Lecture 16: Filter Implementation… Again.

Foundations of Digital Signal Processing

Outline

- Reviewing different types of filters
- Designing the phase response
- Implementation of FIR Filters
- Implementation of IIR Filters

News

- **Homework #7**
  - Due *this Thursday*
  - Submit via canvas
- **Coding Problem #4**
  - Due *this Thursday*
  - Submit via canvas
- **In two weeks**
  - Exam #2 (yay!)

News

- **Next week**
  - Guest lectures
  - Tuesday: Resampling
  - Thursday: Review for Exam #2

- **Practice Exam**
  - Will come out early next week

Lecture 16: Filter Implementation… Again.

Foundations of Digital Signal Processing

Outline

- **Reviewing different types of filters**
- Designing the phase response
- Implementation of FIR Filters
- Implementation of IIR Filters

Designing the Magnitude Response

- **Question: What types of filters are there?**
  - Low pass
  - High pass
  - Band pass
  - All pass

Designing the Magnitude Response

- **Question: How do I turn a low pass filter into other filters?**
  - Low pass (prototype filter) $H(\omega)$
  - High pass
  - Band pass
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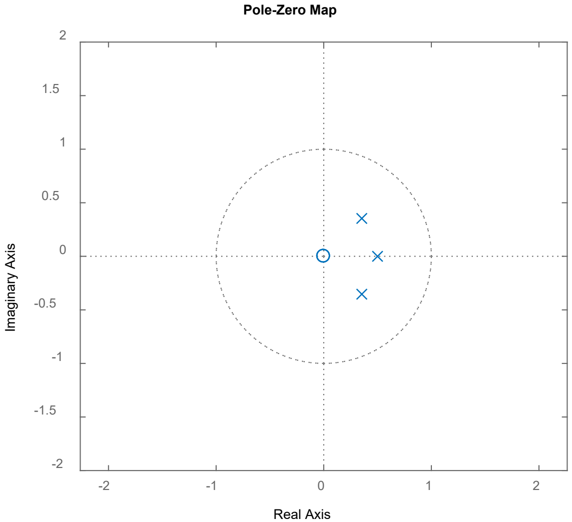

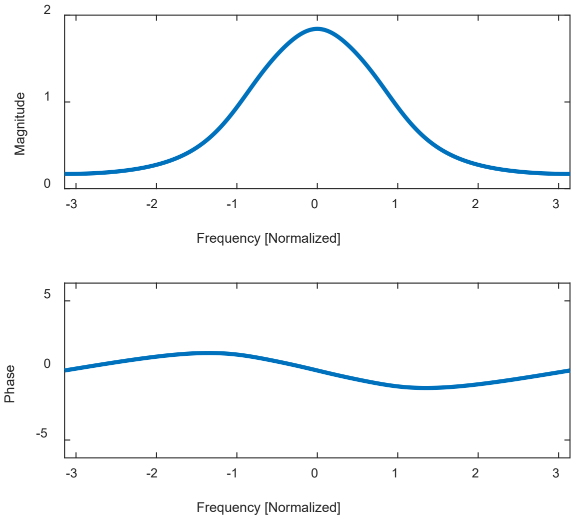

Designing the Magnitude Response

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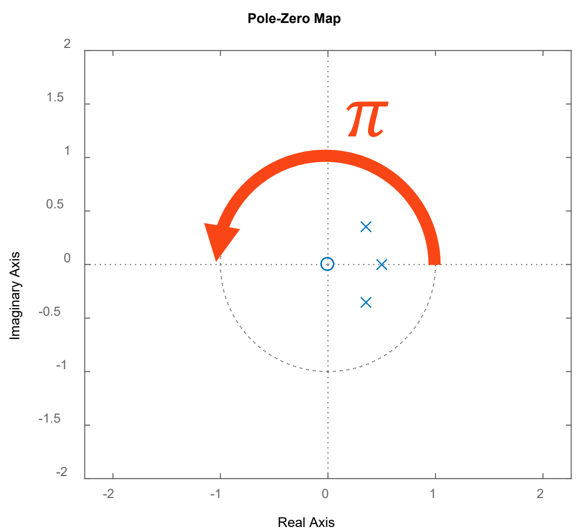

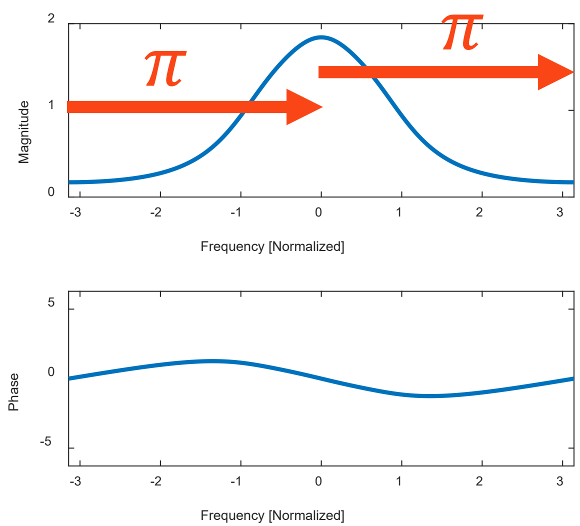

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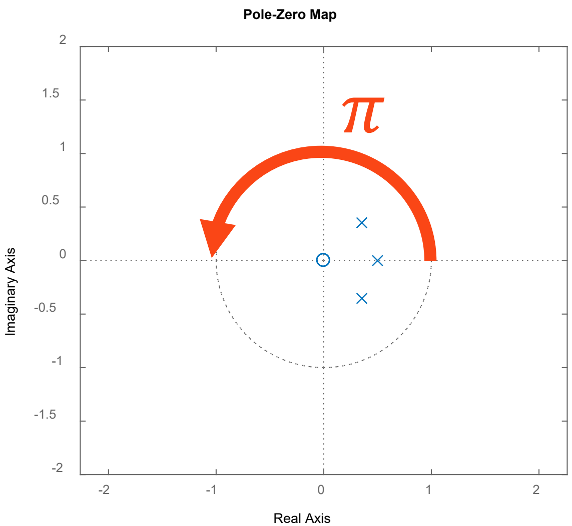

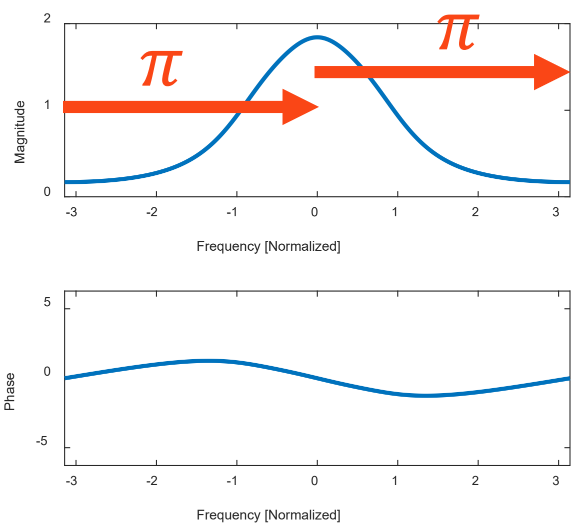

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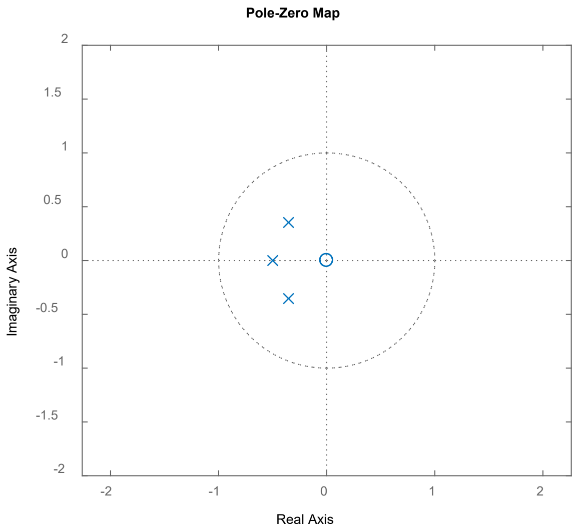
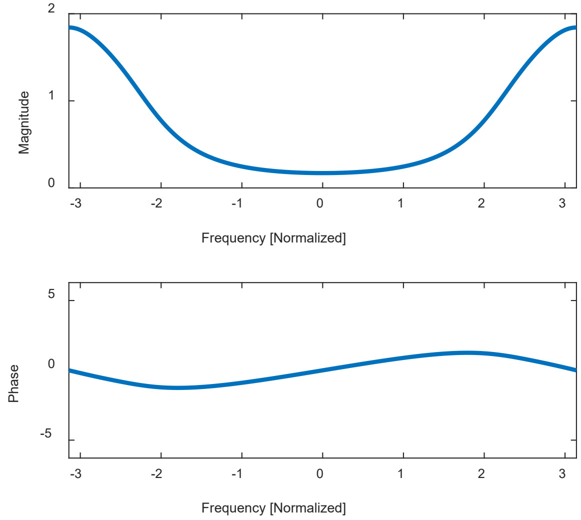
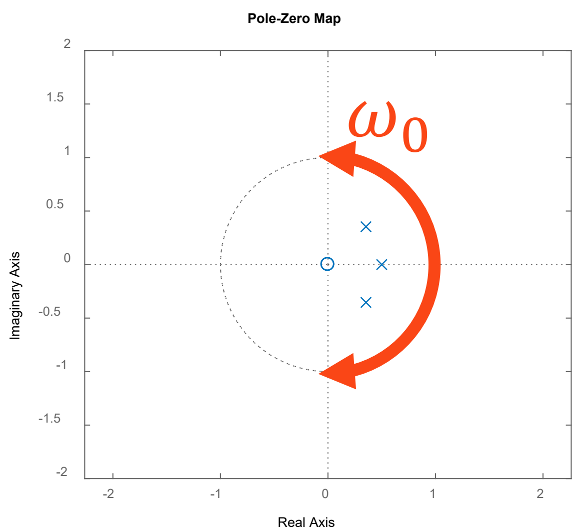
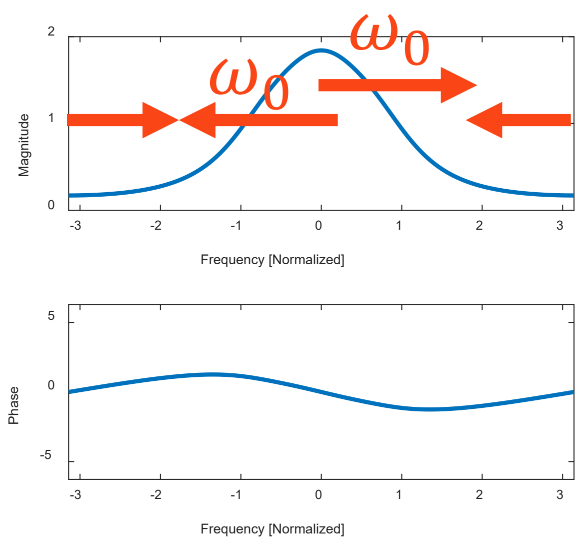




Designing the Magnitude Response

- **Question: How do I make a notch filter?**
  - Low pass (prototype filter) $H(\omega)$
  - High pass $H(\omega - \pi)$
  - Band pass $H(\omega - \omega_0) + H(\omega + \omega_0)$
  - All pass



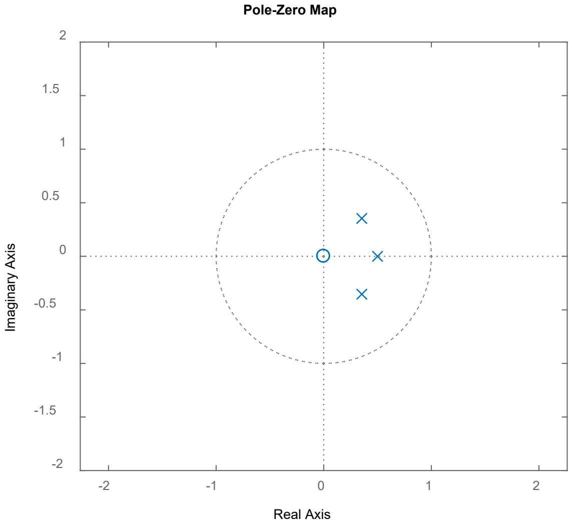


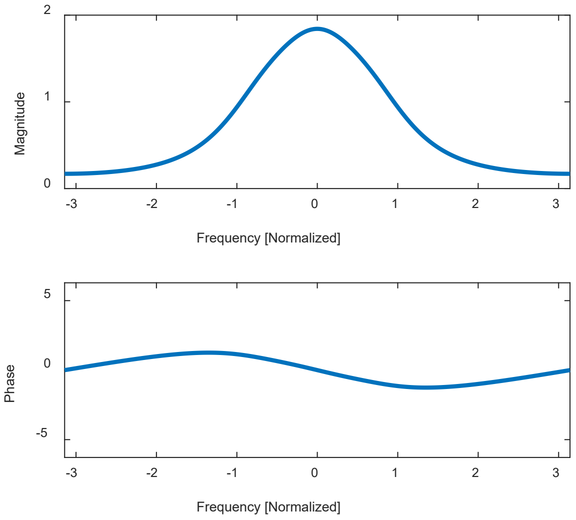
Designing the Magnitude Response

- **Question: How do I make an all pass filter?**
  - Low pass (prototype filter) $H(\omega)$
  - High pass $H(\omega - \pi)$
  - Band pass $H(\omega - \omega_0) + H(\omega + \omega_0)$
  - All pass $H(\omega) = z^{-N} \frac{A(z^{-1})}{A(z)}$

Flip poles / zeros around the unit circle

Swap Poles and Zeros





Designing the Magnitude Response

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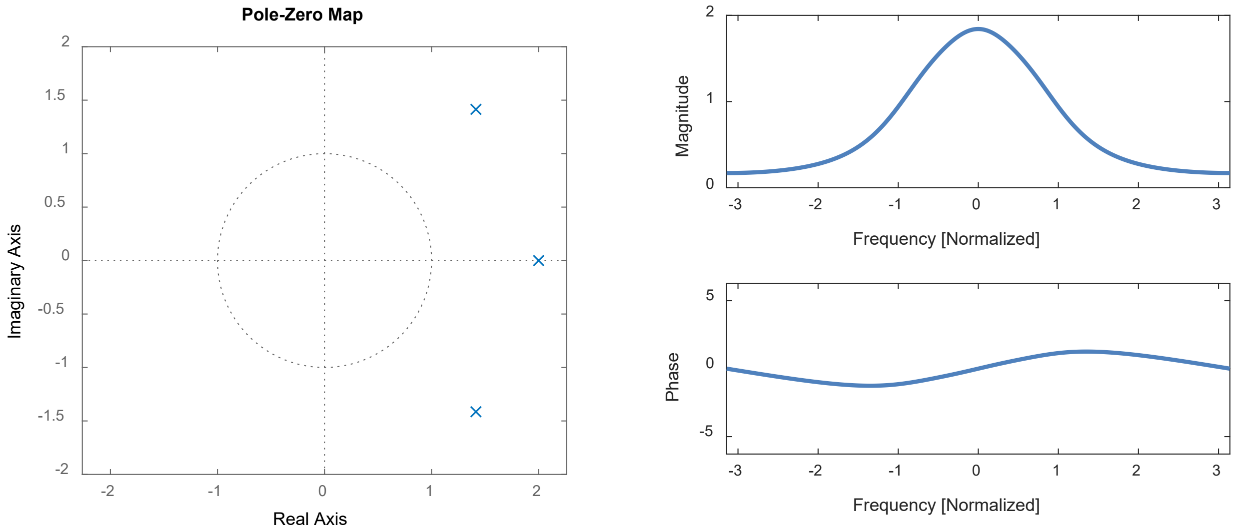

Designing the Magnitude Response

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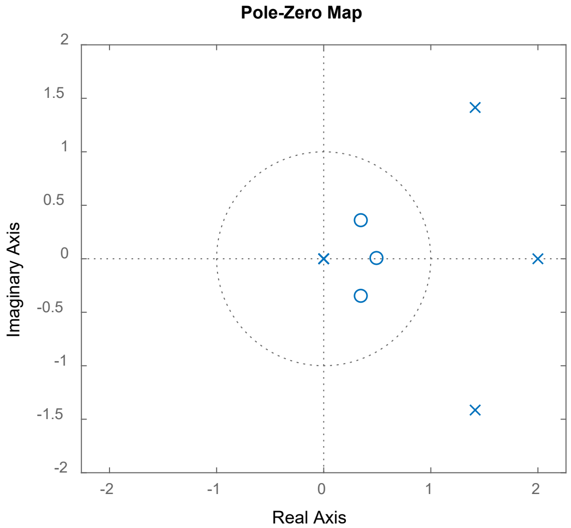
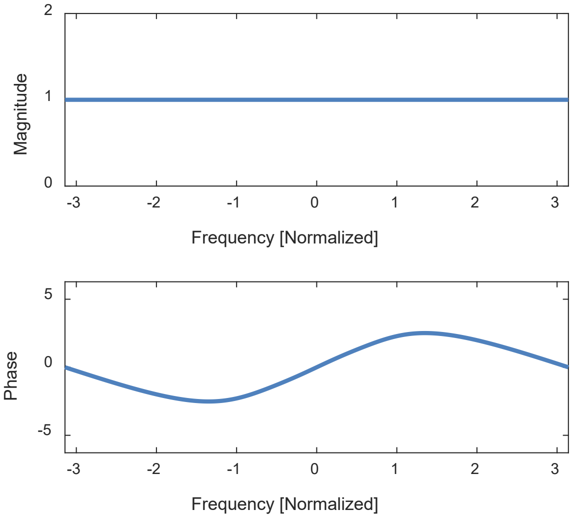




Designing the Magnitude Response

- **Question: What are these operations in time?**
  - Low pass (prototype filter)
    - ◊ $H(\omega)$
  - High pass
    - ◊ $H(\omega - \pi)$
  - Band pass
    - ◊ $H(\omega - \omega_0) + H(\omega + \omega_0)$
  - All pass
    - ◊ $H(\omega) = z^{-N} \frac{A(z^{-1})}{A(z)}$

Designing the Magnitude Response

- **Question: What are these operations in time?**
  - Low pass (prototype filter)
    - $H(\omega)$
  - High pass
    - $H(\omega - \pi)$
    - $h[n]\cos(\pi n)$
  - Band pass
    - $H(\omega - \omega_0) + H(\omega + \omega_0)$
    - $2\, h[n]\cos(\omega_0 n)$
  - All pass
    - $H(\omega) = z^{-N}\dfrac{A(z^{-1})}{A(z)}$

Time-reversal

De-convolution (inverse convolution)

Designing the Magnitude Response

- **Question: What is a de-convolution in frequency?**

Designing the Magnitude Response

- **Question: What is a de-convolution in frequency?**

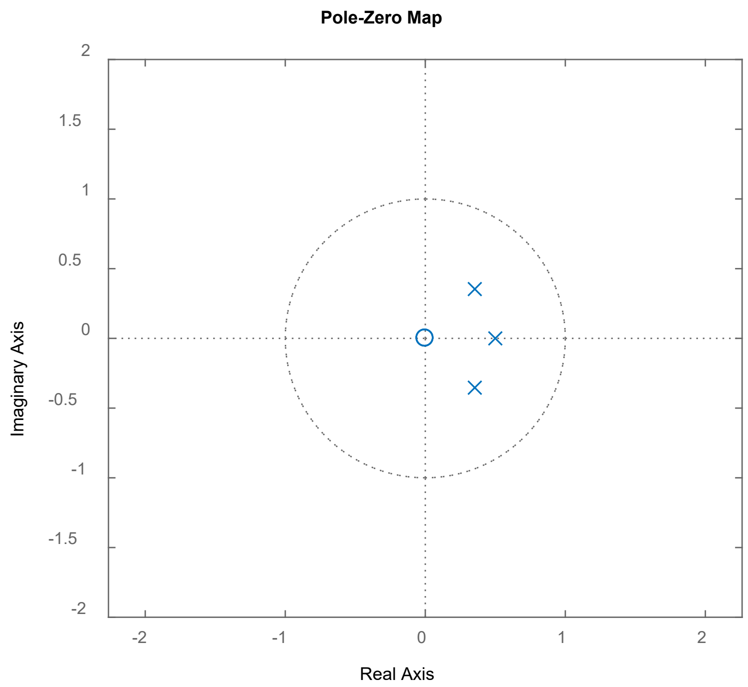
- **Answer: Inverse**

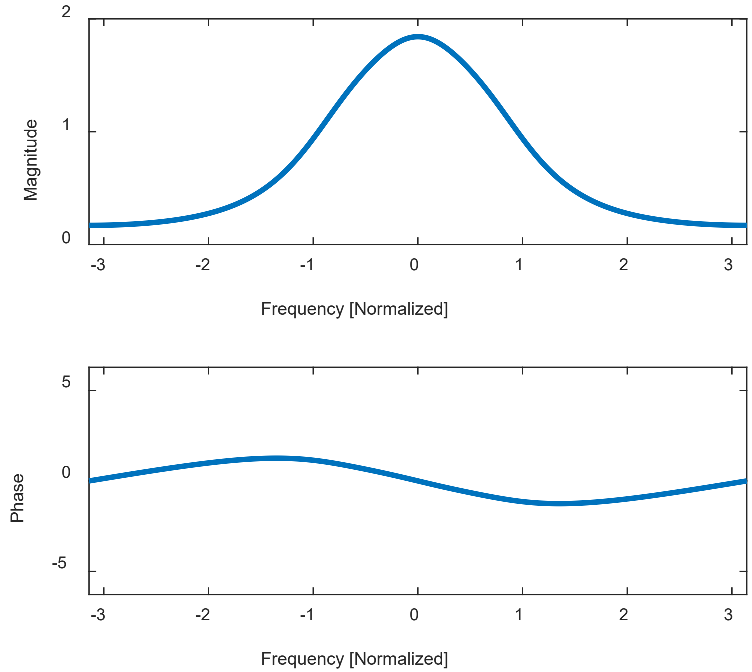
- $Y(\omega) = H(\omega)X(\omega)$

- $X(\omega) = \frac{Y(\omega)}{H(\omega)}$

Designing the Magnitude Response

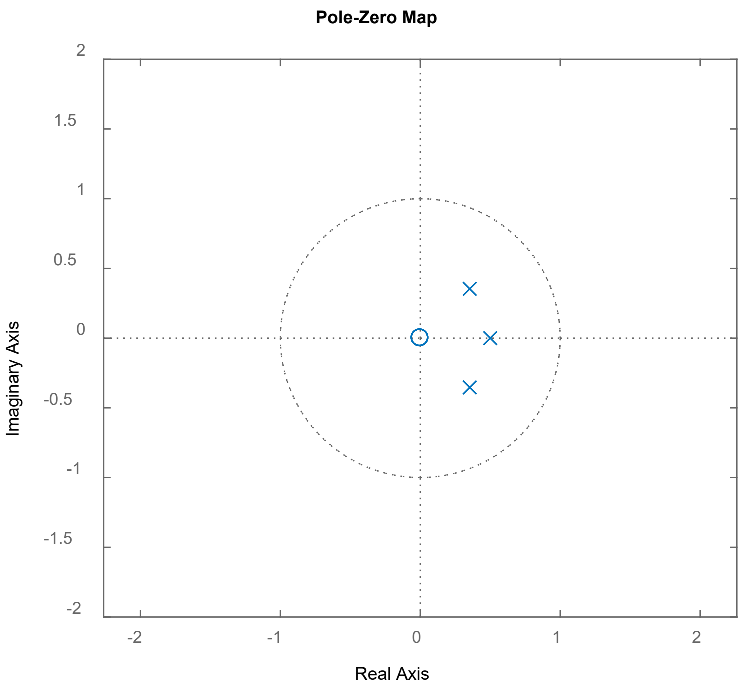
- **Question: If all <u>poles</u> are inside the unit circle, what does this mean?**

