Lecture 9: Exam 1 Review

Foundations of Digital Signal Processing

Outline

- Exam 1 Review

News

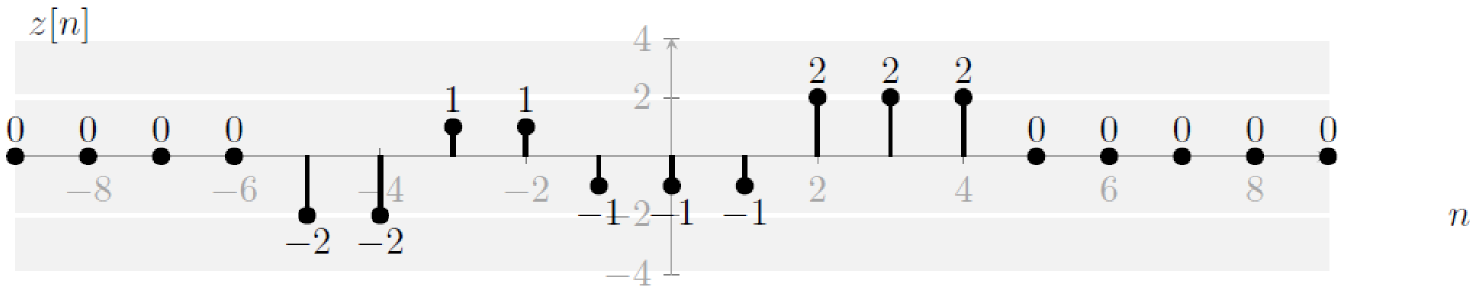
- **Homework #4**
  - Due *Today* by 11:59 PM
  - Submit via canvas
- **Coding Problem #2**
  - Due *Today* by 11:59 PM
  - Submit via canvas

News

- **Exam #1**
  - September 25th (1 week away)
  - Will cover all material up to today… such as
    - Signal properties
    - System properties
    - LTI Systems
    - Difference equations
    - Discrete-time convolution
    - The Z-transform and its properties
    - The Discrete-time Fourier Transform and its properties
    - Etc.

Example Exam Question

Question #2: Let the discrete-time signal $z[n]$ be defined by (additional values are zero)



(a) *(5 pts)* Express $z[n]$ as a sum of step functions.

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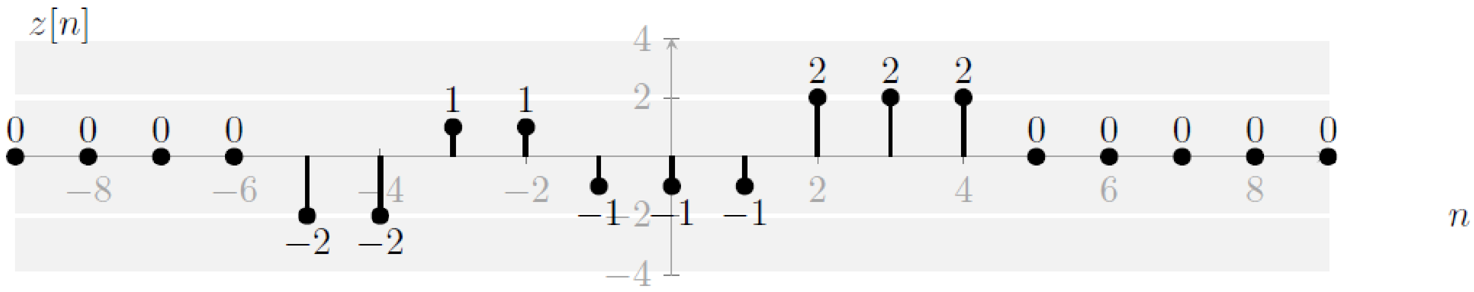
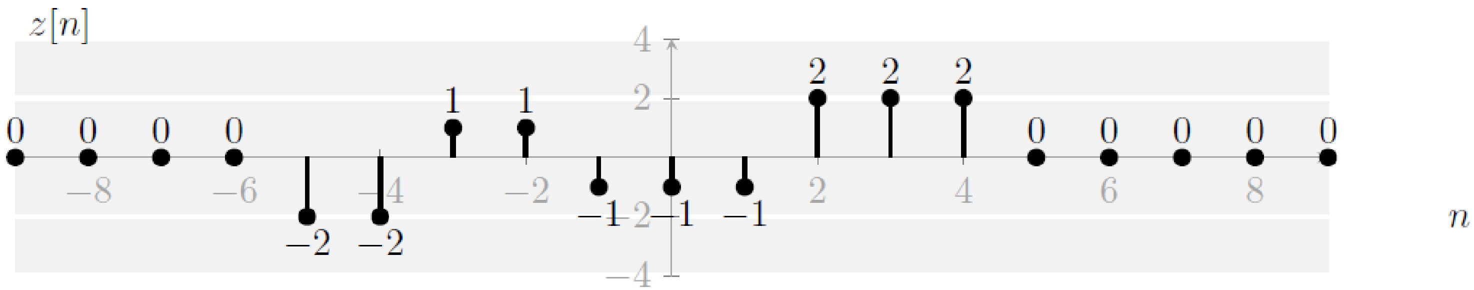


(a) *(5 pts)* Express $z[n]$ as a sum of step functions.

$$z[n] = -2u[n+5] + 3u[n+3] - 2u[n+1] + 3u[n-2] - 2u[n-5]$$

Example Exam Question

Question #2: Let the discrete-time signal $z[n]$ be defined by (additional values are zero)



(b) *(5 pts)* Is $z[n]$ an energy signal, a power signal, or neither? If $z[n]$ is an energy signal, compute its energy. If $z[n]$ is a power signal, compute its power. If $z[n]$ is neither, explain why.

