Lecture 4: Discrete-Time LTI Systems

Foundations of Digital Signal Processing

Outline

- Input-Output Representation Review
- Discrete-Time Convolution
- Properties of Discrete-Time Convolution
- Combining Systems
- Properties of the Impulse Response
- General Form for LTI Systems

News

- **Homework #2**
  - Due *Thursday* by 11:59 PM
  - Submit via canvas
- **Coding Assignment #1**
  - Due *Thursday* by 11:59 PM
  - Submit via canvas
    - Submit answers as a PDF
    - Submit code as .m files

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Convolution

- **Consider the system defined by the input-output relationship**

$$y[n] = \sum_{m=-\infty}^{n} x[m]$$

  - Compute the output for input $x[n] = \delta[n-1] - 3\delta[n-3]$

Convolution

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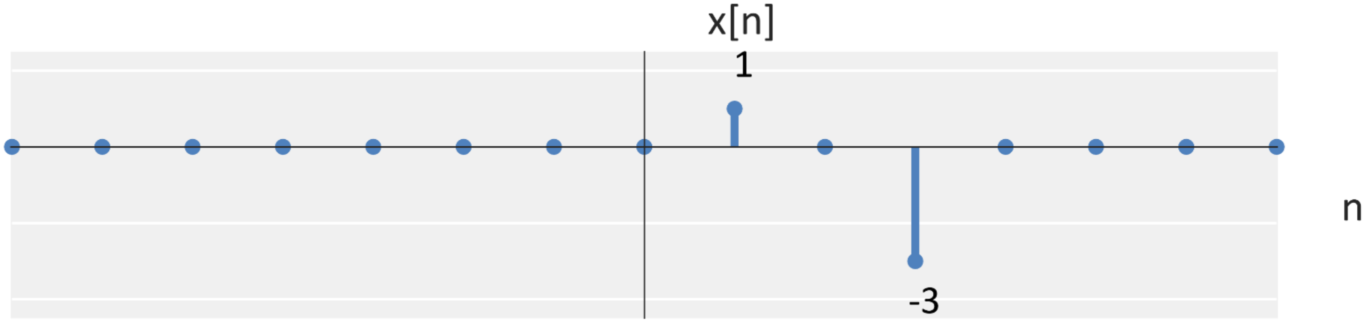

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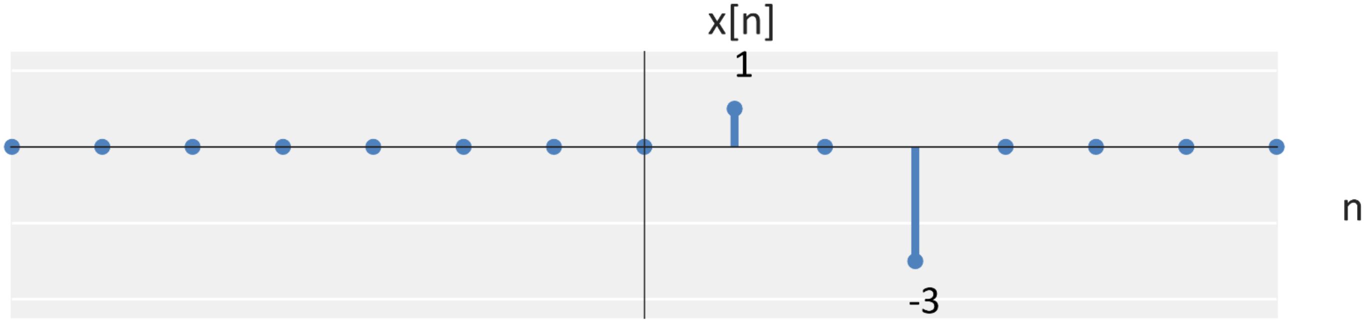

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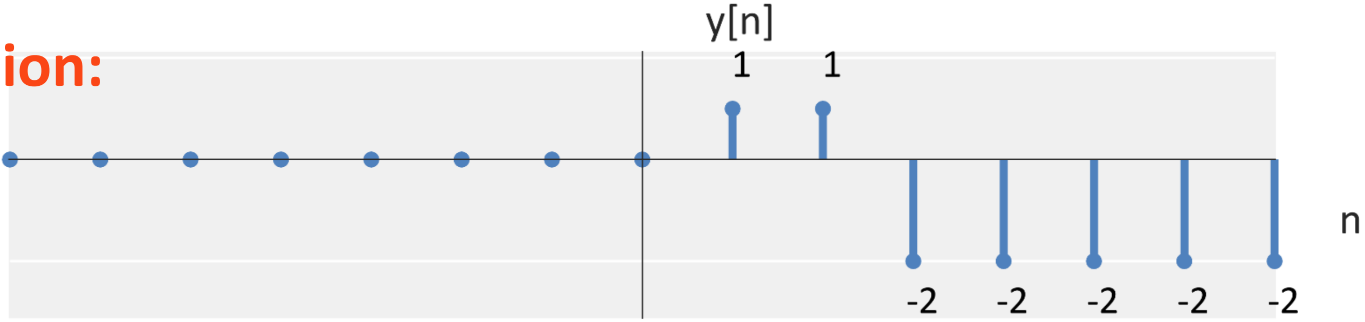

Convolution

- **Consider the system defined by the input-output relationship**

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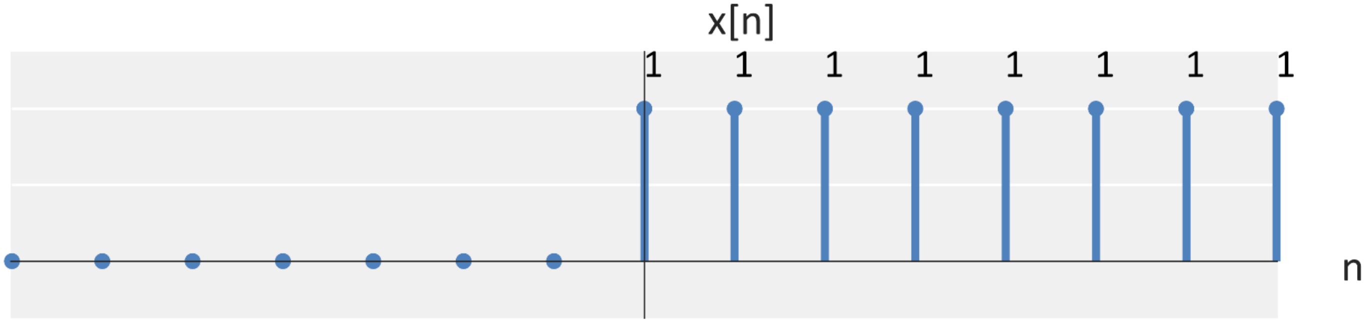
  - Compute the output for input $x[n] = u[n]$



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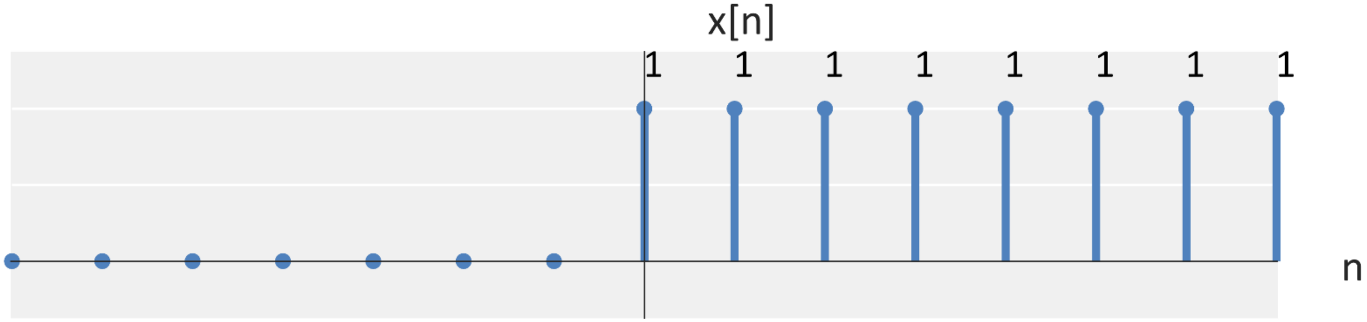

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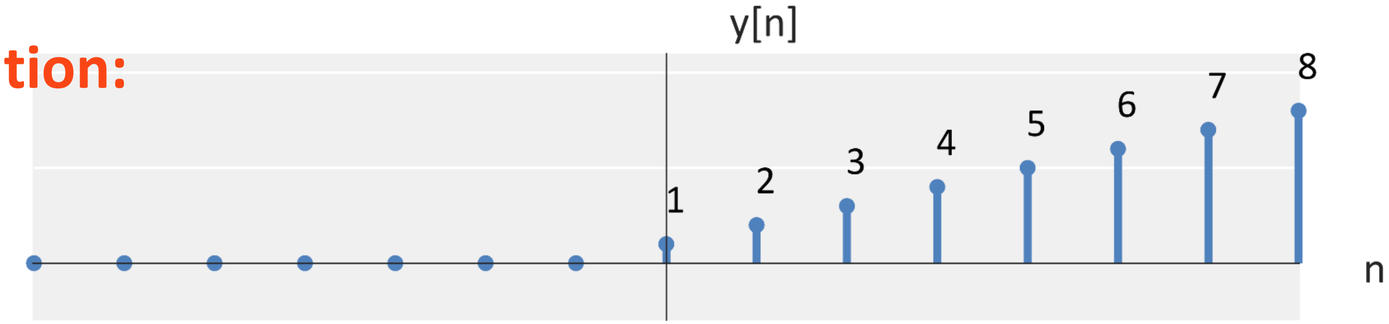

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Convolution

- **Definition of convolution**

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

Convolution

- **Definition of convolution**

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

Shift that corresponds to the final point

Domain that we multiply and sum over

Signal Operations

- **Consider this**
  - Plotting $x[3-n]$
  - We follow this procedure
    - ◊ $x[n] \underset{\substack{\uparrow \\ \textbf{Shift} \\ \underline{\textbf{left 3}}}}{\rightarrow} x[n+3] \underset{\substack{\uparrow \\ \textbf{Time} \\ \textbf{reverse}}}{\rightarrow} x[-n+3] = x[3-n]$

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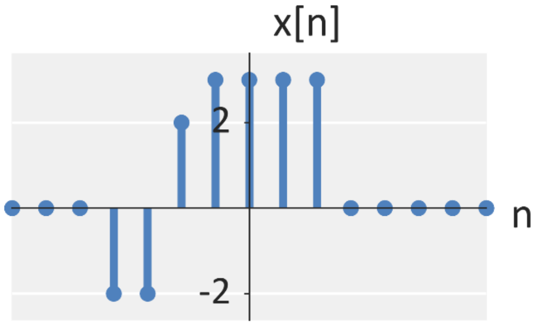
Shift left 3

Time reverse



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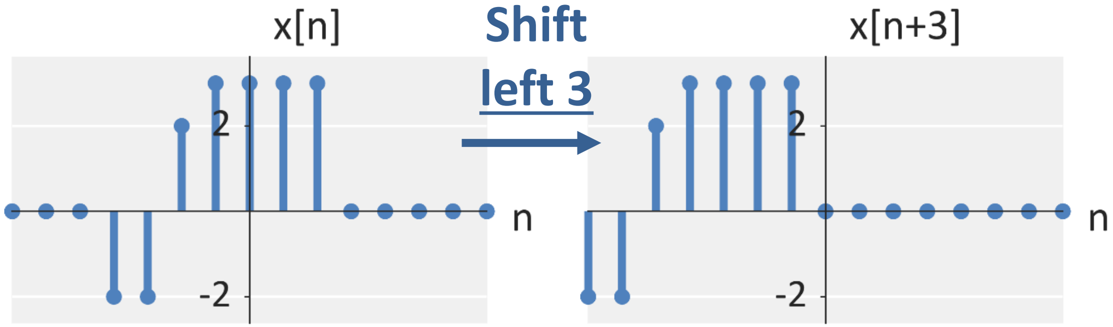

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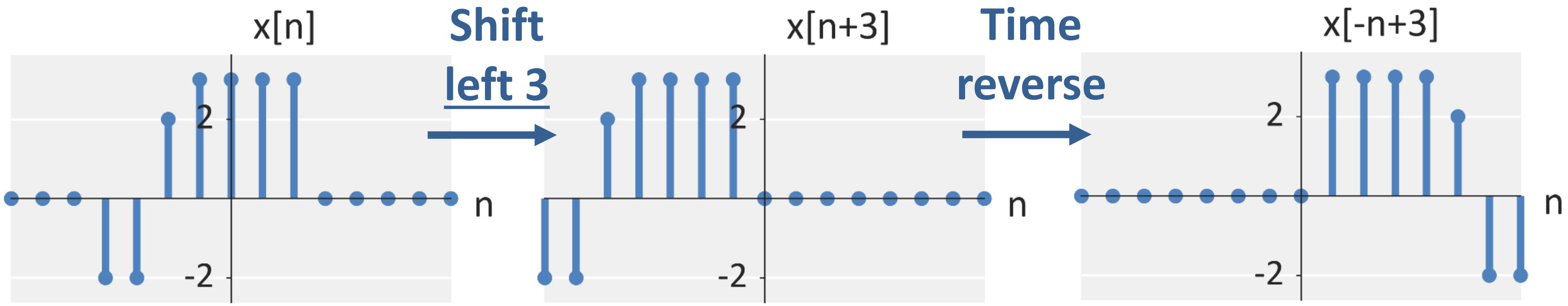

Signal Operations

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    - $$x[n] \underset{\text{Shift left 3}}{\rightarrow} x[n+3] \underset{\text{Time reverse}}{\rightarrow} x[-n+3] = x[3-n]$$
  - We can go in the other direction (used to illustrate convolution)
    - $$x[n] \underset{\text{Time reverse}}{\rightarrow} x[-n] \underset{\text{Shift right 3}}{\rightarrow} x[-(n-3)] = x[-n+3] = x[3-n]$$

Signal Operations

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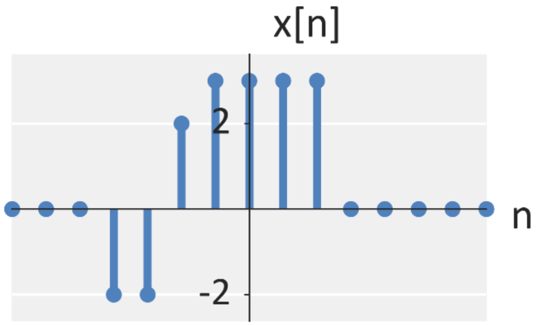

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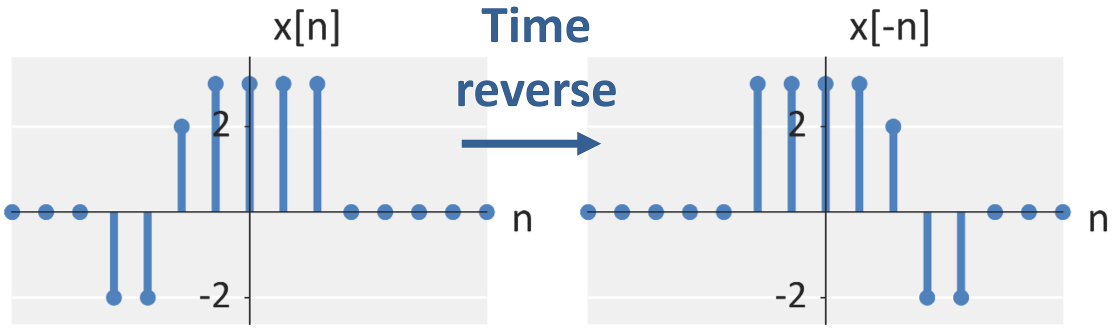

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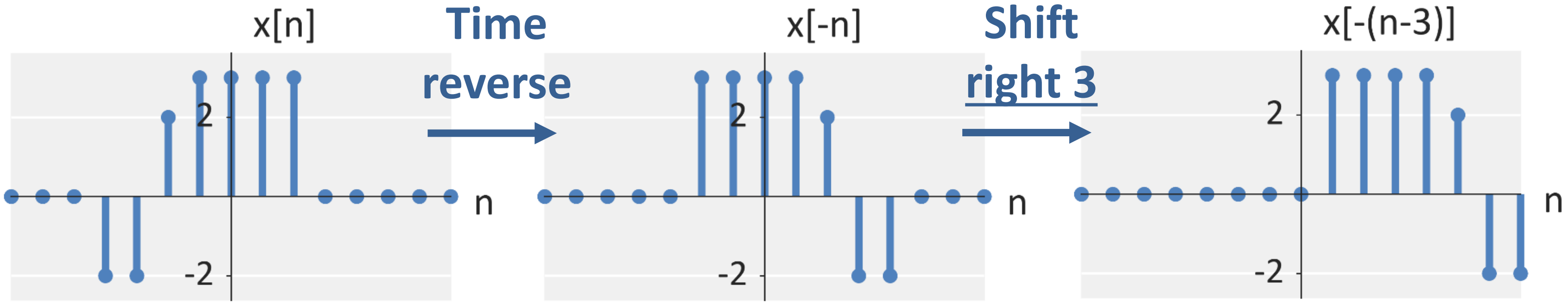

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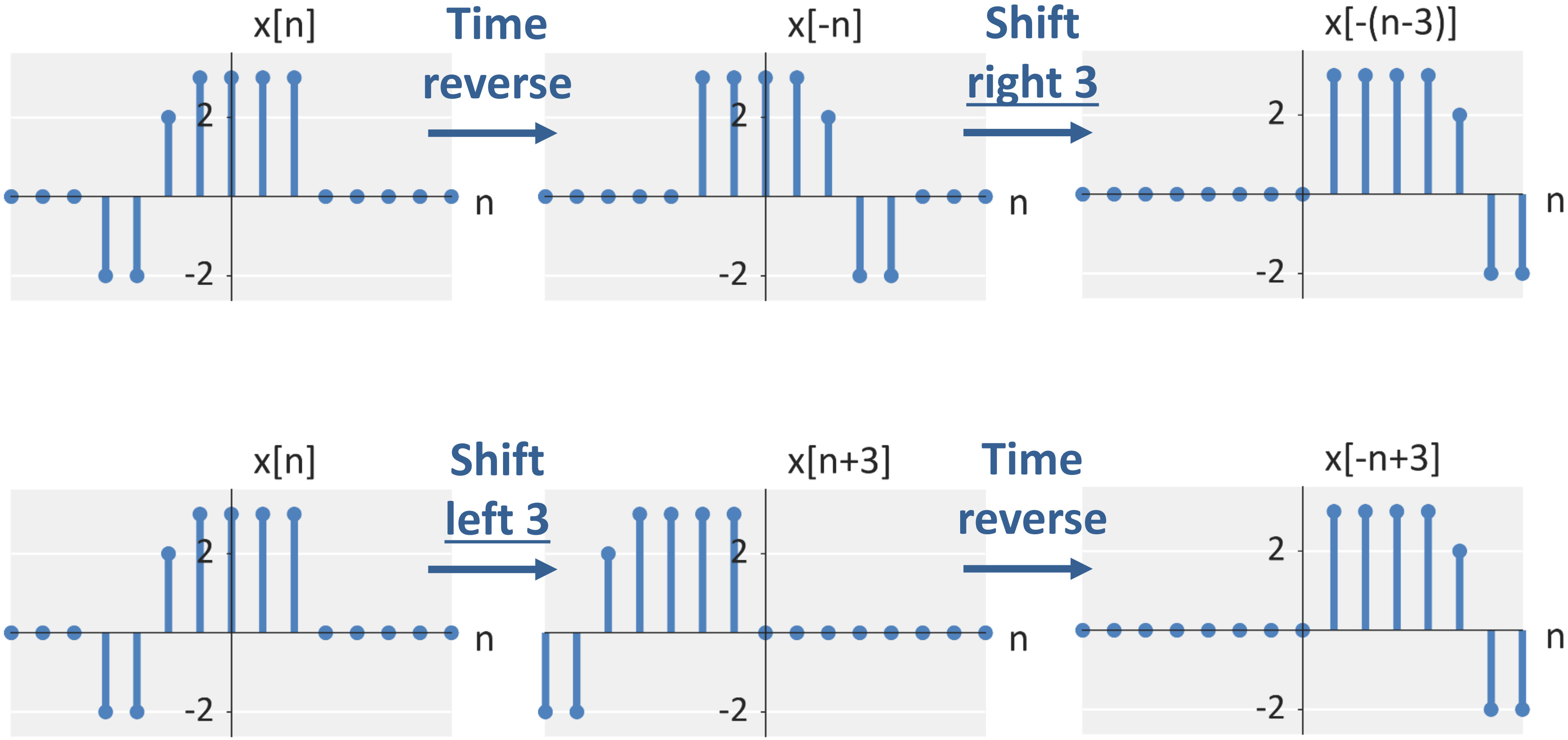