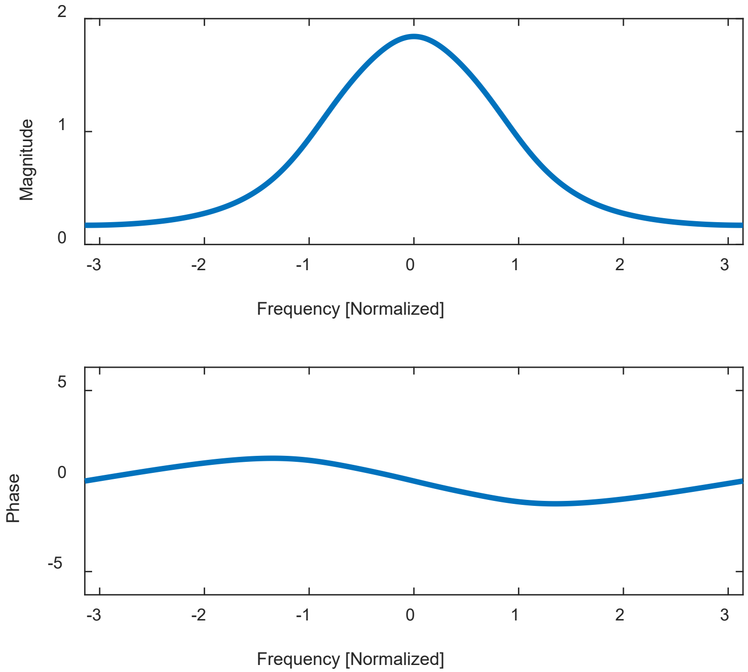




Designing the Magnitude Response

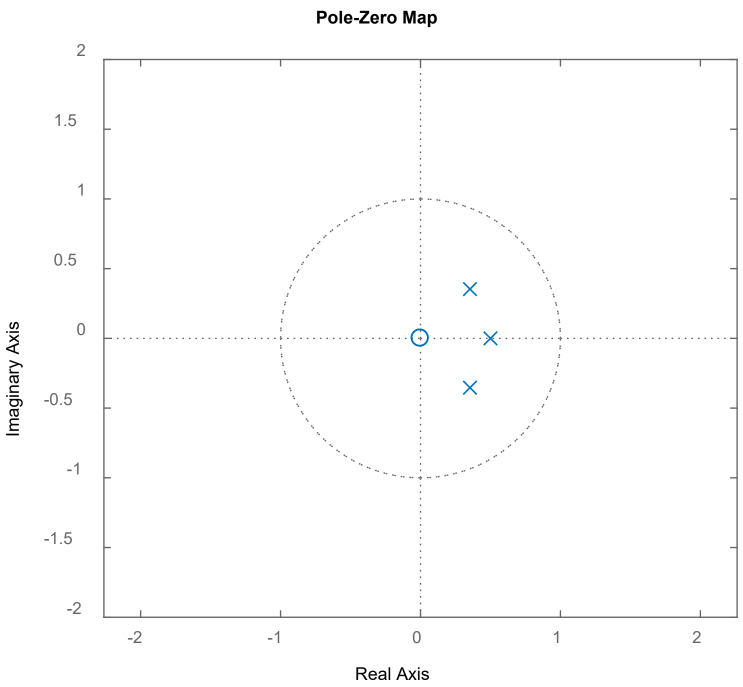
- **Question:** **If all <u>poles</u> are inside the unit circle, what does this mean?** **System is stable**

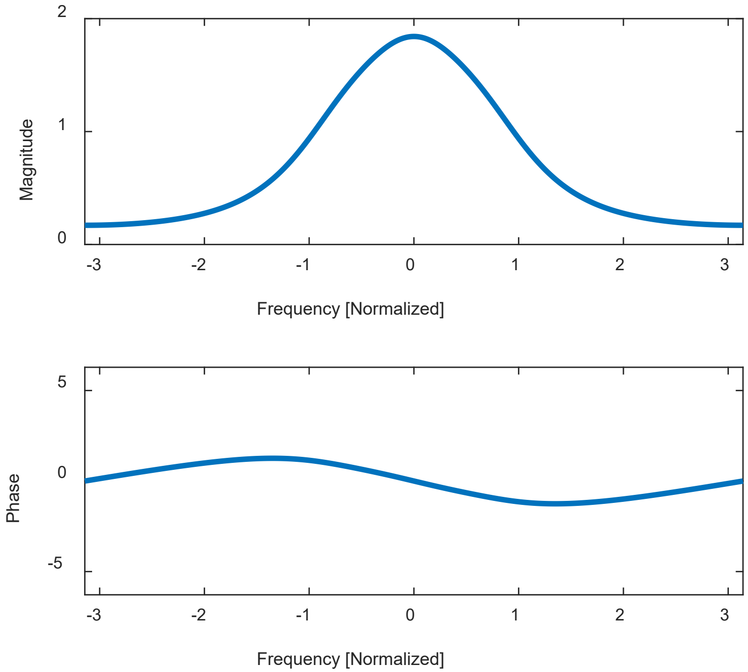




Designing the Magnitude Response

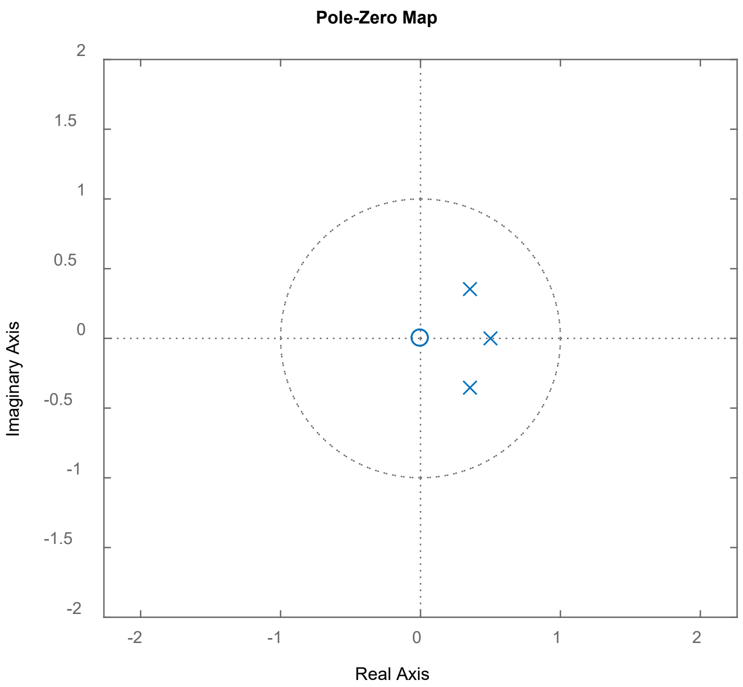
- **Question: If all <u>zeros</u> are inside the unit circle, what does this mean?**

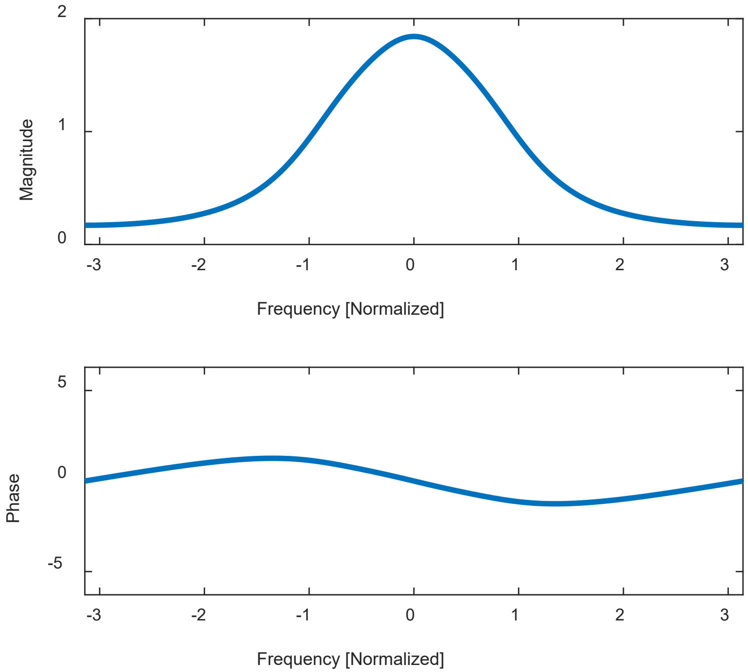




Designing the Magnitude Response

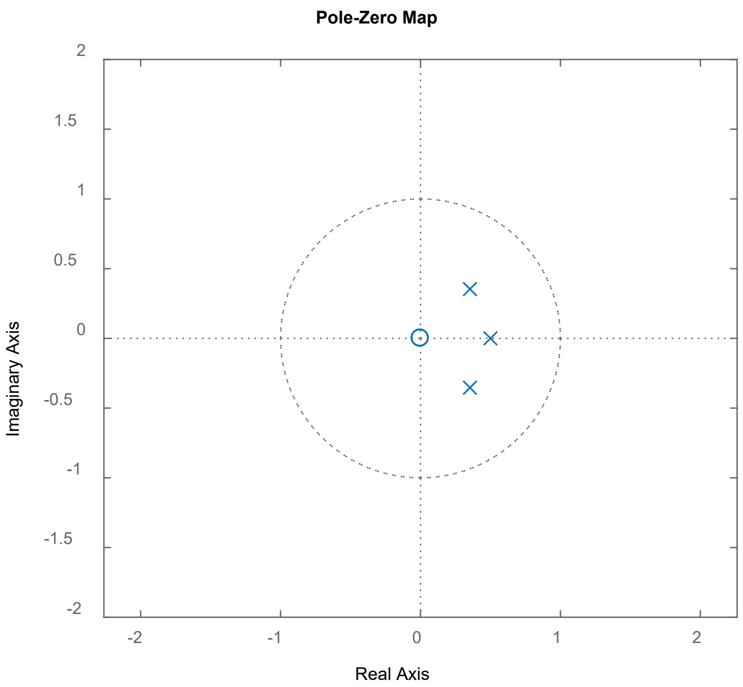
- **Question: If all <u>zeros</u> are inside the unit circle, what does this mean? System inverse is stable**

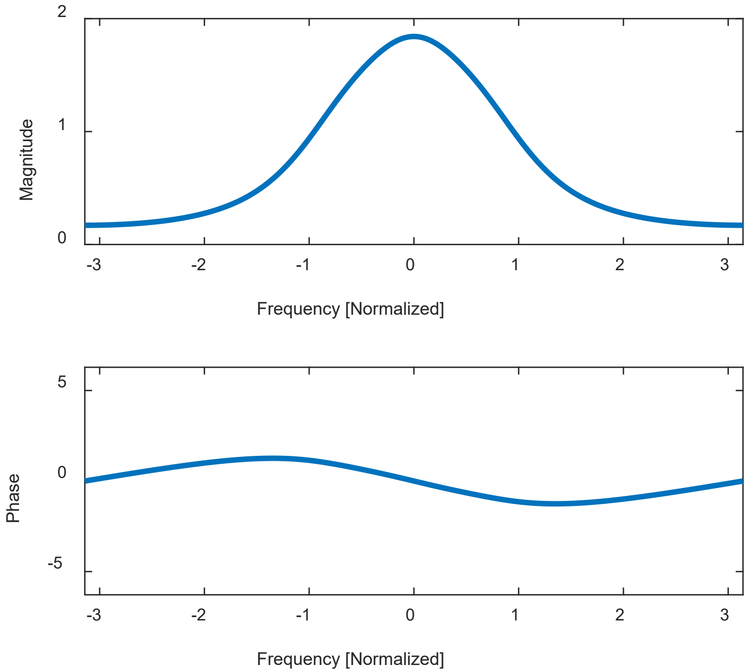




Designing the Magnitude Response

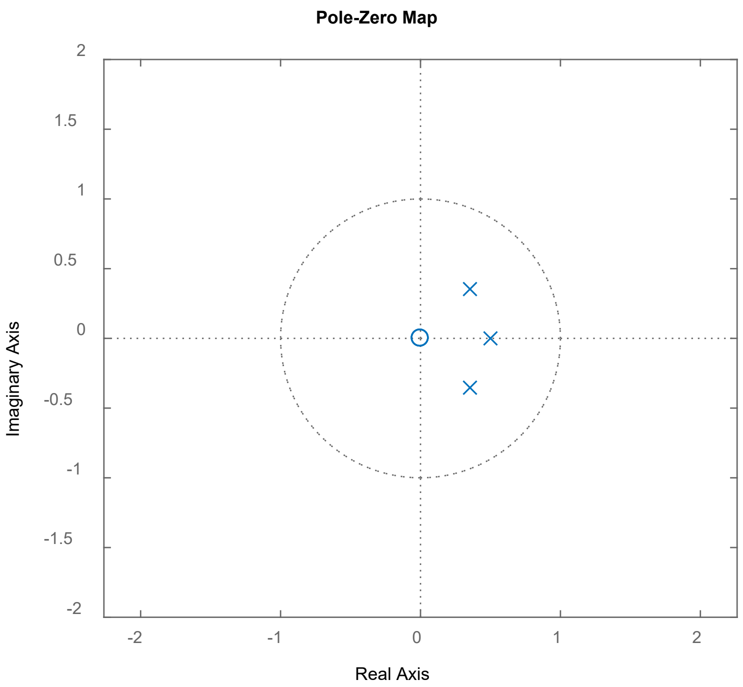
- **Question: If all poles and zeros are inside the unit circle, what does this mean?**

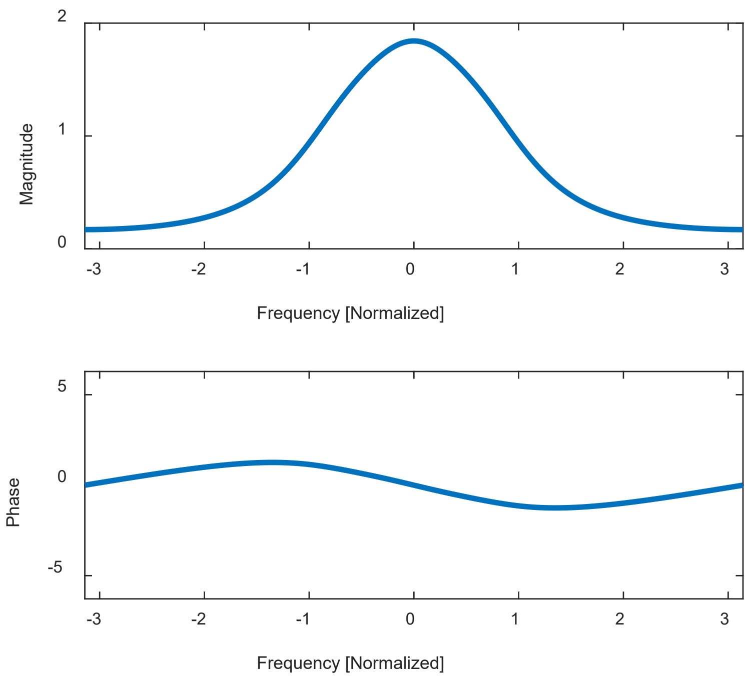




Designing the Magnitude Response

- **Question: If all poles and zeros are inside the unit circle, what does this mean? System and its inverse are stable**

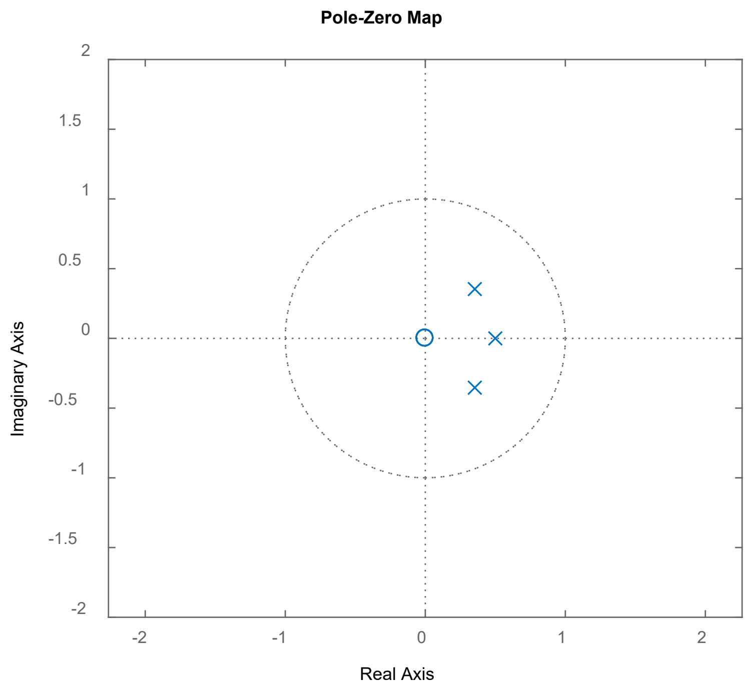


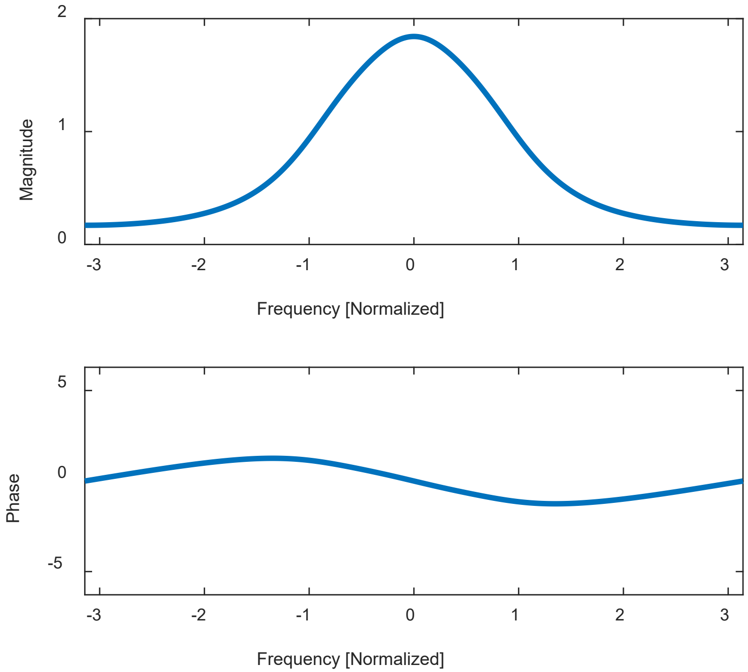


Designing the Magnitude Response

- **Question: If all poles and zeros are inside the unit circle, what does this mean?**

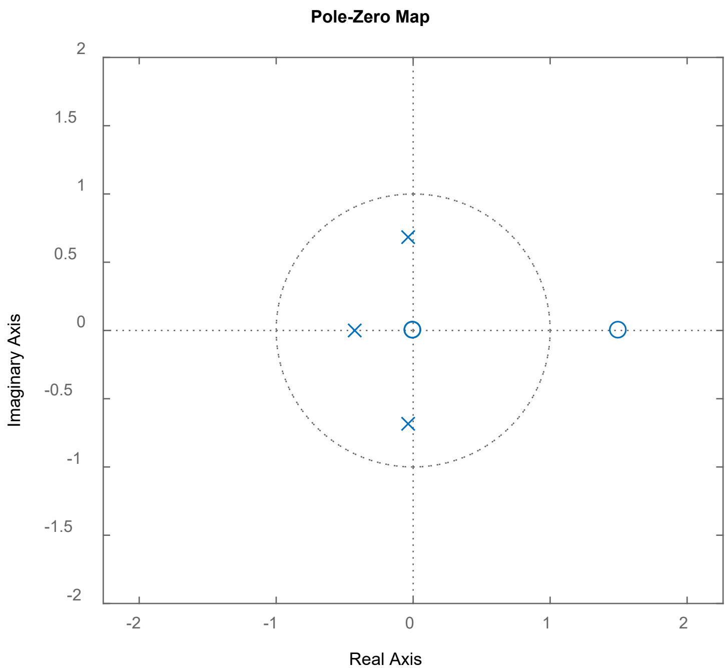
- **We call these types of filters minimum phase filters.**





Designing the Magnitude Response

- **Example: Is the filter defined by the following pole-zero plot…**
  - A low pass, high pass, or band pass filter?
  - Stable?
  - Have a Stable Inverse?