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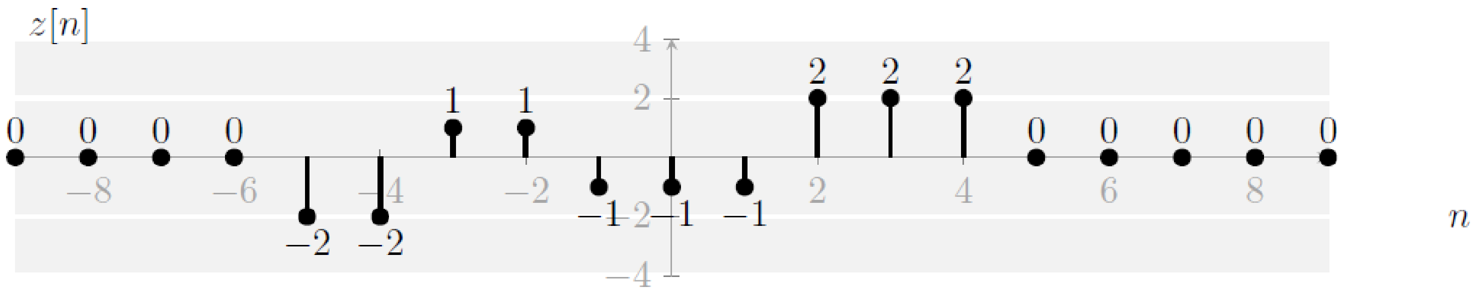

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Energy Signal

$$E_x = (-2)^{2}*2+(1)^{2}*2+(-1)^{2}*3+(2)^{2}*3 = 8+2+3+12 = 25$$

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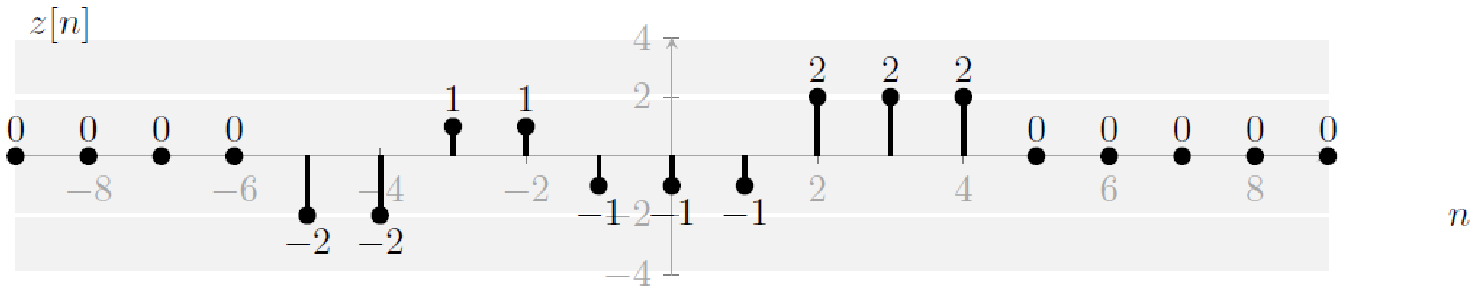

(c) *(4 pts)* Is $z[-n+7]$ causal?

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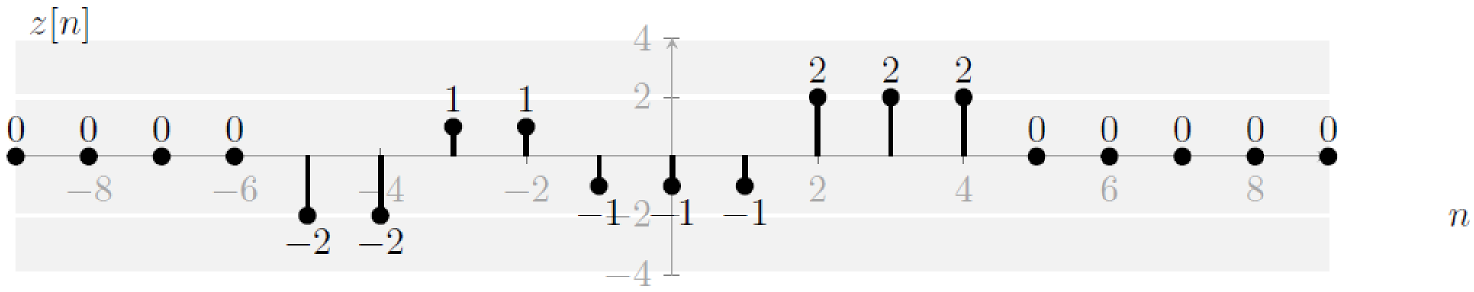

(c) *(4 pts)* Is $z[-n+7]$ causal?

Yes! Shift left by 7 and then time-reverse.

