Full Name: ____________________

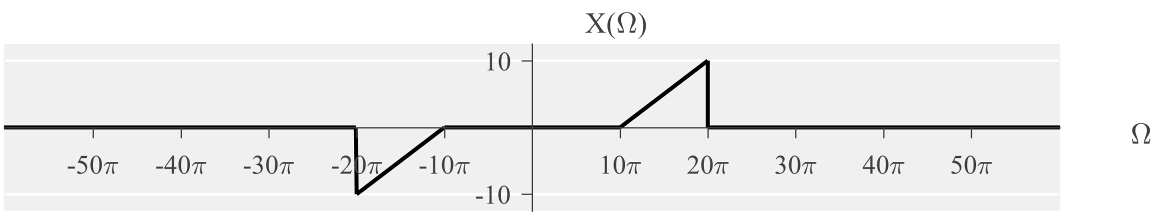
EEL 4750 / EEE 5502 (Fall 2018) – HW #06 **Due Date:** **Oct. 12, 2018**

Homework learning objectives: By the end of this homework, you should be able to:

- Plot the Fourier transform of a sampled signal
- Determine the Nyquist sampling rate of a signal
- Reconstruct sampled signal
- Apply an anti-aliasing filter before sampling
- Perform circular convolution

Question #1: *(1 pts)* How many hours did you spend on this homework?

Question #2: *(10 pts)* Consider the Fourier Transform of a continuous-time signal $x(t)$ below.



(a) Determine the Nyquist sampling rate for $X(\Omega)$.

(b) Determine the Nyquist sampling rate for $X(\Omega/4)$.

(c) Sketch (for $\Omega = -60\pi$ to $\Omega = +60\pi$) the Fourier transform $X_s(\Omega)$ of the sampled signal $x(t)$ (with Fourier transform $X(\Omega)$) for a sampling rate of $\Omega_s = 20\pi$.
Do we experience aliasing?

(d) Sketch (for $\Omega = -60\pi$ to $\Omega = +60\pi$) the Fourier transform $X_s(\Omega)$ of the sampled signal $x(t)$ (with Fourier transform $X(\Omega)$) for a sampling rate of $\Omega_s = 50\pi$.
Do we experience aliasing?

(e) Sketch (for $\Omega = -60\pi$ to $\Omega = +60\pi$) the Fourier transform $X_s(\Omega)$ of the time-sampled signal $x(t)$ (with Fourier transform $X(\Omega)$) with a sampling rate of $\Omega_s = 30\pi$ **after** applying a low-pass anti-aliasing filter with cut-off frequency of $\Omega_c = \Omega_s/2$ and gain $K = T_s$.

Question #3: *(8 pts)* Consider the continuous-time sinusoid signal

$$x(t) = \cos(\Omega_0 t)$$

that is sampled at a rate of $\Omega_s = 50$ rad/s. **After sampling**, the continuous signal is reconstructed with a lowpass reconstruction filter with cut-off frequency of $\Omega_c = \Omega_s/2$ and gain $K = T_s$. Determine the reconstructed sinusoid when

(a) $\Omega_0 = 10$ rad/s

(b) $\Omega_0 = 20$ rad/s

(c) $\Omega_0 = 40$ rad/s

(d) $\Omega_0 = 60$ rad/s

Question #4: *(8 pts)* Consider the continuous time sinusoid signal

$$x(t) = \cos(\Omega_0 t)$$

that is sampled at a rate of $\Omega_s = 50$ rad/s. **Before sampling**, the signal is filtered with a low-pass anti-aliasing filter with cut-off frequency of $\Omega_c = \Omega_s/2$ and gain $K = 1$. **After sampling**, the continuous signal is reconstructed with a lowpass reconstruction filter with cut-off frequency of $\Omega_c = \Omega_s/2$ and gain $K = T_s$. Determine the reconstructed sinusoid when

(a) $\Omega_0 = 10$ rad/s

(b) $\Omega_0 = 20$ rad/s

(c) $\Omega_0 = 40$ rad/s

(d) $\Omega_0 = 60$ rad/s

Question #5: *(10 pts)* For each of the following signals, decide whether the Fourier Series, Fourier Transform, Discrete-time Fourier Transform, or Discrete Fourier Transform (or Discrete-Time Fourier Series) are the most appropriate to use for Fourier analysis.