Time reverse →

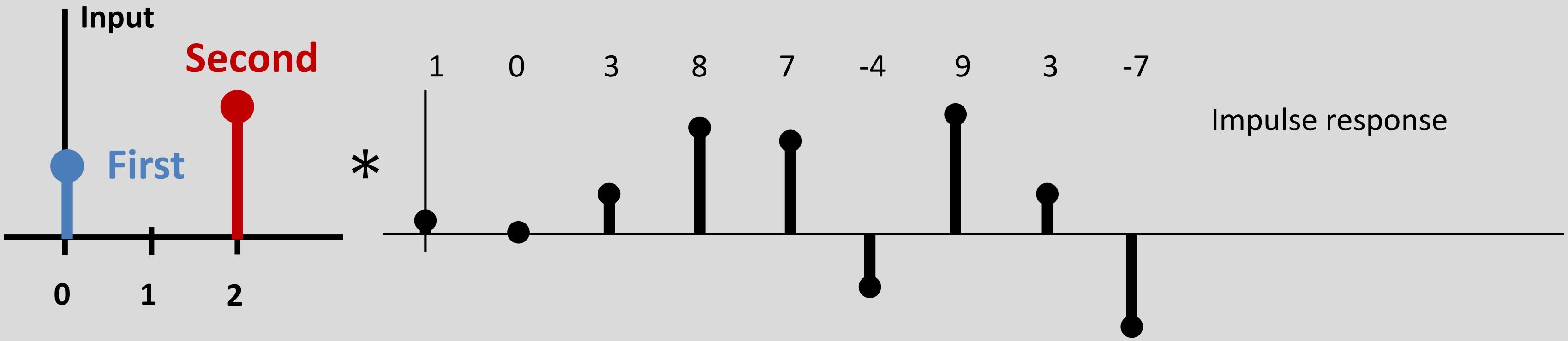
Shift right 3 →

Shift left 3 →

Time reverse →

Convolution Illustration

- **Convolution:** $x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$



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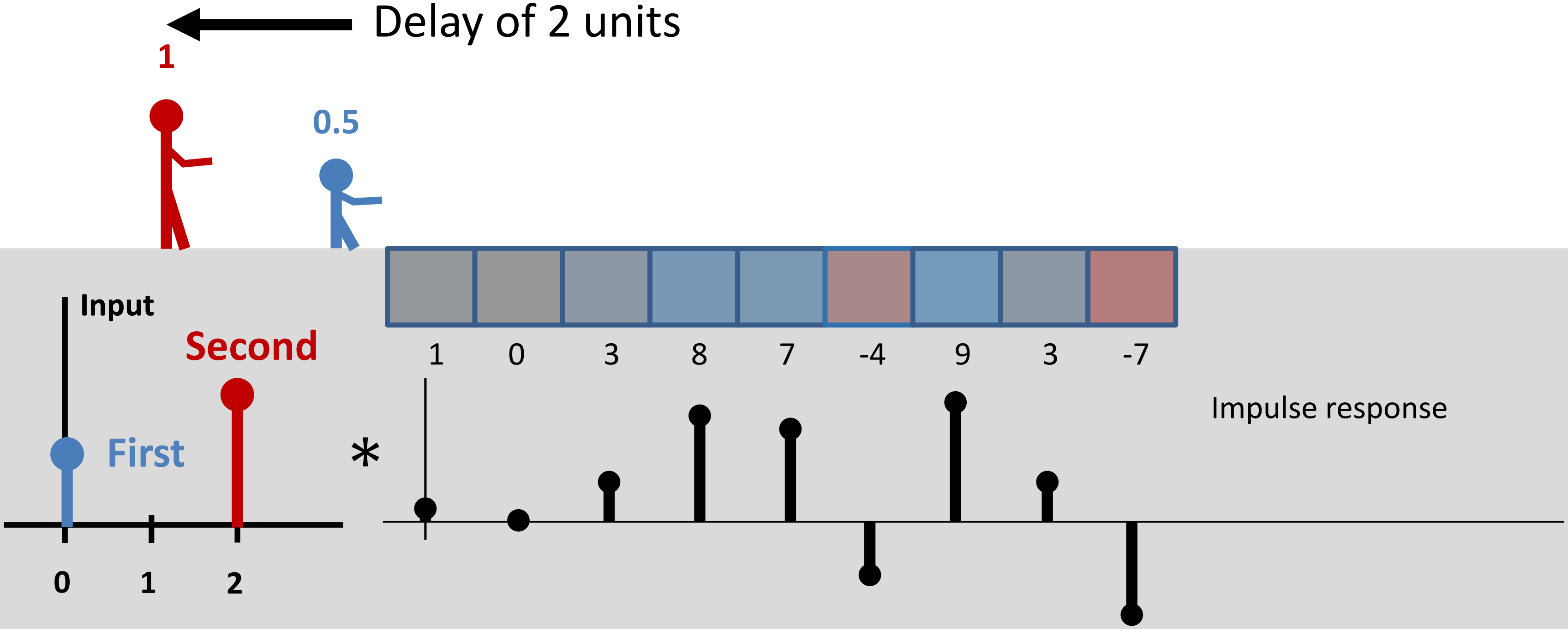

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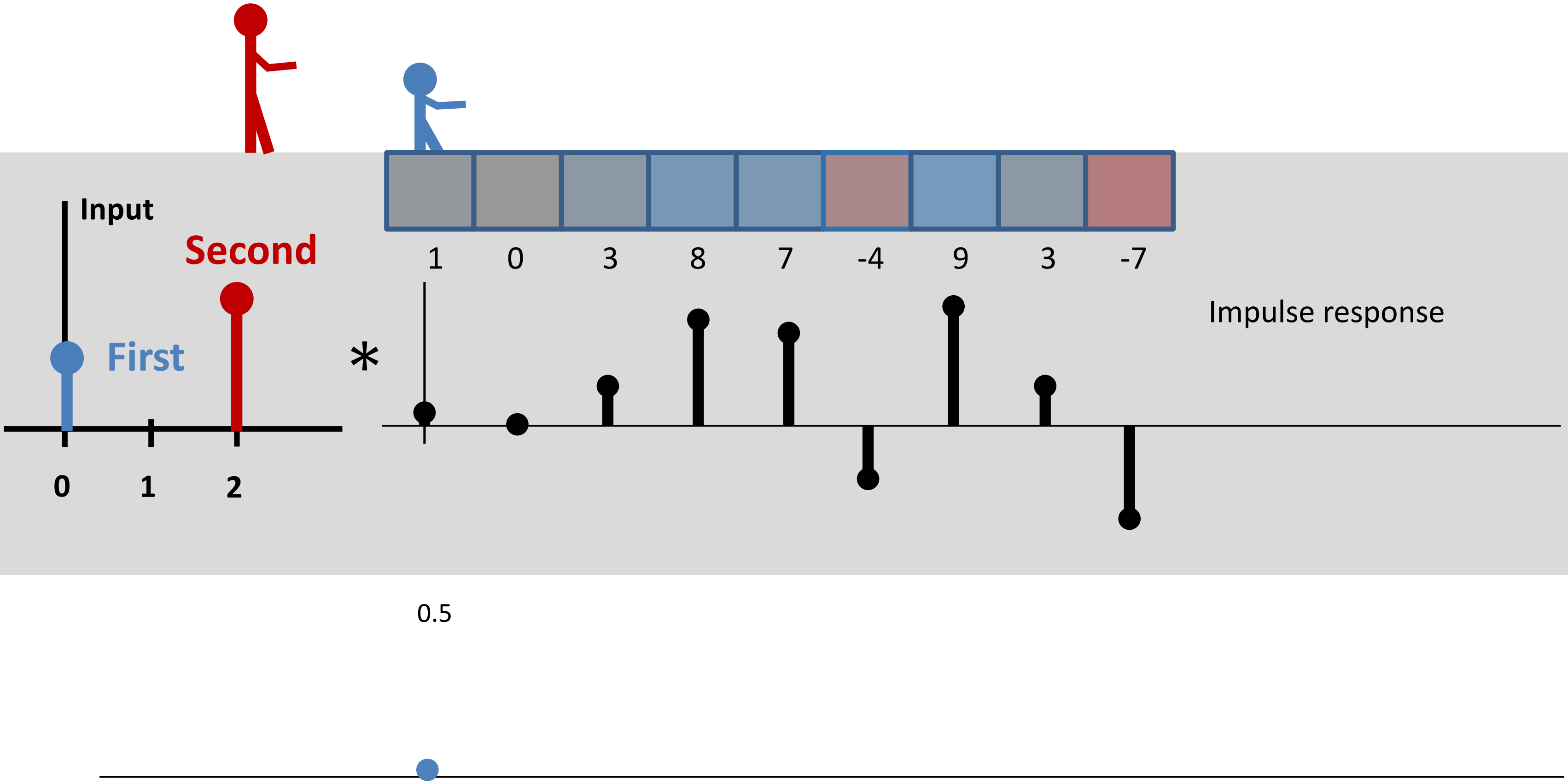

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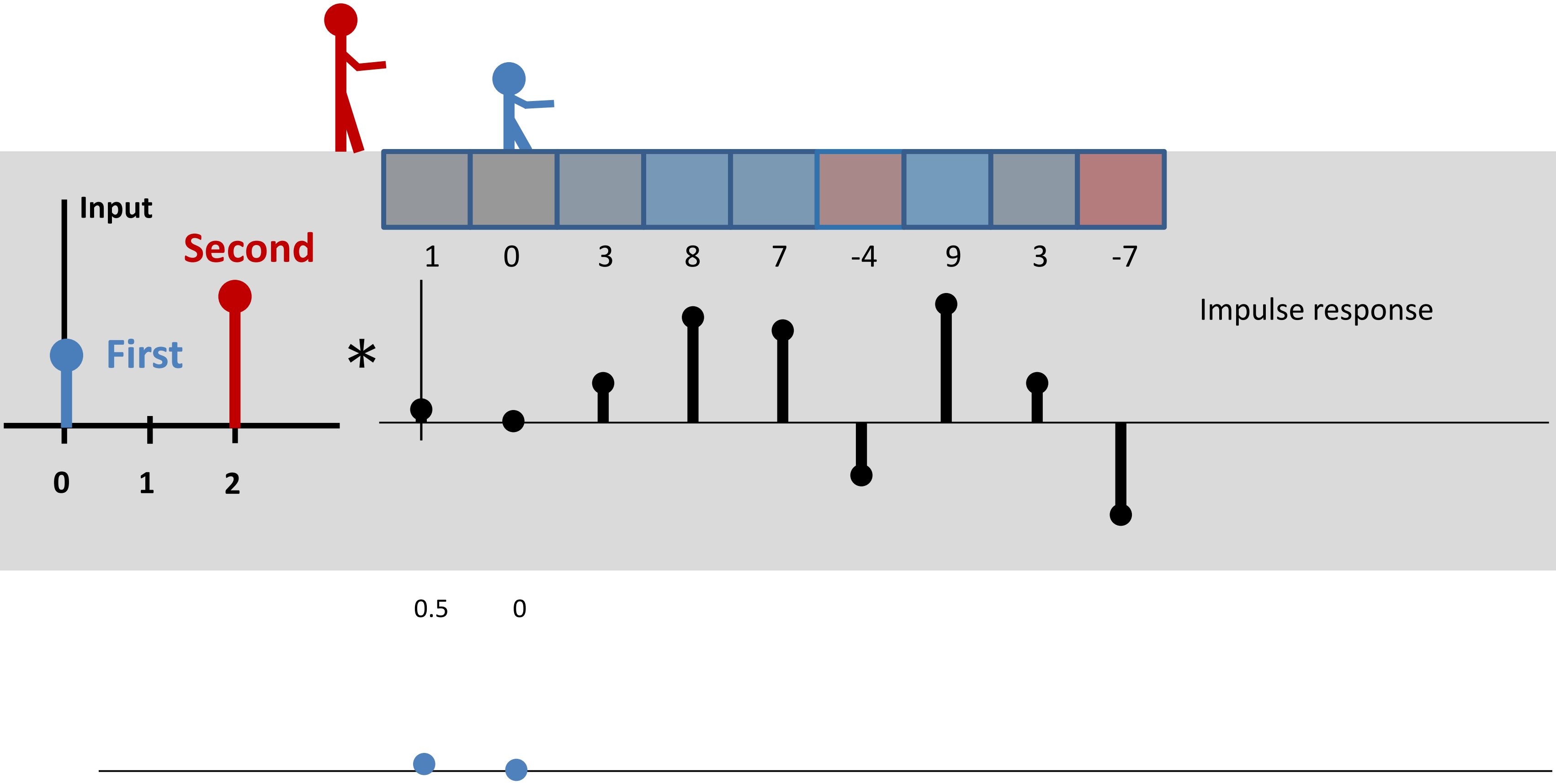

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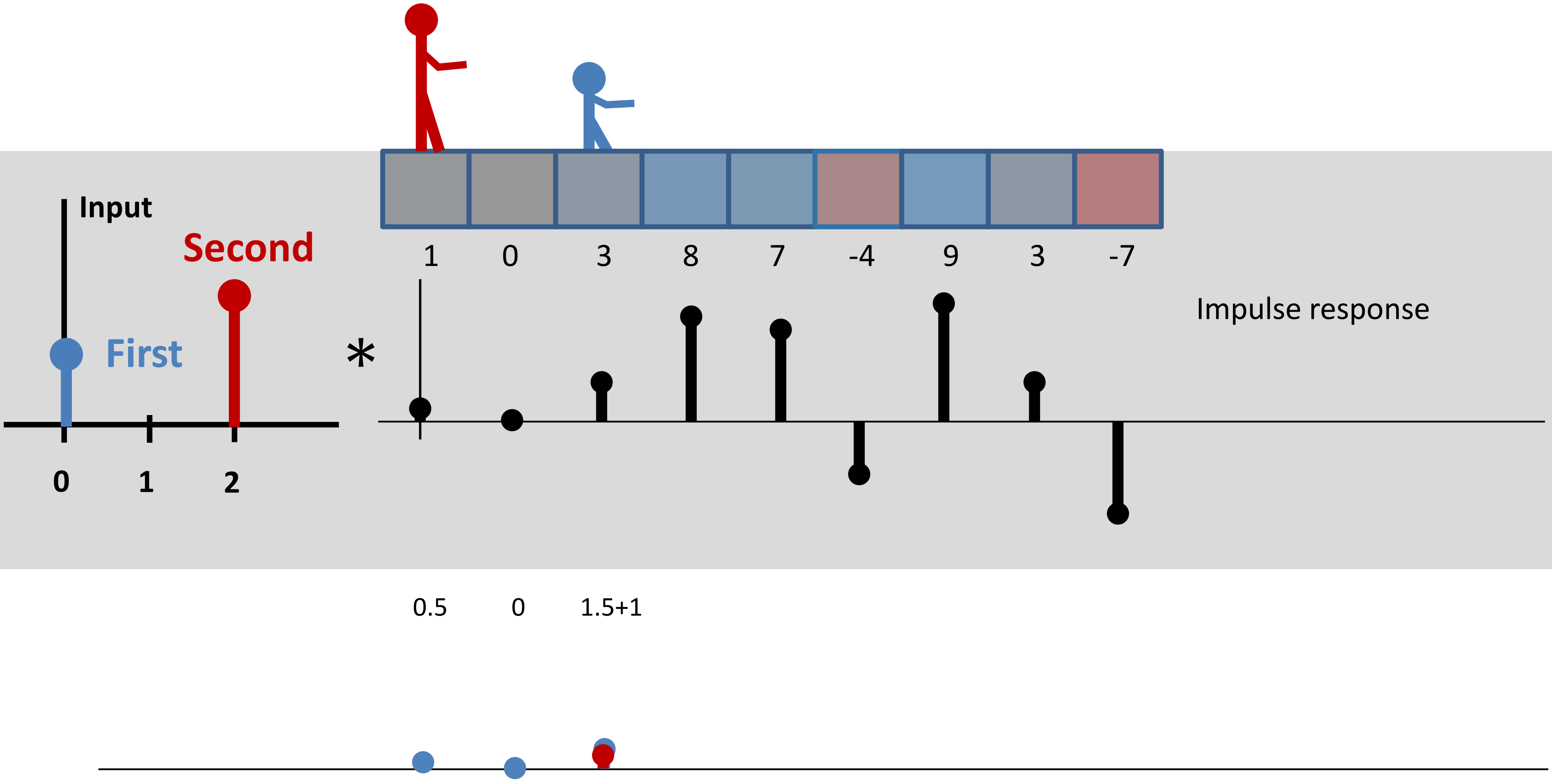

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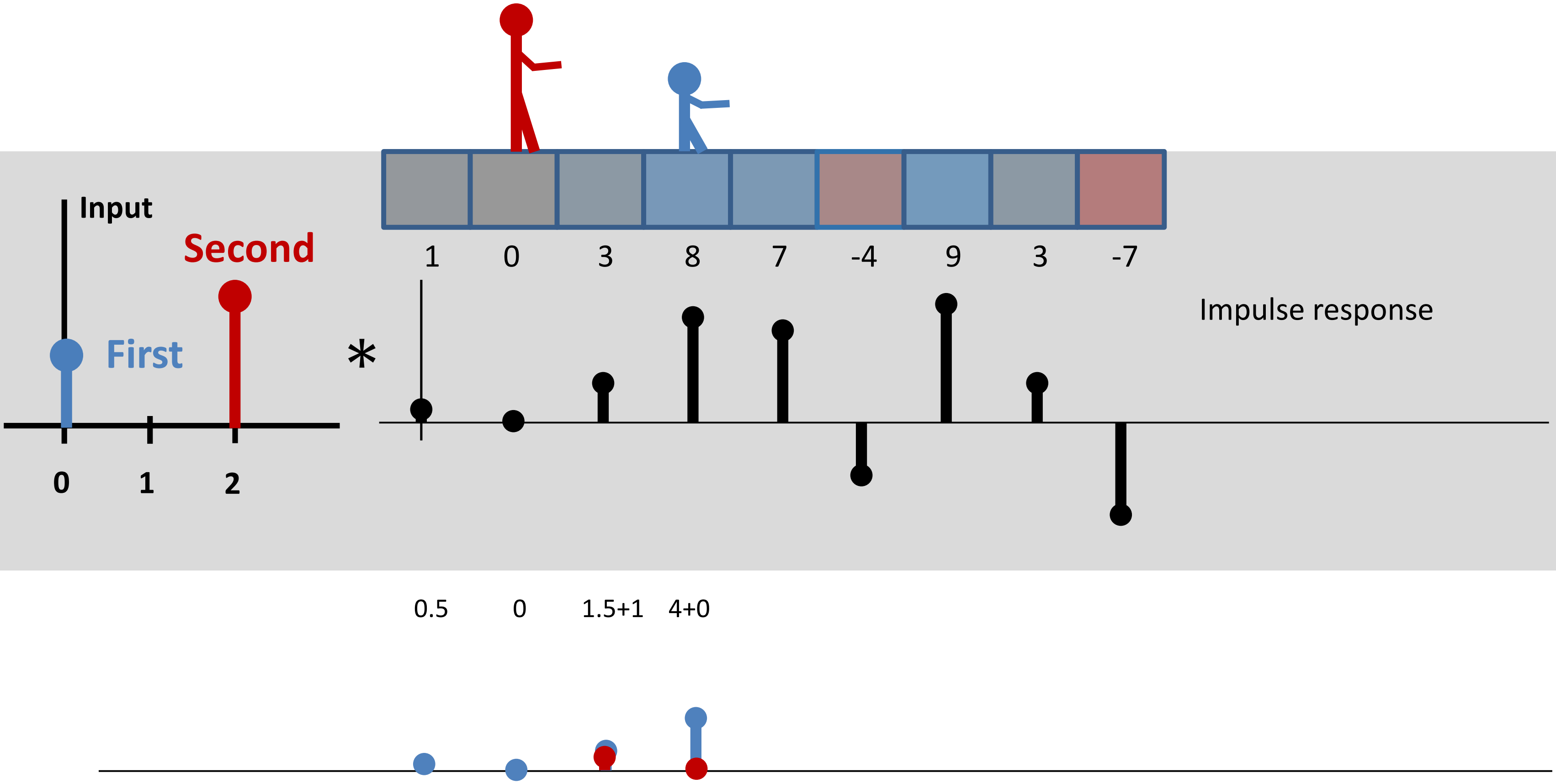

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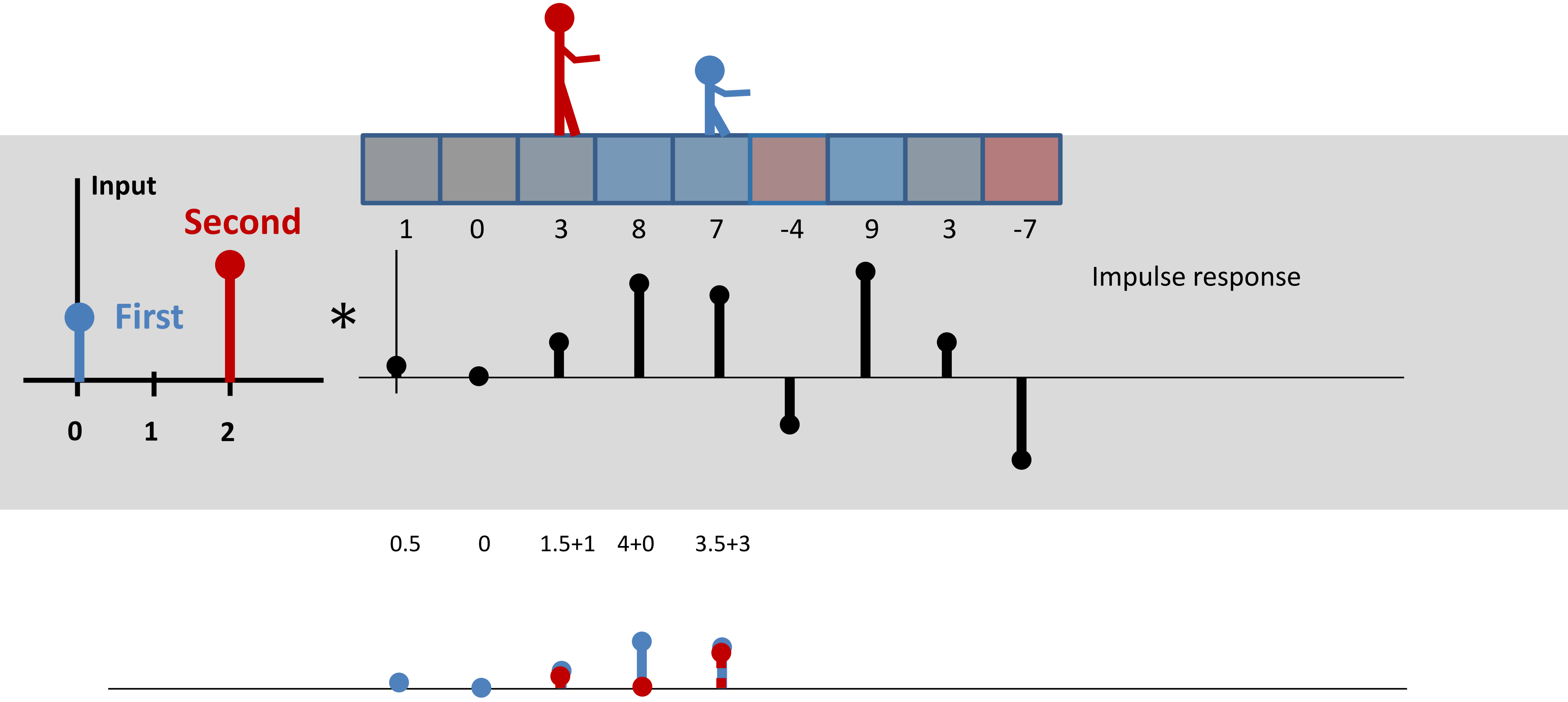

