Lecture 17: Lattice Structures

Foundations of Digital Signal Processing

Outline

- Implementation of FIR Filters
- Implementation of IIR Filters
- Implementation of Lattice Filters

News

- **Homework #7**
  - Due *today*
  - Submit via canvas
- **Coding Problem #4**
  - Due *today*
  - Submit via canvas
- **In 1.5 weeks**
  - Exam #2 (yay!)

Lecture 17: Lattice Structures

Foundations of Digital Signal Processing

Outline

- **Implementation of FIR Filters**
- Implementation of IIR Filters
- Implementation of Lattice Filters

Implementing FIR Filters

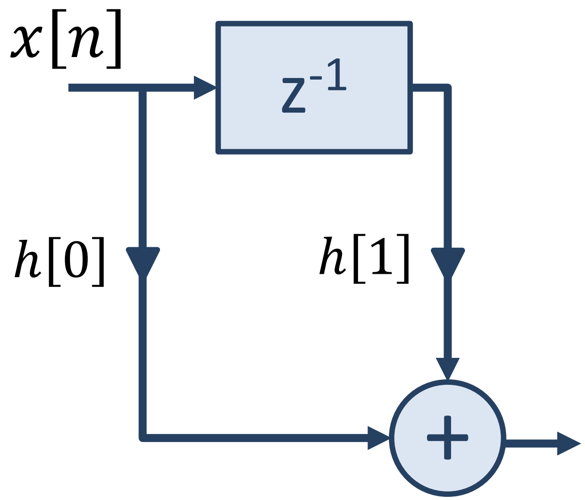
- **FIR Direct Form**

$$y[n] = \sum_{m=0}^{M-1} h[m]x[n-m]$$

Implementing FIR Filters

- **FIR Direct Form (M = 1)**

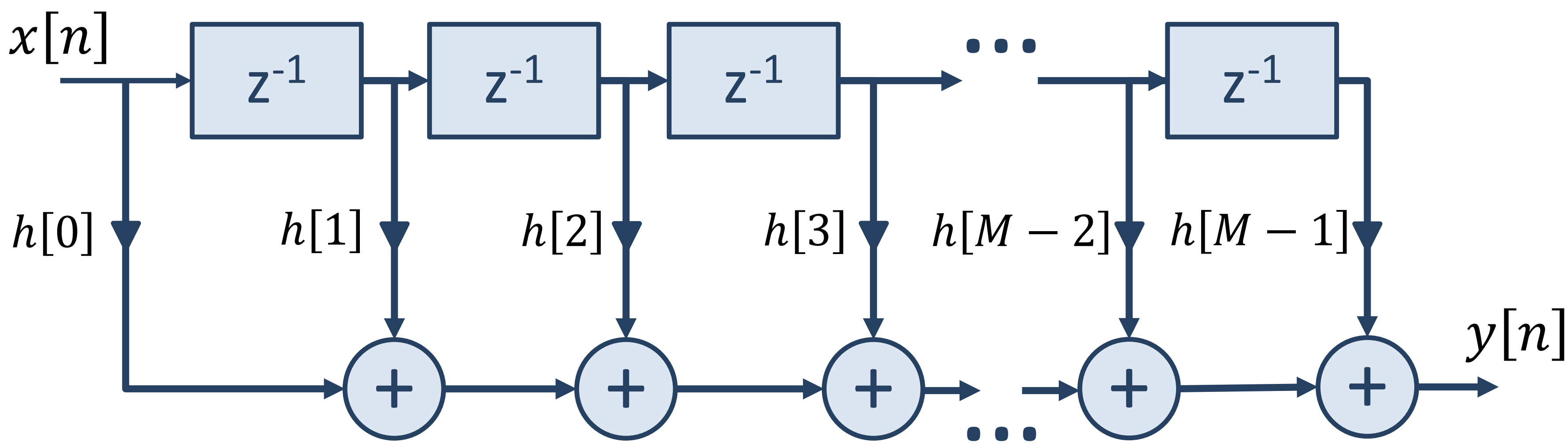
$$y[n] = h[0] + h[1]x[n-1]$$



Implementing FIR Filters

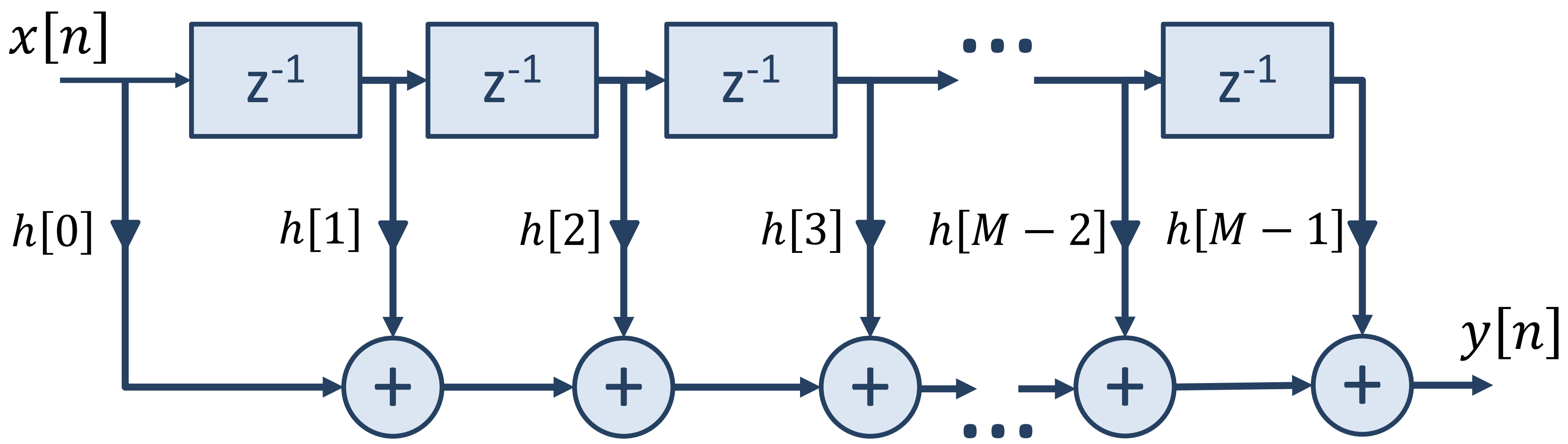
- **FIR Direct Form**

$$y[n] = \sum_{m=0}^{M-1} h[m]x[n-m]$$



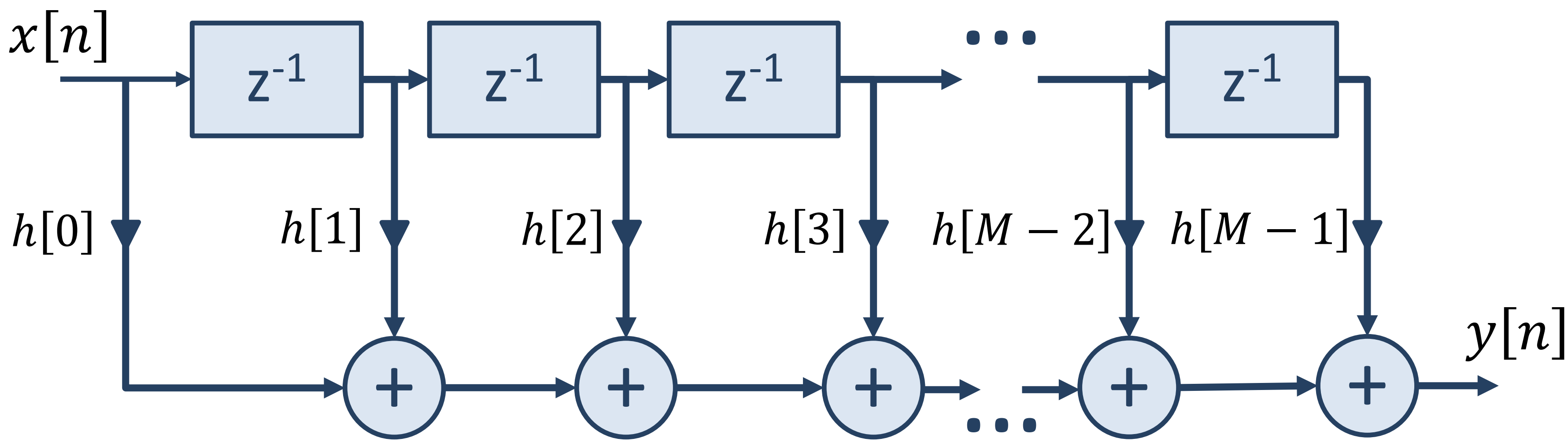
Implementing FIR Filters

- **FIR Direct Form**
  - What are some benefits of using a direct form implementation?



Implementing FIR Filters

- **FIR Direct Form**
  - If the impulse is symmetric, then we can shorten this



Implementing FIR Filters

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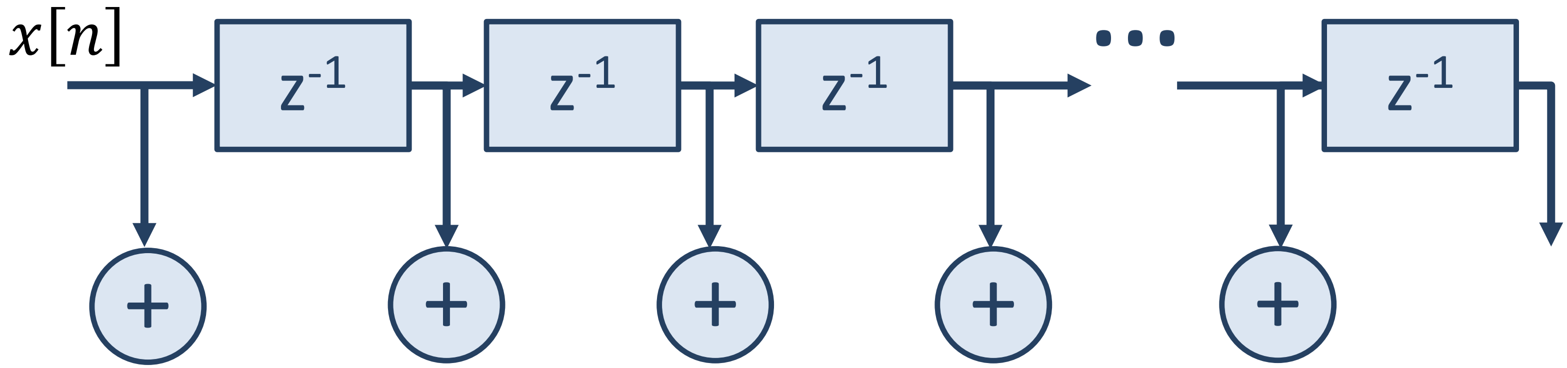

Implementing FIR Filters

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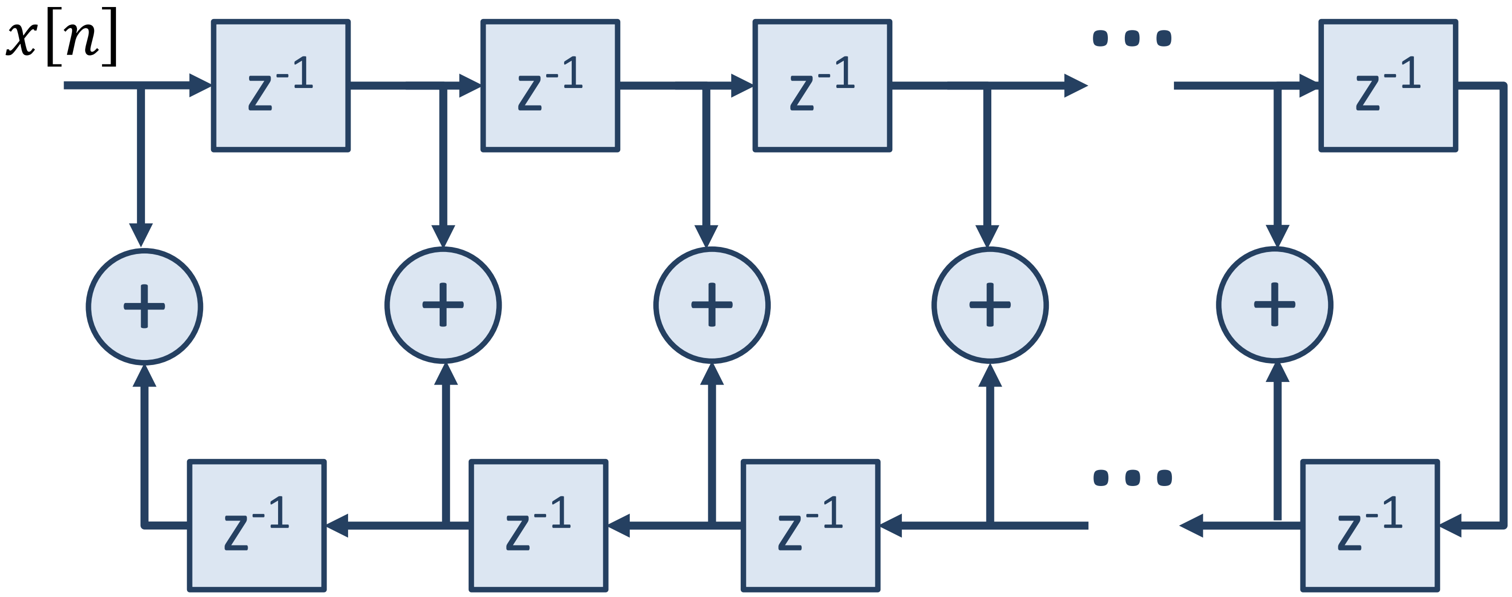

Implementing FIR Filters

- **FIR Direct Form**
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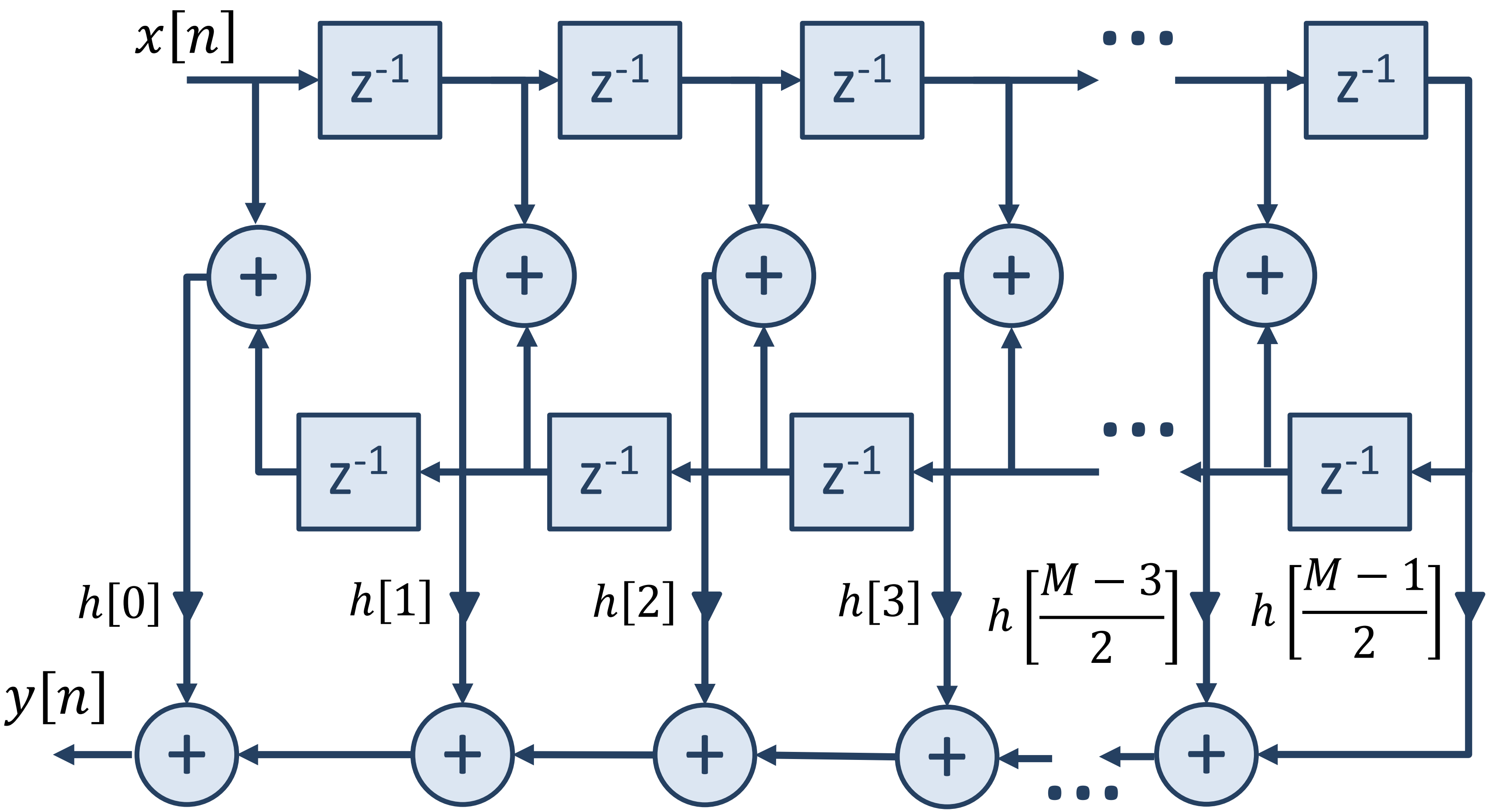
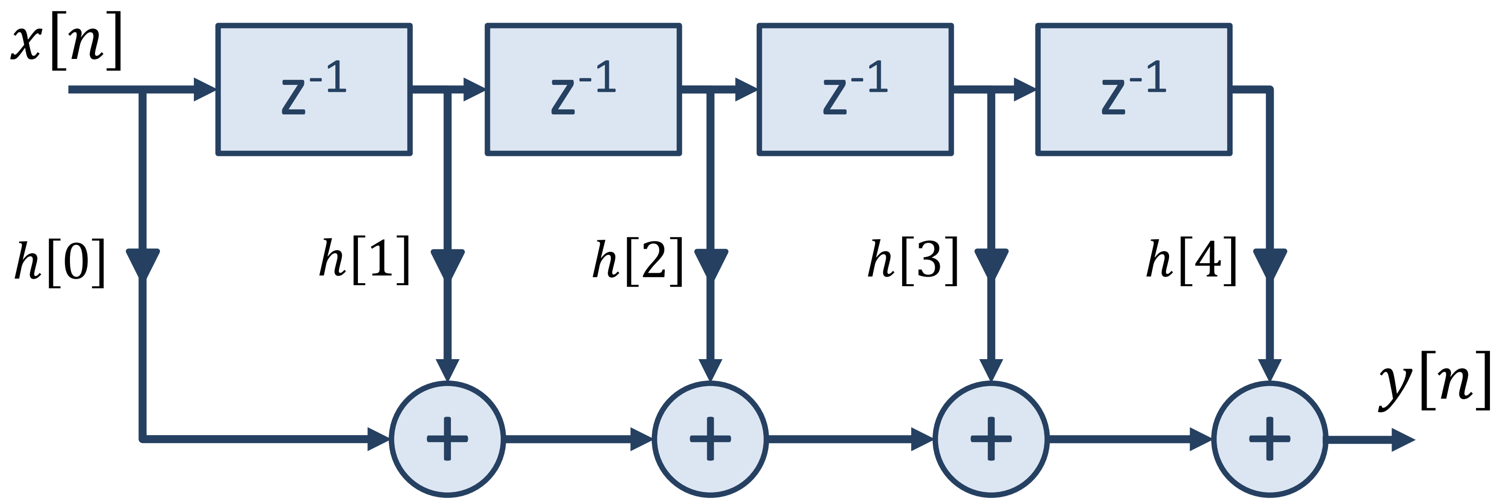