Designing the Magnitude Response

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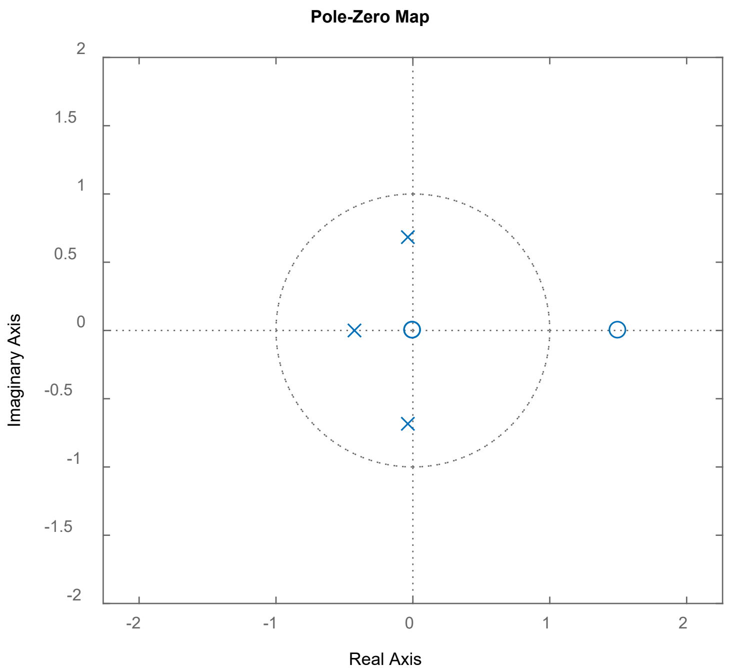

Designing the Magnitude Response

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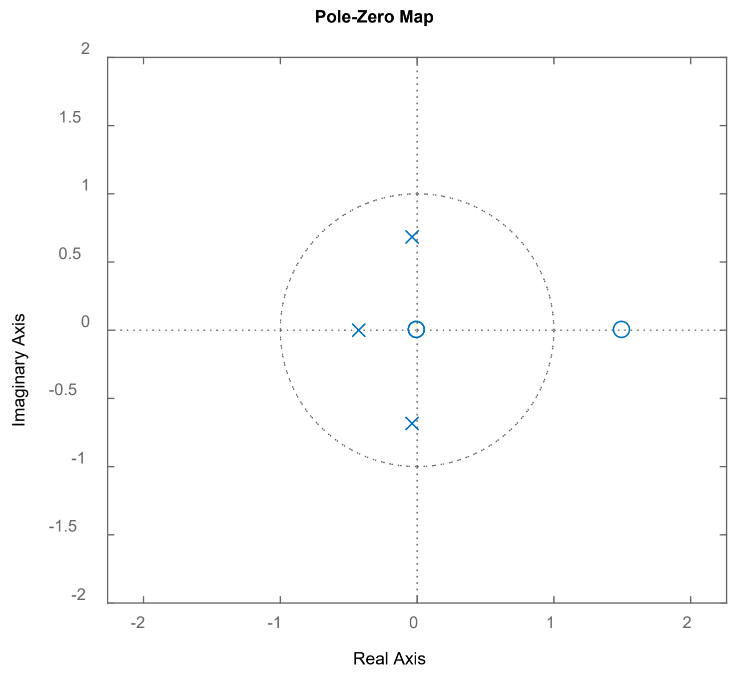

Designing the Magnitude Response

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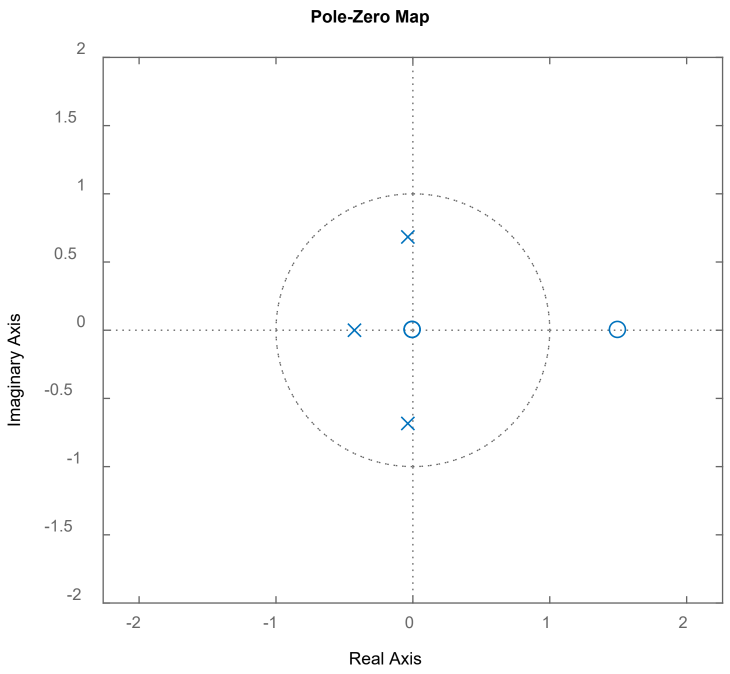

Designing the Magnitude Response

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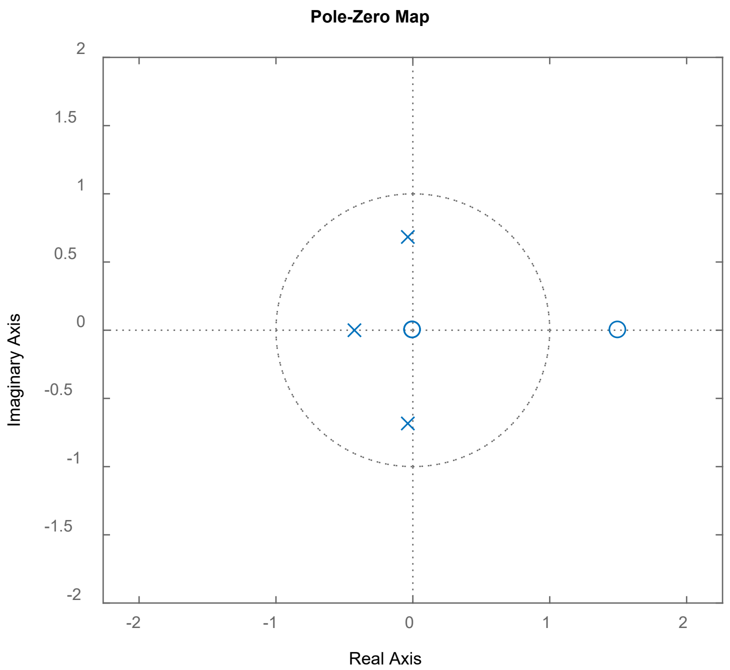
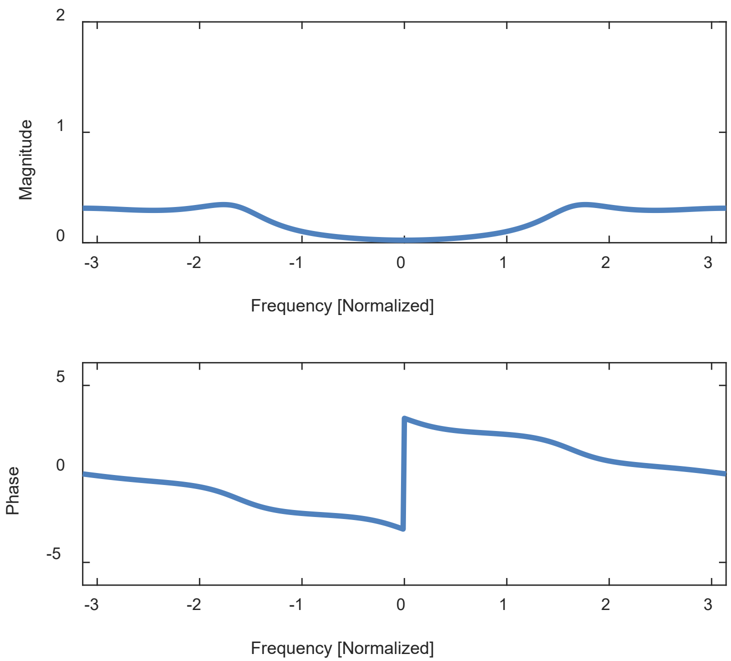




Designing the Magnitude Response

- **Question: Are there other types of filters?**

Lecture 16: Filter Implementation… Again.

Foundations of Digital Signal Processing

Outline

- Reviewing different types of filters
- **Designing the phase response**
- Implementation of FIR Filters
- Implementation of IIR Filters

Designing the Phase Response

- **Question: What is the frequency-domain phase if the impulse response is <u>even</u>?**

$$h[n] = h[-n]$$

Designing the Phase Response

- **Question: What is the frequency-domain phase if the impulse response is <u>even</u>?**

$$h[n] = h[-n]$$

Result: There is no phase.

$$H(\omega) = H^*(-\omega)$$
$$H(\omega) = |H(\omega)|e^{j\angle H(\omega)} = |H(\omega)|$$

Designing the Phase Response

- **Question: What is the frequency-domain phase if the <u>even</u> impulse response is shifted?**

$$h[n] = h[-n]$$
$$h[n - n_0]$$

Designing the Phase Response

- **Question: What is the frequency-domain phase if the <u>even</u> impulse response is shifted?**

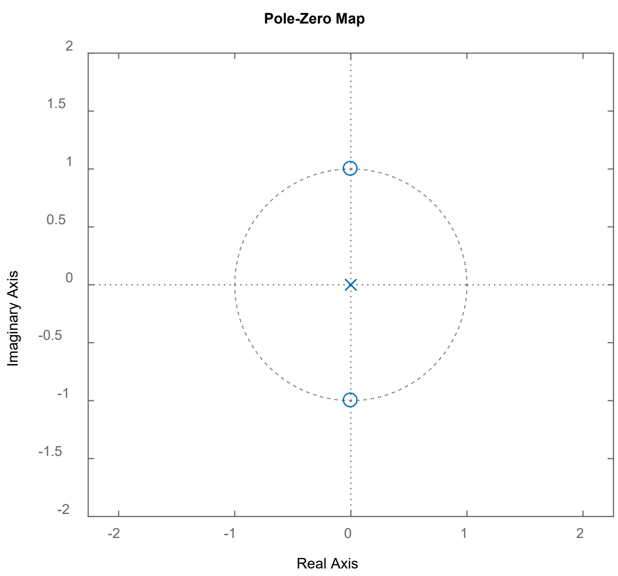
$$h[n] = h[-n]$$
$$h[n - n_0]$$

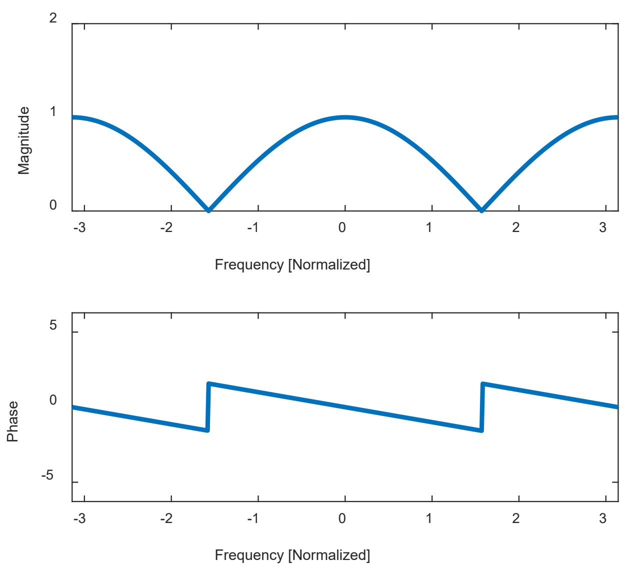
Result: There is a linear phase.

$$H(\omega) = |H(\omega)|e^{-j\omega n_0}$$

Designing the Phase Response

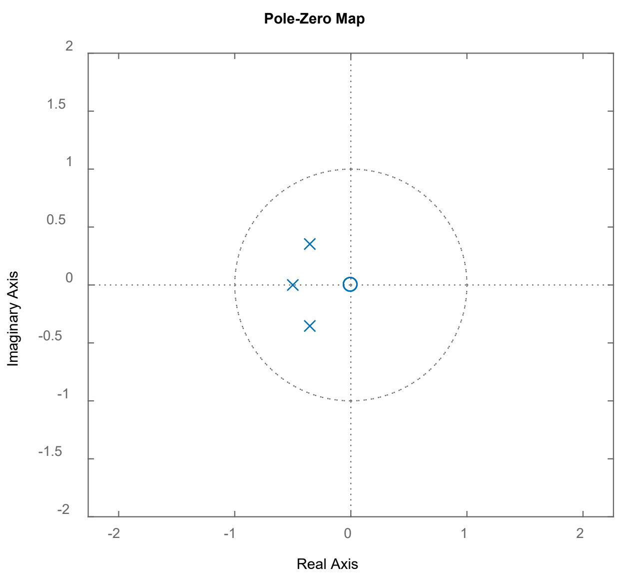
- **Linear phase -> Delay**
  - $H(\omega) = |H(\omega)|e^{-j\omega n_0}$ <- slope = delay

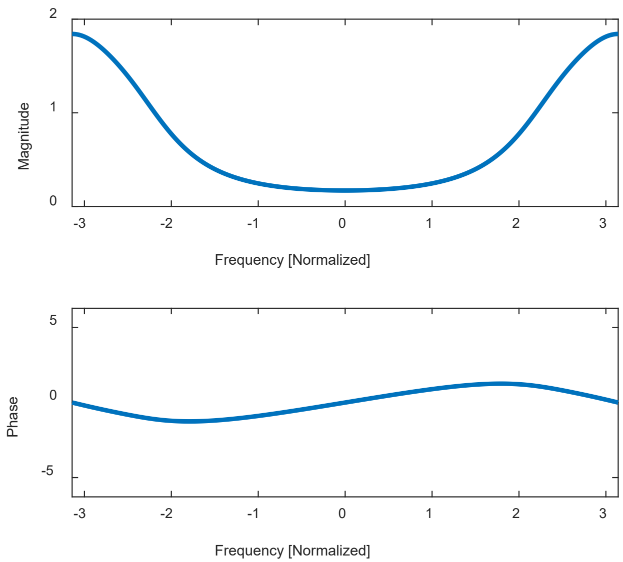




Designing the Phase Response

- **Linear phase -> Delay**
  - $H(\omega) = |H(\omega)|e^{-j\omega n_0}$ <- slope = delay
- More generally
  - $\dfrac{d\,\angle H(\omega)}{d\omega} =$ group delay (delay at each frequency)





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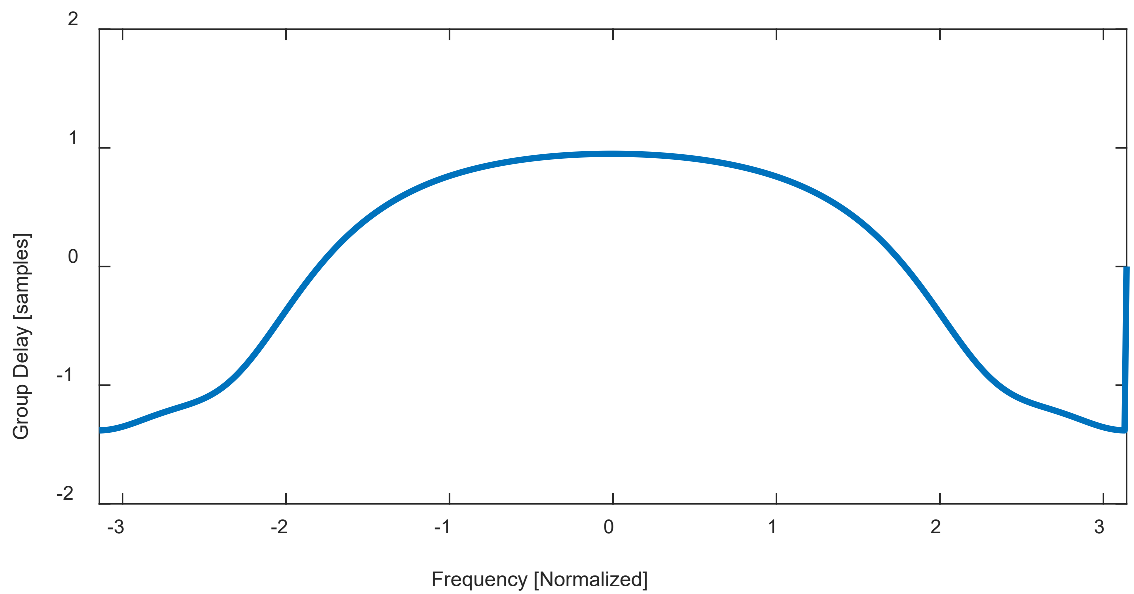

Designing the Phase Response

- **Example: Compute the phase response of**

$$H(z) = \frac{1}{1 - az^{-1}} \Rightarrow H(\omega) = \frac{1}{1 - ae^{-j\omega}}$$

Designing the Phase Response

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$$H(z) = \frac{1}{1 - az^{-1}} \Rightarrow H(\omega) = \frac{1}{1 - ae^{-j\omega}}$$

$$\angle H(\omega) = \angle(1) - \angle\left(1 - e^{-j\omega}\right)$$
$$\angle H(\omega) = 0 - \angle\left(1 - a(\cos(\omega) - j\sin(\omega))\right)$$
$$\angle H(\omega) = -\operatorname{atan}\left(\frac{a\sin(\omega)}{1 - a\cos(\omega)}\right)$$

Designing the Phase Response

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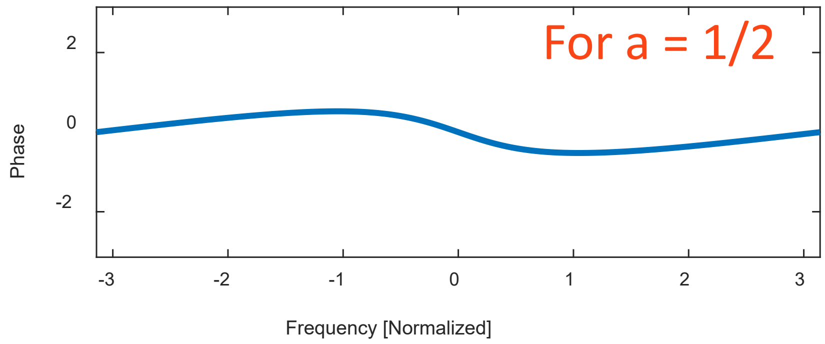

Designing the Phase Response

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- **Question: What happens as $a \to 0$?**

Designing the Phase Response

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$$\angle H(\omega) = -\operatorname{atan}\left(\frac{a\sin(\omega)}{1 - a\cos(\omega)}\right) \rightarrow -\operatorname{atan}(0) = 0$$

- **Question: What happens as $a \rightarrow 0$?**

Designing the Phase Response

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Designing the Phase Response

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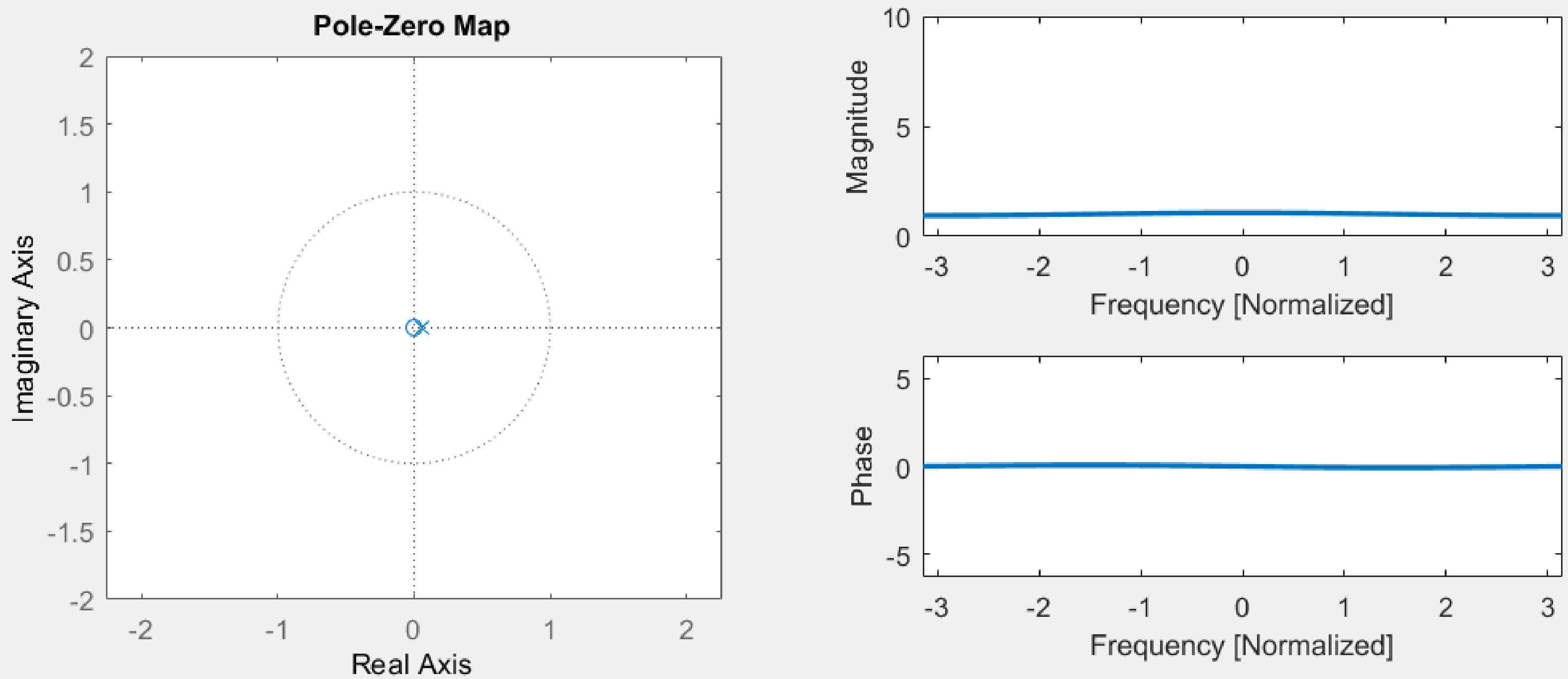

Designing the Phase Response

- **Example:** **Compute the phase response of**

$$H(\omega) = \frac{1}{(1 - ae^{-j\omega})(1 - be^{-j\omega})}$$

Designing the Phase Response

- **Example: Compute the phase response of**

$$H(\omega) = \frac{1}{(1 - ae^{-j\omega})(1 - be^{-j\omega})}$$

$$\angle H(\omega) = \angle\left[\frac{1}{(1 - ae^{-j\omega})}\right] + \angle\left[\frac{1}{(1 - be^{-j\omega})}\right]$$

$$\angle H(\omega) = -\operatorname{atan}\left(\frac{a\sin(\omega)}{1 - a\cos(\omega)}\right) - \operatorname{atan}\left(\frac{b\sin(\omega)}{1 - b\cos(\omega)}\right)$$

Designing the Phase Response

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$$H(\omega) = \frac{1}{(1 - ae^{-j\omega})(1 - be^{-j\omega})}$$

$$\angle H(\omega) = -\operatorname{atan}\left(\frac{a\sin(\omega)}{1 - a\cos(\omega)}\right) - \operatorname{atan}\left(\frac{b\sin(\omega)}{1 - b\cos(\omega)}\right)$$

Question: What happens as *a, b* gets large?

Designing the Phase Response

- **Example: Compute the phase response of**

$$H(\omega) = \frac{1}{(1 - ae^{-j\omega})(1 - be^{-j\omega})}$$

$$\angle H(\omega) = -\operatorname{atan}\left(\frac{a\sin(\omega)}{1 - a\cos(\omega)}\right) - \operatorname{atan}\left(\frac{b\sin(\omega)}{1 - b\cos(\omega)}\right) \rightarrow 2\omega$$

Question: What happens as *a, b* gets large?

Designing the Phase Response

- **Question: Assume I have 46 poles. What is the minimum possible delay? What is the maximum possible delay?**

Designing the Phase Response

- **Question: Assume I have 46 poles. What is the minimum possible delay? What is the maximum possible delay?**

- **Answer:**
  - Minimum delay = 0
  - Maximum delay = 46

Designing the Phase Response

- **Example: Compute the phase response of**

$$H(z) = 1 - az^{-1} \Rightarrow H(\omega) = 1 - ae^{-j\omega}$$

Designing the Phase Response

- **Example: Compute the phase response of**

$$H(z) = 1 - az^{-1} \Rightarrow H(\omega) = 1 - ae^{-j\omega}$$

$$\angle H(\omega) = \angle\Big(1 - a(\cos(\omega) - j\sin(\omega))\Big)$$

$$\angle H(\omega) = \operatorname{atan}\left(\frac{a\sin(\omega)}{1 - a\cos(\omega)}\right)$$

Designing the Phase Response

- **Example: Compute the phase response of**

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Designing the Phase Response

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Question: What happens as *a* gets small?

Designing the Phase Response

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Designing the Phase Response

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$$\angle H(\omega) = \operatorname{atan}\left(\frac{a\sin(\omega)}{1 - a\cos(\omega)}\right) \rightarrow -\omega$$

Question: What happens as *a* gets large?

