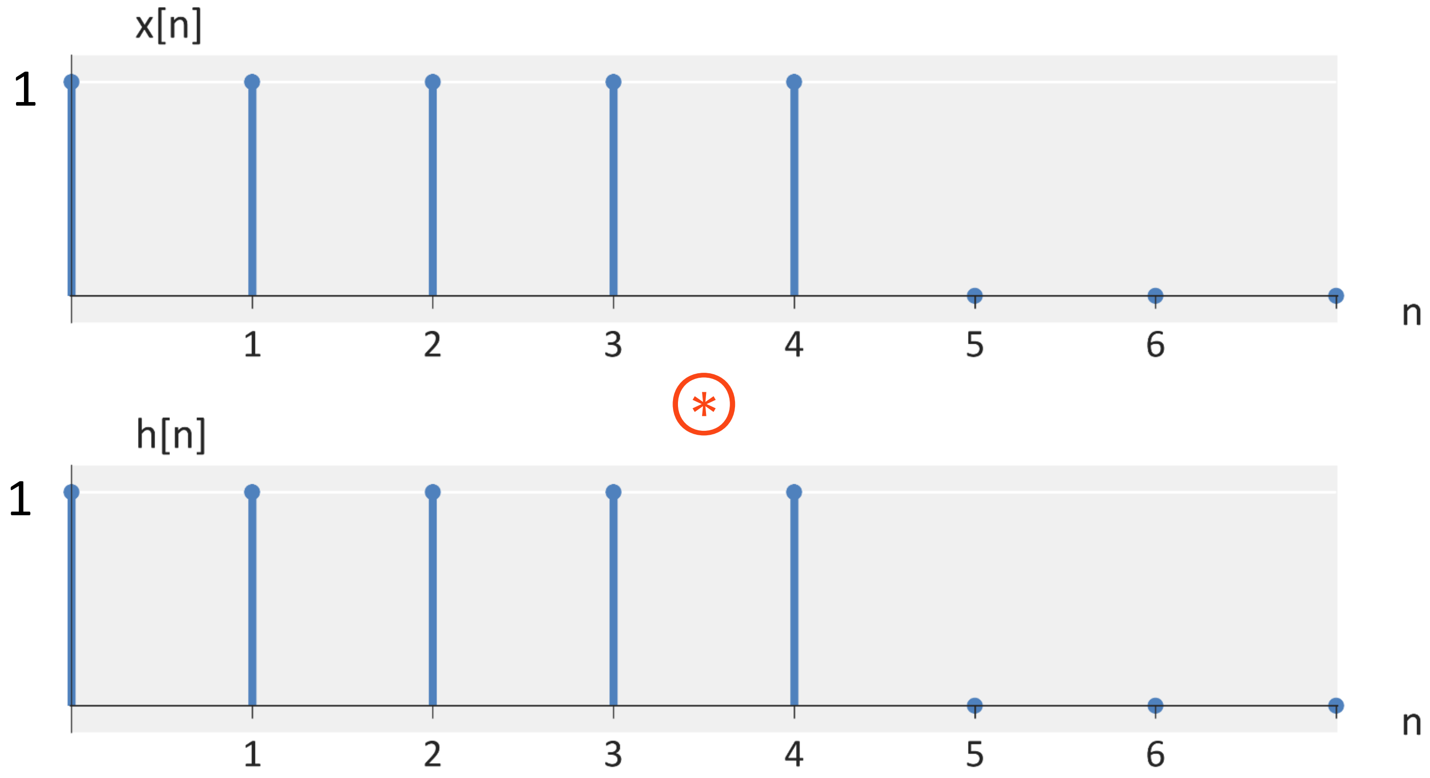
Circular Convolution

- **Example: Compute the circular convolution**
  - Assume the signals are ***finite*** length-8 signals



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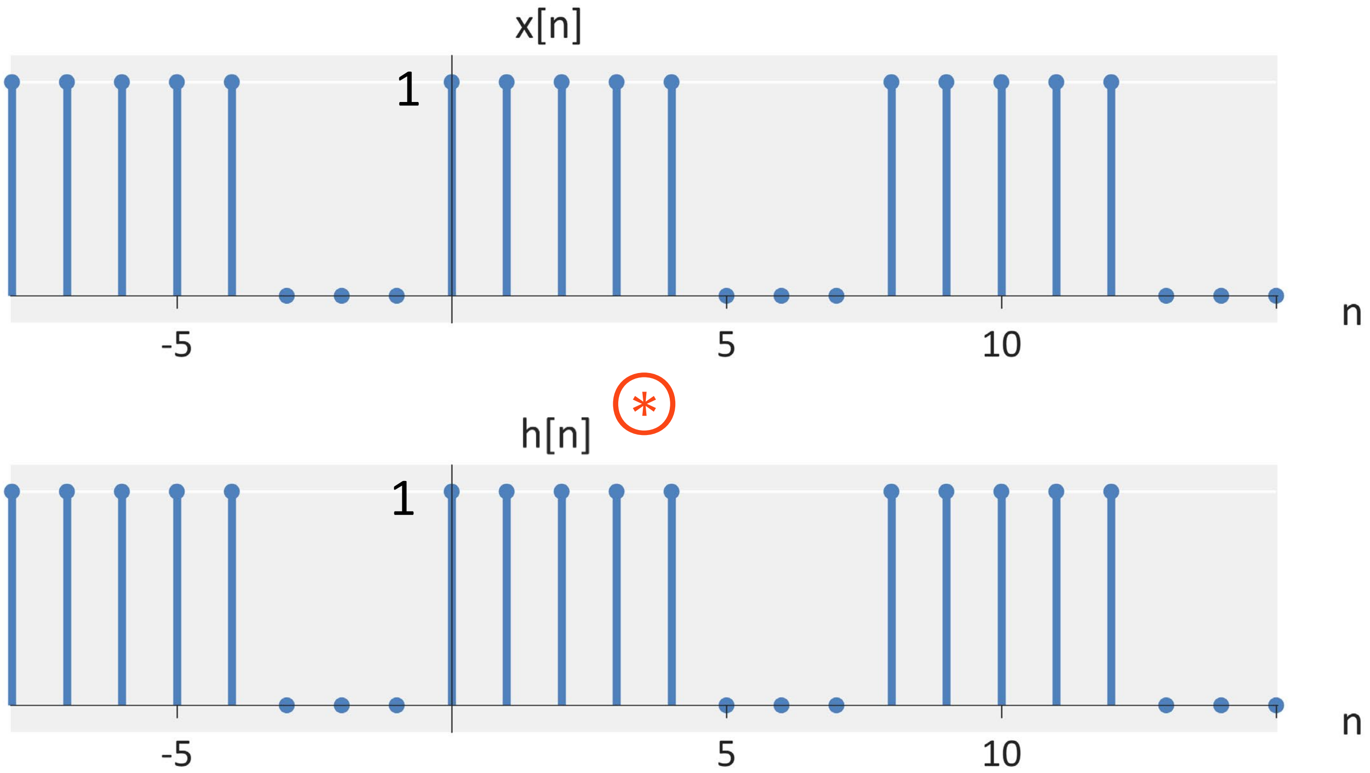

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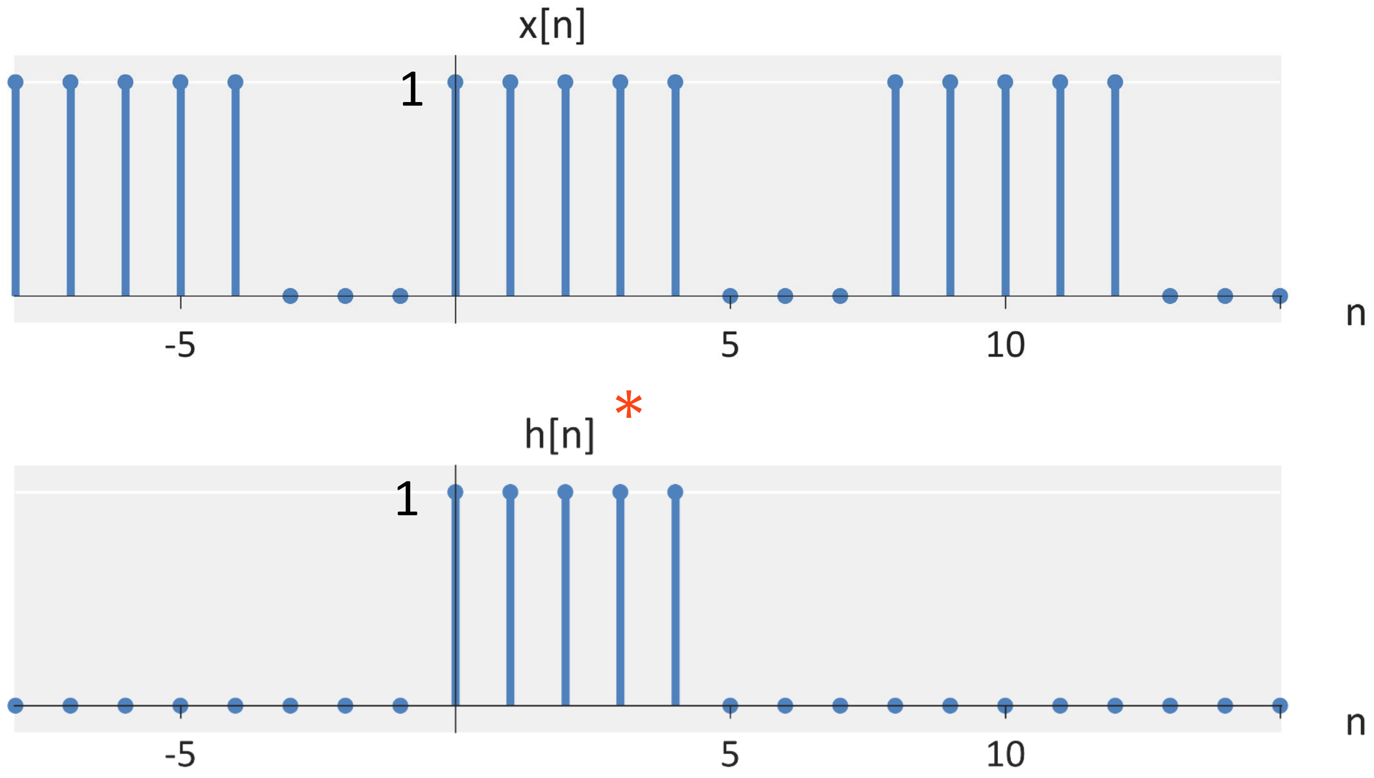

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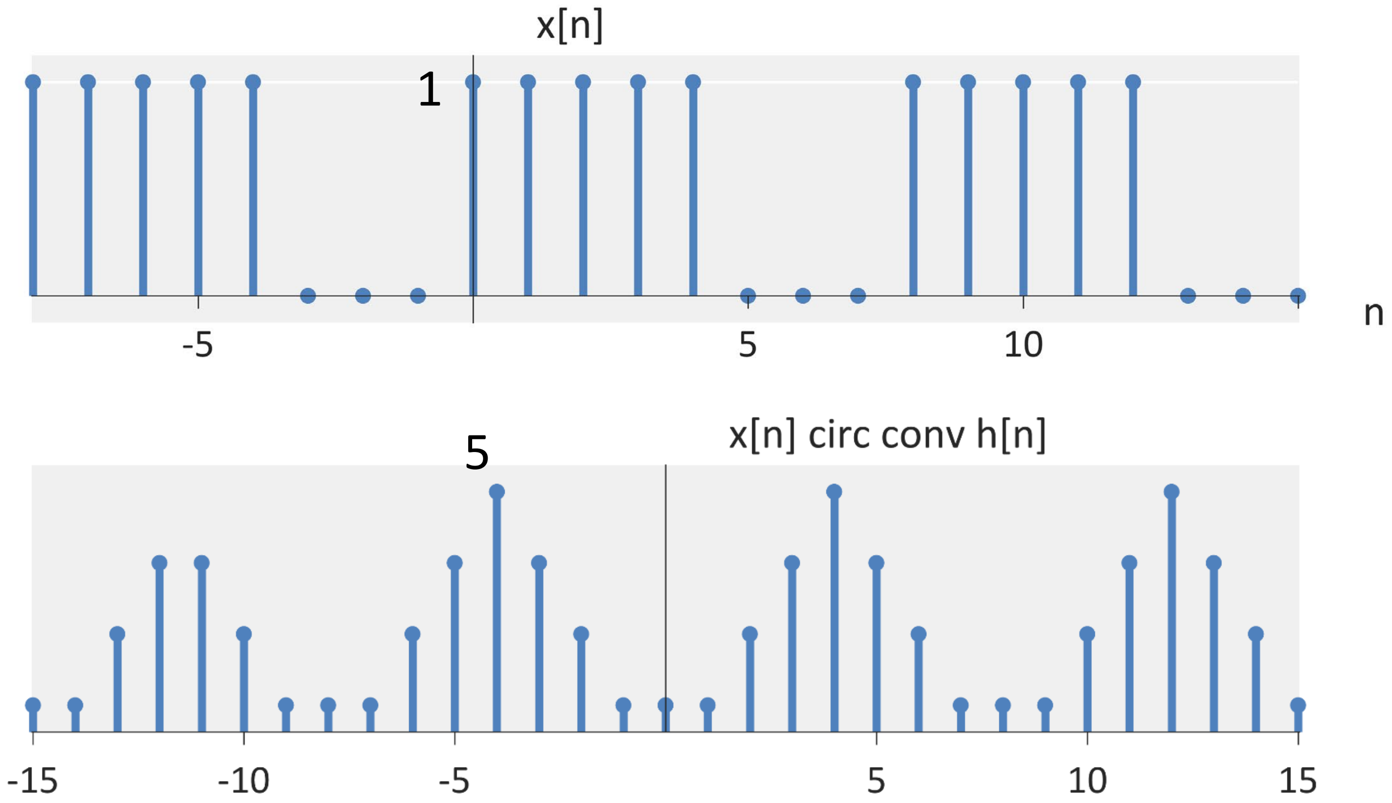

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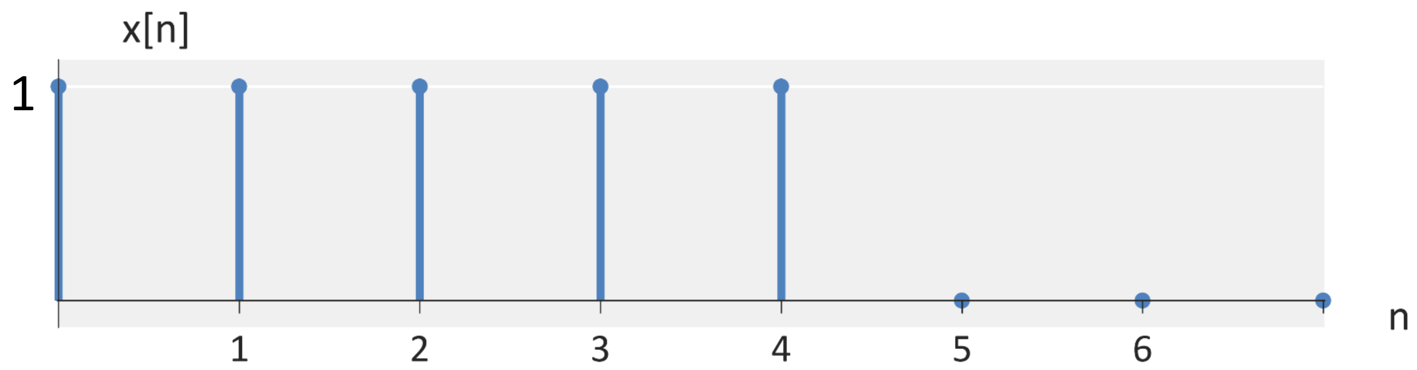

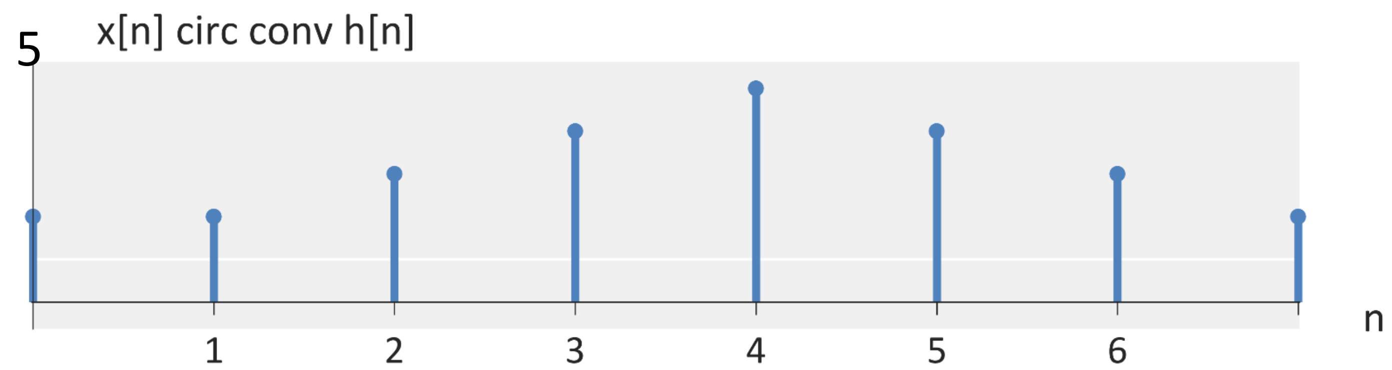

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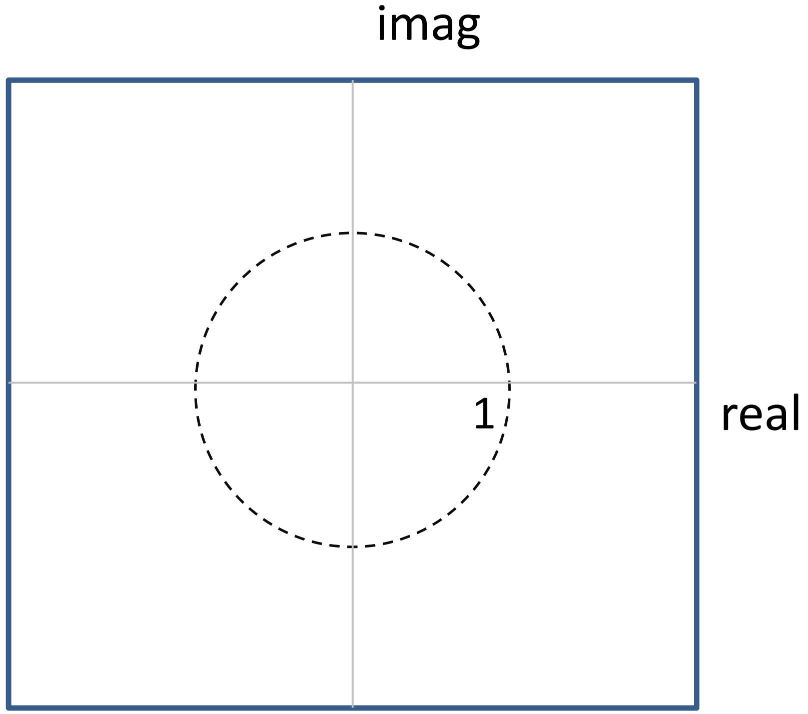
The Fast Fourier Transform

- **Z-Transform**

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$z = |r|e^{j\omega}$$



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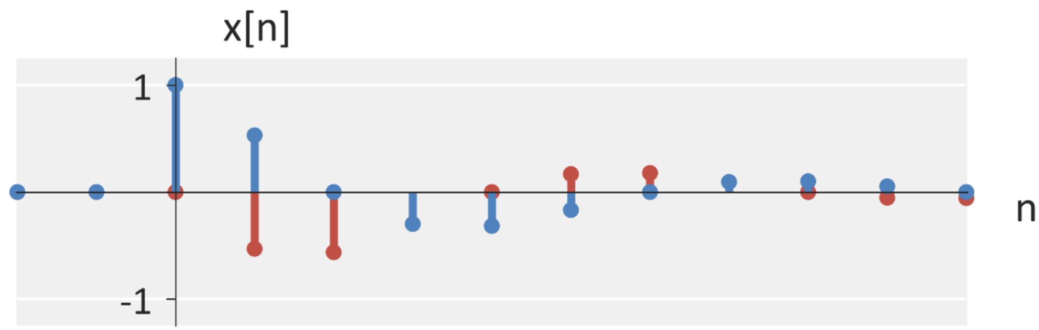

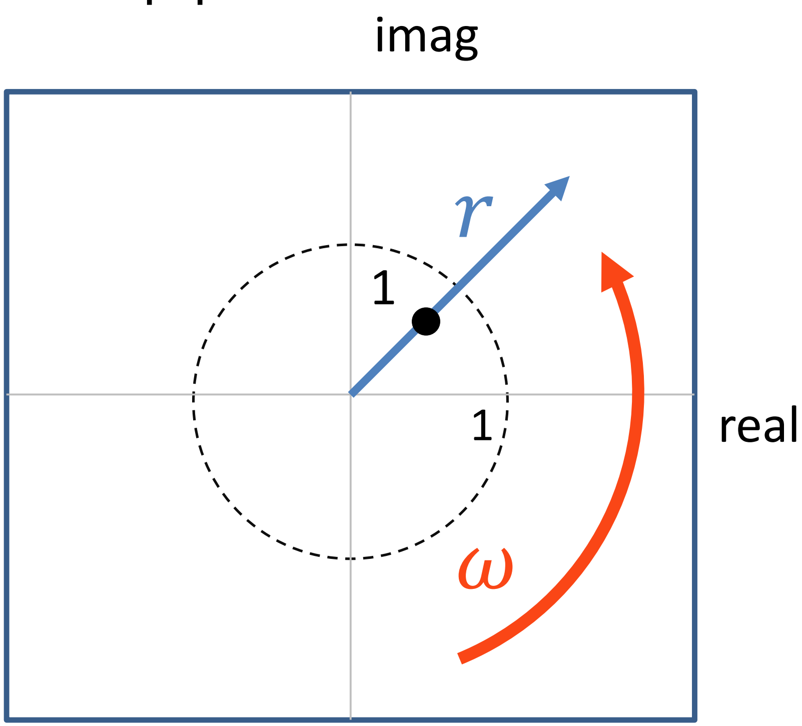

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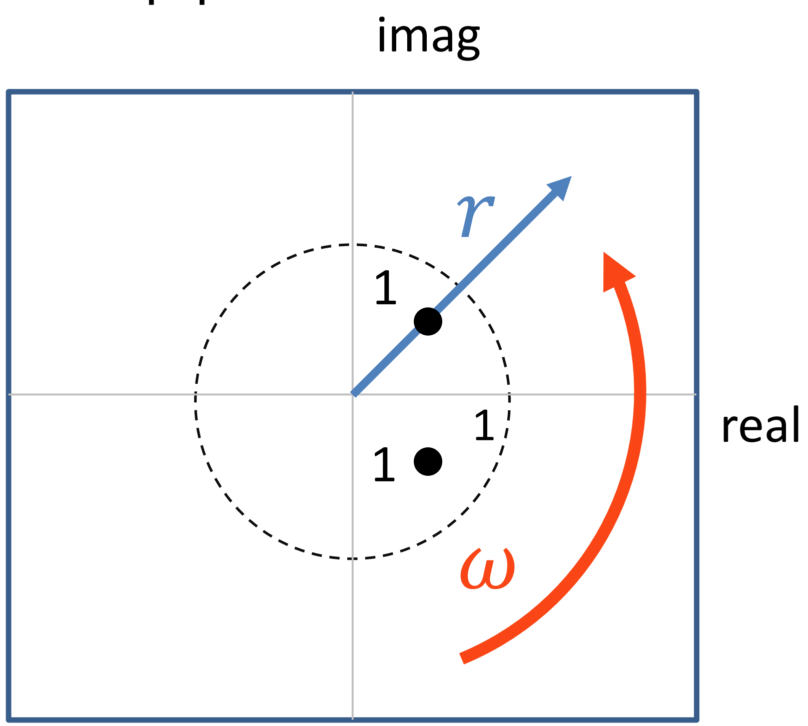

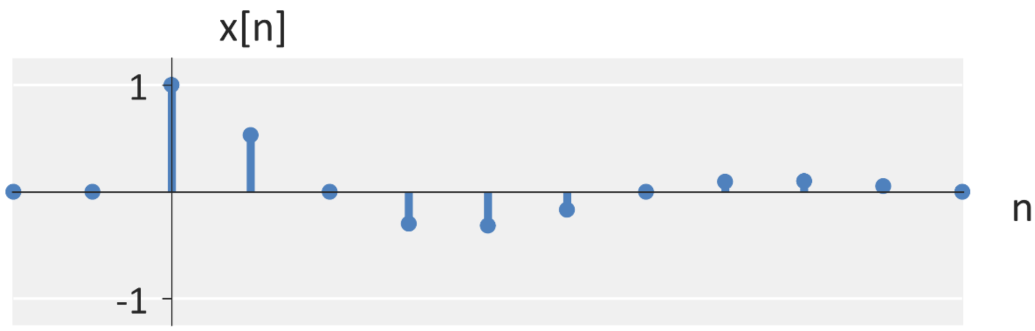

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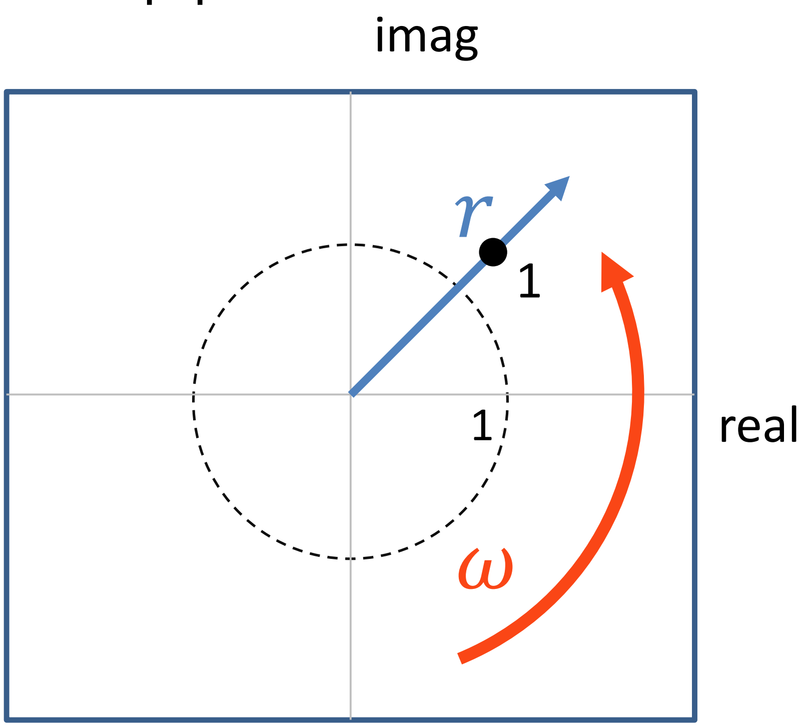