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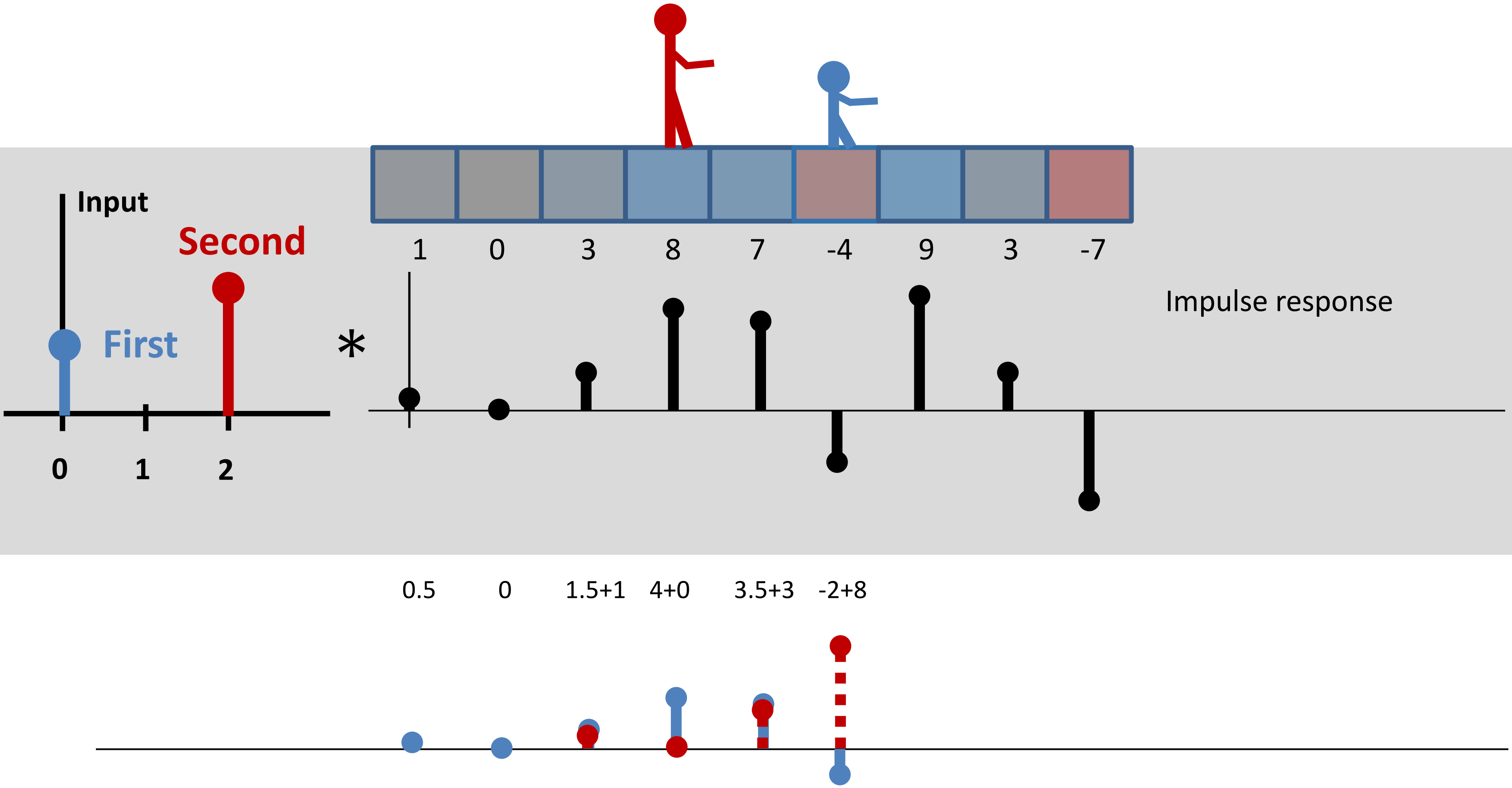

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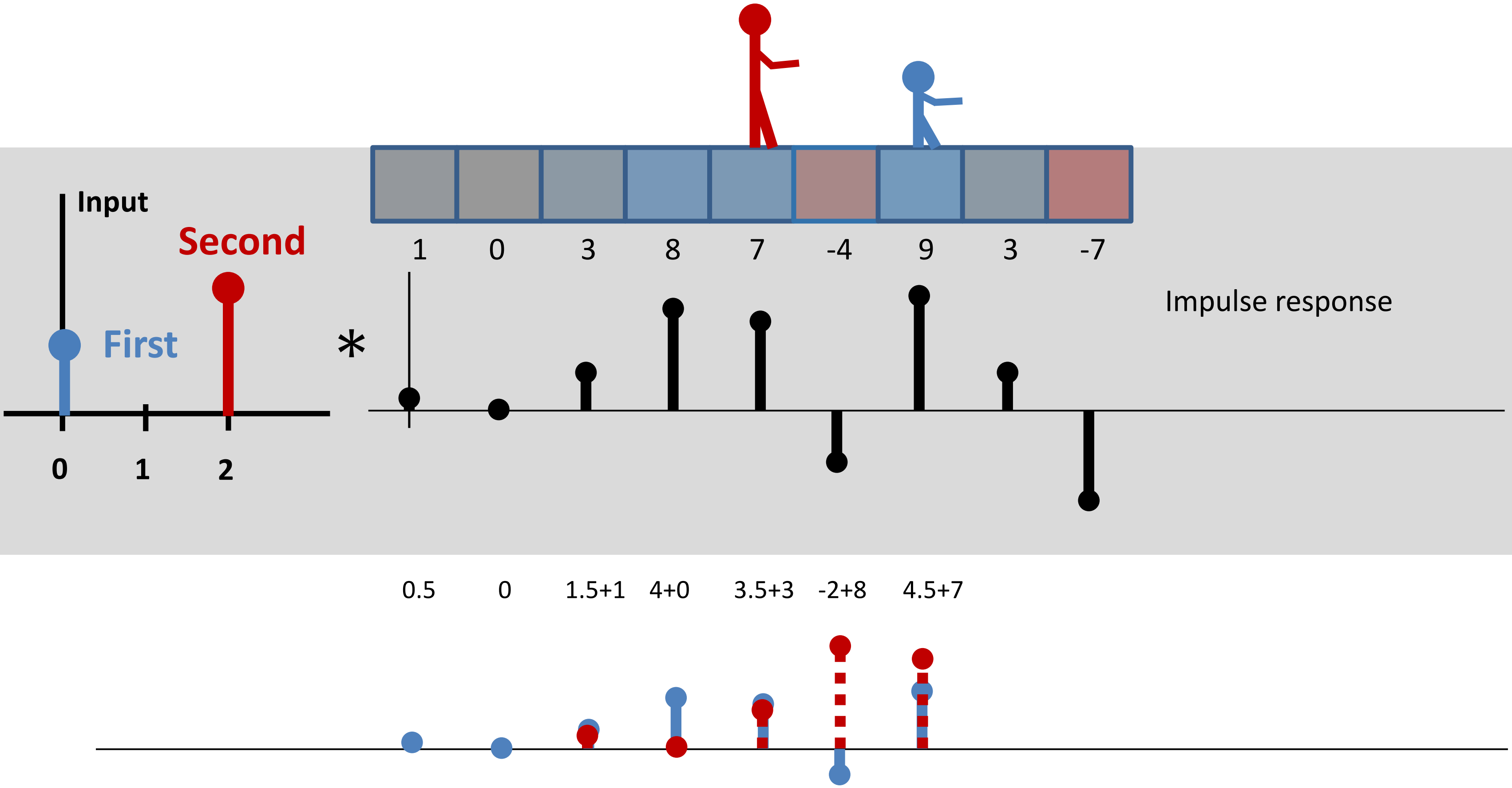

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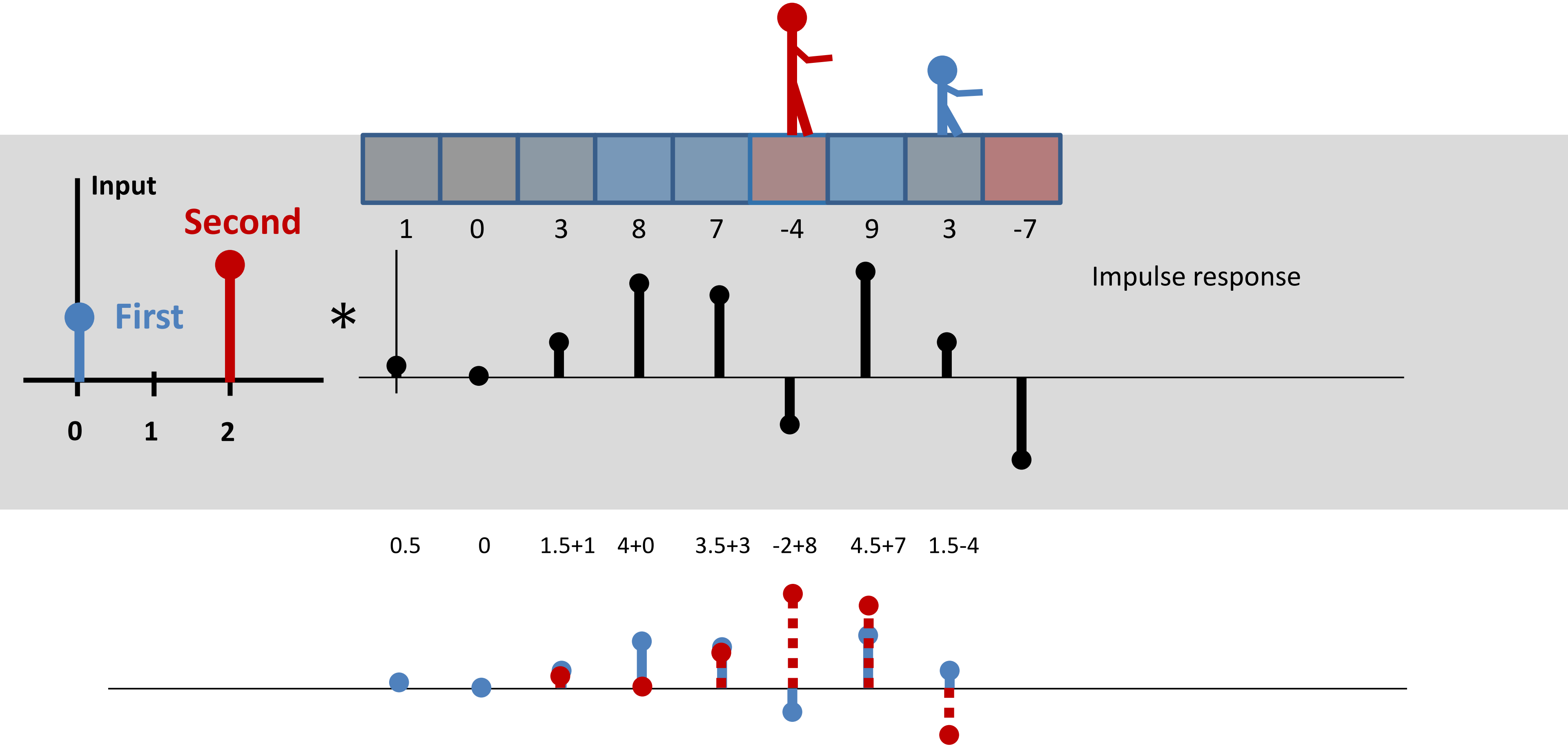

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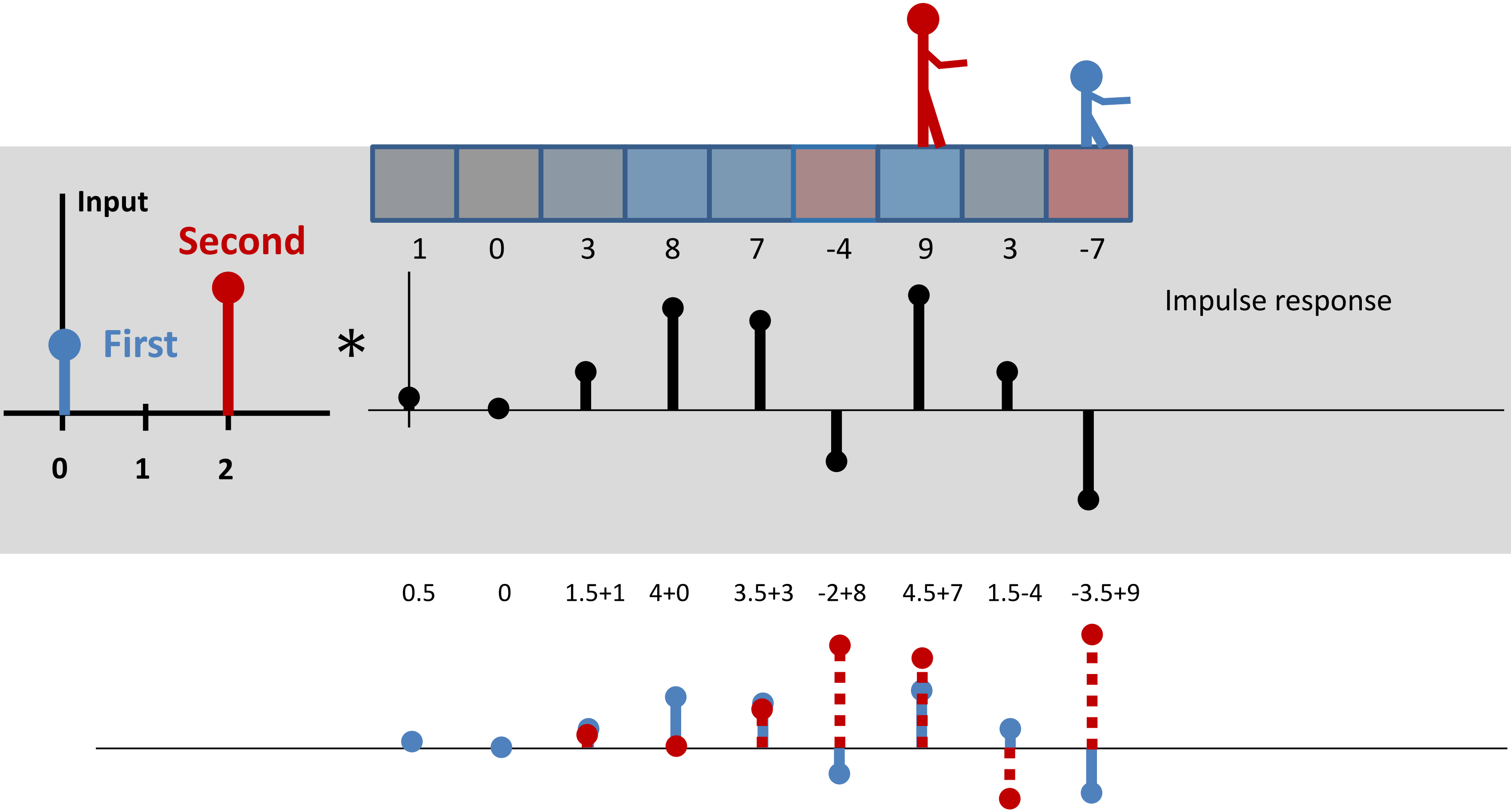

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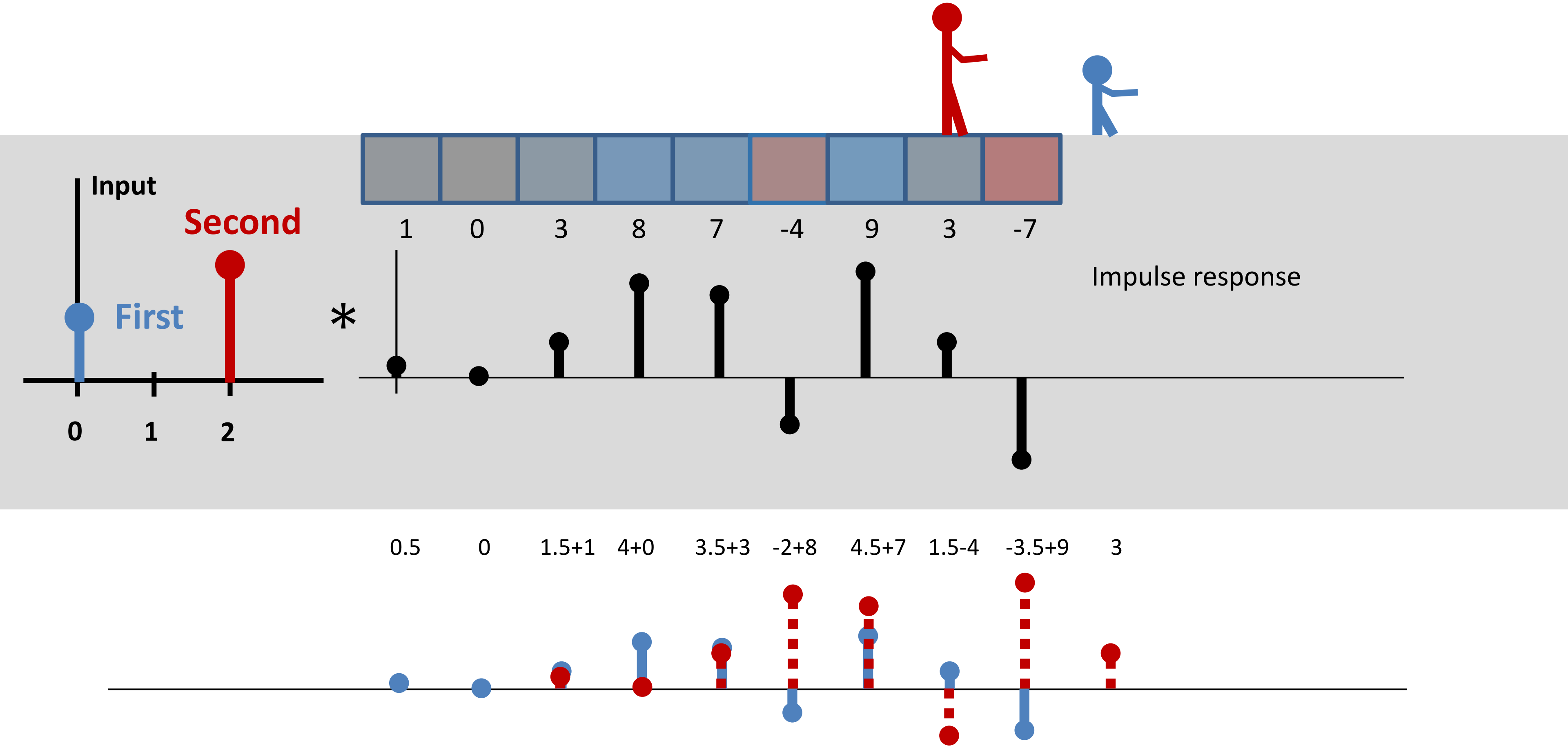

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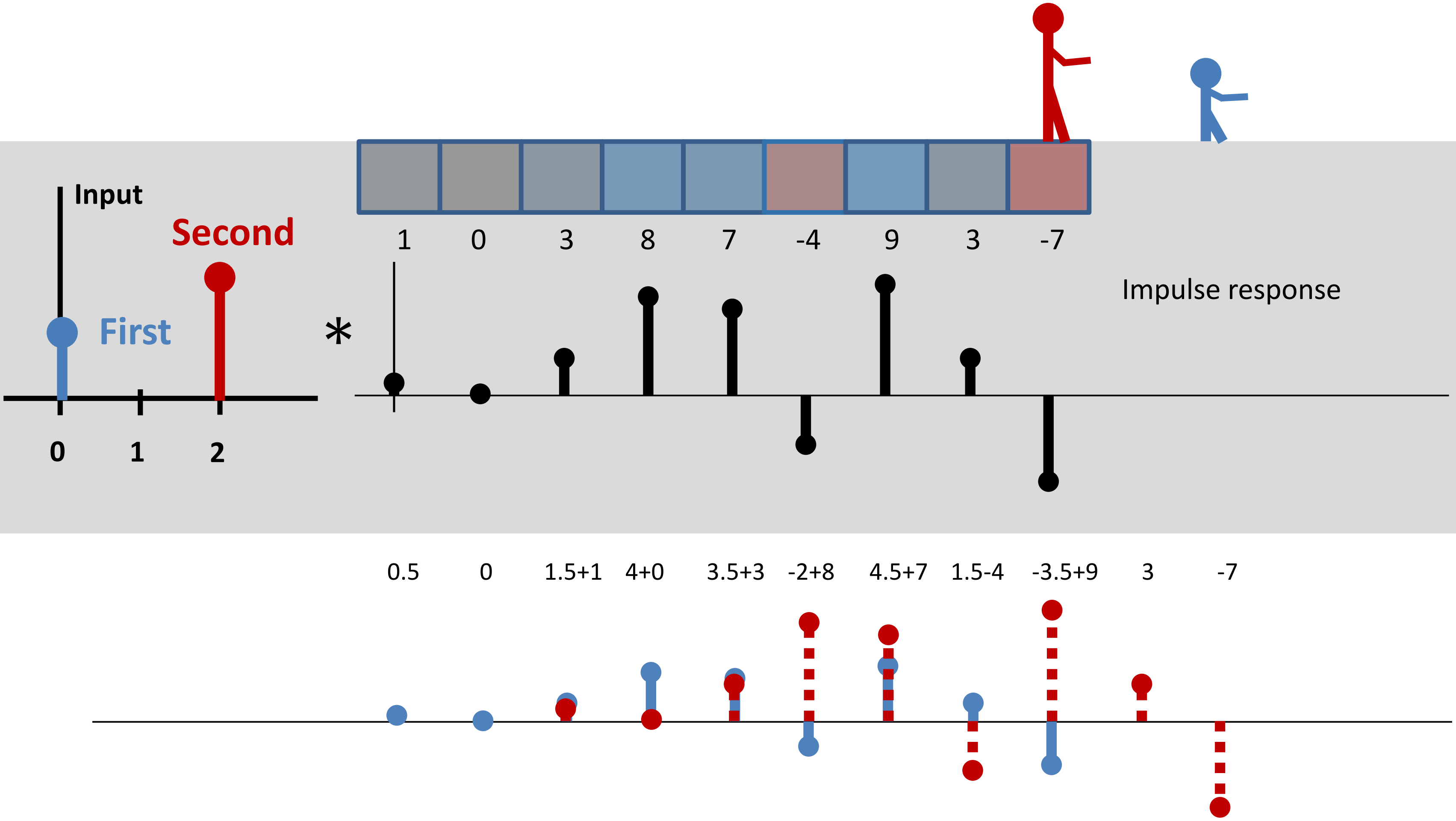

