EEE5502 Code #04 Hudanyun Sheng

Question #1

I spent 6 hours.

Question #2

(a) $x[n] = e^{-jw_x n} = \cos(w_x n) - j\sin(w_x n)$. $y[n] = e^{-jw_y n} = \cos(w_y n) - j\sin(w_y n)$

$C = \sum_{n=-\infty}^{\infty} x[n](y[n])^* = \sum_{n=-\infty}^{\infty} [\cos(w_x n) - j\sin(w_x n)] \cdot [\cos(w_y n) + j\sin(w_y n)]$

$\because w_x = w_y$

$\therefore C = \sum_{n=-\infty}^{\infty} \cos^2(w_x n) + j\sin(w_x n)\cos(w_x n) - j\sin(w_x n)\cos(w_x n) - j^2\sin^2(w_x n)$

$= \sum_{n=-\infty}^{\infty} \cos^2(w_x n) + \sin^2(w_x n) = \sum_{n=-\infty}^{\infty} 1 = \infty \neq 0$

$\therefore$ These vectors are not orthogonal.

(b) $C = \sum_{n=-\infty}^{\infty} \cos(w_x n)\cos(w_y n) + j\cos(w_x n)\sin(w_y n) - j\sin(w_x n)\cos(w_y n) - j^2\sin(w_x n)\sin(w_y n)$

$= \sum_{n=-\infty}^{\infty} \cos(w_x n)\cos(w_y n) + \sin(w_x n)\sin(w_y n) - j\sin(w_x n - w_y n)$

$= \sum_{n=-\infty}^{\infty} \cos(w_x n - w_y n) - j\sin(w_x n - w_y n) = \sum_{n=-\infty}^{\infty} \cos(w_x n - w_y n) \neq 0$

$\therefore$ These vectors are not orthogonal.

(c) $x[n] * (y[n])^* = \mathcal{F}^{-1}[X(w)Y(w)]$ ~~$= \mathcal{F}^{-1}[2$~~

$= \mathcal{F}^{-1}[2\pi \sum_{k=-\infty}^{\infty} \delta(w + w_x - 2\pi k) \cdot 2\pi \sum_{k=-\infty}^{\infty} \delta(w - w_y - 2\pi k)]$

$= \mathcal{F}^{-1}\{4\pi^2 \sum_{k=-\infty}^{\infty} \delta[w - (-w_x + 2\pi k)] \cdot \sum_{k=-\infty}^{\infty} \delta[w - (w_y + 2\pi k)]\}$

$\because w_x \neq w_y$.

If $w_x \cdot w_y \geq 0$

$x[n] * (y[n])^* = 0$

$\therefore$ If $w_x \cdot w_y \geq 0$, these vectors are orthogonal.

(d) $\sum_{n=-\infty}^{\infty} x[n](y[n])^* = \sum_{n=-\infty}^{\infty} e^{-jw_x n}(u[n] - u[n-N]) \cdot e^{jw_y n}(u[n] - u[n-N])$

$= \sum_{n=-\infty}^{\infty} e^{-j(w_x - w_y)n} (u[n] - u[n-N])$

$= \sum_{n=0}^{N-1} e^{-j(w_x - w_y)n} (u[n] - u[n-N])$

$$\xrightarrow[\omega_y=\frac{2\pi}{N}k_y]{\omega_x=\frac{2\pi}{N}k_x} \sum_{n=0}^{N-1} e^{-j(k_x-k_y)\frac{2\pi n}{N}} (u[n]-u[n-N]).$$

$$= \underbrace{\sum_{n=0}^{N-1} \cos\left[(k_x-k_y)\frac{2\pi n}{N}\right]}_{\text{range: }[0,2K\pi]} - j\sum_{n=0}^{N-1} \sin\left[(k_x-k_y)\frac{2\pi n}{N}\right] = 0 \quad \text{QED.}$$

, where $K = k_x - k_y$ is an integer.

(e) Similar to what have been discussed above, we got

$$\sum_{n=-\infty}^{\infty} x[n](y[n])^* = \sum_{n=0}^{N-1} e^{-j(k_x-k_y)\frac{2\pi n}{K}} (u[n]-u[n-N])$$

$$= \sum_{n=0}^{N-1} \cos\left[\underbrace{(k_x-k_y)}_{(k_x \neq k_y)}\frac{2\pi n}{K}\right] - j\sum_{n=0}^{N-1} \sin\left[(k_x-k_y)\frac{2\pi n}{K}\right] \neq 0$$

$\because N \neq K \therefore \frac{2\pi n}{K}$ not in range $[0, 2\pi]$

$\therefore$ The summation from 0 to $N-1$ ~~may be 0~~ $\neq 0$

$\therefore x[n]$ and $y[n]$ are not orthogonal.

(f) $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{N-1} \cos\left(\frac{2\pi}{N}k_x n\right) e^{-j\frac{2\pi}{N}kn}$

$$= \sum_{n=0}^{N-1} \frac{1}{2}\left[e^{j\frac{2\pi}{N}k_x n} + e^{-j\frac{2\pi}{N}k_x n}\right] e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k_x-k)n} + e^{-j\frac{2\pi}{N}(k_x+k)n} \quad \cancel{=0}$$

If $k_x = k$, $x[k] = \frac{N}{2}$;

If $k_x = -k$ $x[k] = \frac{N}{2}$

If $k_x \neq k$, $k_x \neq -k$, $x[k] = 0$

$$\therefore X[k] = \begin{cases} \frac{N}{2}, & \text{if } k_x = k \pm 2\pi \text{ or } k_x = -k \pm 2\pi \\ 0, & \text{otherwise.} \end{cases}$$