Full Name: Solutions **ExamID: 010001**

EEL 4750 / EEE 5502 (Fall 2018) – Practice Exam #02 **Date: Oct. 30, 2018**

Question	# of Points Possible	# of Points Obtained	Grader
# 1	18		
# 2	17		
# 3	17		
# 4	16		
# 5	16		
# 6	16		
Total	100		

For full credit when sketching: remember to label axes and make locations and amplitudes clear.

Before starting the exam, read and sign the following agreement.

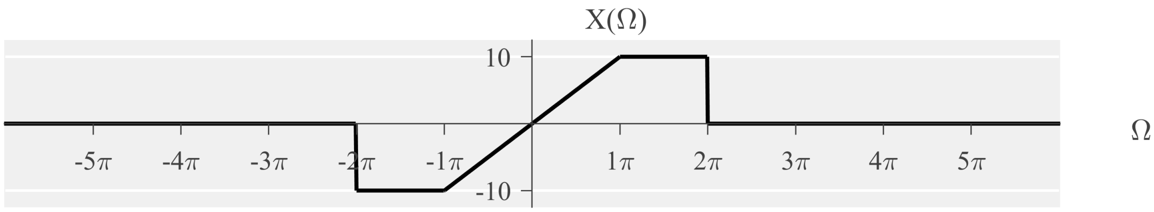
By signing this agreement, I agree to solve the problems of this exam while adhering to the policies and guidelines of the University of Florida and EEL 4750 / EEE 5502 and without additional external help. The guidelines include, but are not limited to,

- The University of Florida honor pledge: "On my honor, I have neither given nor received unauthorized aid in doing this assignment."
- Only one 8.5 by 11 inch cheat sheet (double-sided) may be used
- No calculators or computers may be used
- No textbooks or additional notes may be used
- No collaboration is allowed
- No cheating is allowed

Student

Date

Question #1: Consider the Fourier Transform of a continuous-time signal $x(t)$ shown below.



(a) *(4 pts)* Determine the Nyquist sampling rate for $z(t) = 2x(t)\cos(12\pi t)$.