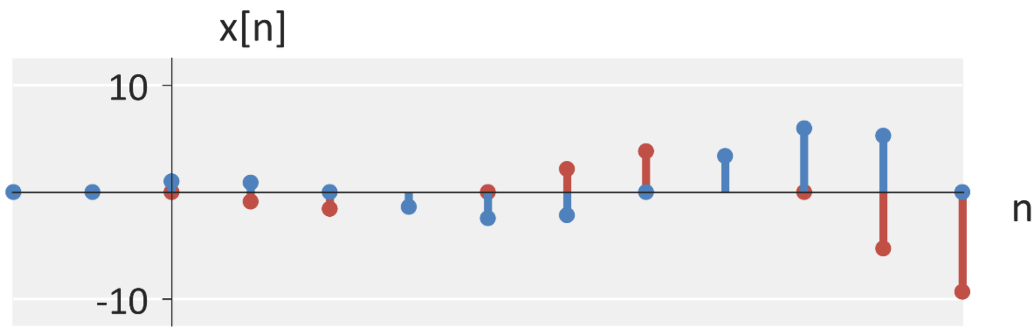

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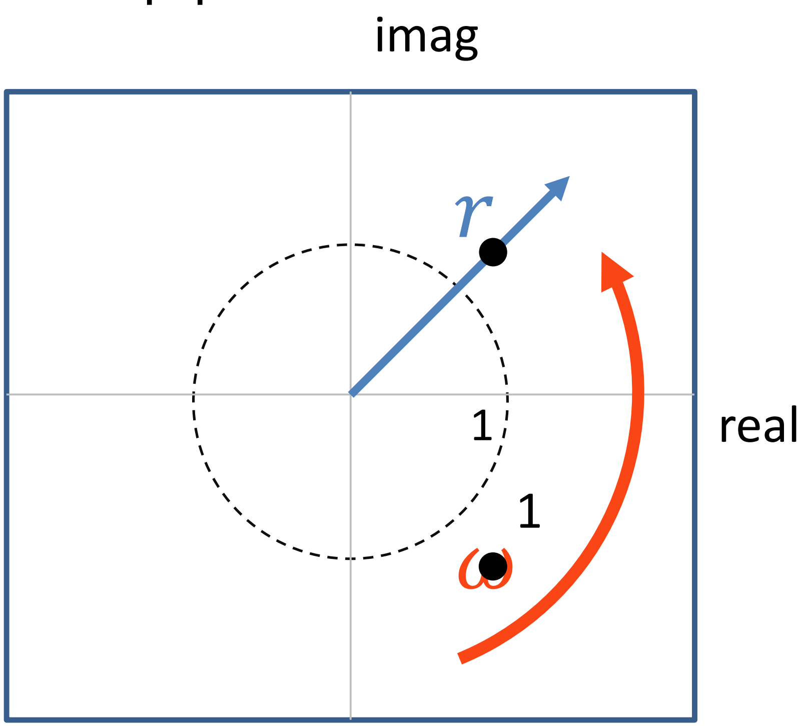

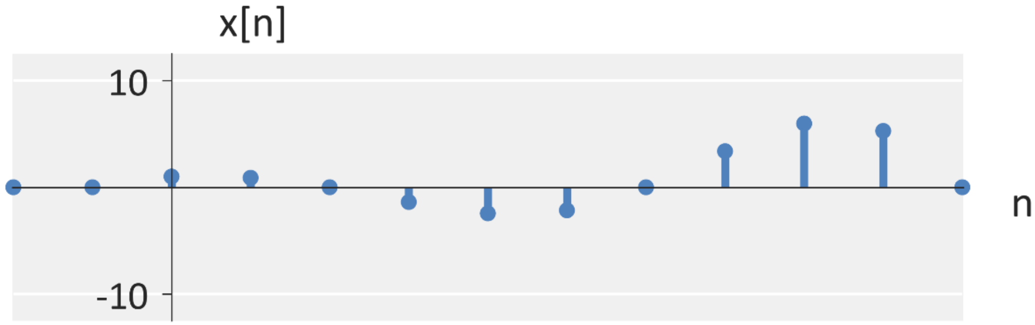

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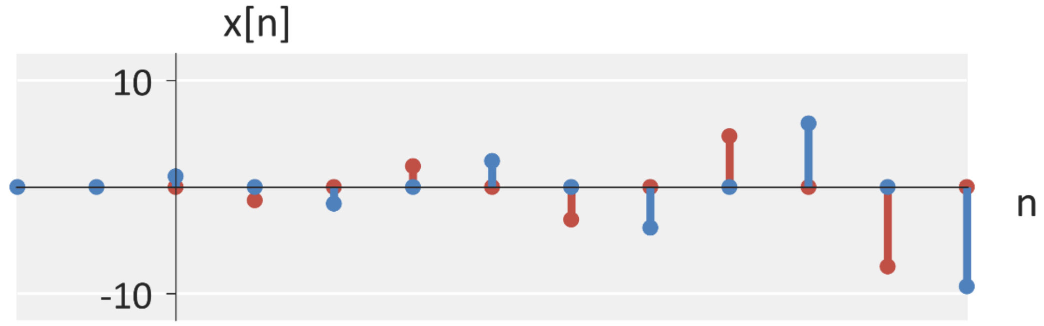

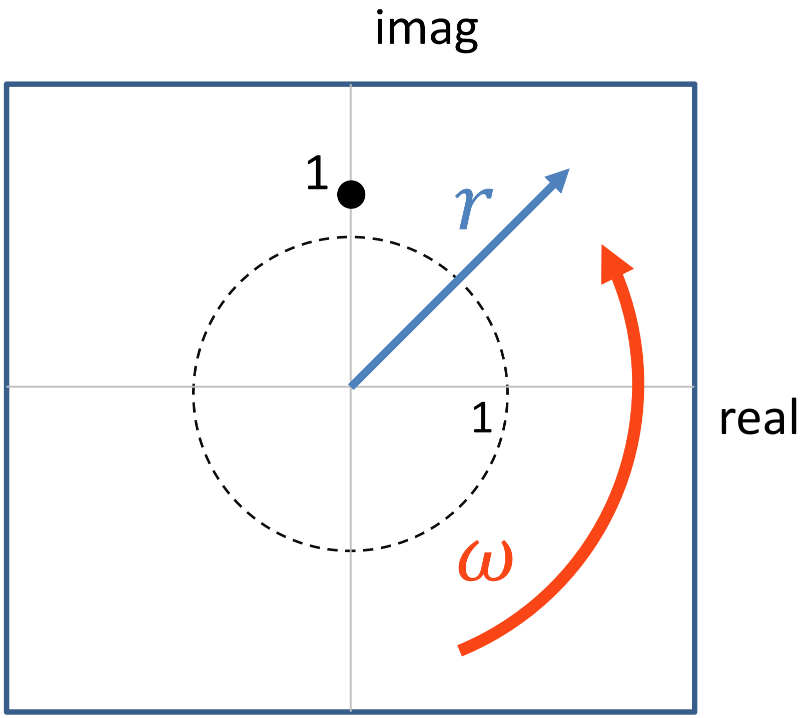

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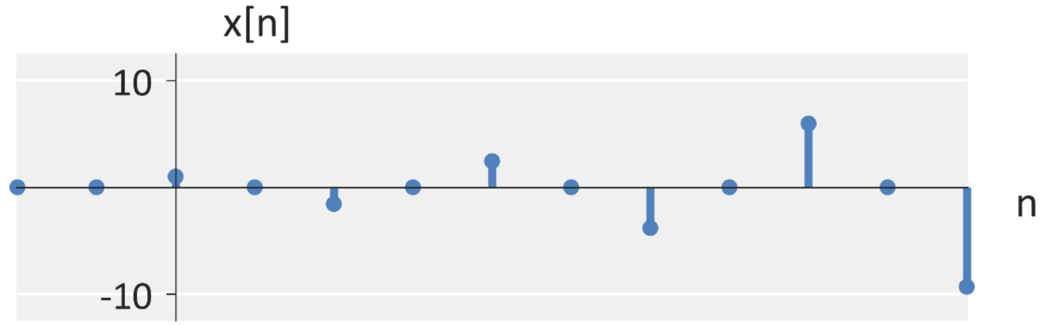

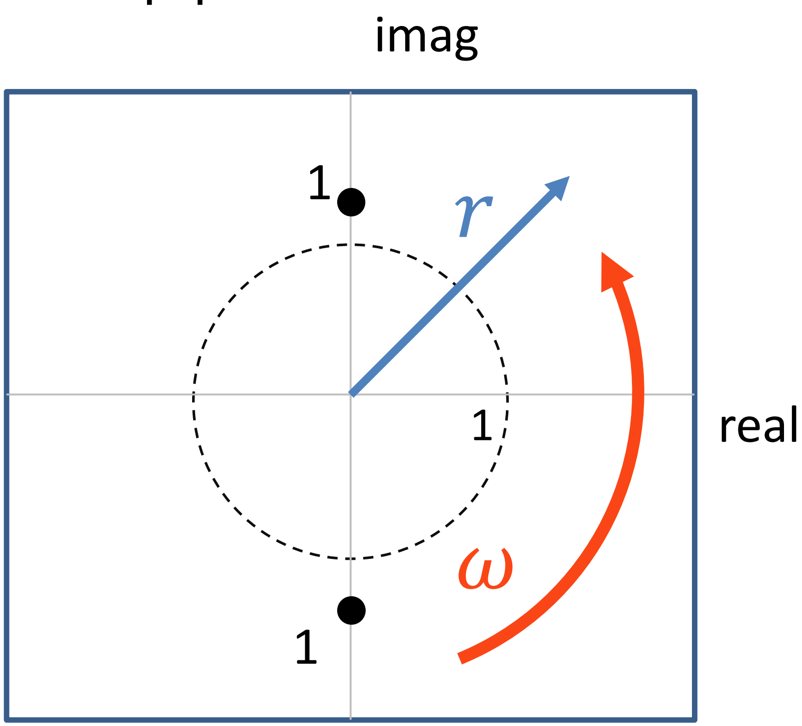

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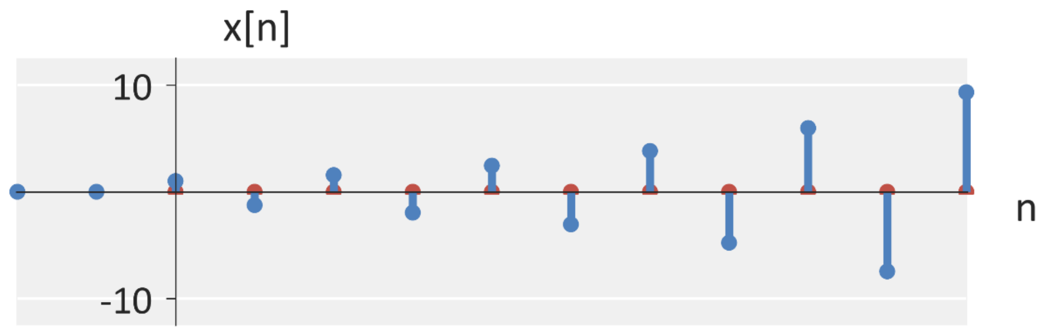

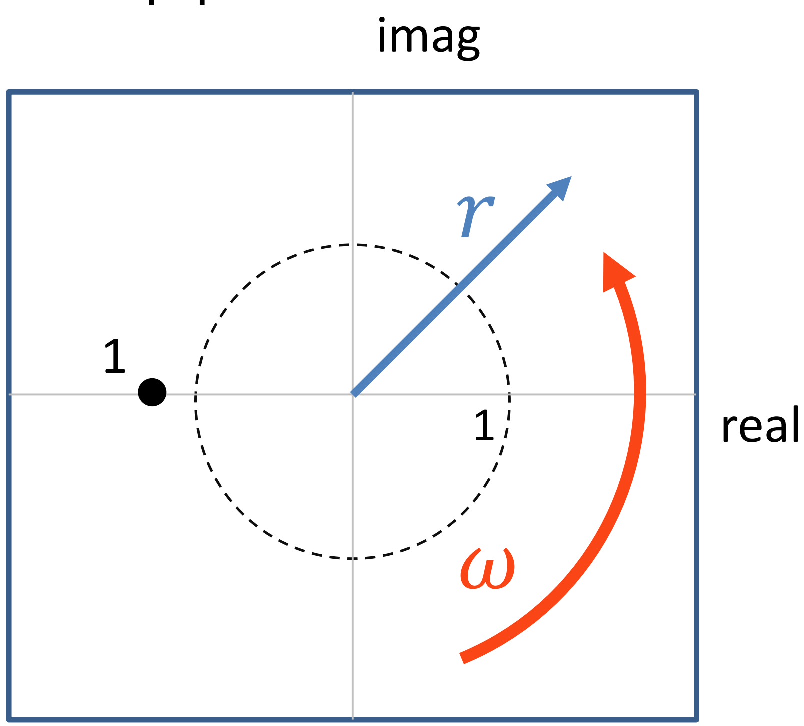

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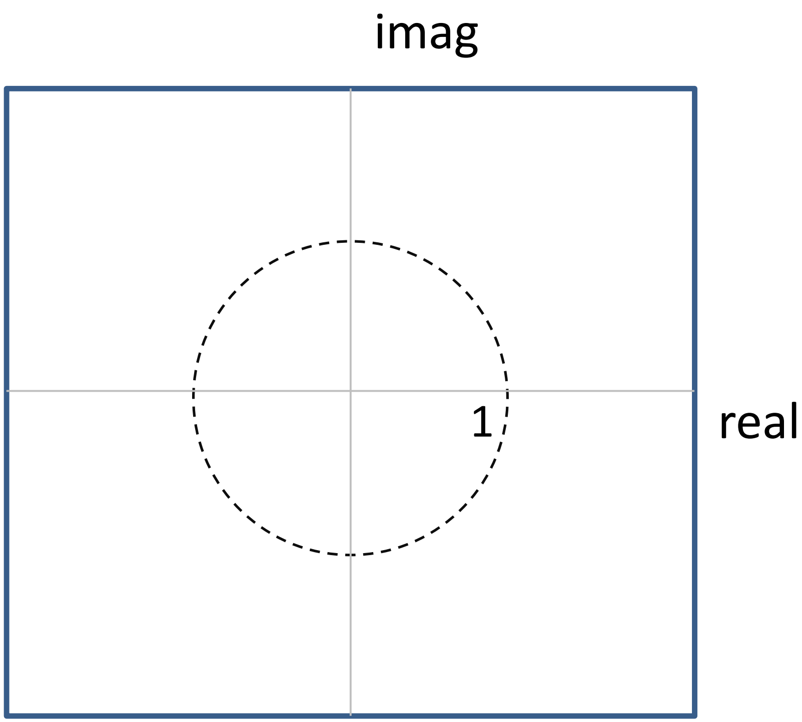
- **DTFT**

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$



Effectively:

$$z = |1|e^{j\omega}$$

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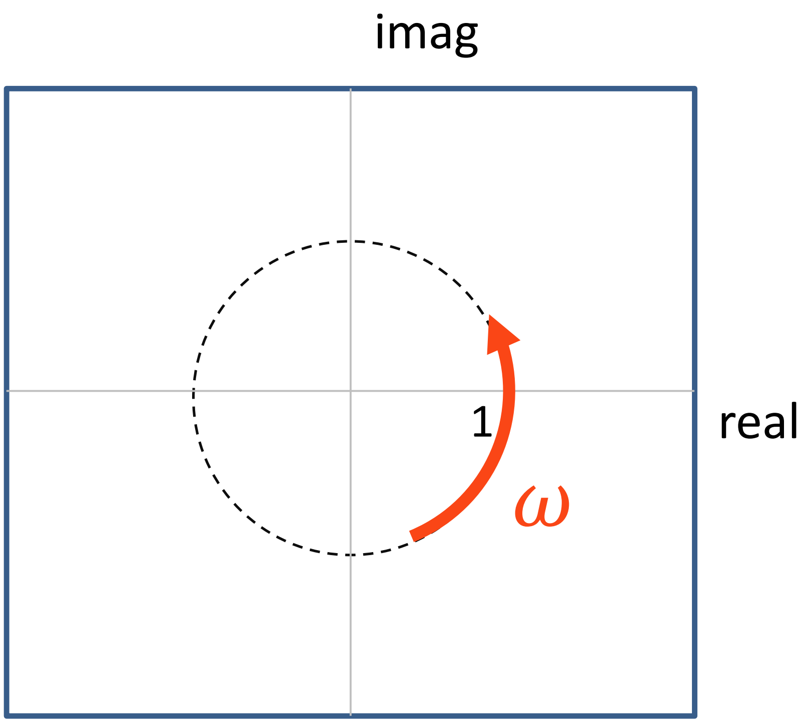

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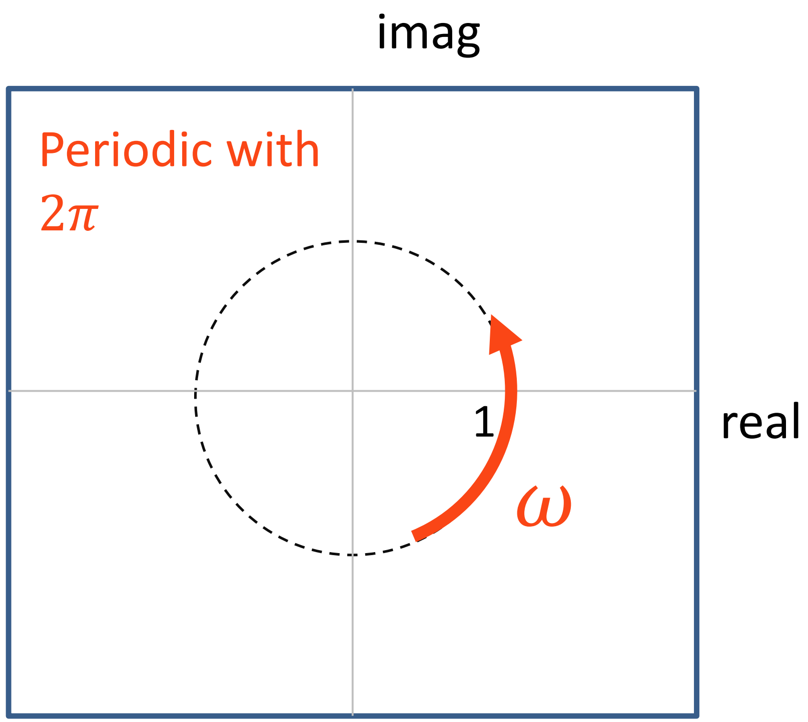

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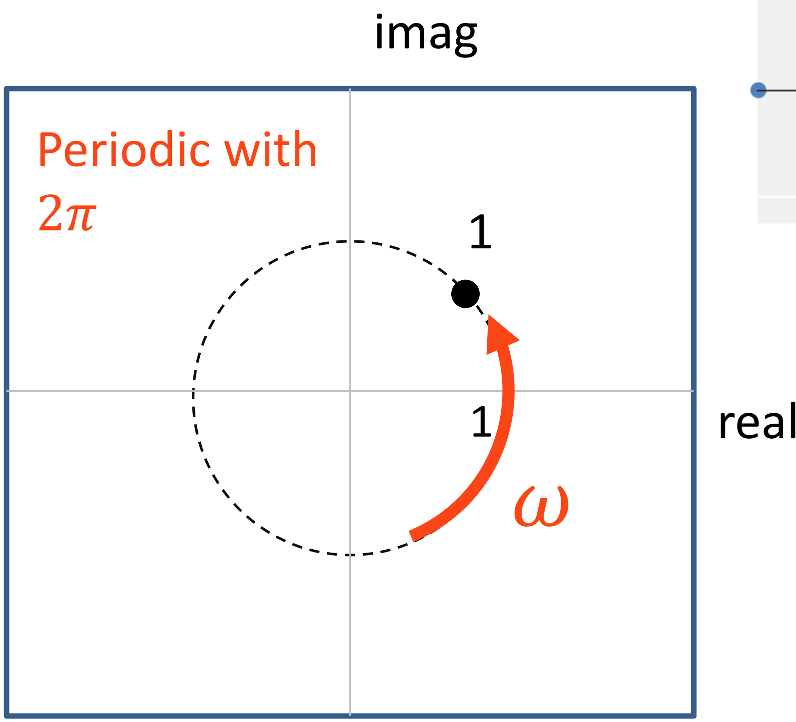

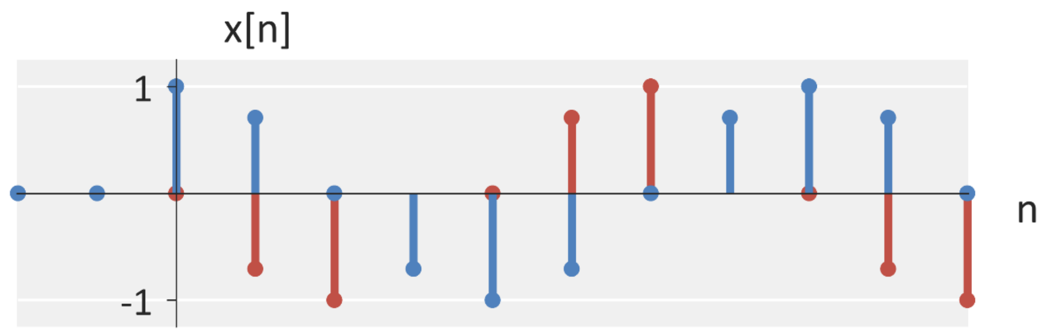

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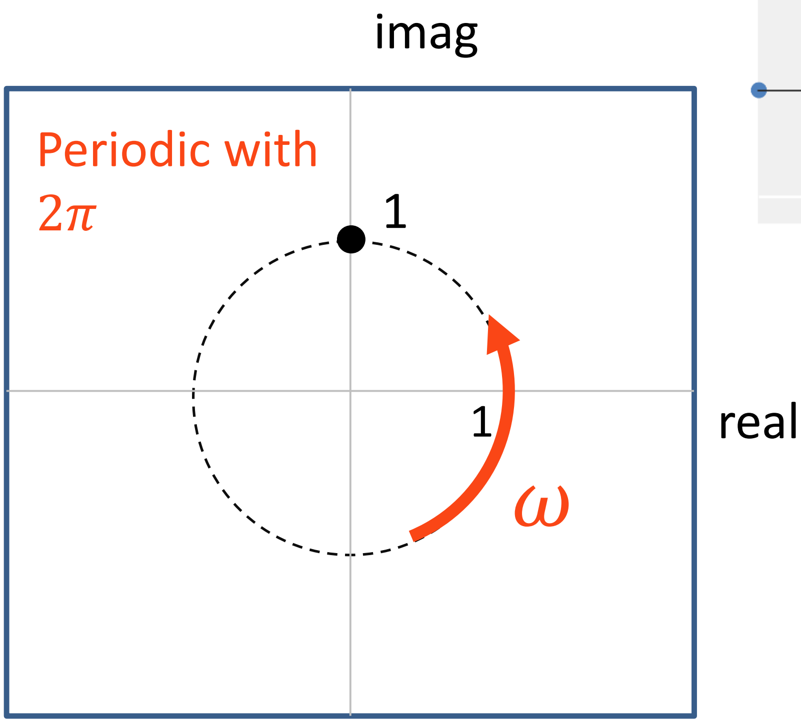

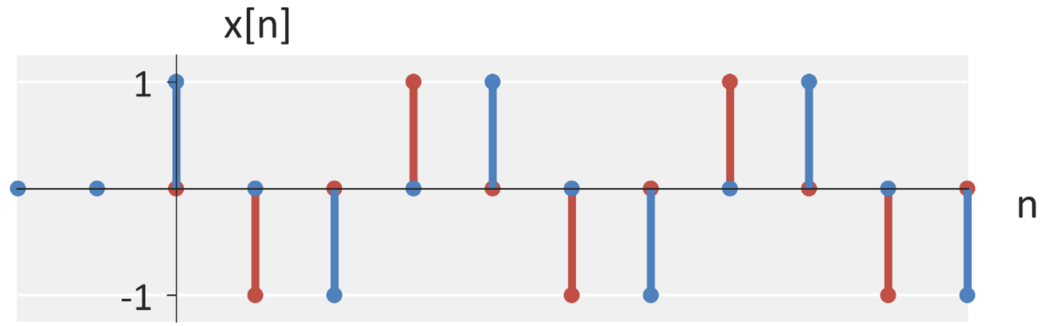

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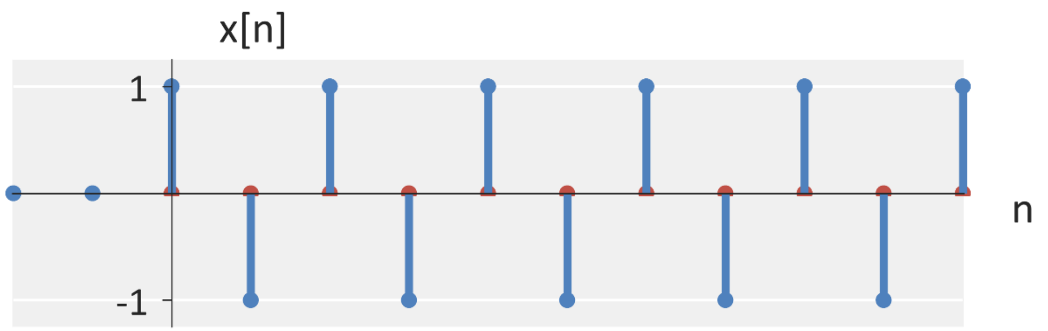