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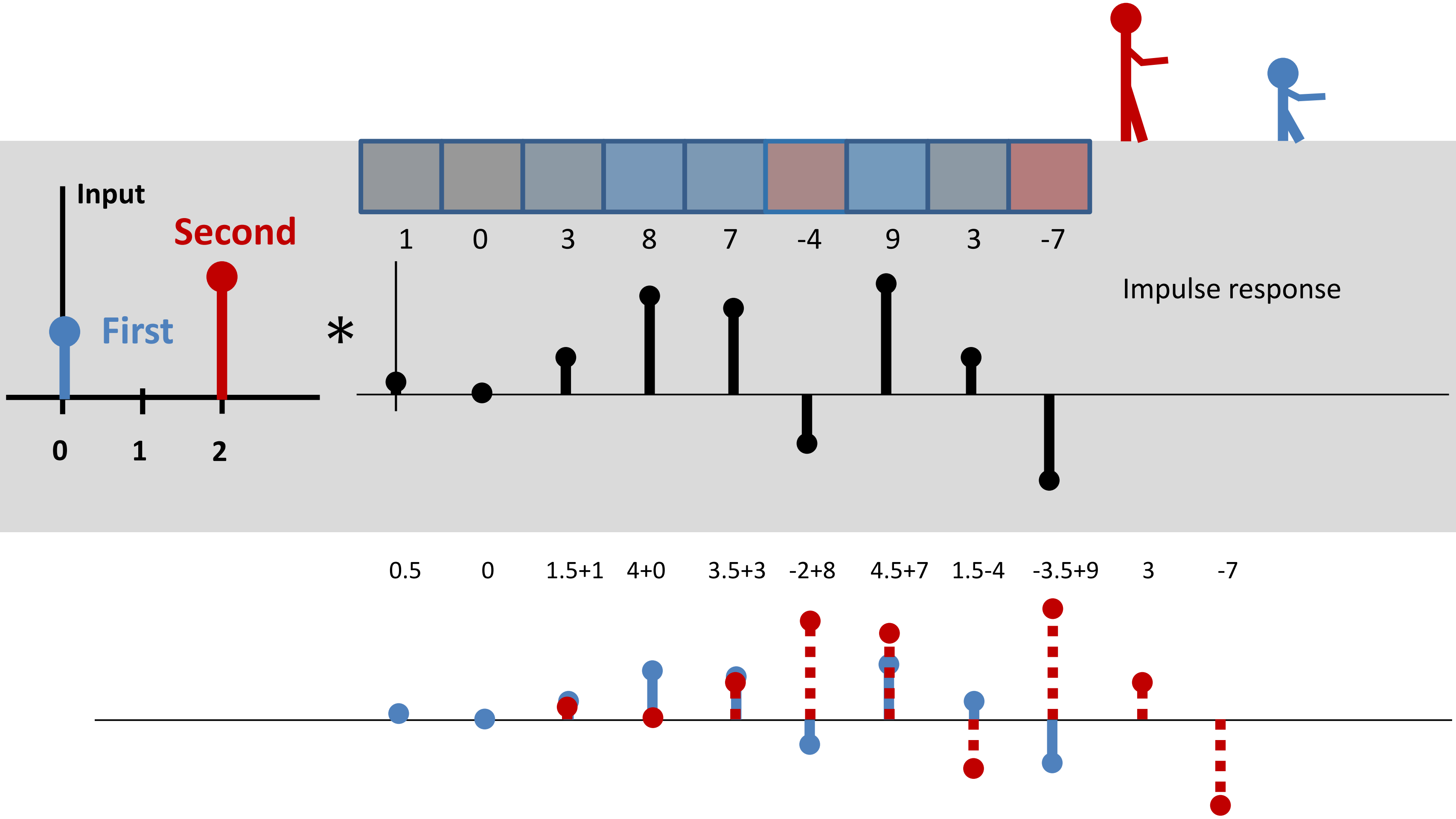

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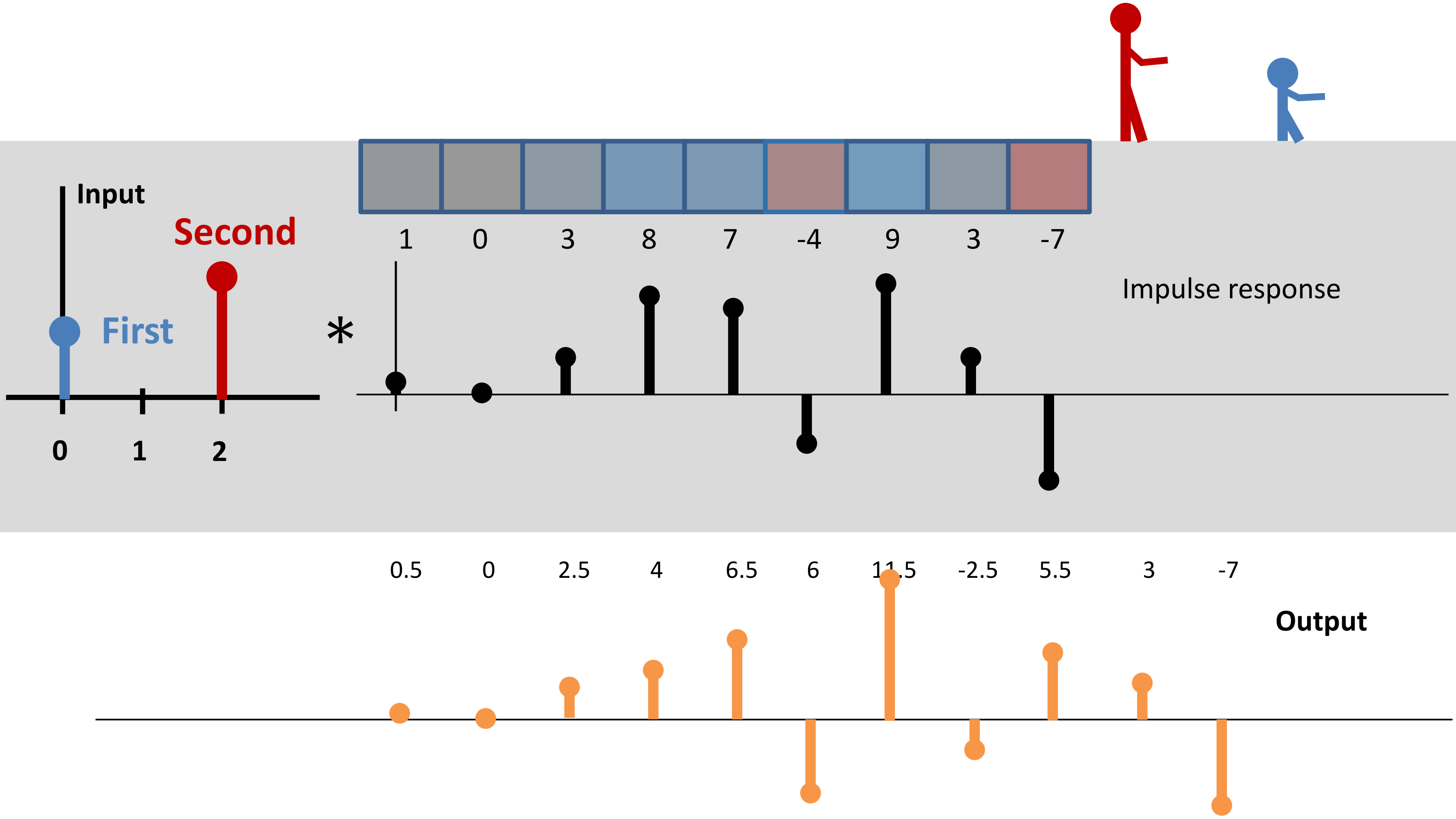

Example Problem #1

Convolution

- **Linear and Time-Invariant (LTI) System**
  - Consider the system with input-output relationship:

$$y[n] = \frac{1}{2}(x[n] + x[n-1])$$

  - Compute response of the system to input of

$$x[n] = 2\,\delta[n-1] + 2\,\delta[n-2]$$

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  - **Solution:**

$$y[n] = \frac{1}{2}(2\,\delta[n-1] + 2\,\delta[n-2] + 2\,\delta[n-2] + 2\,\delta[n-3])$$

$$= \frac{1}{2}(2\,\delta[n-1] + 4\,\delta[n-2] + 2\,\delta[n-3])$$

$$= \delta[n-1] + 2\,\delta[n-2] + \delta[n-3]$$

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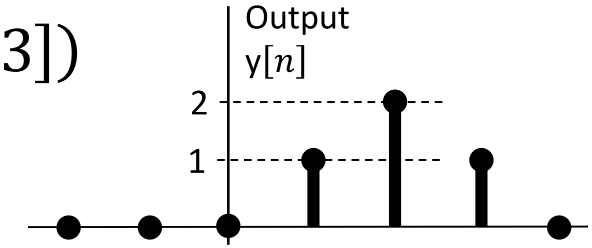

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  - **Solution:**

$$x[n] = \delta[n]$$

$$h[n] = \frac{1}{2}(\delta[n] + \delta[n-1])$$

Convolution

- **Linear and Time-Invariant (LTI) System**
  - Consider the system with input-output relationship:

$$y[n] = \frac{1}{2}(x[n] + x[n-1])$$

  - What do you think this system does?

Convolution

- **Linear and Time-Invariant (LTI) System**
  - Consider the system with input-output relationship:

$$y[n] = \frac{1}{2}(x[n] + x[n-1])$$

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$$y[n] = h[n] * x[n]$$

$$h[n] = (1/2)(\delta[n] + \delta[n-1])$$

$$x[n] = 2\,\delta[n-1] + 2\,\delta[n-2]$$

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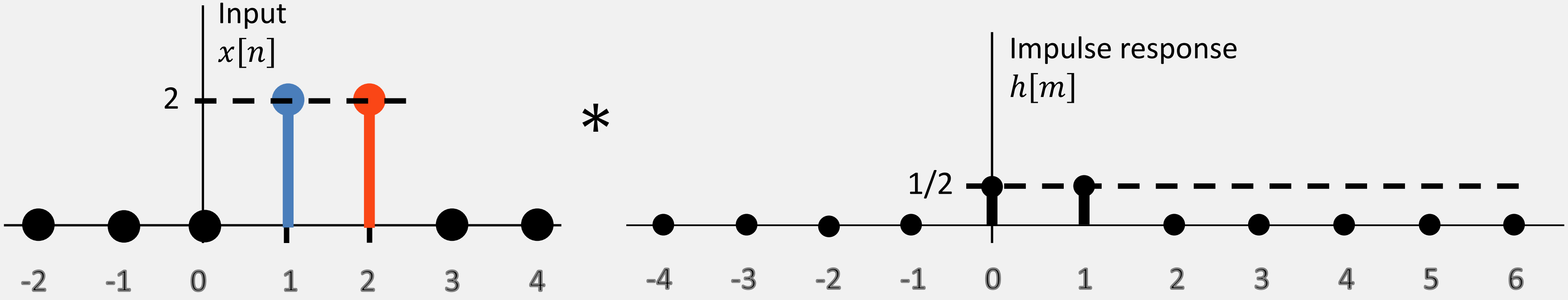

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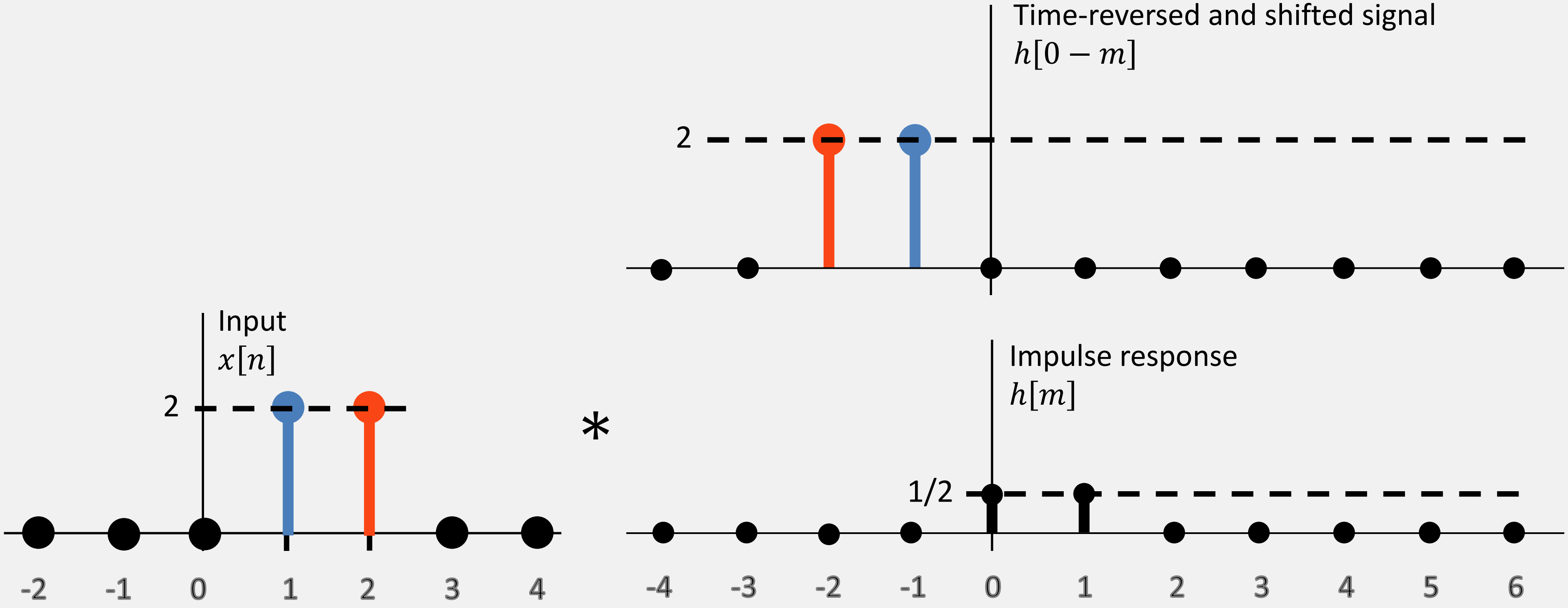
Step 1: Time-reverse a signal



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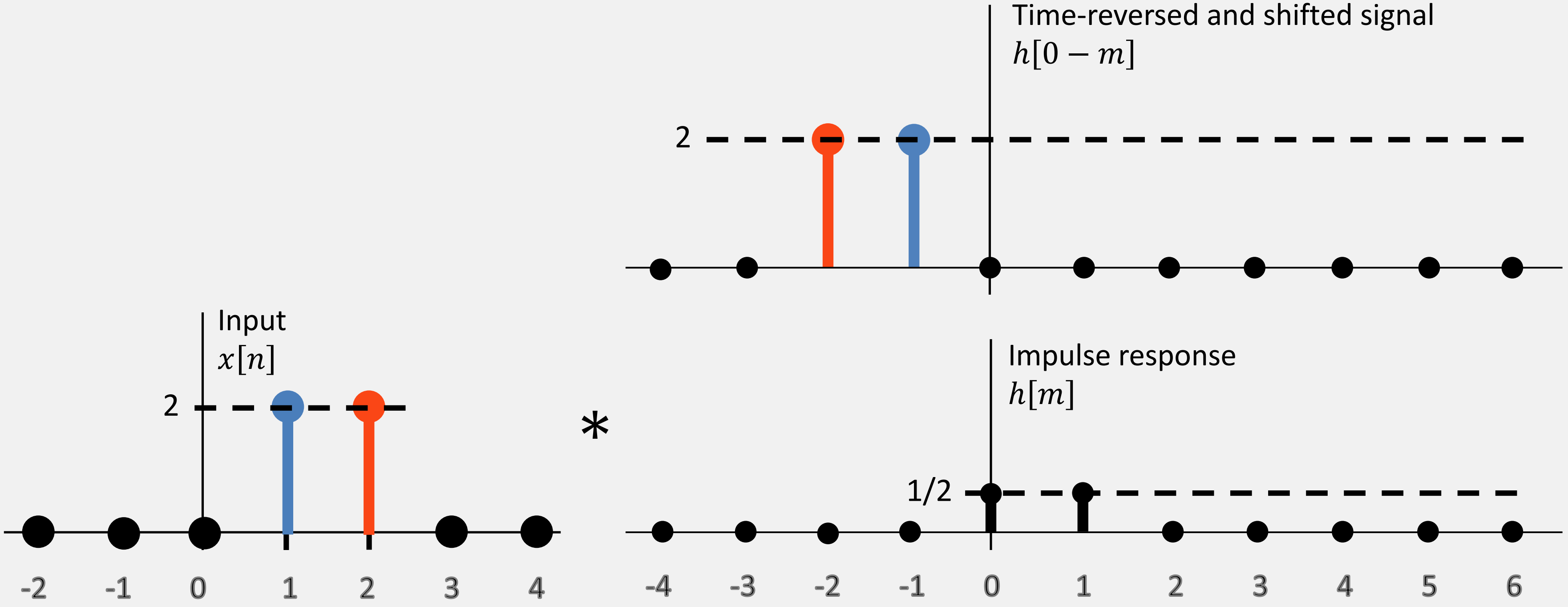
Step 1: Time-reverse a signal
Step 2: Shift that signal by *n*
Step 3: Multiply the signals & sum the result



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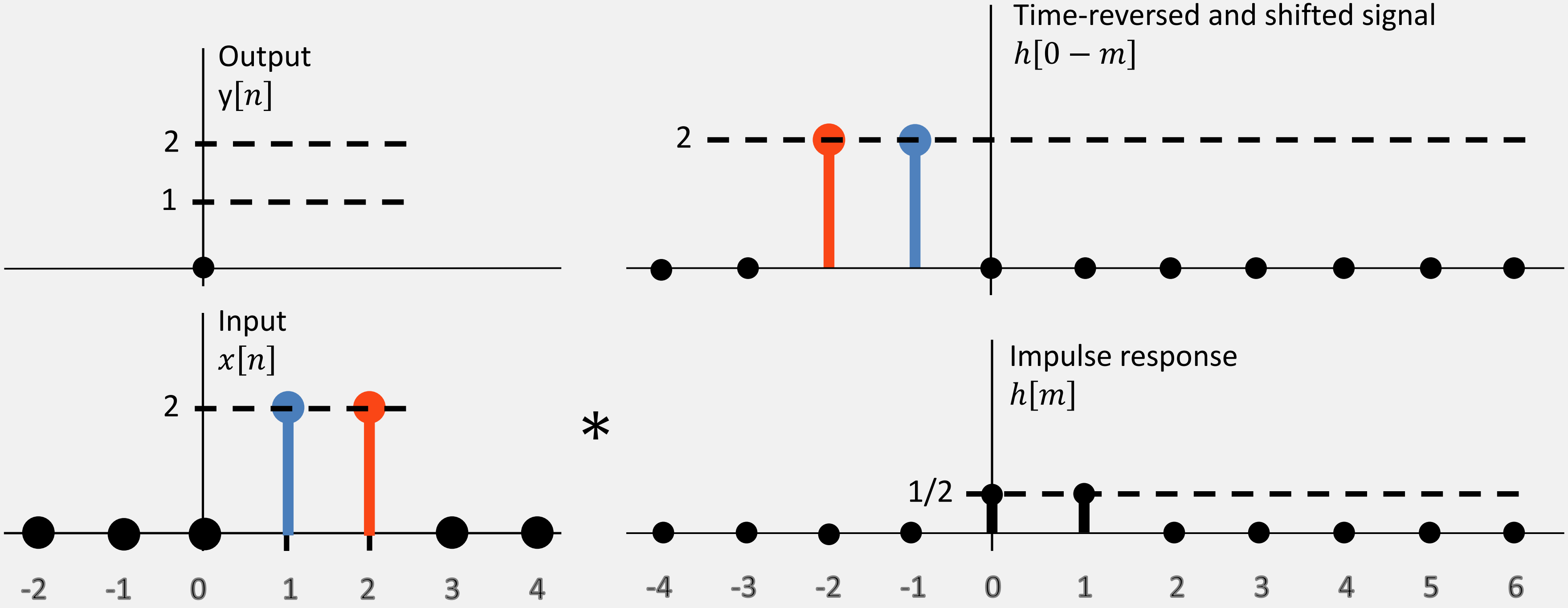
Step 4: Assign the sum to *y[n]* for shift *n*



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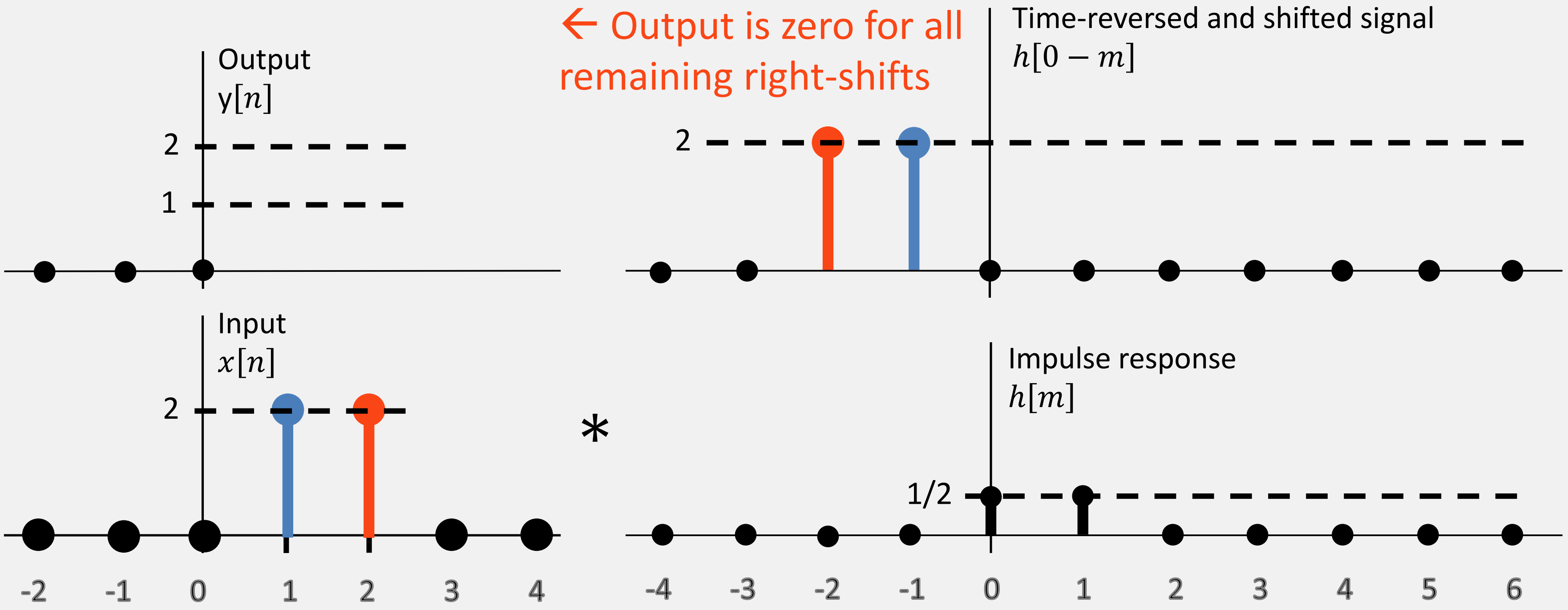