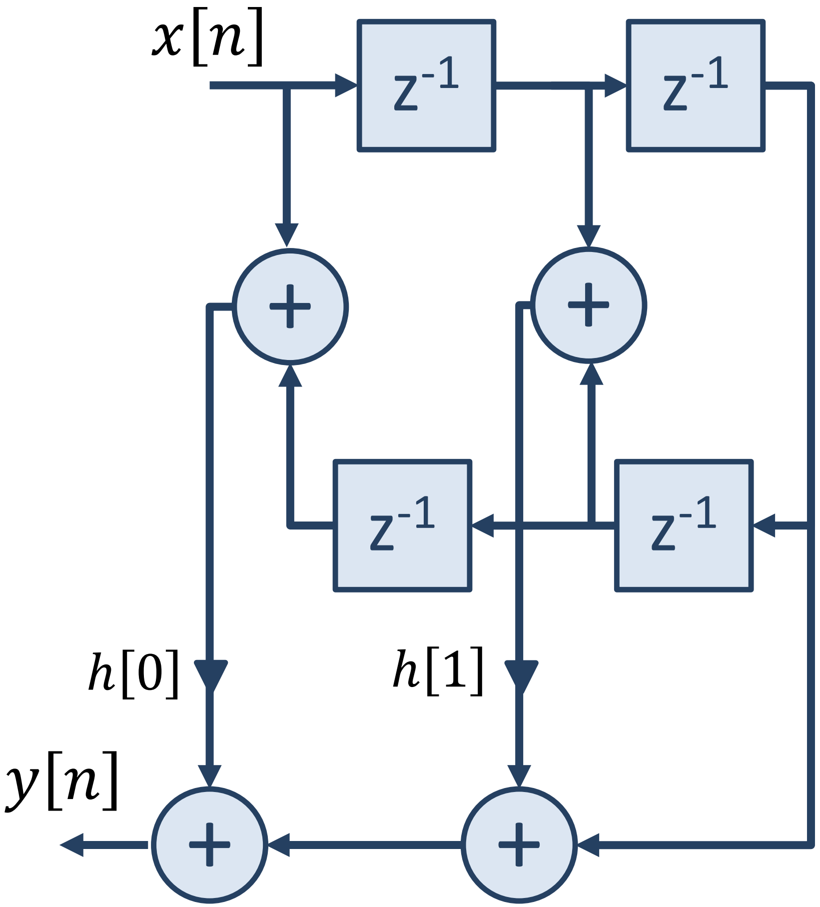
Implementing FIR Filters

- **FIR Direct Form**
  - Non-symmetric impulse response (4 multiplications)



Implementing FIR Filters

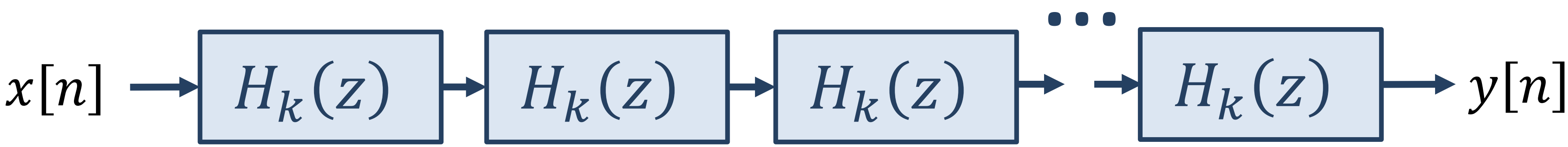
- **FIR Direct Form**
  - Symmetric impulse response (2 multiplications)



Implementing FIR Filters

- **FIR Cascade Form**

$$Y[z] = X(z)\sum_{m=0}^{M-1} h[m]z^{-m} = X(z)\prod_{k=1}^{K} H_k(z)$$



Implementing FIR Filters

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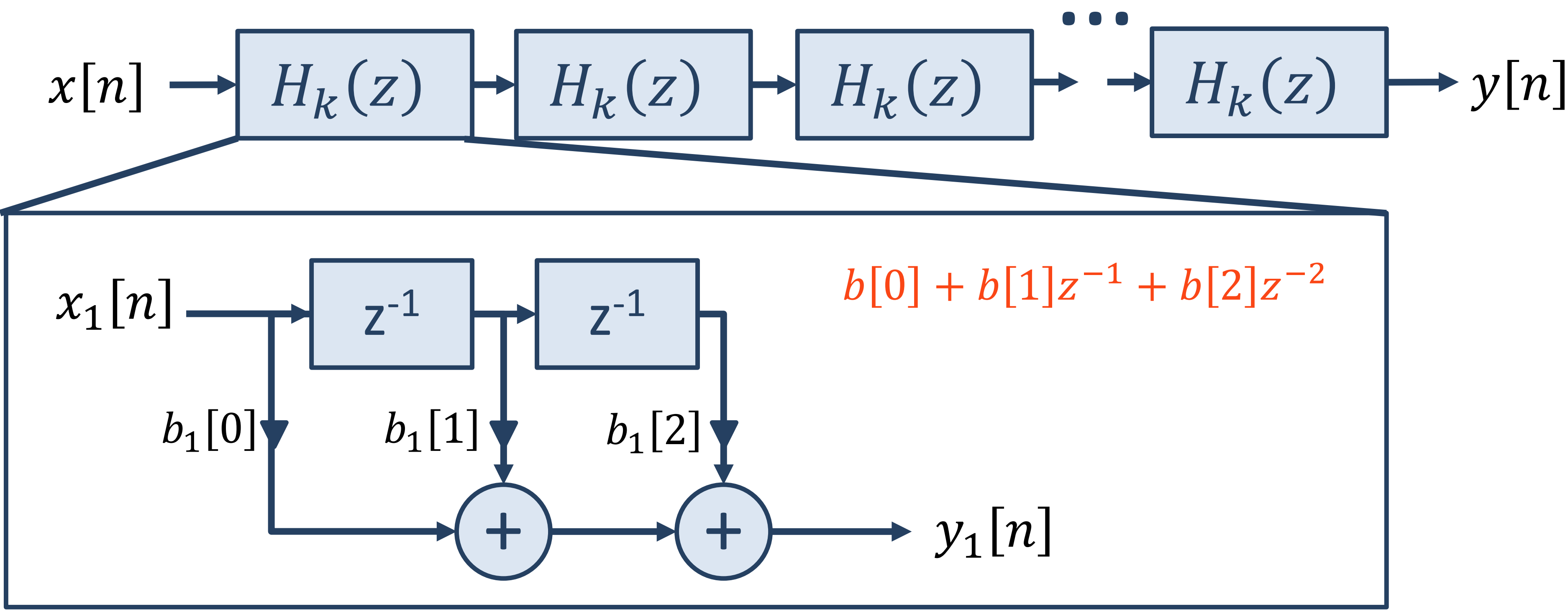

Implementing FIR Filters

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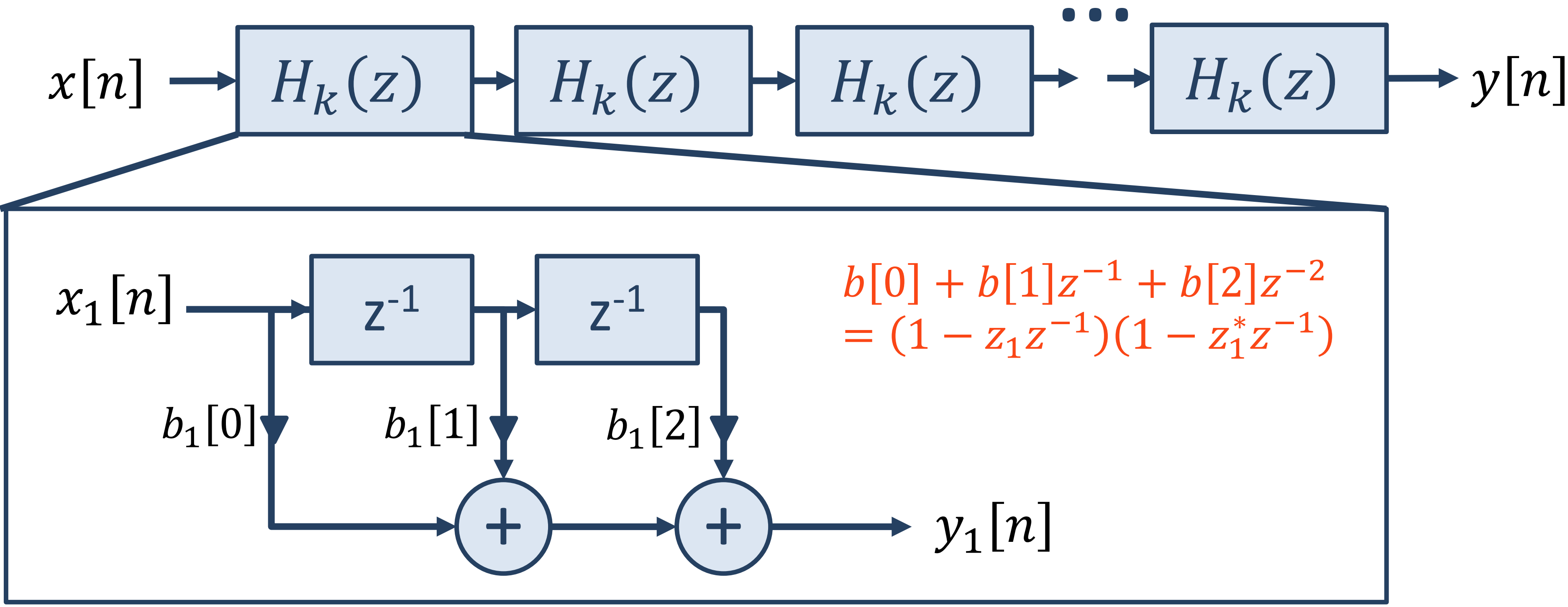

Implementing FIR Filters

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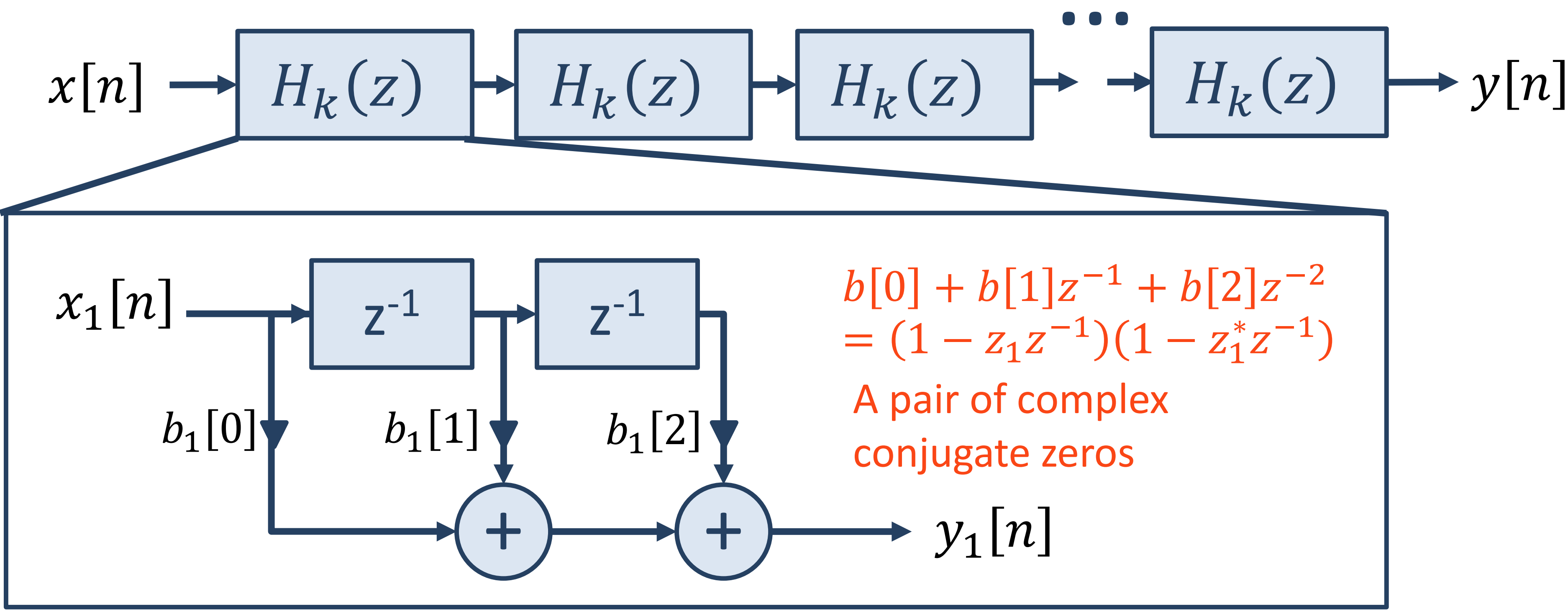
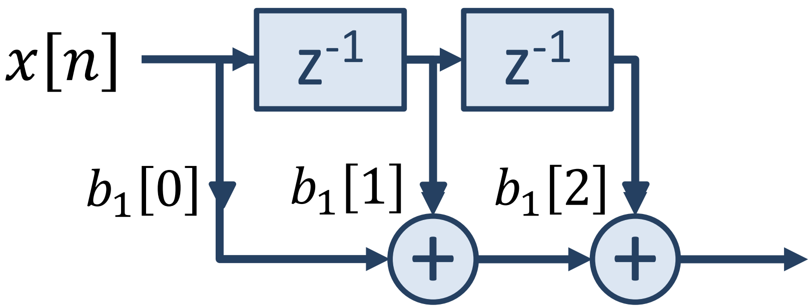


Implementing FIR Filters

- **FIR Cascade Form**

  - What are some benefits of using a cascade form implementation?



Implementing FIR Filters

- **FIR Cascade Form**
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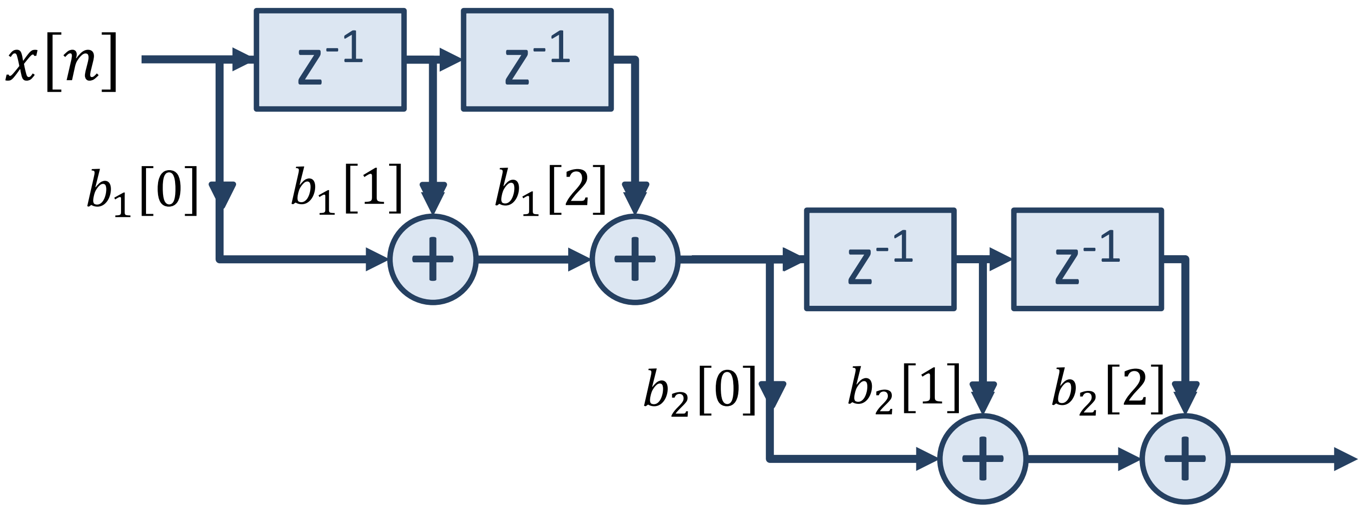

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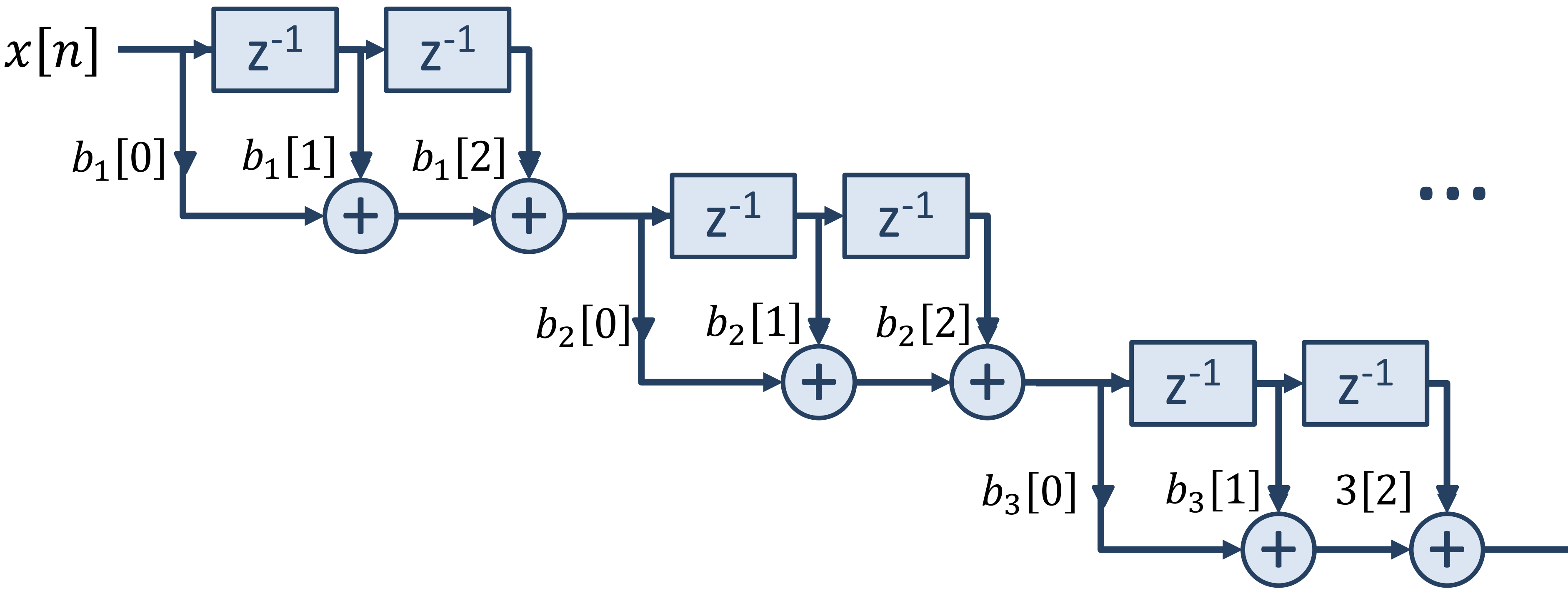
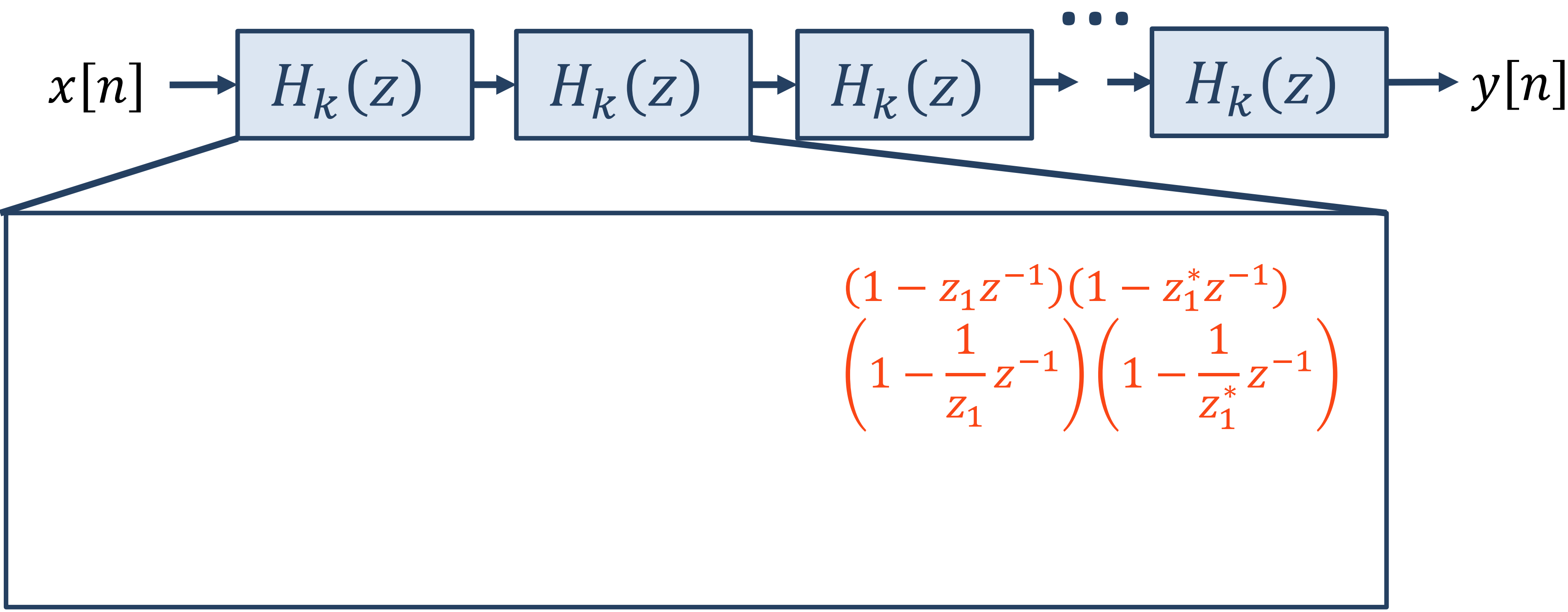