Designing the Phase Response

- **Question: How can I force a linear phase response?**

Designing the Phase Response

- **Question : How can I force a linear phase response?**
  - $h[n] = h[-n]$ or $h[n] = -h[-n]$

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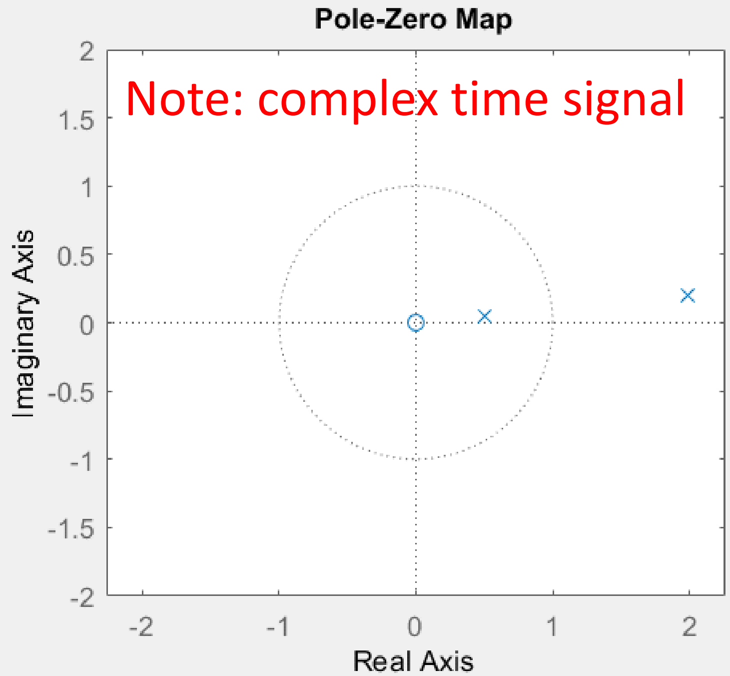

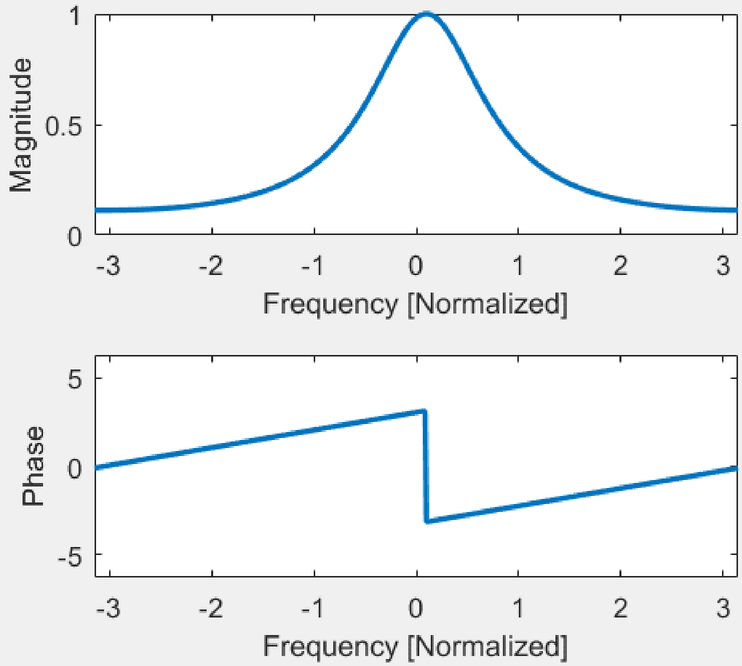

Designing the Phase Response

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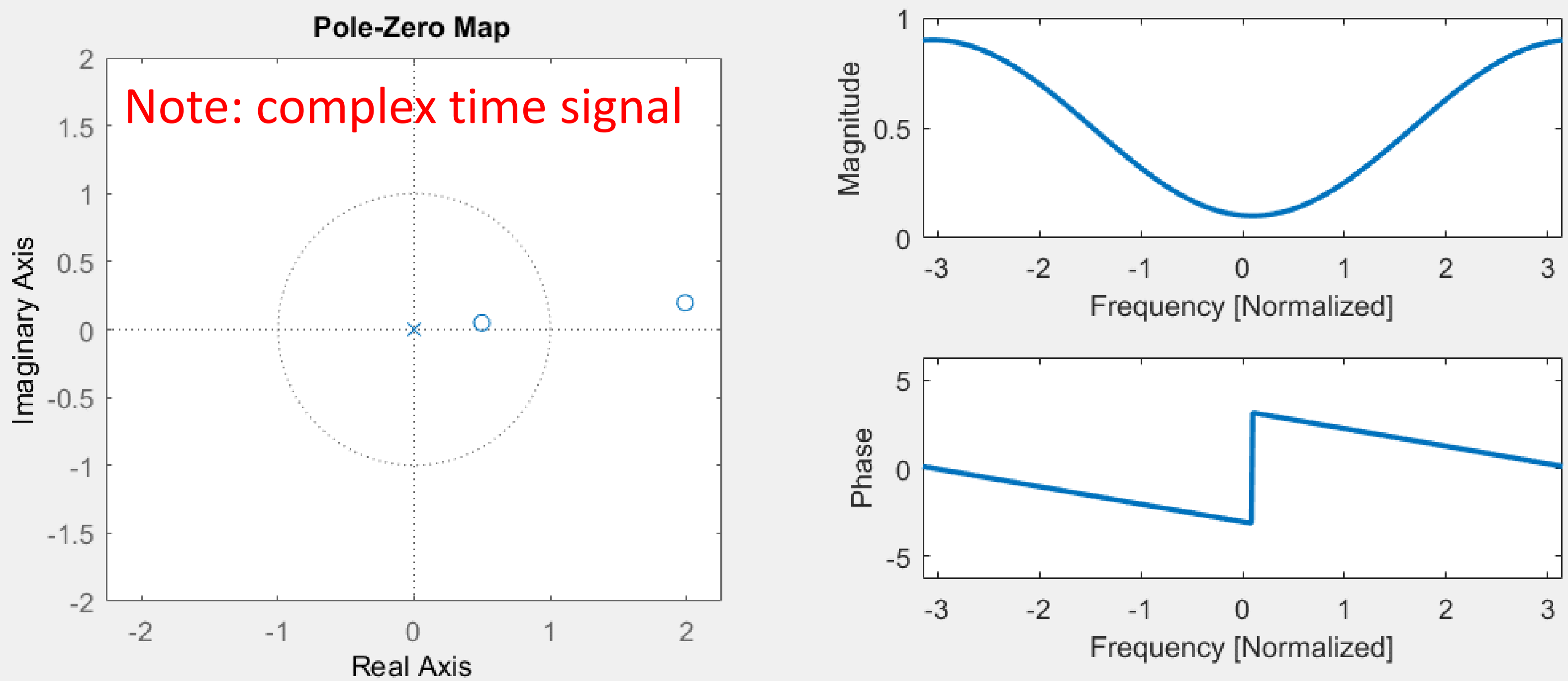

Designing the Phase Response

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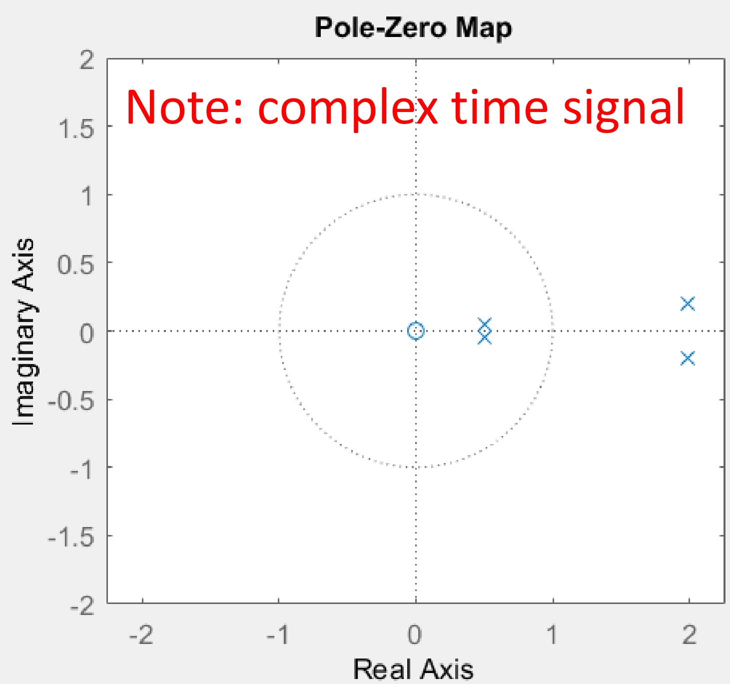

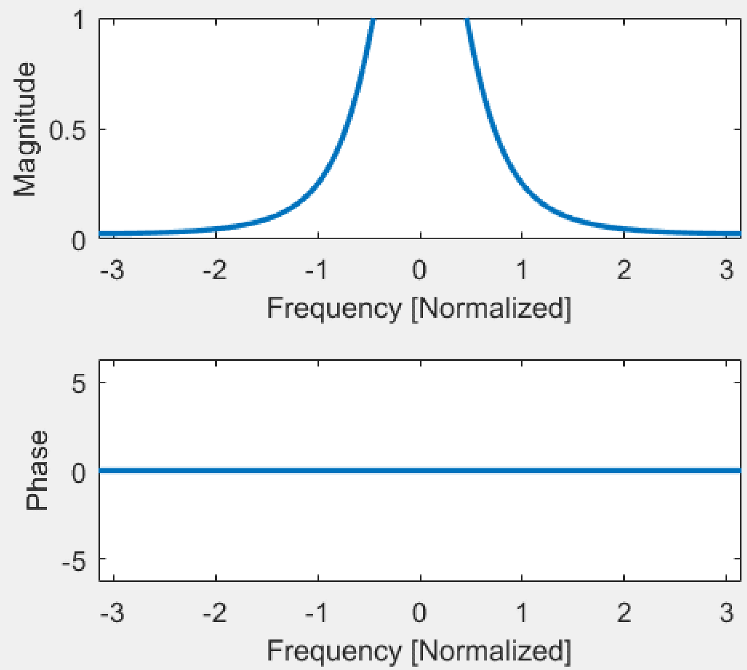

Designing the Phase Response

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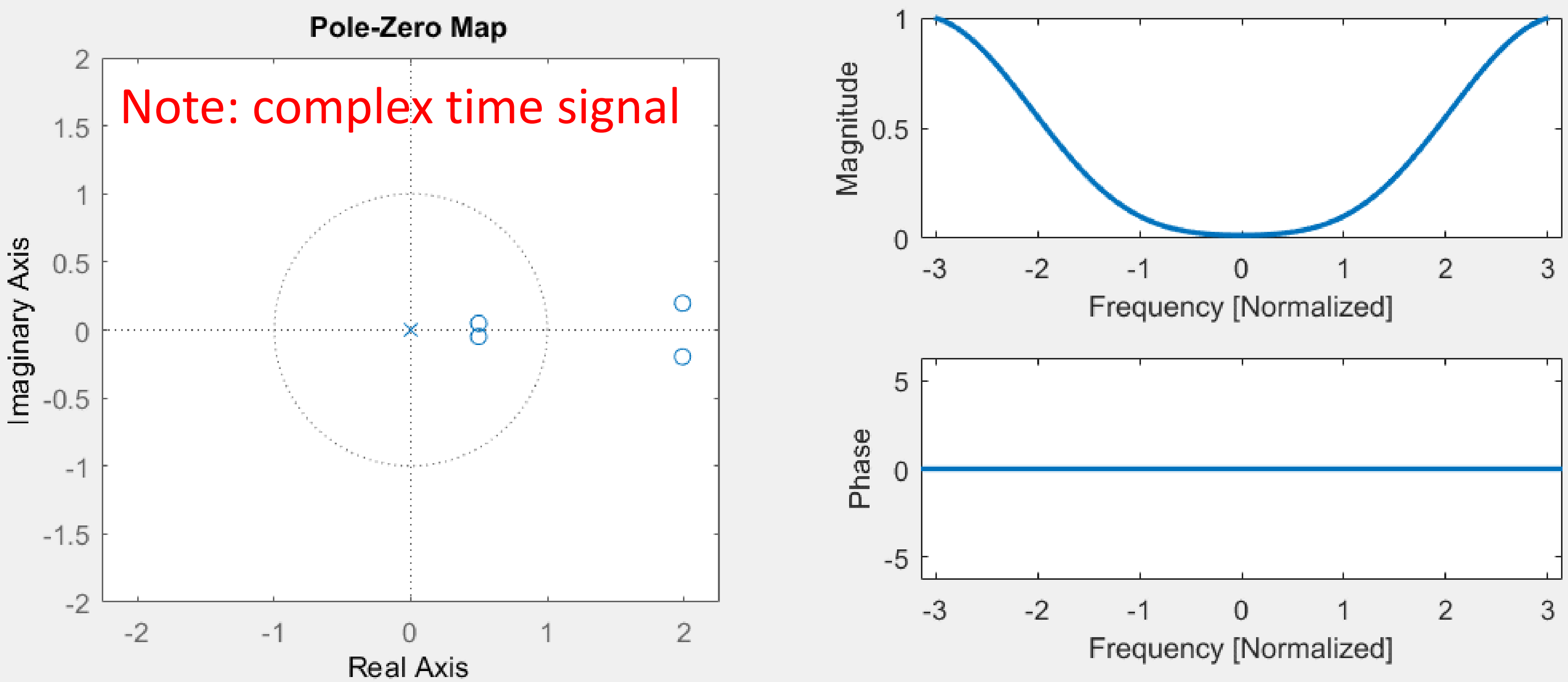


Designing the Phase Response

- **Question: So why do I care about phase?**

Lecture 16: Filter Implementation… Again.

Foundations of Digital Signal Processing

Outline

- Reviewing different types of filters
- Designing the phase response
- **Implementation of FIR Filters**
- Implementation of IIR Filters

Implementing FIR Filters

- **Question: What do we mean by implementation? Why do we care?**

Implementing FIR Filters

- **Two ways to look at FIR filters**
  - Convolution perspective

$$y[n] = \sum_{m=0}^{M-1} h[m]x[n-m]$$

  - Filtering perspective

$$Y(Z) = X(z) \sum_{m=0}^{M-1} h[m]z^{-m}$$

Implementing FIR Filters

- **FIR Direct Form**

$$y[n] = \sum_{m=0}^{M-1} h[m]x[n-m]$$

- **Question: What are the defining characteristics of the pole-zero plot?**

Implementing FIR Filters

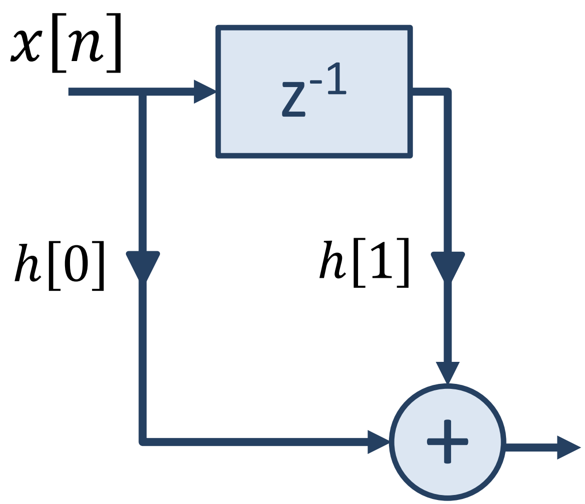
- **FIR Direct Form**

$$y[n] = \sum_{m=0}^{M-1} h[m]x[n-m]$$

Implementing FIR Filters

- **FIR Direct Form (M = 1)**

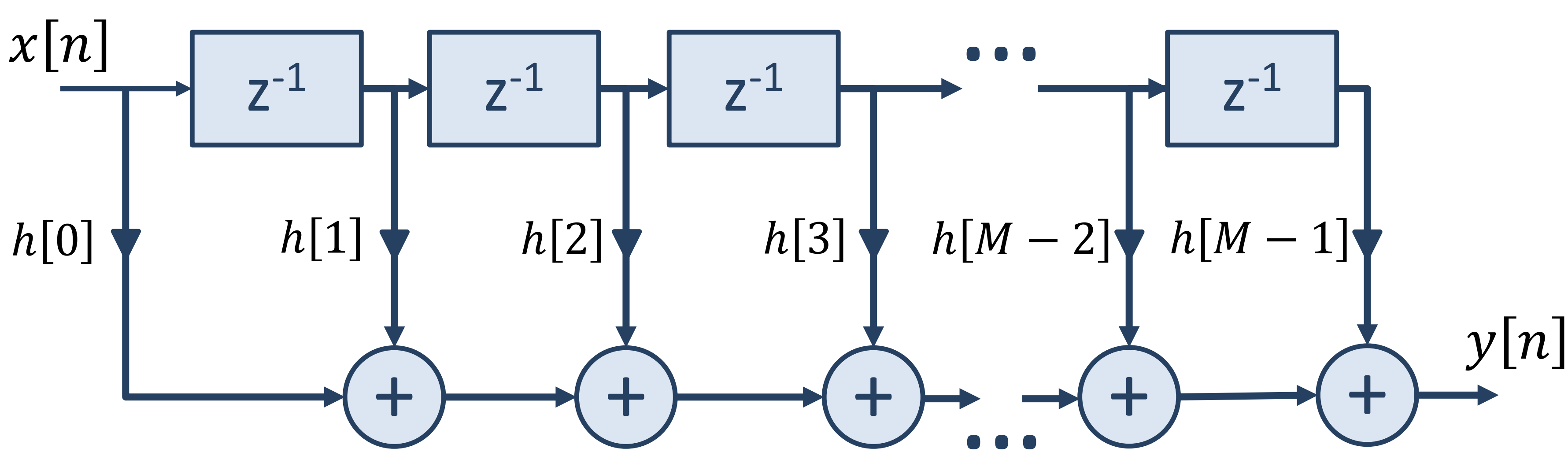
$$y[n] = h[0] + h[1]x[n-1]$$



Implementing FIR Filters

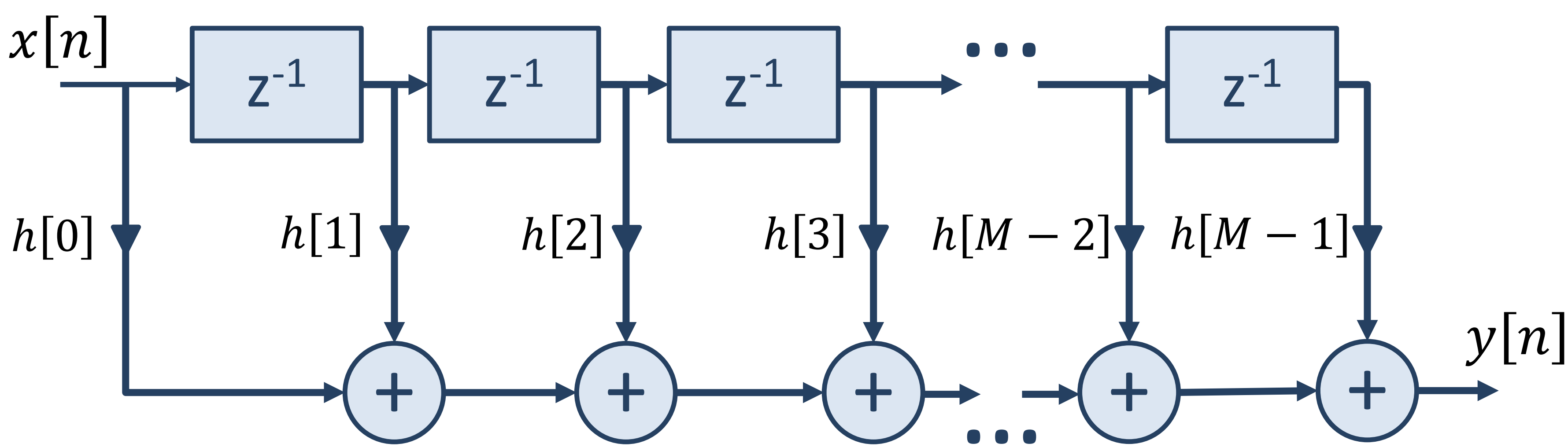
- **FIR Direct Form**

$$y[n] = \sum_{m=0}^{M-1} h[m]x[n-m]$$



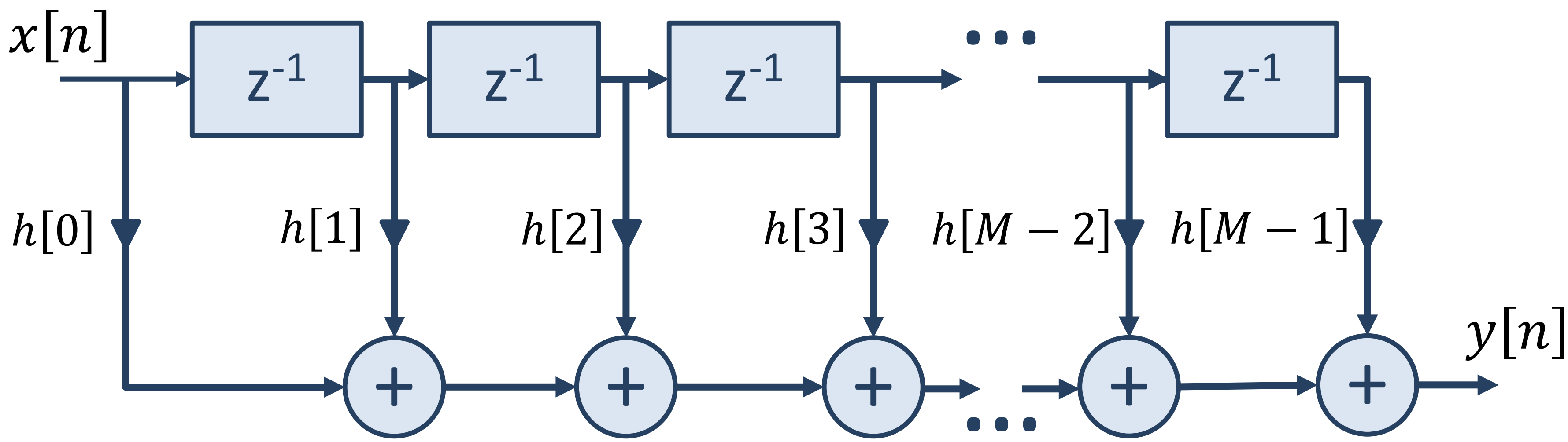
Implementing FIR Filters

- **FIR Direct Form**
  - What are some benefits of using a direct form implementation?