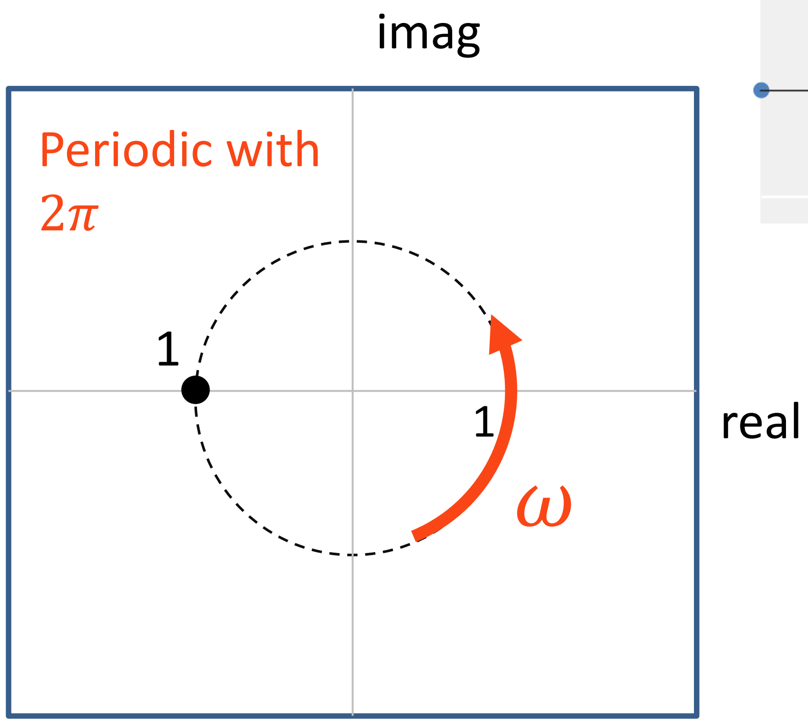

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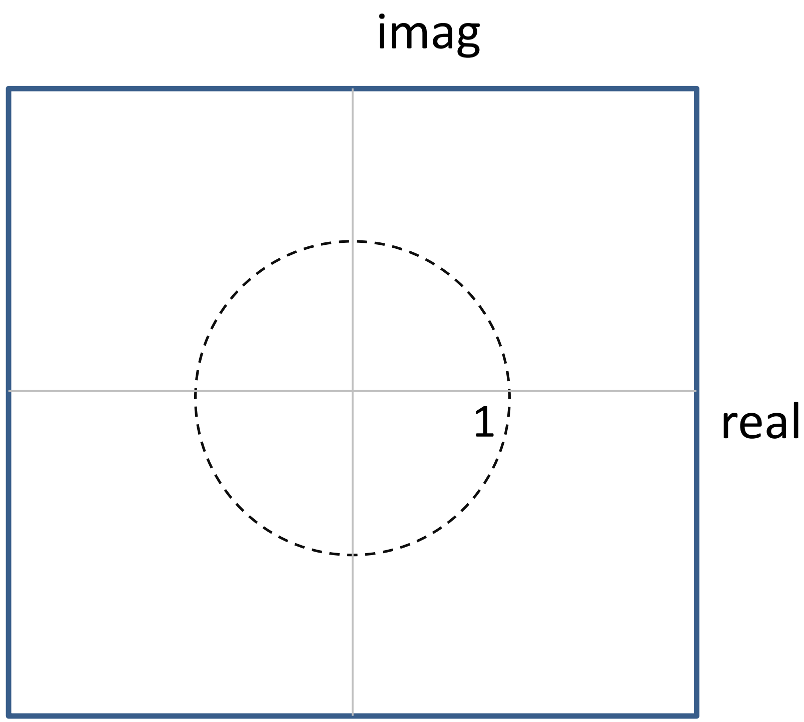
Effectively:

$$z = |1|e^{j\omega}$$

$$\omega = \frac{2\pi}{N}k$$



The Fast Fourier Transform

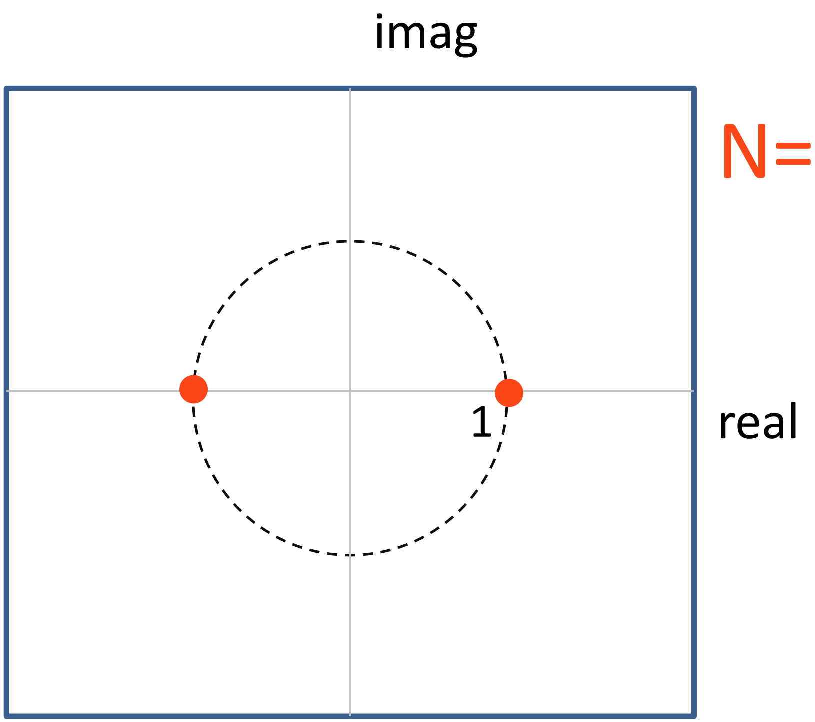
- **DTFT**

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$$

Effectively:

$$z = |1| e^{j\omega}$$

$$\omega = \frac{2\pi}{N} k$$



N=2

The Fast Fourier Transform

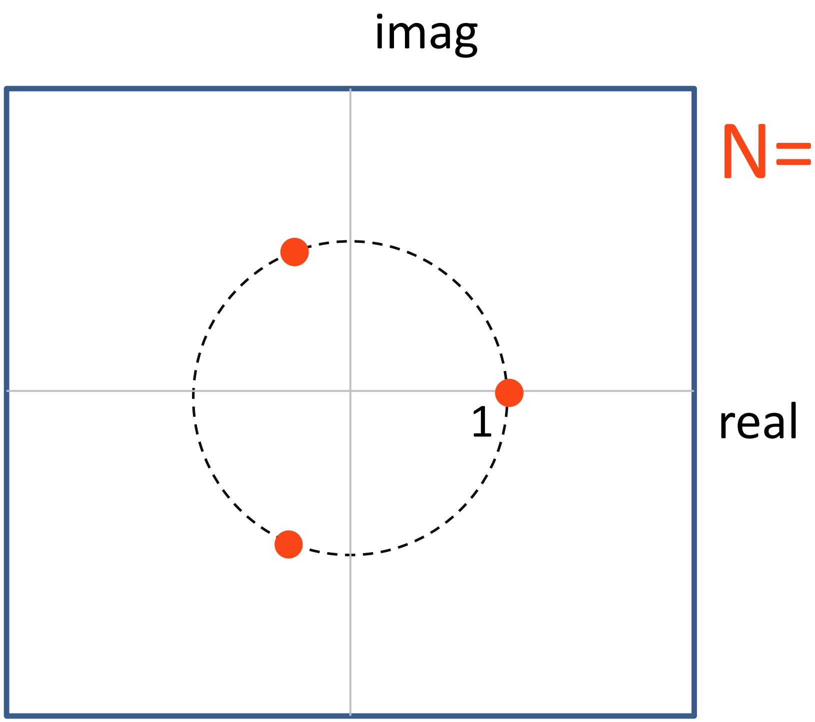
- **DTFT**

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$$

Effectively:

$$z = |1| e^{j\omega}$$

$$\omega = \frac{2\pi}{N} k$$



N=3

The Fast Fourier Transform

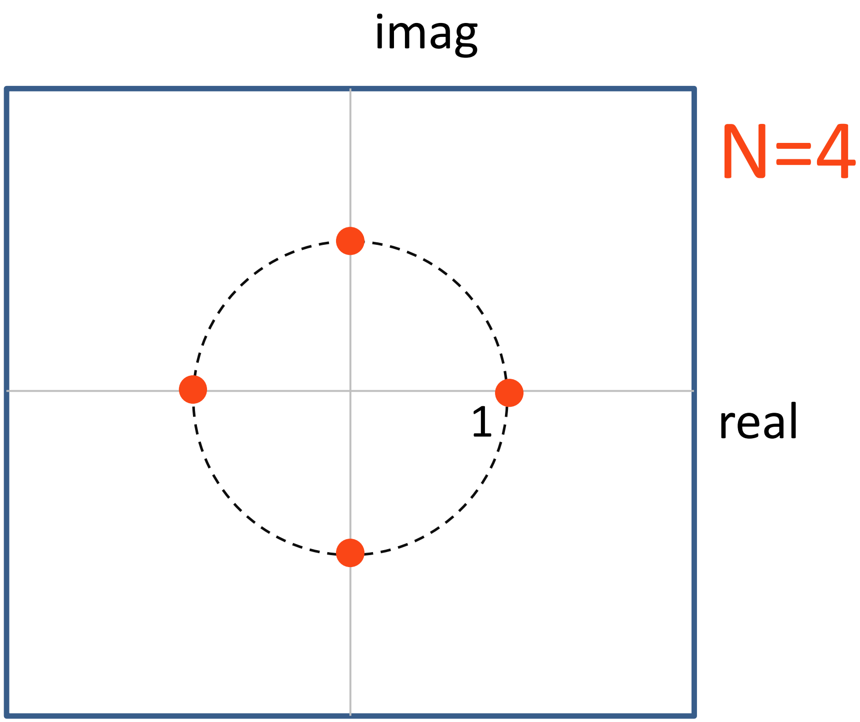
- **DTFT**

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nk}$$

Effectively:

$$z = |1|e^{j\omega}$$

$$\omega = \frac{2\pi}{N}k$$



The Fast Fourier Transform

- **DTFT**

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$$

Effectively:

$$z = |1| e^{j\omega}$$

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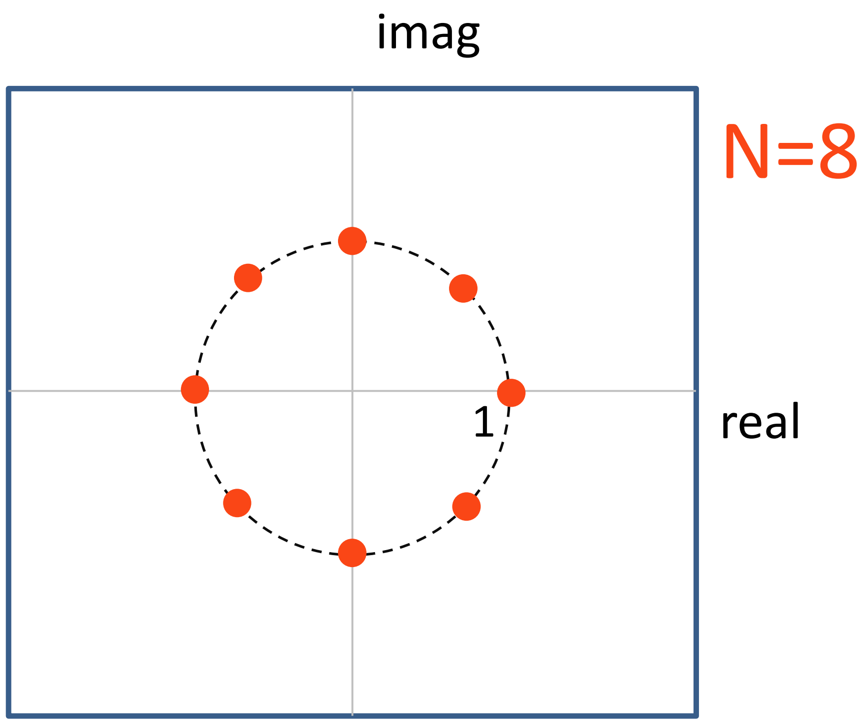


The Fast Fourier Transform

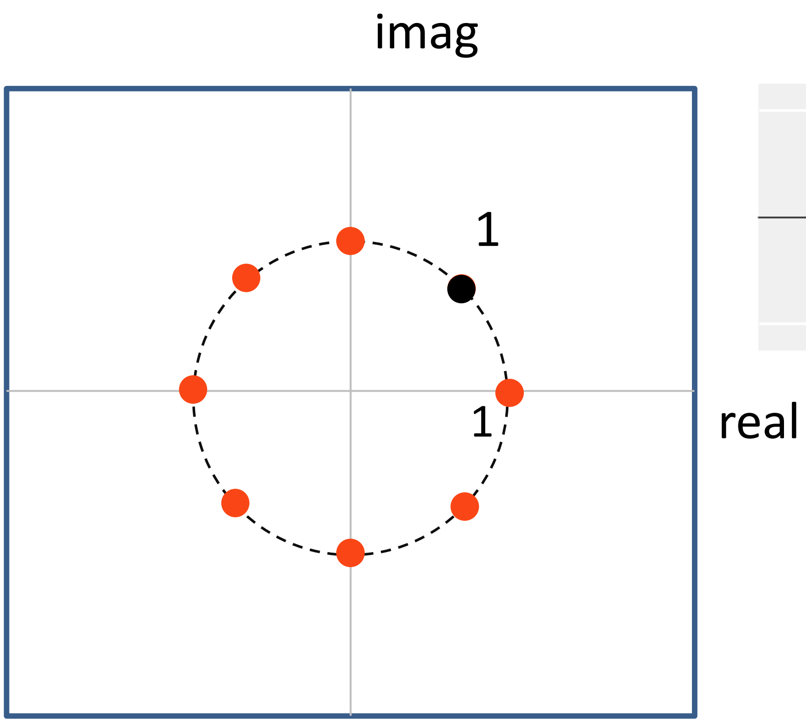
- **DTFT**

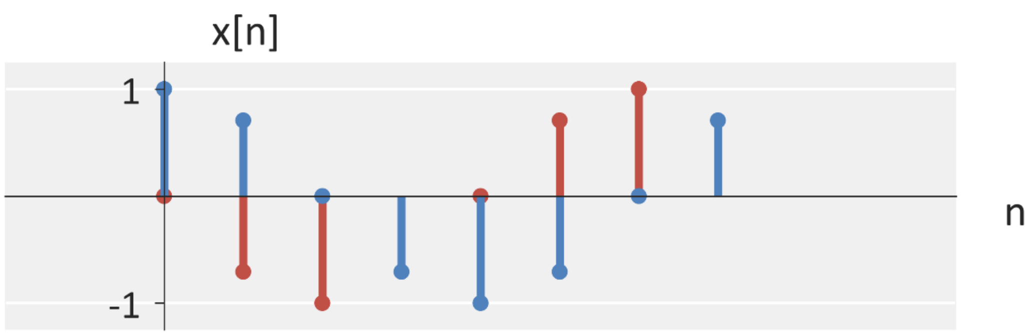
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$$

Effectively:

$$z = |1| e^{j\omega}$$

$$\omega = \frac{2\pi}{N} k$$





The Fast Fourier Transform

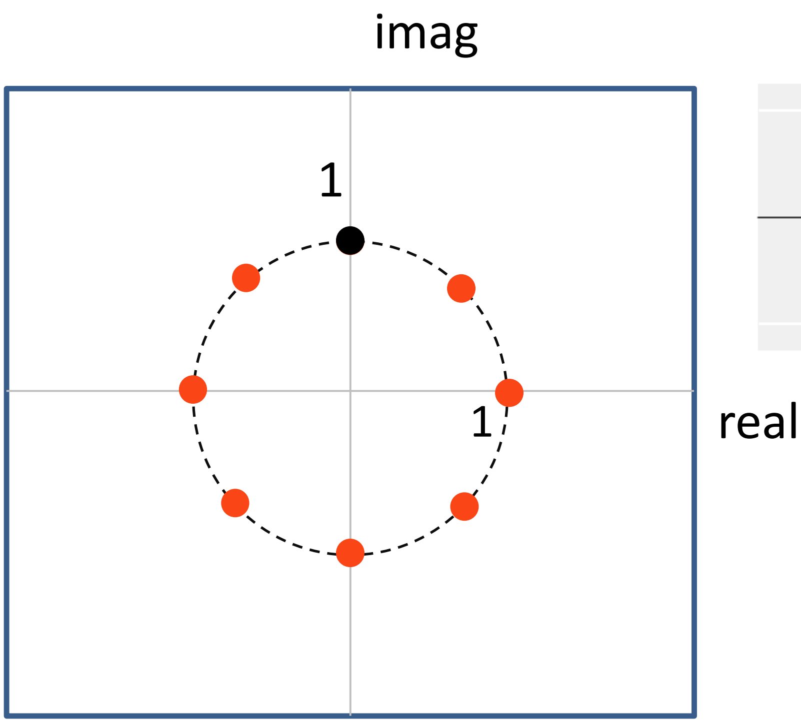
- **DTFT**

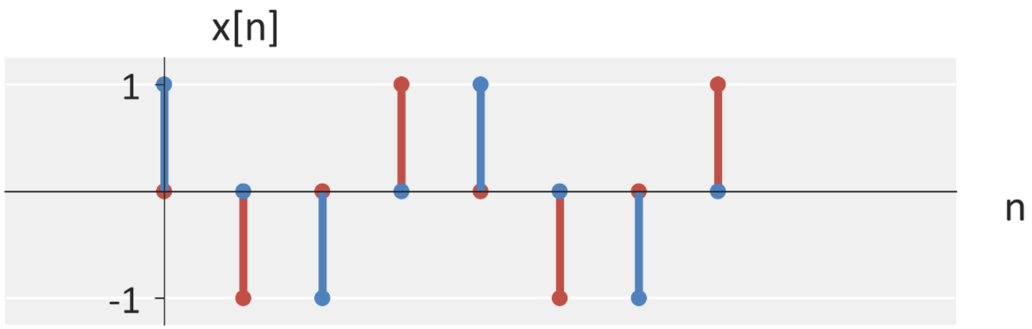
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$$

Effectively:

$$z = |1| e^{j\omega}$$

$$\omega = \frac{2\pi}{N} k$$





The Fast Fourier Transform

- **DTFT**

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$$

Effectively:

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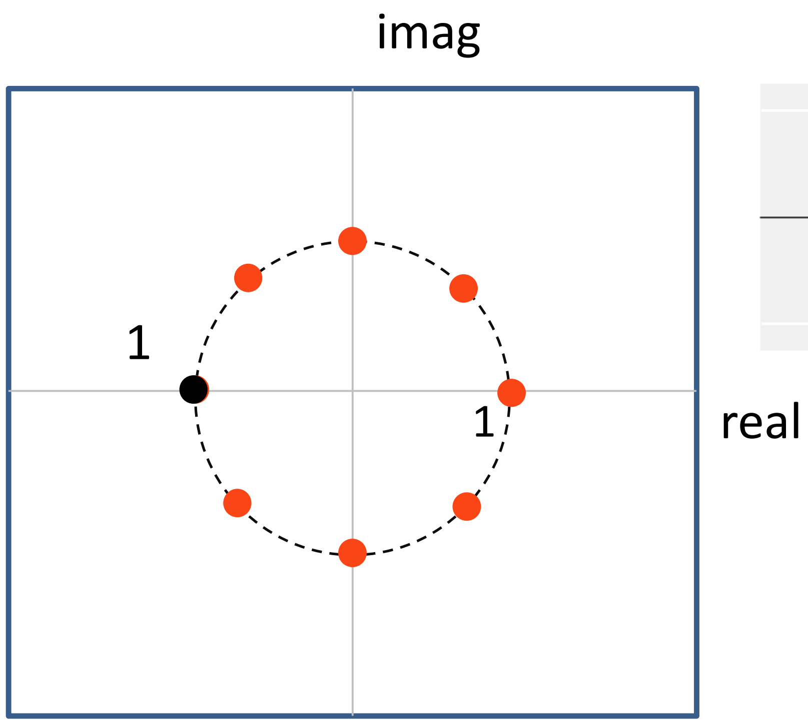
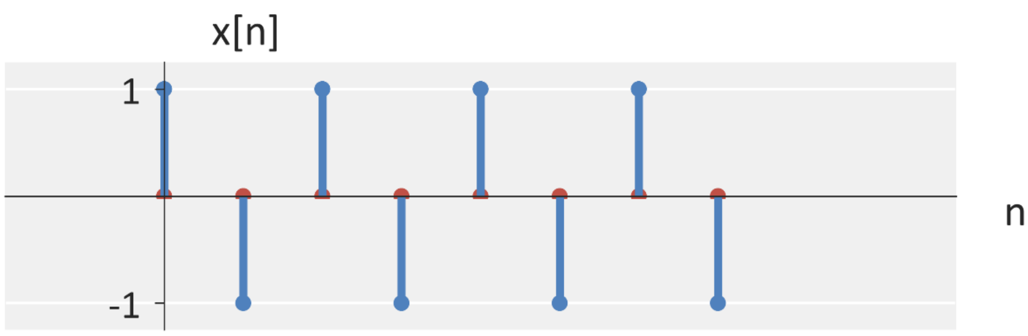




Lecture 14: Discrete -Time Filters

Foundations of Digital Signal Processing

Outline

- Circular Convolution Review
- The DTFT and the DFT: The Relationship
- **The Fast Fourier Transform**
- Designing the magnitude response
- Designing the phase response

The Fast Fourier Transform

- **Question: How many multiplications are used in convolution?**
  - $\approx N(2N-1)$

- **Convolution with two length-N signals**

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

The Fast Fourier Transform

- **Question: How many multiplications are used in convolution?**
  - $\approx N(2N-1)$ (Computational complexity is $\mathcal{O}(N^2)$)

- **Convolution with two length-N signals**

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

The Fast Fourier Transform

- **Question: How many multiplications are used in the DFT?**

- **The Discrete Fourier Transform (DFT)**
  - Analysis Equations

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

  - Synthesis Equations

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j\frac{2\pi}{N}kn}$$

The Fast Fourier Transform

- **Question: How many multiplications are used in the DFT?**
  - N^2

- **The Discrete Fourier Transform (DFT)**
  - Analysis Equations

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

  - Synthesis Equations

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j\frac{2\pi}{N}kn}$$

The Fast Fourier Transform

- **Question: How many multiplications are used in the DFT?**
  - N^2 (Computational complexity is $\mathcal{O}(N^2)$)

- **The Discrete Fourier Transform (DFT)**
  - Analysis Equations

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

  - Synthesis Equations

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j\frac{2\pi}{N}kn}$$

The Fast Fourier Transform

- **Question: How many multiplications are used in the DFT?**
  - No clear speed gain from using the DFT ☹

- **The Discrete Fourier Transform (DFT)**
  - Analysis Equations

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

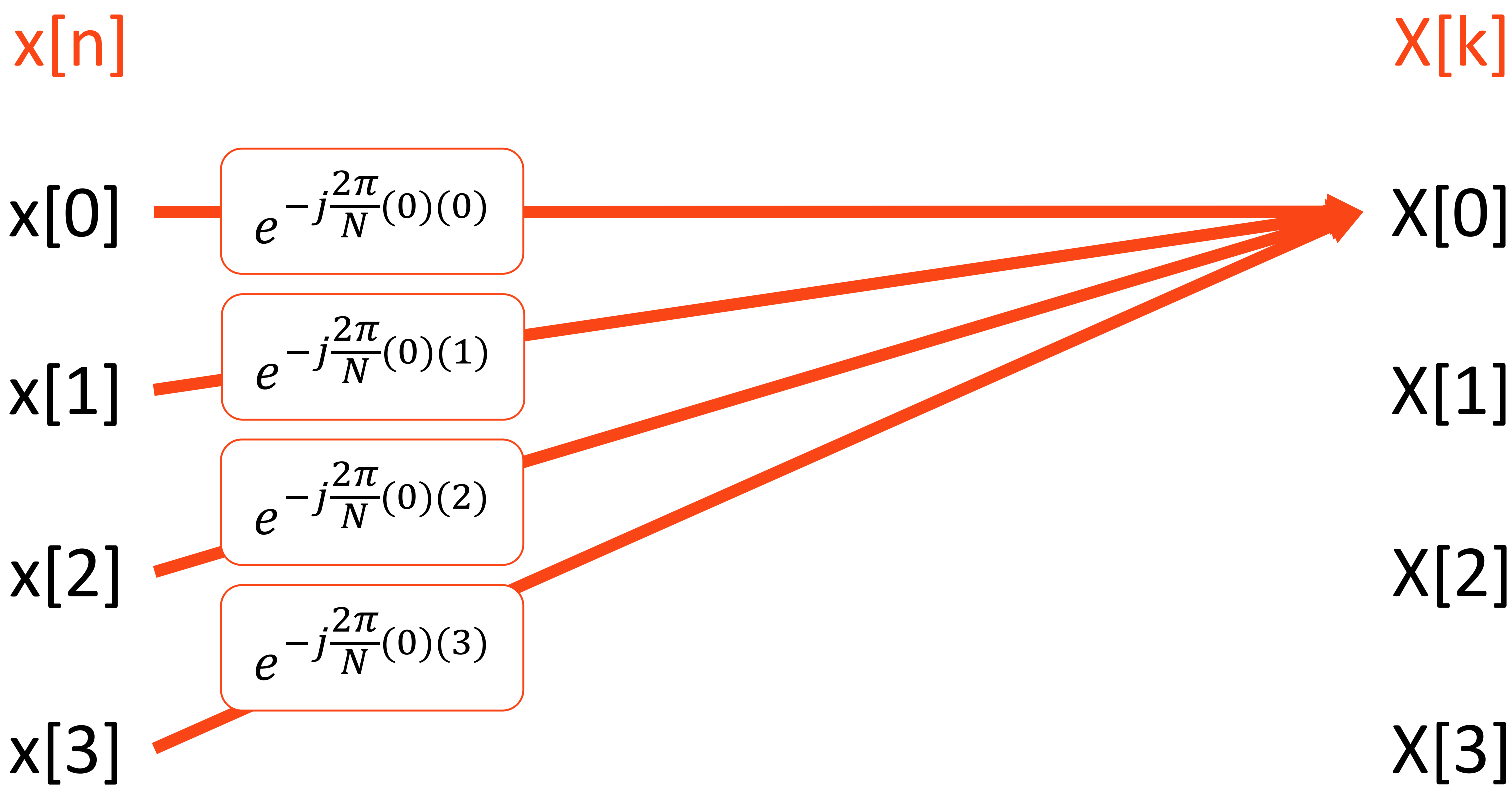
  - Synthesis Equations

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j\frac{2\pi}{N}kn}$$

The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

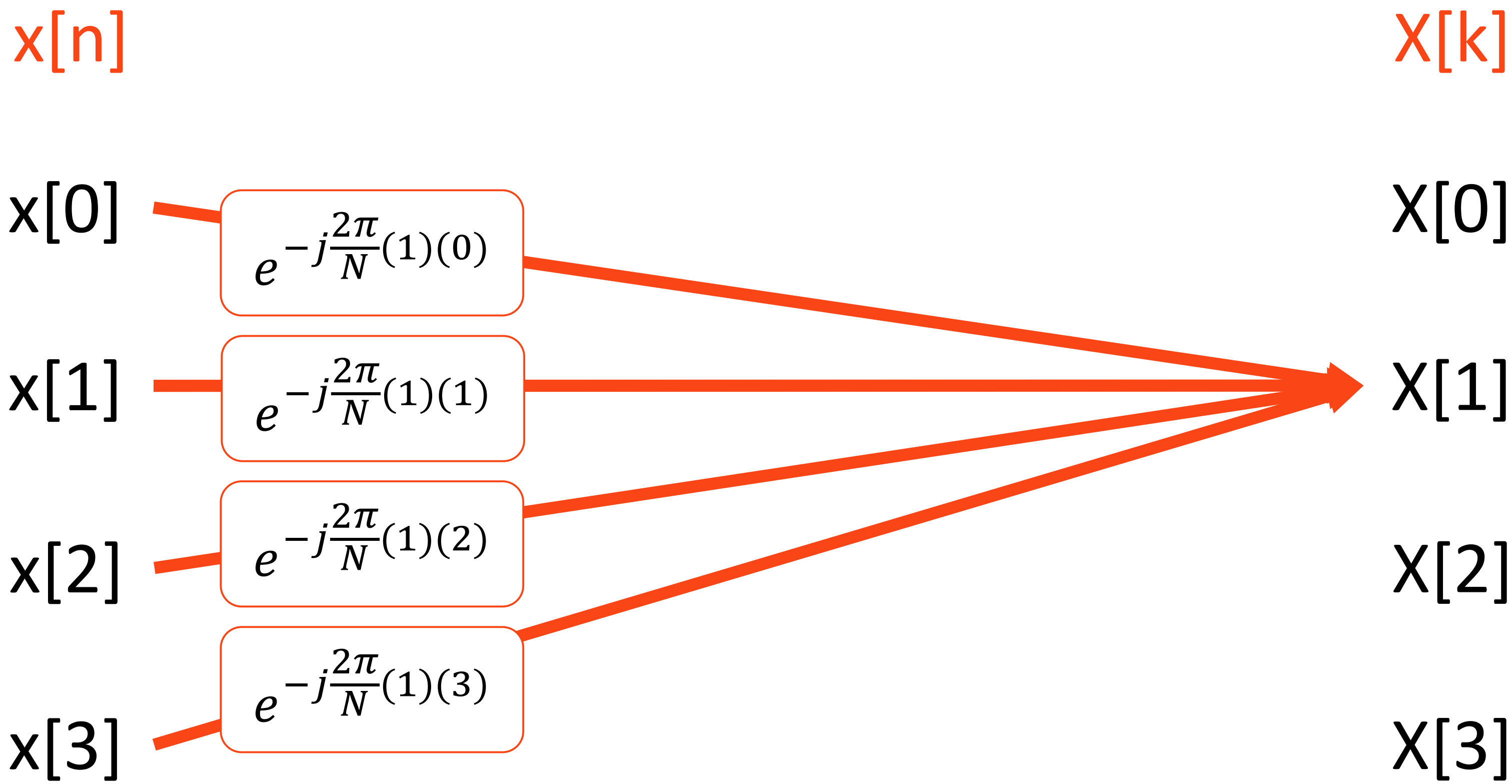
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$