Implementing FIR Filters

- **FIR Cascade Form (if the impulse response is symmetric)**

$$Y[z] = X(z)\sum_{m=0}^{M-1} h[m]z^{-m} = X(z)\prod_{k=1}^{K} H_k(z)$$



$$(1 - z_1 z^{-1})(1 - z_1^* z^{-1})$$

$$\left(1 - \frac{1}{z_1} z^{-1}\right)\left(1 - \frac{1}{z_1^*} z^{-1}\right)$$

Implementing FIR Filters

- **FIR Cascade Form (if the impulse response is symmetric)**

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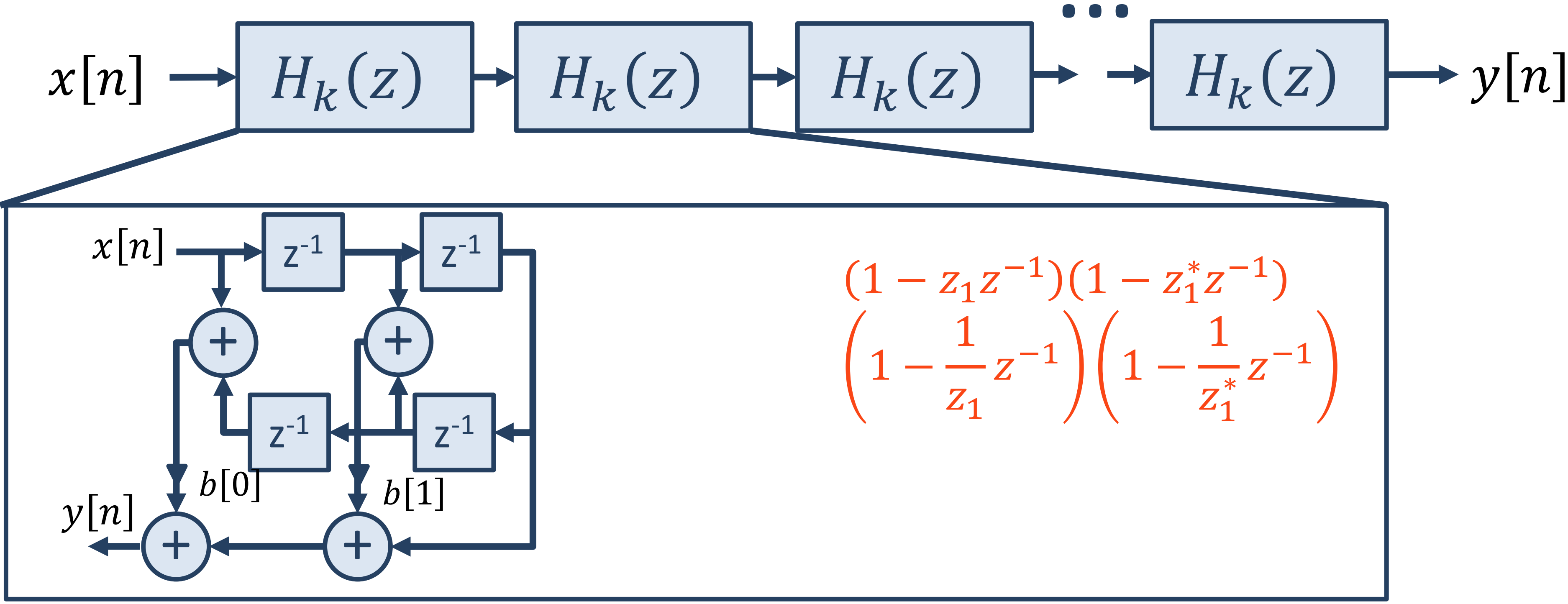

$$(1 - z_1 z^{-1})(1 - z_1^* z^{-1})$$
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Implementing FIR Filters

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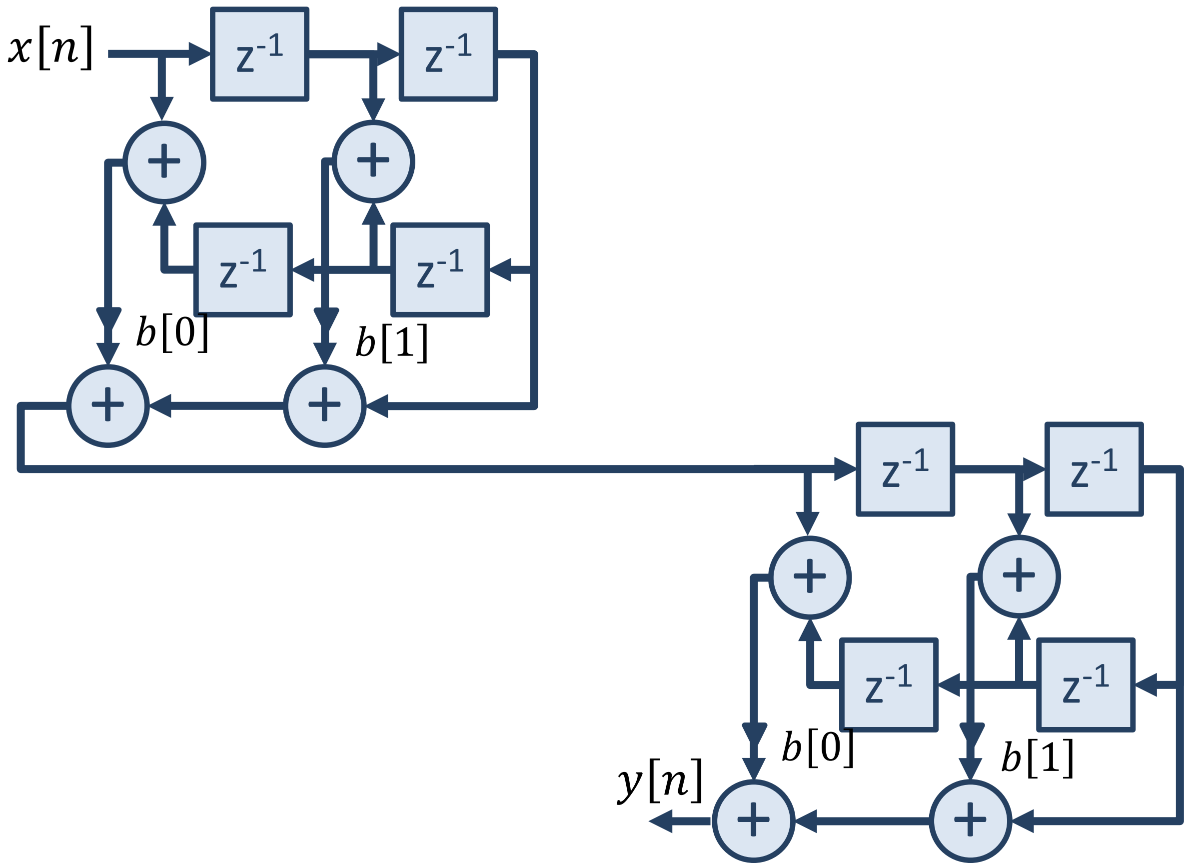

…

Implementing FIR Filters

- **Frequency Sampling Form**

$$H[k] = \sum_{n=0}^{N-1} h[n] e^{-j\frac{2\pi}{N}kn} \quad , \qquad h[n] = \frac{1}{N}\sum_{k=0}^{N-1} H[k] e^{j\frac{2\pi}{N}kn}$$

Implementing FIR Filters

- **Frequency Sampling Form**

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$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n} = \sum_{n=0}^{N-1}\left(\frac{1}{N}\sum_{k=0}^{N-1} H[k] e^{j\frac{2\pi}{N}kn}\right) z^{-n}$$

$$= \sum_{n=0}^{N-1}\left(\frac{1}{N}\sum_{k=0}^{N-1} H[k] e^{j\frac{2\pi}{N}kn}\right) z^{-n}$$

$$= \sum_{k=0}^{N-1}\left(H[k]\frac{1}{N}\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}kn} z^{-n}\right)$$

Implementing FIR Filters

- **Frequency Sampling Form**

$$H[k] = \sum_{n=0}^{N-1} h[n] e^{-j\frac{2\pi}{N}kn} \quad , \qquad h[n] = \frac{1}{N}\sum_{k=0}^{N-1} H[k] e^{j\frac{2\pi}{N}kn}$$

$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n} = \sum_{n=0}^{N-1}\left(\frac{1}{N}\sum_{k=0}^{N-1} H[k] e^{j\frac{2\pi}{N}kn}\right) z^{-n}$$

$$= \sum_{k=0}^{N-1}\left(H[k]\frac{1}{N}\frac{1-e^{j\frac{2\pi}{N}kN}z^{-N}}{1-e^{j\frac{2\pi}{N}kn}z^{-n}}\right)$$

$$= \frac{1-e^{j\frac{2\pi}{N}kN}z^{-N}}{N}\sum_{k=0}^{N-1}\left(\frac{H[k]}{1-e^{j\frac{2\pi}{N}kn}z^{-n}}\right)$$

Implementing FIR Filters

- **Frequency Sampling Form**

$$H(z) = \frac{1 - e^{j\frac{2\pi}{N}kN} z^{-N}}{N} \sum_{k=0}^{N-1} \left(\frac{H[k]}{1 - e^{j\frac{2\pi}{N}kn} z^{-n}} \right)$$

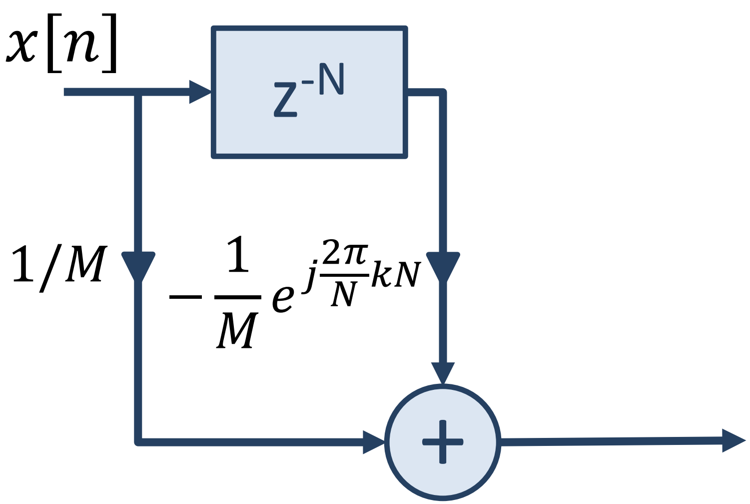
Implementing FIR Filters

- **Frequency Sampling Form**

$$H(z) = \underbrace{\frac{1 - e^{j\frac{2\pi}{N}kN} z^{-N}}{N}}_{\text{FIR Filter}} \sum_{k=0}^{N-1} \underbrace{\left(\frac{\overset{\text{Weights}}{H[k]}}{1 - e^{j\frac{2\pi}{N}kn} z^{-n}} \right)}_{\text{IIR Filter}}$$

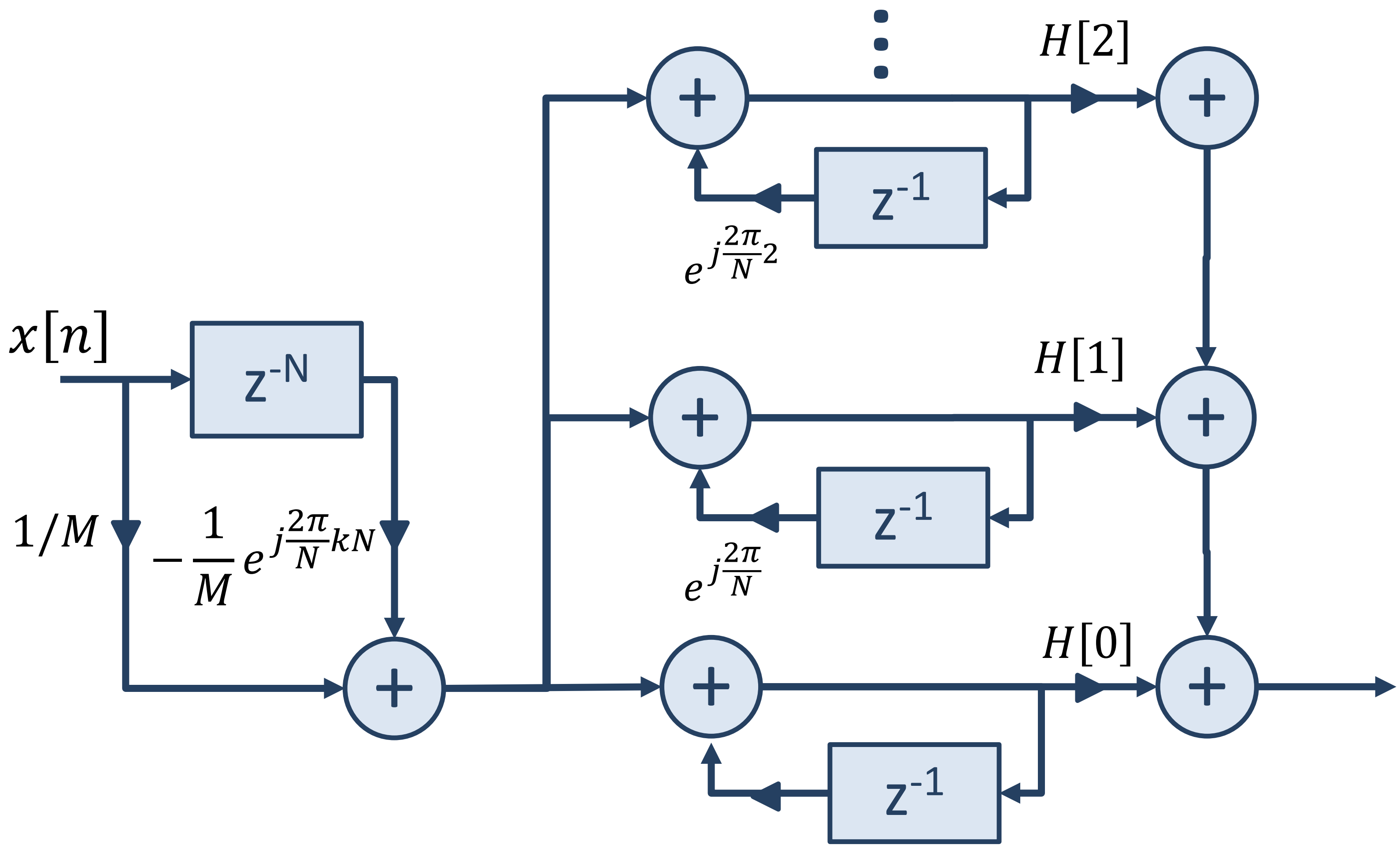
- **Frequency Sampling Form**

$$H(z) = \frac{1 - e^{j\frac{2\pi}{N}kN} z^{-N}}{N} \sum_{k=0}^{N-1} \left(\frac{H[k]}{1 - e^{j\frac{2\pi}{N}kn} z^{-n}} \right)$$



Implementing FIR Filters

- **Frequency Sampling Form**



Implementing FIR Filters

- **Frequency Sampling Form**
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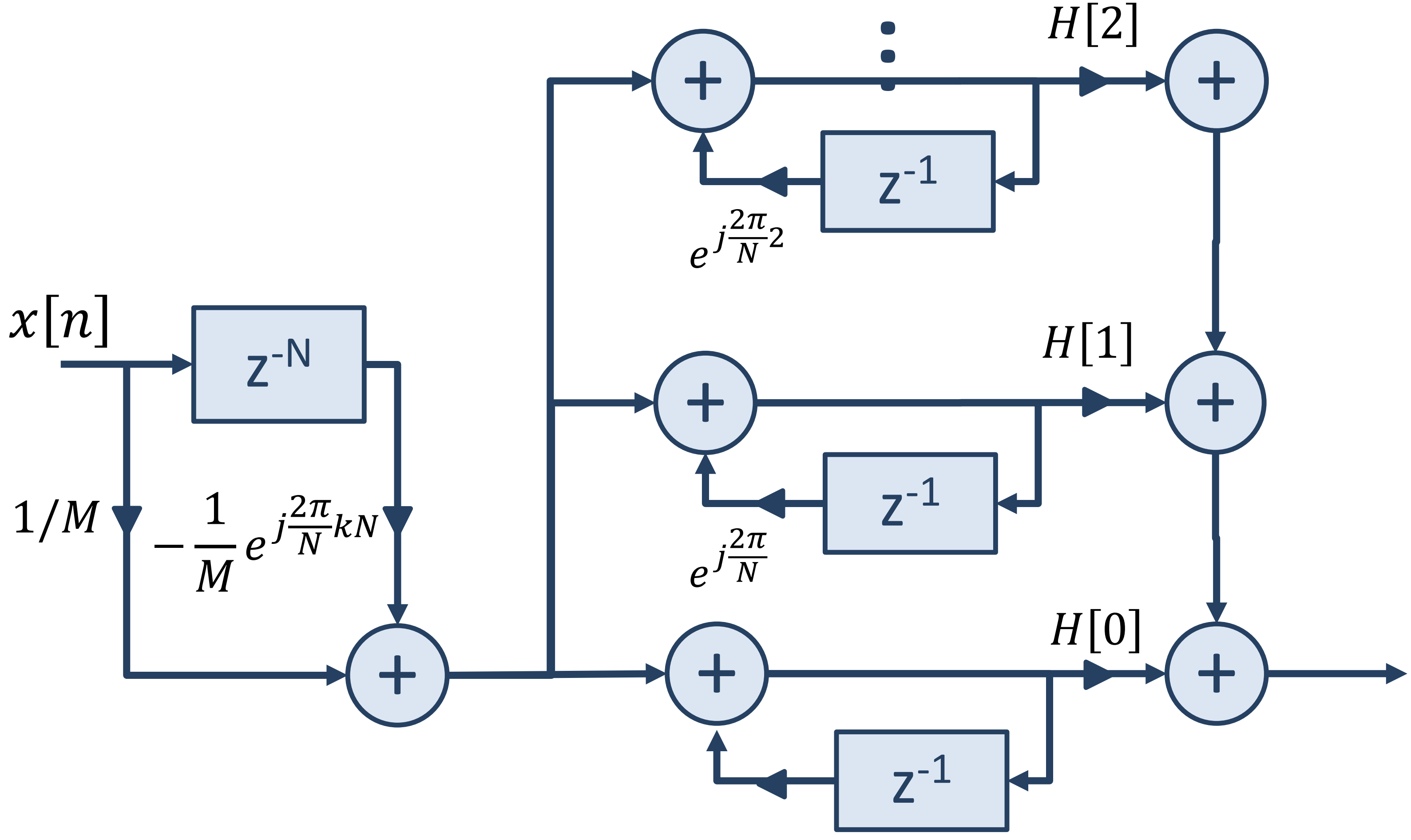


Lecture 17: Lattice Structures

Foundations of Digital Signal Processing

Outline

- Implementation of FIR Filters
- **Implementation of IIR Filters**
- Implementation of Lattice Filters

