Full Name: ______________________________ **ExamID: 010001**

EEL 4750 / EEE 5502 (Fall 2018) – Practice Exam #03 **Date: Nov. 28, 2018**

Question	# of Points Possible	# of Points Obtained	Grader
# 1	17		
# 2	15		
# 3	16		
# 4	18		
# 5	18		
# 6	16		
Total	100		

For full credit when sketching: remember to label axes and make locations and amplitudes clear.

Before starting the exam, read and sign the following agreement.

By signing this agreement, I agree to solve the problems of this exam while adhering to the policies and guidelines of the University of Florida and EEL 4750 / EEE 5502 and without additional external help. The guidelines include, but are not limited to,

- The University of Florida honor pledge: "On my honor, I have neither given nor received unauthorized aid in doing this assignment."
- Only one 8.5 by 11 inch cheat sheet (double-sided) may be used
- No calculators or computers may be used
- No textbooks or additional notes may be used
- No collaboration is allowed
- No cheating is allowed

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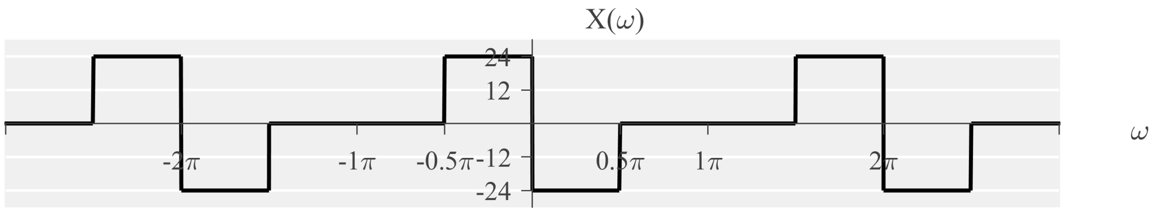
Student Date

Question #1: Consider the DTFT of the signal $x[n]$ (i.e., $X(\omega)$) shown below.

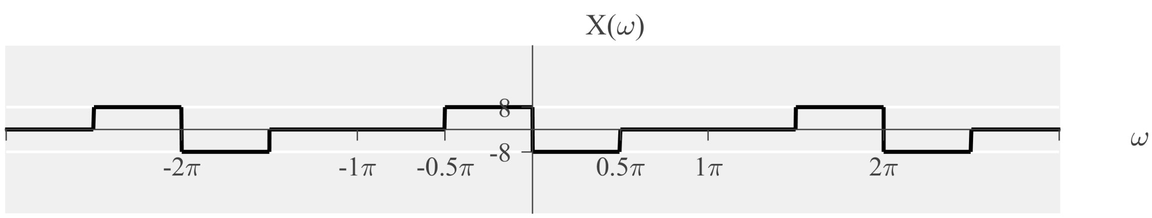


(a) *(4 pts)* What is the maximum achievable downsampling factor for $5x[n]$ without aliasing?

Solution: The maximum downsampling factor is 2

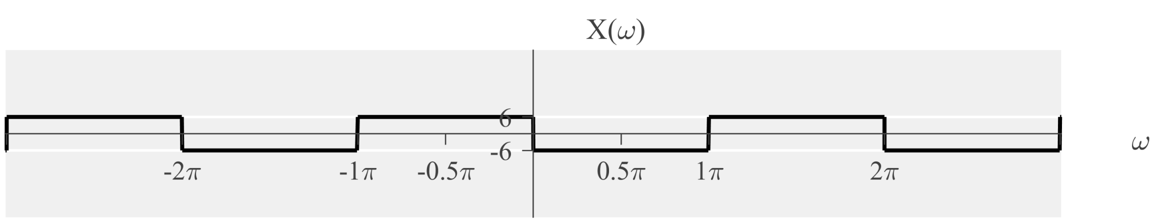
(b) *(7 pts)* Sketch the DTFT (from $\omega = -3\pi$ to $\omega = 3\pi$) of $x[n]$ after downsampling by 3 (**with no anti-aliasing filter**). Remember to label important locations / values.

