EEE 5502 Code #03 Hudanyun Sheng

Question #1

I spent 4 hours

Question #2

(a) $\mathcal{F}\{x[n]*y[n]\} = \sum_{n=-\infty}^{\infty}[x[n]*y[n]]e^{-j\omega n} = \sum_{n=-\infty}^{\infty}[\sum_{m=-\infty}^{\infty} x[m]y[n-m]]e^{-j\omega n}$

$= \sum_{m=-\infty}^{\infty} x[m][\sum_{n=-\infty}^{\infty} y[n-m]e^{-j(n-m)\omega}]e^{-jm\omega} = X(\omega)Y(\omega)$

(b) $X(\omega) = \sum_{n=-\infty}^{\infty} x[n](\cos(\omega n) - j\sin(\omega n))$

If $x[n]$ is real, the real part of $X(\omega)$ will be only $\sum_{n=-\infty}^{\infty} x[n]\cos(\omega n)$

~~Let~~

$X(-\omega) = \sum_{n=-\infty}^{\infty} x[n]\cos(-\omega n) = \sum_{n=-\infty}^{\infty} x[n]\cos(\omega n) = X(\omega)$ □

If $x[n]$ is complex, there will be imaginary part in $x[n]$. The real part of $X(\omega)$ would have the format: $\sum_{n=-\infty}^{\infty} x_{real}[n]\cos(\omega n) + m\, x_{imaginary}[n]\sin(\omega n)$, which is not even.

(c) If $x[n]$ is real, the imaginary part of $X(\omega)$ will be $j\sum_{n=-\infty}^{\infty} x[n]\sin(\omega n)$.

$\therefore X_{ima}(-\omega) = j\sum_{n=-\infty}^{\infty} x[n]\sin(-\omega n) = -j\sum_{n=-\infty}^{\infty}\sin(\omega n) = -X_{ima}(\omega)$

$\therefore X(\omega) = -X(-\omega)$ □

(d) $\mathcal{F}\{x[-n]\} = X(-\omega)$

$\because x[n]$ is real

$\therefore$ The real part of $X(\omega) = \mathcal{F}\{x[n]\}$ is even, the imaginary part of $X(\omega)$ is odd

$\therefore X(-\omega) = X^*(\omega)$. (complex conjugate).

$\therefore \mathcal{F}\{x[-n]\} = X^*(\omega)$

(e) $\mathcal{F}\{c[n]\} = \mathcal{F}\{x[-n] * y[n]\}$

$= \mathcal{F}\{x[-n]\} \cdot \mathcal{F}\{y[n]\}$

$= X^*(w) \cdot Y(w).$