Implementing IIR Filters

- **Two ways to look at IIR filters with only recursive components**
  - Convolution perspective

$$y[n] + \sum_{m=1}^{M-1} g[m]y[n-m] = x[n]$$

  - Filtering perspective

$$Y(z) = \frac{X(z)}{1 + \sum_{m=Q}^{M-1} g[m]z^{-m}}$$

Implementing IIR Filters

- **Two ways to look at IIR filters with only recursive components**
  - Convolution perspective

$$y[n] = x[n] - \sum_{m=1}^{M-1} g[m]y[n-m]$$

  - Filtering perspective

$$Y(z) = \frac{X(z)}{1 + \sum_{m=Q}^{M-1} g[m]z^{-m}}$$

Implementing IIR Filters

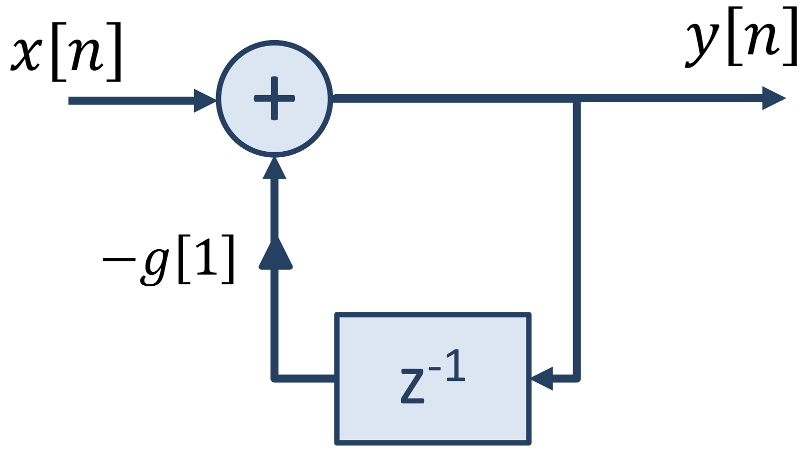
- **IIR Direct Form**

$$y[n] = x[n] - \sum_{m=1}^{M-1} g[m]y[n-m]$$

Implementing IIR Filters

- **IIR Direct Form (M = 1)**

$$y[n] = x[n] - g[1]y[n-1]$$



Implementing IIR Filters

- **IIR Direct Form**

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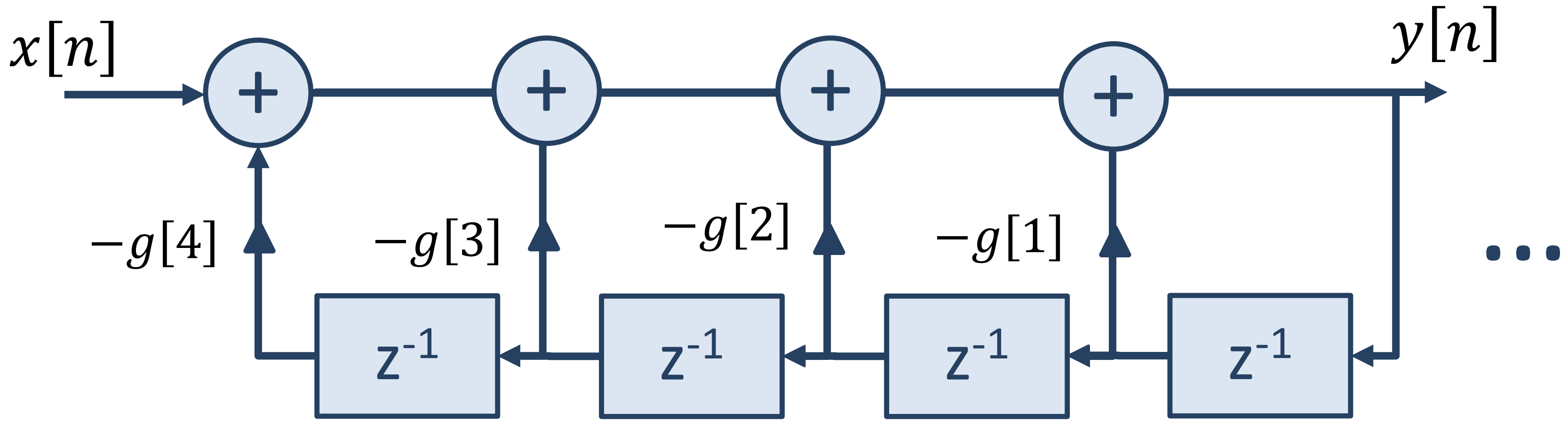


Implementing IIR Filters

- **Two ways to look at general IIR filters**
  - Convolution perspective

$$y[n] + \sum_{m=1}^{M-1} g[m]y[n-m] = \sum_{k=1}^{M-1} h[k]x[n-k]$$

  - Filtering perspective

$$\frac{Y(z)}{X(z)} = \frac{\sum_{m=Q}^{M-1} h[m]z^{-m}}{1 + \sum_{m=Q}^{M-1} g[m]z^{-m}}$$

Implementing IIR Filters

- **Two ways to look at general IIR filters**
  - Convolution perspective

$$y[n] = \sum_{k=1}^{M-1} h[k]x[n-k] - \sum_{m=1}^{M-1} g[m]y[n-m]$$

  - Filtering perspective

$$\frac{Y(z)}{X(z)} = \frac{\sum_{m=Q}^{M-1} h[m]z^{-m}}{1 + \sum_{m=Q}^{M-1} g[m]z^{-m}}$$

Implementing IIR Filters

- **Two ways to look at IIR filters with only recursive components**
  - Convolution perspective

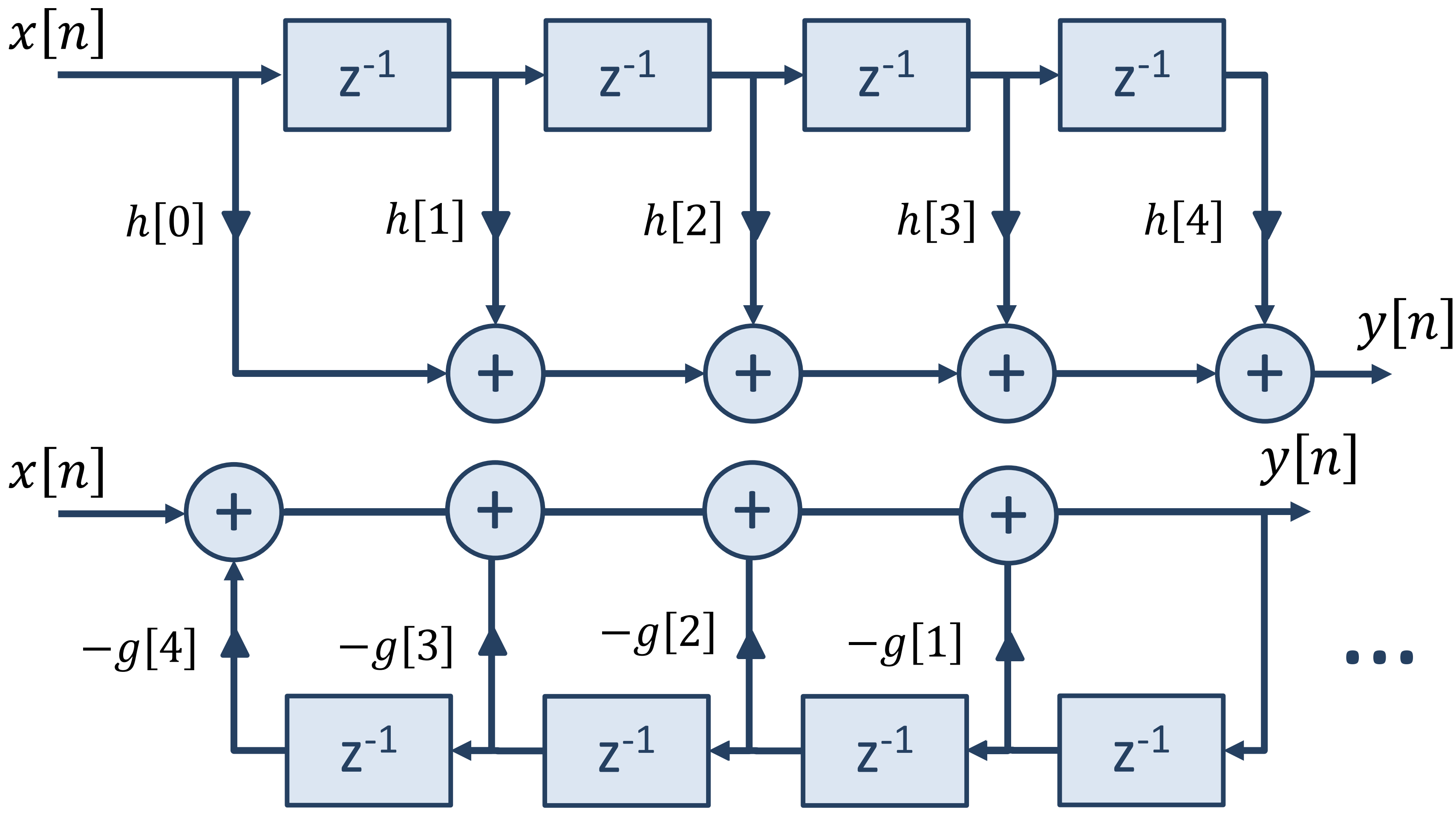
$$y[n] = x[n] - \sum_{m=1}^{M-1} g[m]y[n-m]$$

  - Filtering perspective

$$\frac{Y(z)}{X(z)} = \frac{\sum_{m=Q}^{M-1} h[m]z^{-m}}{1 + \sum_{m=Q}^{M-1} g[m]z^{-m}}$$