Implementing FIR Filters

- **FIR Direct Form**
  - If the impulse is symmetric, then we can shorten this



Implementing FIR Filters

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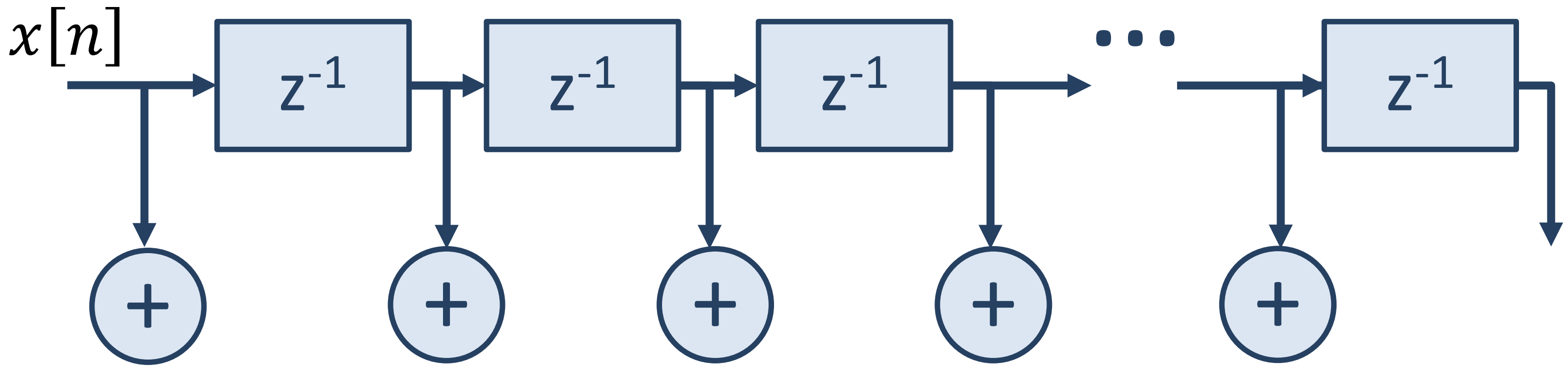

Implementing FIR Filters

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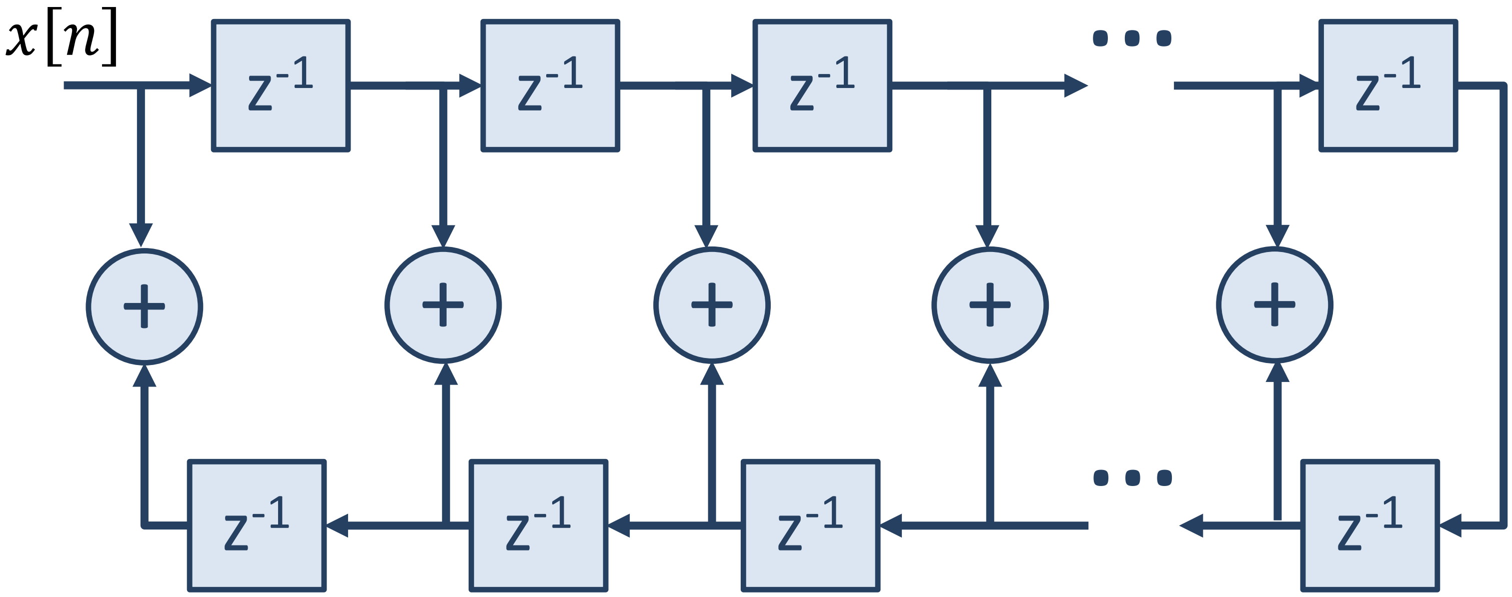

Implementing FIR Filters

- **FIR Direct Form**
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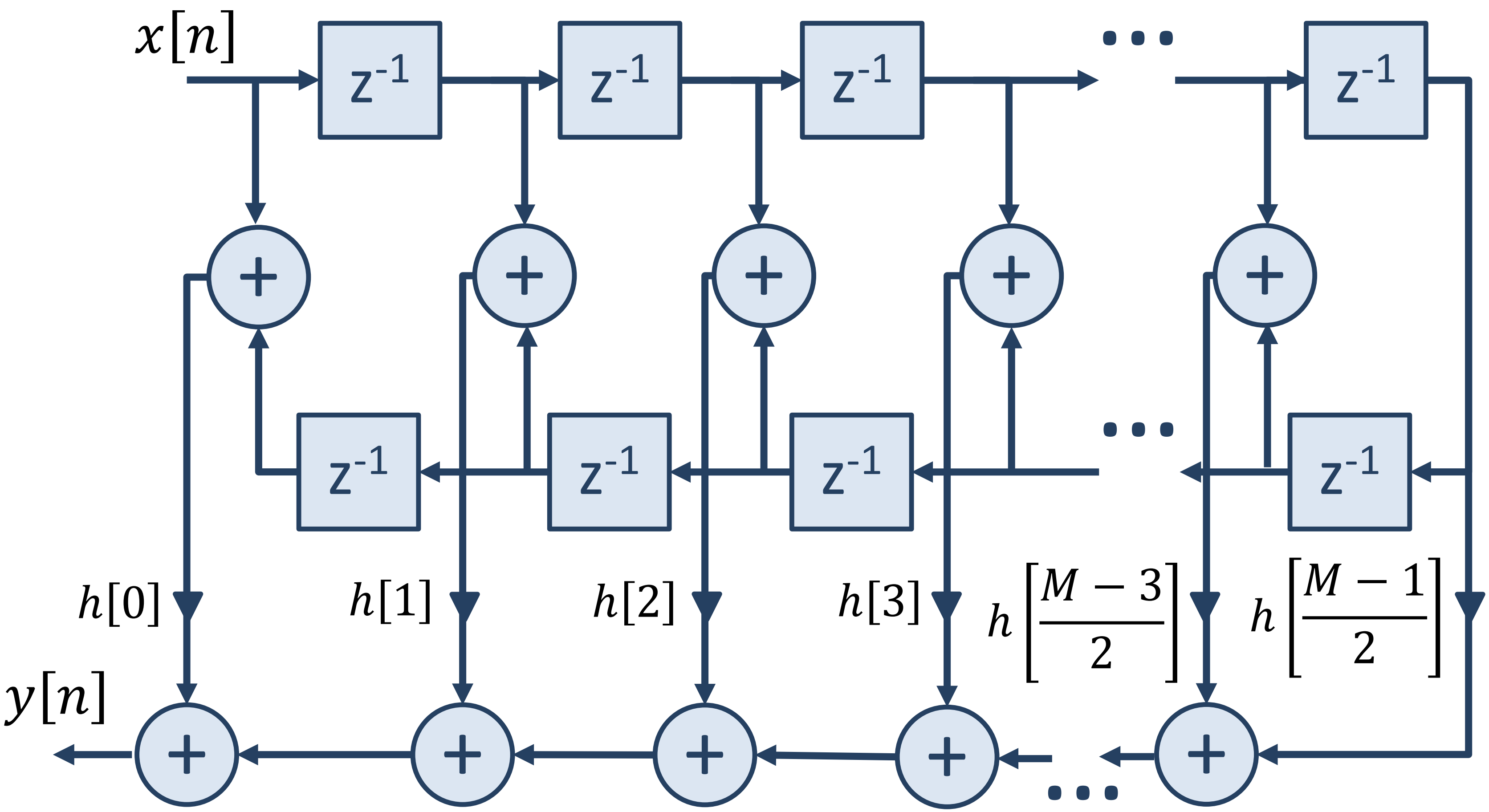
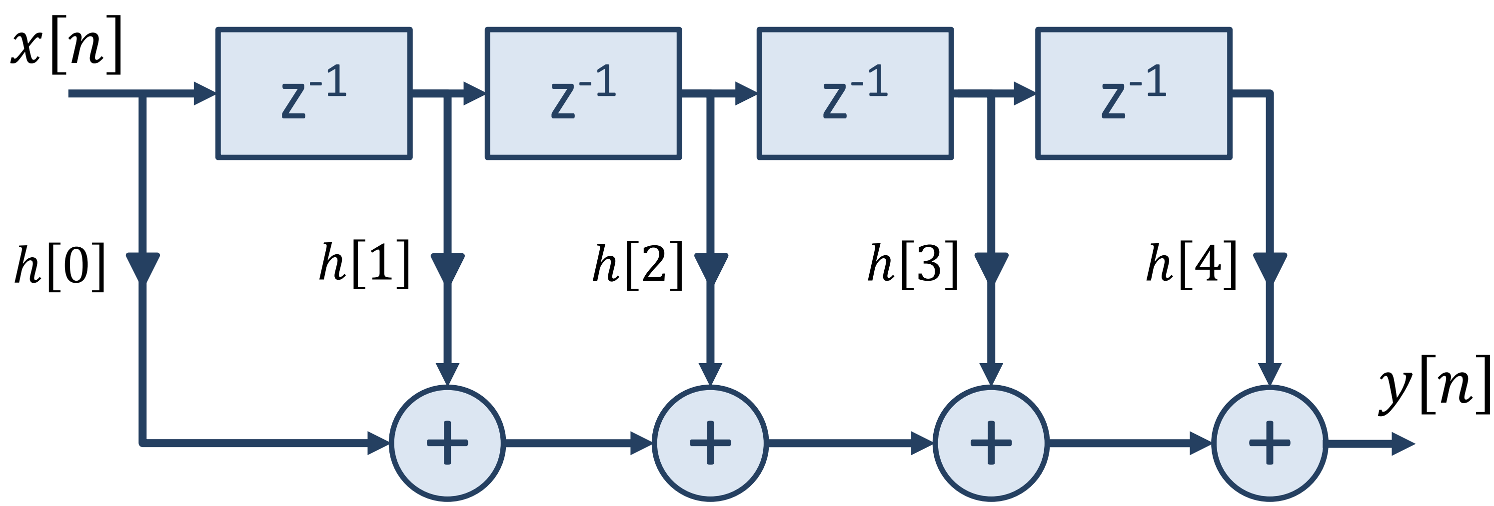


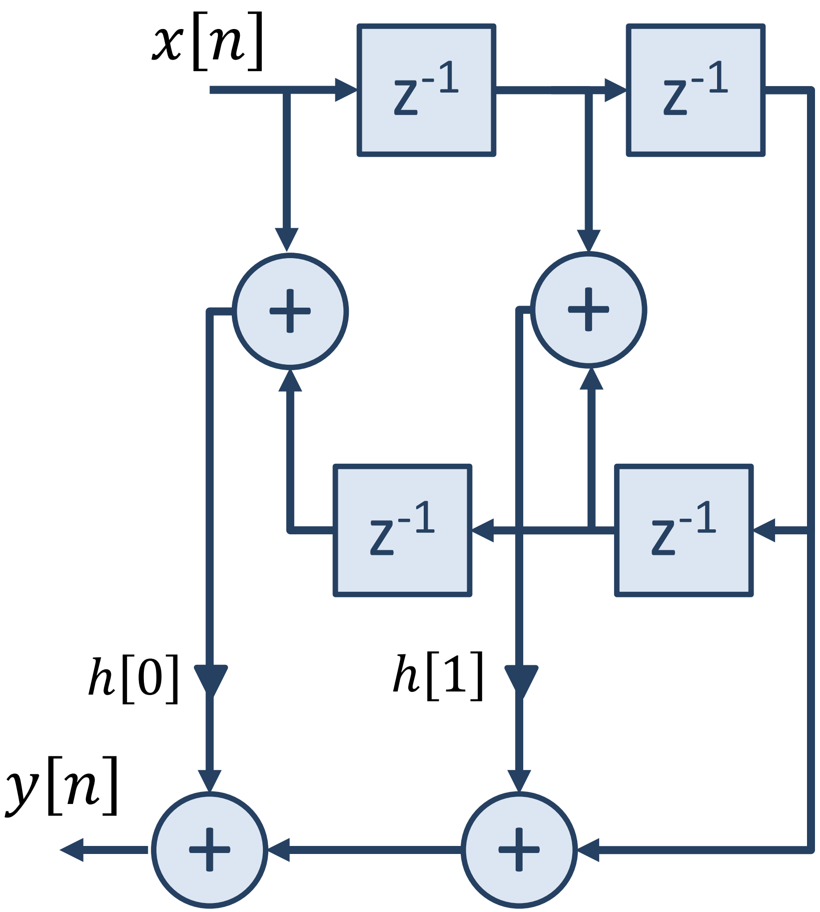
Implementing FIR Filters

- **FIR Direct Form**
  - Non-symmetric impulse response (4 multiplications)



Implementing FIR Filters

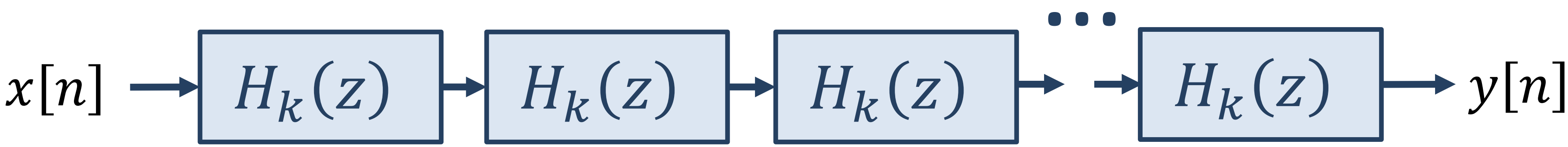
- **FIR Direct Form**
  - Symmetric impulse response (2 multiplications)



Implementing FIR Filters

- **FIR Cascade Form**

$$Y[z] = X(z)\sum_{m=0}^{M-1} h[m]z^{-m} = X(z)\prod_{k=1}^{K} H_k(z)$$



Implementing FIR Filters

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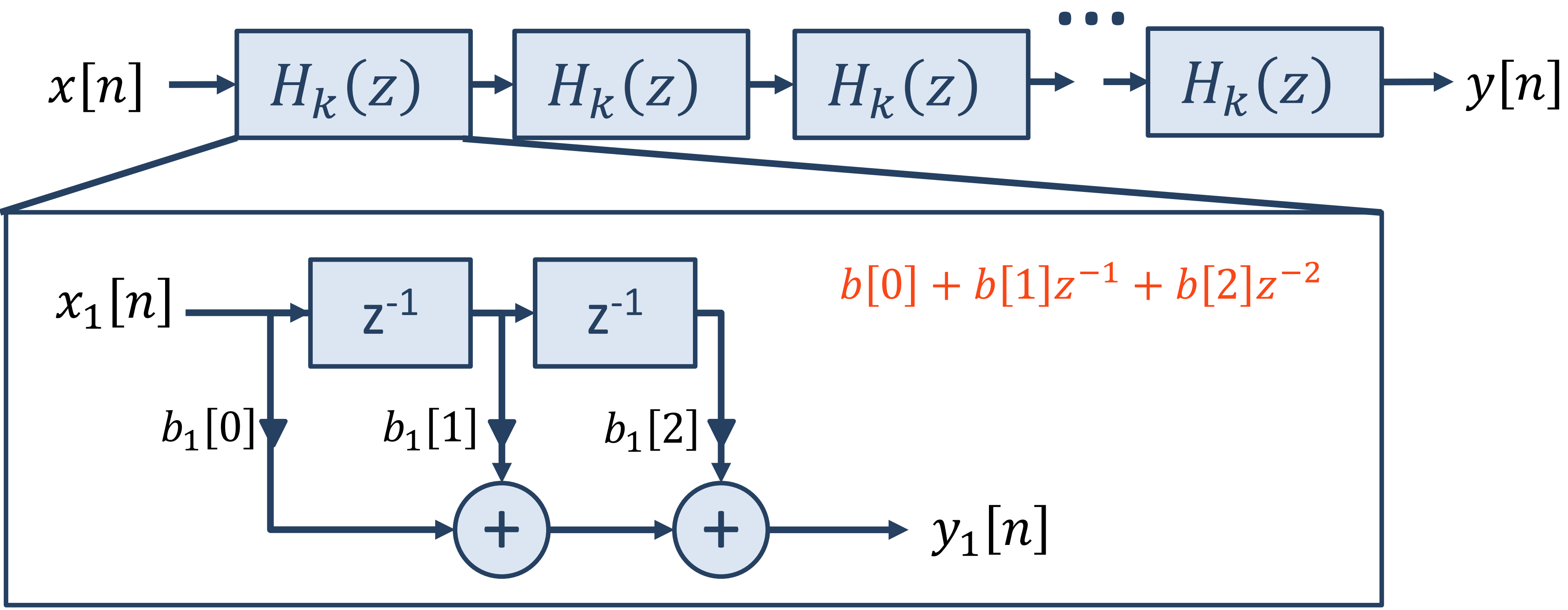

Implementing FIR Filters

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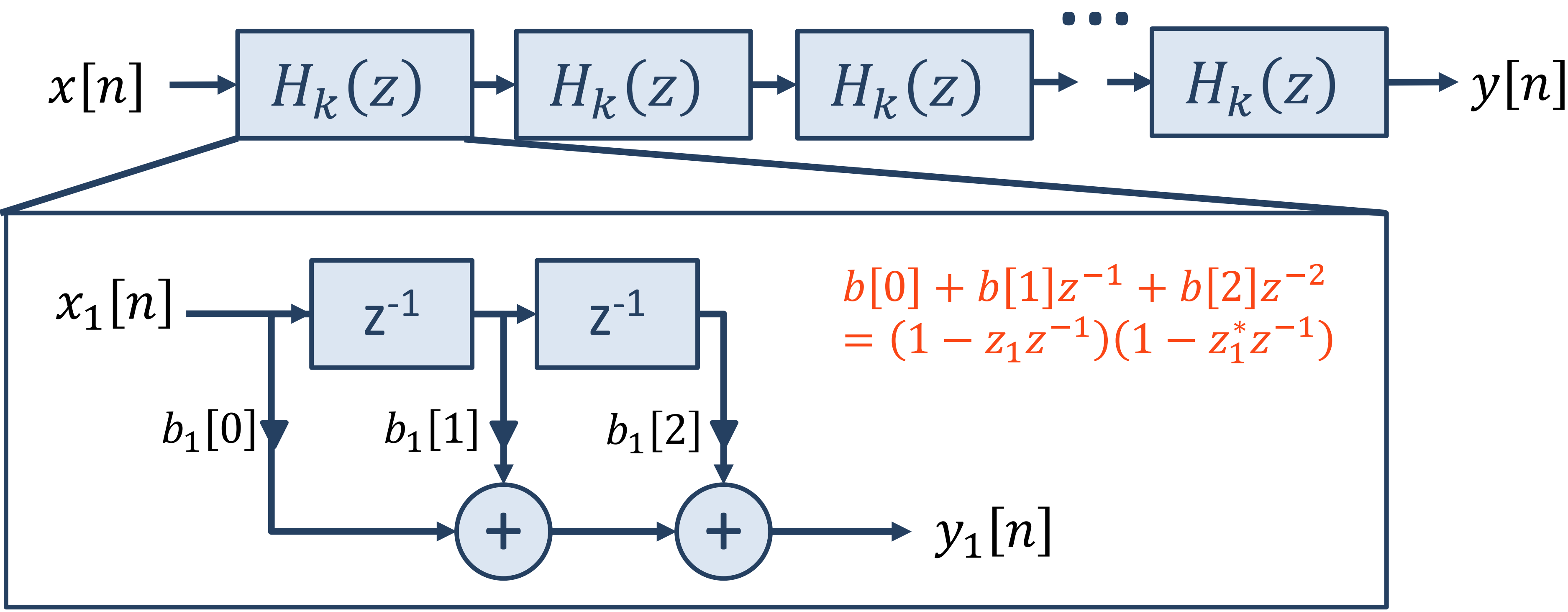

Implementing FIR Filters

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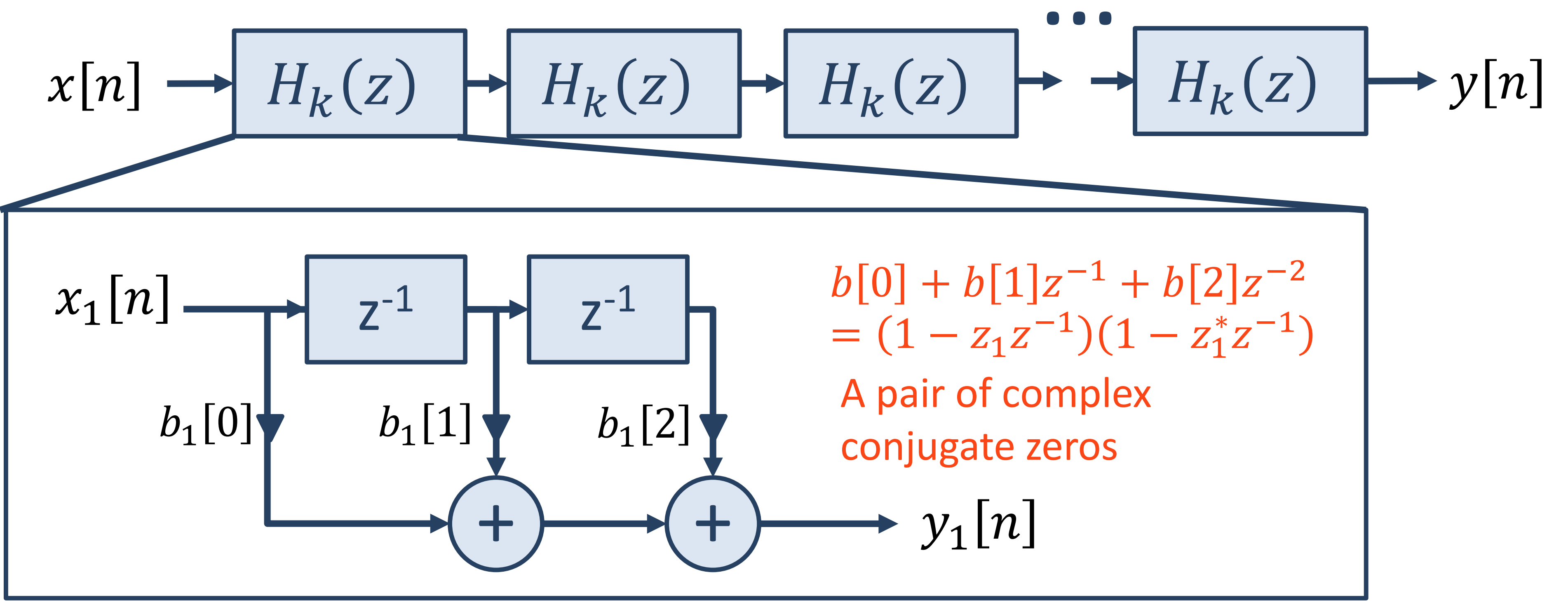
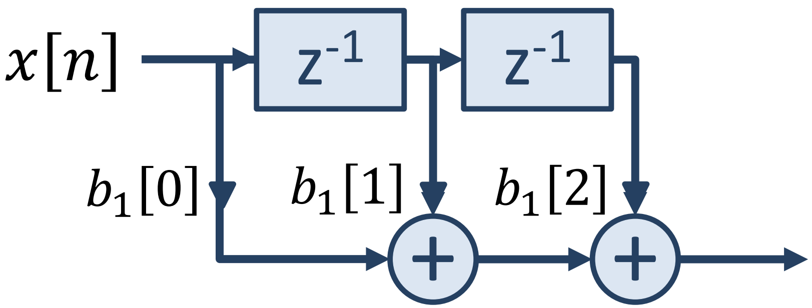


Implementing FIR Filters

- **FIR Cascade Form**

  - What are some benefits of using a cascade form implementation?