← Output is zero for all remaining right-shifts

Convolution

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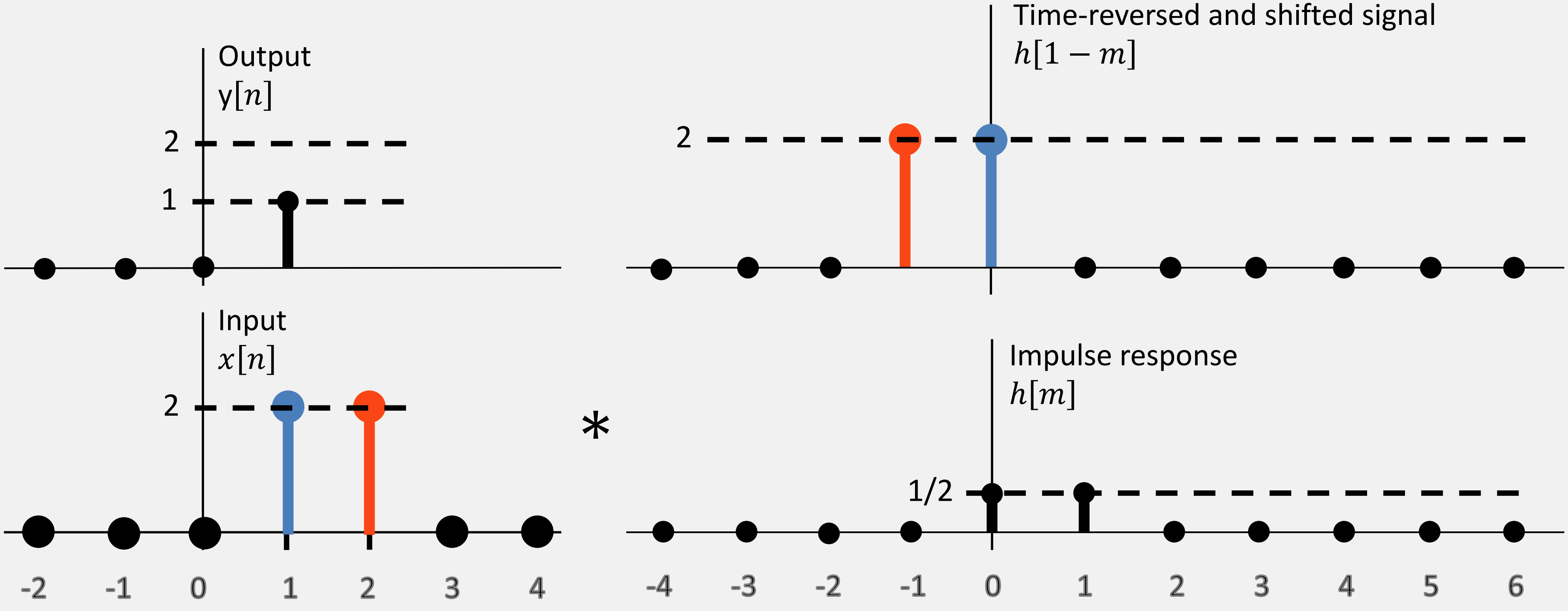

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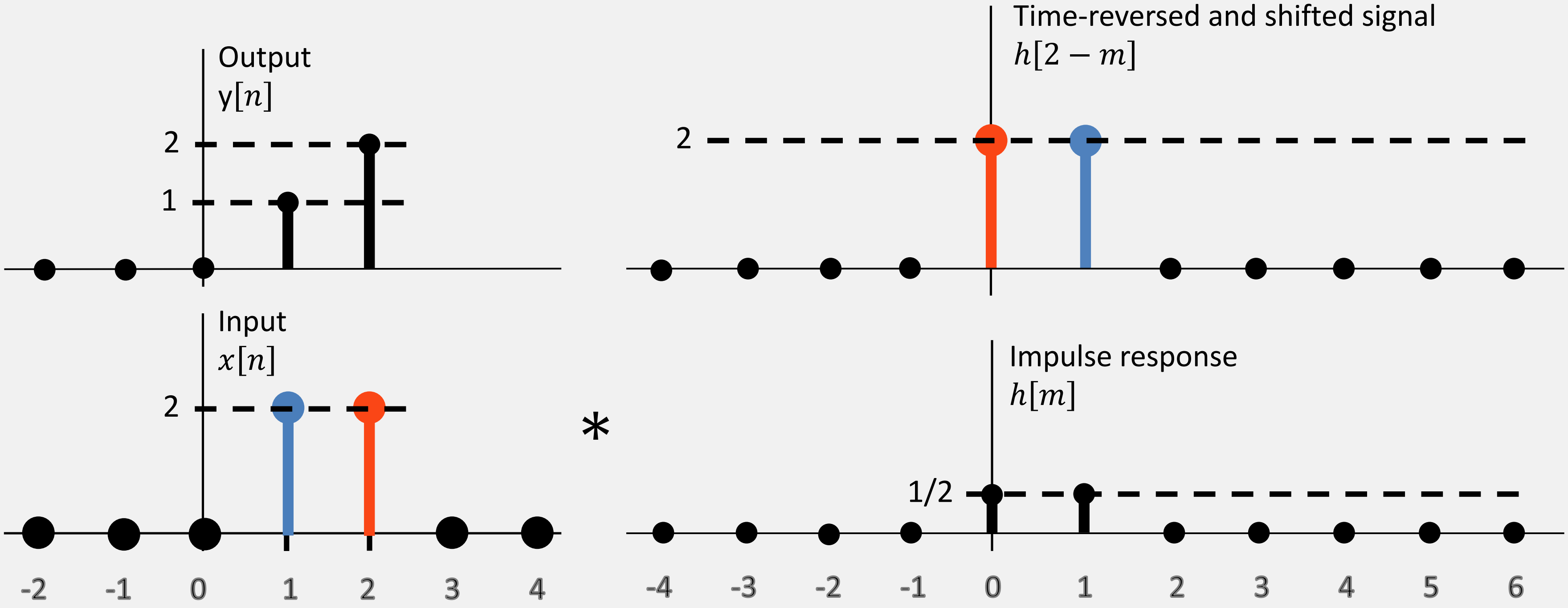

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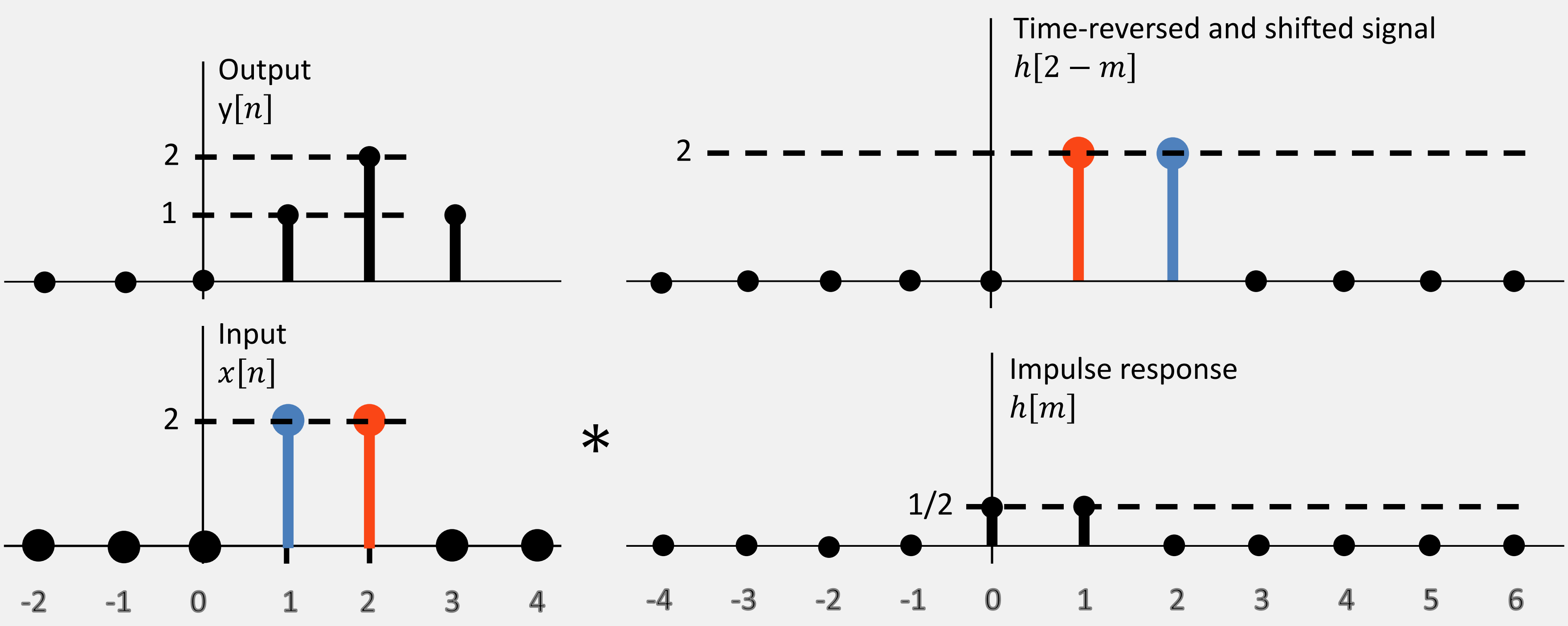


Convolution

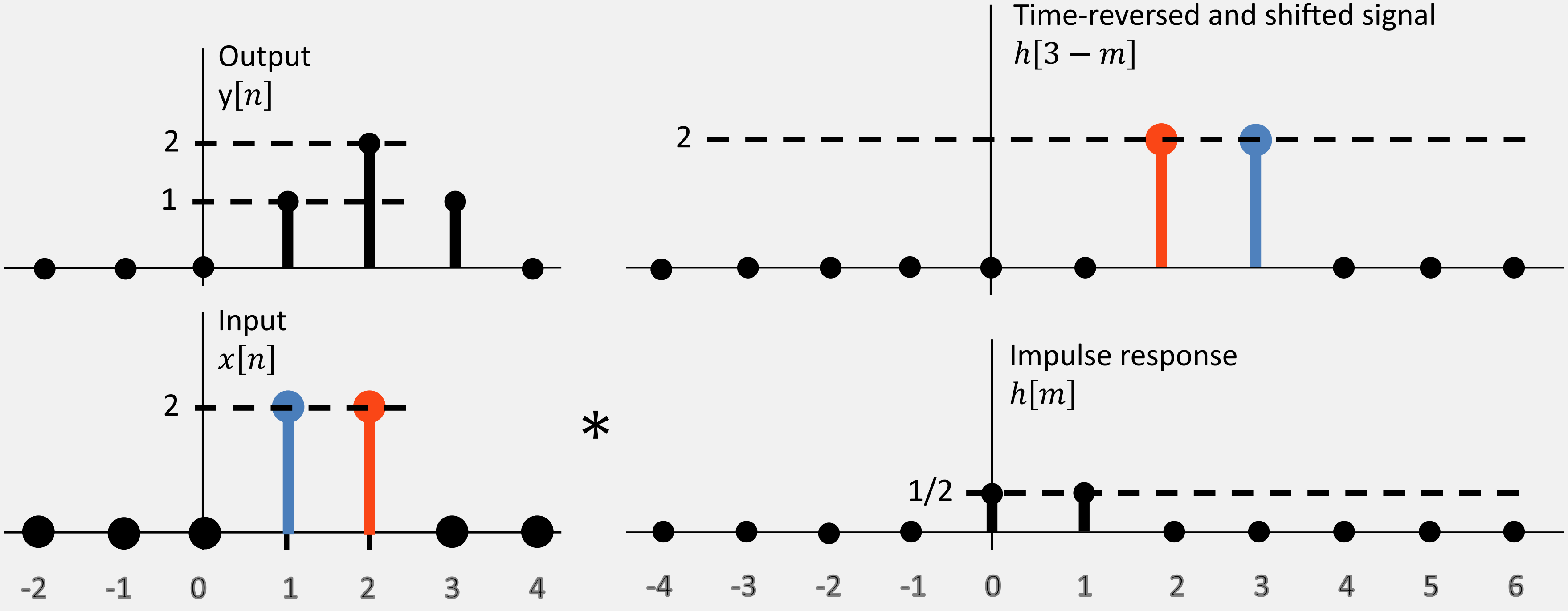
- **Compute the convolution of:**

$$y[n] = h[n] * x[n]$$

$$h[n] = (1/2)(\delta[n] + \delta[n-1])$$

$$x[n] = 2\,\delta[n-1] + 2\,\delta[n-2]$$

Step 1: Time-reverse a signal
Step 2: Shift that signal by *n*
Step 3: Multiply the signals & sum the result
Step 4: Assign the sum to *y[n]* for shift *n*
Step 5*:* Repeat for all shifts $-\infty < n < \infty$



Convolution

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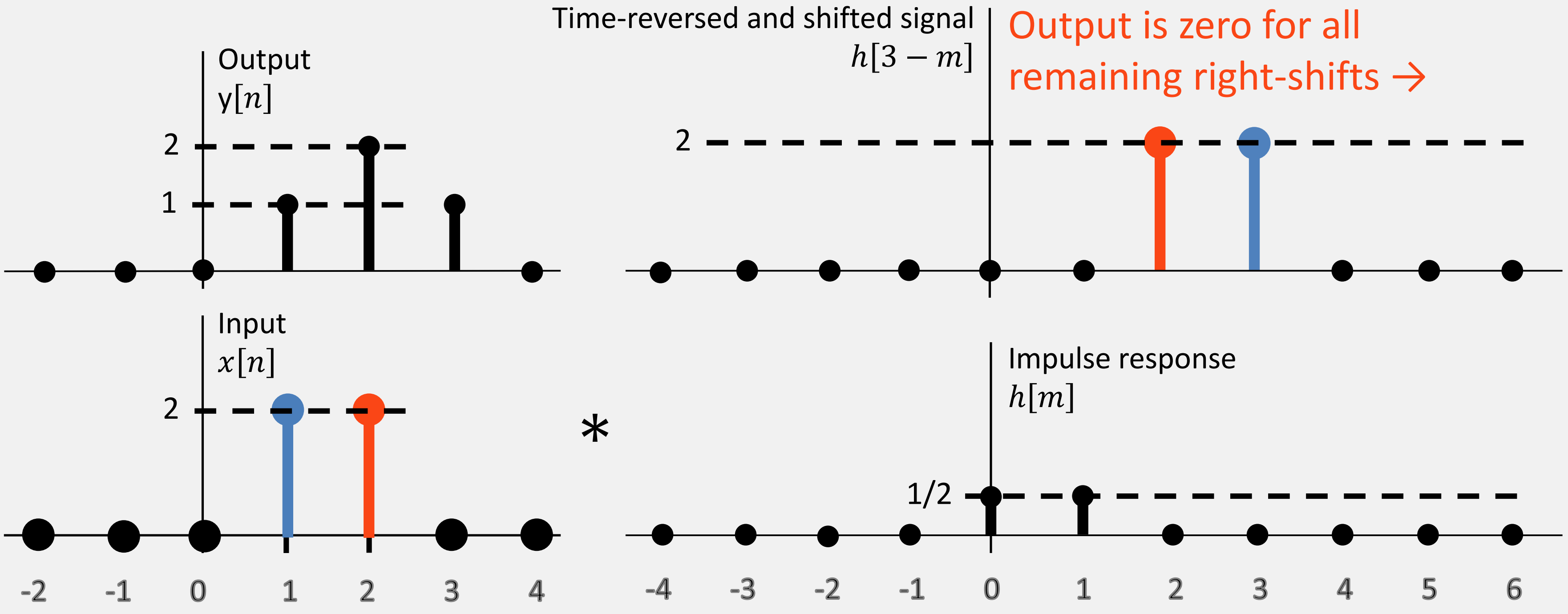

Output is zero for all remaining right-shifts →

Convolution

- **Compute the convolution of:**

$$y[n] = h[n] * x[n]$$

$$h[n] = (1/2)(\delta[n] + \delta[n-1])$$

$$x[n] = 2\,\delta[n-1] + 2\,\delta[n-2]$$

Step 1: Time-reverse a signal
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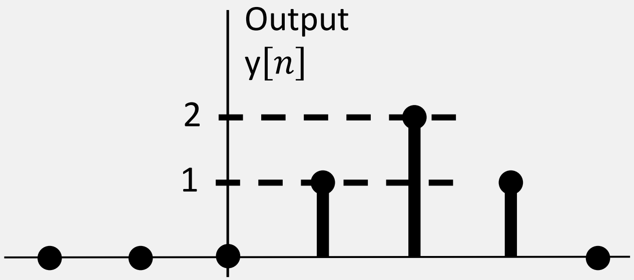


$$y[n] = \delta[n-1] + 2\,\delta[n-2] + \delta[n-3]$$

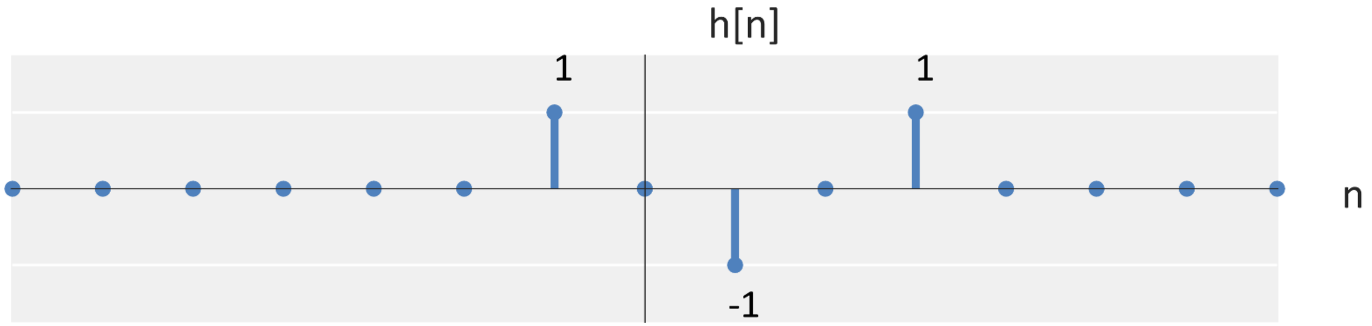
Same as our first result!

We can compute the system output via convolution!

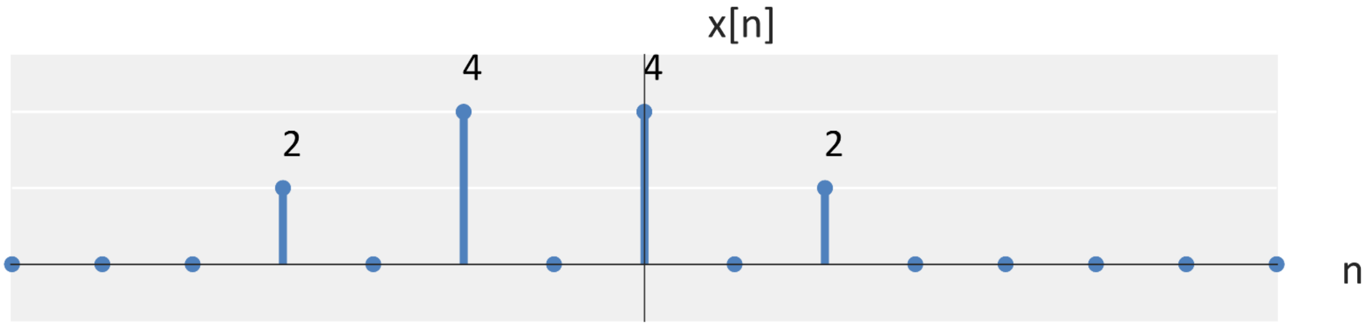
Example Problem #2

Convolution

- **Linear and Time-Invariant (LTI) System**
  - Consider the system with impulse response $h[n]$

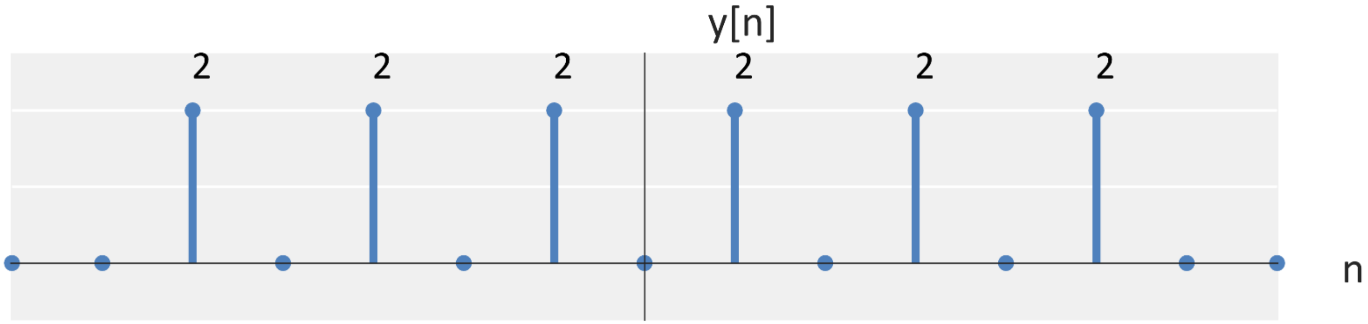


  - Compute response of the system to input $x[n]$ below



Convolution

- **Linear and Time-Invariant (LTI) System**
  - **Solution:**



Example Problem #3: Correlation

Convolution / Correlation

- **Definition of convolution**

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

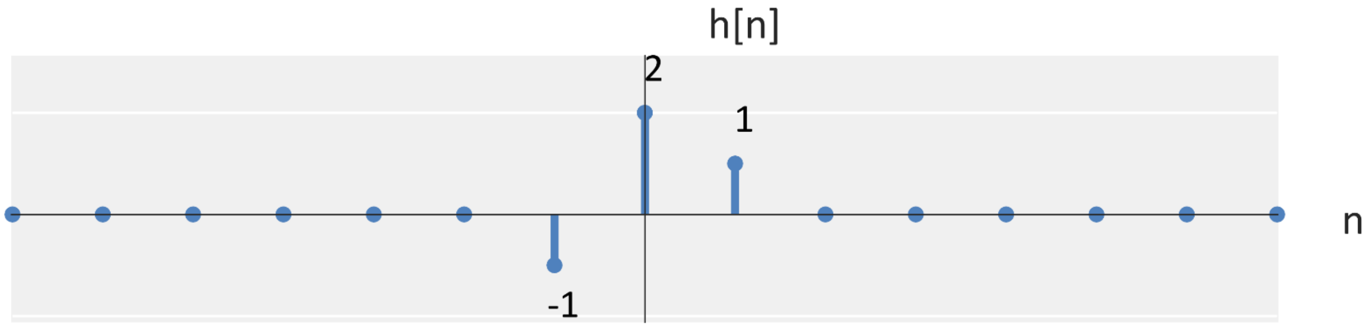
- **Definition of correlation**

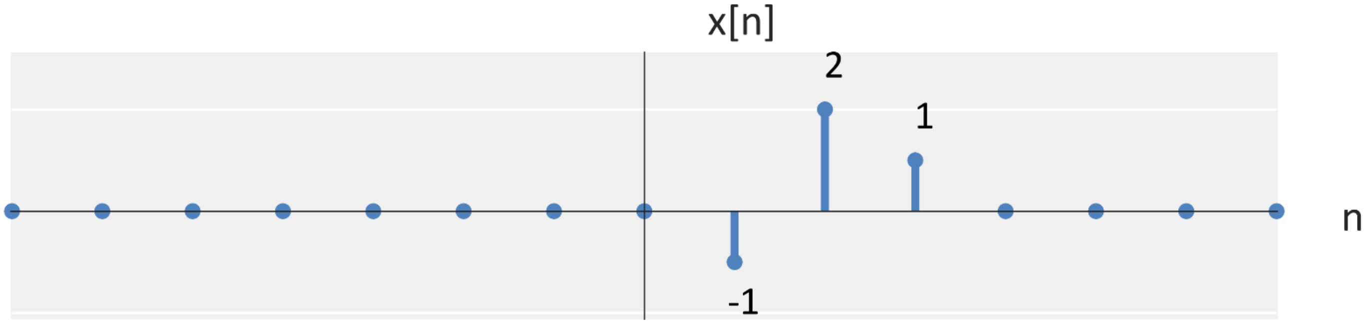
$$y[n] = x[-n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n+m]$$

Convolution / Correlation

- **Linear and Time-Invariant (LTI) System**
  - Consider $h[n]$ and $\mathrm{x}[n]$ below. Compute their <u>correlation</u>.