The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

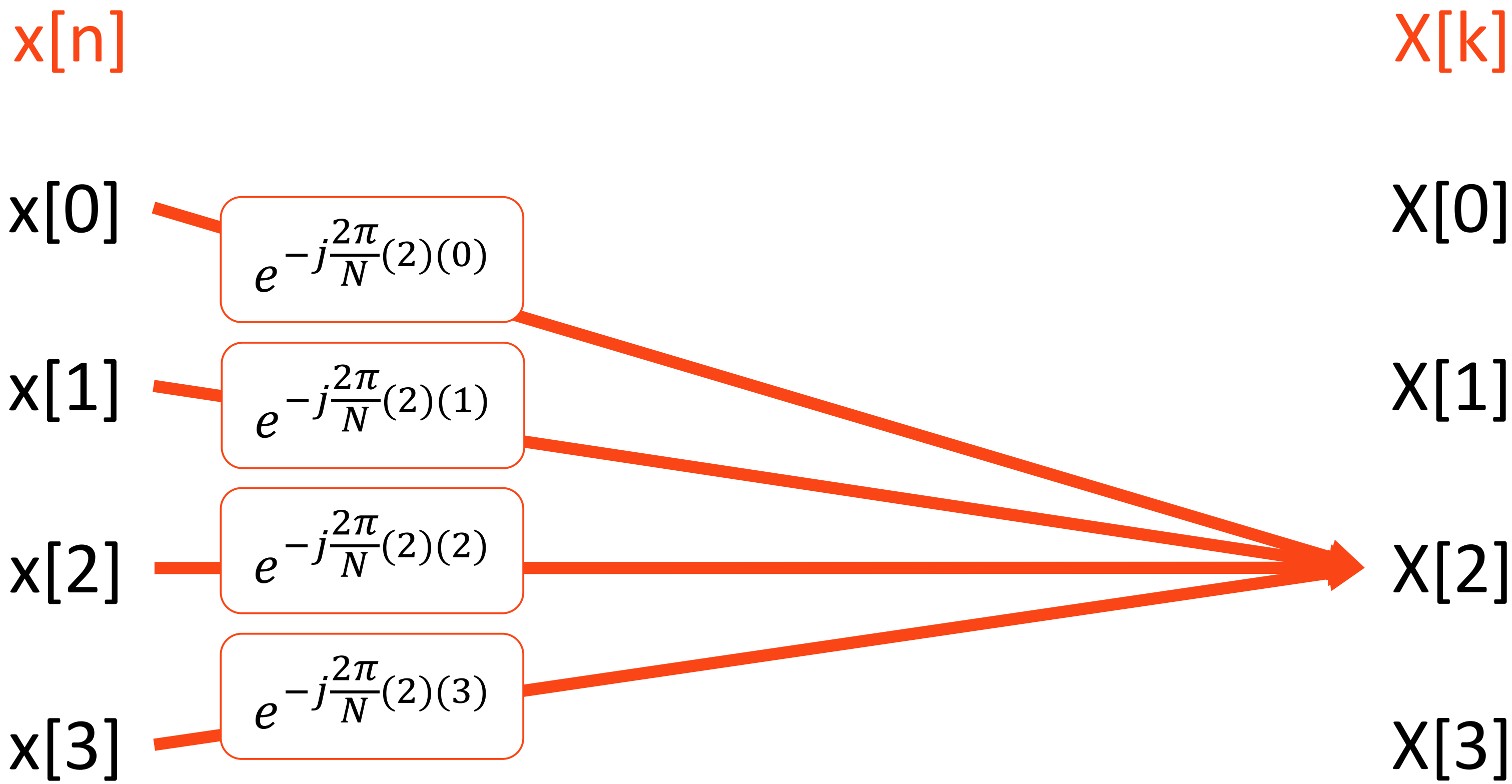
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$



The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

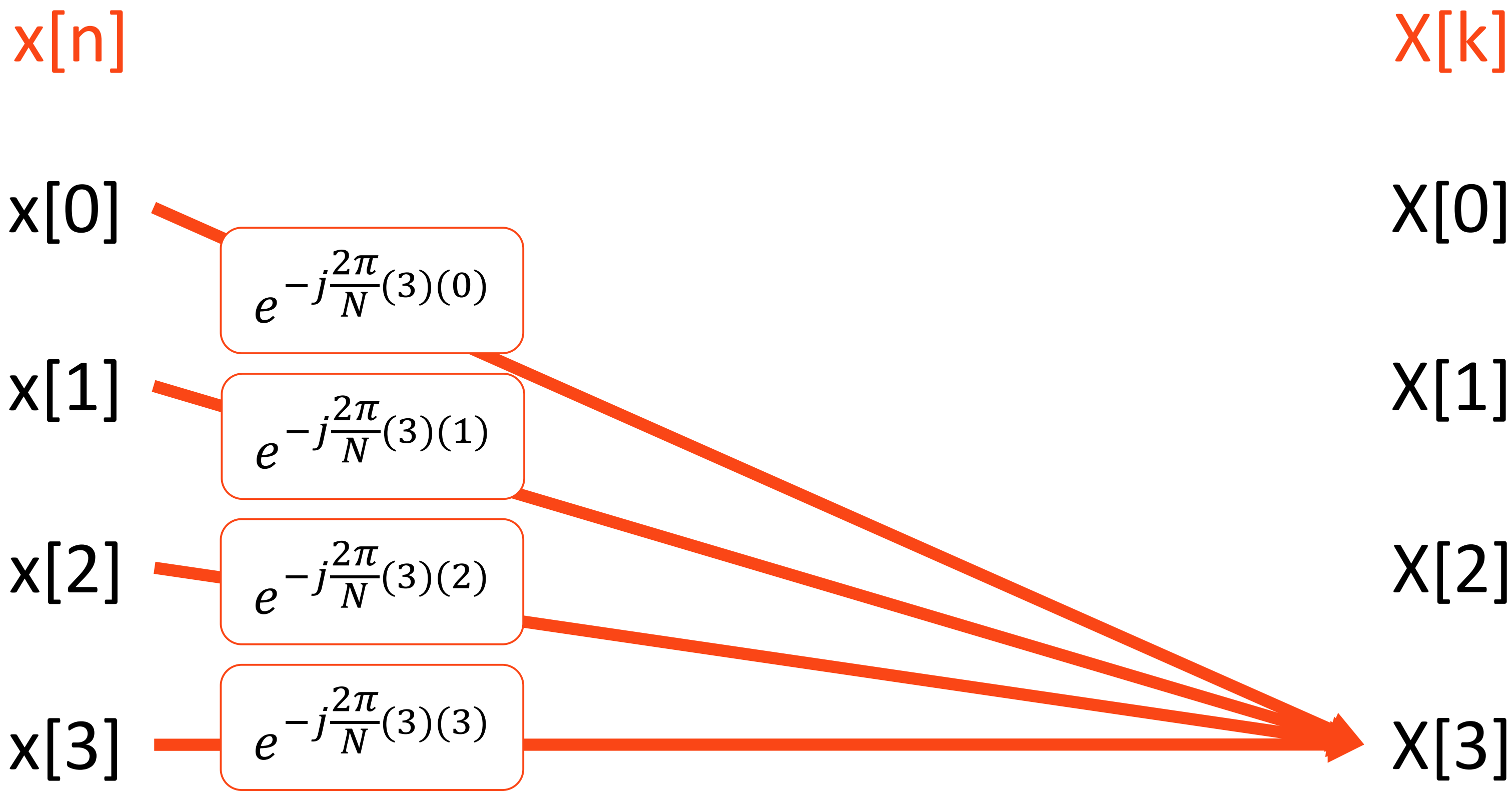
$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$



The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

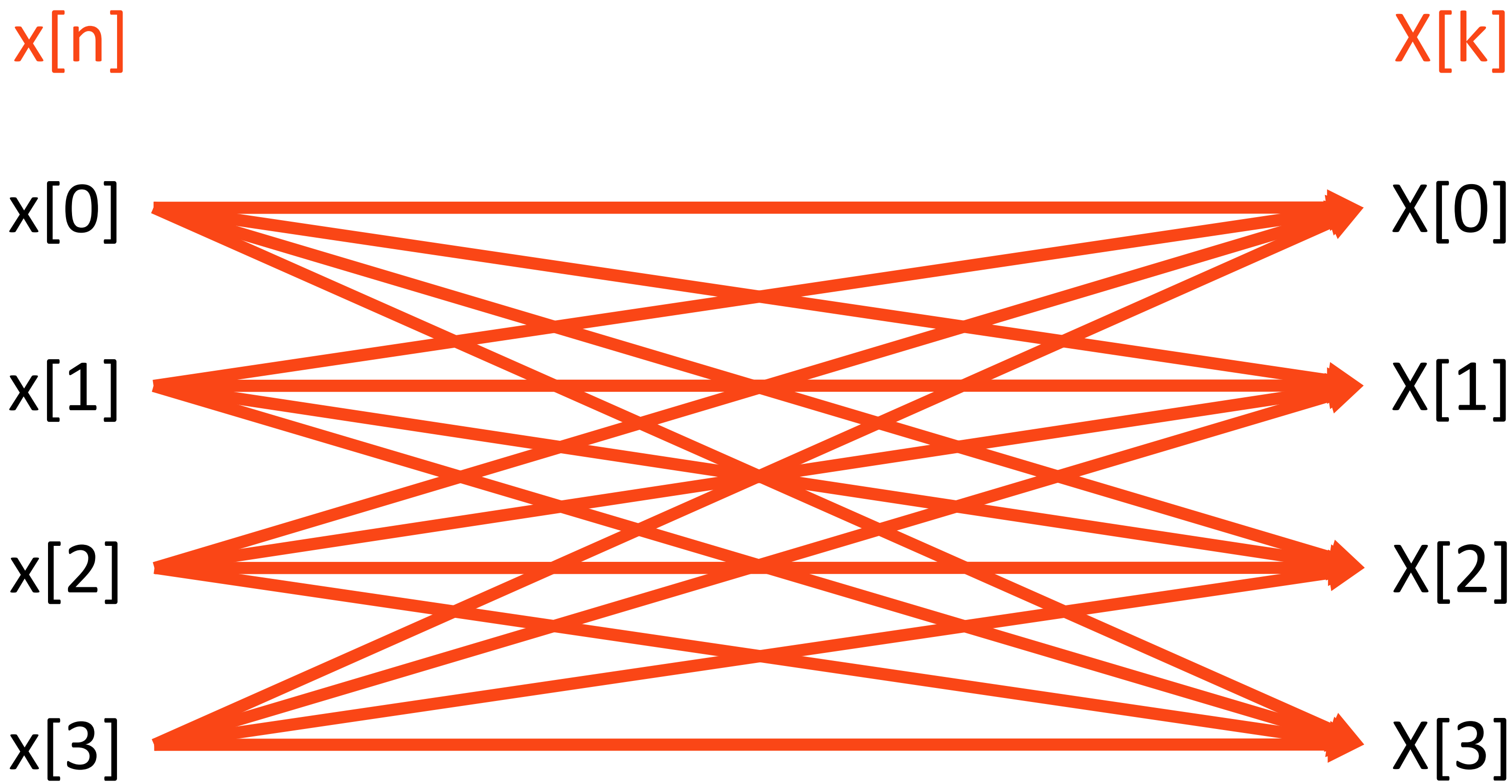
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$



The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$



The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

x[n] X[k]

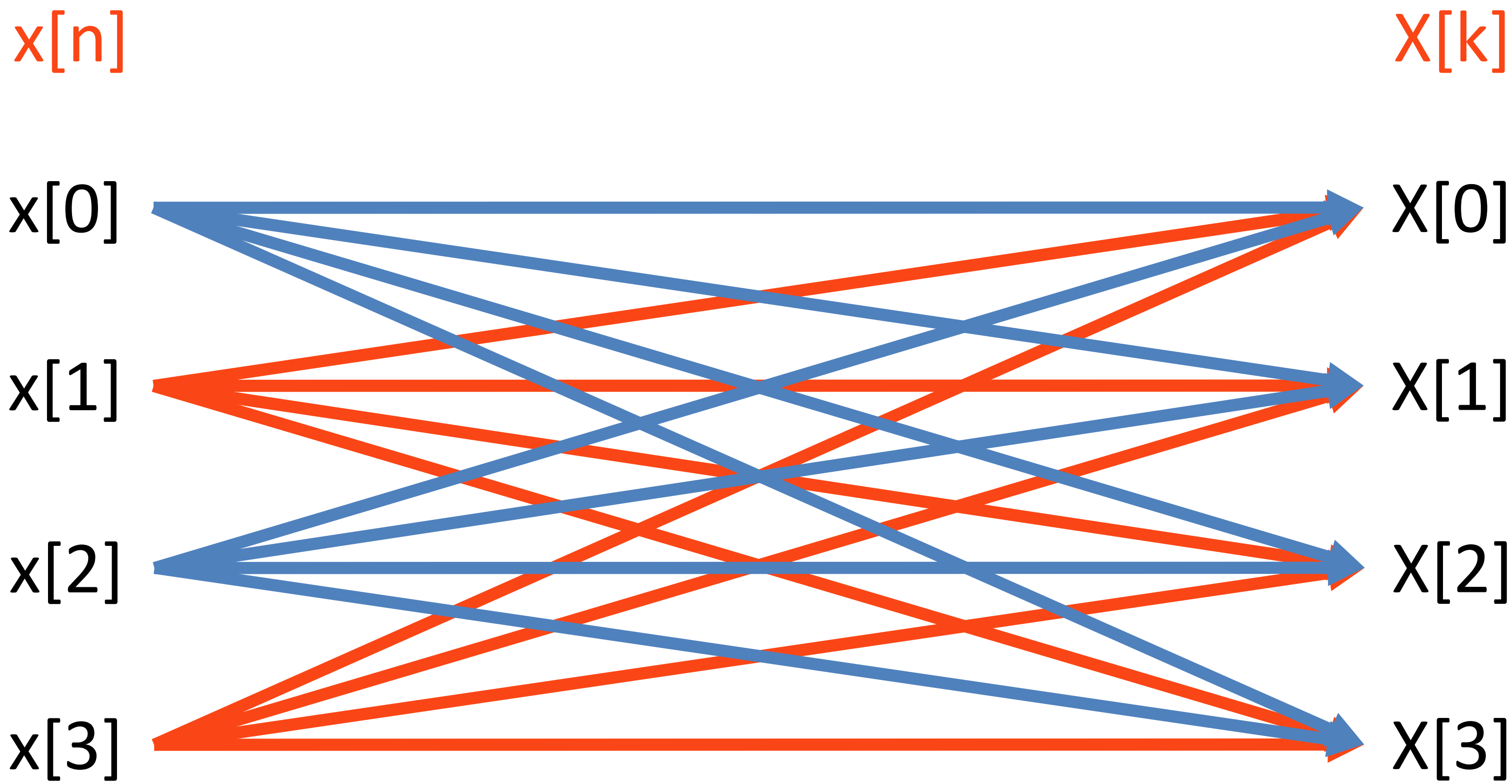
But wait!!

$$e^{-j\frac{2\pi}{N}n(k+N/2)} = e^{-j\frac{2\pi}{N}nk} e^{-j\pi n} = \begin{cases} e^{-j\frac{2\pi}{N}nk} & \text{for} \quad n \text{ is even} \\ -e^{-j\frac{2\pi}{N}nk} & \text{for} \quad n \text{ is odd} \end{cases}$$

The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

$$X[k] = \sum_{n=0}^{N/2-1} x[2n]e^{-j\frac{2\pi}{N}k(2n)} + \sum_{n=0}^{N/2-1} x[2n+1]e^{-j\frac{2\pi}{N}k(2n+1)}$$



The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

$$X[k] = \sum_{n=0}^{N/2-1} x[2n]e^{-j\frac{2\pi}{N}k(2n)} + e^{-j\frac{2\pi}{N}k} \sum_{n=0}^{N/2-1} x[2n+1]e^{-j\frac{2\pi}{N}k(2n)}$$

$$X\left[k+\frac{N}{2}\right] = \sum_{n=0}^{N/2-1} x[2n]e^{-j\frac{2\pi}{N}k(2n)} - e^{-j\frac{2\pi}{N}k} \sum_{n=0}^{N/2-1} x[2n+1]e^{-j\frac{2\pi}{N}k(2n)}$$

$$e^{-j\frac{2\pi}{N}\left(k+\frac{N}{2}\right)(2n)} = e^{-j\frac{2\pi}{N}k(2n)}e^{-j2\pi}$$

$$= e^{-j\frac{2\pi}{N}k(2n)}$$

The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

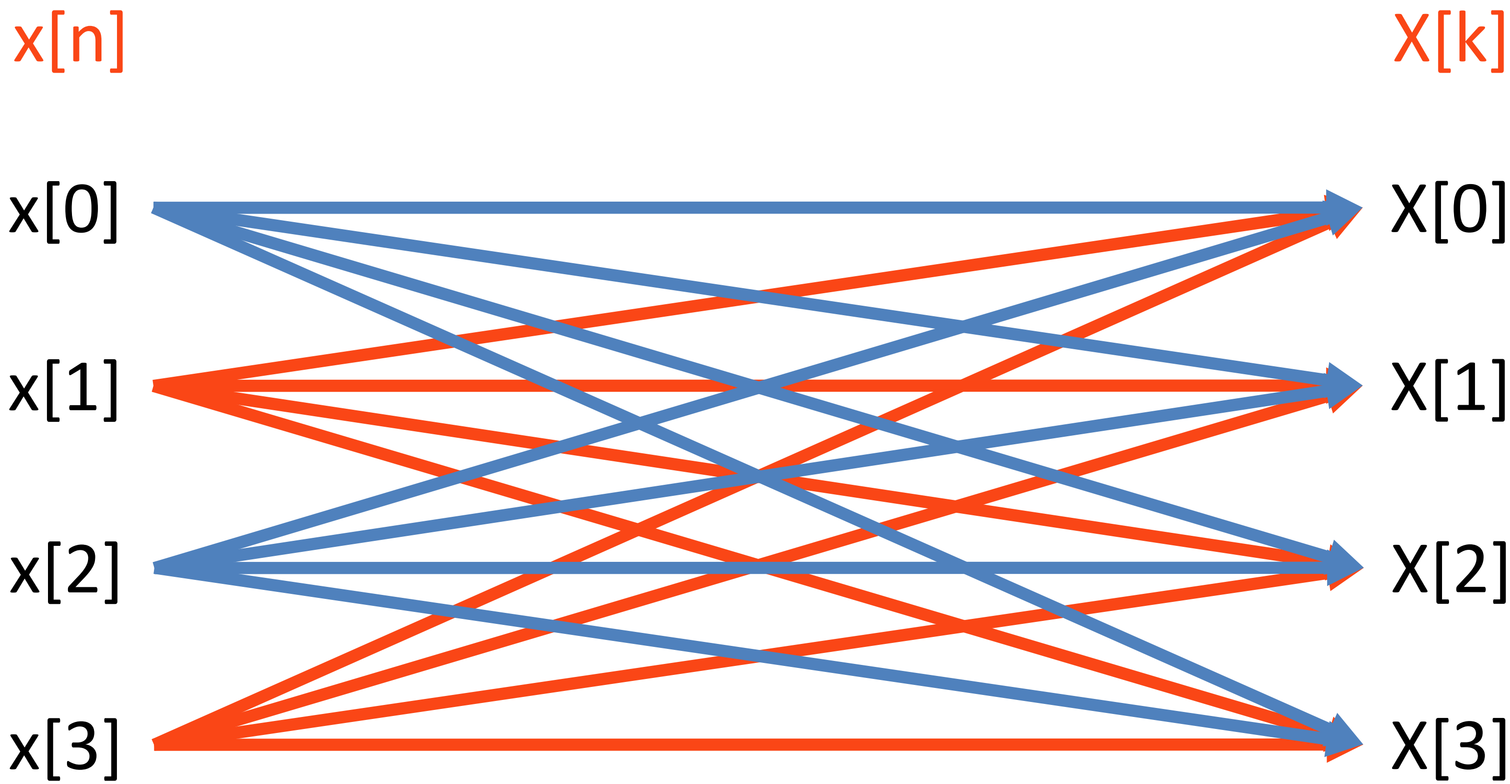
$$X[k] = E[k] + e^{-j\frac{2\pi}{N}k} O[k]$$

$$X\left[k + \frac{N}{2}\right] = E[k] - e^{-j\frac{2\pi}{N}k} O[k]$$

The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

$$X[k] = E[k] + e^{-j\frac{2\pi}{N}k} O[k] \qquad X\left[k + \frac{N}{2}\right] = E[k] - e^{-j\frac{2\pi}{N}k} O[k]$$

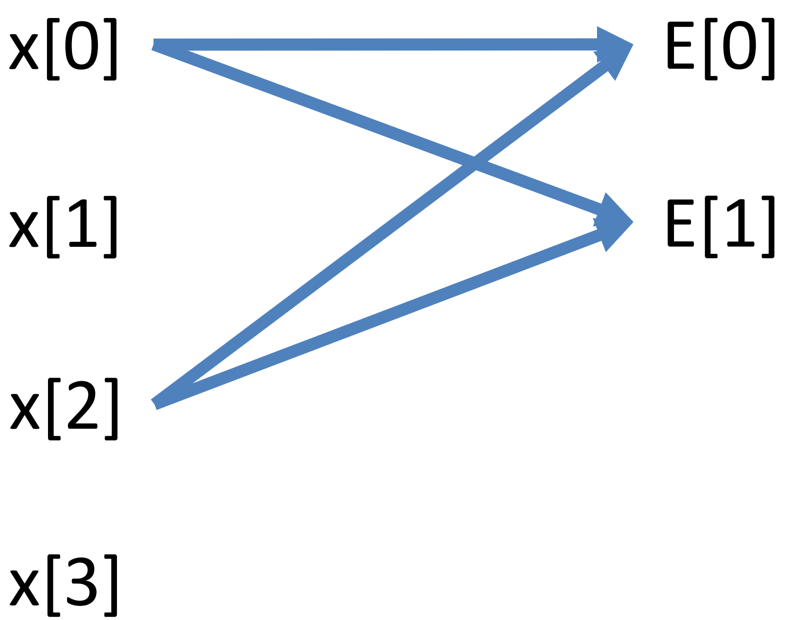


- **The Discrete Fourier Transform (DFT)**

$$X[k] = E[k] + e^{-j\frac{2\pi}{N}k} O[k] \qquad X\left[k + \frac{N}{2}\right] = E[k] - e^{-j\frac{2\pi}{N}k} O[k]$$

x[n]

X[k]



The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

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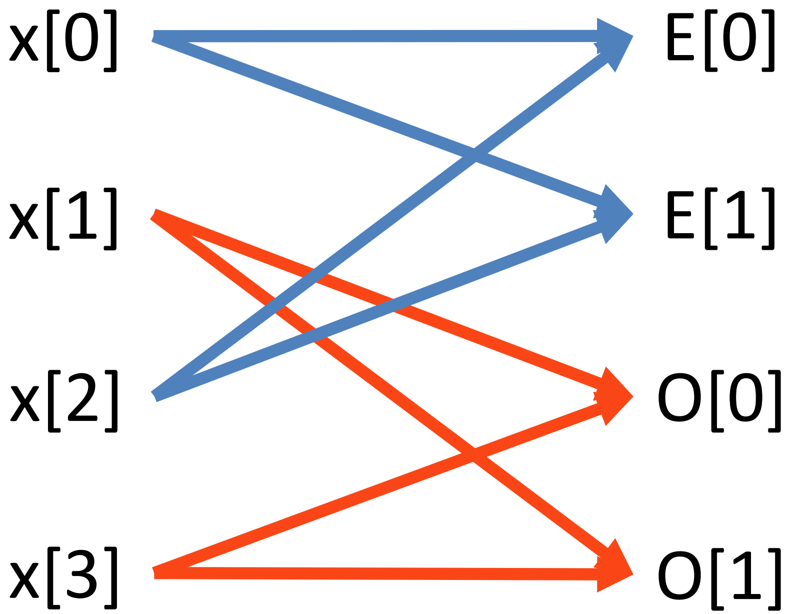
x[n]

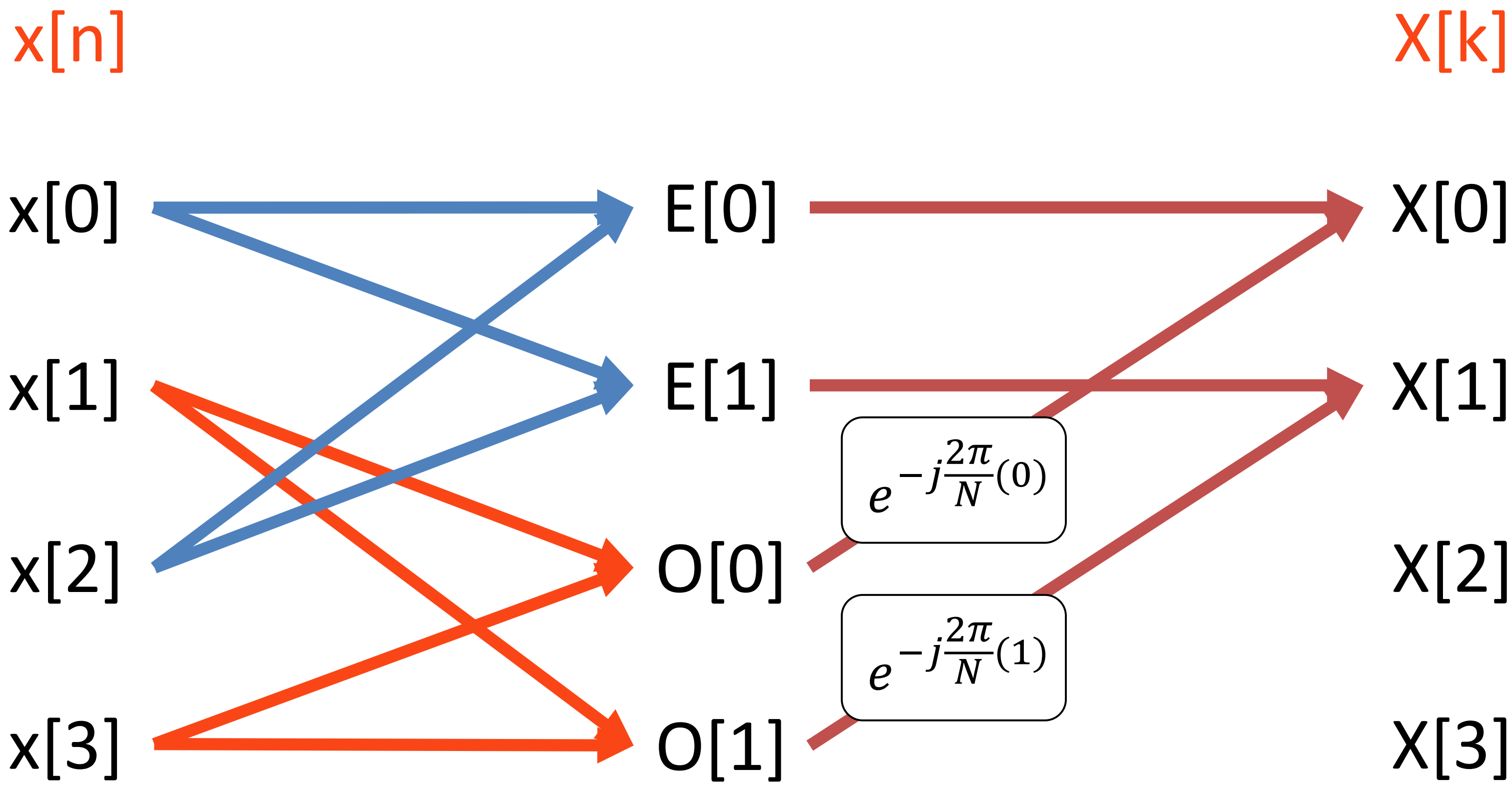
X[k]