Example Exam Question

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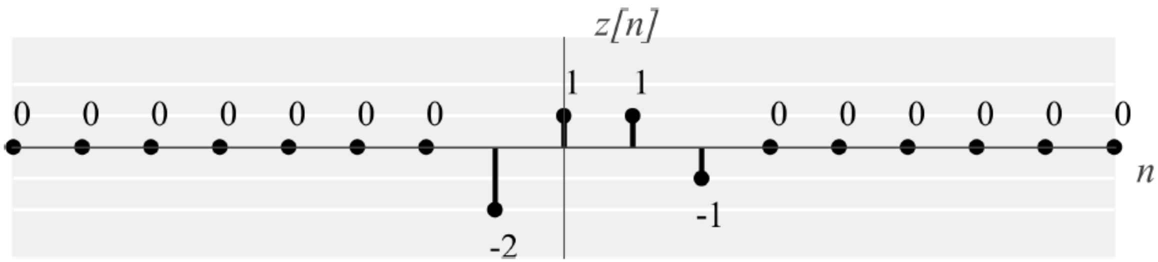

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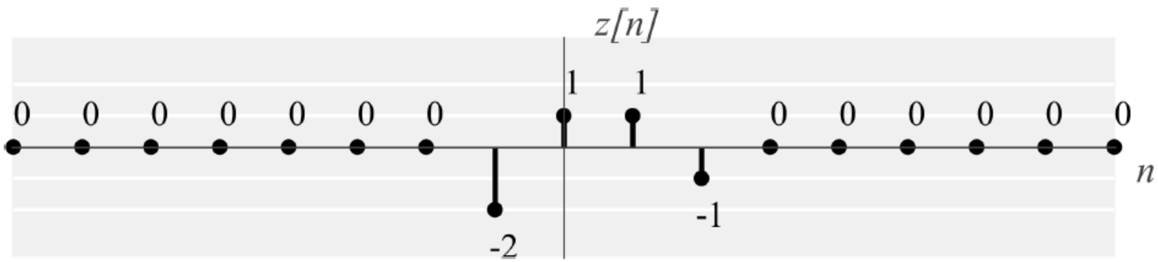

(a) *(5 pts)* Express $z[n]$ as a sum of step functions $u[n]$.

$$\boxed{z[n] = -2u[n+1] + 3u[n] - 2u[n-2] + u[n-3]}$$

Example Exam Question

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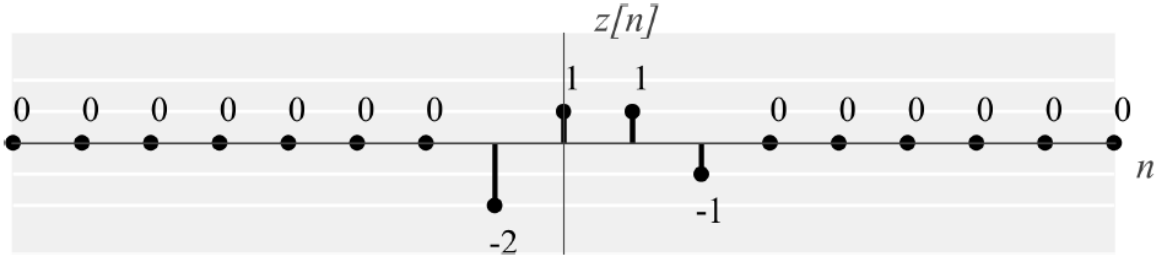
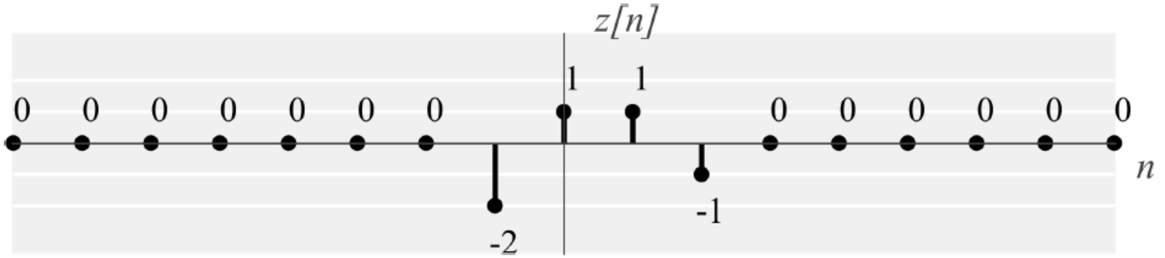


(b) *(6 pts)* Sketch $y[n] = \sum_{m=-1}^{1}(m+1)z[n-3m]$. Remember to label your axes.