Solution:

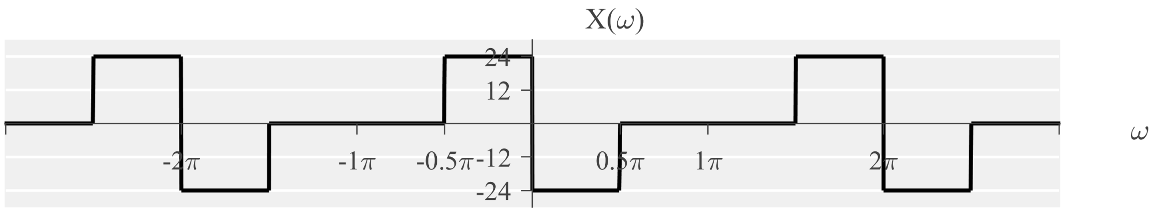


(c) *(8 pts)* Sketch the DTFT (from $\omega = -3\pi$ to $\omega = 3\pi$) of $x[n]$ after downsampling by 4 (**with an anti-aliasing filter**). Remember to label important locations / values.

Solution:

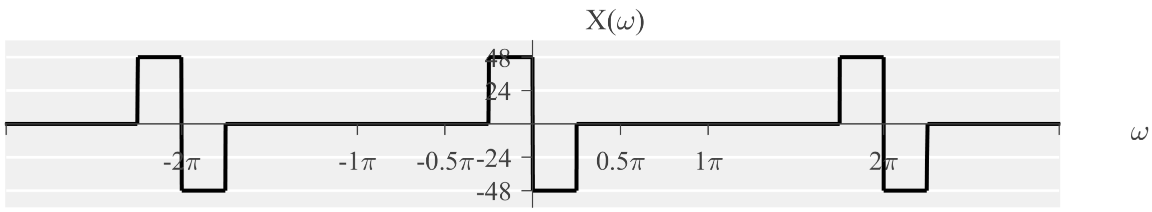


Question #2: Consider the DTFT of the signal $x[n]$ (i.e., $X(\omega)$) shown below.



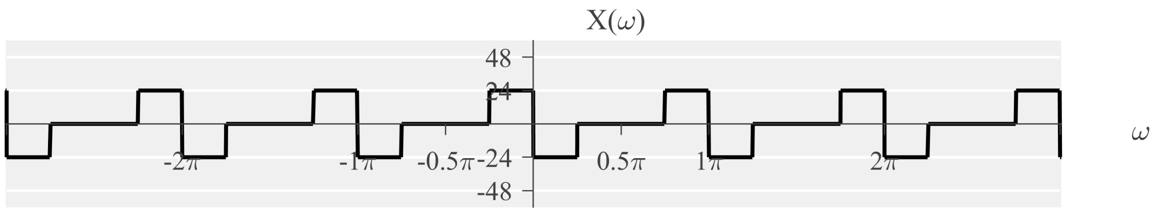
(a) *(8 pts)* Sketch the DTFT (from $\omega = -3\pi$ to $\omega = 3\pi$) of $x[n]$ after upsampling by 2 (**with an interpolation filter**). Remember to label important locations / values.

Solution:



(b) *(7 pts)* Sketch the DTFT (from $\omega = -3\pi$ to $\omega = 3\pi$) of $x[n]$ after upsampling by 2 (**with no interpolation filter**). Remember to label important locations / values.

Solution:



Question #3: Consider the desired frequency response

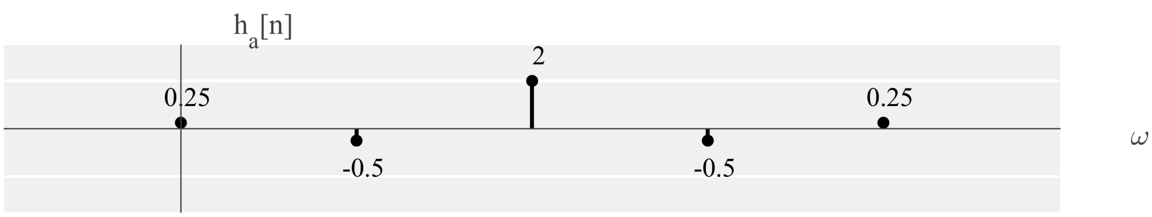
$$H_d(\omega) = \frac{1}{1+(1/2)e^{-j\omega}} + \frac{1}{1+(1/2)e^{+j\omega}}$$

(a) *(8 pts)* Approximate $H_d(\omega)$ with a length $N = 5$ windowing method. Use a rectangular window. **Force the resulting filter to be causal and linear phase.** Sketch the time-domain filter coefficients $h_a[n]$ with these requirements.