(a) Our signal is continuous and is periodic in time

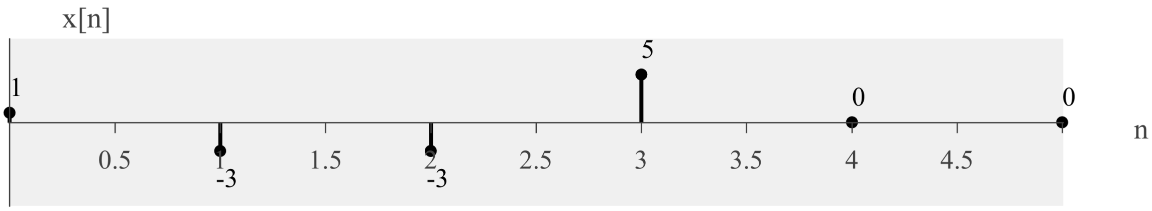
(b) Our signal is continuous and is aperiodic in time

(c) Our signal is discrete and is aperiodic in time

(d) Our signal is continuous and has a finite energy in time

(e) Our signal is discrete and has a finite power in time

Question #6: *(10 pts)* Consider discrete-time signal $x[n]$ of length $N_0 = 6$ shown below.



When processing data on the computer, we compute the Fourier transform with the Discrete Fourier Transform (DFT). For the DFT, multiplication in frequency is circular convolution in time. That is,

$$x_1[n] \circledast x_2[n] \overset{DFT}{\longleftrightarrow} DFT\left\{x_1[n]\right\} DFT\left\{x_2[n]\right\}$$

where $\circledast$ denotes circular convolution.

(a) Compute $x[n] \circledast \delta[n-2]$

(b) Compute $x[n] \circledast \delta[n-4]$

(c) Compute $x[n] \circledast \delta[n-6]$

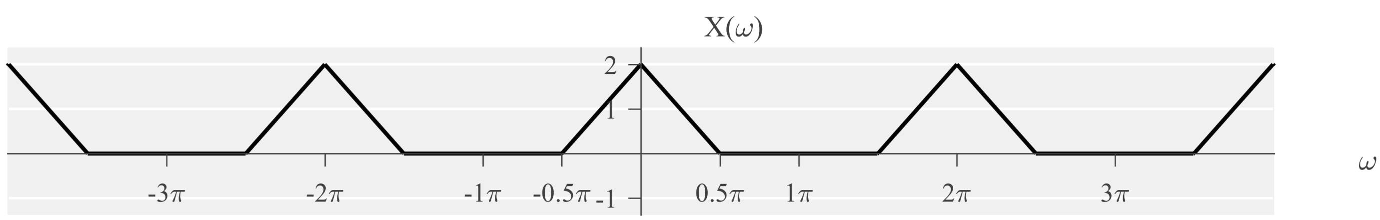
(d) Compute $x[n] \circledast \left(\frac{1}{3}\left(\delta[n-1] + \delta[n-2] + \delta[n-3]\right)\right)$

(e) Compute $x[n] \circledast x[5-n]$

Question #7: *(8 pts)* Consider the DTFT of $x[n]$, shown below and consider the multiplication property of the DTFT

$$x[n]y[n] \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi} X(\omega) \circledast Y(\omega)$$

where $\circledast$ denotes circular convolution.



(a) Sketch the DTFT of $y_1[n] = x[n]\cos((\pi n)/2)$

(b) Sketch the DTFT of $y_2[n] = x[n]\cos((3\pi n)/4)$

(c) Sketch the DTFT of $y_3[n] = x[n]\cos((3\pi n)/2)$

(d) Sketch the DTFT of $y_1[n] = x[n]\cos(102\pi n)$