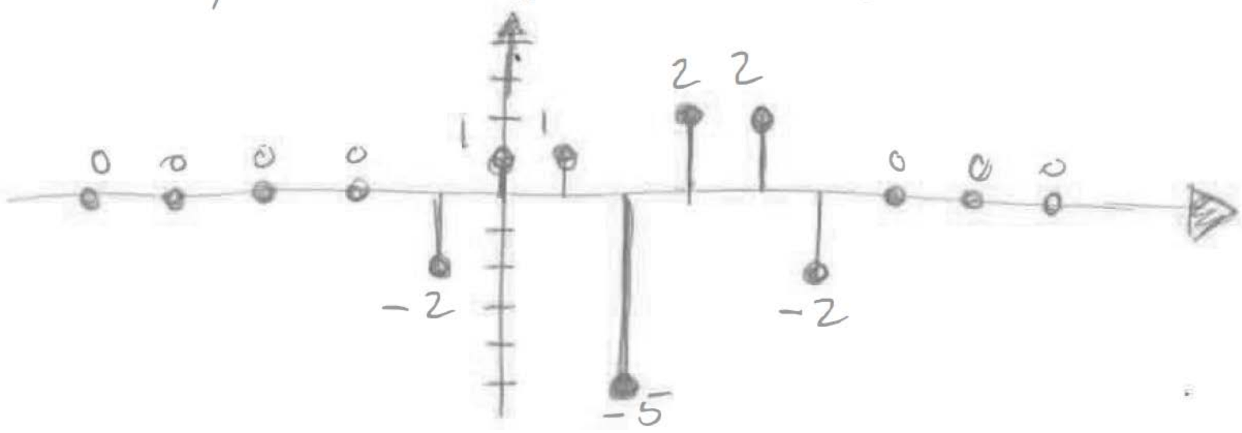
Example Exam Question

Question #2: Let the discrete-time signal $z[n]$ be shown below



(b) *(6 pts)* Sketch $y[n] = \sum_{m=-1}^{1}(m+1)z[n-3m]$. Remember to label your axes.

$y[n] = (1)z[n] + (2)z[n-3]$



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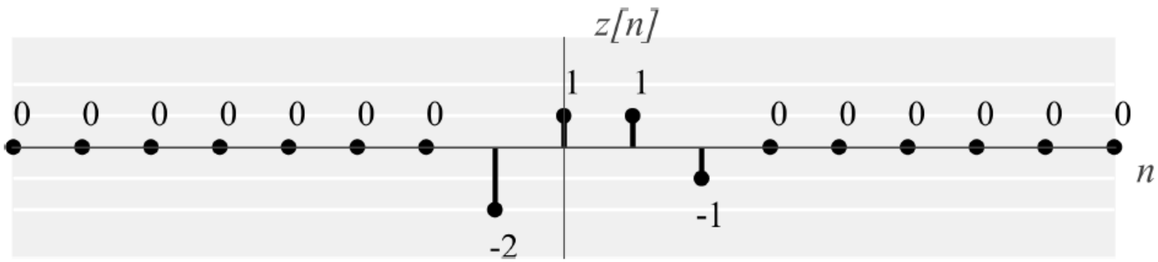

(c) *(5 pts)* Is $z[n]$ an energy signal, a power signal, or neither? If $z[n]$ is an energy signal, compute its energy. If $z[n]$ is a power signal, compute its power. If $z[n]$ is neither, explain why.

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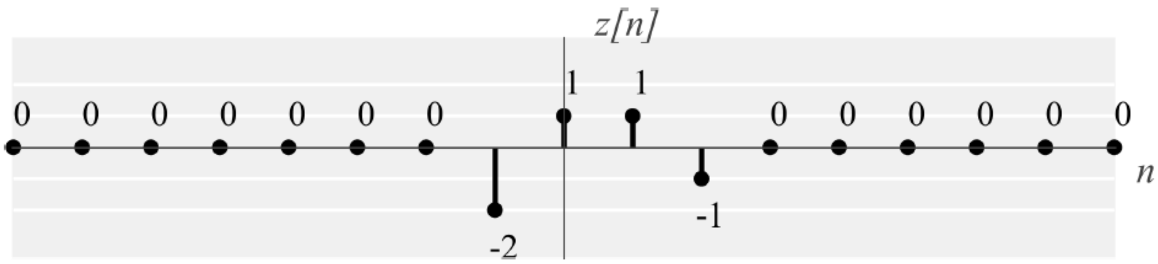

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z[n] is an energy signal

$$E_z = (-2)^2 + (1)^2 + (1)^2 + (-1)^2 = 4 + 1 + 1 + 1$$

$$\boxed{E_z = 7}$$

Example Exam Question

Question #3: Consider the continuous-time system

$$y[n] = \sum_{k=-2}^{\infty} (k)^{-n} x[n-k]$$

(a) *(5 pts)* Is this system linear? **Justify why.**

Example Exam Question

Question #3: Consider the continuous-time system

$$y[n] = \sum_{k=-2}^{\infty} (k)^{-n} x[n-k]$$

(a) *(5 pts)* Is this system linear? **Justify why.**

H{a x_1[n] + b x_2[n]} = $\text{sum}_{k=-2}^{\text{inf}}$ $(k)^{-n}$ [a x_1[n-k] + b x_2[n-k]]

a H{x_1[n]} + b H{x_2[n]} = a $\text{sum}_{k=-2}^{\text{inf}}$ $(k)^{-n}$ x_1[n-k] + b $\text{sum}_{k=-2}^{\text{inf}}$ $(k)^{-n}$ x_2[n-k]

= $\text{sum}_{k=-2}^{\text{inf}}$ $(k)^{-n}$ [a x_1[n-k] + b x_2[n-k]]

The results are the same so the system is **linear.**

Example Exam Question

Question #3: Consider the continuous-time system

$$y[n] = \sum_{k=-2}^{\infty} (k)^{-n} x[n-k]$$

(b) *(5 pts)* Is this system time-invariant? **Justify why.**

Example Exam Question

Question #3: Consider the continuous-time system

$$y[n] = \sum_{k=-2}^{\infty} (k)^{-n} x[n-k]$$

(b) *(5 pts)* Is this system time-invariant? **Justify why.**

$H\{x[n-N]\} = \mathrm{sum}_{k=-2}^{\mathrm{inf}} (k)^{-n}\, x[n-N-k]$

$y[n-N] = \mathrm{sum}_{k=-2}^{\mathrm{inf}} (k)^{-[n-N]}\, x[n-N-k]$

The results are not the same so the system is **not time-invariant.**

Example Exam Question

Question #3: Consider the continuous-time system

$$y[n] = \sum_{k=-2}^{\infty} (k)^{-n} x[n-k]$$

(c) *(4 pts)* Is this system causal? **Justify why.**

Example Exam Question

Question #3: Consider the continuous-time system

$$y[n] = \sum_{k=-2}^{\infty} (k)^{-n} x[n-k]$$

(c) *(4 pts)* Is this system causal? **Justify why.**

No. When k = -2, we use input x[n+2], a future input.