The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

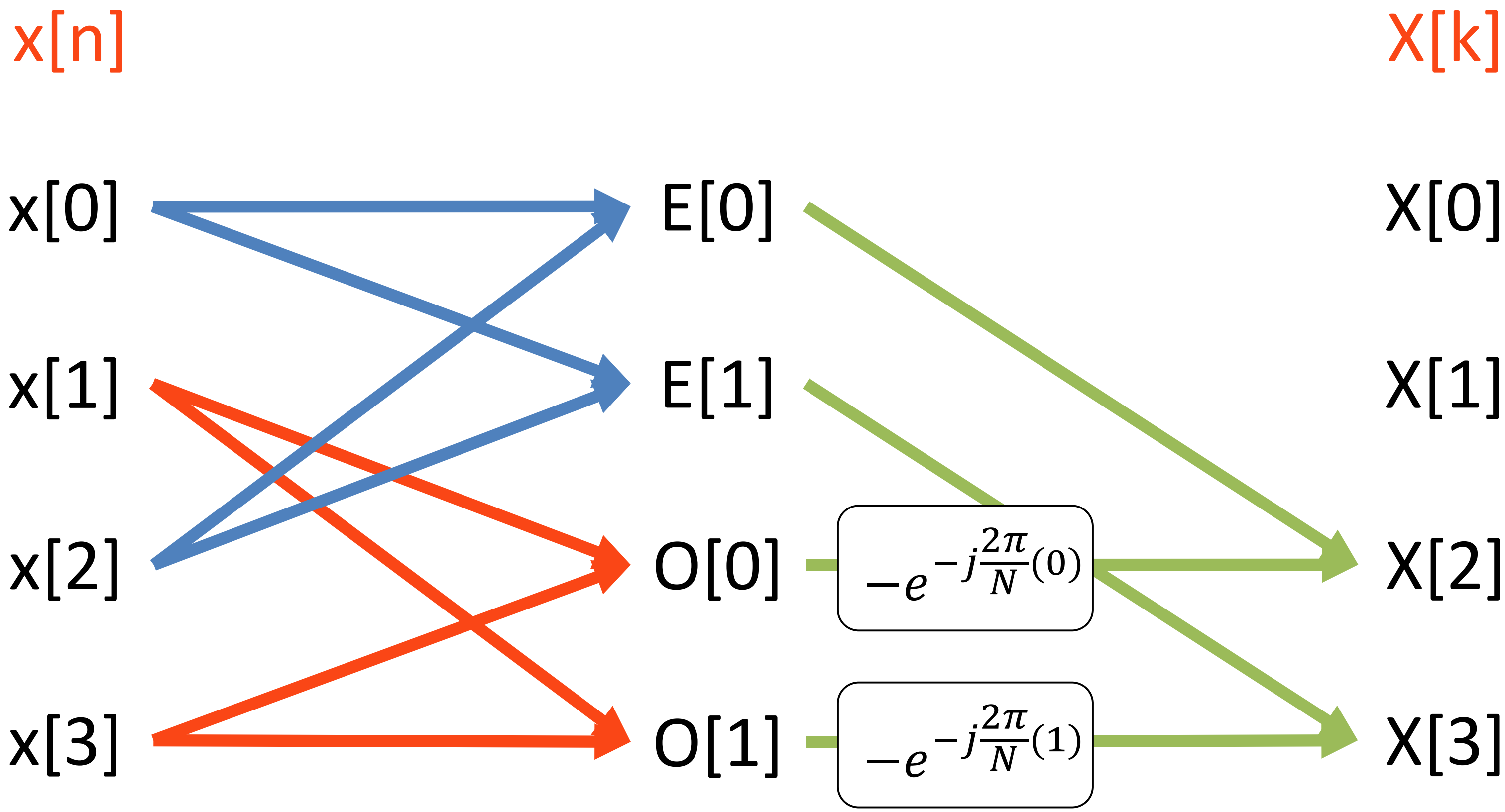
$$X[k] = E[k] + e^{-j\frac{2\pi}{N}k} O[k] \qquad X\left[k + \frac{N}{2}\right] = E[k] - e^{-j\frac{2\pi}{N}k} O[k]$$



The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

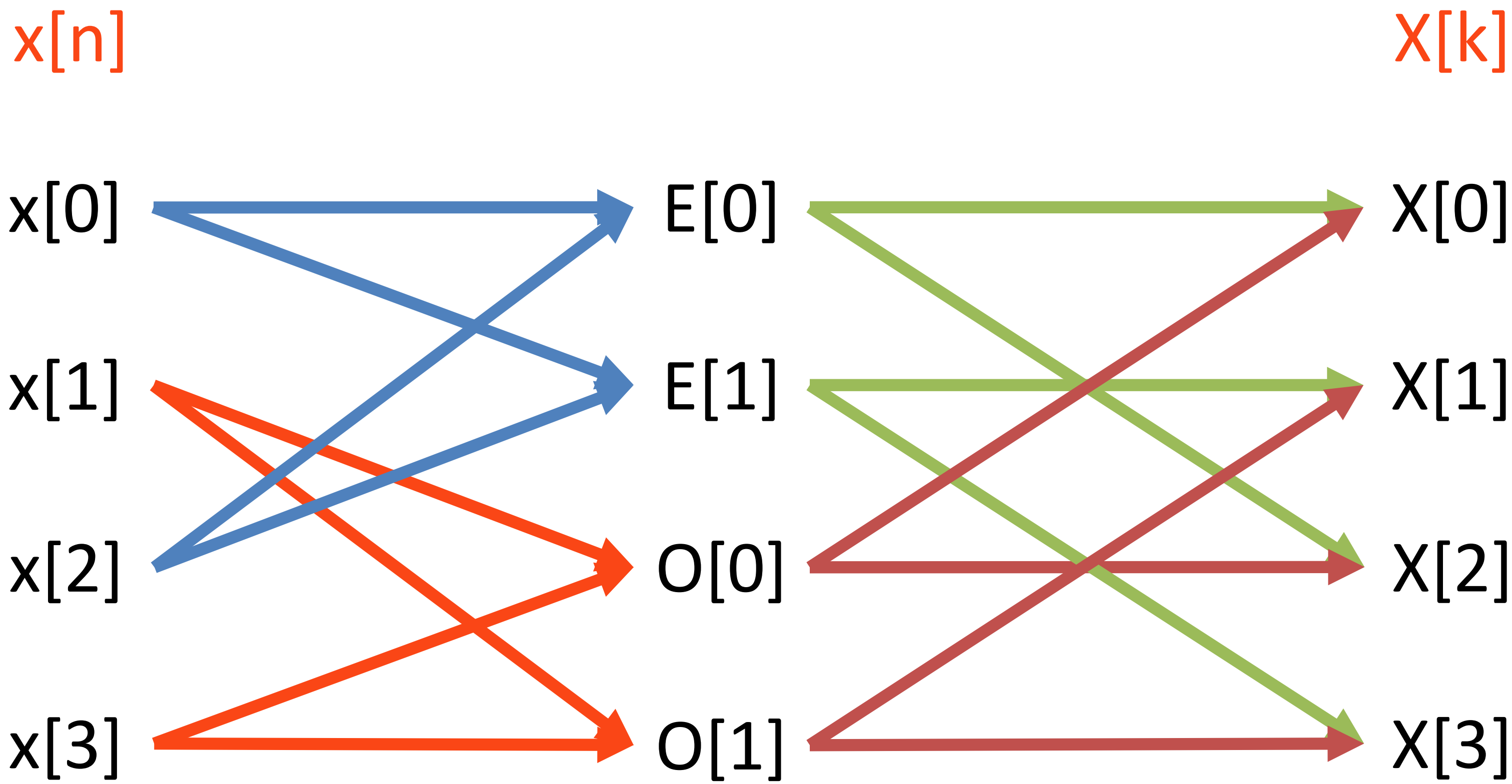
$$X[k] = E[k] + e^{-j\frac{2\pi}{N}k}O[k] \qquad X\left[k+\frac{N}{2}\right] = E[k] - e^{-j\frac{2\pi}{N}k}O[k]$$



The Fast Fourier Transform

- **The Discrete Fourier Transform (DFT)**

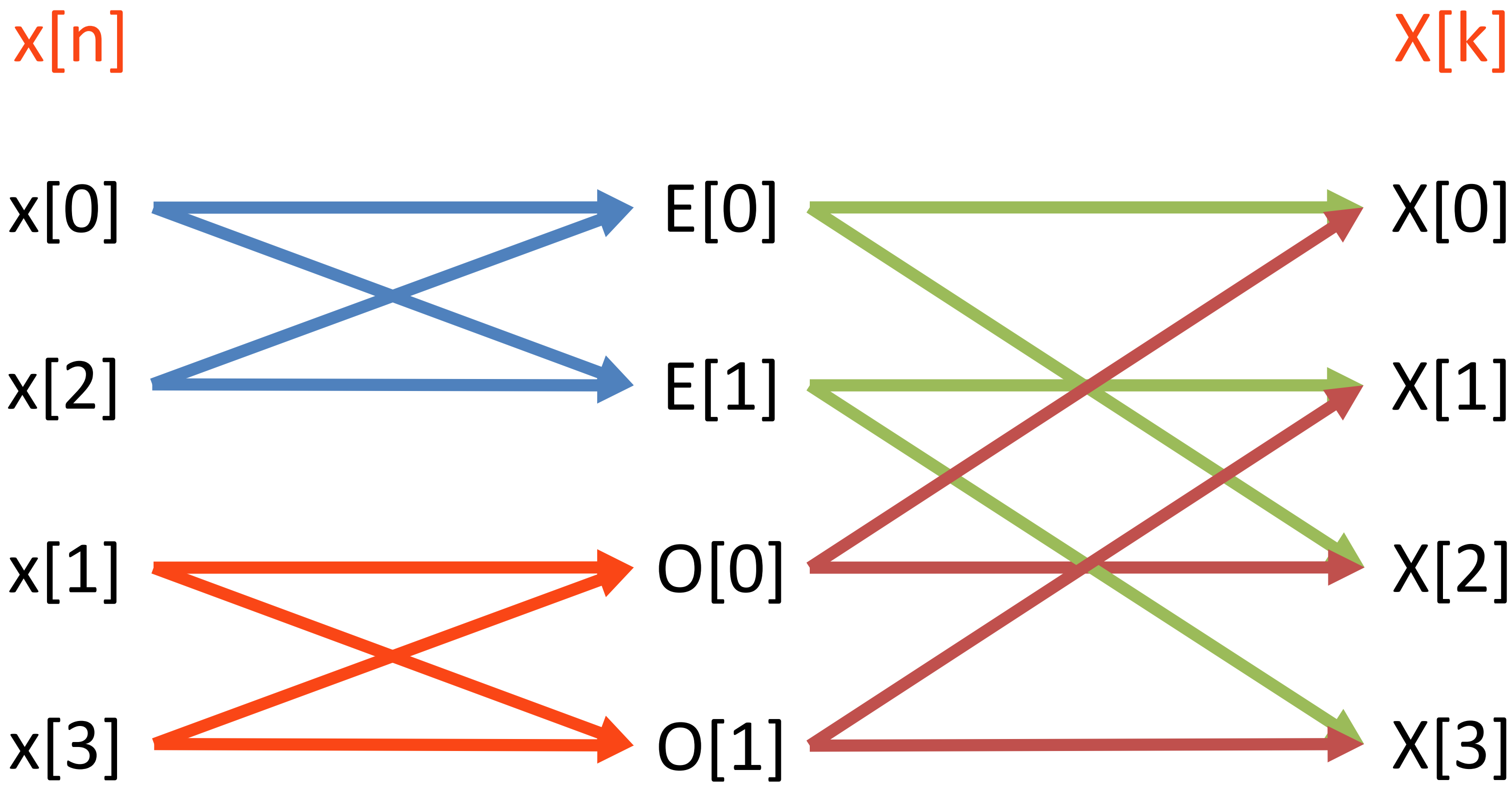
$$X[k] = E[k] + e^{-j\frac{2\pi}{N}k} O[k] \qquad X\left[k + \frac{N}{2}\right] = E[k] - e^{-j\frac{2\pi}{N}k} O[k]$$



The Fast Fourier Transform

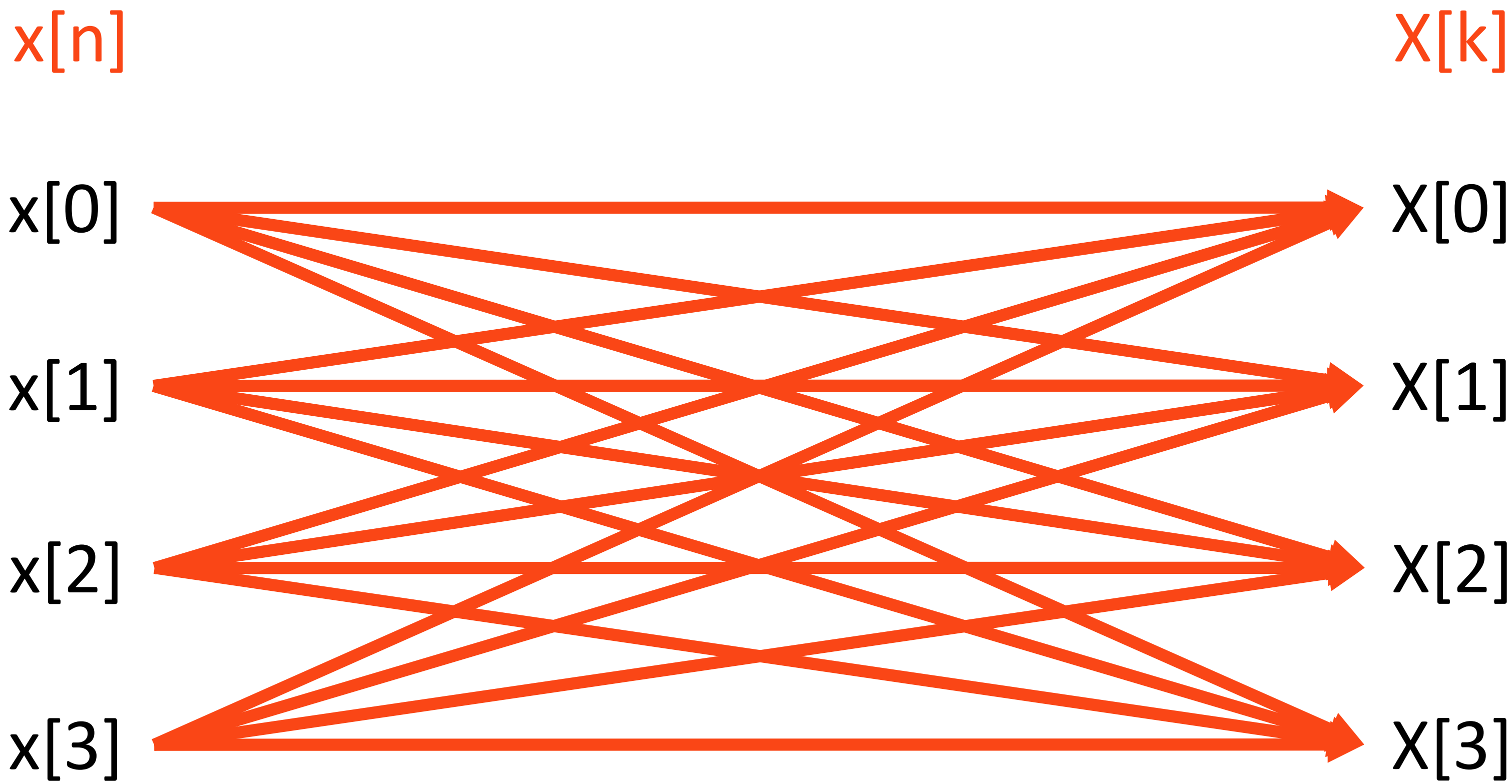
- **The Discrete Fourier Transform (DFT)**

$$X[k] = E[k] + e^{-j\frac{2\pi}{N}k}O[k] \qquad X\left[k + \frac{N}{2}\right] = E[k] - e^{-j\frac{2\pi}{N}k}O[k]$$



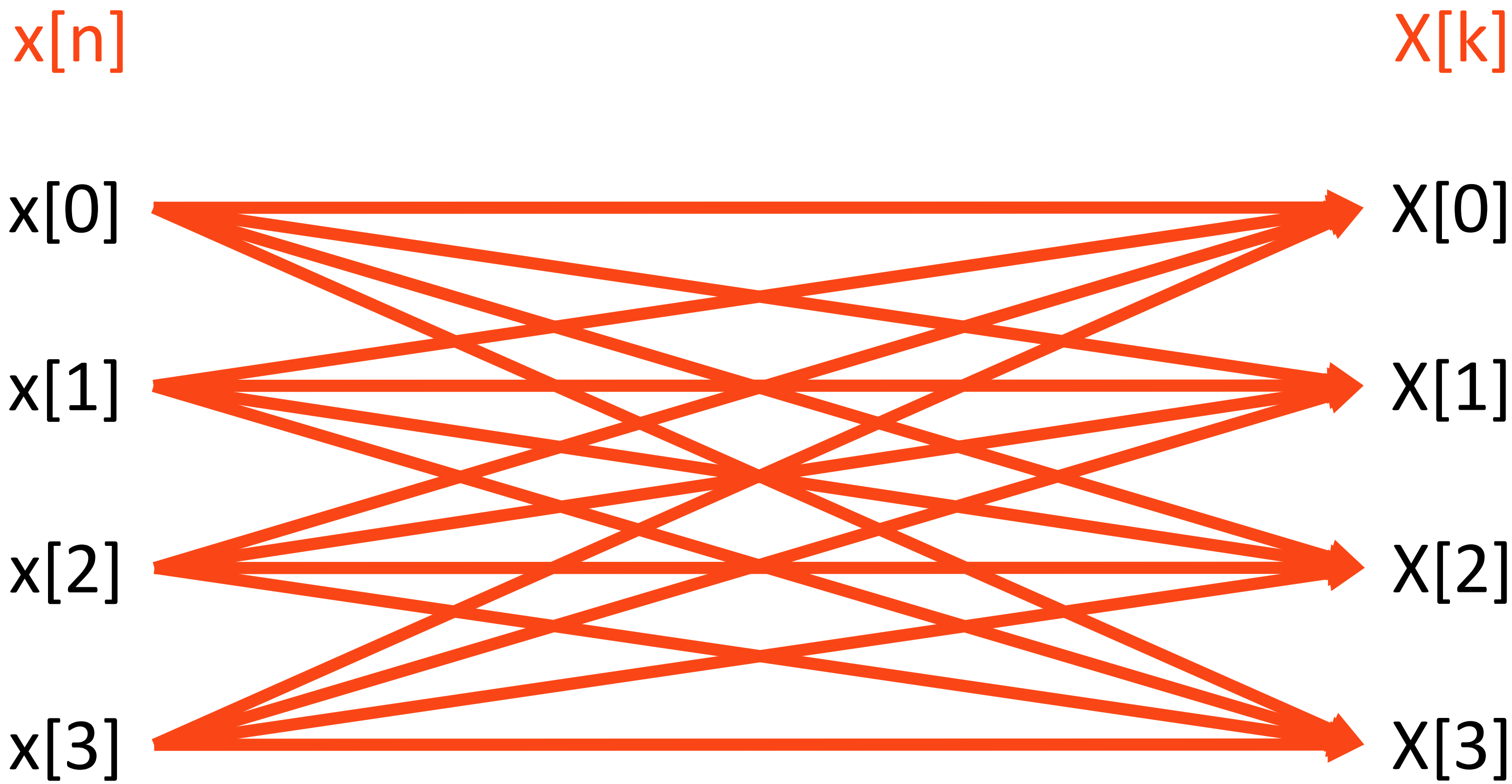
The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - For DFT



The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - $N^2 = 4^2 = 16$



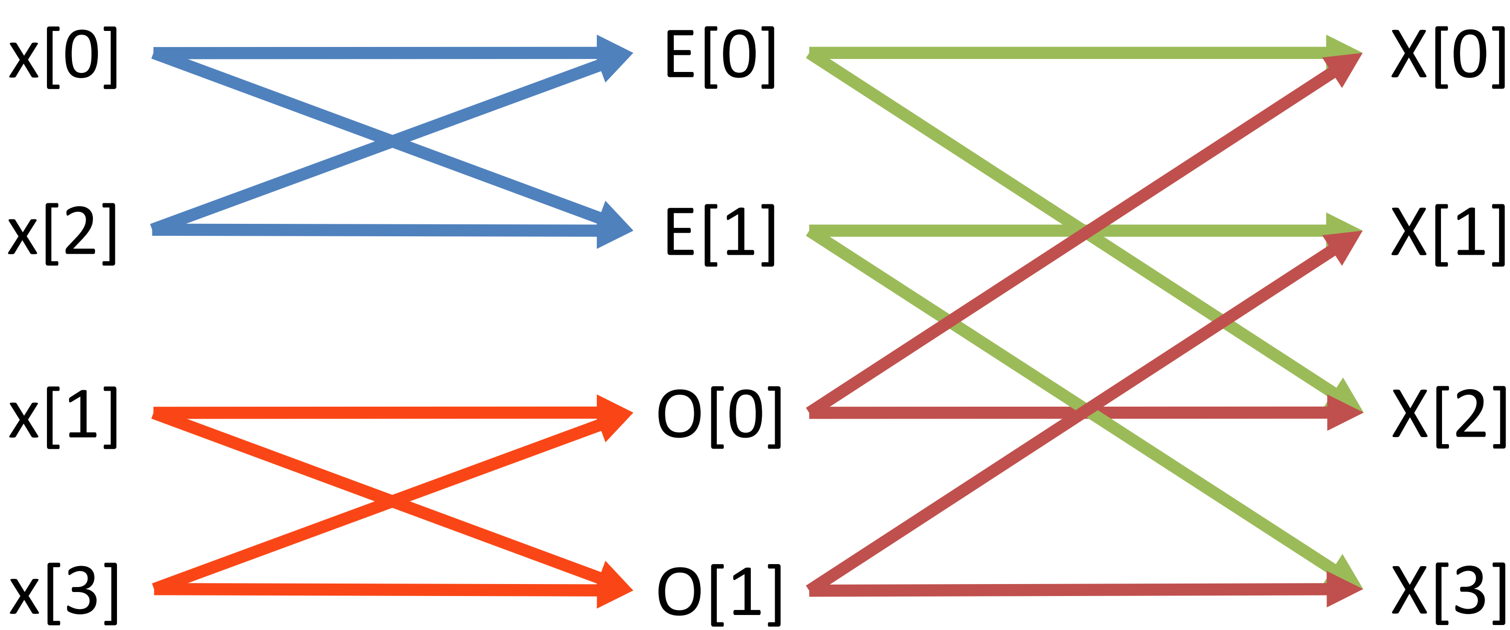
- **Number of Multiplications (including multiplication by 1)?**
  - $2(4) + 2(4) = 2(2)(4) = 16$

Number of samples

Number of layers

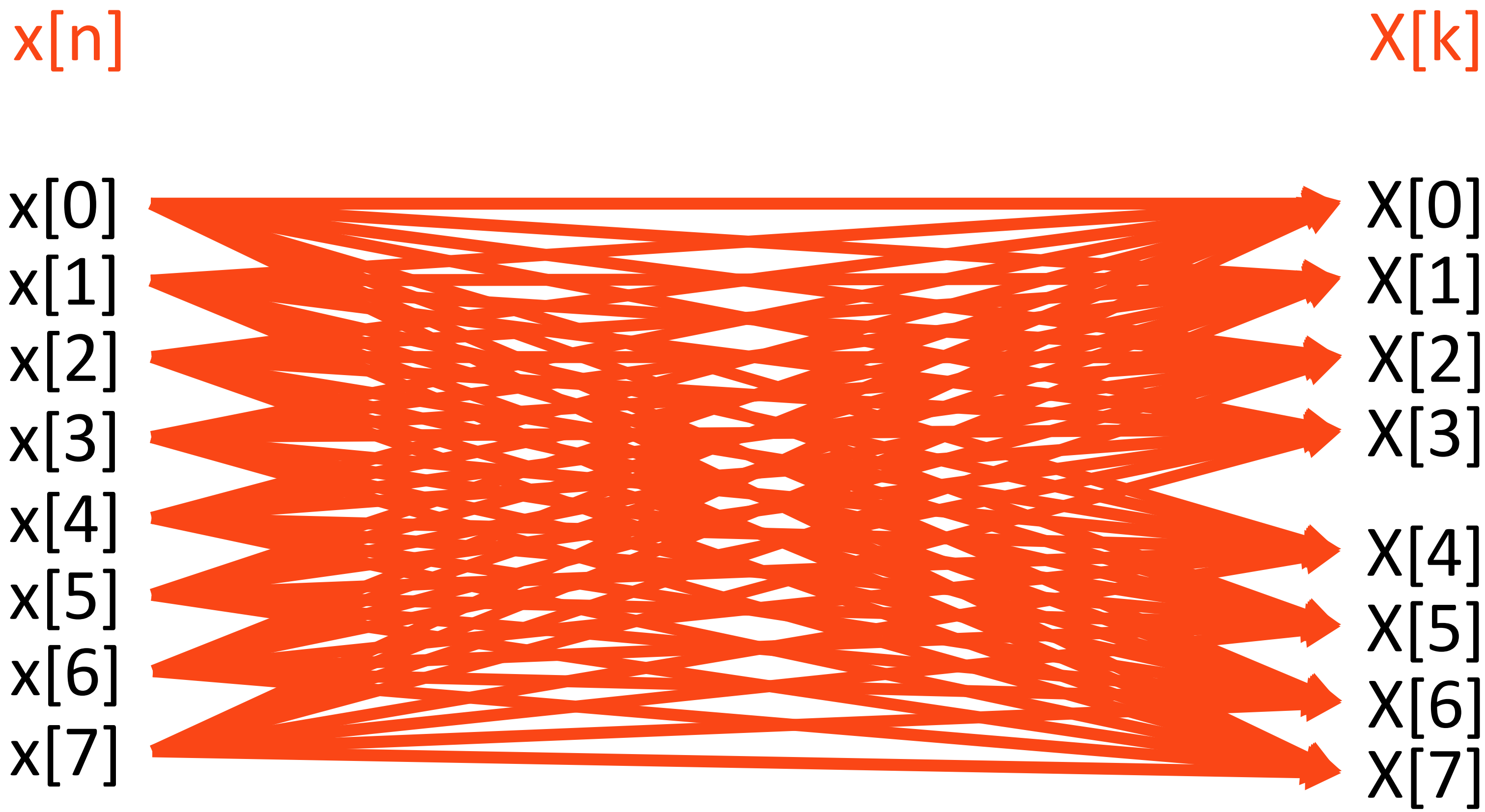
x[n]