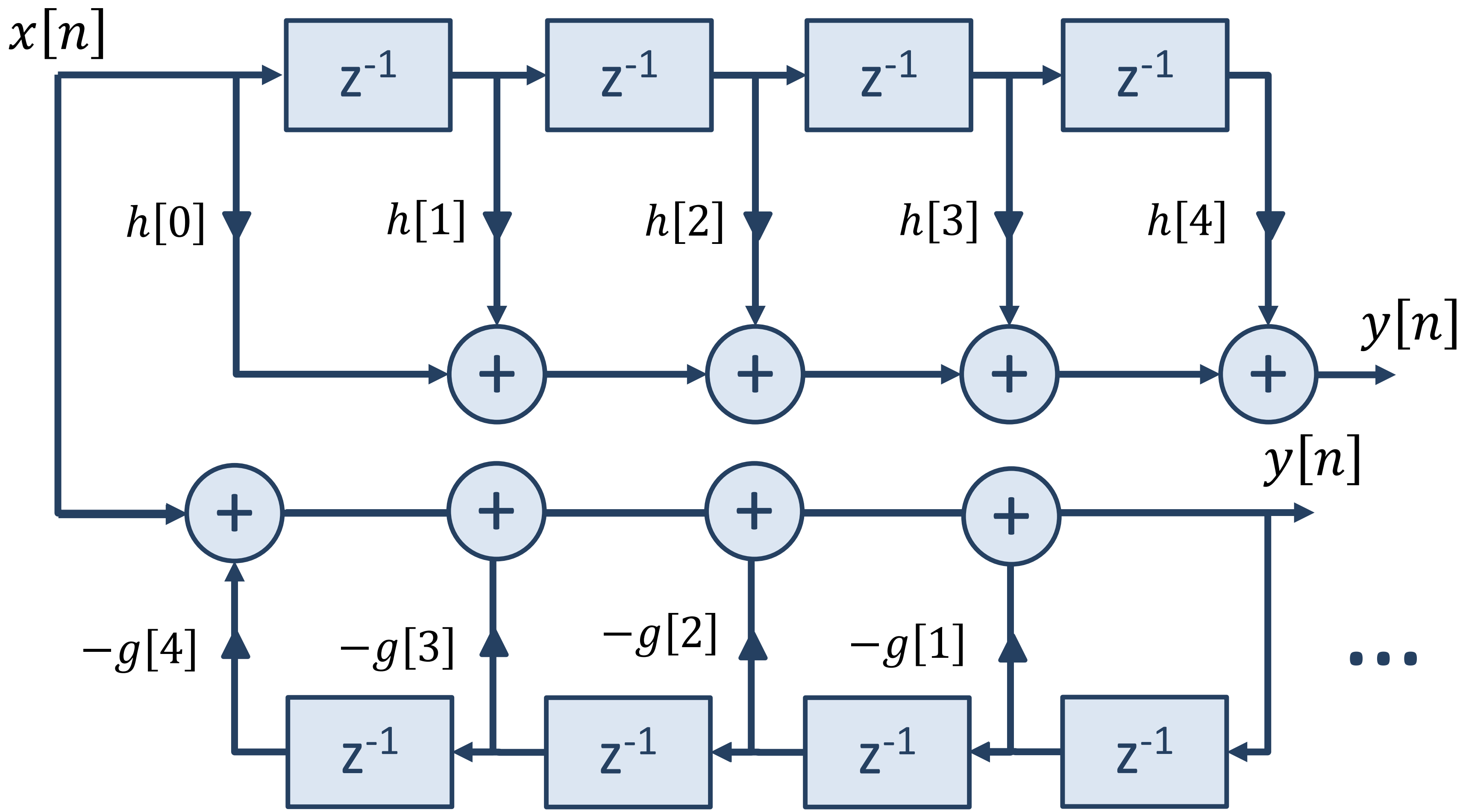
Implementing IIR Filters

- **IIR Direct Form**



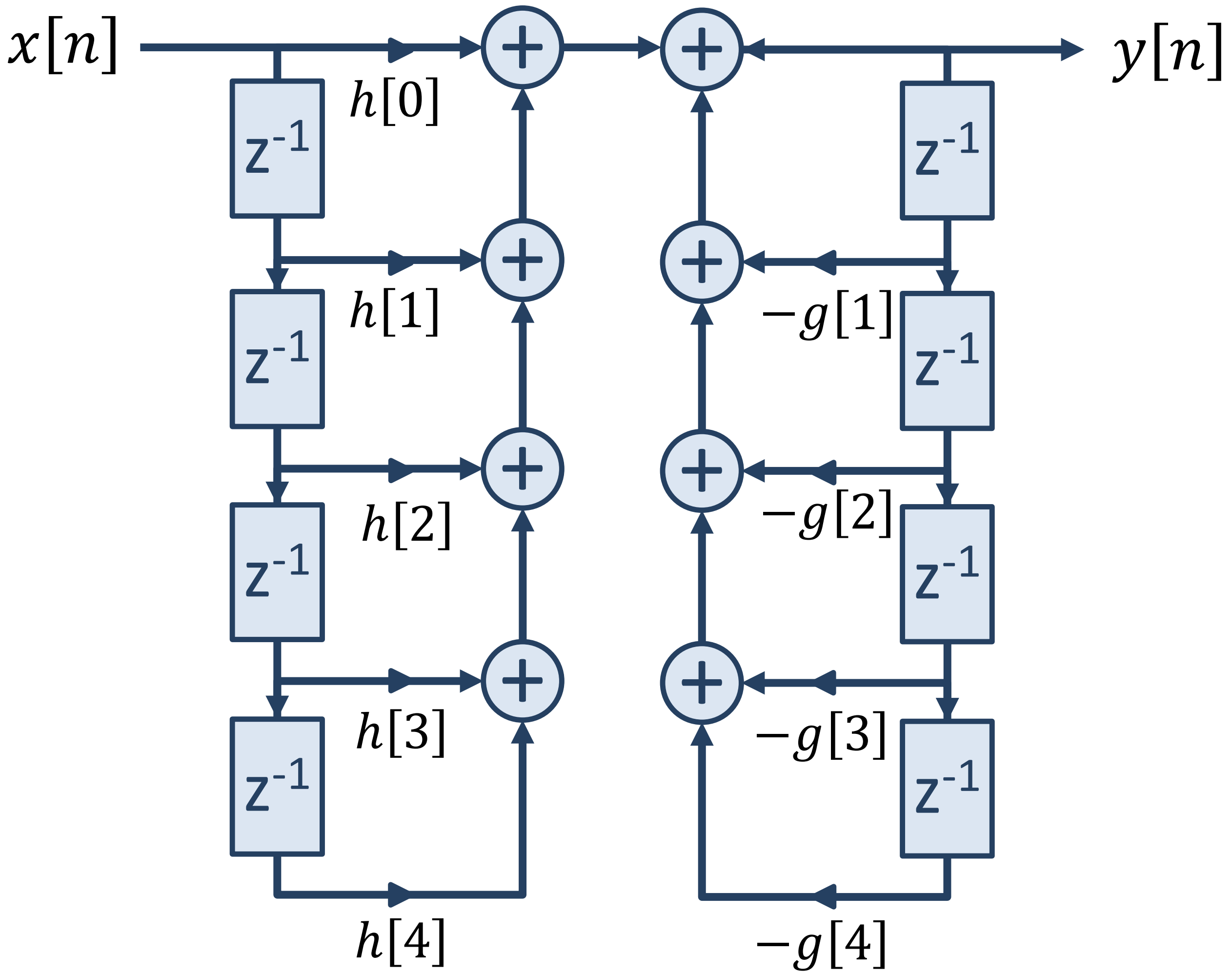
Implementing IIR Filters

- **IIR Direct Form**



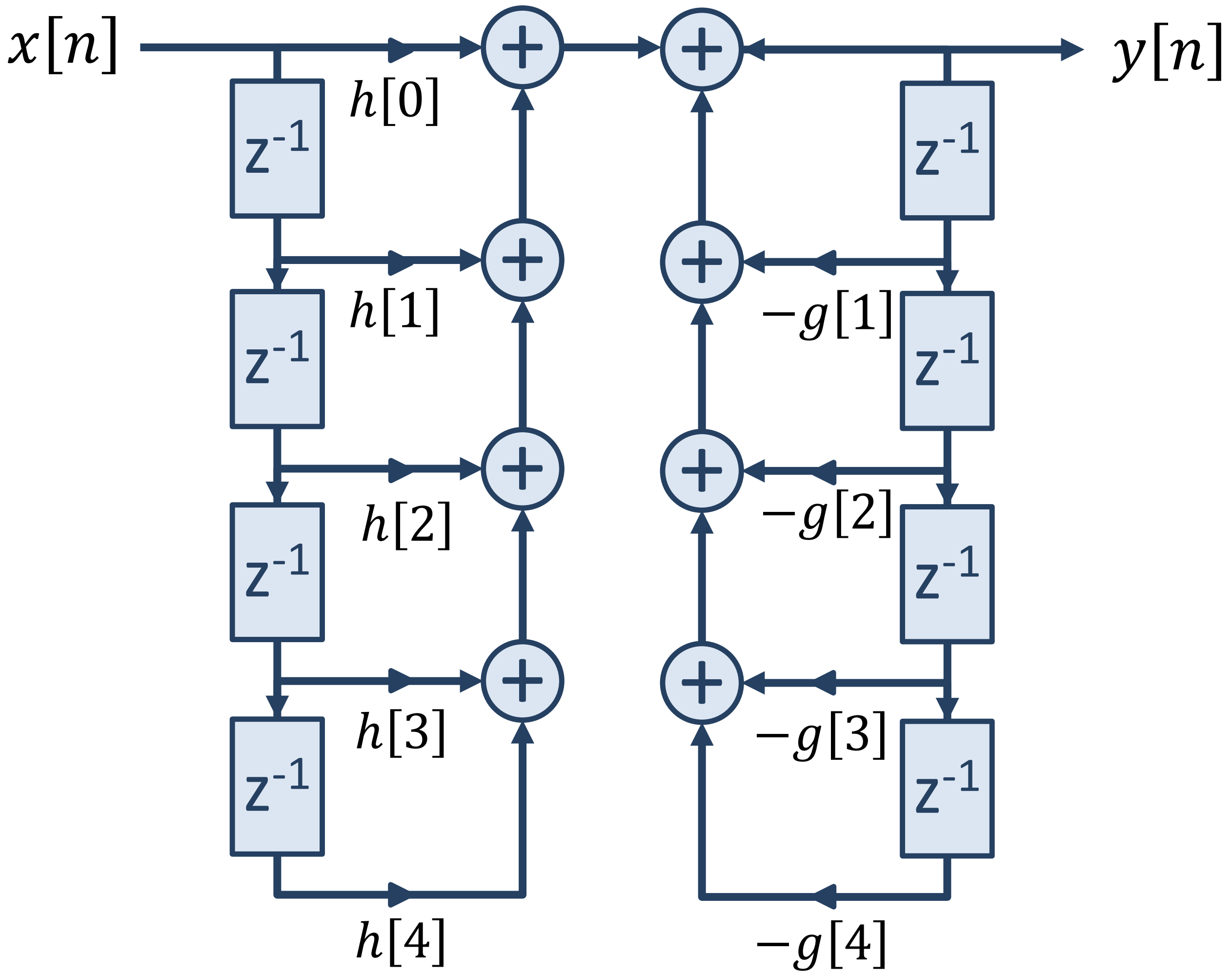
Implementing IIR Filters

- **IIR Direct Form**



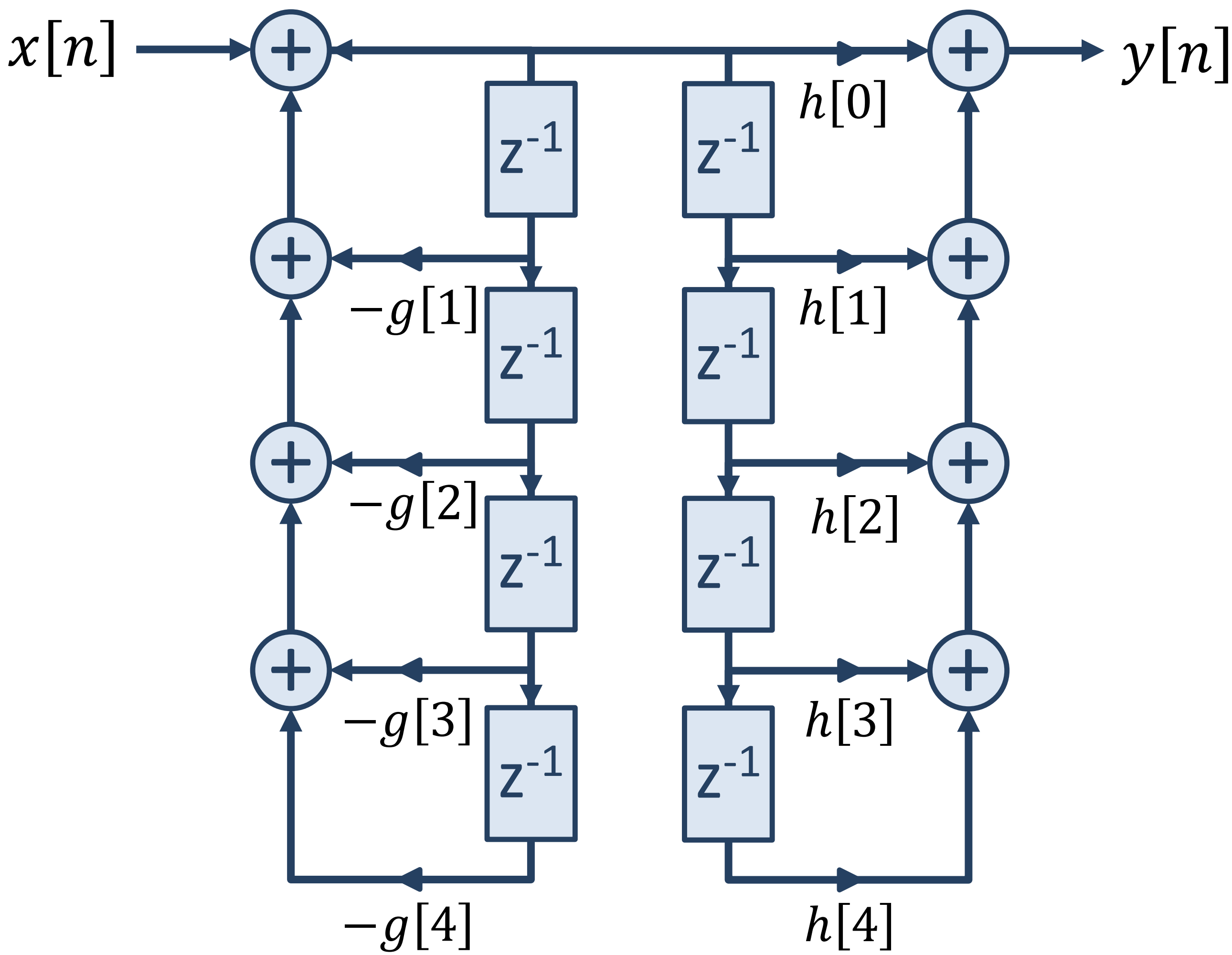
Implementing IIR Filters

- **IIR Direct Form I**



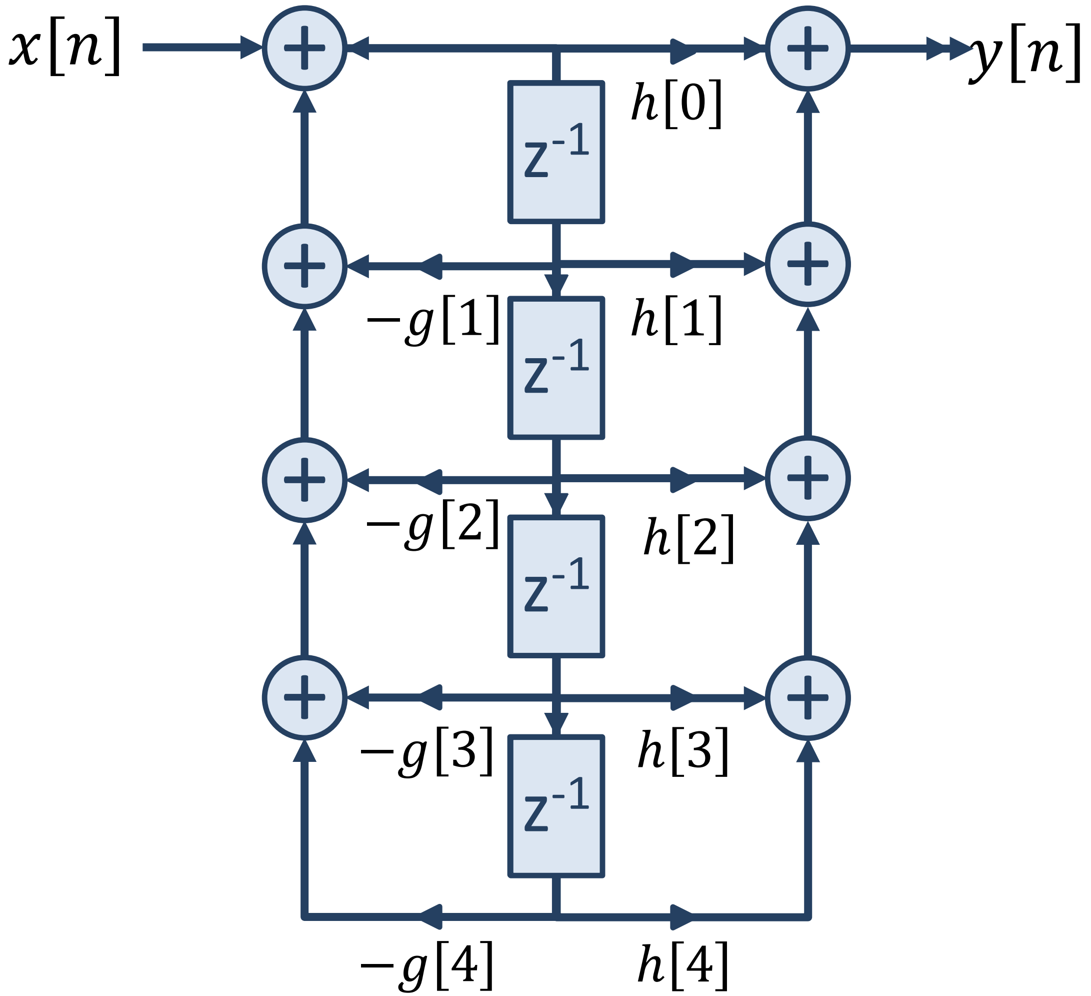
Implementing IIR Filters

- **IIR Direct Form II (transition)**



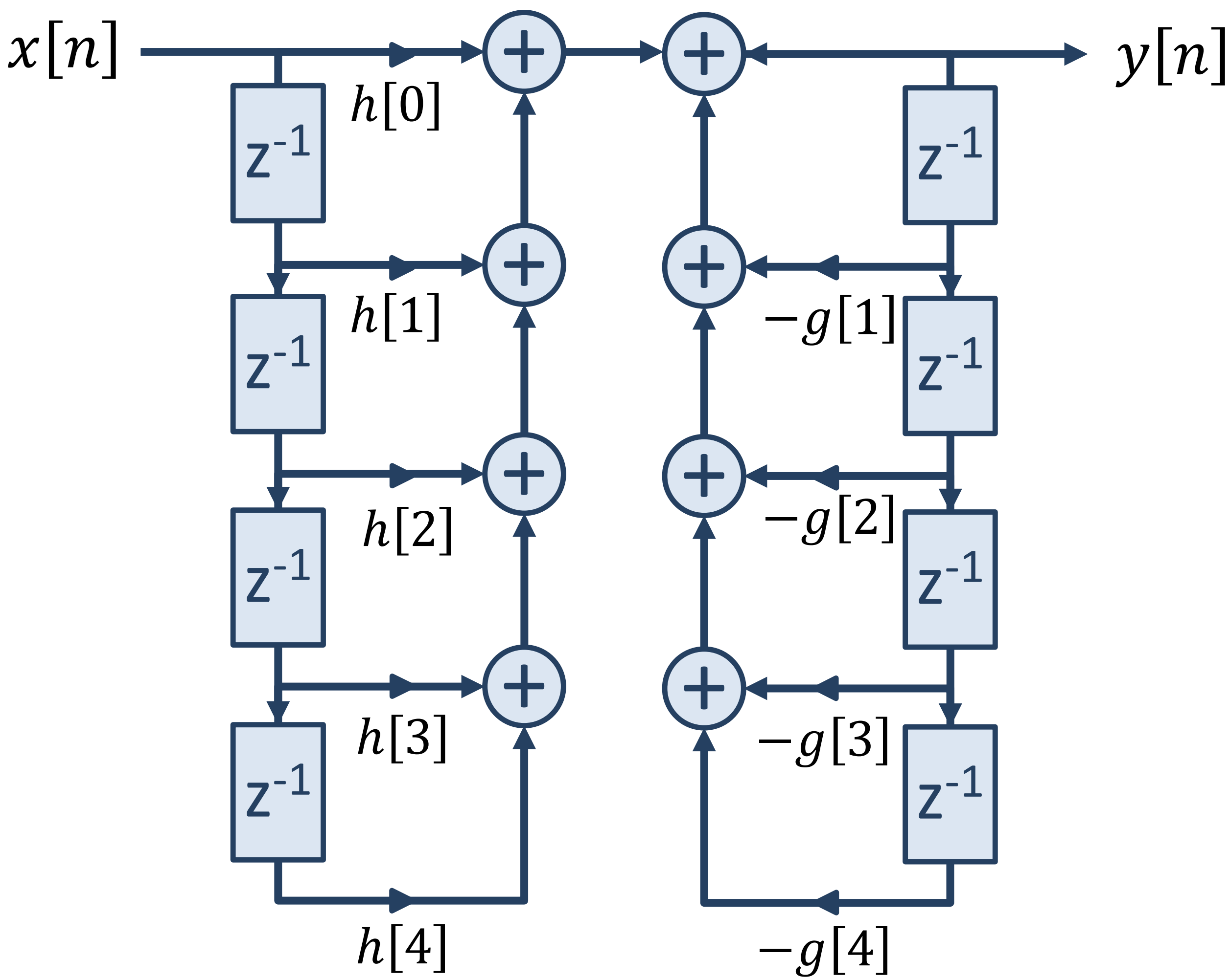
Implementing IIR Filters

- **IIR Direct Form II**



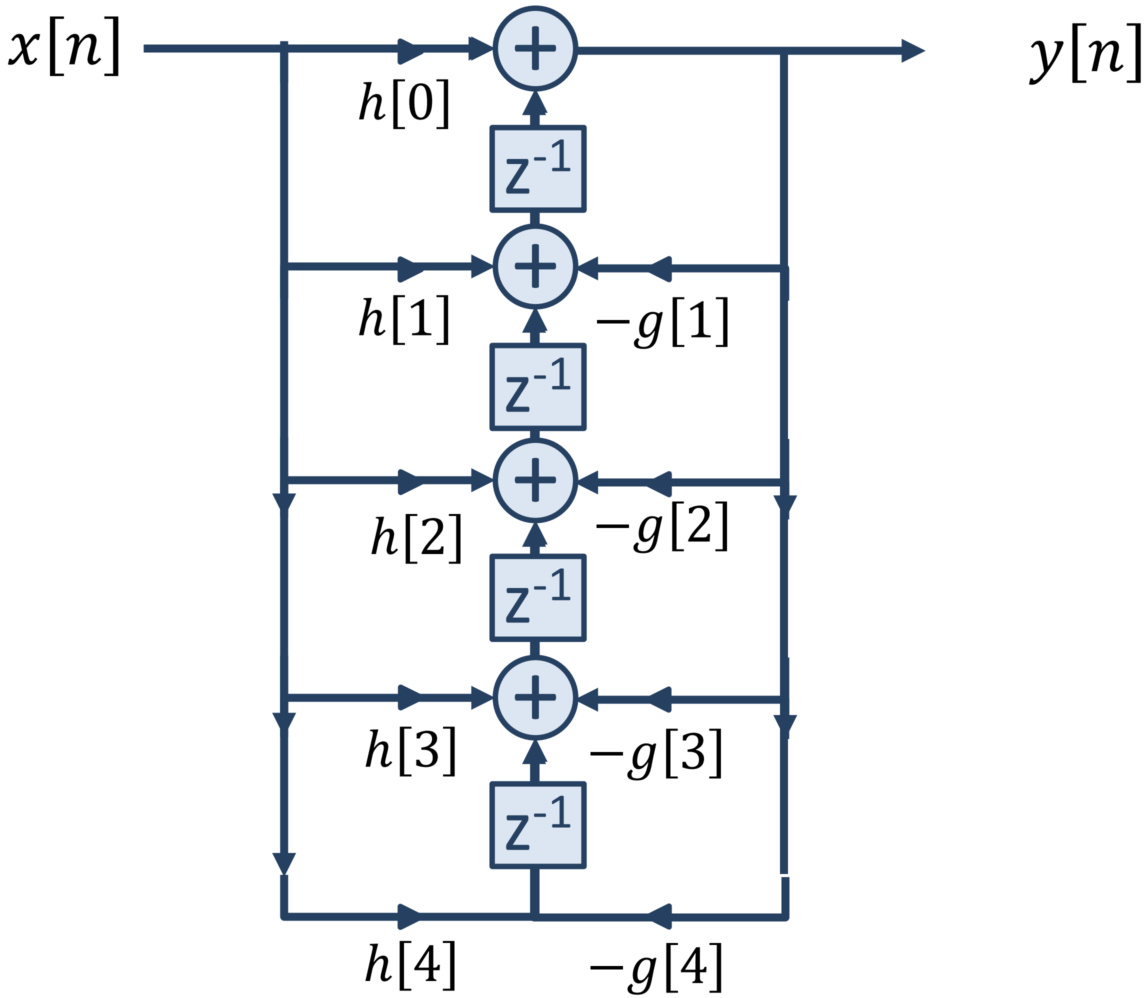
Implementing IIR Filters

- **IIR Direct Form I**



Implementing IIR Filters

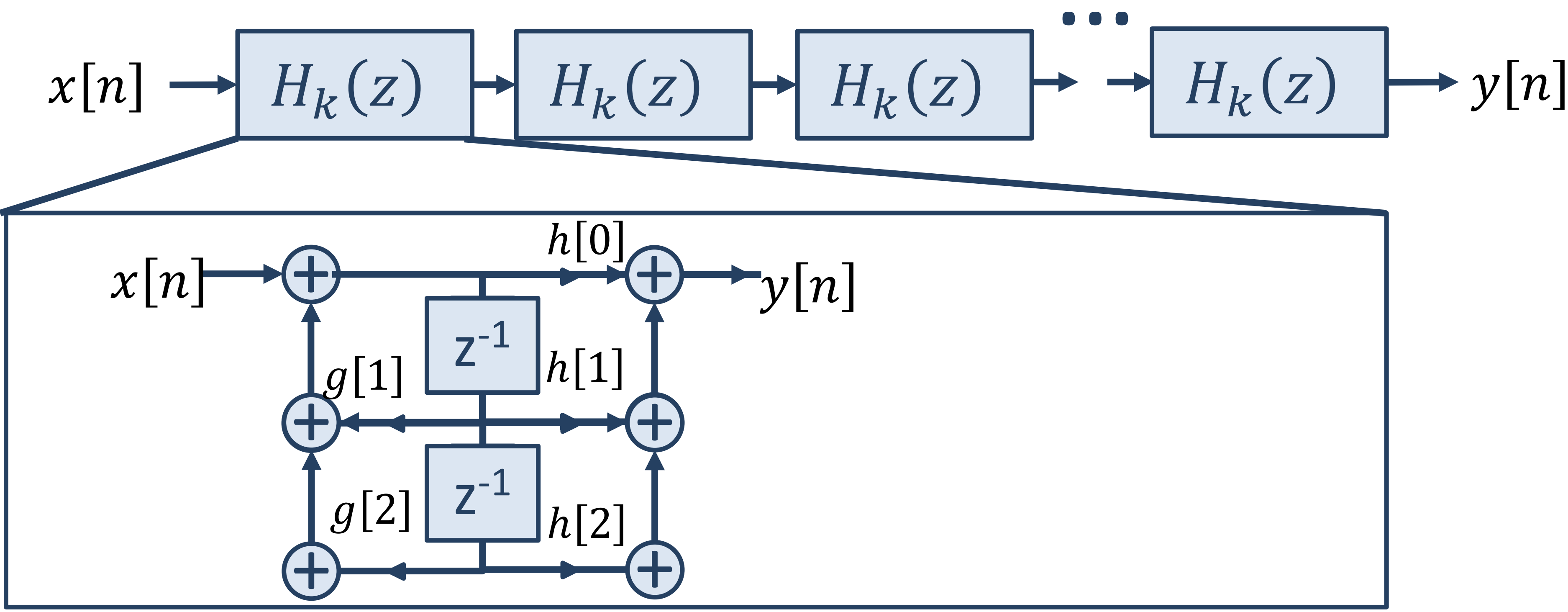
- **Transposed IIR Direct Form II**



Implementing FIR Filters

- **IIR Cascade Form**

$$Y(z) = X(z) \sum_{m=0}^{M-1} h[m] z^{-m} = X(z) \prod_{k=1}^{K} H_k(z)$$



Implementing FIR Filters

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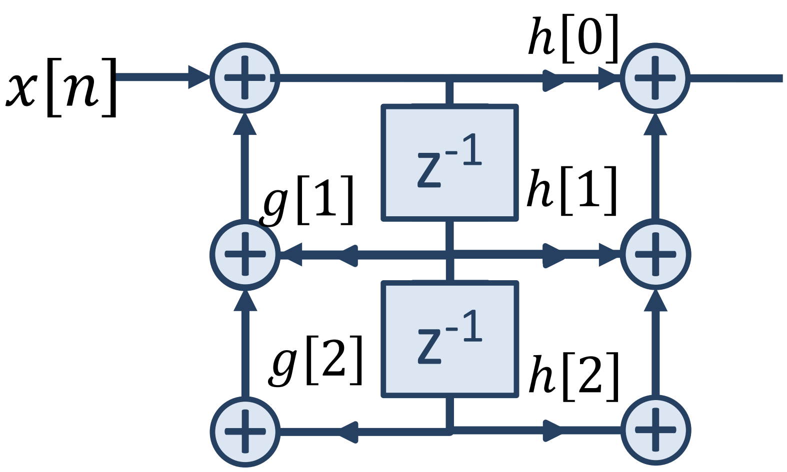

