

Question #1:

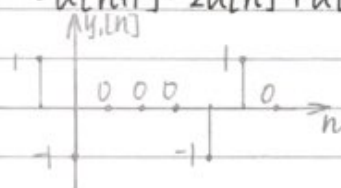
I spent 2 hours.

Question #2:

$$(a) y_1[n] = x_1[n+1] - 2x_1[n] + x_1[n-1]$$

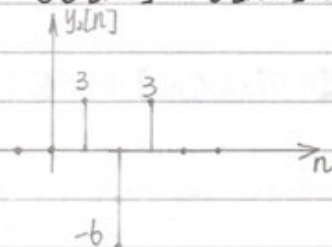
$$= u[n+1] - u[n-4] - 2u[n] + 2u[n-5] + u[n-1] - u[n-6]$$

$$= u[n+1] - 2u[n] + u[n-1] - u[n-4] + 2u[n-5] - u[n-6]$$



$$(b) y_2[n] = x_2[n+1] - 2x_2[n] + x_2[n-1]$$

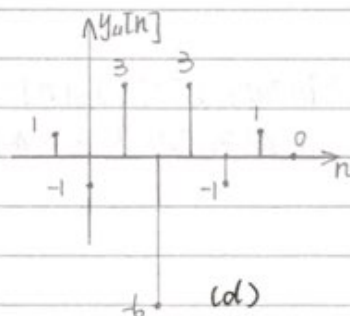
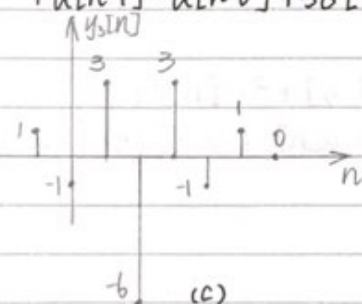
$$= 3\delta[n+1] - 6\delta[n-2] + 3\delta[n-3]$$



$$(c) \text{Input: } x_1[n] + x_2[n] = u[n] - u[n-5] + 3\delta[n-2] = x_3[n]$$

$$y_3[n] = u[n+1] - u[n-4] + 3\delta[n-1] - 2u[n] + 2u[n-5] - 6\delta[n-2]$$

$$+ u[n-1] - u[n-6] + 3\delta[n-3]$$



$$(d) y_4[n] = H\{x_1[n]\} + H\{x_2[n]\} = y_1[n] + y_2[n]$$

The plot is shown above on the right)

(e) Based on the discussion above, the system is linear. The proof is also straightforward and is shown below:

$$y_1[n] = H\{a x_1[n]\}, y_2[n] = H\{b x_2[n]\}$$

$$H\{ax_1[n] + bx_2[n]\} = \cancel{ax_1[n+1]} - 2ax_1[n] + \cancel{ax_1[n-1]} + bx_2[n+1]$$

$$ax_1[n+1] + bx_2[n+1] - 2ax_1[n] - 2bx_2[n] + ax_1[n-1] + bx_2[n-1]$$

$$= a[x_1[n+1] - 2x_1[n] + x_1[n-1]] + b[x_2[n+1] - 2x_2[n] + x_2[n-1]]$$

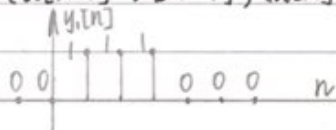
$$= ay_1[n] + by_2[n]$$

\therefore This system is linear.

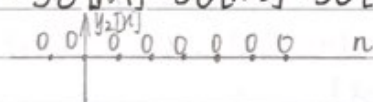
(f) It calculates the difference between "the summation of the past and future signal" and two times the current input signal.

Question #3:

$$(a) y_1[n] = (u[n+1] - u[n-4])(u[n] - u[n-5])(u[n-1] - u[n-6])$$

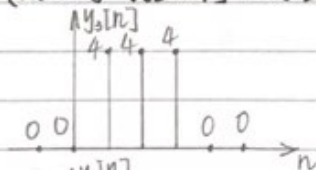


$$(b) y_2[n] = 3\delta[n-1] \cdot 3\delta[n-2] \cdot 3\delta[n-3] = 27\delta[n-1] \cdot \delta[n-2] \cdot \delta[n-3]$$

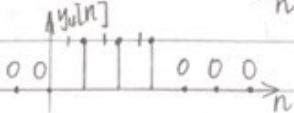


$$(c) \text{Input: } x_1[n] + x_2[n] = u[n] - u[n-5] + 3\delta[n-2]$$

$$y_3[n] = (u[n+1] - u[n-4] + 3\delta[n-1])(u[n] - u[n-5] + 3\delta[n-2])(u[n-1] - u[n-6] + 3\delta[n-3])$$



(d)



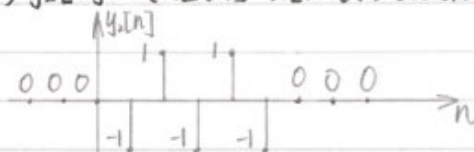
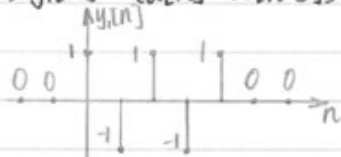
(e) Based on part (c) and (d), the system is not ^alinear system.

(f) It calculates the production of the past, current and future input

signal.

Question #4:

(a) $y_1[n] = (u[n] - u[n-5]) \cos(\pi n)$ (b) $y_2[n] = (u[n-1] - u[n-6]) \cos(\pi n)$



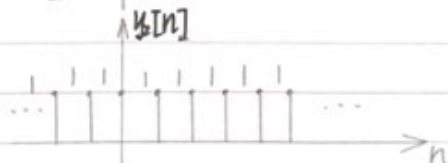
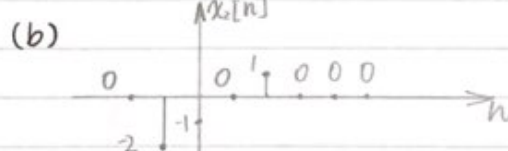
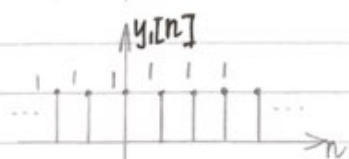
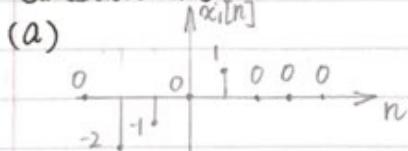
(c) This system is ^{not} time-invariant. because when the input is $x[n-1]$, the output of ~~only on the current input signal.~~ the system is not $y[n-1]$

~~$H\{x[n-1]\} = x[n-1] \cos(\pi(n-1))$~~

(d) This system is memoryless, because the output of the system depends only on $x[n]$ at current time.

(e) ~~The system gives only 1 or -1 outputs.~~ The $\cos(\pi n)$ gives only -1 and 1. So the system either ~~at~~ outputs the input signal, or outputs the negative values of the input signals.

Question #5:



(c) The system is time-invariant based on the discussions above. When the input is $x[n-1]$, the output of the system becomes $y[n-1]$.

(e) This system can be used to filter out any input ~~the~~ signal with values smaller than 1. outputs values ~~are~~ are all not less than 1.