Solution:

$$H_d(\omega) = A(\omega) + A(-\omega)$$
$$A(\omega) = \frac{1}{1+(1/2)e^{-j\omega}}$$
$$h_d[n] = (-1/2)^n u[n] + (-1/2)^{-n} u[-n]$$

After shifting by $(N-1)/2 = 2$ samples to force casually and a linear phase, the solution is:



(b) *(8 pts)* Approximate $H_d(\omega)$ with a length $N = 4$ frequency sampling method. **Force the resulting filter to be causal and linear phase.** Compute the time-domain filter coefficients $h_b[n]$ with these requirements.

Solution: The frequency coefficients up to (and including) π are:

$$\omega_0 = 0 \qquad H_d(0) = \frac{1}{1+(1/2)} + \frac{1}{1+(1/2)} = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

$$\omega_1 = \frac{2\pi(1)}{4} = \frac{\pi}{2} \qquad H_d(\pi/2) = \frac{1}{1-(1/2)j} + \frac{1}{1+(1/2)j} = \frac{1-(1/2)j+1+(1/2)j}{(1-(1/2)j)(1+(1/2)j)} = \frac{2}{1+(1/4)} = \frac{8}{5}$$

$$\omega_2 = \frac{2\pi(2)}{4} = \pi \qquad H_d(\pi) = \frac{1}{1-(1/2)} + \frac{1}{1-(1/2)} = \frac{1}{1/2} + \frac{1}{1/2} = 4$$

Therefore,

$$h_b[n] = \frac{4}{3} + \frac{16}{5}\cos\left(\frac{\pi}{2}\left(n - \frac{3}{2}\right)\right) + 8\cos\left(\pi\left(n - \frac{3}{2}\right)\right) \quad \text{for} \quad 0 \le n \le 3$$

Question #4: Consider a desired filter frequency response (with a causal impulse response)

$$H_d(s) = \frac{60}{s + 1/2}$$

(a) *(6 pts)* Approximate $H_d(s)$ as a discrete-time IIR filter by approximating the differential equation with a sampling rate $T = 2$. Compute the time-domain filter coefficients $h_a[n]$ of this filter. **Force the resulting filter to be causal.**

Solution: $s \to \frac{1}{T}(1 - z^{-1})$

$$\begin{aligned} H_a(z) &= \frac{60}{\frac{1}{2}(1 - z^{-1}) + 1/2} \\ &= \frac{60}{1/2 - (1/2)z^{-1} + 1/2} \\ &= \frac{60}{1 - (1/2)z^{-1}} \\ h_a[n] &= 60(1/2)^n u[n] \end{aligned}$$

(b) *(6 pts)* Approximate $H_d(s)$ as a discrete-time IIR filter using the impulse invariance method with a sampling rate $T = 2$. Compute the time-domain filter coefficients $h_b[n]$ of this filter. **Force the resulting filter to be causal.**

Solution: Poles: $s = -1/2$
Convert s-poles into z-poles: $s_0 \to e^{s_0 T}$

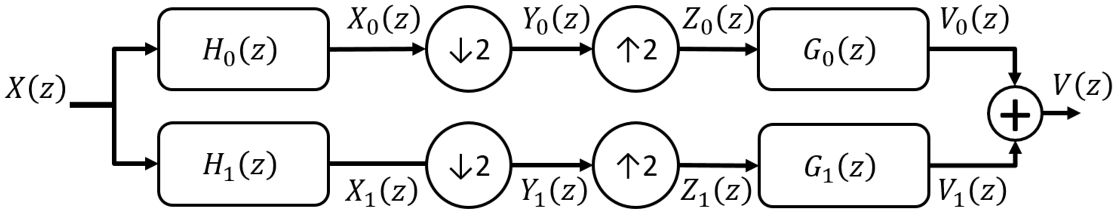
$$\begin{aligned} H_b(z) &= \frac{60}{1 - e^{(2)(-1/2)} z^{-1}} \\ &= \frac{60}{1 - e^{-1} z^{-1}} \\ h_b[n] &= 60 e^{-n} u[n] \end{aligned}$$

(c) *(6 pts)* Approximate $H_d(s)$ as a discrete-time IIR filter using the bilinear transform with a sampling rate $T = 2$. Compute the time-domain filter coefficients $h_c[n]$ of this filter. **Force the resulting filter to be causal.**

Solution: $s \rightarrow \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

$$\begin{aligned}
H_c(z) &= \frac{60}{\frac{1-z^{-1}}{1+z^{-1}} + 1/2} \\
&= \frac{60(1+z^{-1})}{1 - z^{-1} + 1/2(1+z^{-1})} \\
&= \frac{60(1+z^{-1})}{3/2 - (1/2)z^{-1}} \\
&= \frac{40(1+z^{-1})}{1 - (1/3)z^{-1}} \\
h_c[n] &= 40(1/3)^n u[n] + 40(1/3)^{n-1} u[n-1]
\end{aligned}$$

Question #5: Consider a 2-channel filter bank shown below.