Implementing FIR Filters

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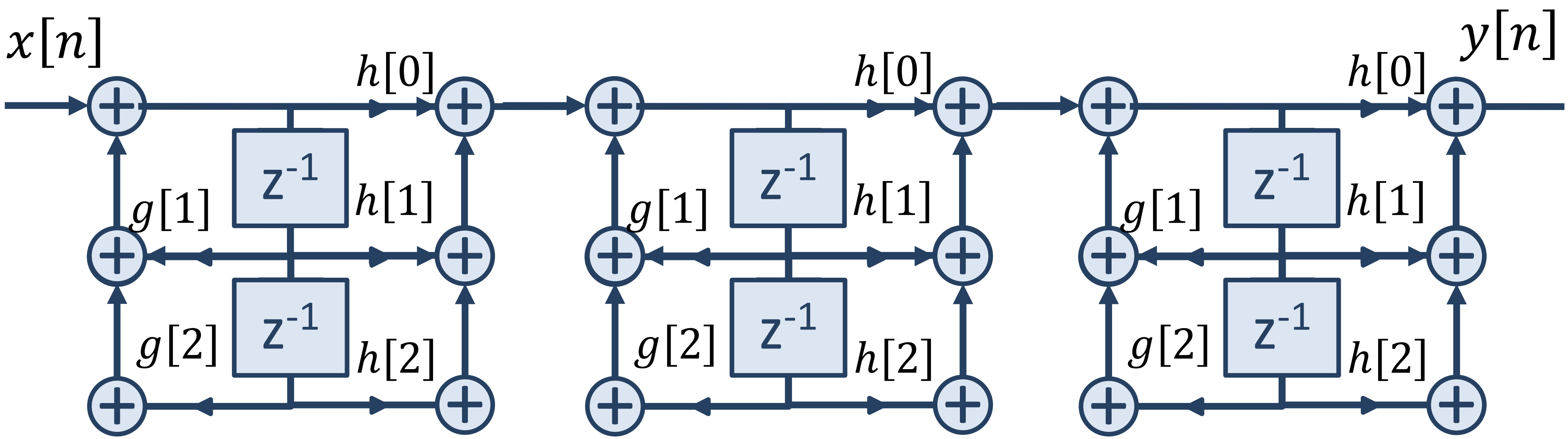
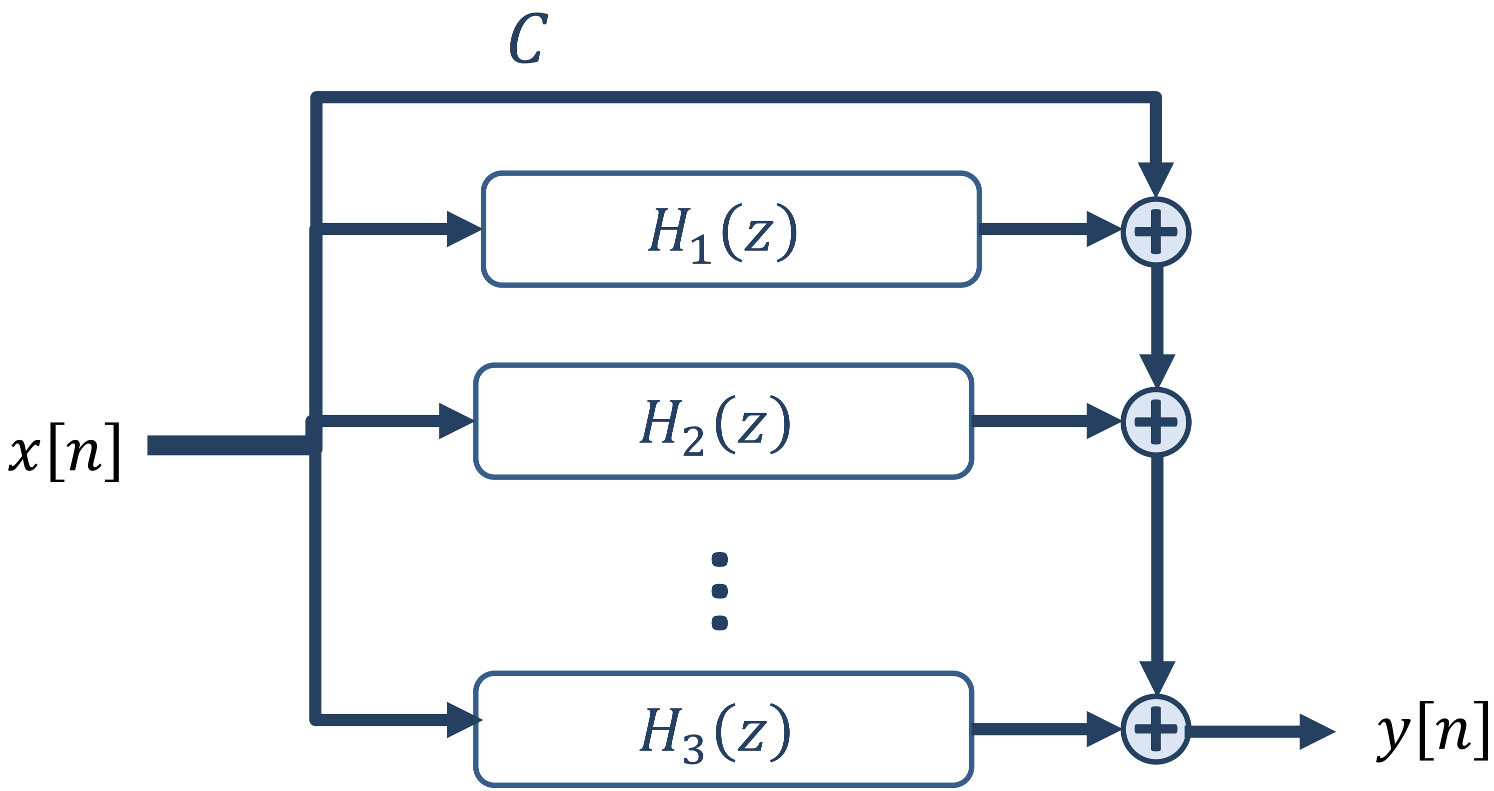


Implementing IIR Filters

- **Parallel Form**

$$H(z) = C + \sum_{m=0}^{M-1} \frac{a[m]}{1 - p_m z^{-1}}$$

From partial fraction decomposition



Lecture 17: Lattice Structures

Foundations of Digital Signal Processing

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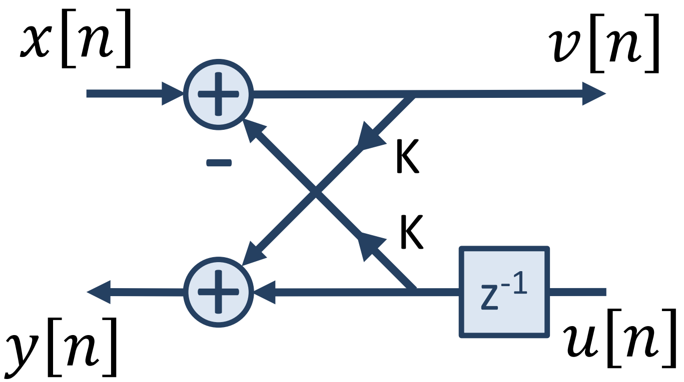
- Implementation of FIR Filters
- Implementation of IIR Filters
- **Implementation of Lattice Filters**

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?

$$y[n] = Kv[n] + u[n-1]$$
$$v[n] = x[n] - Ku[n-1]$$



Implementing Lattice Filters

- **Lattice Stage**
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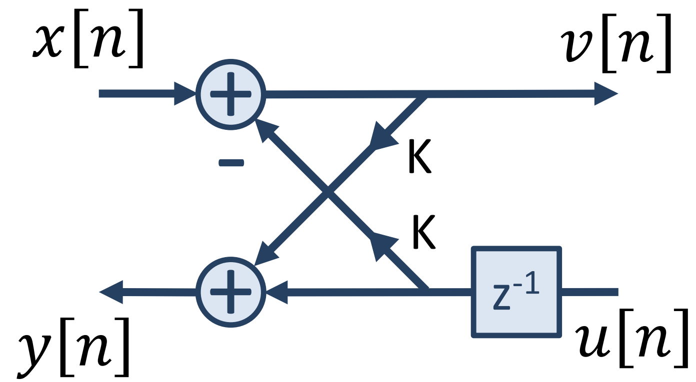

$$Y(z) = KV(z) + U(z)z^{-1}$$

$$V(z) = X(z) - KU(z)z^{-1}$$

Implementing Lattice Filters

- **Lattice Stage**
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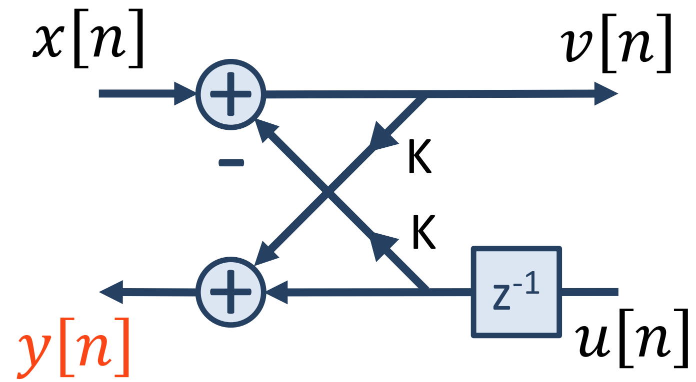

$$Y(z) = KV(z) + U(z)z^{-1}$$

$$V(z) = X(z) - KU(z)z^{-1}$$

$$\frac{Y(z)}{X(z)} = ?$$

Implementing Lattice Filters

- **Lattice Stage**
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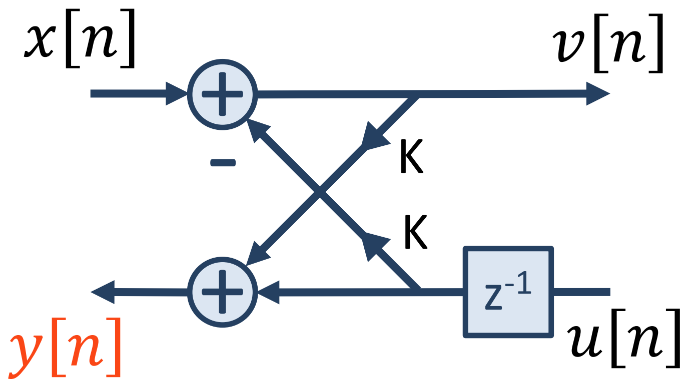


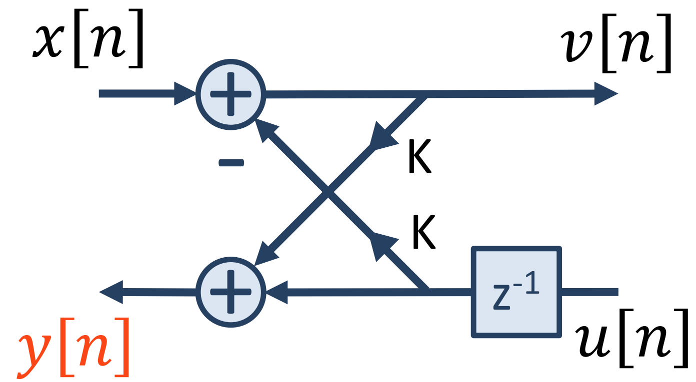
$$Y(z) = KV(z) + U(z)z^{-1}$$

$$X(z) = V(z) + KU(z)z^{-1}$$

$$\frac{Y(z)}{X(z)} = ?$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?

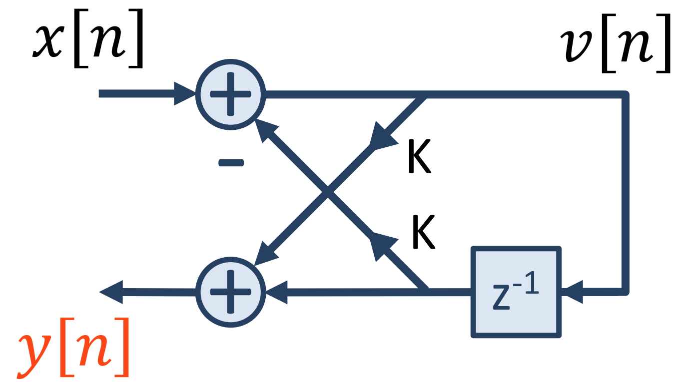


$$Y(z) = KV(z) + U(z)z^{-1}$$

$$X(z) = V(z) + KU(z)z^{-1}$$

$$\frac{Y(z)}{X(z)} = \frac{KV(z) + U(z)z^{-1}}{V(z) + KU(z)z^{-1}}$$

- **Lattice Stage**
  - **Question:** What is the transfer function?



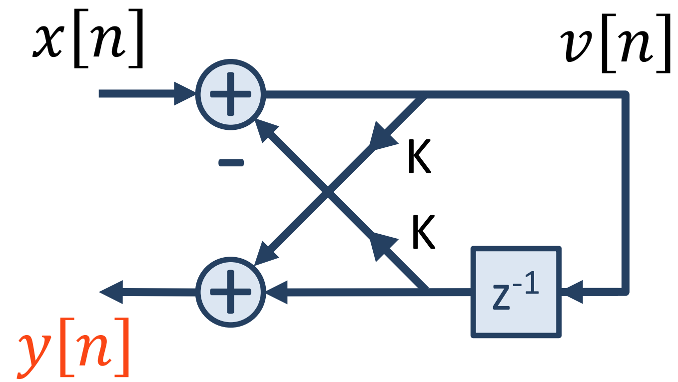
$$Y(z) = KV(z) + V(z)z^{-1}$$

$$X(z) = V(z) + KV(z)z^{-1}$$

$$\frac{Y(z)}{X(z)} = \frac{KV(z) + V(z)z^{-1}}{V(z) + KV(z)z^{-1}}$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?

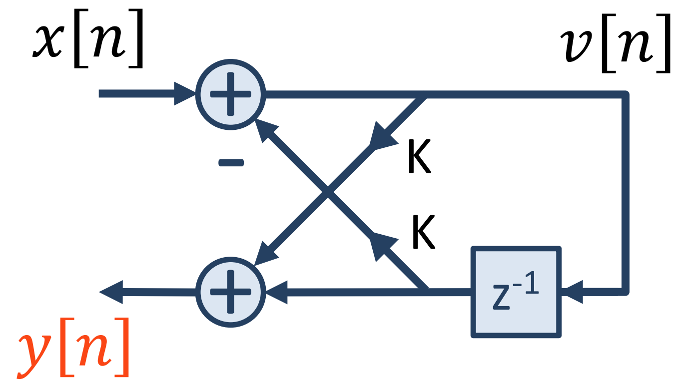


$$Y(z) = [K + z^{-1}]V(z)$$
$$X(z) = [1 + Kz^{-1}]V(z)$$

$$\frac{Y(z)}{X(z)} = \frac{KV(z) + V(z)z^{-1}}{V(z) + KV(z)z^{-1}}$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



$$Y(z) = [K + z^{-1}]V(z)$$

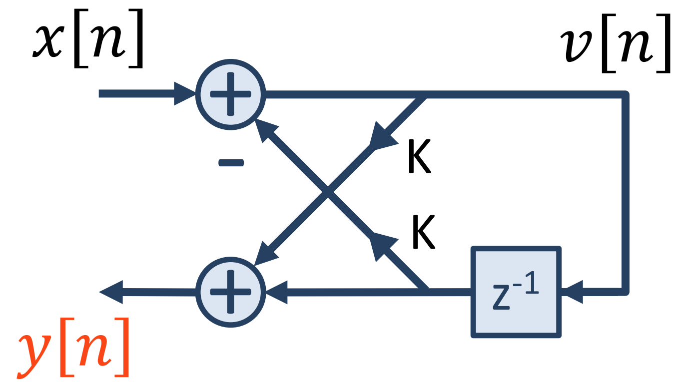
$$X(z) = [1 + Kz^{-1}]V(z)$$

$$\frac{Y(z)}{X(z)} = \frac{KV(z) + V(z)z^{-1}}{V(z) + KV(z)z^{-1}} = \frac{K + z^{-1}}{1 + Kz^{-1}}$$

Question: What kind of filter is this?

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



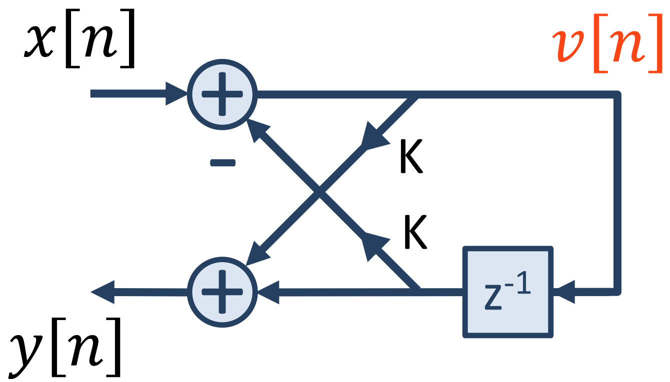
$$Y(z) = [K + z^{-1}]V(z)$$
$$X(z) = [1 + Kz^{-1}]V(z)$$

$$\frac{Y(z)}{X(z)} = \frac{KV(z) + V(z)z^{-1}}{V(z) + KV(z)z^{-1}} = \frac{K + z^{-1}}{1 + Kz^{-1}}$$

Question: What kind of filter is this? **All-Pass Filter**

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



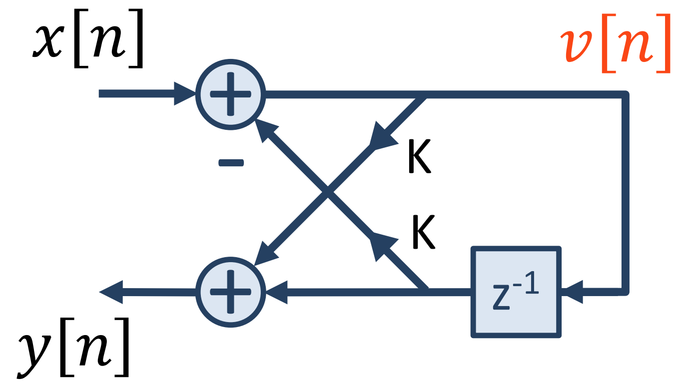
$$Y(z) = [K + z^{-1}]V(z)$$

$$X(z) = [1 + Kz^{-1}]V(z)$$

$$\frac{V(z)}{X(z)} = ?$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



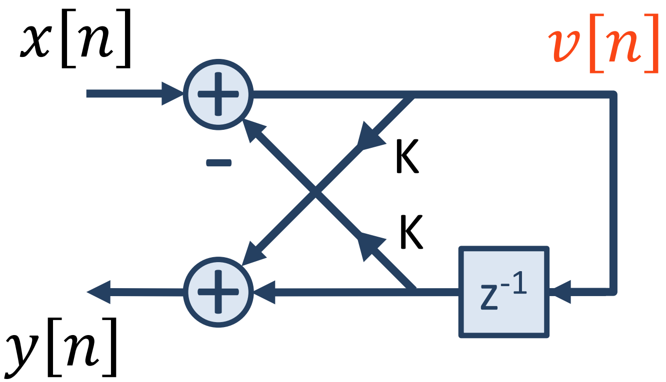
$$Y(z) = [K + z^{-1}]V(z)$$
$$X(z) = [1 + Kz^{-1}]V(z)$$

$$\frac{V(z)}{X(z)} = \frac{1}{1 + Kz^{-1}}$$

Question: What kind of filter is this?