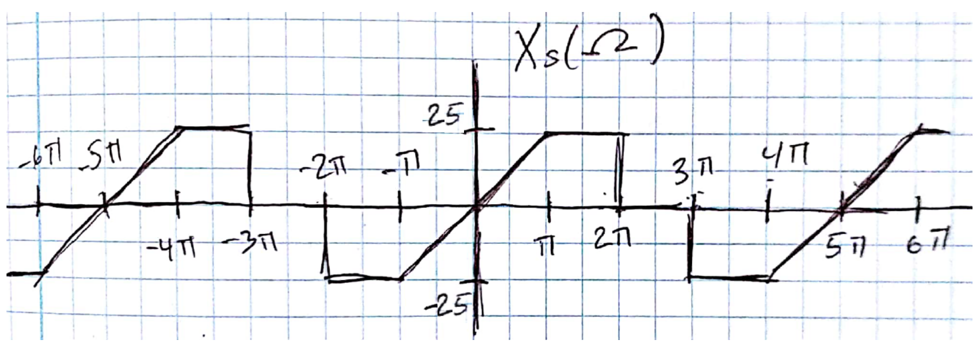
The Nyquist rule for $x(t)$ is

$\Omega_s > 4\pi$ since $\Omega_{max} = 2\pi$

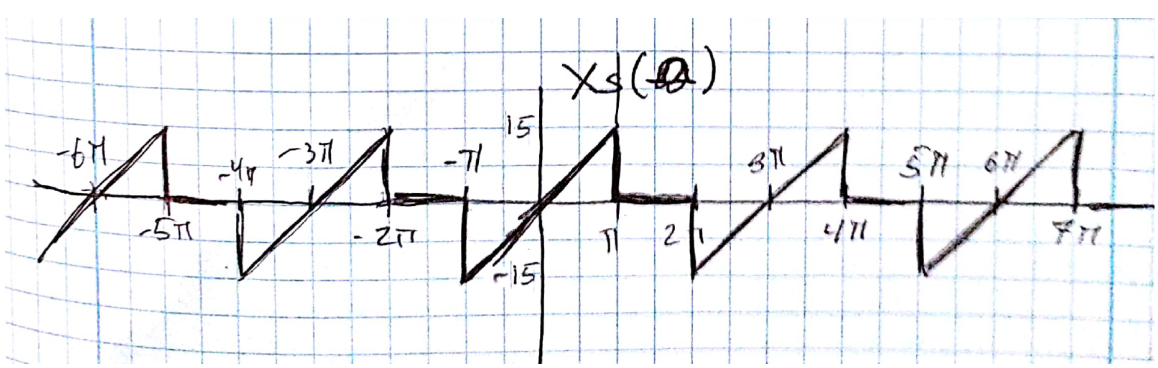
The $\cos(12\pi t)$ shifts the frequency by 12π, so the new Nyquist rate for $z(t)$ is

$\boxed{\Omega_s > 28\pi}$

(b) *(6 pts)* Sketch (for $\Omega = -6\pi$ to $\Omega = +6\pi$) the Fourier transform $X_s(\Omega)$ of the sampled $X(\Omega)$ with a sampling rate of $\Omega_s = 5\pi$. **Do we experience aliasing?**



(c) *(6 pts)* Sketch (for $\Omega = -6\pi$ to $\Omega = +6\pi$) the Fourier transform $X_s(\Omega)$ of the sampled $X(\Omega)$ with a sampling rate of $\Omega_s = 3\pi$. **Do we experience aliasing?**



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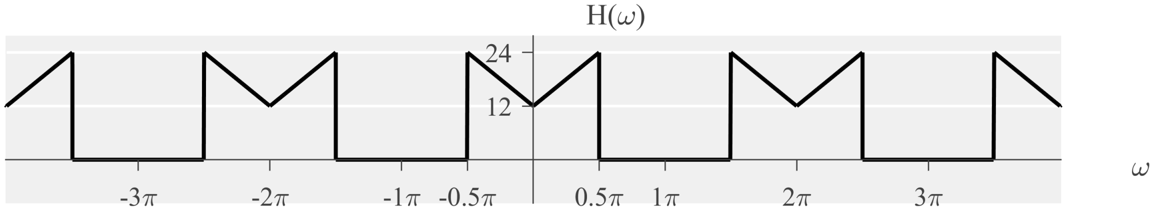
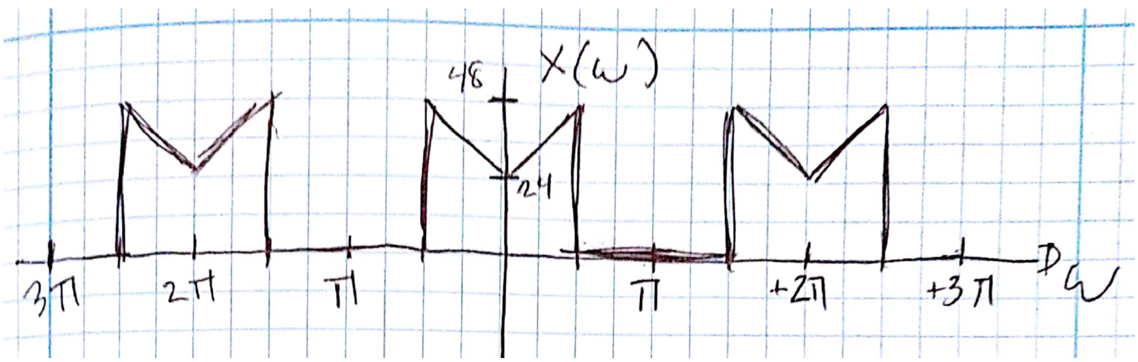
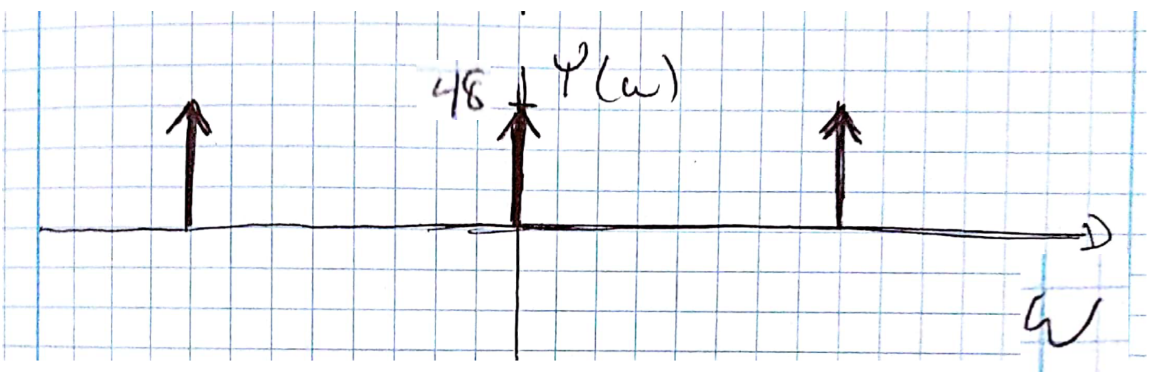
Question #2: Consider the impulse response $h[n]$ below with frequency response $H(\omega)$.