Example Exam Question

Question #3: Consider the continuous-time system

$$y[n] = \sum_{k=-2}^{\infty} (k)^{-n} x[n-k]$$

(d) *(4 pts)* Is this system memoryless? **Justify why.**

Example Exam Question

Question #3: Consider the continuous-time system

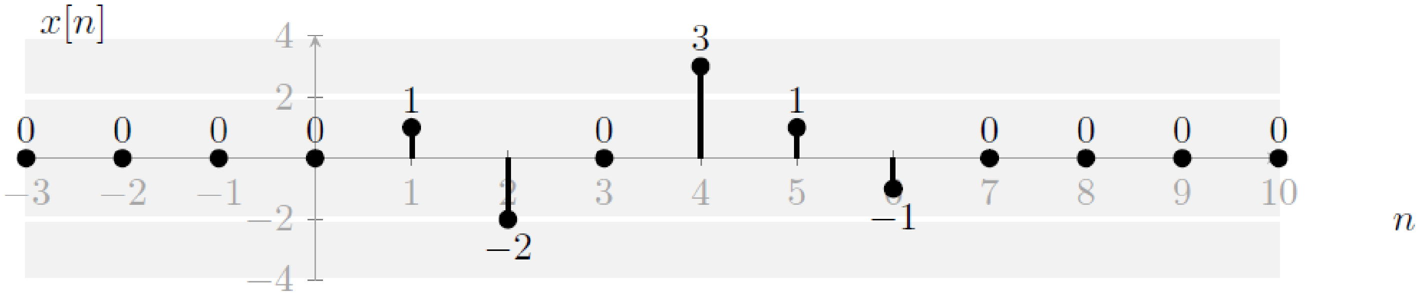
$$y[n] = \sum_{k=-2}^{\infty} (k)^{-n} x[n-k]$$

(d) *(4 pts)* Is this system memoryless? **Justify why.**

No. When k = 2, we use input x[n-2], a past input (i.e., we need memory).

Example Exam Question

Question #6: Consider discrete-time signal $x[n]$ defined below (additional values are zero)

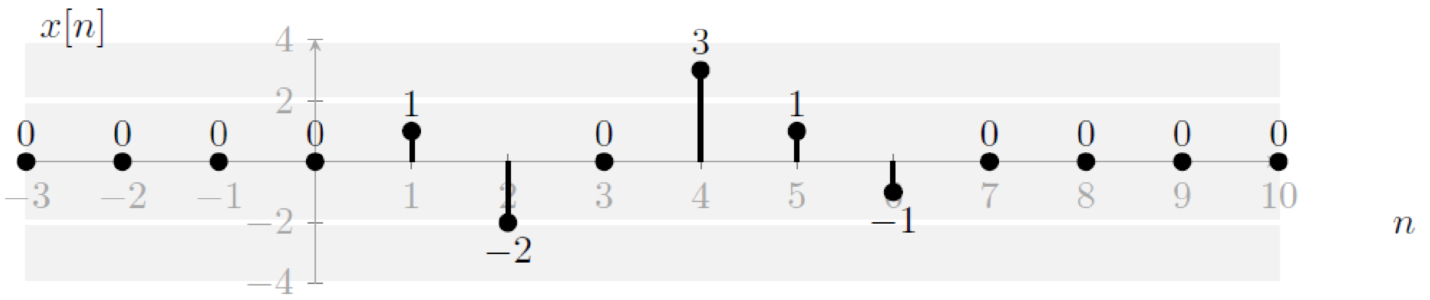


(a) *(9 pts)* Sketch $y[n] = x[n] * h[n]$ for the impulse response

$$h[n] = \delta[n-4] + \delta[n+2]$$

Example Exam Question

Question #6: Consider discrete-time signal $x[n]$ defined below (additional values are zero)