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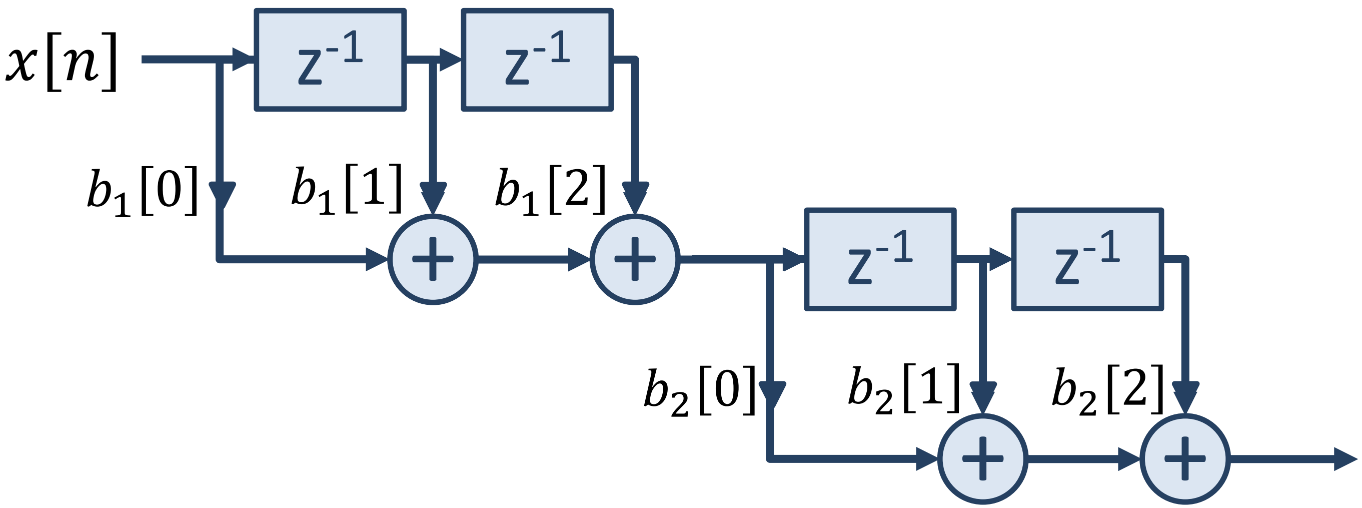

Implementing FIR Filters

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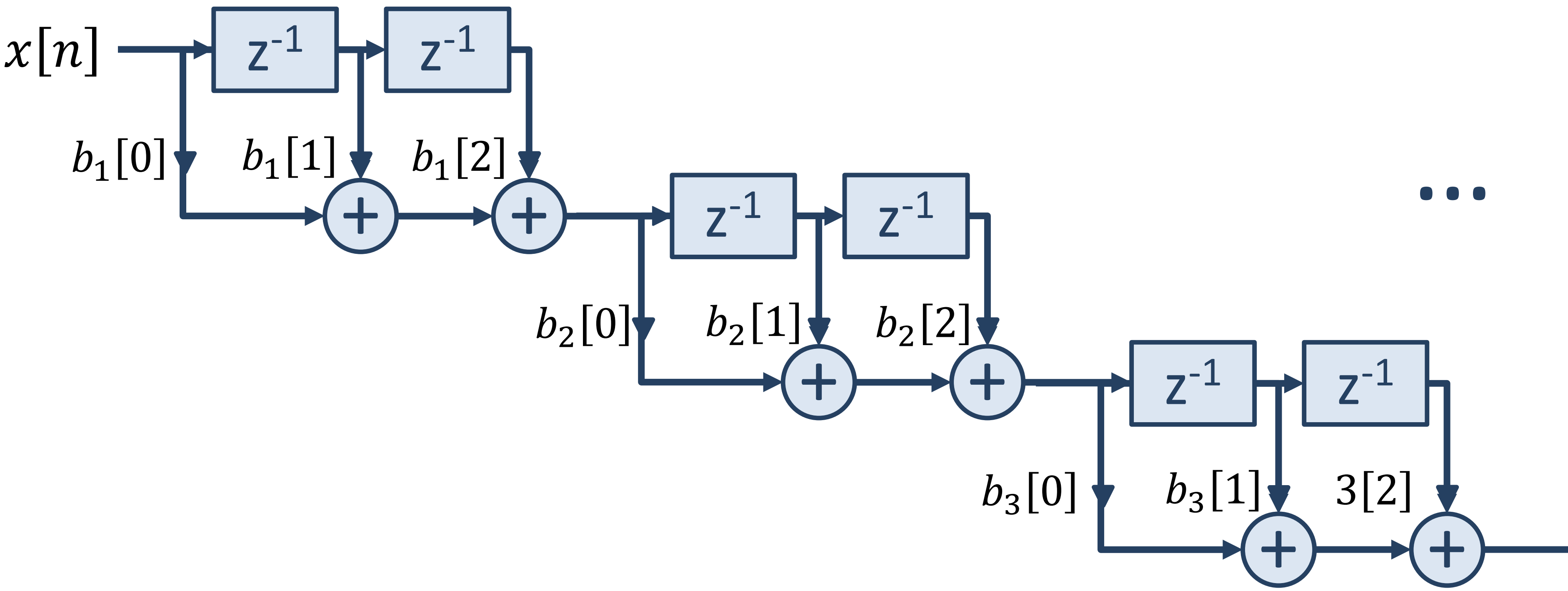
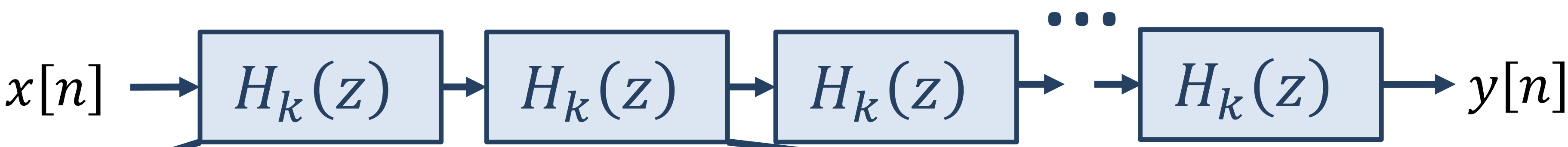


Implementing FIR Filters

- **FIR Cascade Form (if the impulse response is symmetric)**

$$Y[z] = X(z)\sum_{m=0}^{M-1} h[m]z^{-m} = X(z)\prod_{k=1}^{K} H_k(z)$$

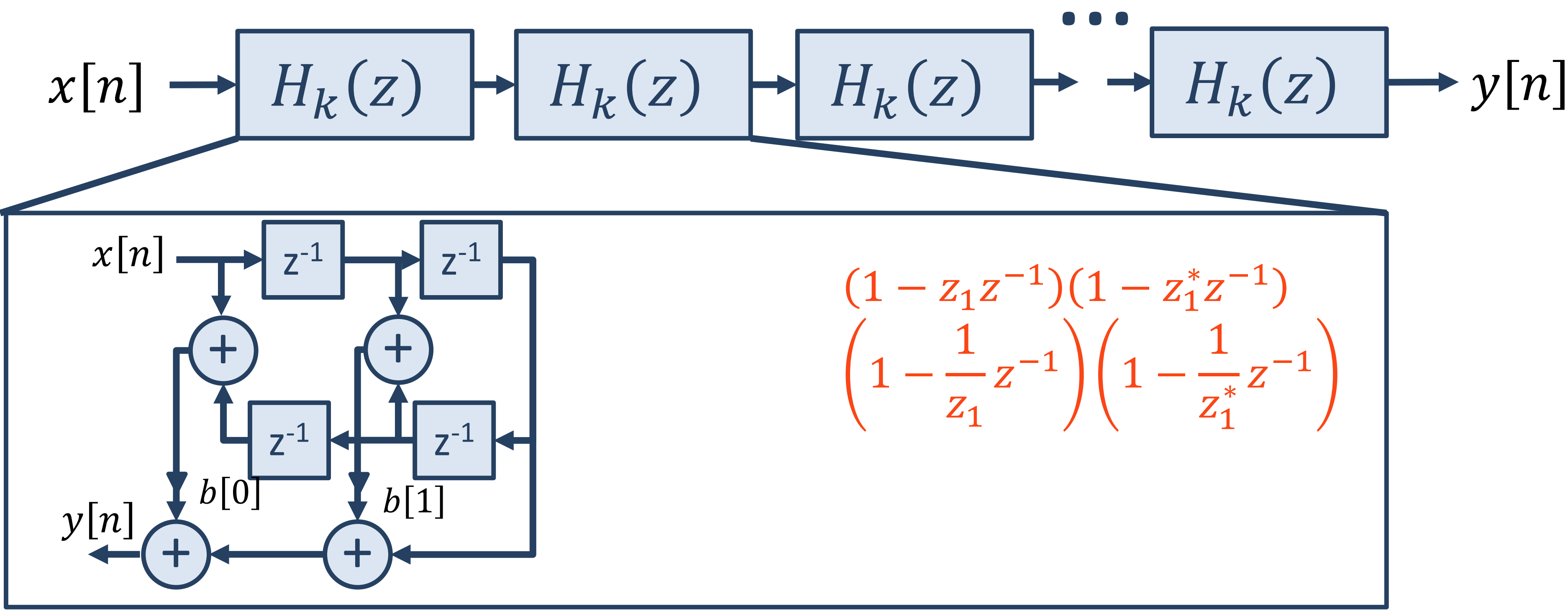


$$(1 - z_1 z^{-1})(1 - z_1^* z^{-1})$$
$$\left(1 - \frac{1}{z_1} z^{-1}\right)\left(1 - \frac{1}{z_1^*} z^{-1}\right)$$

Implementing FIR Filters

- **FIR Cascade Form (if the impulse response is symmetric)**

$$Y[z] = X(z)\sum_{m=0}^{M-1} h[m]z^{-m} = X(z)\prod_{k=1}^{K} H_k(z)$$



Implementing FIR Filters

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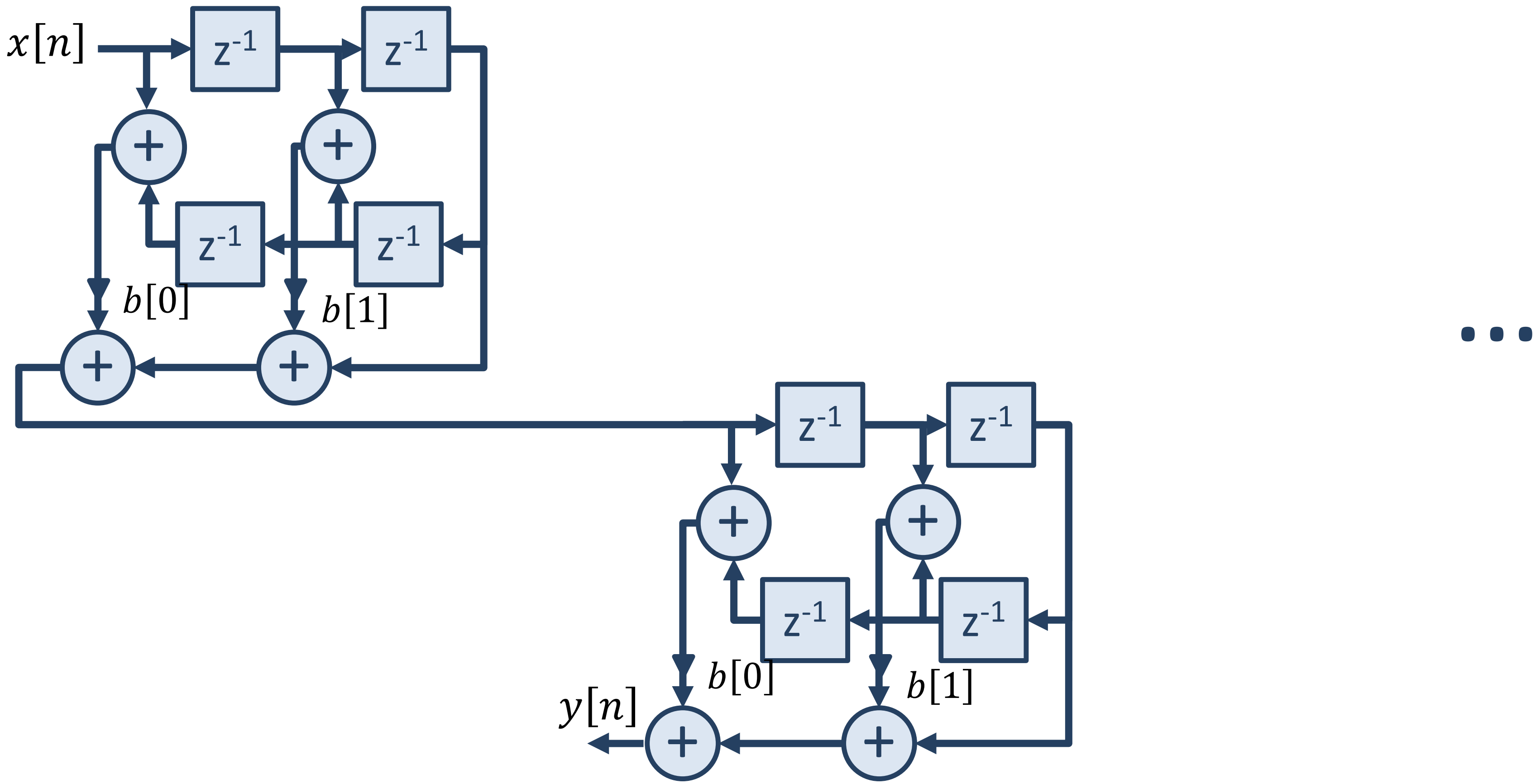

Implementing FIR Filters

- **Frequency Sampling Form**

$$H[k] = \sum_{n=0}^{N-1} h[n] e^{-j\frac{2\pi}{N}kn} \quad , \qquad h[n] = \frac{1}{N}\sum_{k=0}^{N-1} H[k] e^{j\frac{2\pi}{N}kn}$$

Implementing FIR Filters

- **Frequency Sampling Form**

$$H[k] = \sum_{n=0}^{N-1} h[n] e^{-j\frac{2\pi}{N}kn} \quad , \qquad h[n] = \frac{1}{N}\sum_{k=0}^{N-1} H[k] e^{j\frac{2\pi}{N}kn}$$

$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n} = \sum_{n=0}^{N-1}\left(\frac{1}{N}\sum_{k=0}^{N-1} H[k] e^{j\frac{2\pi}{N}kn}\right) z^{-n}$$

$$= \sum_{n=0}^{N-1}\left(\frac{1}{N}\sum_{k=0}^{N-1} H[k] e^{j\frac{2\pi}{N}kn}\right) z^{-n}$$

$$= \sum_{k=0}^{N-1}\left(H[k]\frac{1}{N}\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}kn}\, z^{-n}\right)$$

Implementing FIR Filters

- **Frequency Sampling Form**

$$H[k] = \sum_{n=0}^{N-1} h[n] e^{-j\frac{2\pi}{N}kn} \quad , \qquad h[n] = \frac{1}{N}\sum_{k=0}^{N-1} H[k] e^{j\frac{2\pi}{N}kn}$$

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$$= \sum_{k=0}^{N-1}\left(H[k]\frac{1}{N}\frac{1-e^{j\frac{2\pi}{N}kN}z^{-N}}{1-e^{j\frac{2\pi}{N}kn}z^{-n}}\right)$$

$$= \frac{1-e^{j\frac{2\pi}{N}kN}z^{-N}}{N}\sum_{k=0}^{N-1}\left(\frac{H[k]}{1-e^{j\frac{2\pi}{N}kn}z^{-n}}\right)$$

Implementing FIR Filters

- **Frequency Sampling Form**

$$H(z) = \frac{1 - e^{j\frac{2\pi}{N}kN} z^{-N}}{N} \sum_{k=0}^{N-1} \left(\frac{H[k]}{1 - e^{j\frac{2\pi}{N}kn} z^{-n}} \right)$$

Implementing FIR Filters

- **Frequency Sampling Form**

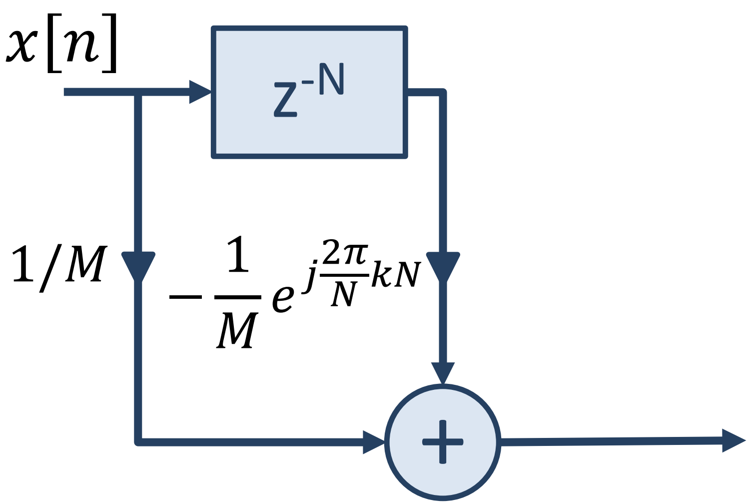
Weights

$$H(z) = \underbrace{\frac{1 - e^{j\frac{2\pi}{N}kN} z^{-N}}{N}}_{\text{FIR Filter}} \sum_{k=0}^{N-1} \underbrace{\left(\frac{H[k]}{1 - e^{j\frac{2\pi}{N}kn} z^{-n}}\right)}_{\text{IIR Filter}}$$

Implementing FIR Filters

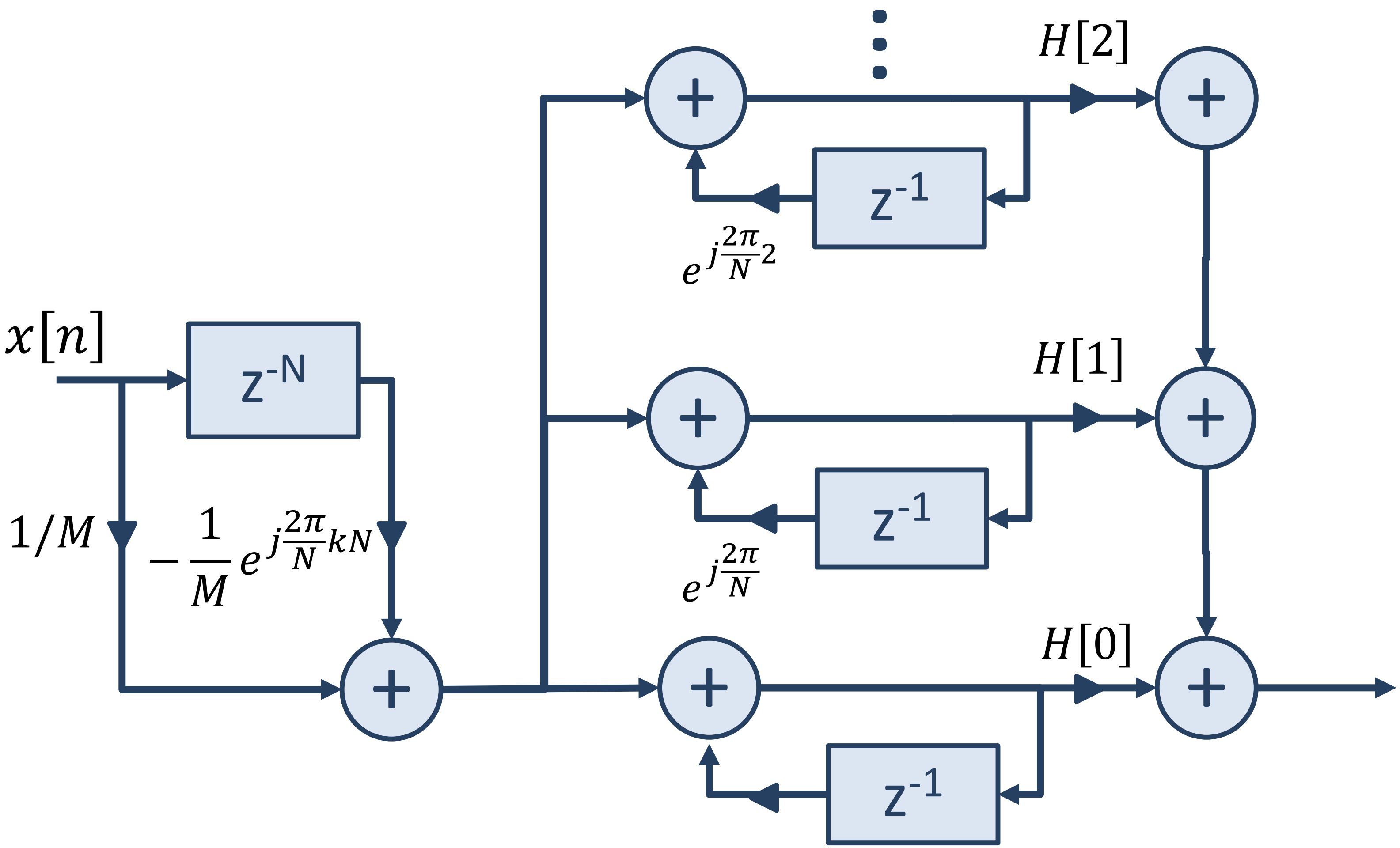
- **Frequency Sampling Form**

$$H(z) = \frac{1 - e^{j\frac{2\pi}{N}kN} z^{-N}}{N} \sum_{k=0}^{N-1} \left(\frac{H[k]}{1 - e^{j\frac{2\pi}{N}kn} z^{-n}} \right)$$



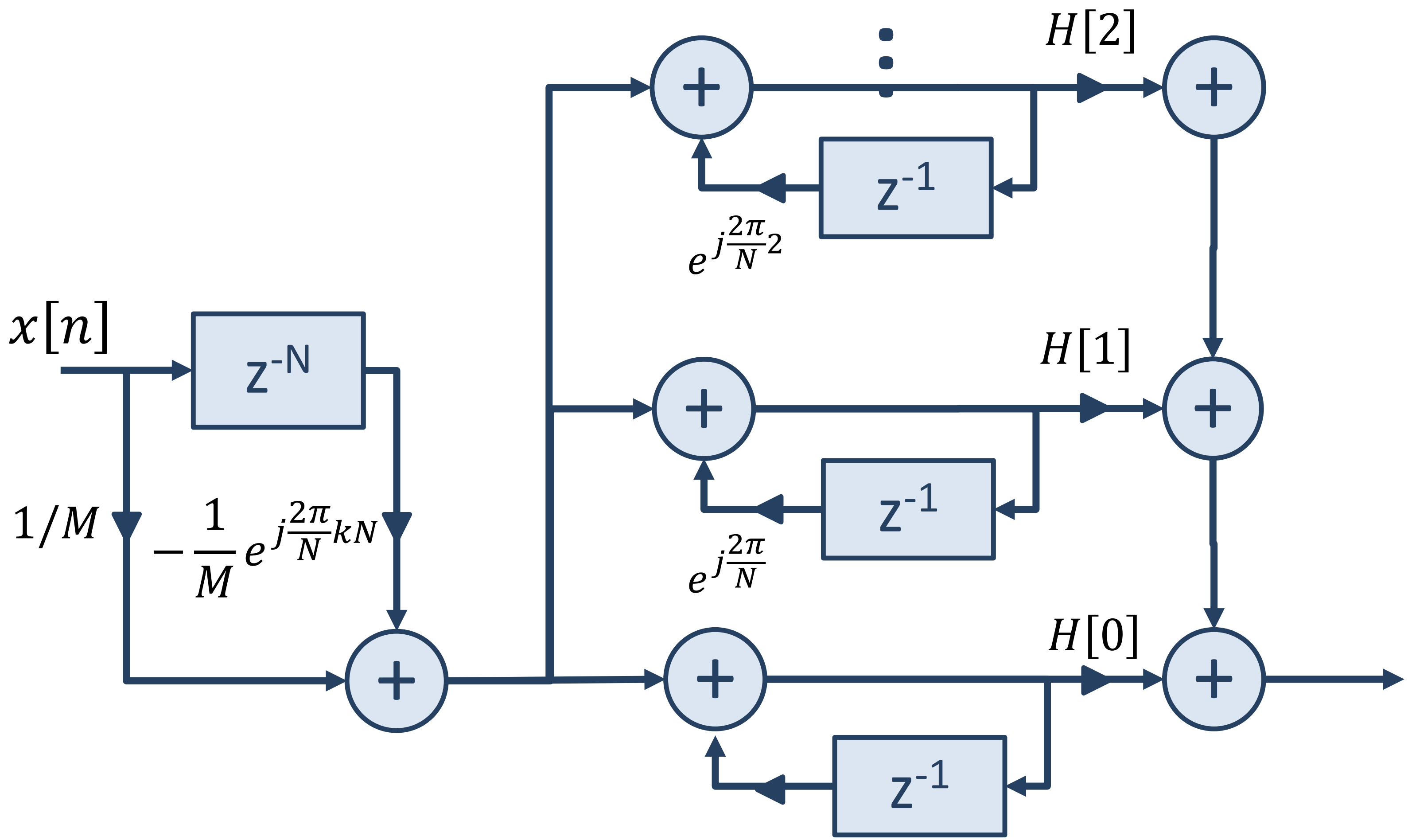
Implementing FIR Filters

- **Frequency Sampling Form**



Implementing FIR Filters

- **Frequency Sampling Form**
  - What are some benefits of using a cascade form implementation?