(a) *(6 pts)* Sketch (for $-3\pi < \omega < 3\pi$) the magnitude of the DTFT of $x[n] = -2h[n-10]$

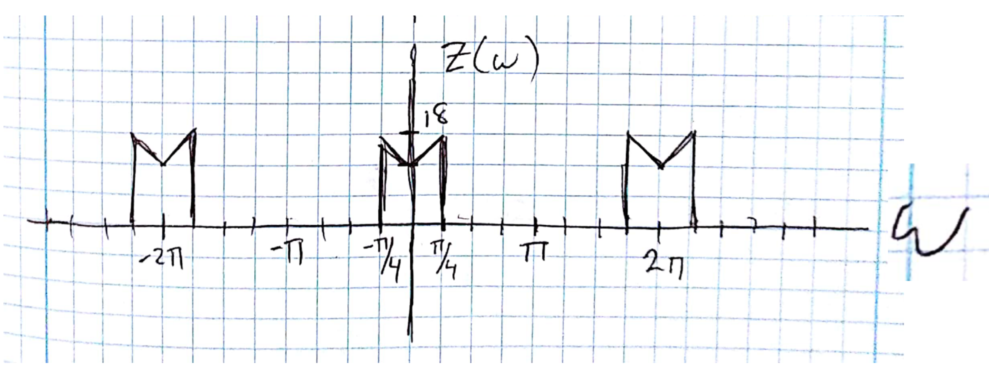


(b) *(5 pts)* Sketch (for $-3\pi < \omega < 3\pi$) the magnitude of the DTFT of $y[n] = h[n] * (2/\pi)$