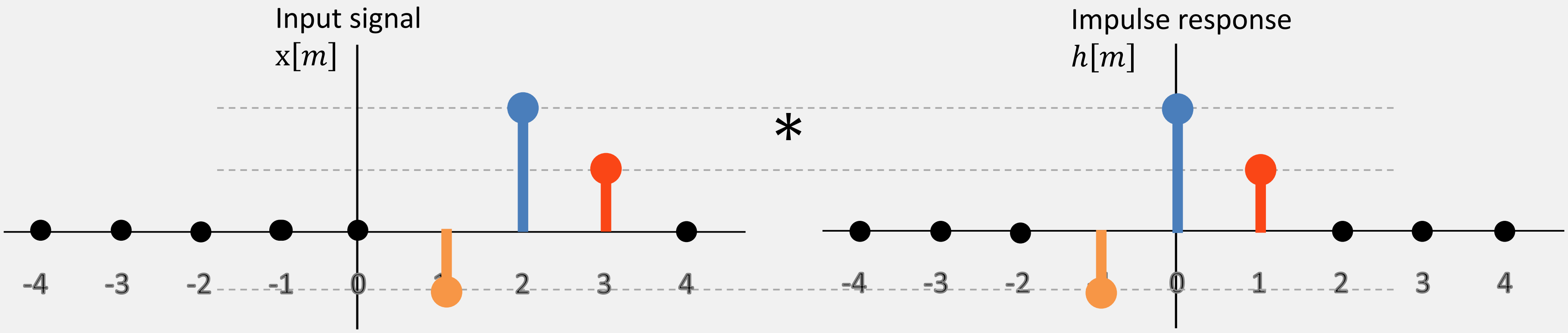
$$y[n] = x[-n] * h[n]$$





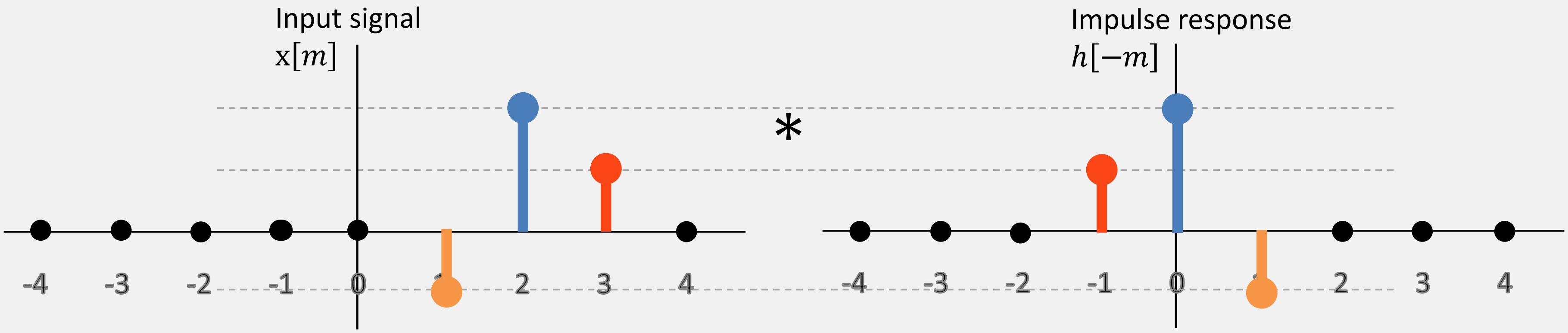
Convolution / Correlation

- **Compute the correlation:**
  $y[n] = x[-n] * h[n]$



Convolution / Correlation

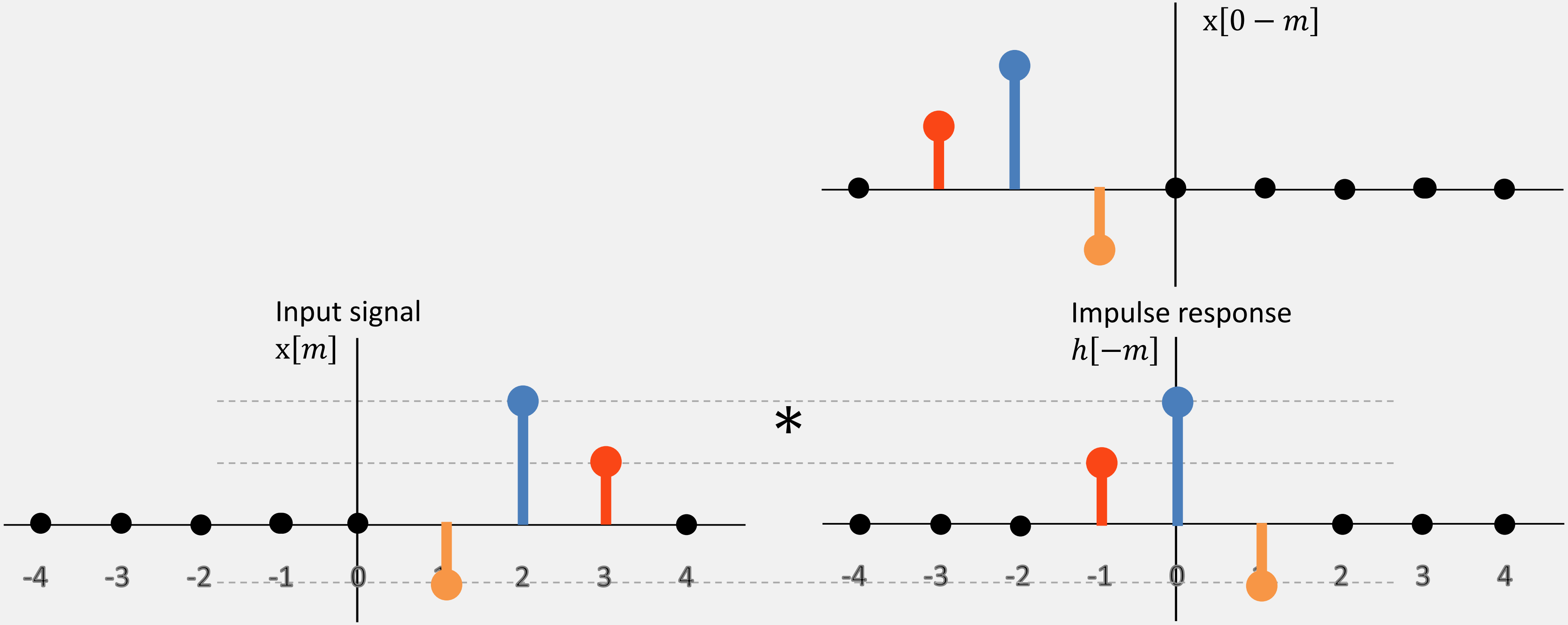
- **Compute the correlation:**
  $y[n] = x[-n] * h[n]$



Convolution / Correlation

- **Compute the correlation:**
  $y[n] = x[-n] * h[n]$

Step 1: Time-reverse a signal



Convolution / Correlation

- **Compute the correlation:**
  $y[n] = x[-n] * h[n]$

Step 1: Time-reverse a signal
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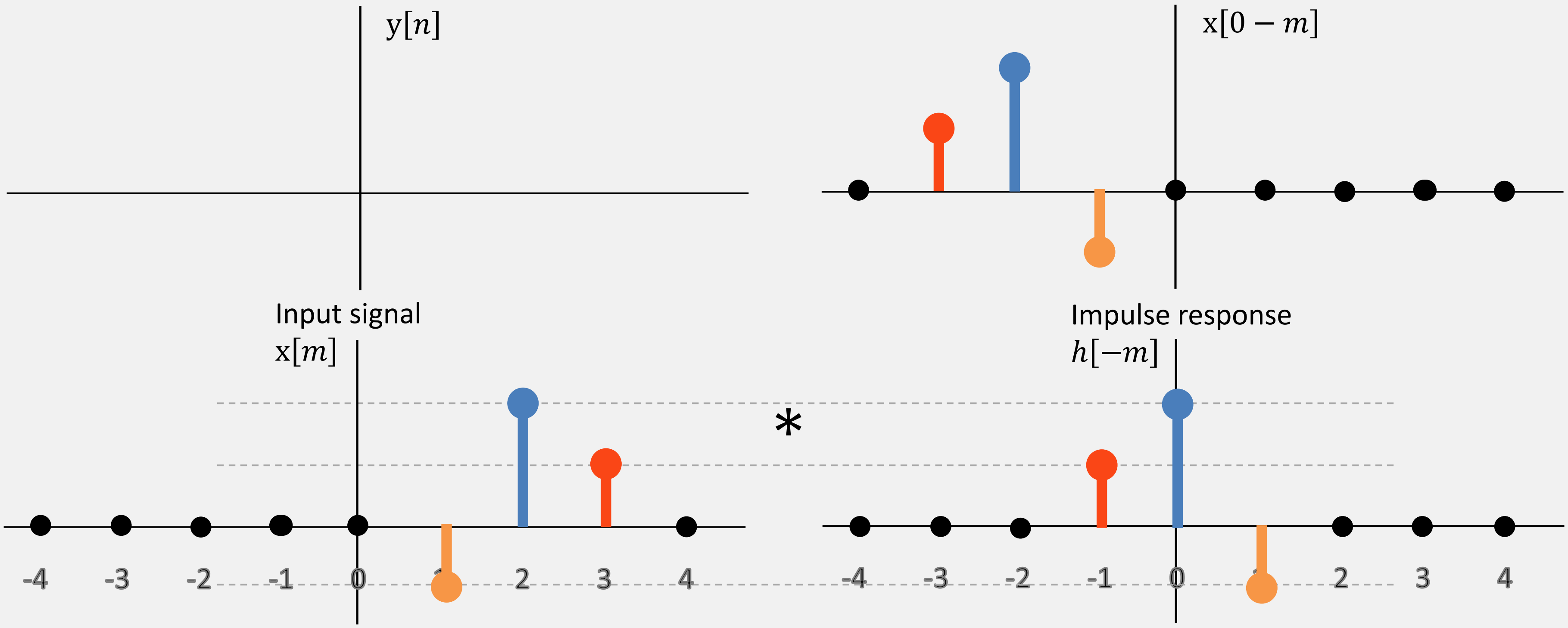

Convolution / Correlation

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Step 1: Time-reverse a signal
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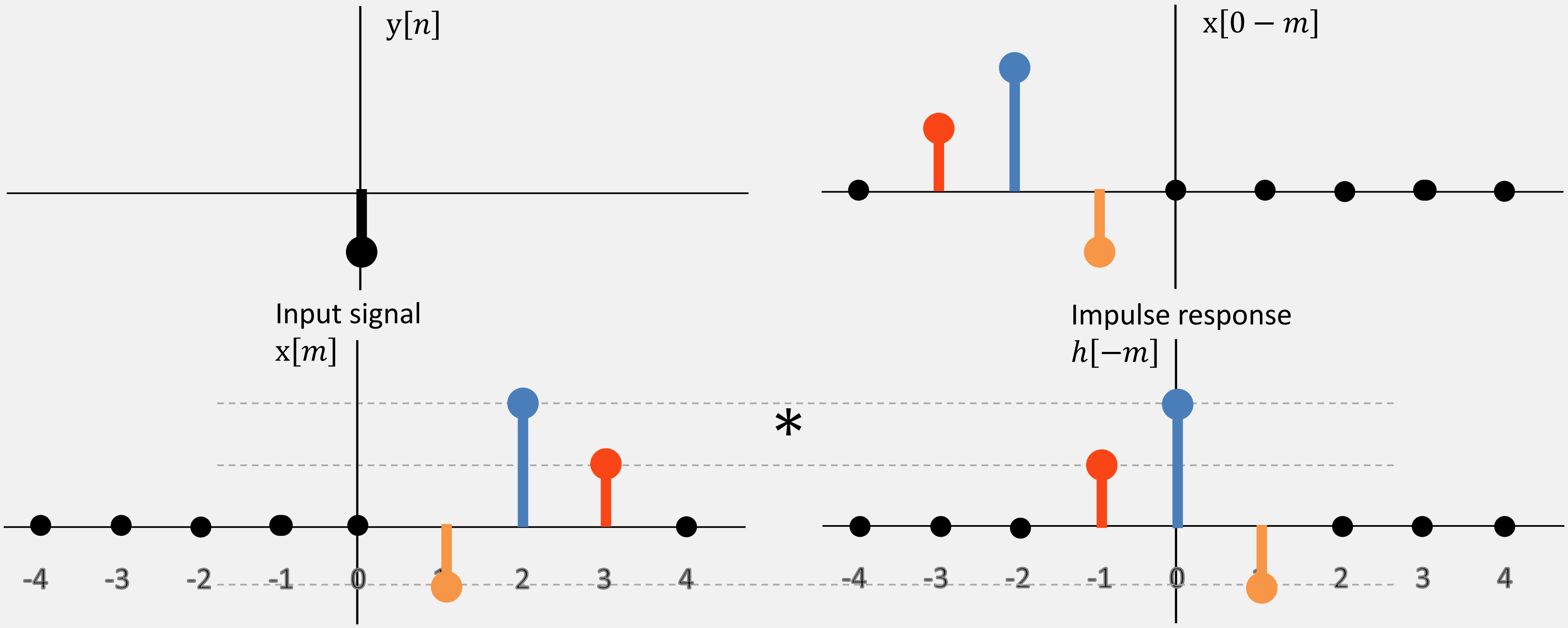

Convolution / Correlation

- **Compute the correlation:**
  $y[n] = x[-n] * h[n]$

Step 1: Time-reverse a signal
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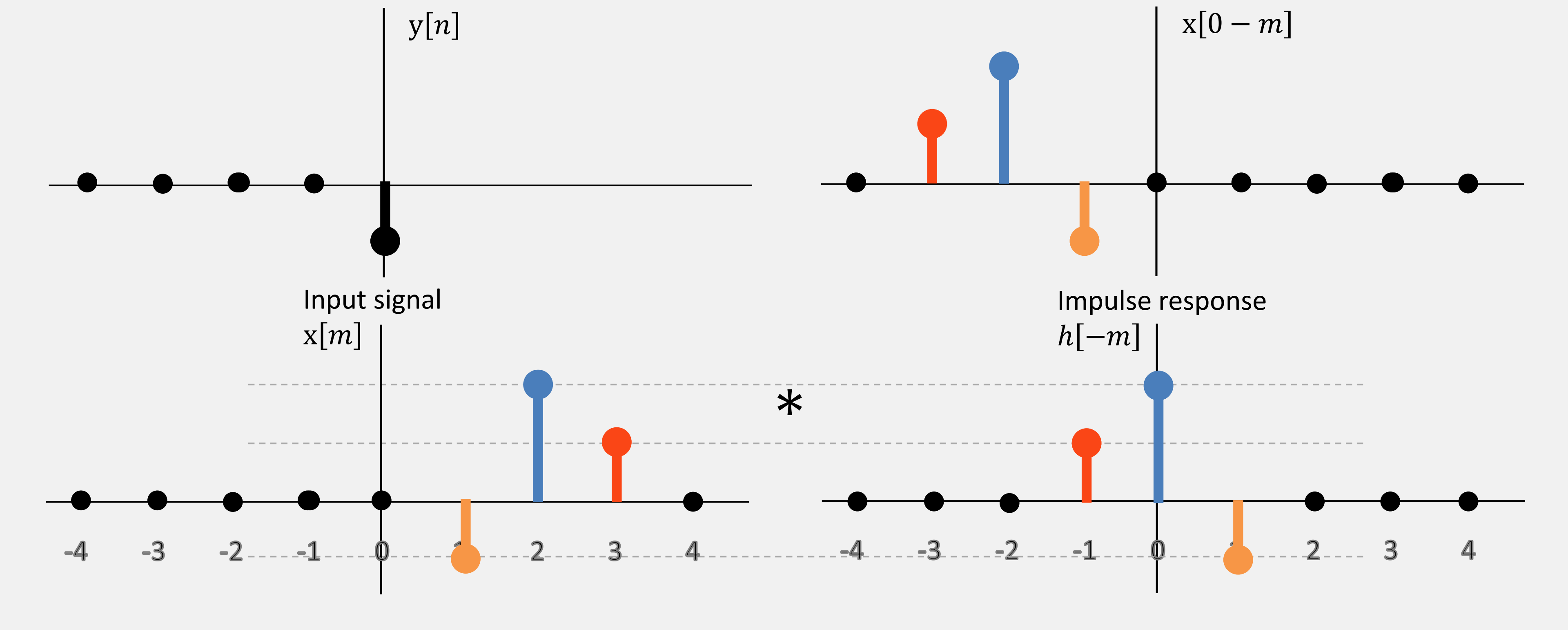

Convolution / Correlation

- **Compute the correlation:**
  $y[n] = x[-n] * h[n]$

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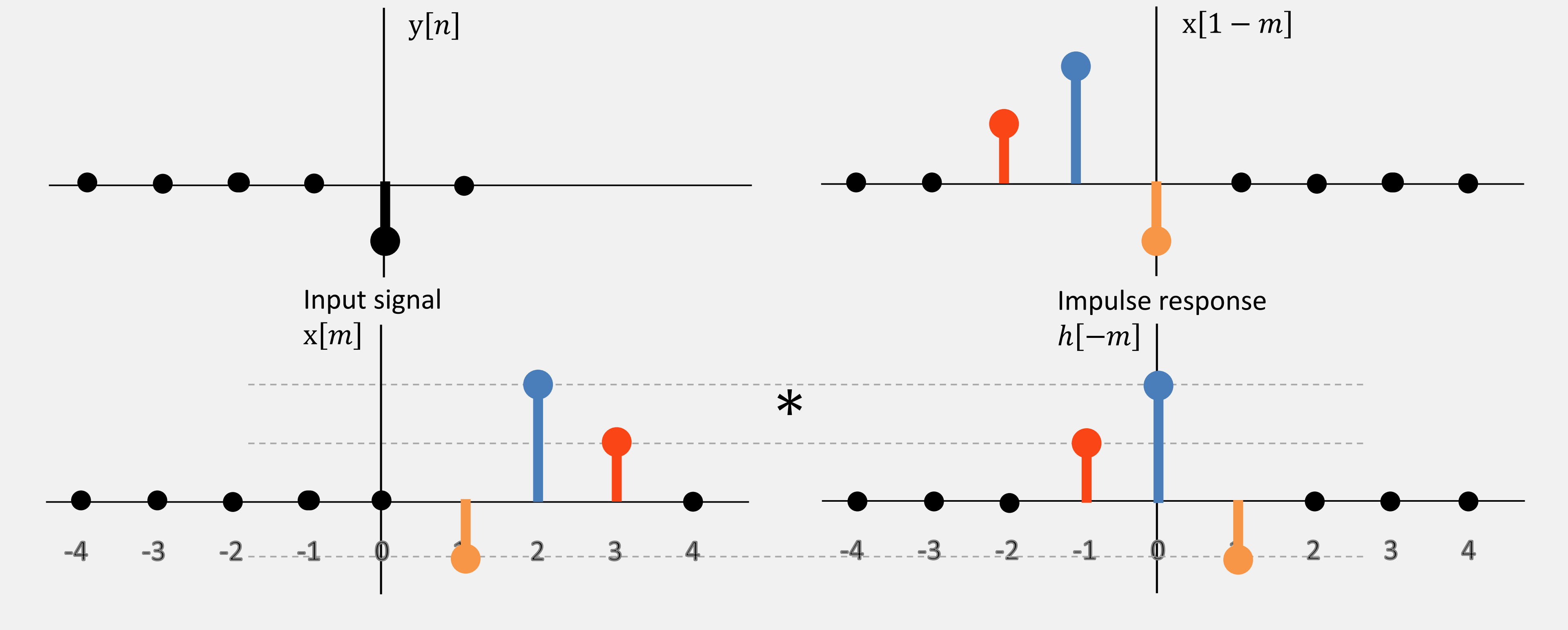