Lecture 16: Filter Implementation… Again.

Foundations of Digital Signal Processing

Outline

- Reviewing different types of filters
- Designing the phase response
- Implementation of FIR Filters
- **Implementation of IIR Filters**

Implementing IIR Filters

- **Two ways to look at IIR filters with only recursive components**
  - Convolution perspective

$$y[n] + \sum_{m=1}^{M-1} g[m]y[n-m] = x[n]$$

  - Filtering perspective

$$Y(z) = \frac{X(z)}{1 + \sum_{m=Q}^{M-1} g[m]z^{-m}}$$

Implementing IIR Filters

- **Two ways to look at IIR filters with only recursive components**
  - Convolution perspective

$$y[n] = x[n] - \sum_{m=1}^{M-1} g[m]y[n-m]$$

  - Filtering perspective

$$Y(z) = \frac{X(z)}{1 + \sum_{m=Q}^{M-1} g[m]z^{-m}}$$

Implementing IIR Filters

- **IIR Direct Form**

$$Y(Z) = \frac{X(z)}{1 + \sum_{m=Q}^{M-1} g[m] z^{-m}}$$

- **Question: What are the defining characteristics of the pole-zero plot?**

Implementing IIR Filters

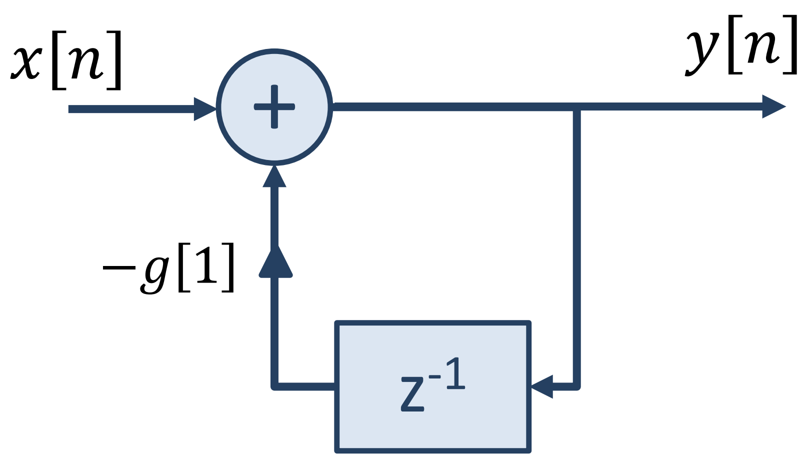
- **IIR Direct Form**

$$y[n] = x[n] - \sum_{m=1}^{M-1} g[m]y[n-m]$$

Implementing IIR Filters

- **IIR Direct Form (M = 1)**

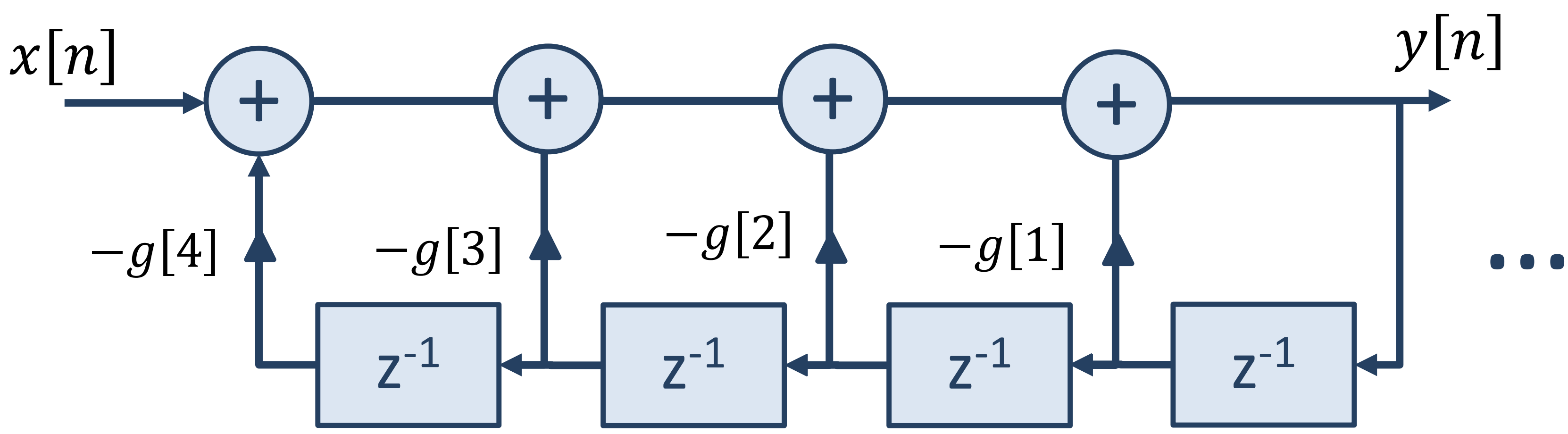
$$y[n] = x[n] - g[1]y[n-1]$$



Implementing IIR Filters

- **IIR Direct Form**

$$y[n] = x[n] - \sum_{m=1}^{M-1} g[m]y[n-m]$$



Implementing IIR Filters

- **Two ways to look at general IIR filters**
  - Convolution perspective

$$y[n] + \sum_{m=1}^{M-1} g[m]y[n-m] = \sum_{k=1}^{M-1} h[k]x[n-k]$$

  - Filtering perspective

$$\frac{Y(z)}{X(z)} = \frac{\sum_{m=Q}^{M-1} h[m]z^{-m}}{1 + \sum_{m=Q}^{M-1} g[m]z^{-m}}$$

Implementing IIR Filters

- **Two ways to look at general IIR filters**
  - Convolution perspective

$$y[n] = \sum_{k=1}^{M-1} h[k]x[n-k] - \sum_{m=1}^{M-1} g[m]y[n-m]$$

  - Filtering perspective

$$\frac{Y(z)}{X(z)} = \frac{\sum_{m=Q}^{M-1} h[m]z^{-m}}{1 + \sum_{m=Q}^{M-1} g[m]z^{-m}}$$

Implementing IIR Filters

- **Two ways to look at IIR filters with only recursive components**
  - Convolution perspective

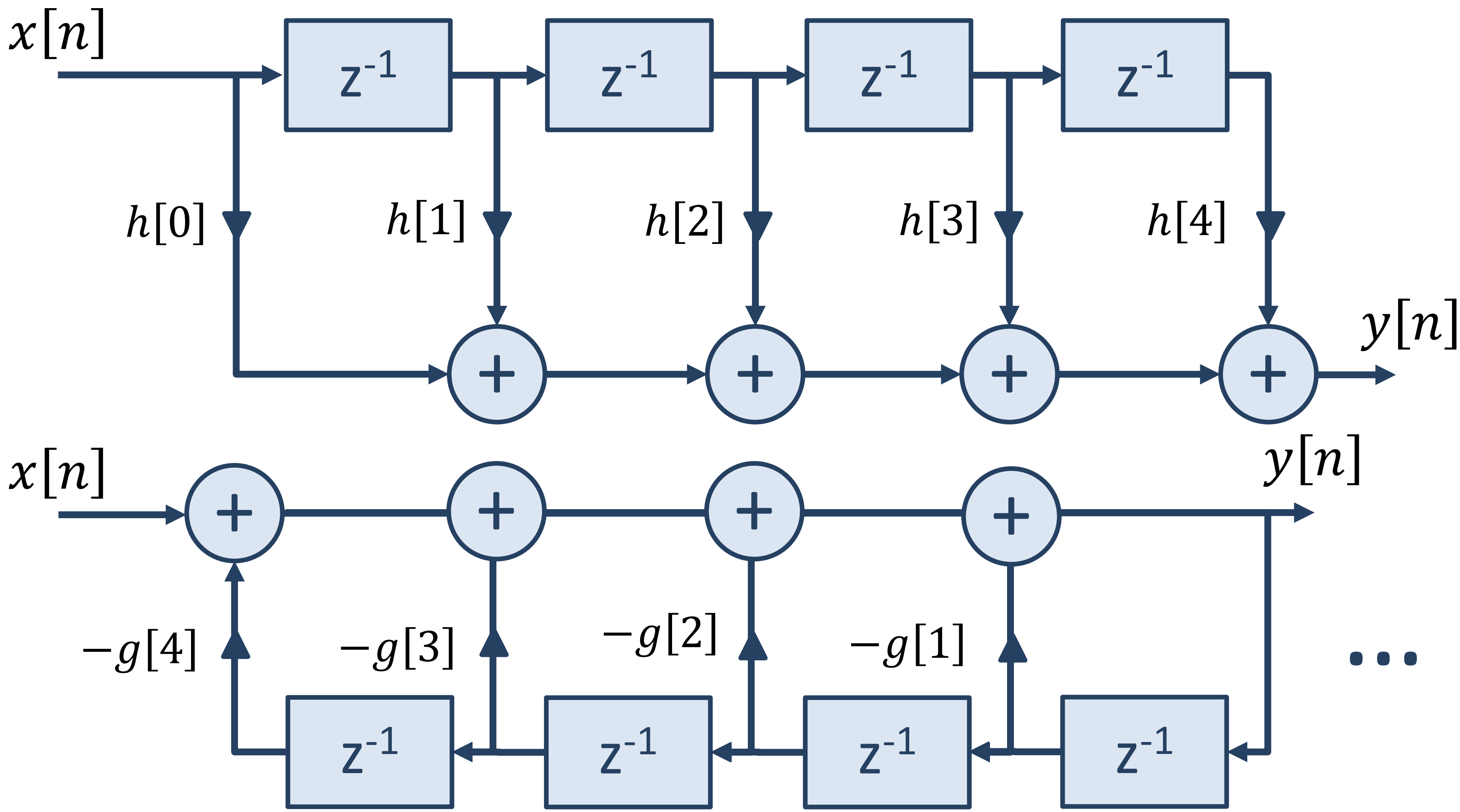
$$y[n] = x[n] - \sum_{m=1}^{M-1} g[m]y[n-m]$$

  - Filtering perspective

$$\frac{Y(z)}{X(z)} = \frac{\sum_{m=Q}^{M-1} h[m]z^{-m}}{1 + \sum_{m=Q}^{M-1} g[m]z^{-m}}$$