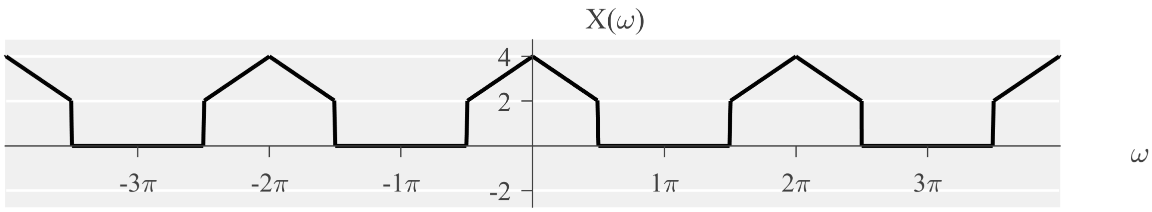


(c) *(6 pts)* Sketch (for $-3\pi < \omega < 3\pi$) the magnitude of the DTFT of

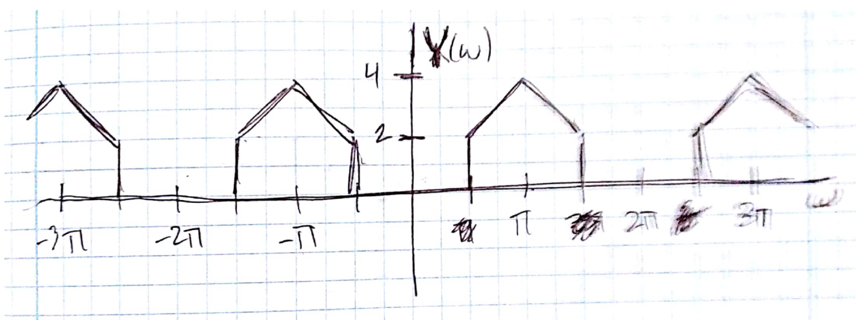
$$Z(\omega) = H(\omega)\left[\sum_{k=-\infty}^{\infty} u(\omega + \pi/4 + 2\pi k) - u(\omega - \pi/4 + 2\pi k)\right]$$



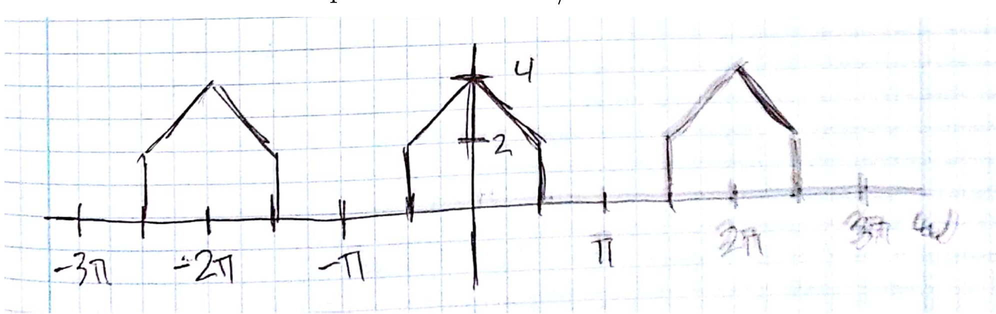
Question #3: Consider the DTFT signal $X(\omega)$ shown below.



(a) *(5 pts)* Sketch the DTFT (from $\omega = -3\pi$ to $\omega = 3\pi$) of $y[n] = x[n]\cos(\pi n)$. Remember to label important locations / values.