X[k]



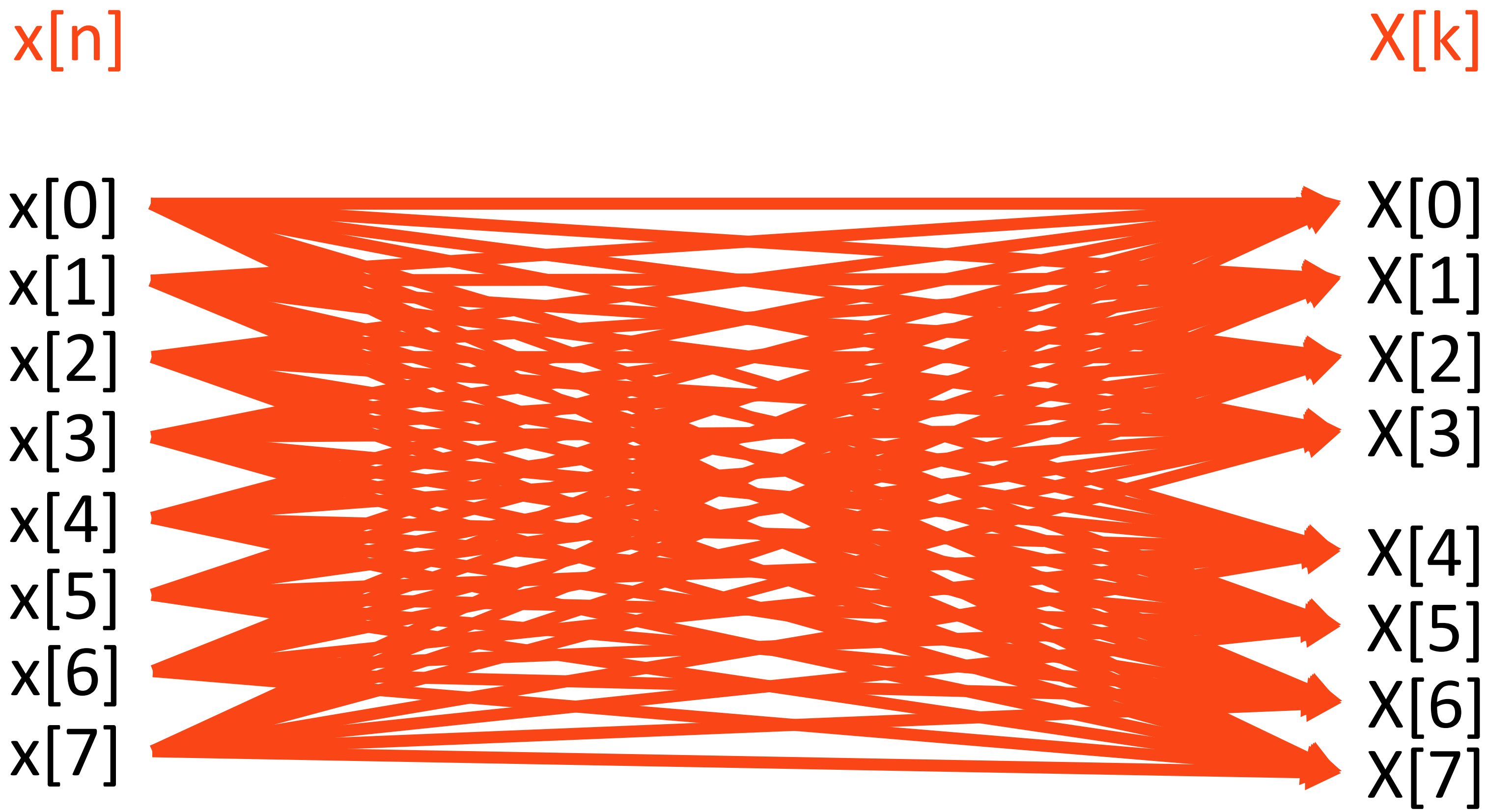
The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - For DFT



The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - $N^2 = 8^2 = 64$



The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - For FFT

$x[n]$ $X[k]$

$x[0]$	$X[0]$
$x[1]$	$X[1]$
$x[2]$	$X[2]$
$x[3]$	$X[3]$
$x[4]$	$X[4]$
$x[5]$	$X[5]$
$x[6]$	$X[6]$
$x[7]$	$X[7]$

The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - For FFT

x[n]

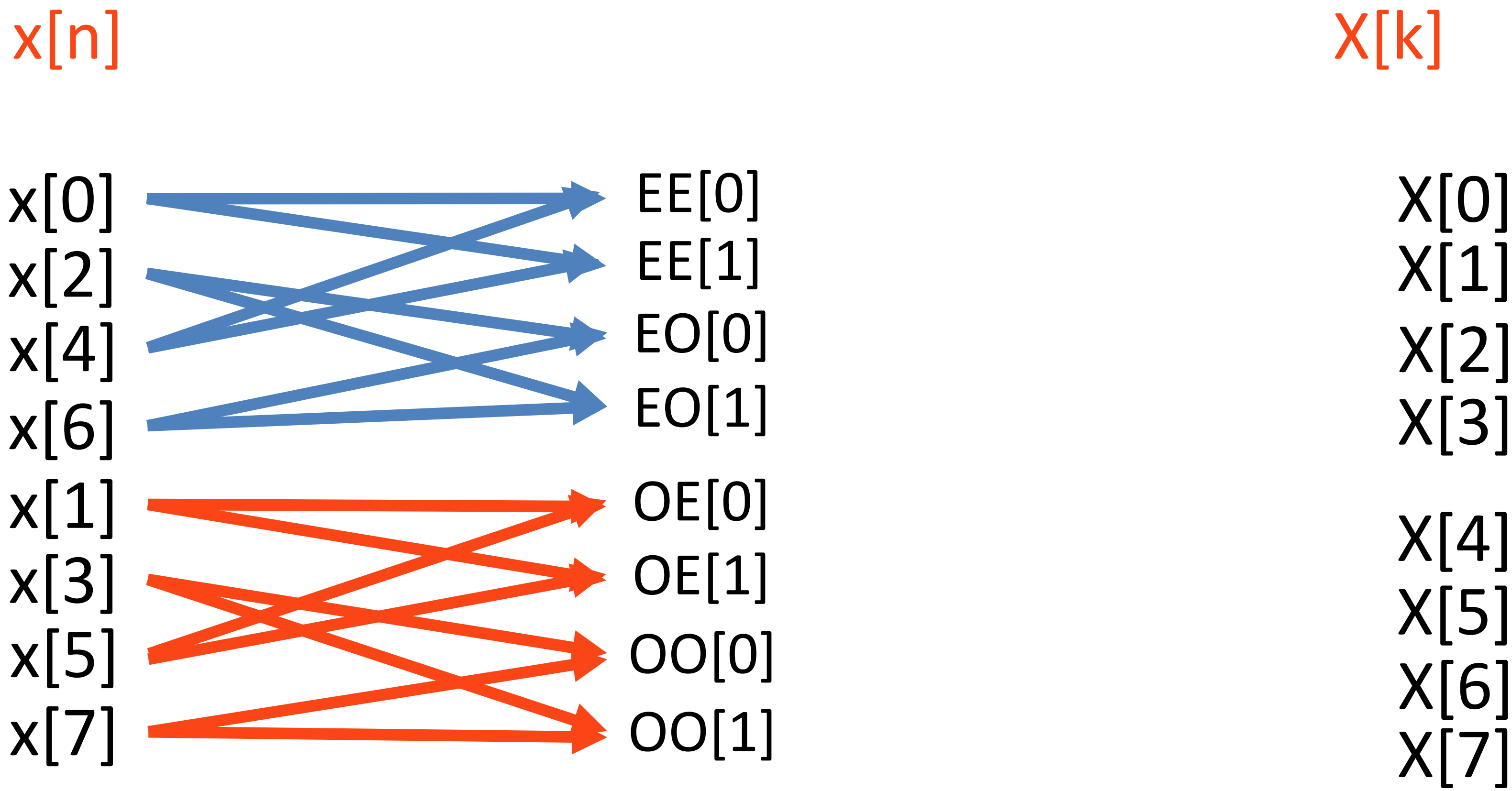
x[0]
x[2]
x[4]
x[6]
x[1]
x[3]
x[5]
x[7]

X[k]

X[0]
X[1]
X[2]
X[3]
X[4]
X[5]
X[6]
X[7]

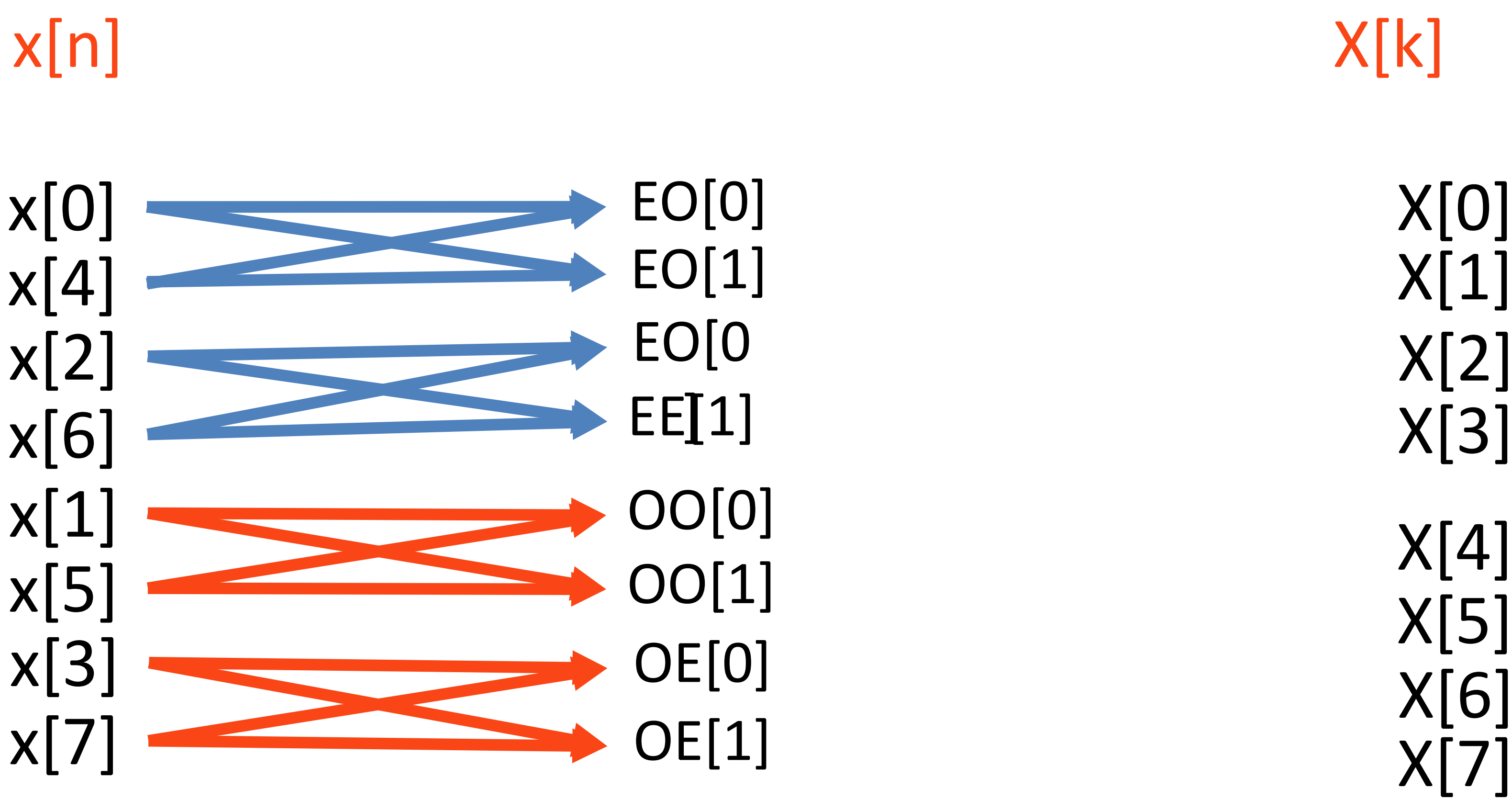
The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - For FFT

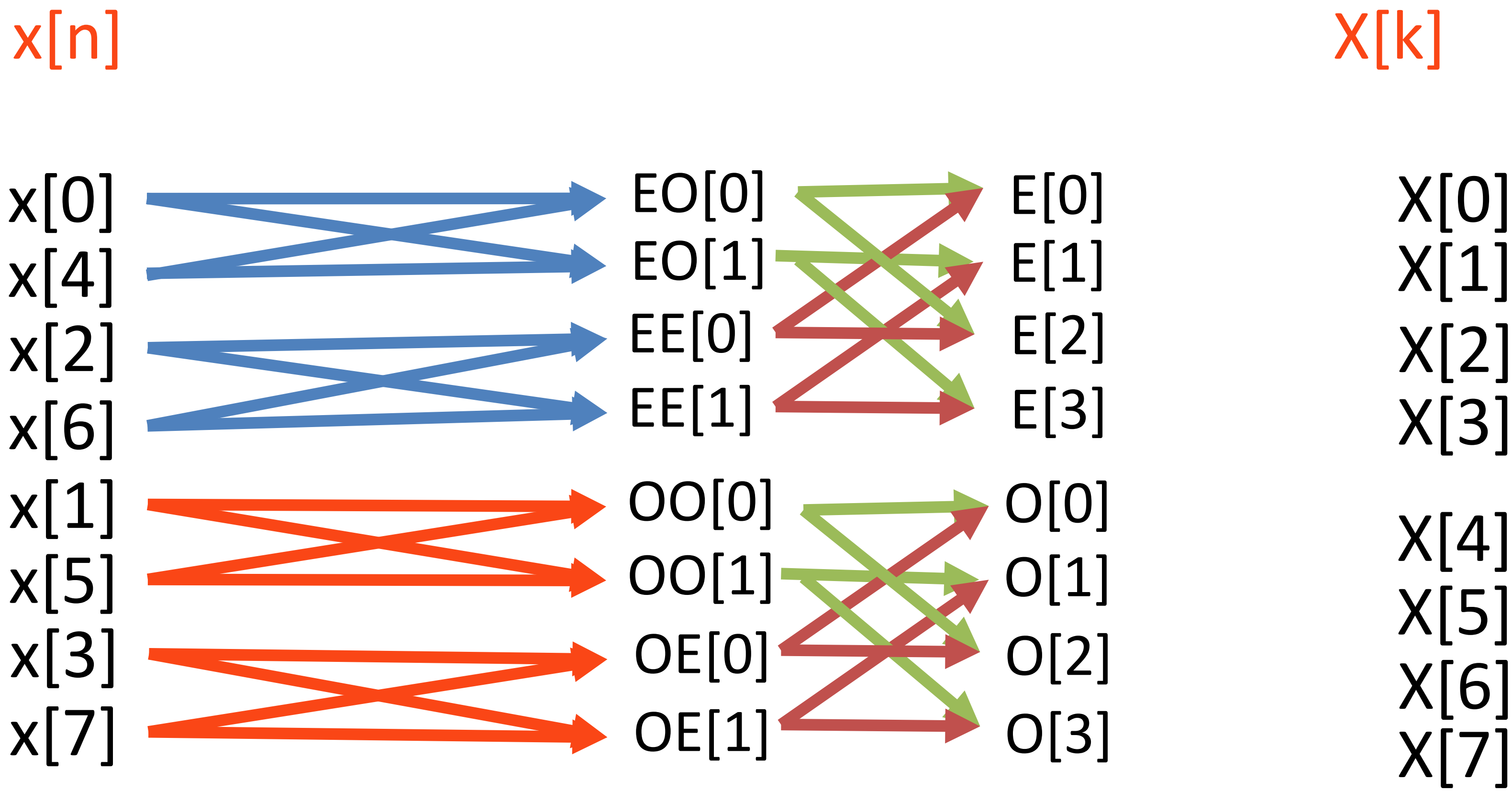


The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - For FFT

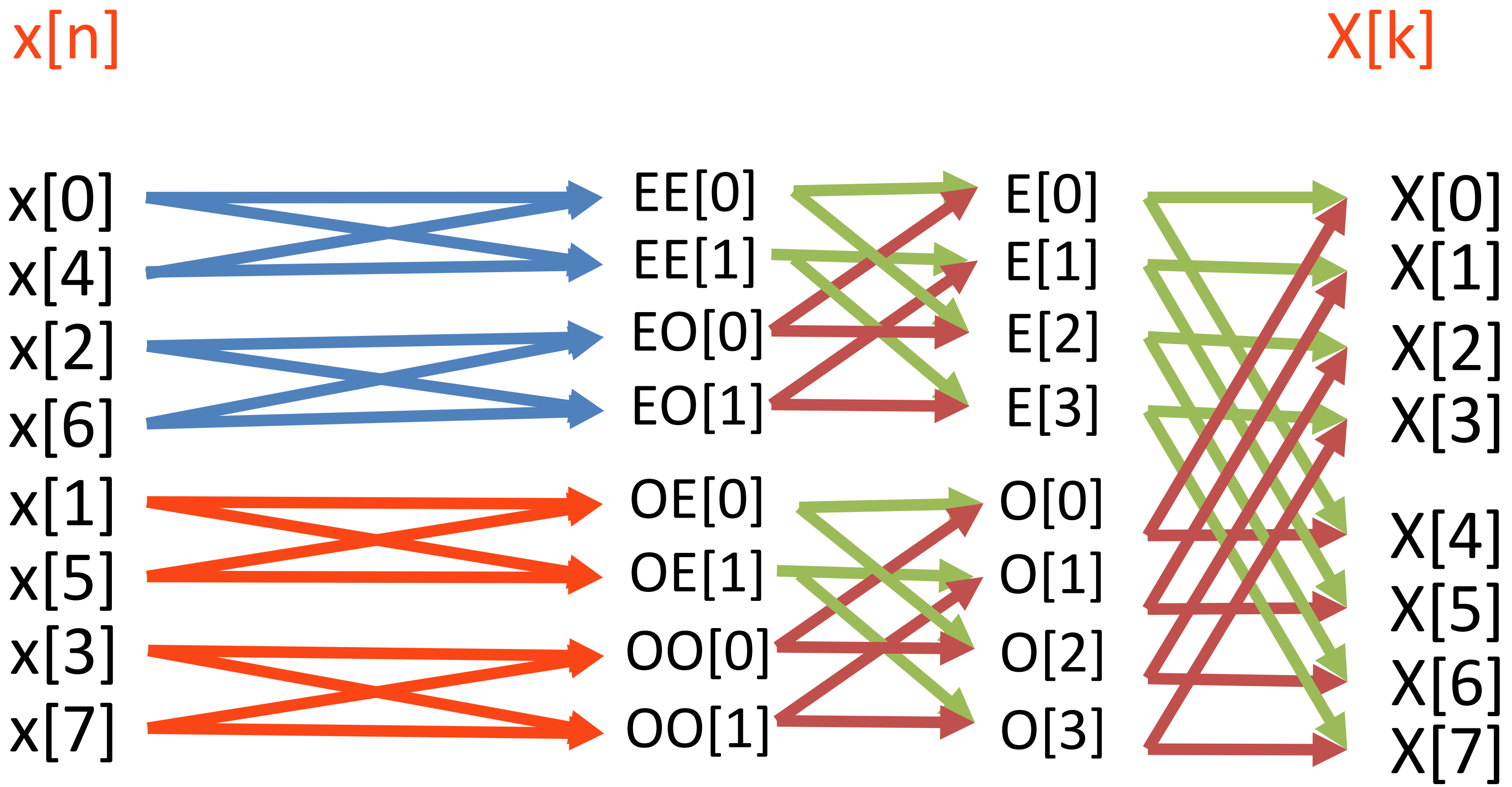


- **Number of Multiplications (including multiplication by 1)?**
  - For FFT



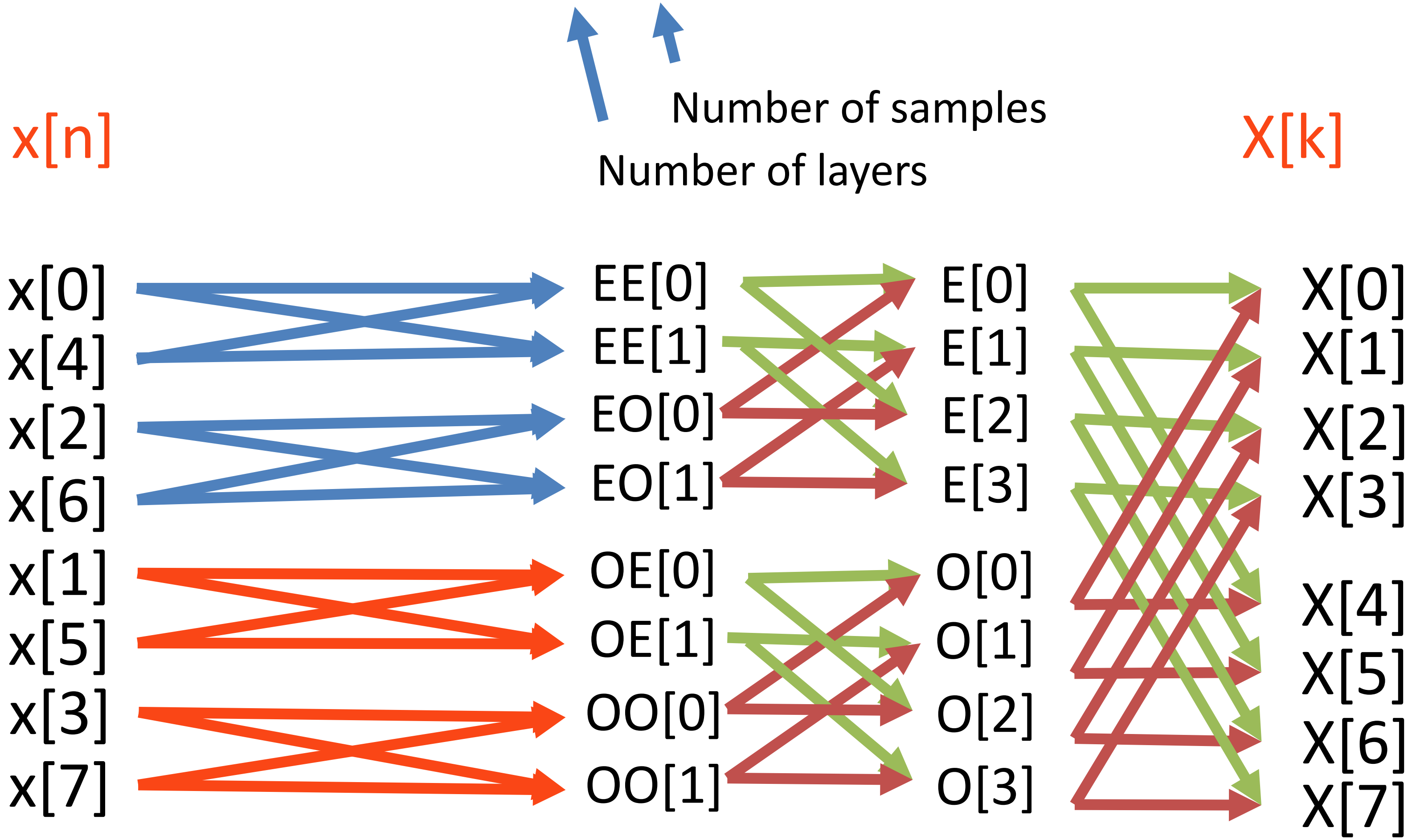
The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - For FFT



The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - $2(8) + 2(8) + 2(8) = 2(3)(8) = 48$

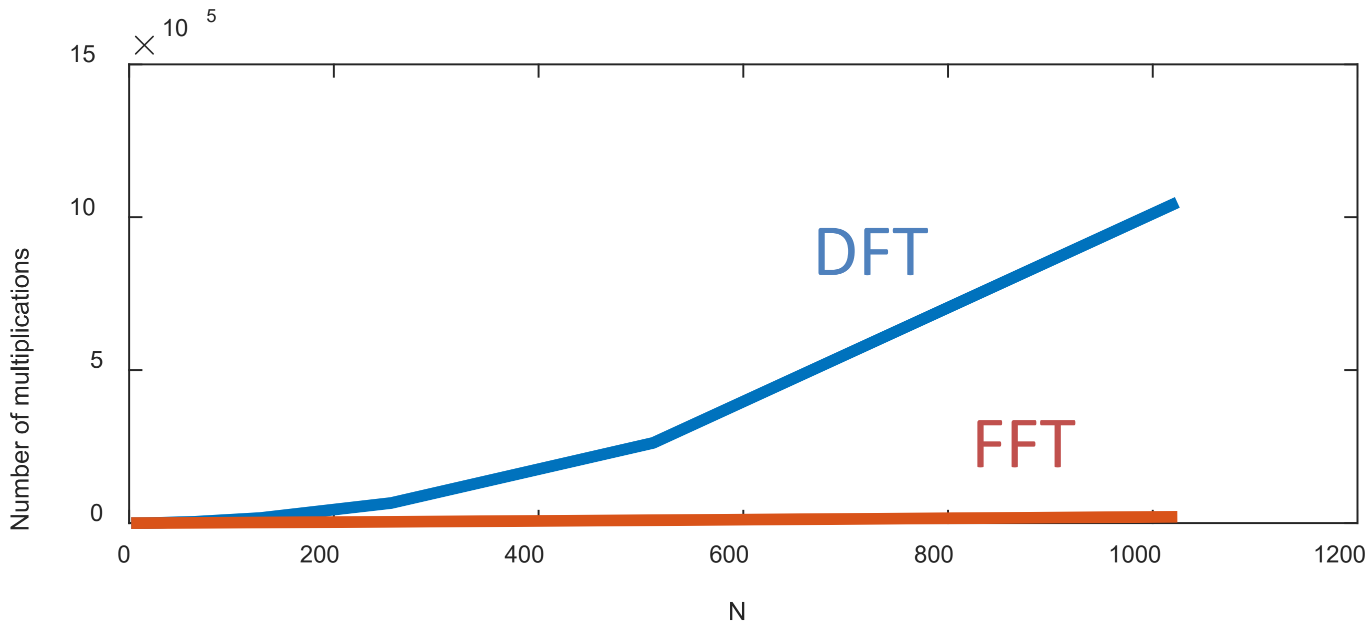


The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - DFT
    - $N = 4$, Multiplications: 16
    - $N = 8$, Multiplications: 64
    - $N = 16$, Multiplications: 256
    - $N = 32$, Multiplications: 1024
  - FFT
    - $N = 4$, Multiplications: 16
    - $N = 8$, Multiplications: 48
    - $N = 16$, Multiplications: 128
    - $N = 32$, Multiplications: 320