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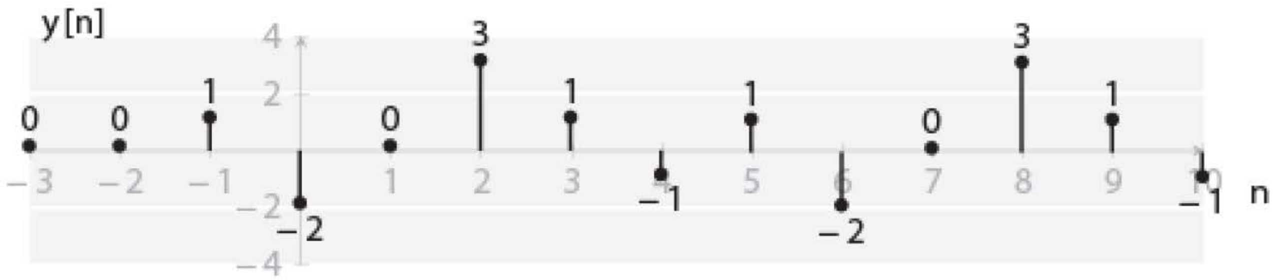
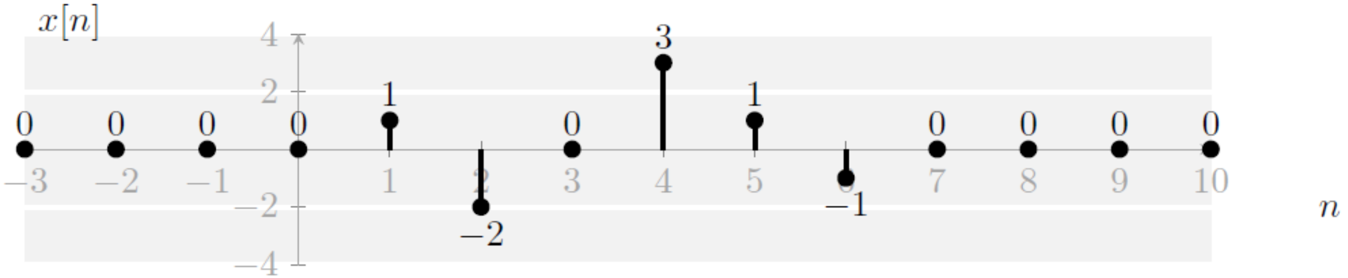


Example Exam Question

Signal repeated for convenience.



(b) *(9 pts)* Sketch $y[n] = x[n] * h[n]$ for impulse response

$$h[n] = \sum_{k=-1}^{1} (-2)^k \delta(n-k)$$

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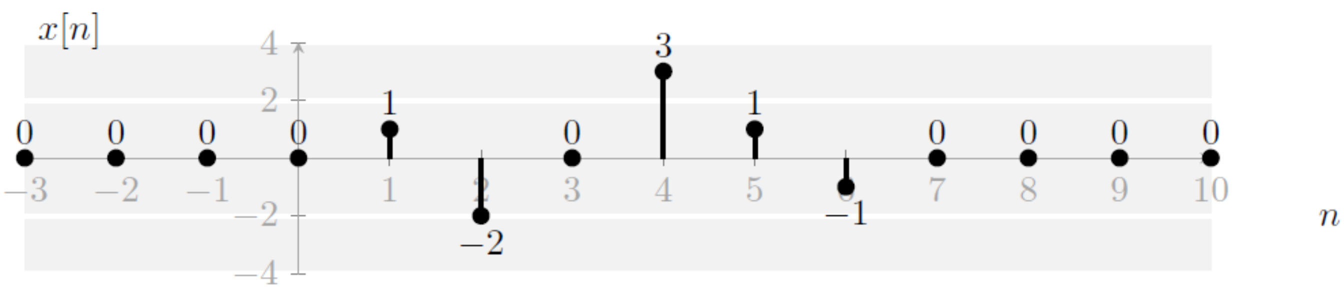

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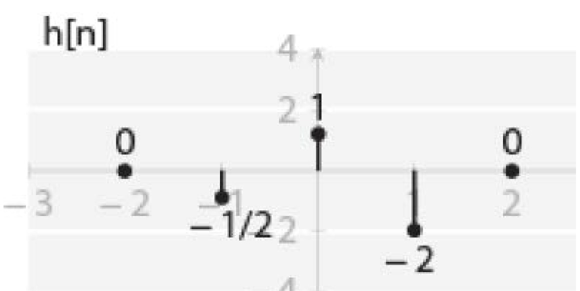

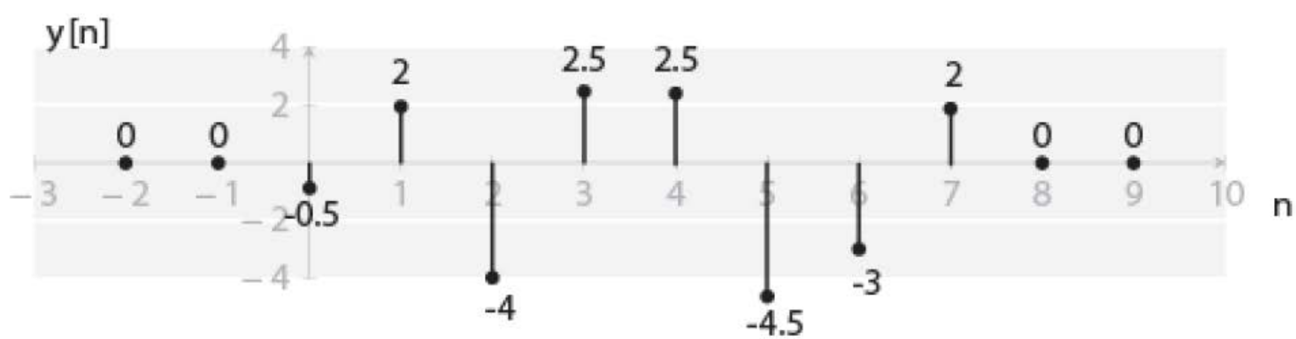

Example Exam Question

Question #2: Answer the following questions.