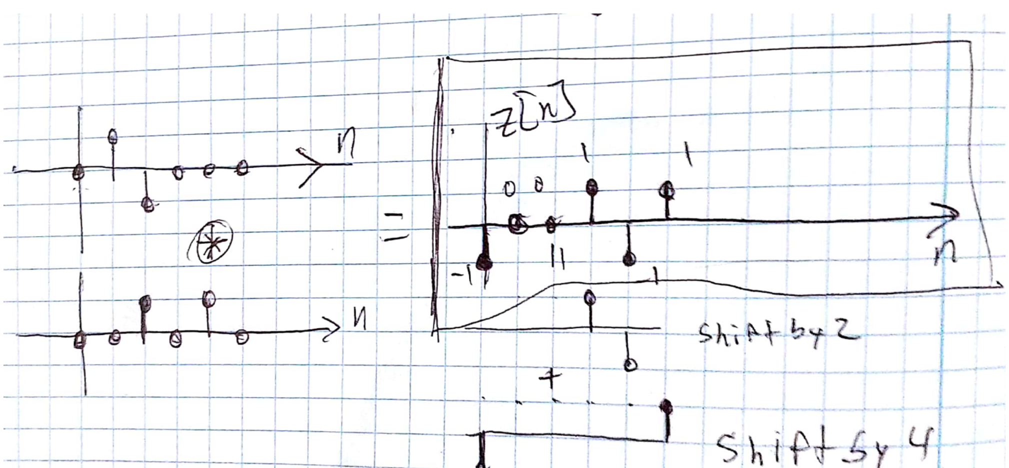


(b) *(5 pts)* Sketch the DTFT (from $\omega = -3\pi$ to $\omega = 3\pi$) of $y[n] = x[n]\cos(10\pi n)$. Remember to label important locations / values.



(c) *(7 pts)* Sketch the result of the length-6 discrete-time circular convolution defined by

$$z[n] = [\delta[n-2] + \delta[n-4]] \circledast [\delta[n-1] - \delta[n-2]] \ .$$



Question #4: Consider the z-transform causal transfer function $H(z)$ defined by

$$H(z) = z^{-1} + 2z^{-3} + z^{-5}$$

(a) *(6 pts)* Compute the phase response for $\angle H(\omega)$.

$$H(z) = z^{-1} + 2z^{-3} + z^{-5}$$

$$= z^{-3}\underbrace{\left[z^{+2} + 2 + z^{-2}\right]}_{\text{Even symmetry} \Rightarrow \text{phase} = 0}$$

$$H(\omega) = e^{-j3\omega}\left[e^{+j2\omega} + 2 + e^{-j2\omega}\right]$$

$$\boxed{\angle H(\omega) = -3\omega}$$

(b) *(5 pts)* Compute the phase response for the DTFT of $v[n] = h[n] * h[-n]$.

$$V(z) = H(z)H(z^{-1})$$

$$= \left[z^{-1} + 2z^{-3} + z^{-5}\right]\left[z + 2z^{3} + z^{5}\right]$$

$$= 1 + 2z^{2} + z^{4} + 2z^{-2} + 4 + 2z^{2} + z^{-4} + 2z^{-2} + 1$$

$$= 6 + 4z^{2} + z^{4} + 4z^{-2} + z^{-4}$$

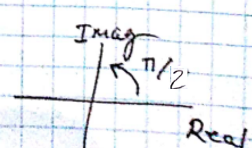
$$= \underbrace{z^{4} + 4z^{2} + 6 + 4z^{-2} + z^{-4}}_{\text{Even symmetry} \Rightarrow \text{phase} = 0}$$

$$\boxed{\angle V(\omega) = 0}$$

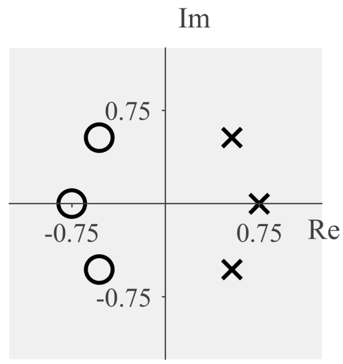
(c) *(5 pts)* Compute the phase response for $G(\omega) = j$.

$$G(\omega) = j = e^{j\pi/2}$$

$$\boxed{\angle G(\omega) = \pi/2}$$



Question #5: Consider the following pole-zero plot, representing a causal LTI system.



(a) *(4 pts)* Is this a low pass filter, bandpass filter, high pass filter, all-pass filter, or none-of-the above? **Explain why.**

Low pass filter since poles are near $\omega = 0$ and zeros are near $\omega = \pi$.

(b) *(4 pts)* Is the system stable? **Explain why.**

Yes, the system is stable since all poles are within the unit circle.