Convolution / Correlation

- **Compute the correlation:**
  $y[n] = x[-n] * h[n]$

Step 1: Time-reverse a signal
Step 2: Shift that signal by *n*
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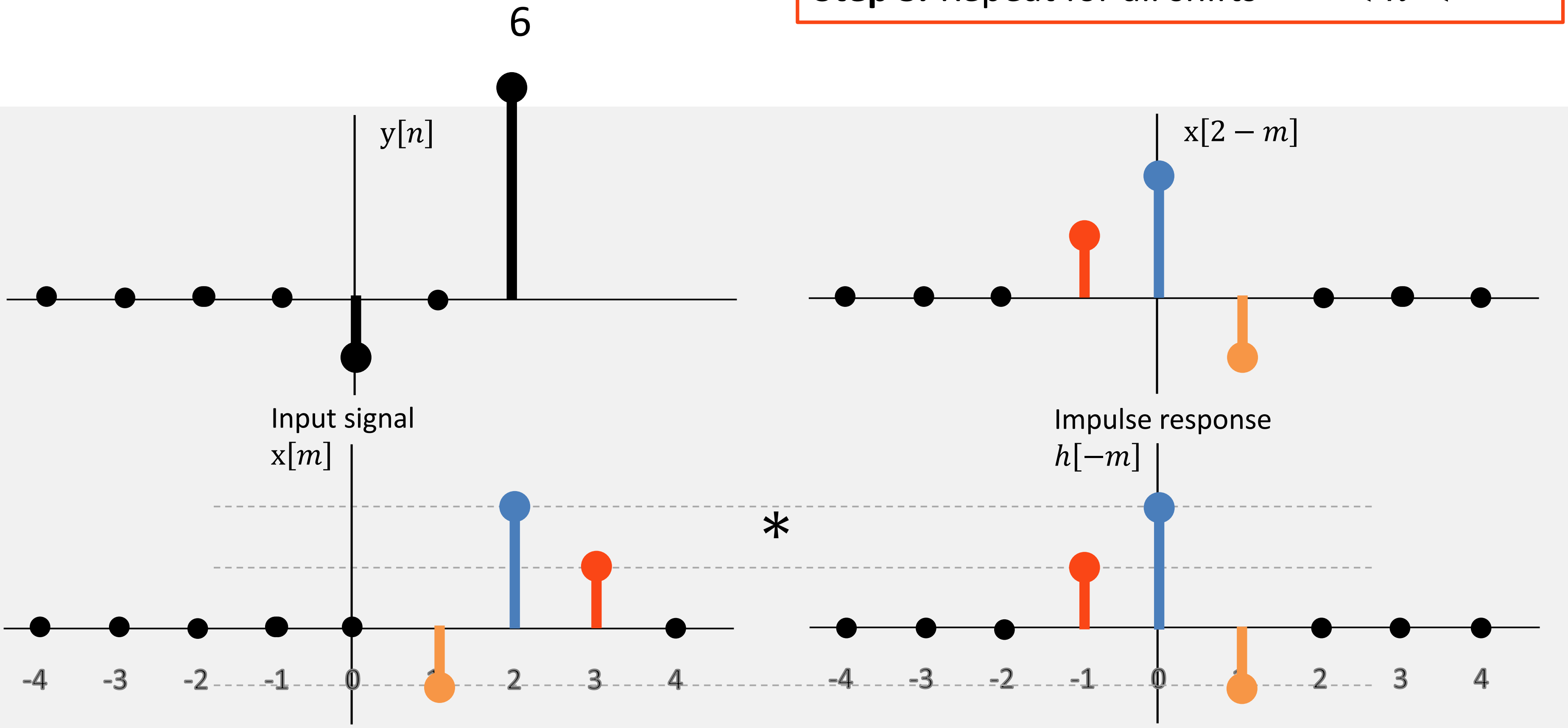

Convolution / Correlation

- **Compute the correlation:**
  $y[n] = x[-n] * h[n]$

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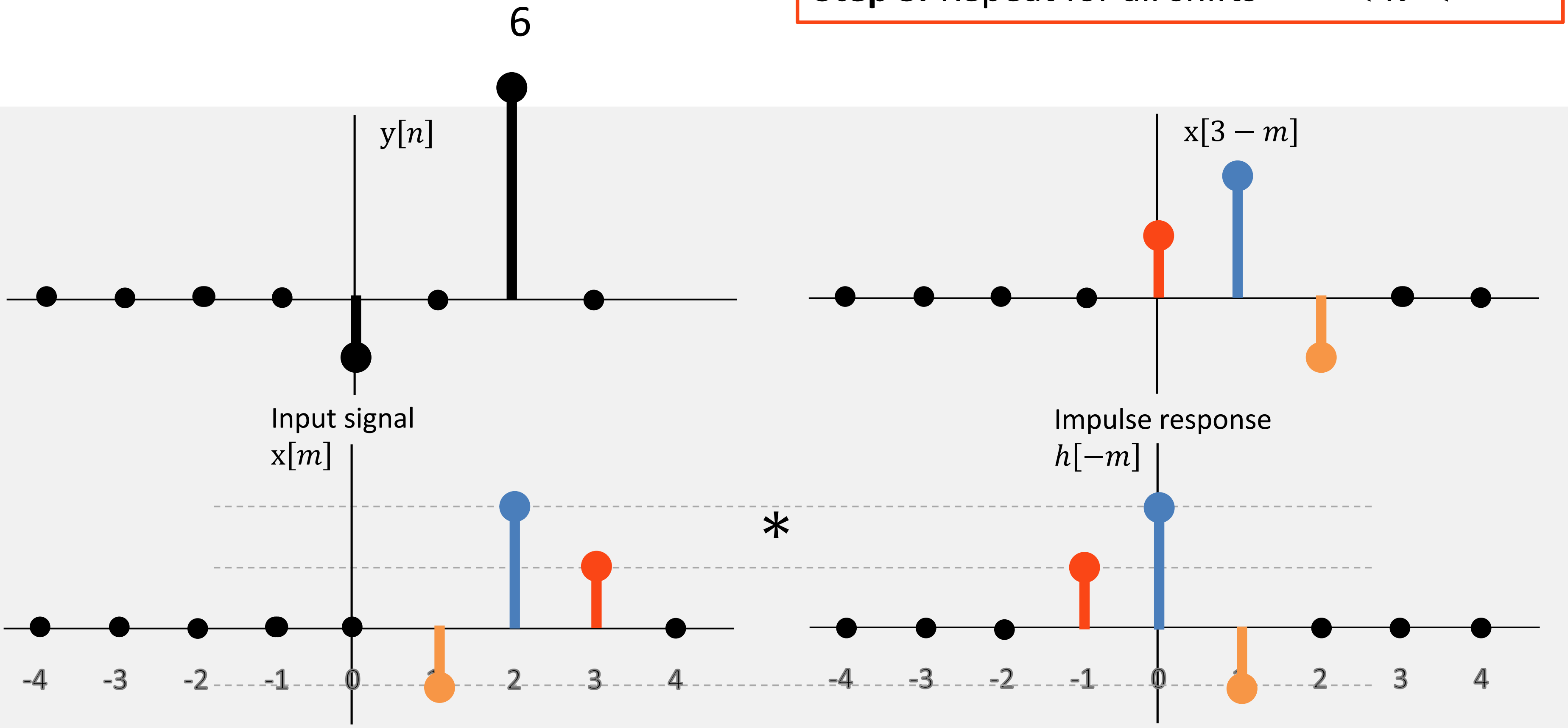

Convolution / Correlation

- **Compute the correlation:**
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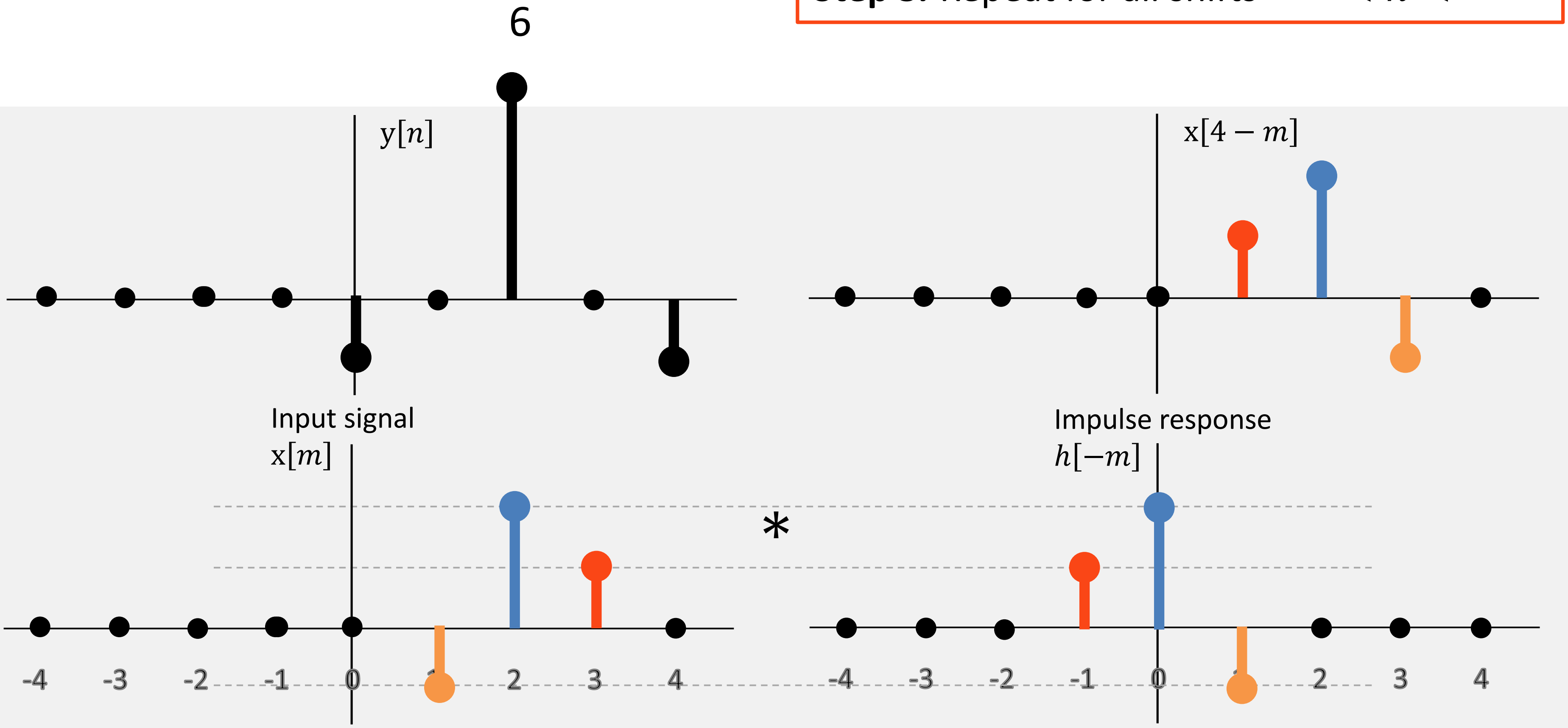

Convolution / Correlation

- **Compute the correlation:**
  $y[n] = x[-n] * h[n]$

Step 1: Time-reverse a signal
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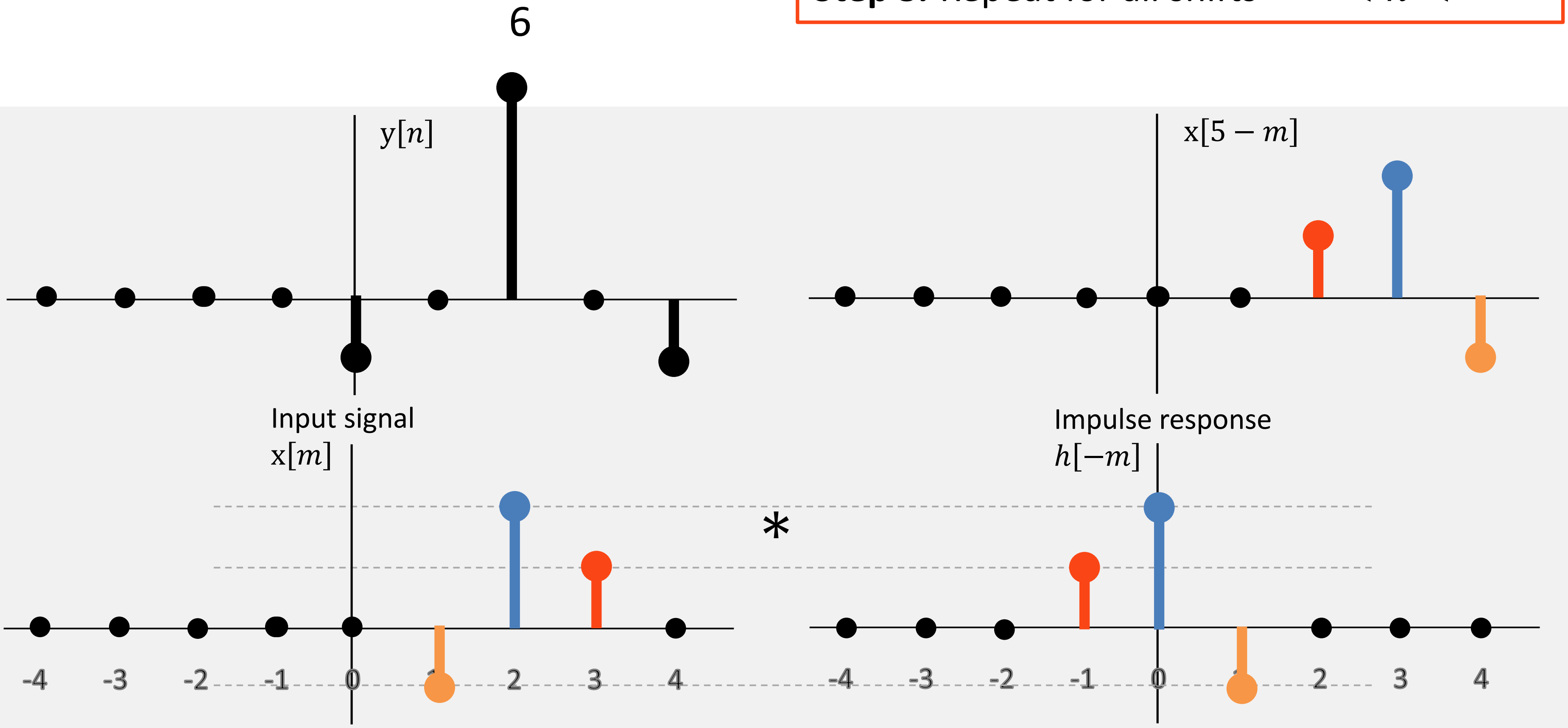
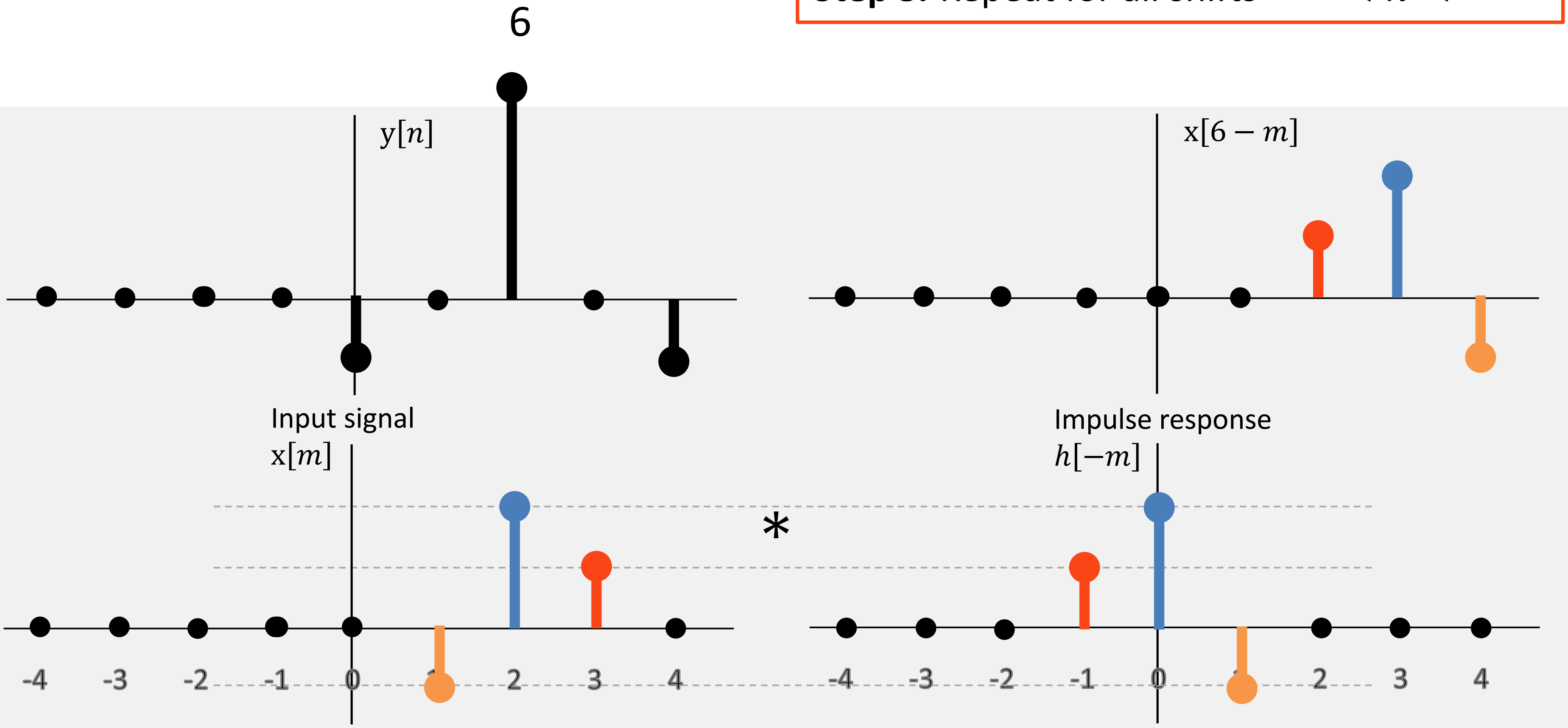


Convolution / Correlation

- **What does this achieve?**

Step 1: Time-reverse a signal
Step 2: Shift that signal by *n*
Step 3: Multiply the signals & sum the result
Step 4: Assign the sum to *y[n]* for shift *n*
Step 5: Repeat for all shifts $-\infty < n < \infty$



Convolution / Correlation

- **Question: What are applications of correlation?**

More Examples on Course Website

Convolution

- **Go to the notes on the course website!**
  - http://smartdata.ece.ufl.edu/eee5502/lecture.html?lecture=03

Lecture 4: Discrete -Time LTI Systems

Foundations of Digital Signal Processing

Outline

- Input-Output Representation Review
- Discrete-Time Convolution
- Properties of Discrete-Time Convolution
- Combining Systems
- Properties of the Impulse Response
- General Form for LTI Systems

Convolution Properties

- **Definition of convolution**

$$x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

  - **Property #1:** Commutativity
    - ◊ $x[n] * h[n] = h[n] * x[n]$
  - **Property #2:** Associativity
    - ◊ $x[n] * (h[n] * g[n]) = (x[n] * h[n]) * g[n]$
  - **Property #3:** Distributivity
    - ◊ $x[n] * (h[n] + g[n]) = (x[n] * h[n]) + (x[n] * g[n])$
  - **Property #4:** Multiplicative identity
    - ◊ $x[n] * \delta[n] = x[n]$

Convolution Properties

- **Definition of convolution**

$$x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

  - **Property #5:** Shifting property
    - ◊ $x[n] * \delta[n-N] = x[n-N]$

Convolution Properties

- **Problem: Let’s prove property #1:**

$$x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]\,h[n-m]$$

  - Show that:
    - $x[n] * h[n] = h[n] * x[n]$

Lecture 4: Discrete -Time LTI Systems

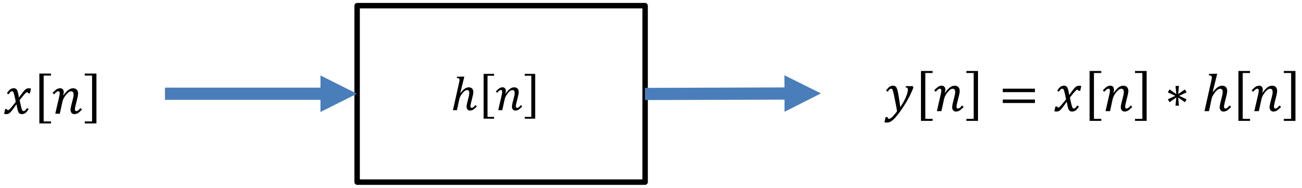
Foundations of Digital Signal Processing

Outline

- Input-Output Representation Review
- Discrete-Time Convolution
- Properties of Discrete-Time Convolution
- **Combining Systems**
- Properties of the Impulse Response
- General Form for LTI Systems

Combining Systems

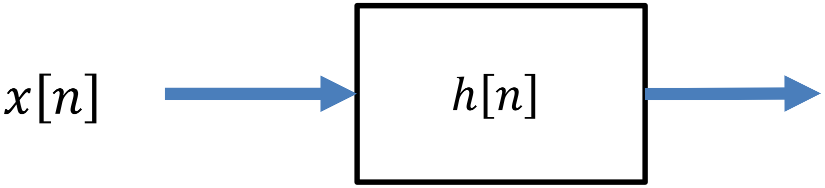
- **Basic System Block Diagram**



$x[n]$ $\xrightarrow{}$ $\boxed{h[n]}$ $\xrightarrow{}$ $y[n] = x[n] * h[n]$

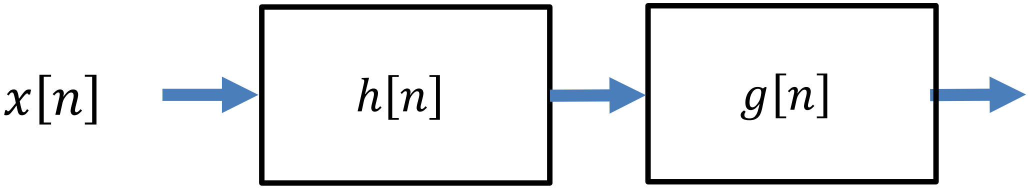
Combining Systems

- **Basic System Block Diagram**



$y[n] = x[n] * h[n]$

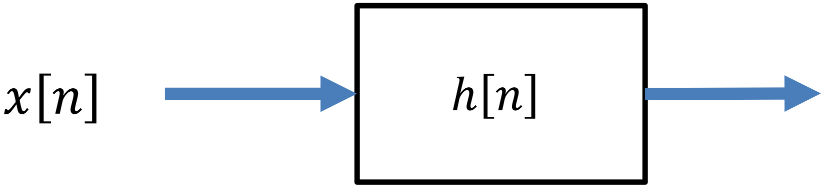
- **Cascading System (Systems in Series)**