(a) *(5 pts)* Compute the DTFT of

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$$x[n] = (-2)(-0.5)(-0.5)^n u[n-1]$$

$$= (-0.5)(-0.5)^{n-1} u[n-1]$$

$$\boxed{X(\Omega) = \frac{-0.5}{1+0.5e^{-j\Omega}} e^{-j\Omega}}$$

Example Exam Question

(b) *(5 pts)* Compute inverse DTFT of

$$X(\Omega) = \begin{cases} 2\pi & \text{when } \Omega = k\pi \quad , \quad \text{where } k \text{ is an integer (i.e., } \ldots\text{,-2,-1,0,1,2,}\ldots\text{)} \\ 0 & \text{otherwise.} \end{cases}$$

Example Exam Question

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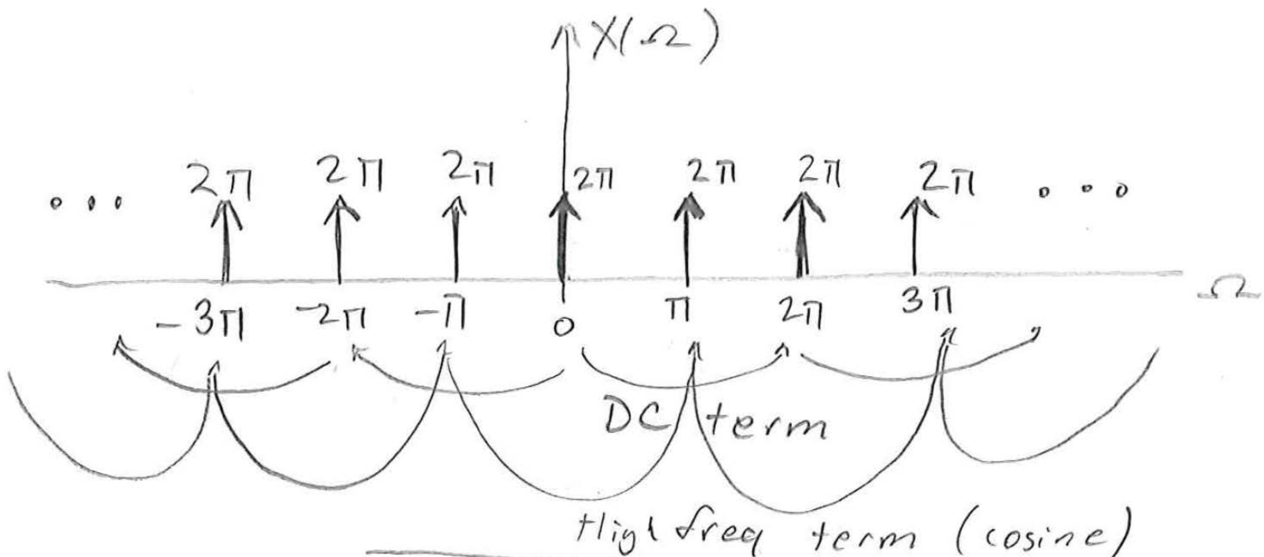


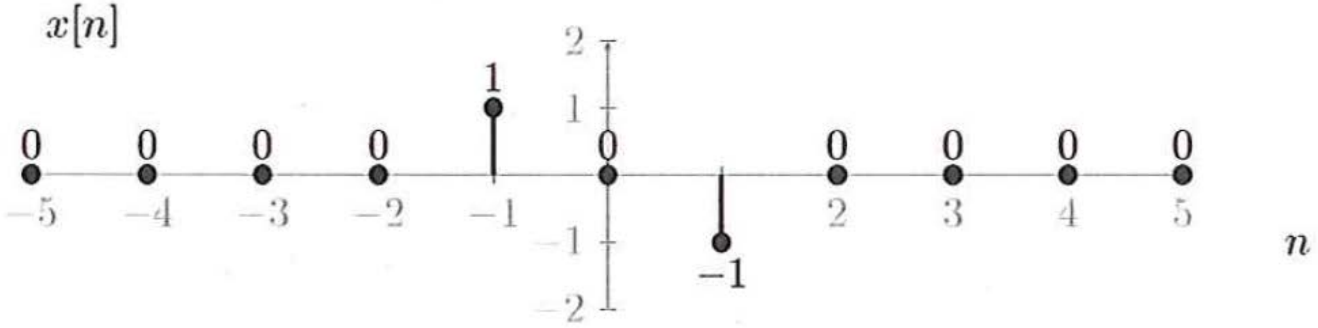
$$\boxed{x[n] = 1 + \cos(\pi n)}$$

3

$$\cos(\pi n) \overset{DTFT}{\longleftrightarrow} \pi \sum_{k=-\infty}^{\infty} [\delta(\Omega - \pi - 2\pi k) + \delta(\Omega + \pi - 2\pi k)]$$

Example Exam Question

Now consider the signal $x[n]$ below.