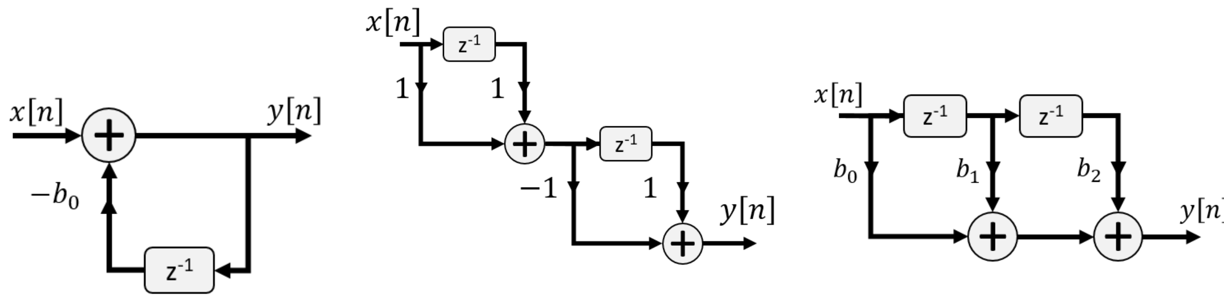
(c) *(4 pts)* Is this a minimum phase system? **Explain why.**

Yes, the system is minimum phase since all poles and zeros are within the unit circle.

(d) *(4 pts)* Is this a linear phase system? **Explain why.**

No, the system is not linear phase since the poles and zeros are not symmetric around the unit circle.

Question #6: *(10 pts)* Consider the IIR all-pole direct form (left), FIR cascade form (center), and FIR direct form (right) implementations below.



(a) *(5 pts)* Write the difference equation for the IIR all-pole direct form (left).

$$y[n] = -b_0 y[n-1] + x[n]$$

$$y[n] + b_0 y[n-1] = x[n]$$

(b) *(5 pts)* Write the z-transform transfer function for to the FIR cascade form (center).

$$H(z) = (1 + z^{-1})(-1 + z^{-1})$$

$$= (z^{-1} + 1)(z^{-1} - 1)$$

(c) *(6 pts)* Determine the weights b_0, b_1, b_2 for the FIR direct form (right) transfer function

$$H(z) = [1 - (1/2)z^{-1}][1 - (1/4)z^{-1}]$$

$$= 1 - (3/4)z^{-1} + \left(\frac{1}{8}\right) z^{-2}$$

$$\boxed{b_0 = 1,\ b_1 = -3/4,\ b_2 = 1/8}$$

Table of Discrete-Time Fourier Transform Pairs:

$$\text{Discrete-Time Fourier Transform} \quad : \quad X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$\text{Inverse Discrete-Time Fourier Transform} \quad : \quad x[n] = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega t}\, d\omega\ .$$

$x[n]$	$X(\omega)$	condition
$a^n u[n]$	$\dfrac{1}{1-ae^{-j\omega}}$	$\lvert a\rvert < 1$
$(n+1)a^n u[n]$	$\dfrac{1}{(1-ae^{-j\omega})^2}$	$\lvert a\rvert < 1$
$\dfrac{(n+r-1)!}{n!(r-1)!}a^n u[n]$	$\dfrac{1}{(1-ae^{-j\omega})^r}$	$\lvert a\rvert < 1$
$\delta[n]$	1	
$\delta[n-n_0]$	$e^{-j\omega n_0}$	
$x[n] = 1$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$	
$u[n]$	$\dfrac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega - 2\pi k)$	
$e^{j\omega_0 n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi k)$	
$\cos(\omega_0 n)$	$\pi \sum_{k=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)\}$	
$\sin(\omega_0 n)$	$\dfrac{\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k)\}$	
$\sum_{k=-\infty}^{\infty} \delta[n-kN]$	$\dfrac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \dfrac{2\pi k}{N}\right)$	
$x[n] = \begin{cases} 1 & , \quad \lvert n\rvert \le N \\ 0 & , \quad \lvert n\rvert > N \end{cases}$	$\dfrac{\sin(\omega(N+1/2))}{\sin(\omega/2)}$	
$\dfrac{\sin(Wn)}{\pi n} = \dfrac{W}{\pi}\mathrm{sinc}\left(\dfrac{Wn}{\pi}\right)$	$X(\omega) = \begin{cases} 1 & , \quad 0 \le \lvert\omega\rvert \le W \\ 0 & , \quad W < \lvert\omega\rvert \le \pi \end{cases}$	