$y[n] = x[n] * h[n] * g[n]$

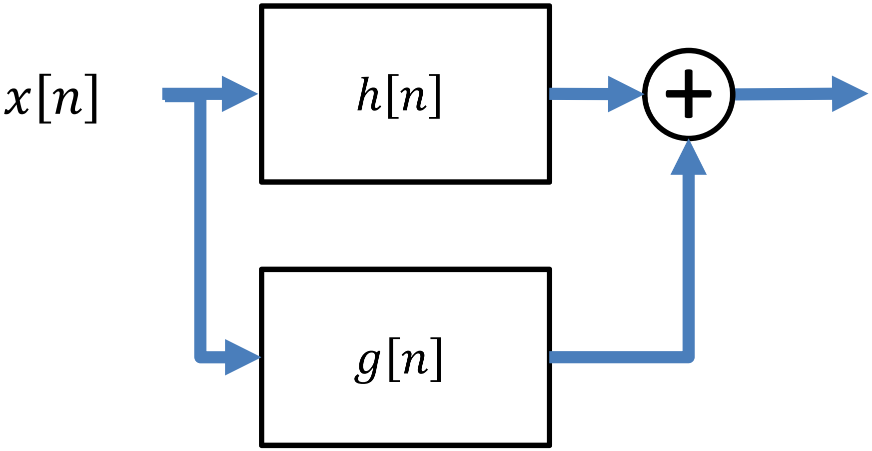
Combining Systems

- **Basic System Block Diagram**



$$y[n] = x[n] * h[n]$$

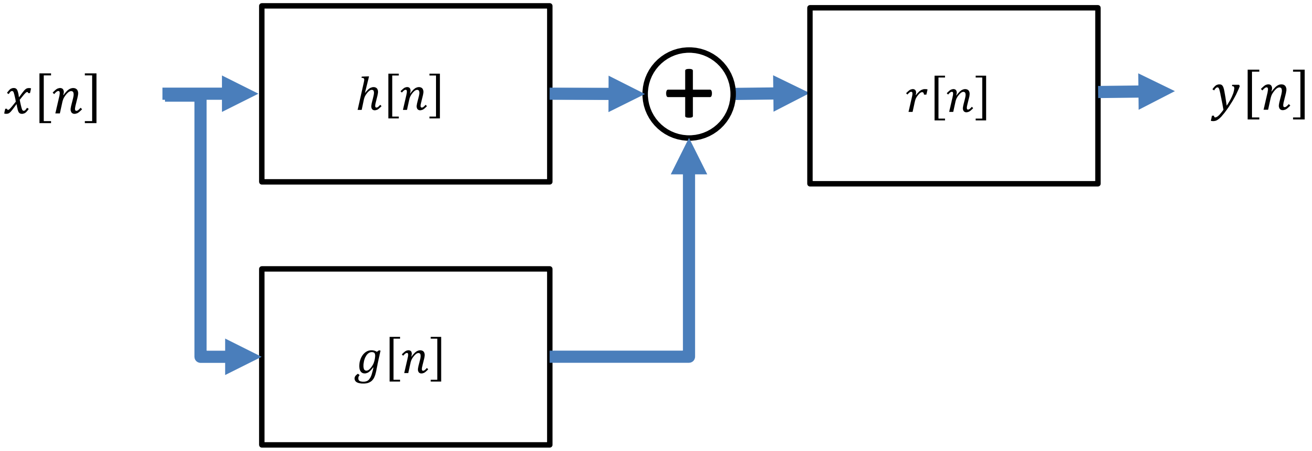
- **Systems in Parallel**



$$\begin{aligned} y[n] &= x[n] * (h[n] + g[n]) \\ &= x[n] * h[n] + x[n] * g[n] \end{aligned}$$

Combining Systems

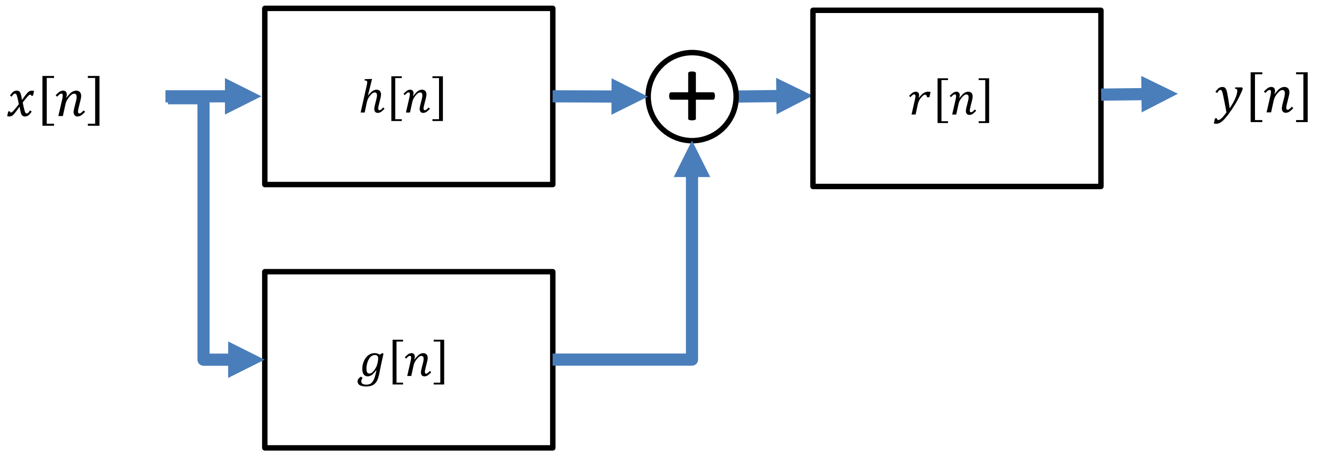
- **Problem: Compute the impulse response of the system below:**



- $h[n] = \delta[n-2], \quad \mathrm{g}[n] = \delta[n-1], \quad \mathrm{r}[n] = \delta[n-1]$

Combining Systems

- **Problem:** **Compute the impulse response of the system below:**



- $h[n] = \delta[n-2], \quad \mathrm{g}[n] = \delta[n-1], \quad \mathrm{r}[n] = \delta[n-1]$

- **Solution**
  - $y[n] = \delta[n] * (\delta[n-2] + \delta[n-1]) * \delta[n-1]$
  - $y[n] = \delta[n-2] + \delta[n-1]$

Lecture 4: Discrete -Time LTI Systems

Foundations of Digital Signal Processing

Outline

- Input-Output Representation Review
- Discrete-Time Convolution
- Properties of Discrete-Time Convolution
- Combining Systems
- **Properties of the Impulse Response**
- General Form for LTI Systems

Impulse Response Properties

- **Let an LTI system be defined by an impulse response $h[n]$**
  - **Property #1:** A system is memoryless if
    - $h[n] = A\,\delta[n]$ for some scalar A
  - **Property #2:** A system is causal if
    - $h[n] = 0 \quad \text{for} \quad n < 0$
    - That is, $h[n]$ is causal
  - **Property #3:** A system is BIBO stable is
    - $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

Impulse Response Properties

- **Linear and Time-Invariant (LTI) System**

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

Convolution!

  - Show that if $h[n] = A\delta[n]$, then the system is memoryless.

Impulse Response Properties

- **Linear and Time-Invariant (LTI) System**

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

Convolution!

  - Show that if $h[n]$ is causal, then the system is causal

Impulse Response Properties

- **Linear and Time-Invariant (LTI) System**

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

Convolution!

  - Show that if $\sum_{n=-\infty}^{\infty} |h[n]| < \infty,$ then the system is BIBO stable

Impulse Response Properties

- **Linear and Time-Invariant (LTI) System**

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

Convolution!

  - Show that if $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$, then the system is BIBO stable

  - **Solution:**

Triangle Inequality

$$|y[n]| = \left| \sum_{m=-\infty}^{\infty} x[m]\, h[n-m] \right| \le \sum_{m=-\infty}^{\infty} |x[m]h[n-m]|$$

$$\le \sum_{m=-\infty}^{\infty} |x[m]||h[n-m]|$$

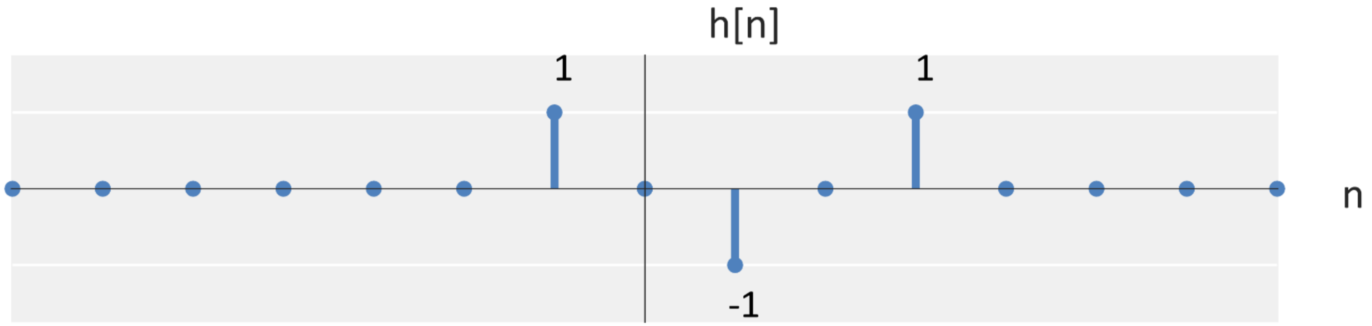
$$\le \sum_{m=-\infty}^{\infty} B_x |h[n-m]| \le B_x B_h$$

x[n] is bounded

Absolute sum of h[n] is bounded

Impulse Response Properties

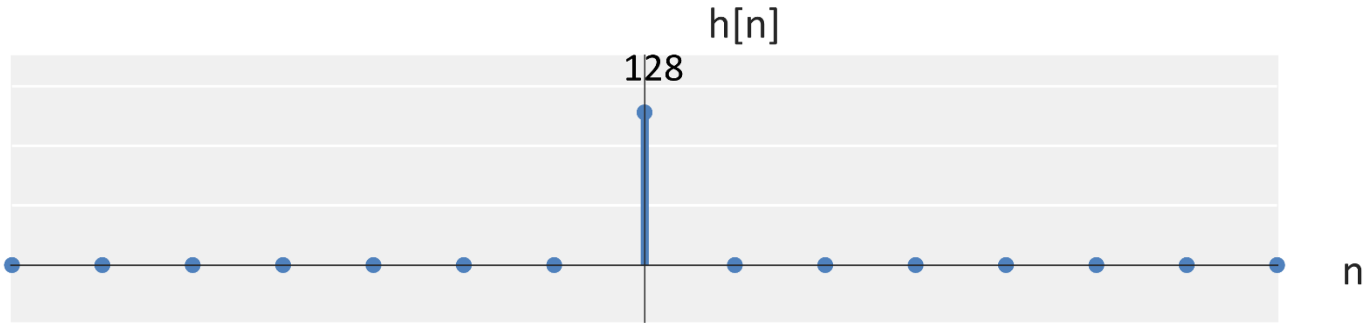
- **Linear and Time-Invariant (LTI) System**
  - Consider the system with impulse response $h[n]$



  - Is the system memoryless?
  - Is the system causal?
  - Is the system BIBO stable?

Impulse Response Properties

- **Linear and Time-Invariant (LTI) System**
  - Consider the system with impulse response $h[n]$



  - Is the system memoryless?
  - Is the system causal?
  - Is the system BIBO stable?
  - What does this system do?

Impulse Response Properties

- **Linear and Time-Invariant (LTI) System**
  - Consider the system with impulse response

$$h[n] = u[n]$$

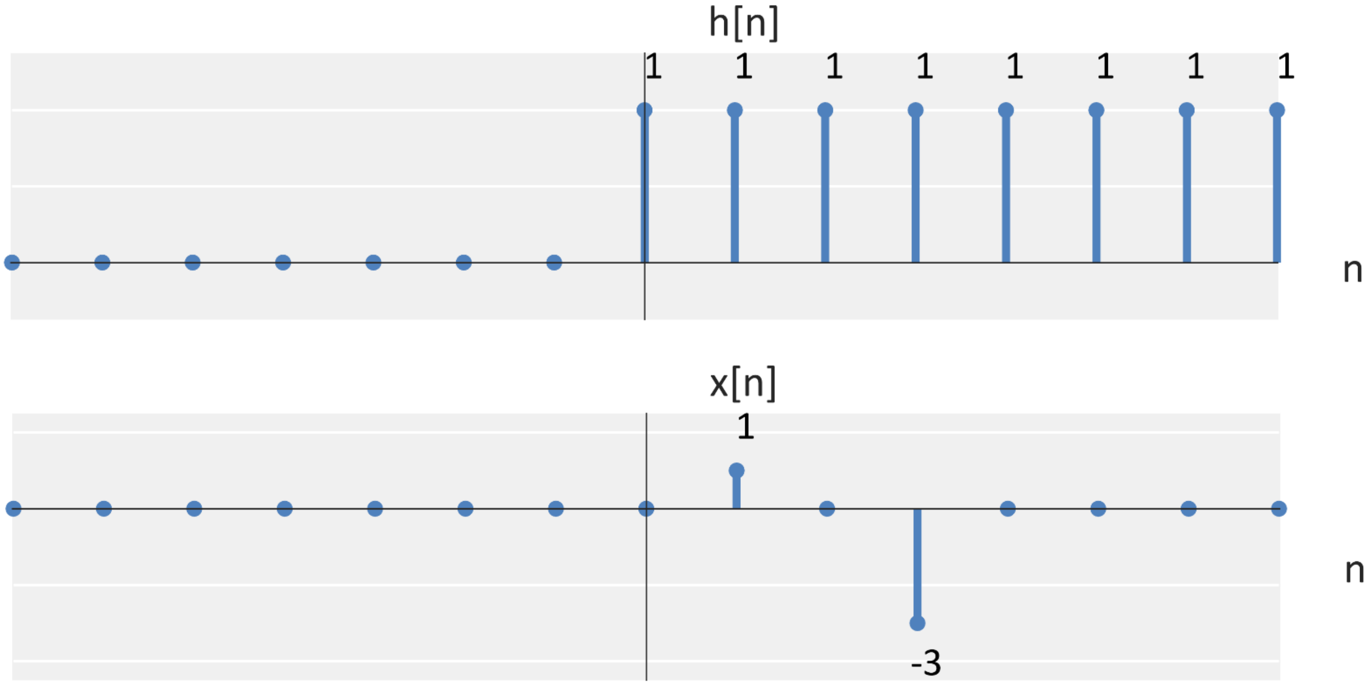
  - Is the system memoryless?
  - Is the system causal?
  - Is the system BIBO stable?
  - What does this system do?

Impulse Response Properties

- **Linear and Time-Invariant (LTI) System**
  - Consider the system with impulse response

$$h[n] = u[n]$$

  - Consider an example input $x[n] = \delta[n-1] - 3\delta[n-3]$

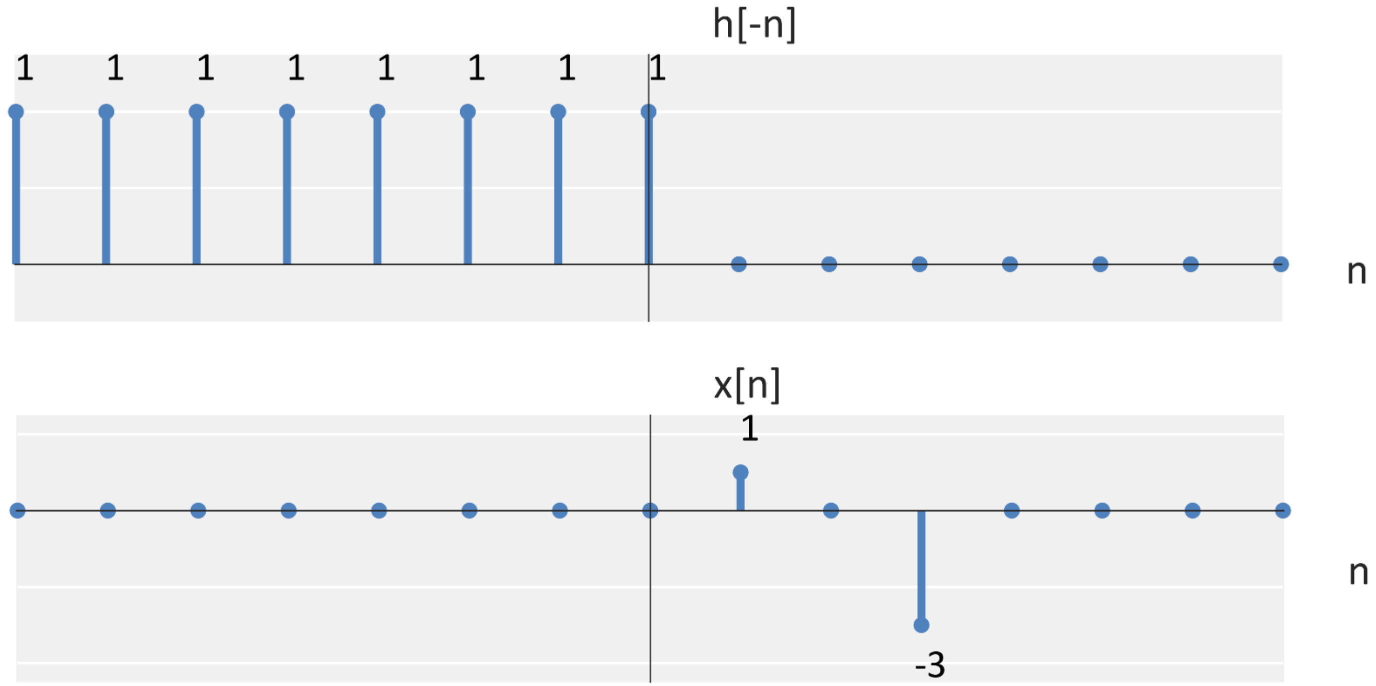


Impulse Response Properties

- **Linear and Time-Invariant (LTI) System**
  - Consider the system with impulse response

$$h[n] = u[n]$$

  - Consider an example input $x[n] = \delta[n-1] - 3\delta[n-3]$

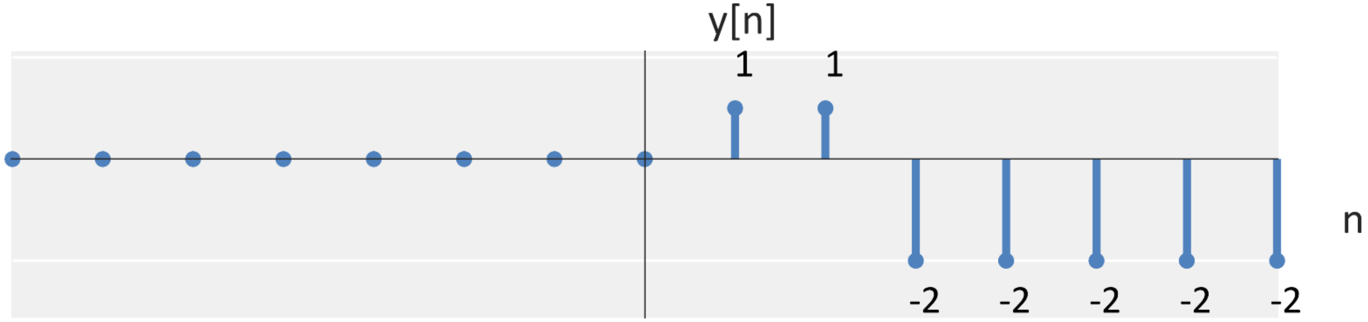


Impulse Response Properties

- **Linear and Time-Invariant (LTI) System**
  - Consider the system with impulse response

$$h[n] = u[n]$$

  - Consider an example input $x[n] = \delta[n-1] - 3\delta[n-3]$



Impulse Response Properties

- **Linear and Time-Invariant (LTI) System**
  - Consider the system with impulse response

$$h[n] = u[n]$$

  - **Hence, a system with this impulse response is equivalent to**

$$y[n] = \sum_{m=-\infty}^{n} x[m]$$

Lecture 4: Discrete -Time LTI Systems

Foundations of Digital Signal Processing

Outline

- Input-Output Representation Review
- Discrete-Time Convolution
- Properties of Discrete-Time Convolution
- Combining Systems
- Properties of the Impulse Response
- **General Form for LTI Systems**

General LTI System

- **Is there a general way to express LTI systems?**
  - Yes, with difference equations.

- **General form for an LTI system is:**

$$\sum_{m=-\infty}^{\infty} a[m]y[n-m] = \sum_{m=-\infty}^{\infty} b[m]x[n-m]$$

$$a[n] * y[n] = b[n] * x[n]$$

General LTI System

- **General form for an LTI system is:**

$$\sum_{m=-\infty}^{\infty} a[m]y[n-m] = \sum_{m=-\infty}^{\infty} b[m]x[n-m]$$

$$a[n] * y[n] = b[n] * x[n]$$

- **Example system: What does this system do?**
  - $y[n] + (-1.1)y[n-1] = x[n]$
  - Or… $y[n] = (1.1)y[n-1] + x[n]$

General LTI System

- **General form for an LTI system is:**

$$\sum_{m=-\infty}^{\infty} a[m]y[n-m] = \sum_{m=-\infty}^{\infty} b[m]x[n-m]$$

$$a[n] * y[n] = b[n] * x[n]$$

- **Example system: How can we analyze these recursive systems?**
  - $y[n] + (-1.1)y[n-1] = x[n]$
  - Or… $y[n] = (1.1)y[n-1] + x[n]$