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



$$Y(z) = [K + z^{-1}]V(z)$$

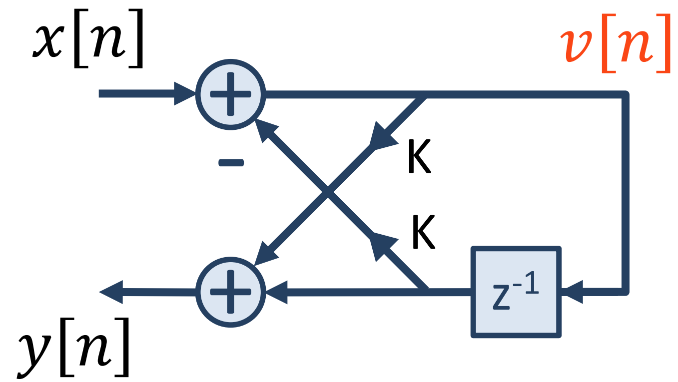
$$X(z) = [1 + Kz^{-1}]V(z)$$

$$\frac{V(z)}{X(z)} = \frac{1}{1 + Kz^{-1}}$$

Question: What kind of filter is this? **IIR Filter**

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



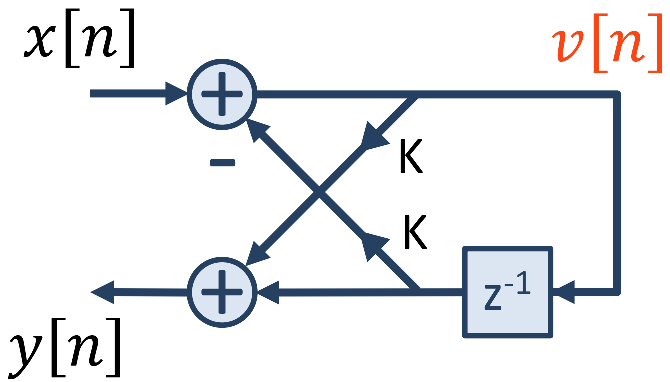
$$Y(z) = [K + z^{-1}]V(z)$$

$$X(z) = [1 + Kz^{-1}]V(z)$$

$$\frac{Y(z)}{V(z)} = ?$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



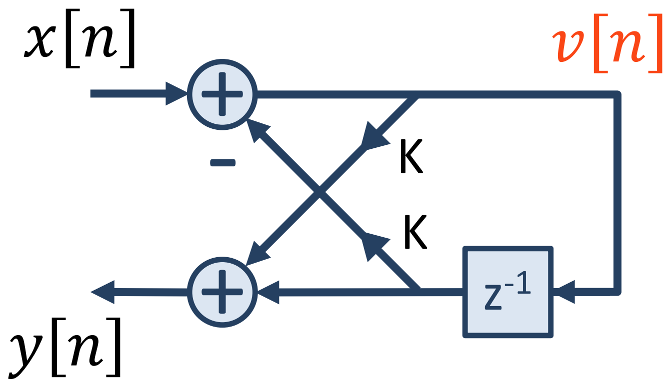
$$Y(z) = [K + z^{-1}]V(z)$$
$$X(z) = [1 + Kz^{-1}]V(z)$$

$$\frac{Y(z)}{V(z)} = 1 + Kz^{-1}$$

Question: What kind of filter is this?

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



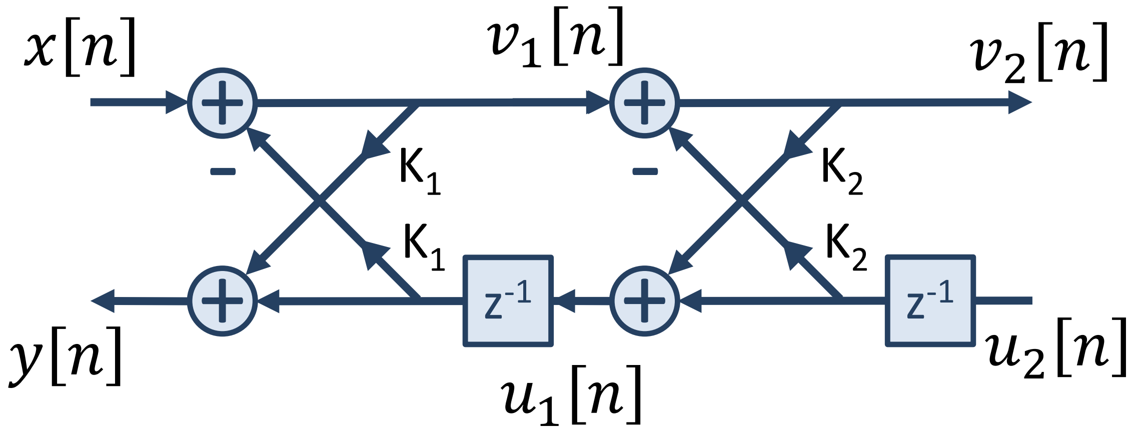
$$Y(z) = [K + z^{-1}]V(z)$$
$$X(z) = [1 + Kz^{-1}]V(z)$$

$$\frac{Y(z)}{V(z)} = 1 + Kz^{-1}$$

Question: What kind of filter is this? **FIR Filter**

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



$$Y(z) = K_1 V_1(z) + U_1(z) z^{-1}$$

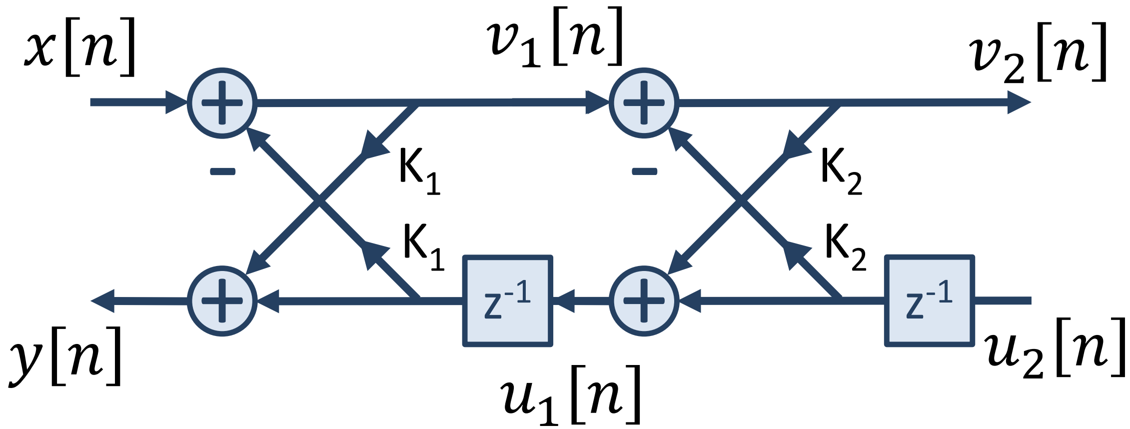
$$X(z) = V_1(z) + K_1 U_1(z) z^{-1}$$

$$U_1(z) = K_2 V_2(z) + U_2(z) z^{-1}$$

$$V_1(z) = V_2(z) + K_2 U_2(z) z^{-1}$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?

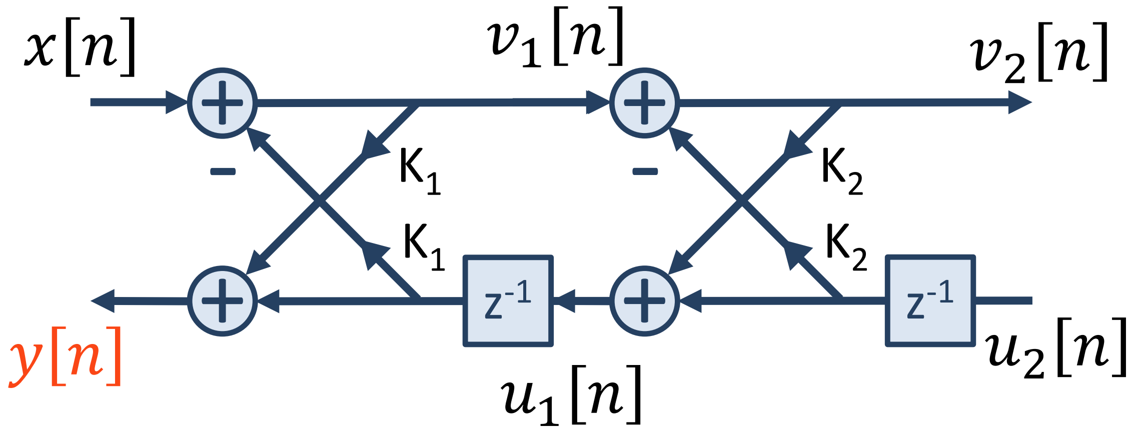


$$Y(z) = K_1[V_2(z) + K_2U_2(z)z^{-1}] + [K_2V_2(z) + U_2(z)z^{-1}]z^{-1}$$

$$X(z) = [V_2(z) + K_2U_2(z)z^{-1}] + K_1[K_2V_2(z) + U_2(z)z^{-1}]z^{-1}$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



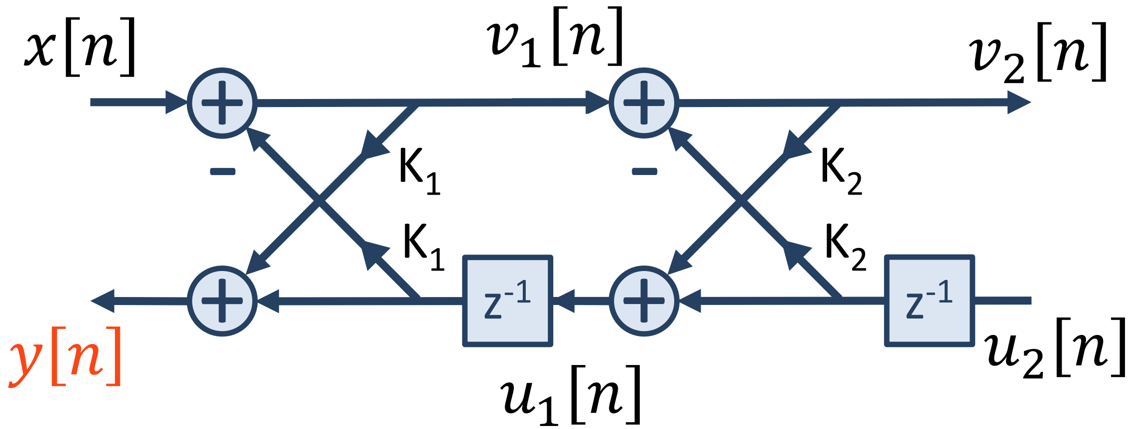
$$Y(z) = K_1[V_2(z) + K_2U_2(z)z^{-1}] + [K_2V_2(z) + U_2(z)z^{-1}]z^{-1}$$

$$X(z) = [V_2(z) + K_2U_2(z)z^{-1}] + K_1[K_2V_2(z) + U_2(z)z^{-1}]z^{-1}$$

$$\frac{Y(z)}{X(z)} = \ ?$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



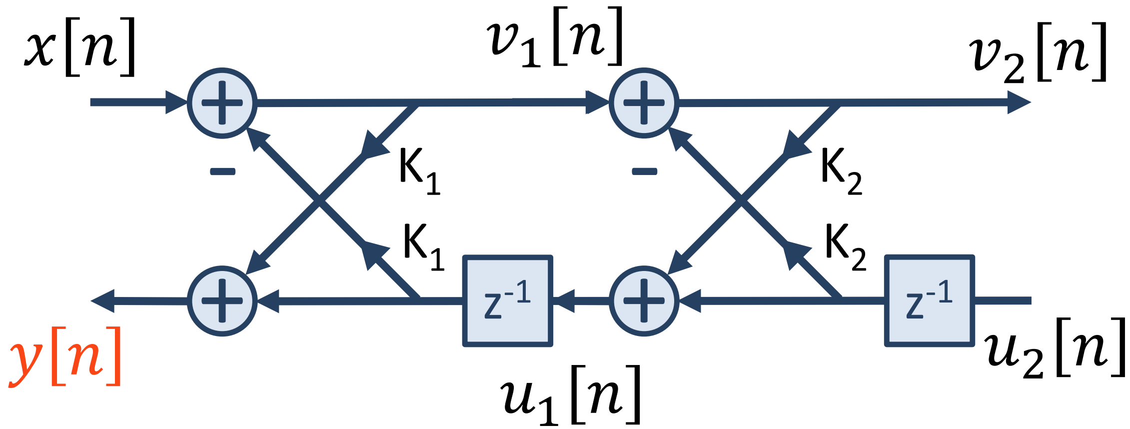
$$Y(z) = K_1 V_2(z) + K_1 K_2 U_2(z) z^{-1} + K_2 V_2(z) z^{-1} + U_2(z) z^{-2}$$

$$X(z) = V_2(z) + K_2 U_2(z) z^{-1} + K_1 K_2 V_2(z) z^{-1} + K_1 U_2(z) z^{-2}$$

$$\frac{Y(z)}{X(z)} = \ ?$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



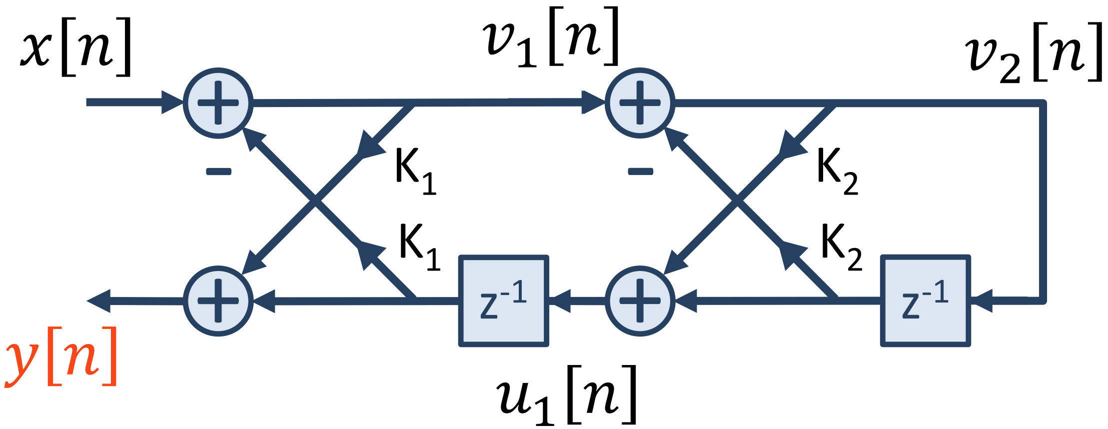
$$Y(z) = K_1 V_2(z) + [K_1 K_2 V_2(z) + K_2 V_2(z)]z^{-1} + V_2(z)z^{-2}$$

$$X(z) = V_2(z) + [K_2 U_2(z) + K_1 K_2 V_2(z)]z^{-1} + K_1 U_2(z)z^{-2}$$

$$\frac{Y(z)}{X(z)} = \ ?$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



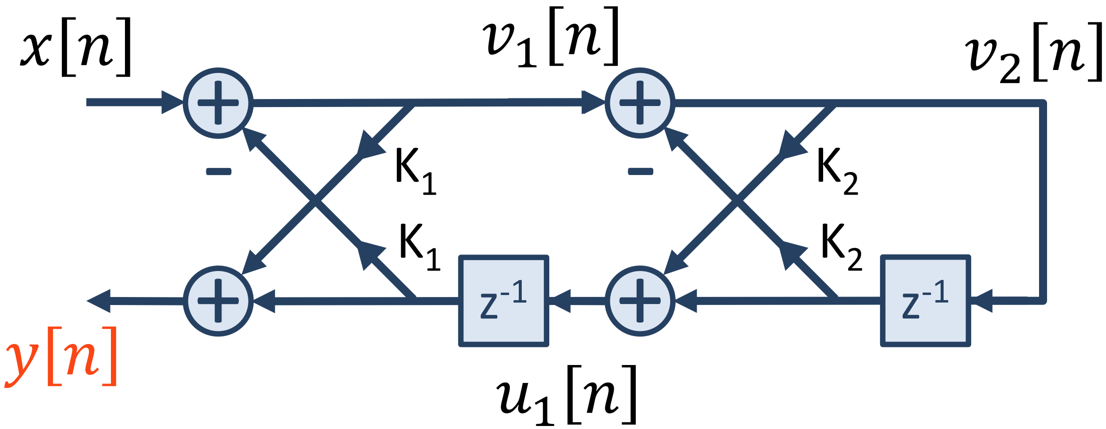
$$Y(z) = V_2(z)[K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}]$$

$$X(z) = V_2(z)[1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}]$$

$$\frac{Y(z)}{X(z)} = \ ?$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



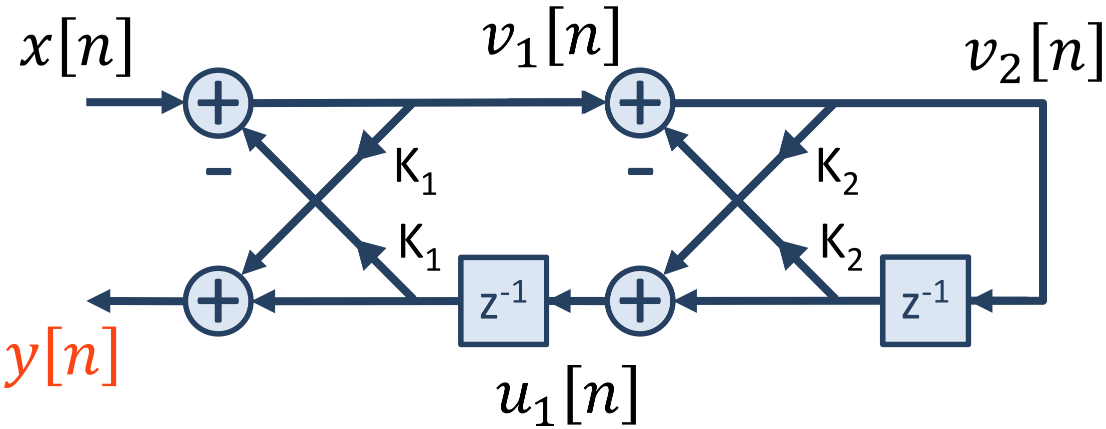
$$Y(z) = V_2(z)[K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}]$$

$$X(z) = V_2(z)[1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}]$$

$$\frac{Y(z)}{X(z)} = \frac{K_1 + (K_2 + K_1K_2)z^{-1} + z^{-2}}{1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}} = \frac{z^{-2}A(z^{-1})}{A(z)}$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



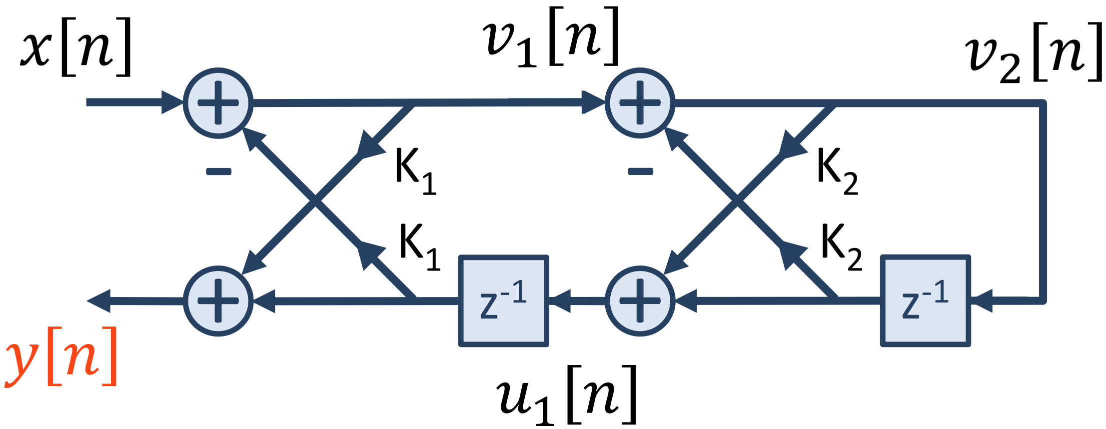
$$Y(z) = V_2(z)[K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}]$$
$$X(z) = V_2(z)[1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}]$$

$$\frac{Y(z)}{X(z)} = \frac{K_1 + (K_2 + K_1K_2)z^{-1} + z^{-2}}{1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}} = \frac{z^{-2}A(z^{-1})}{A(z)}$$

Question: What kind of filter is this?