The Fast Fourier Transform

- **Number of Multiplications (including multiplication by 1)?**
  - DFT
    - Grows at rate: N^2
  - FFT
    - Grows at rate: $N\log_2(N)$



Lecture 14: Discrete -Time Filters

Foundations of Digital Signal Processing

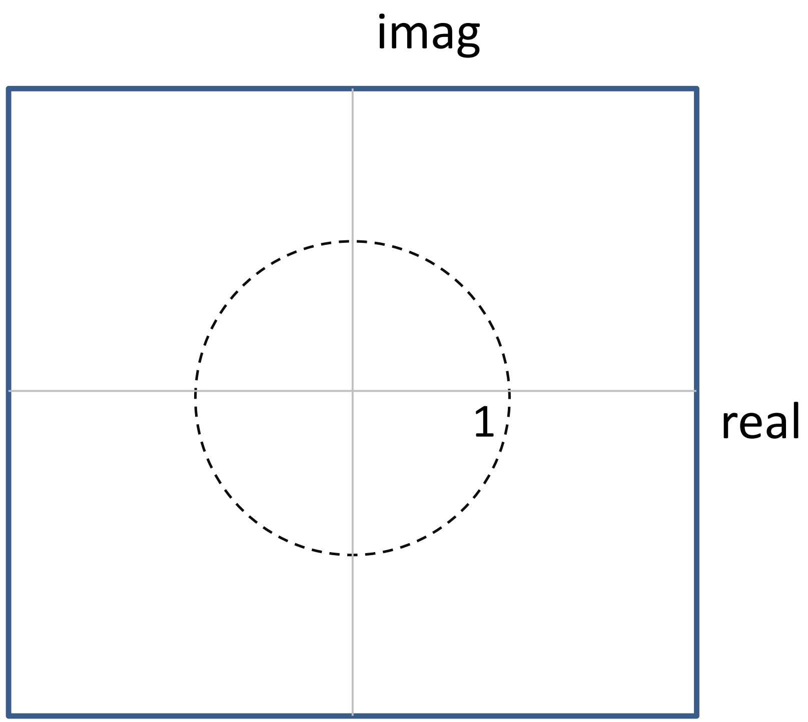
Outline

- Circular Convolution Review
- The DTFT and the DFT: The Relationship
- The Fast Fourier Transform
- **Designing the magnitude response**
- Designing the phase response

Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

$$H(z) = \frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{k=1}^{N}(1 - p_k z^{-1})}$$

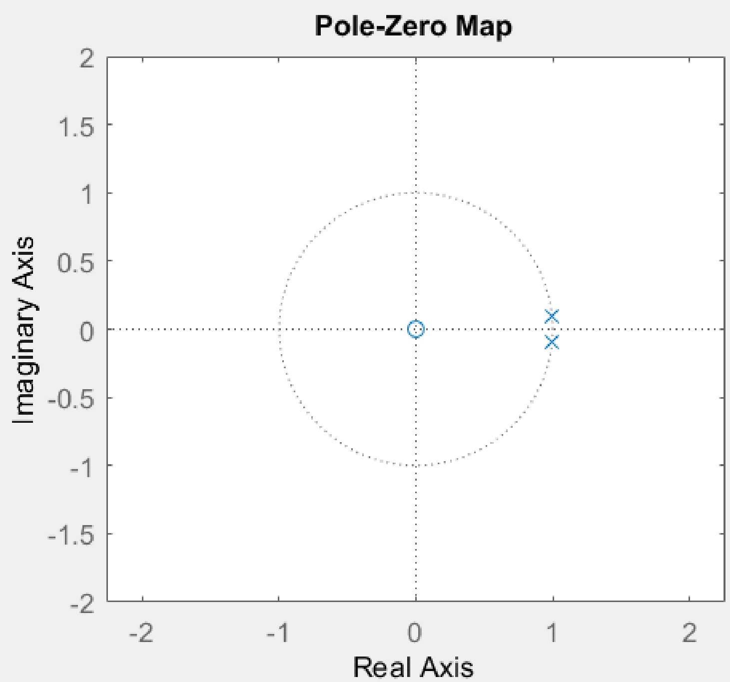


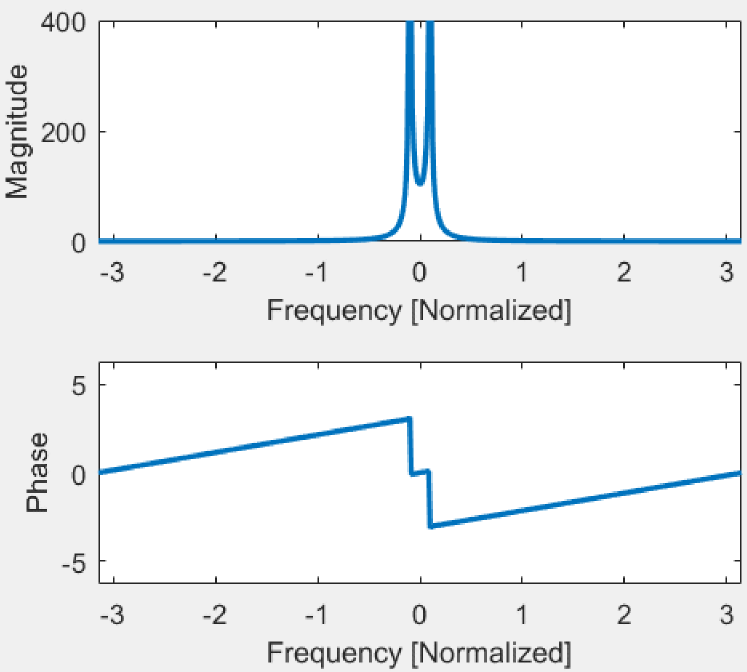
Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $H(z)$ corresponding to this?

$$H(z) = \frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{k=1}^{N}(1 - p_k z^{-1})}$$



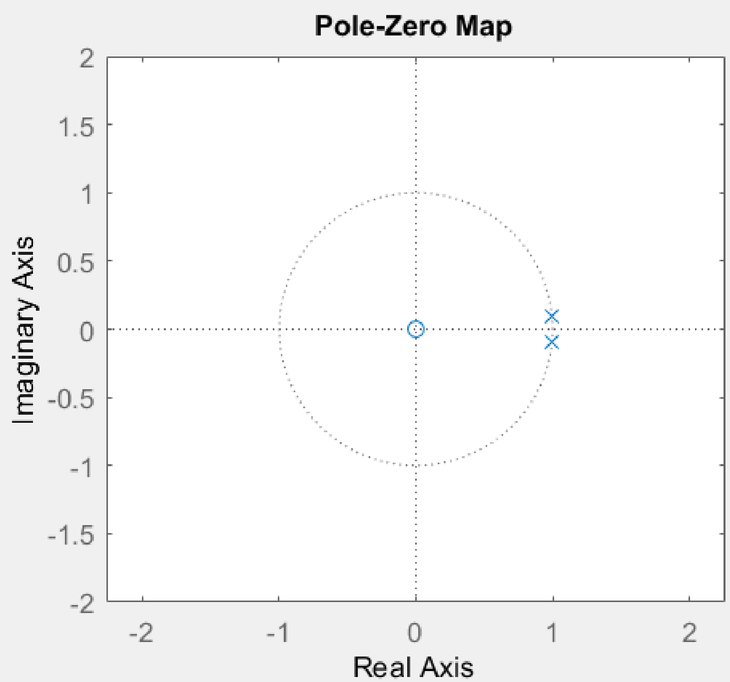


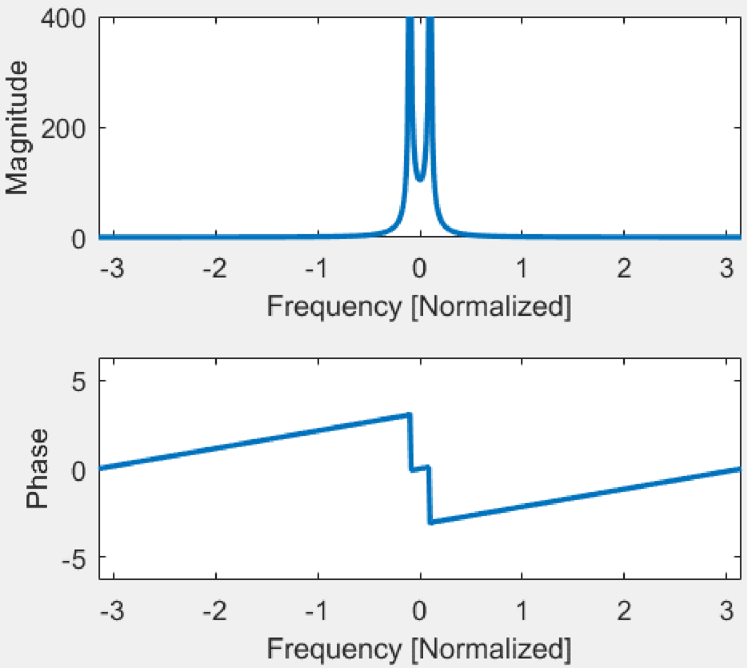
Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $H(z)$ corresponding to this?

$$H(z) = \frac{1}{(1 - e^{+j\phi}z^{-1})(1 - e^{-j\phi}z^{-1})}$$



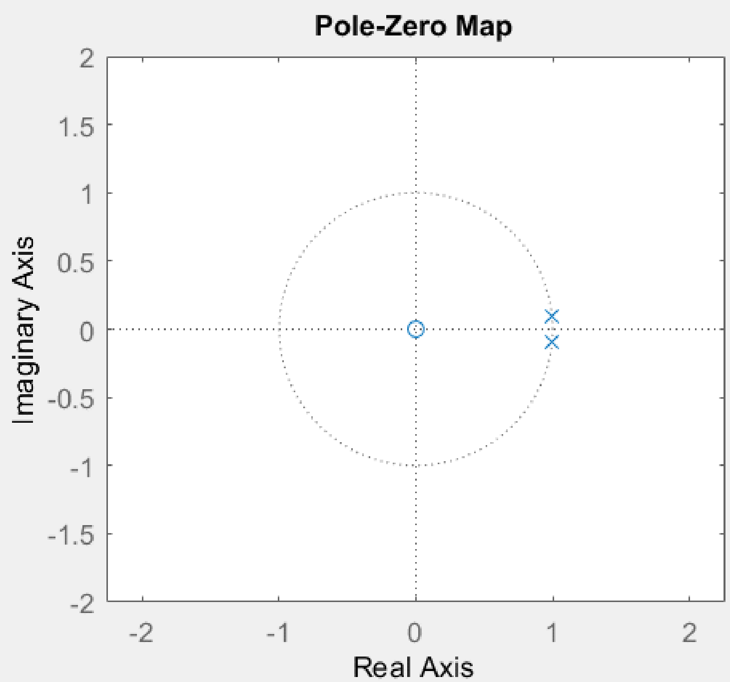


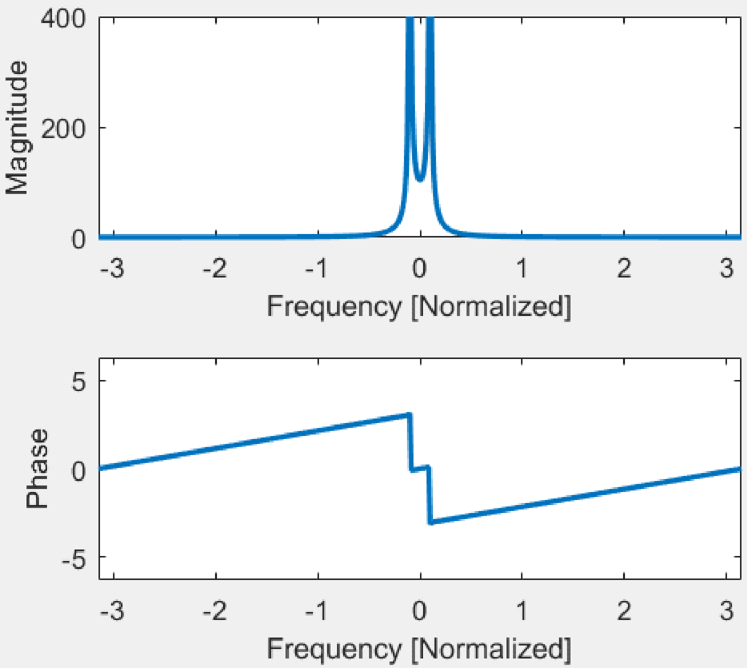
Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $H(z)$ corresponding to this?

$$H(z) = \frac{1}{1 - 2\cos(\phi)\, z^{-1} + z^{-2}}$$





Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $|H(\omega)|$ corresponding to this?

$$H(z) = \frac{1}{1 - 2\cos(\phi)\, z^{-1} + z^{-2}}$$

Designing the Magnitude Response

■ **Question: What happens when we move poles and zeros around for a filter?**

What is the $|\boldsymbol{H}(\omega)|$ corresponding to this?

$$|H(z)| = \left|\frac{1}{1 - 2\cos(\phi)\, z^{-1} + z^{-2}}\right|$$
$$= \left|\frac{1}{z^{+1} - 2\cos(\phi) + z^{-1}}\right|$$

Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $|H(\omega)|$ corresponding to this?

$$|H(\omega)| = \left|\frac{1}{e^{+j\omega} - 2\cos(\phi) + e^{-j\omega}}\right|$$

$$= \left|\frac{1}{e^{+j\omega} + e^{-j\omega} - 2\cos(\phi)}\right|$$

$$= \left|\frac{1}{2\cos(\omega) - 2\cos(\phi)}\right|$$

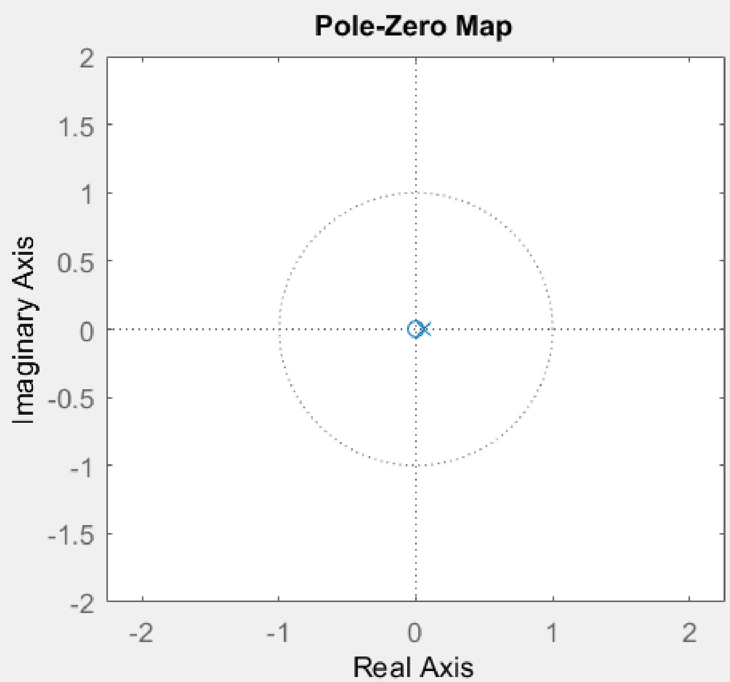
$$= \left|\frac{1/2}{\cos(\omega) - \cos(\phi)}\right|$$

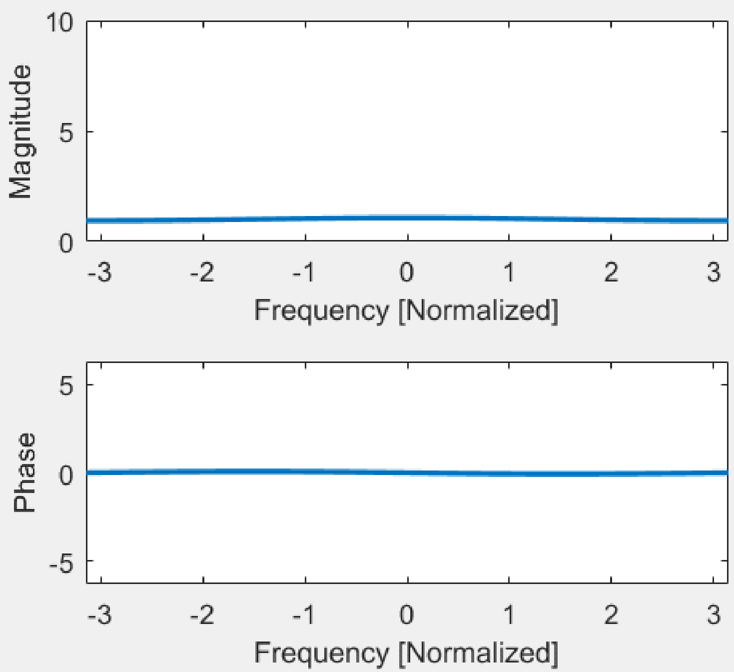
Designing the Magnitude Response

■ **Question: What happens when we move poles and zeros around for a filter?**

What is the $H(z)$ corresponding to this?

$$H(z) = \frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{k=1}^{N}(1 - p_k z^{-1})}$$



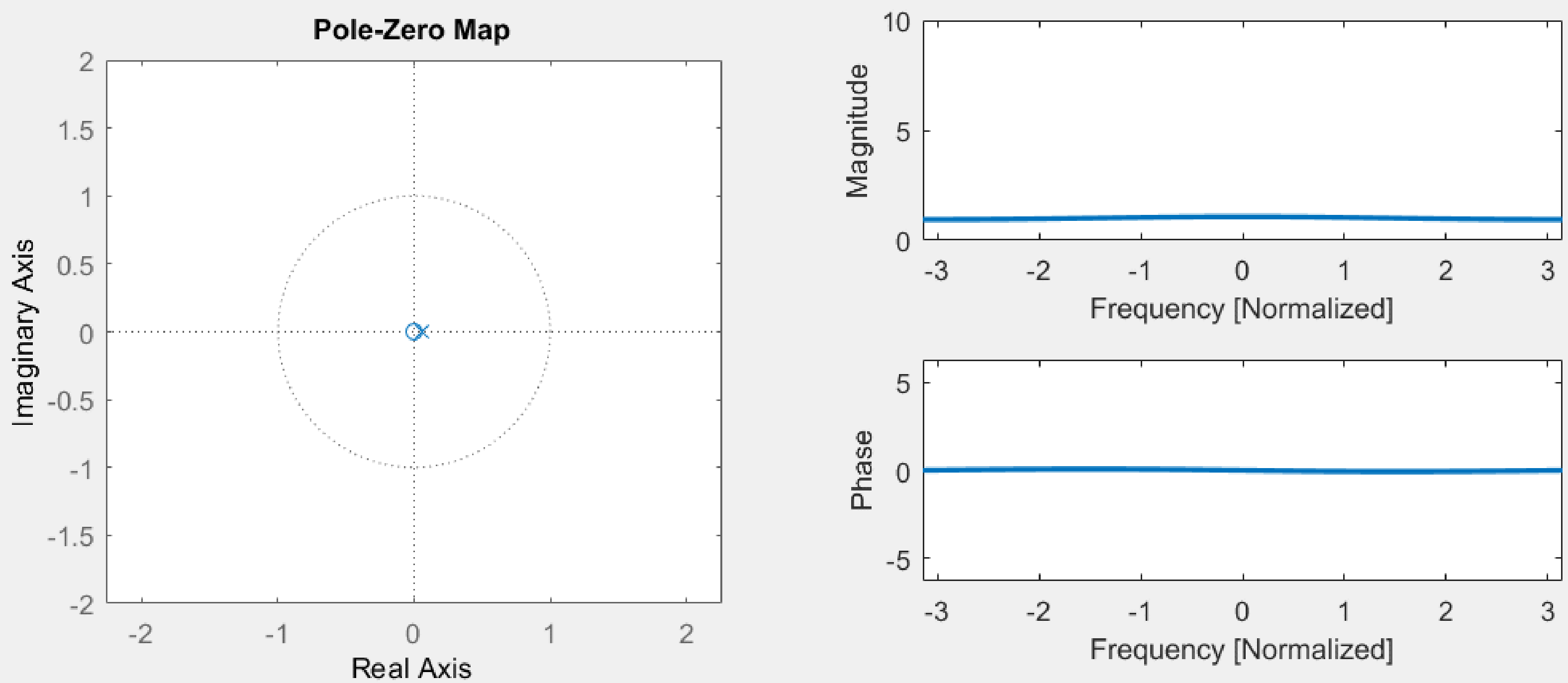


Designing the Magnitude Response

■ **Question: What happens when we move poles and zeros around for a filter?**

What is the $H(z)$ corresponding to this?

$$H(z) = \frac{1}{(1 - az^{-1})}$$



Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $|H(\omega)|$ corresponding to this?

$$|H(\omega)| = \left|\frac{1}{(1 - ae^{-j\omega})}\right|$$

Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

$$|H(\omega)| = \left|\frac{1}{(1 - ae^{-j\omega})}\right|$$

$$= \frac{1}{|1 - ae^{-j\omega}|}$$

$$= \frac{1}{\sqrt{(1 - a\cos(\omega))^2 + a^2\sin^2(\omega)}}$$

$$= \frac{1}{\sqrt{1 - 2\cos(\omega) + a^2\cos^2(\omega) + a^2\sin^2(\omega)}}$$

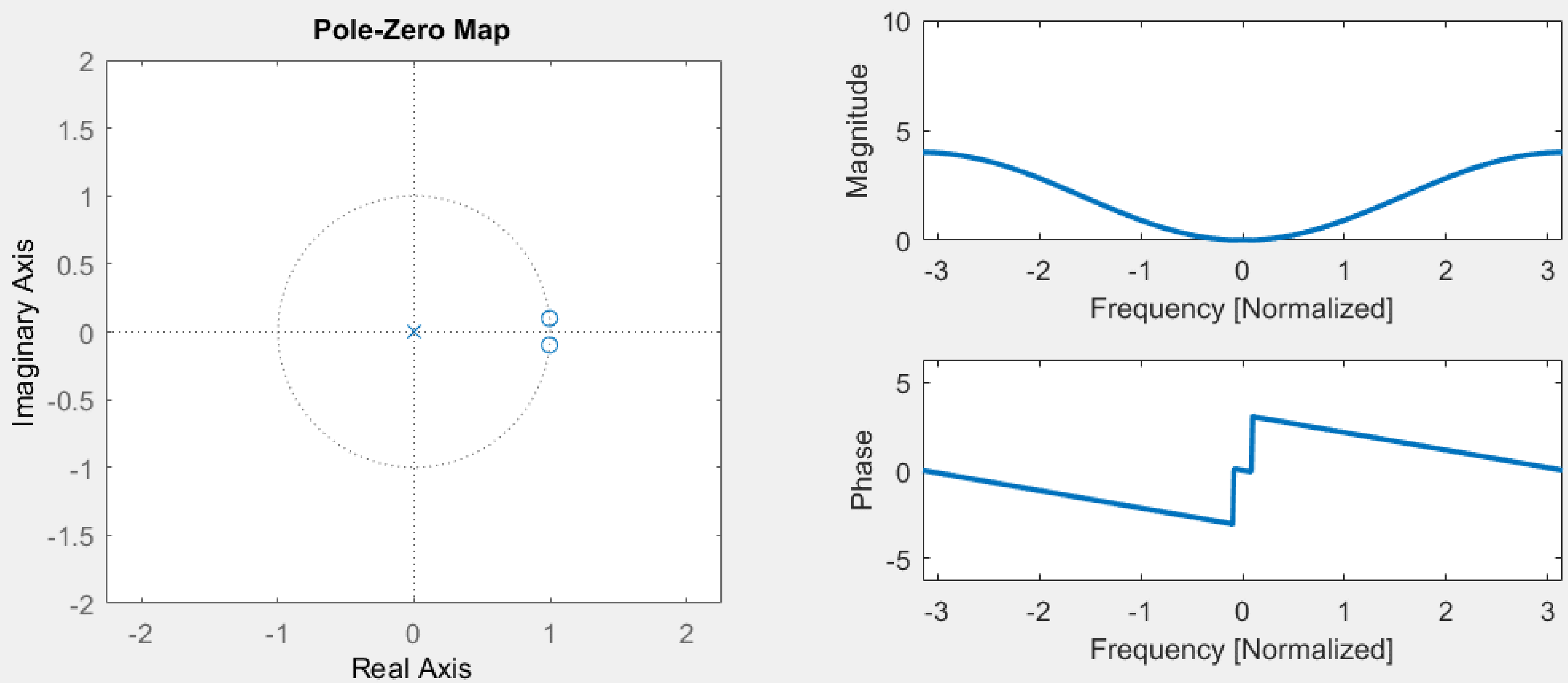
$$= \frac{1}{\sqrt{(1 + a^2) - 2\cos(\omega)}}$$

Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $H(z)$ corresponding to this?

$$H(z) = \frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{k=1}^{N}(1 - p_k z^{-1})}$$

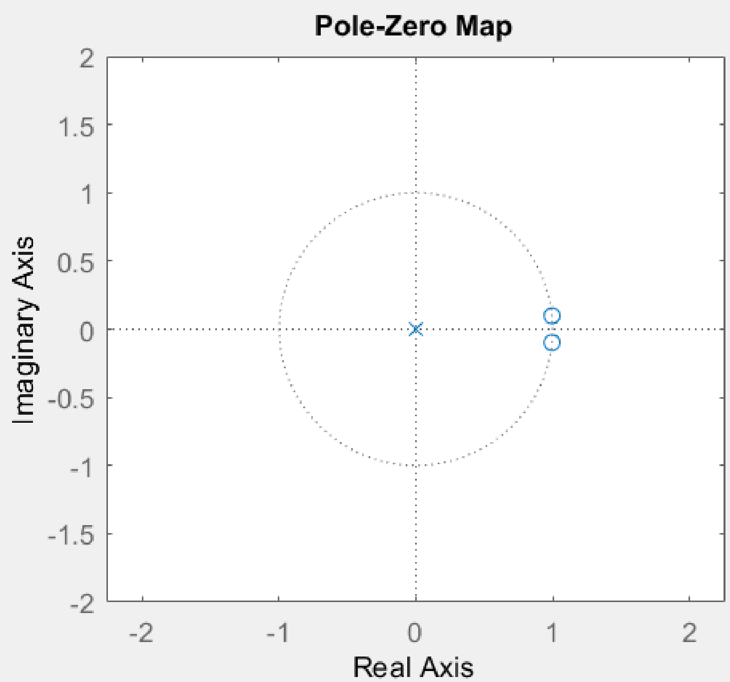


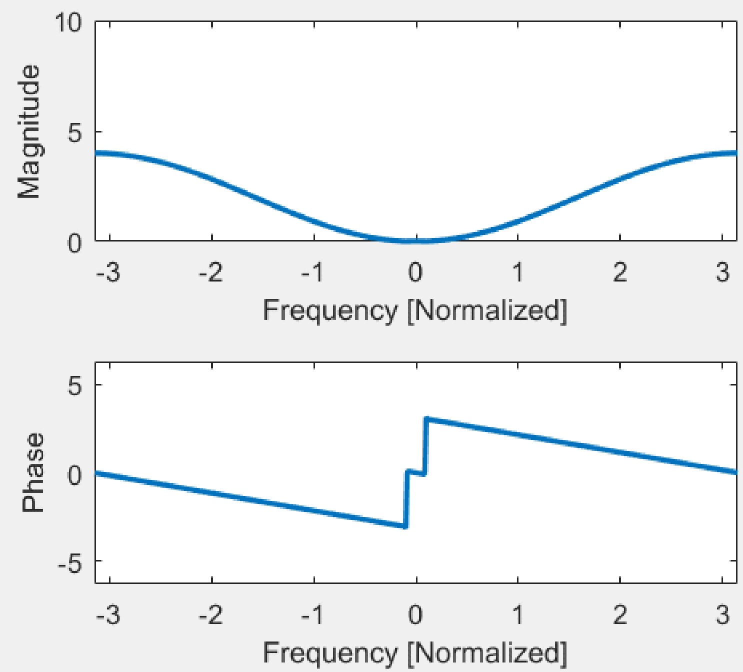
Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $H(z)$ corresponding to this?