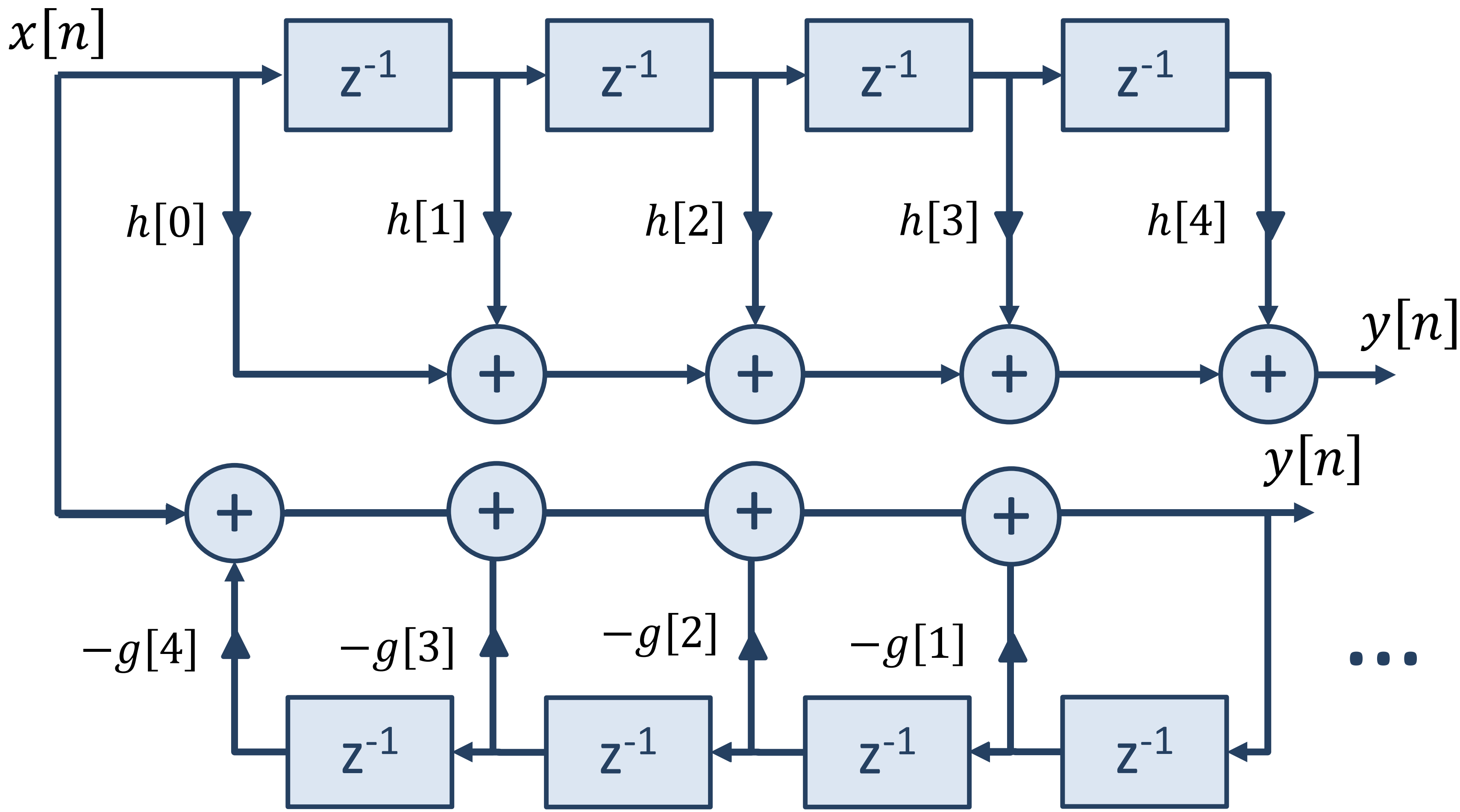
Implementing IIR Filters

- **IIR Direct Form**



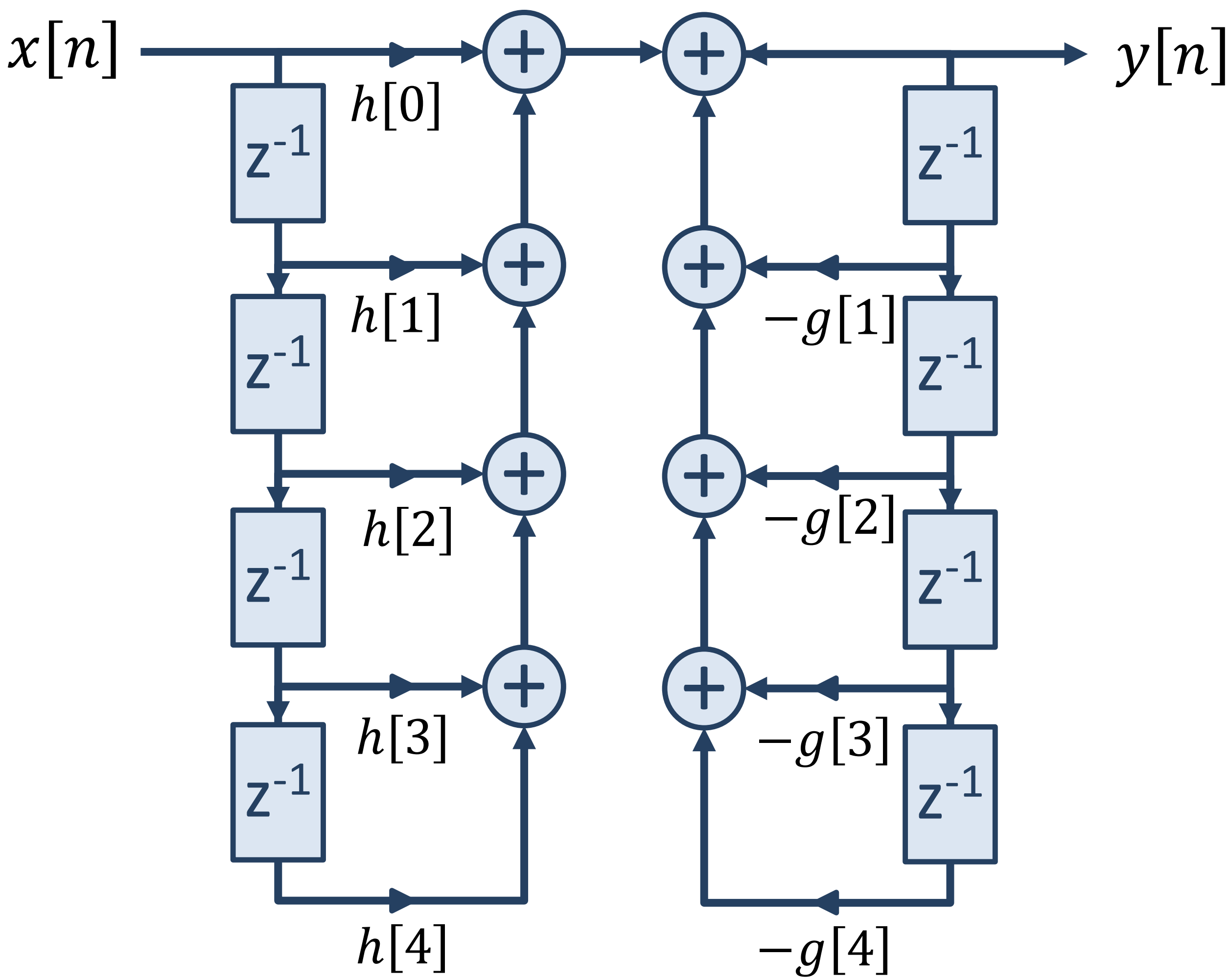
Implementing IIR Filters

- **IIR Direct Form**



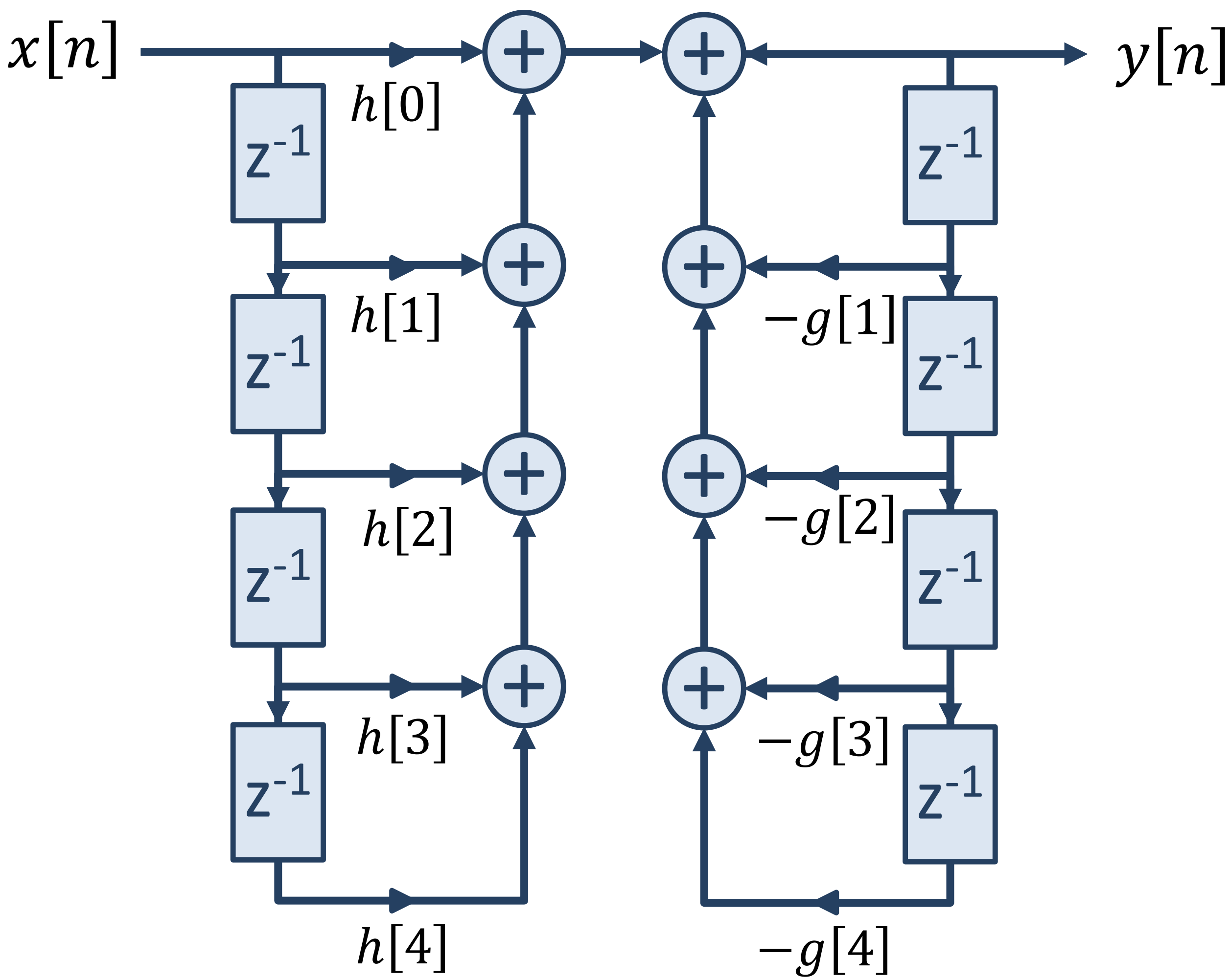
Implementing IIR Filters

- **IIR Direct Form**



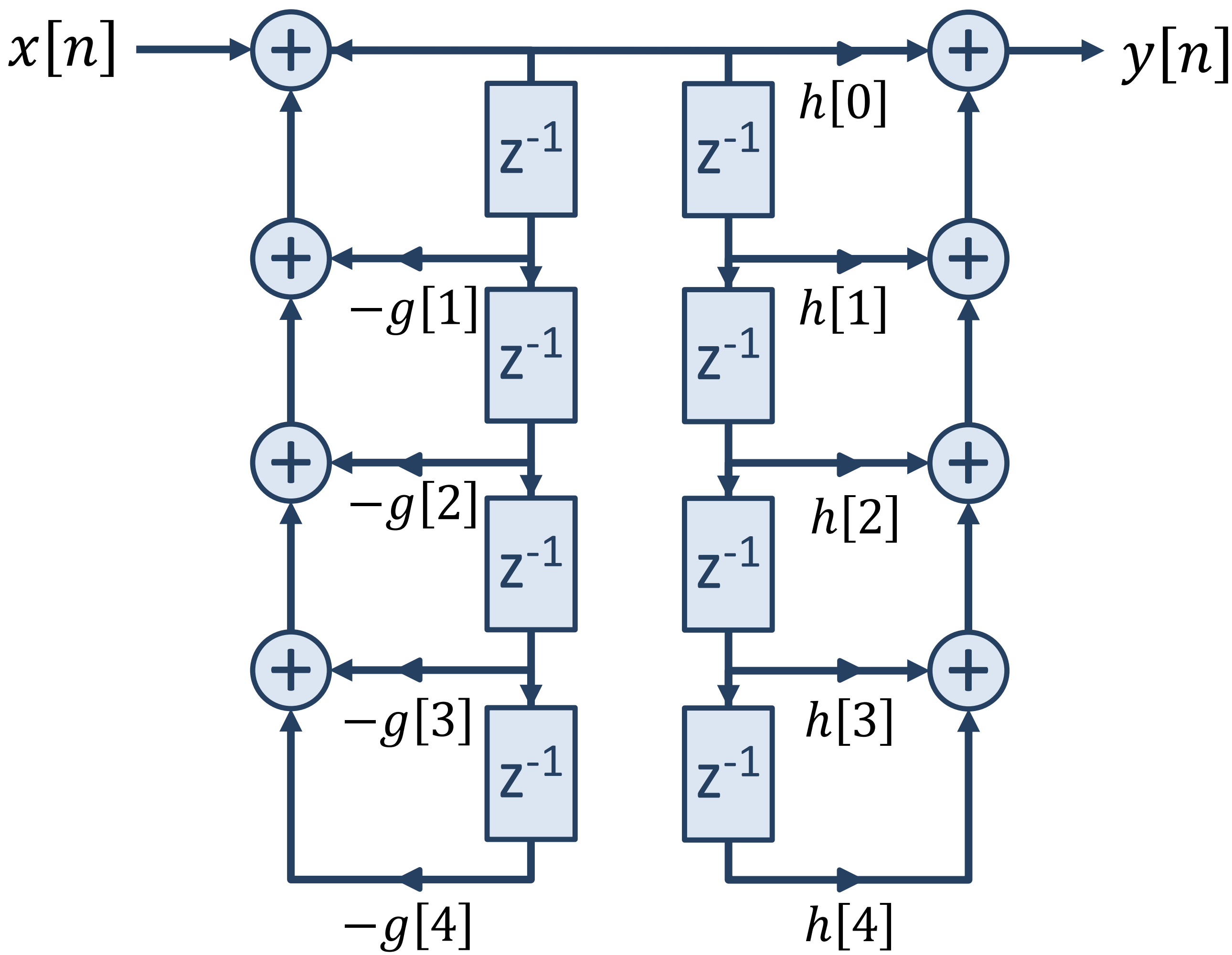
Implementing IIR Filters

- **IIR Direct Form I**



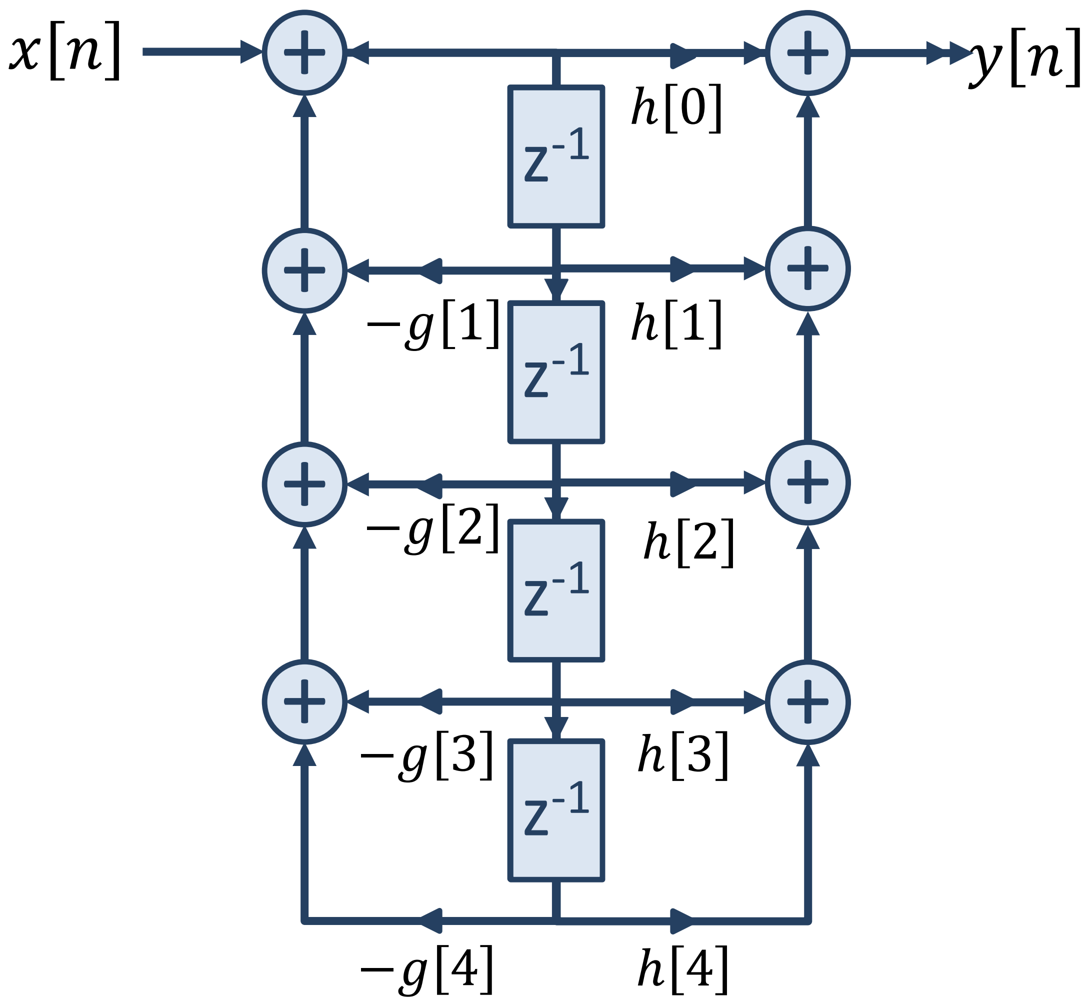
Implementing IIR Filters

- **IIR Direct Form II**



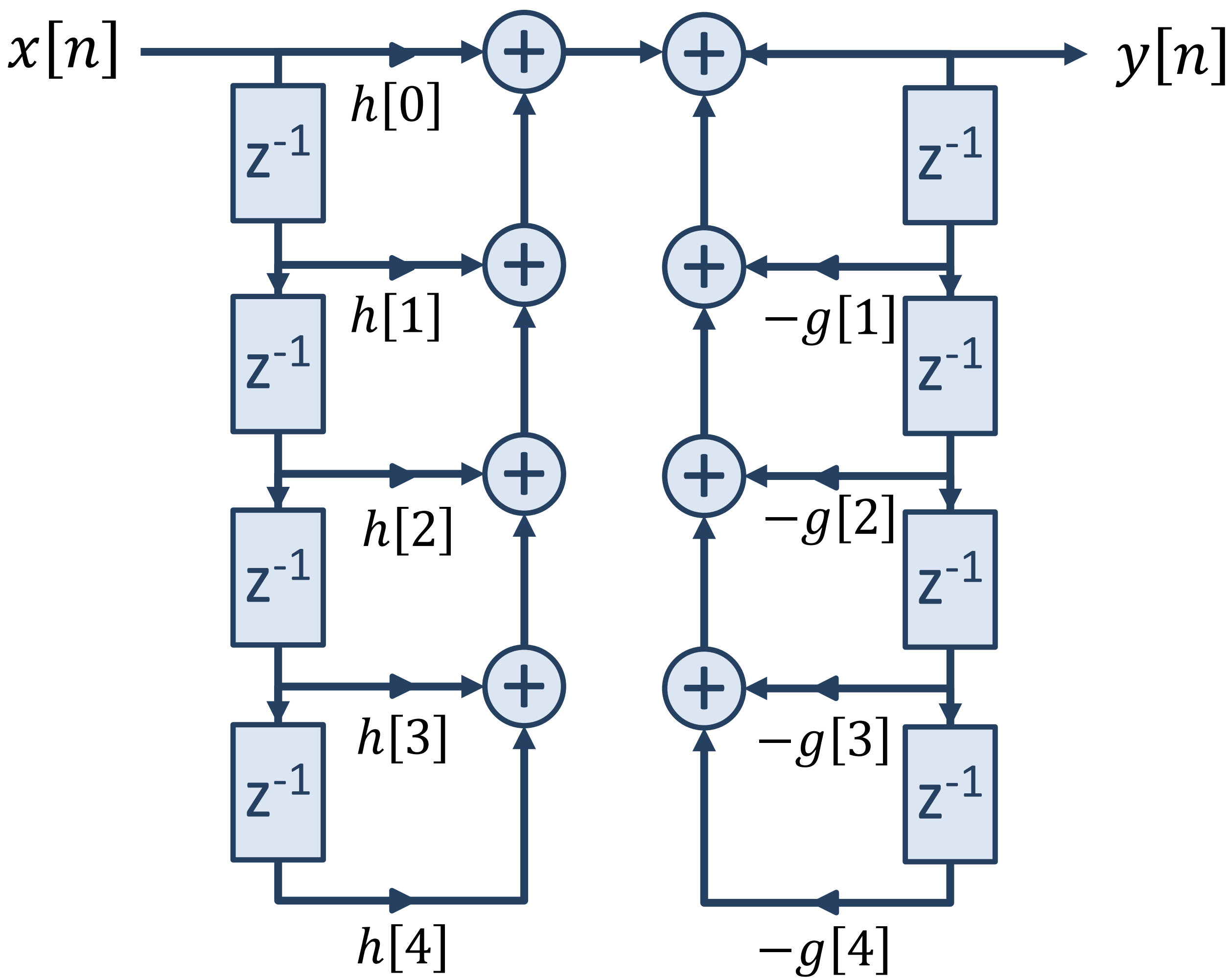
Implementing IIR Filters

- **IIR Direct Form II**



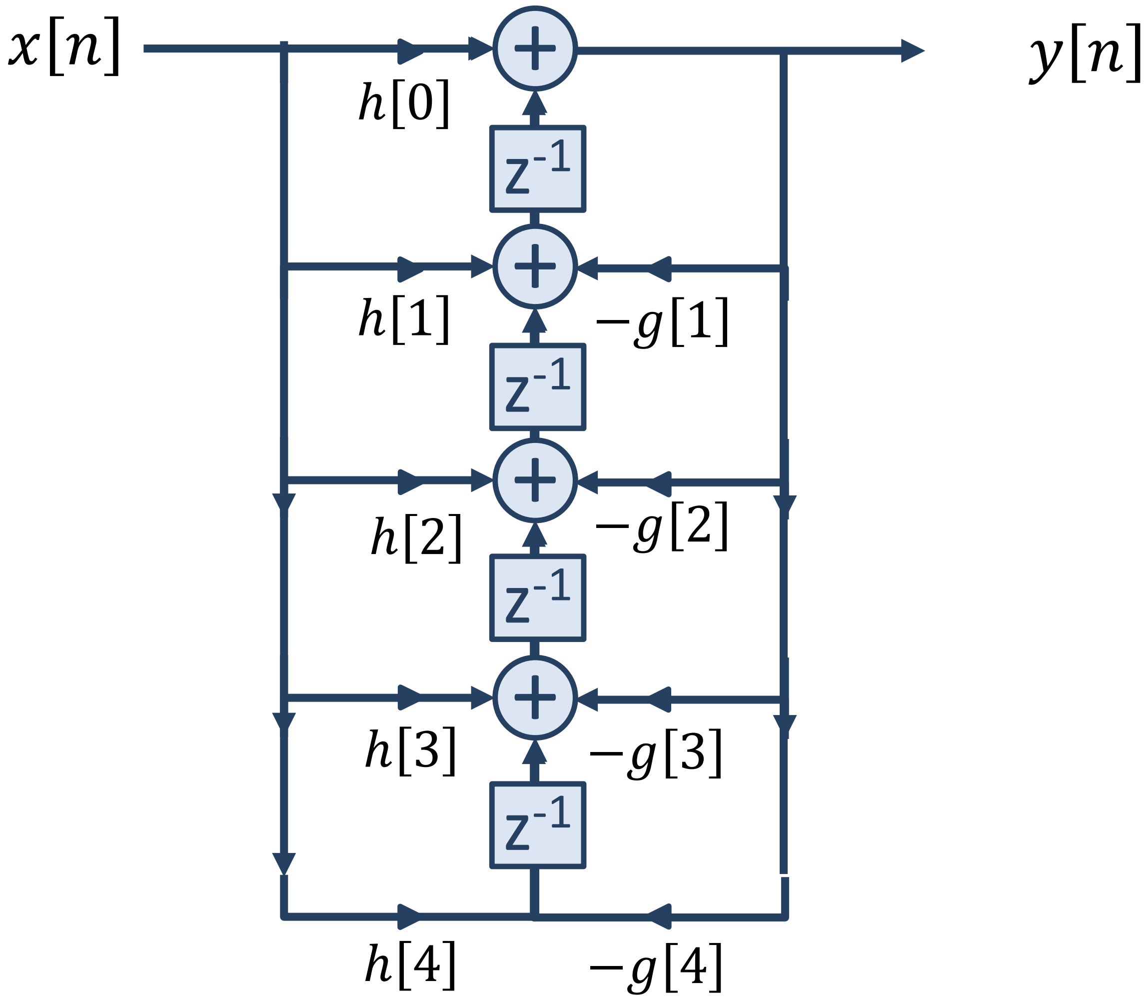
Implementing IIR Filters

- **IIR Direct Form I**



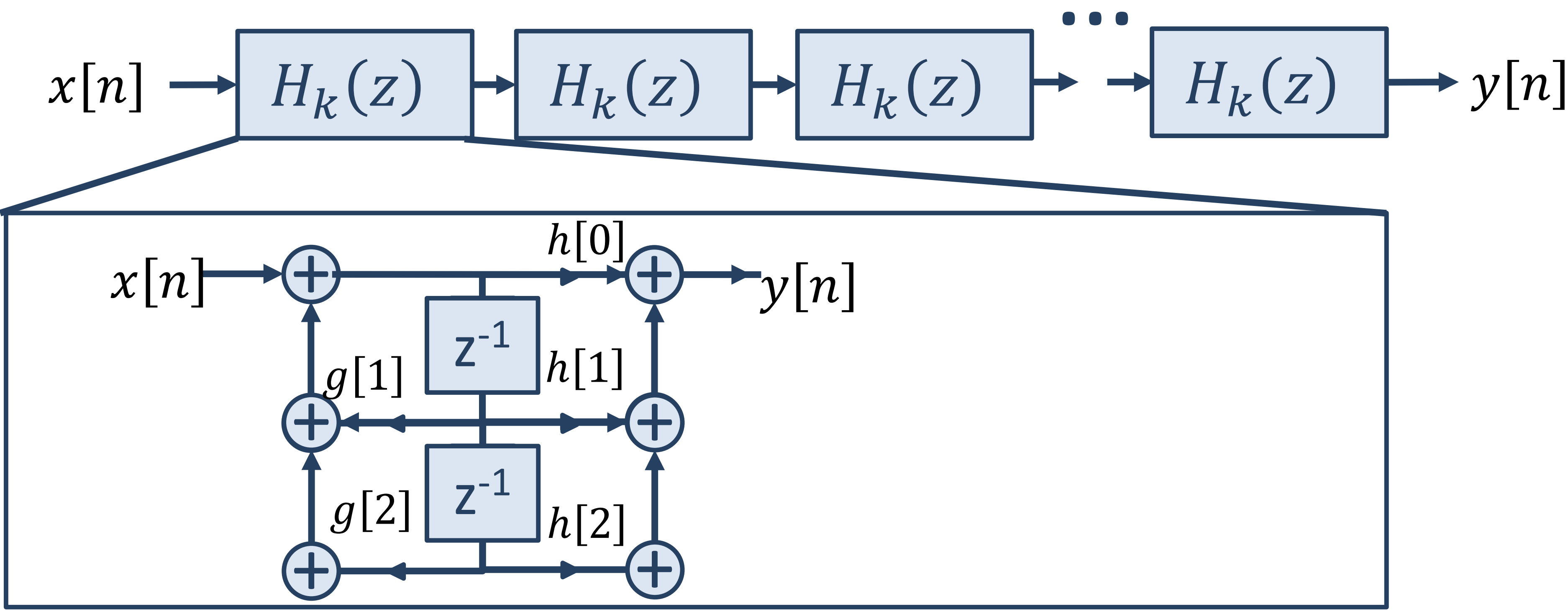
Implementing IIR Filters

- **Transposed IIR Direct Form II**



IIR Cascade Form

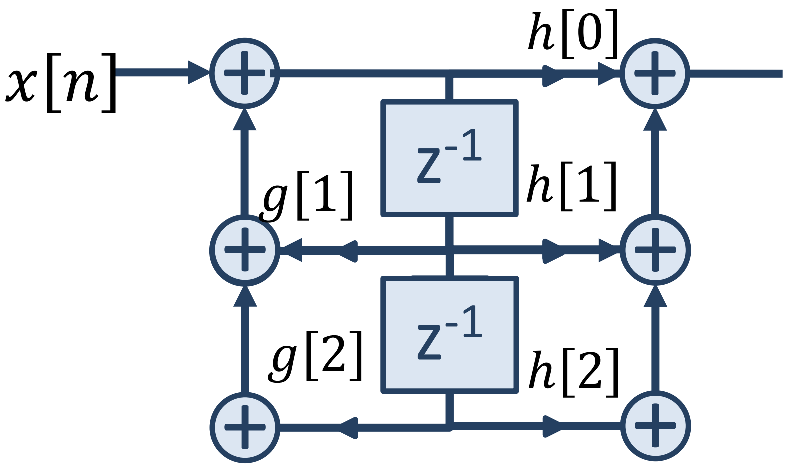
$$Y(z) = X(z)\sum_{m=0}^{M-1} h[m]z^{-m} = X(z)\prod_{k=1}^{K} H_k(z)$$



Implementing FIR Filters

- **IIR Cascade Form**

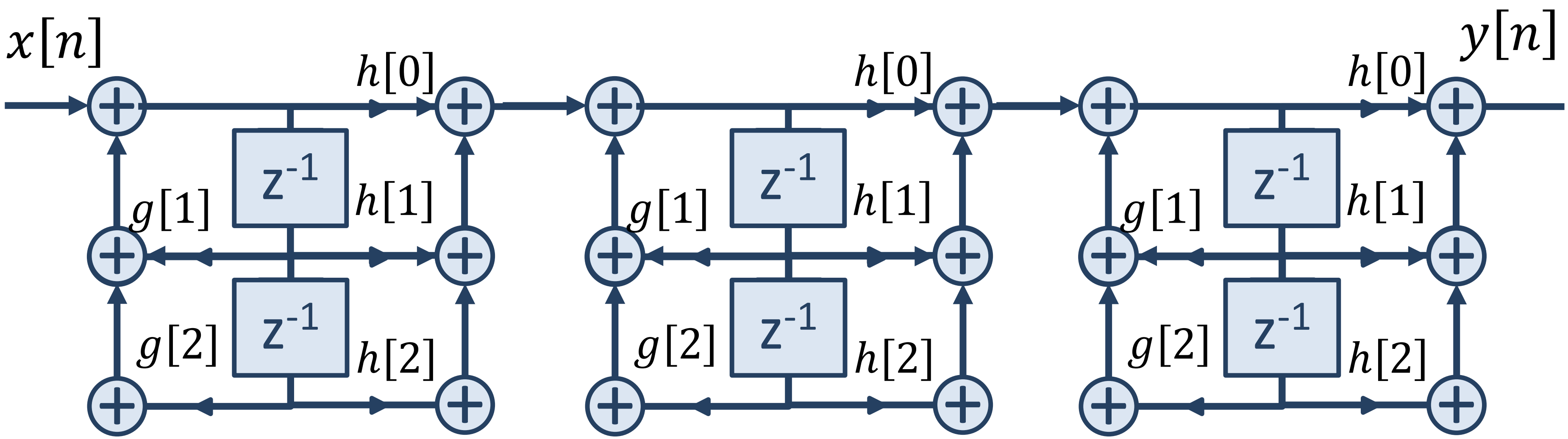
$$Y(z) = X(z) \sum_{m=0}^{M-1} h[m] z^{-m} = X(z) \prod_{k=1}^{K} H_k(z)$$



Implementing FIR Filters

- **IIR Cascade Form**

$$Y(z) = X(z)\sum_{m=0}^{M-1} h[m]z^{-m} = X(z)\prod_{k=1}^{K} H_k(z)$$



Implementing IIR Filters

- **Parallel Form**

$$H(z) = C + \sum_{m=0}^{M-1} \frac{a[m]}{1 - p_m z^{-1}}$$