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



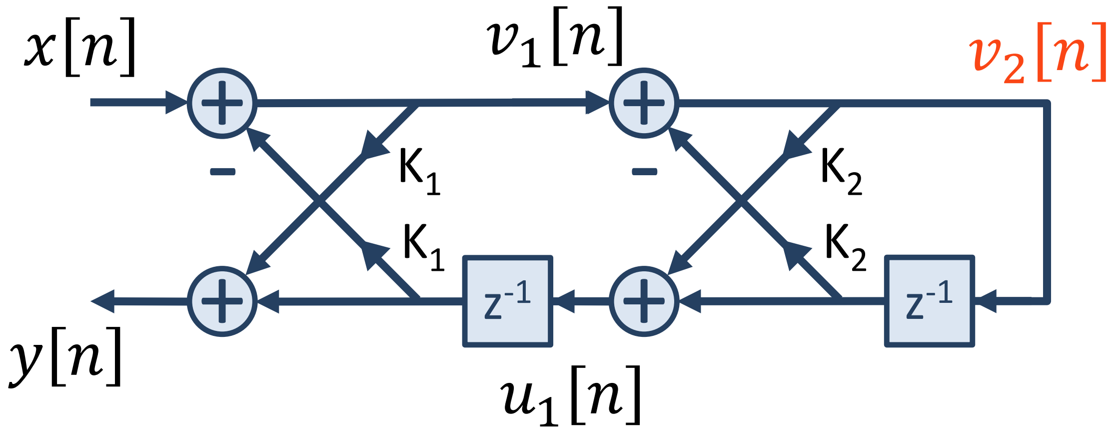
$$Y(z) = V_2(z)[K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}]$$
$$X(z) = V_2(z)[1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}]$$

$$\frac{Y(z)}{X(z)} = \frac{K_1 + (K_2 + K_1K_2)z^{-1} + z^{-2}}{1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}} = \frac{z^{-2}A(z^{-1})}{A(z)}$$

Question: What kind of filter is this? **All-Pass Filter**

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?

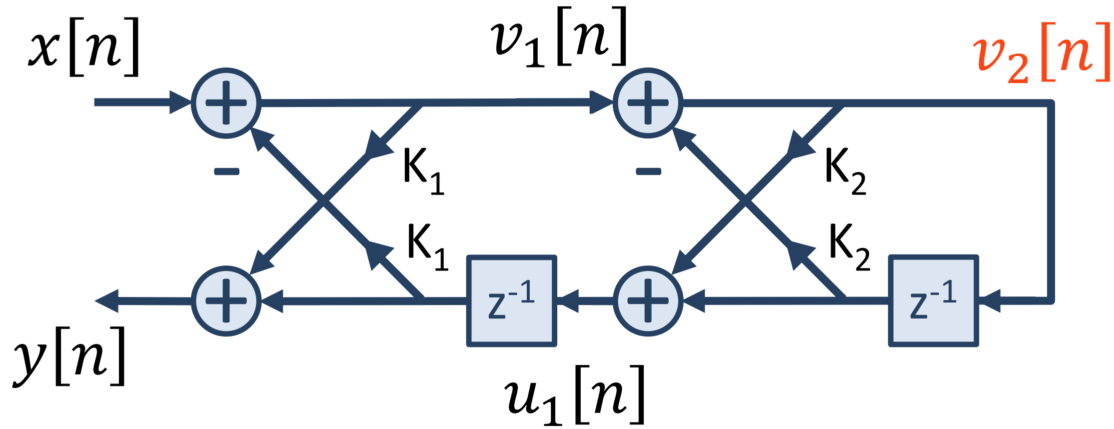


$$Y(z) = V_2(z)[K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}]$$
$$X(z) = V_2(z)[1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}]$$

$$\frac{V_2(z)}{X(z)} = ?$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



$$Y(z) = V_2(z)[K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}]$$

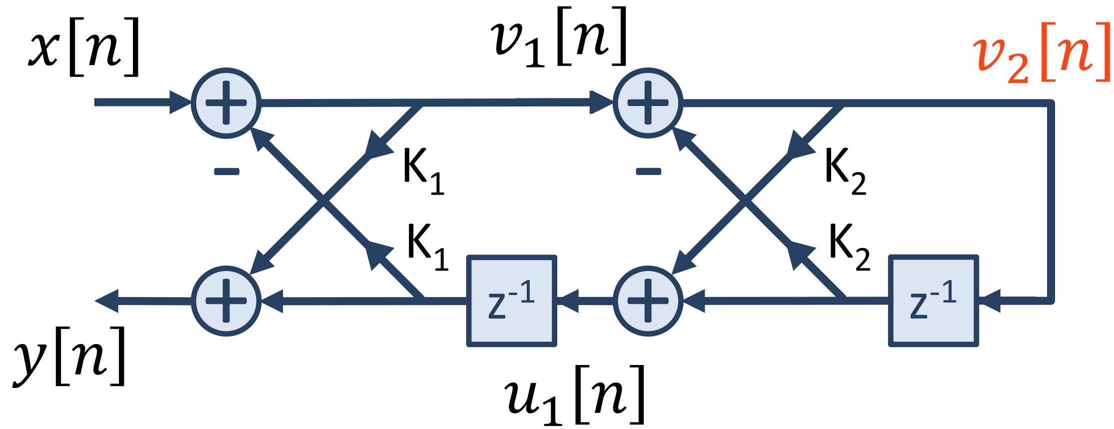
$$X(z) = V_2(z)[1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}]$$

$$\frac{V_2(z)}{X(z)} = \frac{1}{K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}}$$

Question: What kind of filter is this?

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



$$Y(z) = V_2(z)[K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}]$$

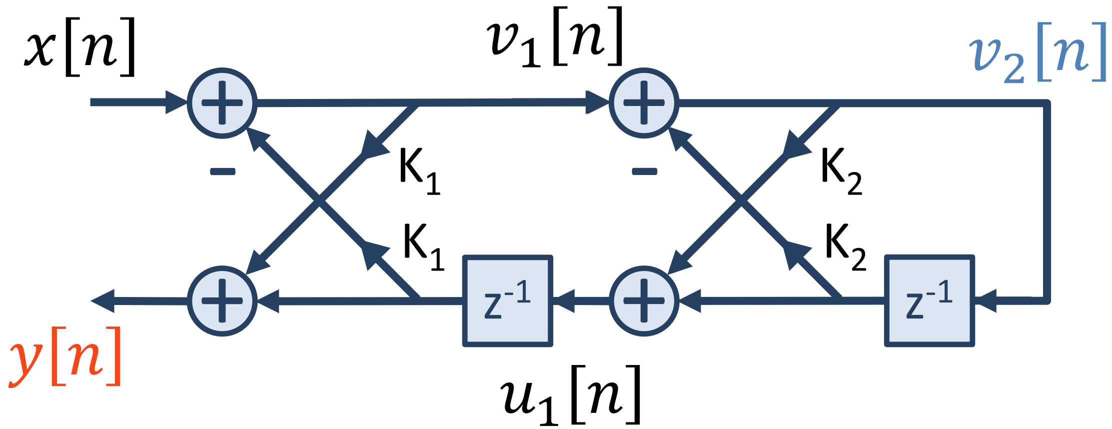
$$X(z) = V_2(z)[1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}]$$

$$\frac{V_2(z)}{X(z)} = \frac{1}{K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}}$$

Question: What kind of filter is this? **IIR Filter**

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



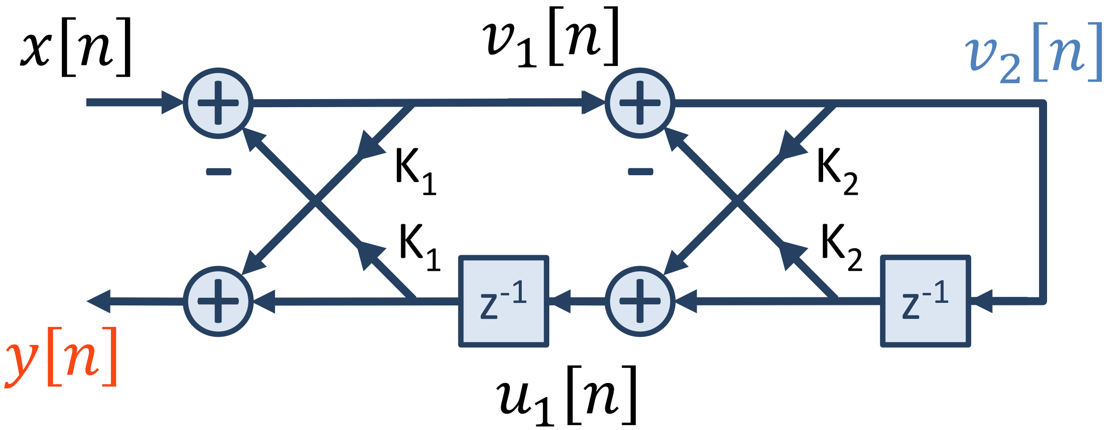
$$Y(z) = V_2(z)[K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}]$$

$$X(z) = V_2(z)[1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}]$$

$$\frac{Y(z)}{V_2(z)} = ?$$

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



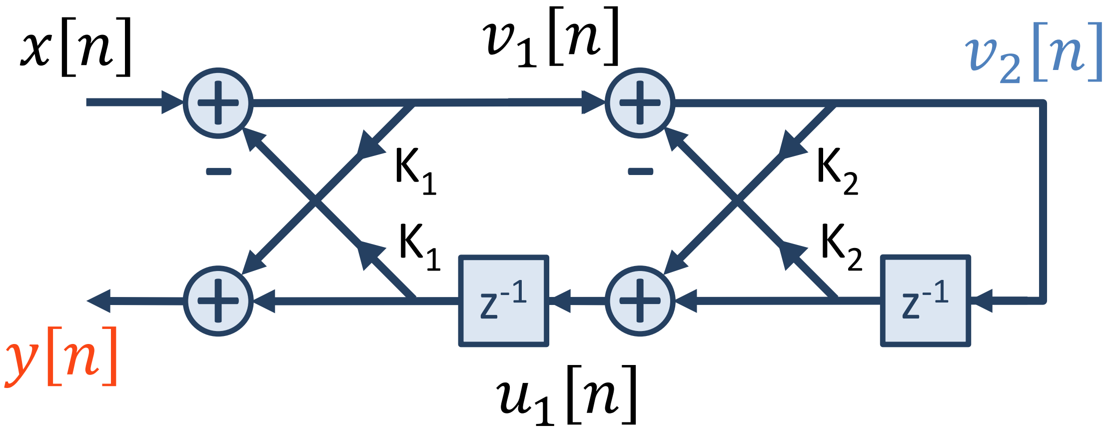
$$Y(z) = V_2(z)[K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}]$$
$$X(z) = V_2(z)[1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}]$$

$$\frac{Y(z)}{V_2(z)} = K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}$$

Question: What kind of filter is this?

Implementing Lattice Filters

- **Lattice Stage**
  - **Question:** What is the transfer function?



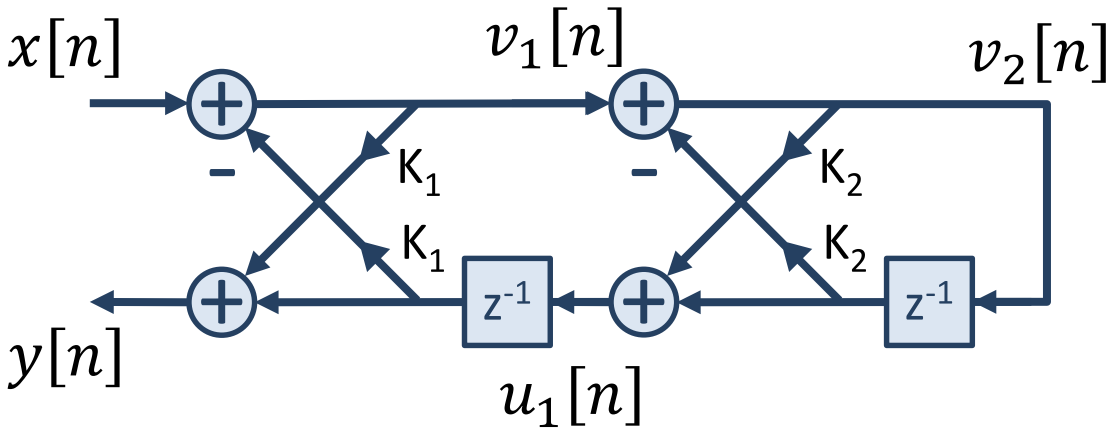
$$Y(z) = V_2(z)[K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}]$$
$$X(z) = V_2(z)[1 + (K_2 + K_1K_2)z^{-1} + K_1z^{-2}]$$

$$\frac{Y(z)}{V_2(z)} = K_1 + (K_1K_2 + K_2)z^{-1} + z^{-2}$$

Question: What kind of filter is this? **FIR Filter**

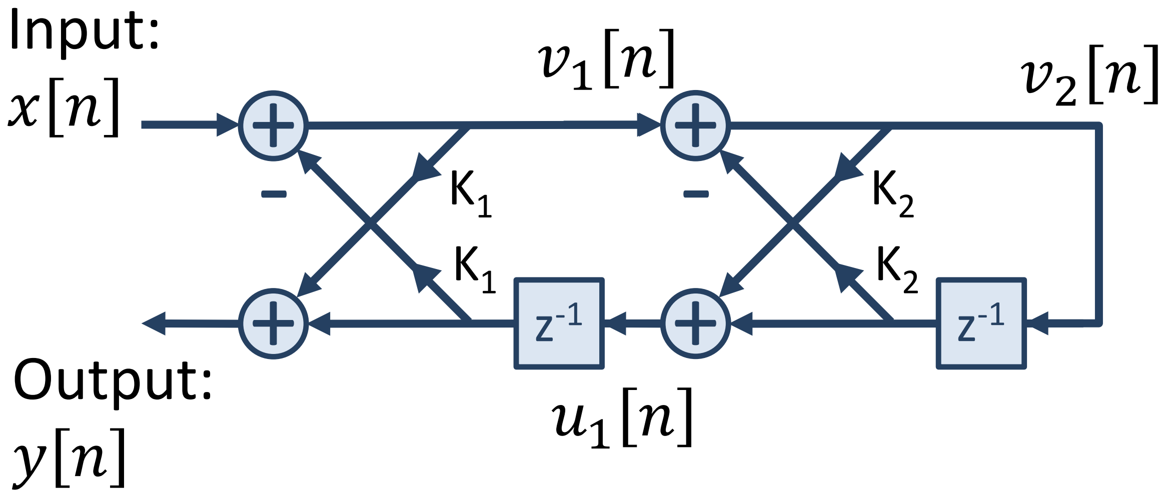
Implementing Lattice Filters

- **Lattice Stage**



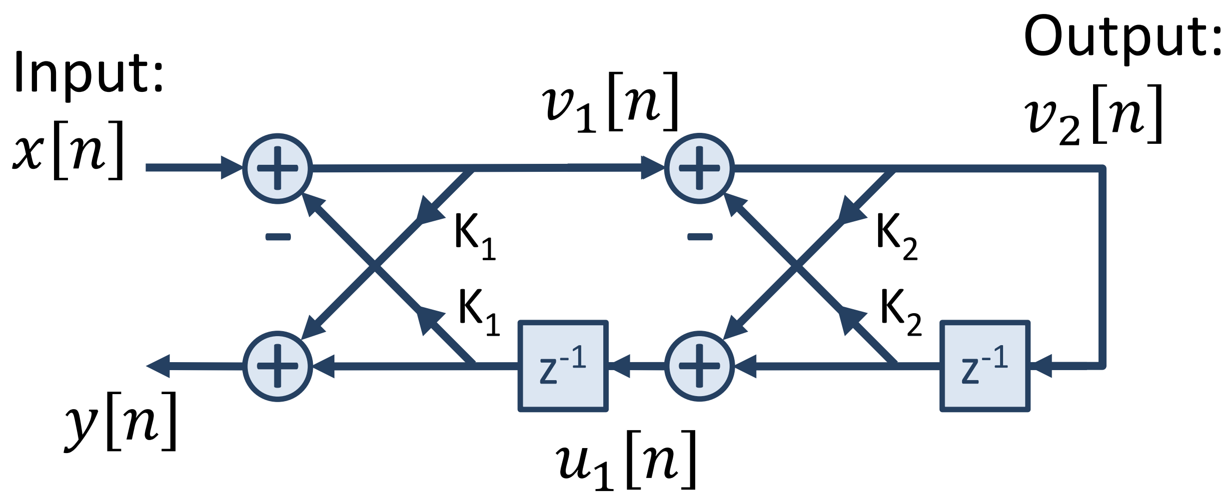
Implementing Lattice Filters

- **Lattice Stage**
  - **All-Pass Filter**



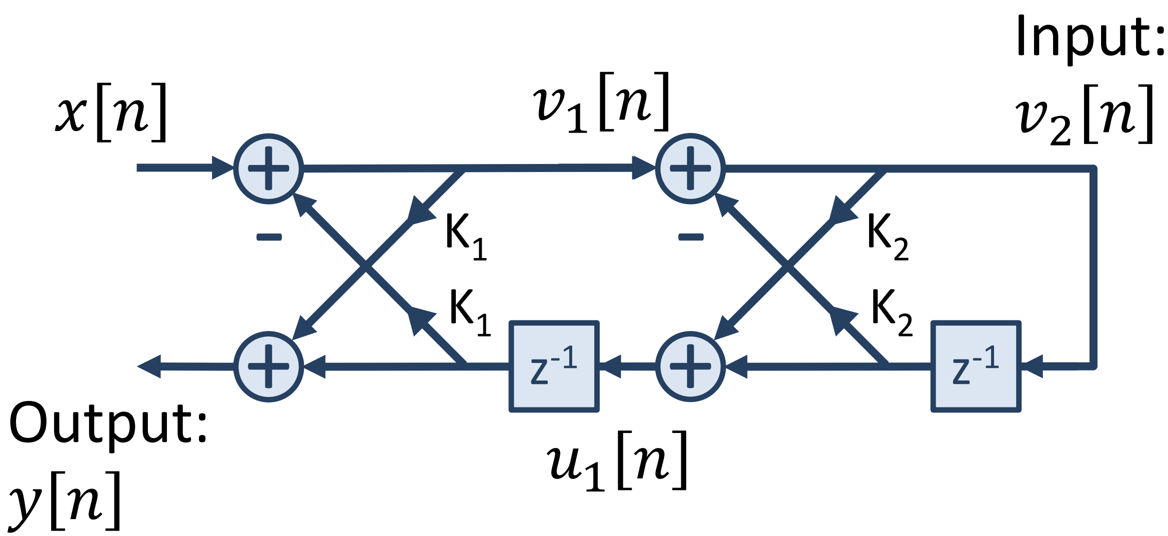
Implementing Lattice Filters

- **Lattice Stage**
  - **IIR Filter**



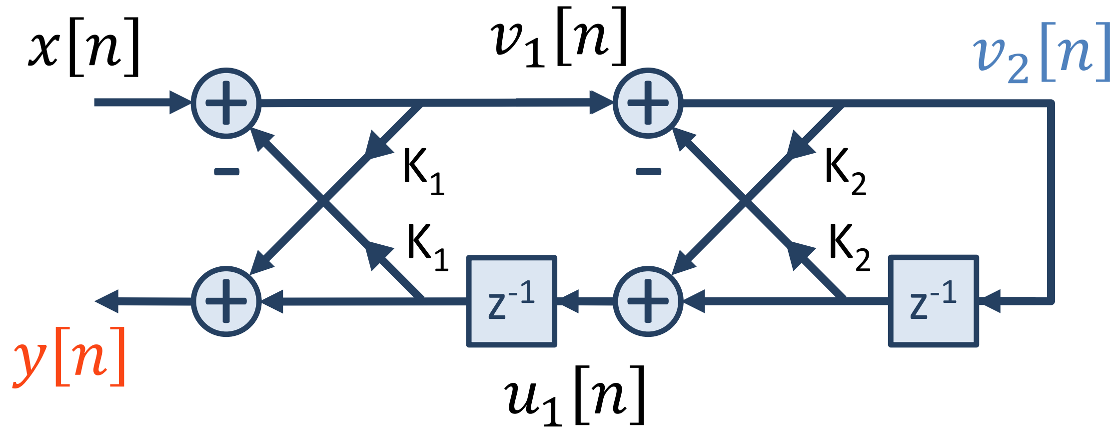
Implementing Lattice Filters

- **Lattice Stage**
  - **FIR Filter**



Implementing Lattice Filters

- **Lattice Stage**
  - **Example:**



Example

- **Example: Sketch the direct form II, parallel form, and cascade form for the system.**
  - $y[n] = \frac{1}{4}y[n-2] + x[n]$