$$H(z) = \left(1 - e^{+j\phi} z^{-1}\right)\left(1 - e^{-j\phi} z^{-1}\right)$$



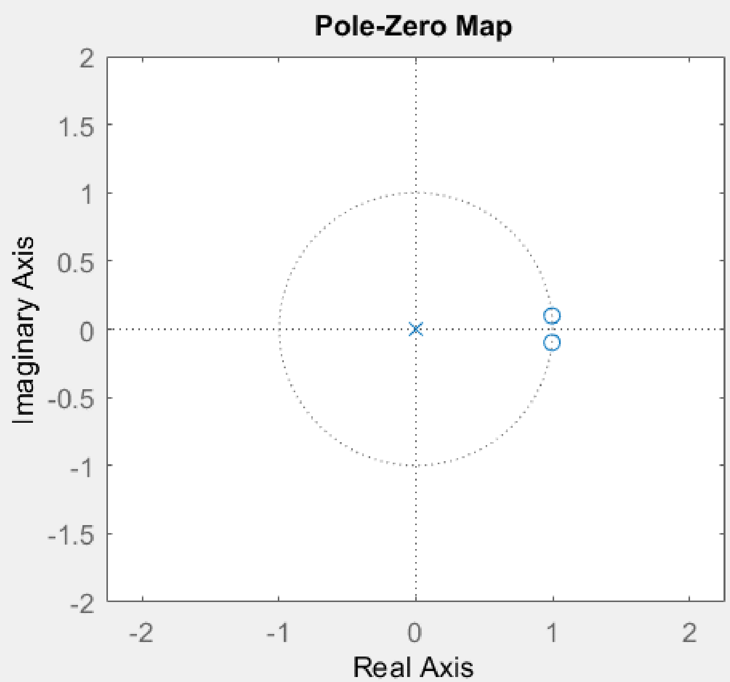


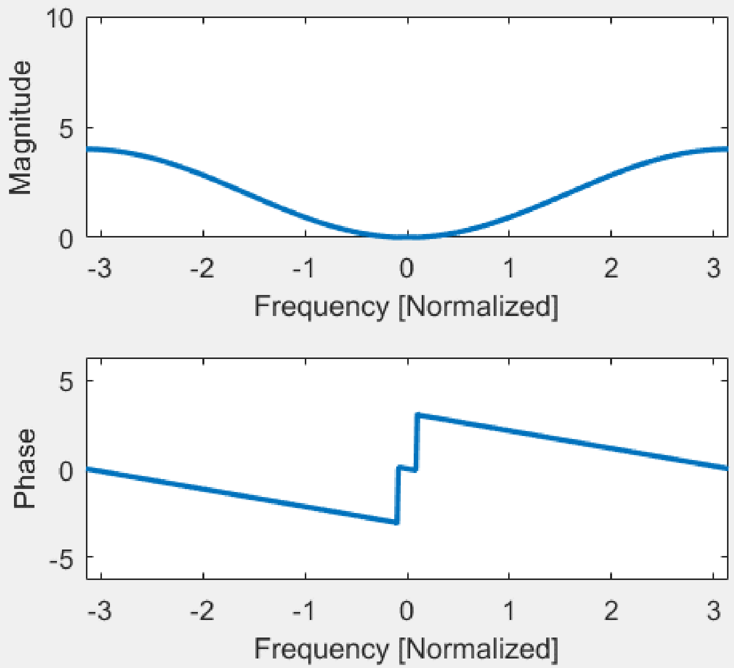
Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $H(z)$ corresponding to this?

$$H(z) = 1 - 2\cos(\phi)\, z^{-1} + z^{-2}$$





Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $|H(\omega)|$ corresponding to this?

$$H(z) = 1 - 2\cos(\phi)\, z^{-1} + z^{-2}$$

Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $|H(\omega)|$ corresponding to this?

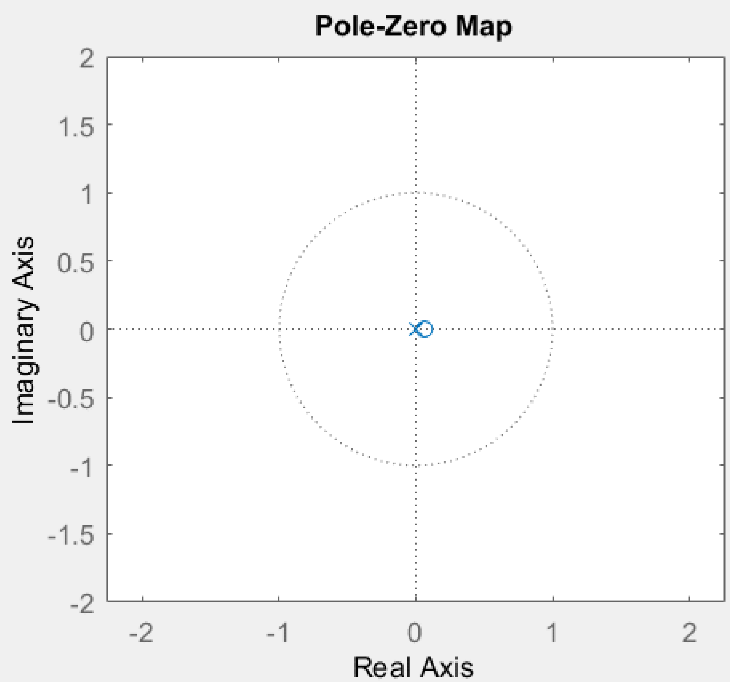
$$|H(\omega)| = 2|\cos(\omega) - \cos(\phi)|$$

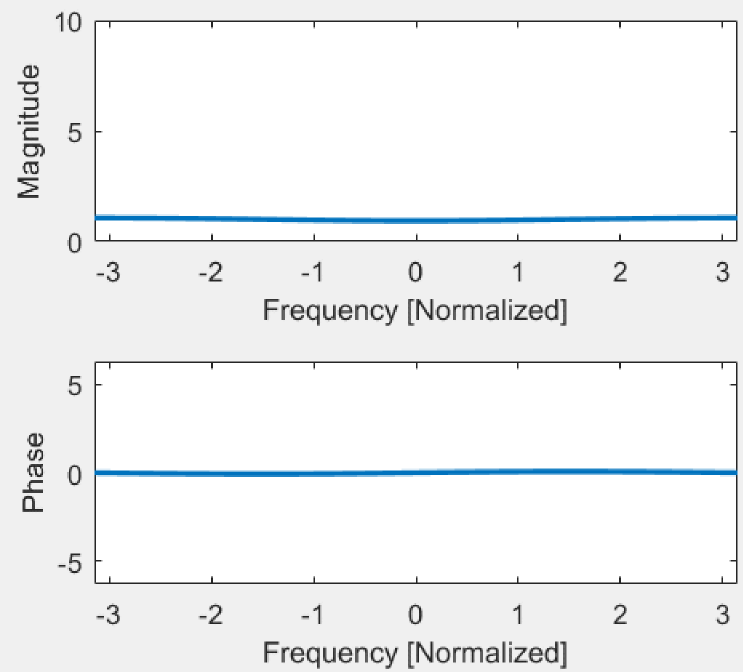
Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

What is the $H(z)$ corresponding to this?

$$H(z) = \frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{k=1}^{N}(1 - p_k z^{-1})}$$



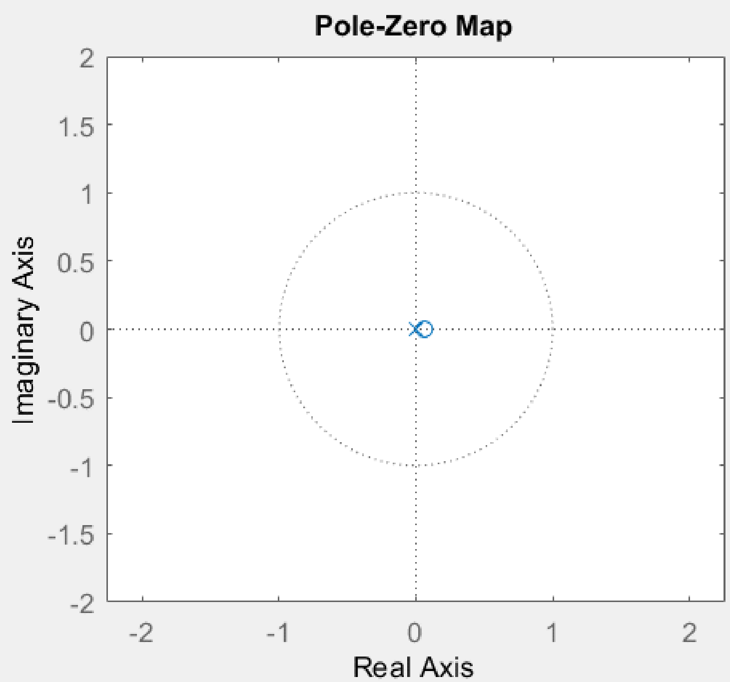


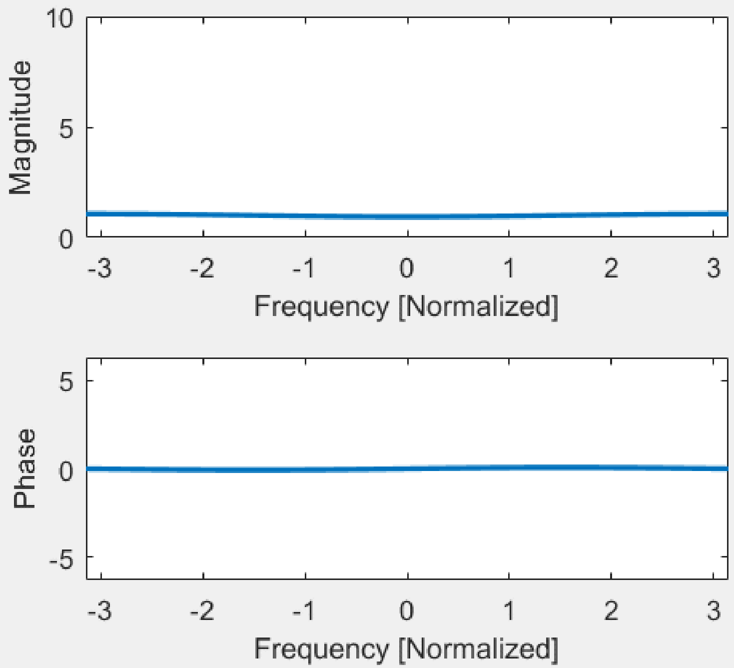
Designing the Magnitude Response

■ **Question: What happens when we move poles and zeros around for a filter?**

What is the $H(z)$ corresponding to this?

$$H(z) = 1 - ae^{-j\omega}$$





Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

$$H(z) = 1 - ae^{-j\omega}$$

What is the $|\boldsymbol{H}(\omega)|$ corresponding to this?

Designing the Magnitude Response

- **Question: What happens when we move poles and zeros around for a filter?**

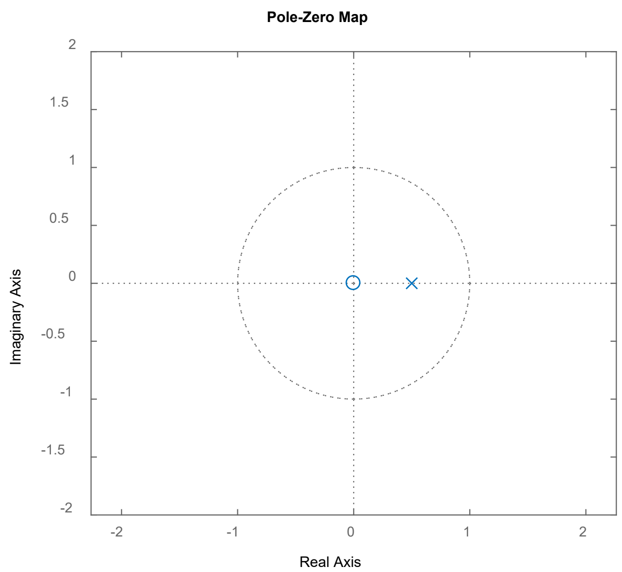
What is the $|H(\omega)|$ corresponding to this?

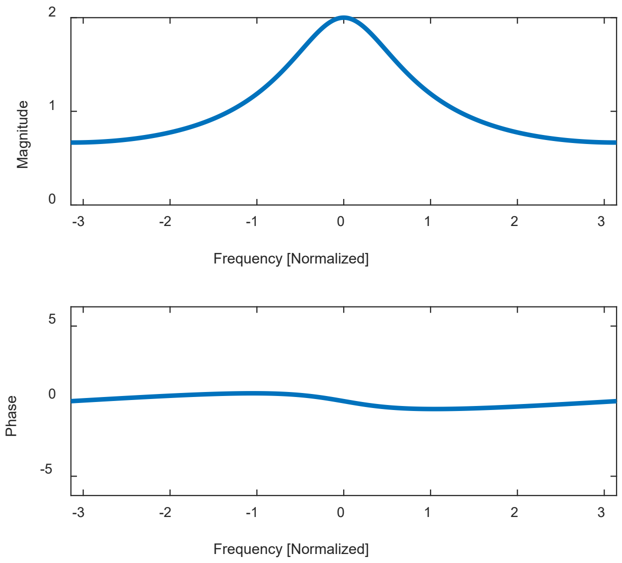
$$|H(\omega)| = \sqrt{(1 + a^2) - 2\cos(\omega)}$$

Designing the Magnitude Response

- **Question: How can I make the high frequencies closer to zero?**

$$H(z) = \frac{1}{(1 - (1/2)z^{-1})}$$



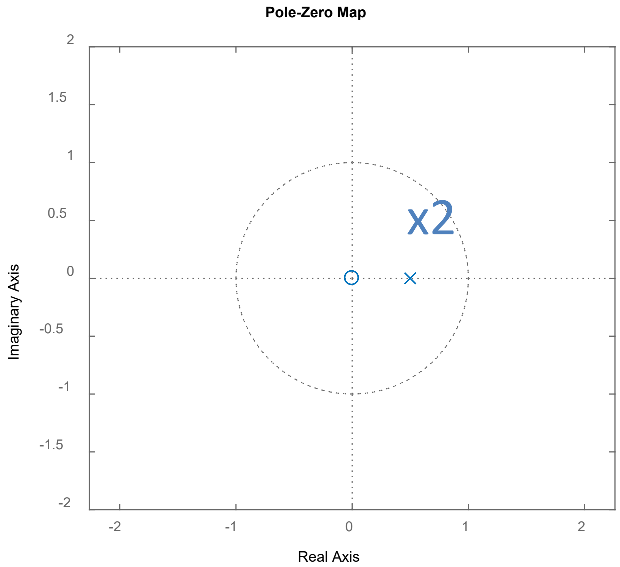


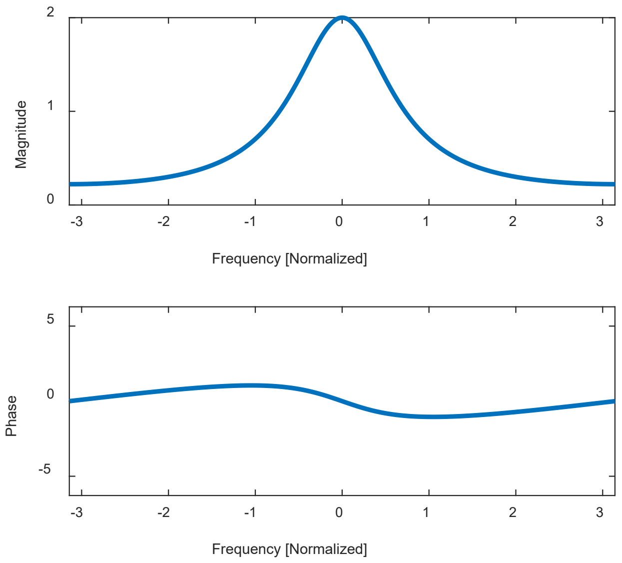
Designing the Magnitude Response

- **Question:** **How can I make the high frequencies closer to zero?**

Option:
Add poles

$$H(z) = \frac{1/2}{(1-(1/2)z^{-1})^2}$$



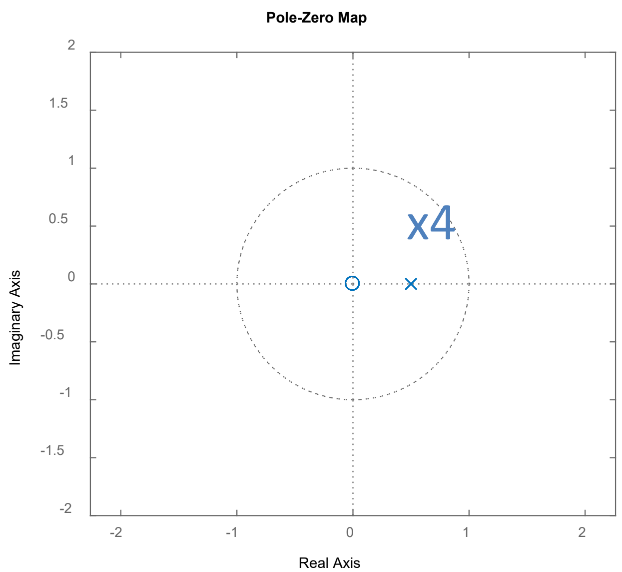


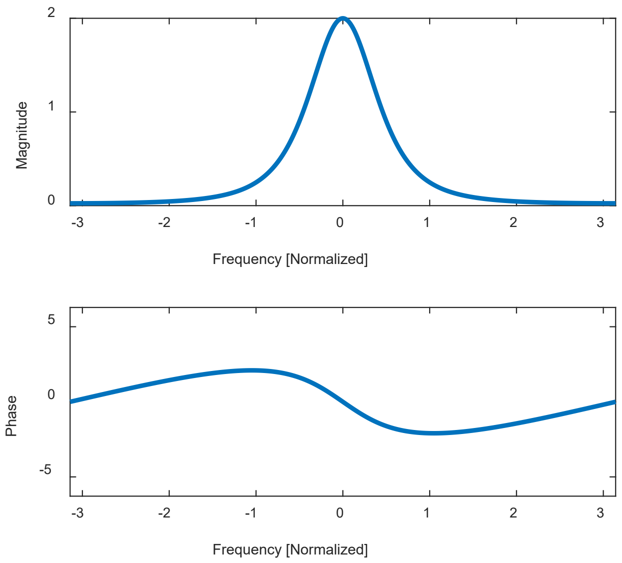
Designing the Magnitude Response

- **Question:** **How can I make the high frequencies closer to zero?**

Option:
Add poles

$$H(z) = \frac{1/8}{(1 - (1/2)z^{-1})^4}$$

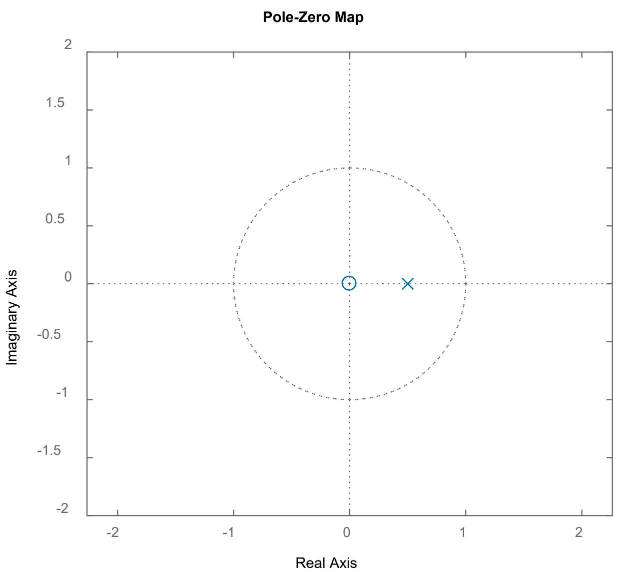


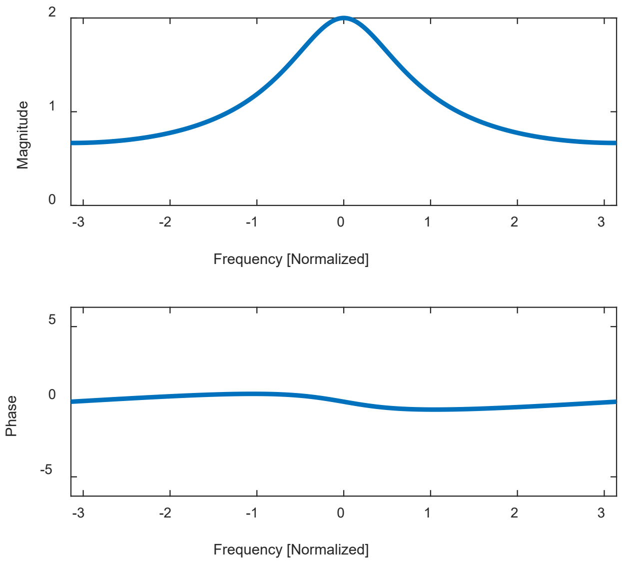


Designing the Magnitude Response

- **Question: How can I make the high frequencies closer to zero?**

$$H(z) = \frac{1}{(1 - (1/2)z^{-1})}$$



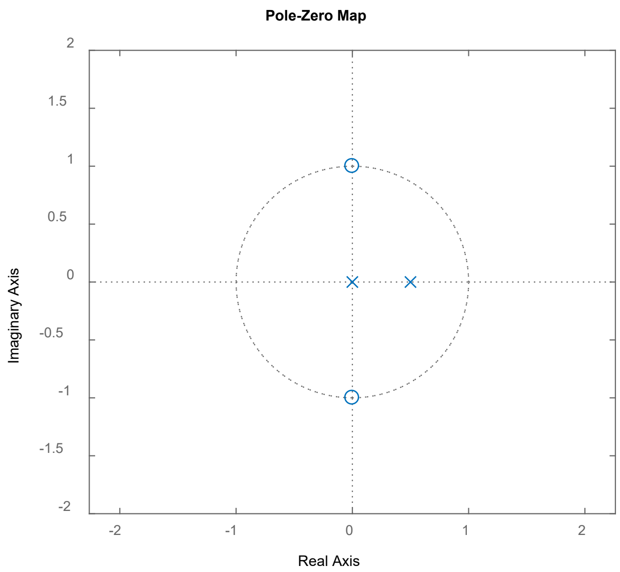


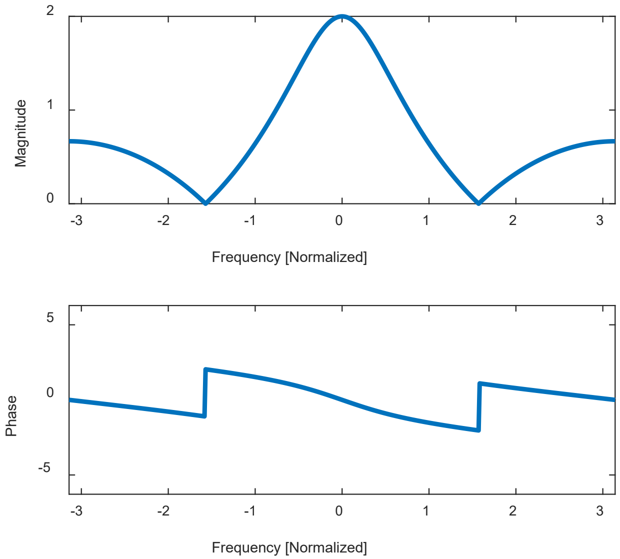
Designing the Magnitude Response

- **Question: How can I make the high frequencies closer to zero?**

Option:
Add zeros

$$H(z) = \frac{(1 - jz^{-1})(1 + jz^{-1})}{2(1 - (1/2)z^{-1})}$$



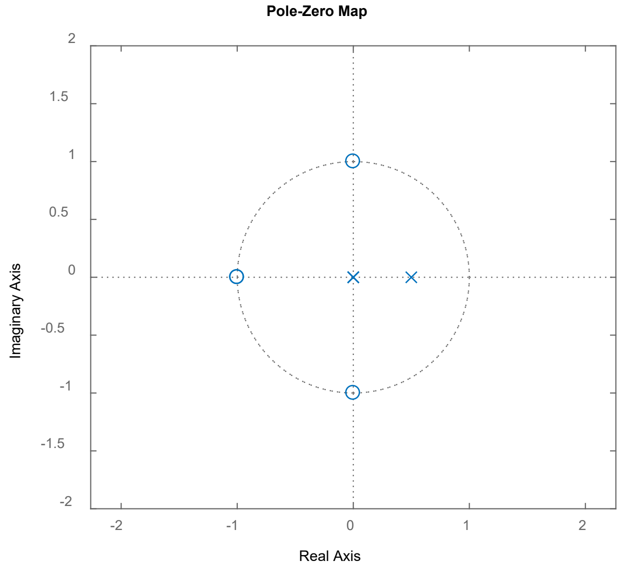


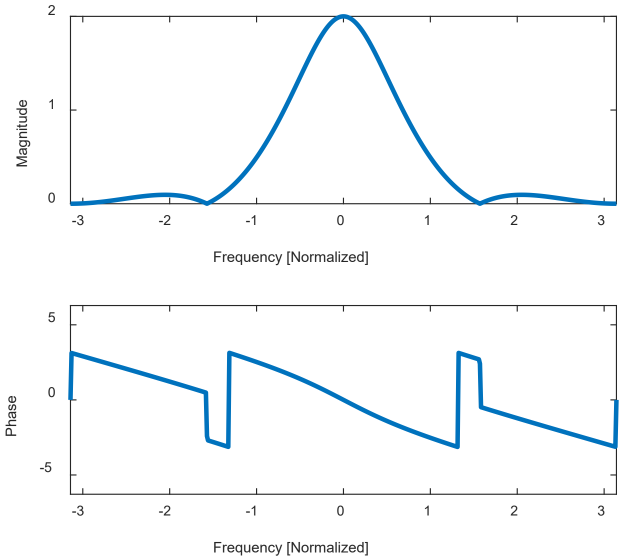
Designing the Magnitude Response

- **Question: How can I make the high frequencies closer to zero?**

Option:
Add zeros

$$H(z) = \frac{(1 - jz^{-1})(1 + jz^{-1})(1 + z^{-1})}{8(1 - (1/2)z^{-1})}$$





Designing the Magnitude Response

- **Question: How can I make the high frequencies closer to zero?**

Option:
Add zeros

$$H(z) = \frac{(1 - jz^{-1})(1 + jz^{-1})(1 + z^{-1})}{8(1 - (1/2)z^{-1})}$$