(c) *(5 pts)* Determine the DTFT of

$$z[n] = x[n] * x[-n]$$

Example Exam Question

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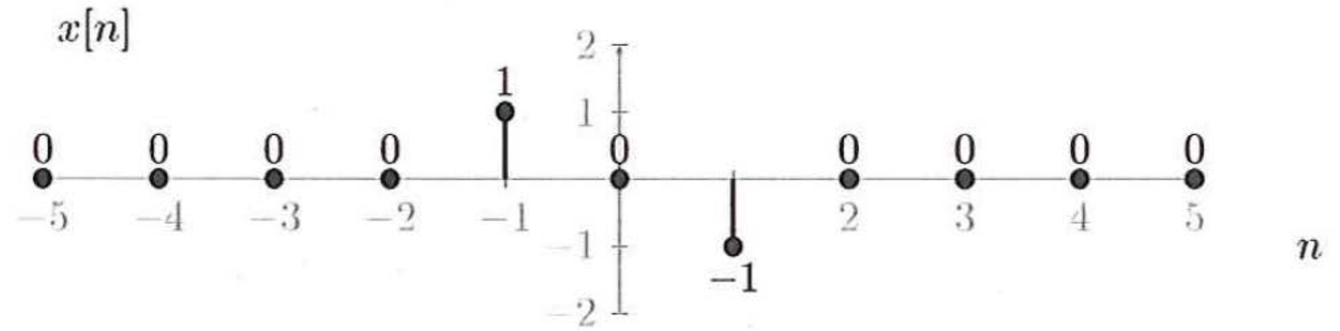

(c) *(5 pts)* Determine the DTFT of

$$x[n] = \delta[n+1] - \delta[n-1]$$

$$z[n] = x[n] * x[-n]$$

Time Reversal Prop.

$$X(\Omega) = \left[e^{+j\Omega} - e^{-j\Omega}\right]\left[e^{-j\Omega} - e^{+j\Omega}\right]$$

$$X(\Omega) = 1 - e^{+2j\Omega} - e^{-2j\Omega} + 1$$

$$\boxed{X(\Omega) = -e^{+2j\Omega} + 2 - e^{-2j\Omega}}$$

Example Exam Question

(b) *(6 pts)* Determine the transfer function $H(z)$ for the given difference equation

$$y[n] = y[n-1] + 2y[n-2] + 3y[n-3] + x[n-100]$$

Example Exam Question

(b) *(6 pts)* Determine the transfer function $H(z)$ for the given difference equation

$$y[n] = y[n-1] + 2y[n-2] + 3y[n-3] + x[n-100]$$

$$Y(z)[1 - z^{-1} - 2z^{-2} - 3z^{-3}] = X(z)z^{-100}$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{z^{-100}}{1 - z^{-1} - 2z^{-2} - 3z^{-3}}$$

Example Exam Question

(c) *(7 pts)* Compute the impulse response $h[n]$ for the given difference equation

$$2y[n-5] = y[n-6] + 5x[n-7]$$

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$$2y[n-5] = y[n-6] + 5x[n-7]$$

$$Y(z)[2z^{-5} - z^{-6}] = 5X(z)z^{-7}$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{5z^{-7}}{2z^{-5} - z^{-6}} = \frac{5z^{-2}}{2 - z^{-1}} = \frac{(5/2)z^{-2}}{1 - (1/2)z^{-1}}$$

$$h[n] = \left(\frac{5}{2}\right)\left(\frac{1}{2}\right)^{n-2} u[n-2]$$

Example Exam Question

Question #6: Consider the z-transforms $H_1(z)$ and $H_2(z)$ below.

$$H_1(z) = \frac{5}{1-(1/2)z^{-1}} + \frac{1}{(1/4)-z^{-1}} \quad , \quad H_2(z) = \frac{z^2-4}{(z-4)^2+16}$$

(a) *(7 pts)* Compute the inverse z-transform of the $H_1(z)$ such that the system is **causal**. Is the system **stable**?

Example Exam Question

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(a) *(7 pts)* Compute the inverse z-transform of the $H_1(z)$ such that the system is **causal**. Is the system **stable**?

$$\begin{aligned} H(z) &= \frac{5}{1-(1/2)z^{-1}} + \frac{4}{1-4z^{-1}} \\ h[n] &= 5(1/2)^n u[n] + 4(4)^n u[n] \end{aligned}$$

The system is **not stable** since one pole is located at $z = 4$ $(4 > 1)$ and the system is causal.

Example Exam Question

Question #6: Consider the z-transforms $H_1(z)$ and $H_2(z)$ below.

$$H_1(z) = \frac{5}{1 - (1/2)z^{-1}} + \frac{1}{(1/4) - z^{-1}} \quad , \quad H_2(z) = \frac{z^2 - 4}{(z-4)^2 + 16}$$

(b) *(7 pts)* Sketch the pole-zero plot and the region-of-convergence for $H_2(z)$. Assume $H_2(z)$ is an **anti-causal** system. Is the system **stable**?

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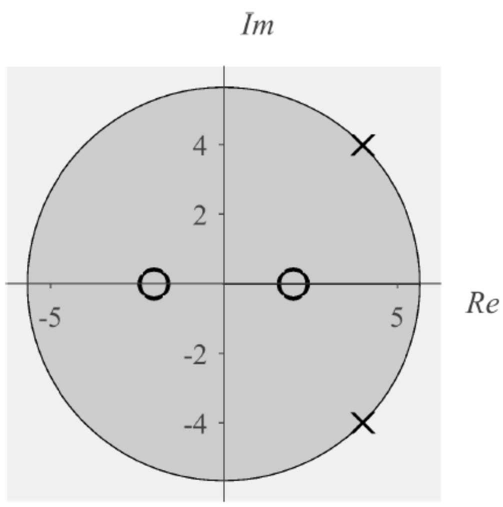

poles: z = 4j + 4, -4j + 4

zeros: z = +2, -2

The system **is stable** since there the ROC contains the unit circle.