$X(\omega)$ is periodic with period 2π

Table of Discrete-Time Fourier Transform Properties: For each property, assume

$$x[n] \overset{DTFT}{\longleftrightarrow} X(\omega) \quad \text{and} \quad y[n] \overset{DTFT}{\longleftrightarrow} Y(\omega)$$

Property	Time domain	DTFT domain
Linearity	$Ax[n] + By[n]$	$AX(\omega) + BY(\omega)$
Time Shifting	$x[n - n_0]$	$X(\omega)e^{-j\omega n_0}$
Frequency Shifting	$x[n]e^{j\omega_0 n}$	$X(\omega - \omega_0)$
Conjugation	$x^*[n]$	$X^*(-\omega)$
Time Reversal	$x[-n]$	$X(-\omega)$
Convolution	$x[n] * y[n]$	$X(\omega)Y(\omega)$
Multiplication	$x[n]y[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{2\pi} X(\theta)Y(\omega - \theta)d\theta$
Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(\omega)$
Accumulation	$\sum_{k=-\infty}^{\infty} x[k]$	$\frac{1}{1-e^{-j\omega}} + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
Frequency Differentiation	$nx[n]$	$j\dfrac{dX(\omega)}{d\omega}$
Parseval's Relation for Aperiodic Signals	$\sum_{k=-\infty}^{\infty} \|x[k]\|^2$	$\frac{1}{2\pi} \int_{2\pi} \|X(\omega)\|^2 d\omega$

Table of Z-Transform Pairs:

$$\text{Z-Transform} \quad : \quad X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$\text{Inverse Z-Transform} \quad : \quad x[n] = \frac{1}{2\pi j} \oint_{\mathcal{C}} X(z) z^{n-1} \, dz \, .$$

$x[n]$	$X(\omega)$	ROC
$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\delta[n]$	1	All z
$\delta[n-n_0]$	z^{-n_0}	All z
$u[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$\cos(\omega_0 n)u[n]$	$\dfrac{1-z^{-1}\cos(\omega_0)}{1-2z^{-1}\cos(\omega_0)+z^{-2}}$	$\lvert z\rvert > 1$
$\sin(\omega_0 n)u[n]$	$\dfrac{z^{-1}\sin(\omega_0)}{1-2z^{-1}\cos(\omega_0)+z^{-2}}$	$\lvert z\rvert > 1$
$a^n\cos(\omega_0 n)u[n]$	$\dfrac{1-az^{-1}\cos(\omega_0)}{1-2az^{-1}\cos(\omega_0)+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$a^n\sin(\omega_0 n)u[n]$	$\dfrac{az^{-1}\sin(\omega_0)}{1-a2z^{-1}\cos(\omega_0)+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

Table of Z-Transform Properties: For each property, assume

$$x[n] \overset{Z}{\longleftrightarrow} X(z) \quad \text{and} \quad y[n] \overset{Z}{\longleftrightarrow} Y(z)$$

Property	Time domain	Z-domain
Linearity	$Ax[n] + By[n]$	$AX(z) + BY(z)$
Time Shifting	$x[n - n_0]$	$X(z)z^{-n_0}$
Z-scaling	$a^n x[n]$	$X(a^{-1}z)$
Conjugation	$x^*[n]$	$X^*(z^*)$
Time Reversal	$x[-n]$	$X(z^{-1})$
Convolution	$x[n] * y[n]$	$X(z)Y(z)$
Differentiation in z-domain	$nx[n]$	$-z\frac{dX(z)}{dz}$
Initial Value Theorem	$x[n]$ is causal	$x(0) = \lim_{z\to\infty} X(z)$