Let the filters be defined by the frequency domain expression

$$H_0(\omega) = G_0(\omega) = \sqrt{2}\sin(\omega/2)$$

(a) *(7 pts)* Choose a filter $H_1(\omega) = G_1(\omega)$ that satisfies the alias canceling conditions.

Solution: The alias canceling conditions:

$$\begin{aligned} H_0(\omega)G_0(\omega) + H_1(\omega)G_1(\omega) &= 2 \\ H_0(\omega-\pi)G_0(\omega) + H_1(\omega-\pi)G_1(\omega) &= 0 \end{aligned}$$

$$\begin{aligned} 2\sin(\omega/2)\sin(\omega/2) + H_1(\omega)G_1(\omega) &= 2 \\ 2\sin((\omega-\pi)/2)\sin(\omega/2) + H_1(\omega-\pi)G_1(\omega) &= 0 \end{aligned}$$

$$\begin{aligned} 2\sin^2(\omega/2) + H_1(\omega)G_1(\omega) &= 2 \\ 2\cos(\omega/2)\sin(\omega/2) + H_1(\omega-\pi)G_1(\omega) &= 0 \end{aligned}$$

If we choose $H_1(\omega) = G_1(\omega) = \sqrt{2}\cos(\omega/2)$,

$$\begin{aligned} 2\sin^2(\omega/2) + 2\cos^2(\omega/2) &= 2 \\ 2\cos(\omega/2)\sin(\omega/2) + 2\cos((\omega-\pi)/2)\cos(\omega) &= 0 \end{aligned}$$

$$\begin{aligned} 2\sin^2(\omega/2) + 2\cos^2(\omega/2) &= 2 \\ 2\cos(\omega/2)\sin(\omega/2) - 2\sin(\omega/2)\cos(\omega) &= 0 \end{aligned}$$

$$\begin{aligned} 2 &= 2 \\ 0 &= 0 \end{aligned}$$

(b) *(7 pts)* Let $X(\omega) = \cos(\omega/2)$. Compute the intermediate signal $V_0(\omega)$.

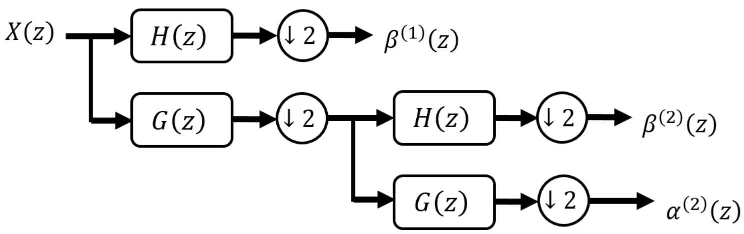
Solution: The frequency response at $V_0(\omega)$ is

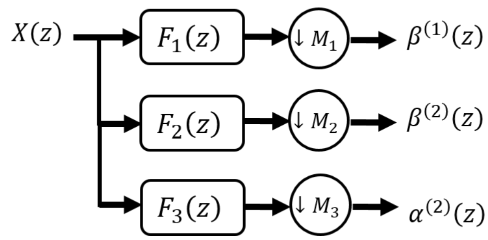
$$\begin{aligned} V_0(\omega) &= [H_0(\omega)X(\omega) + H_0(\omega-\pi)X(\omega-\pi)]\, G_0(\omega) \\ &= [\sin(\omega/2)\cos(\omega/2) + \sin((\omega-\pi)/2)\cos((\omega-\pi)/2)]\, G_0(\omega) \\ &= [\sin(\omega/2)\cos(\omega/2) - \cos(\omega/2)\sin(\omega/2)]\, G_0(\omega) \\ &= 0 \end{aligned}$$

(c) *(4 pts)* **(True or False)** When the alias canceling conditions are met, $V_0(z) = V_1(z)$.

Solution: **False**, alias canceling ensures that $V_0(z) + V_1(z) = X(z)$, which is not guaranteed to be true when $V_0(z) = V_1(z)$.

Question #6: Consider the following wavelet bank and filter bank.





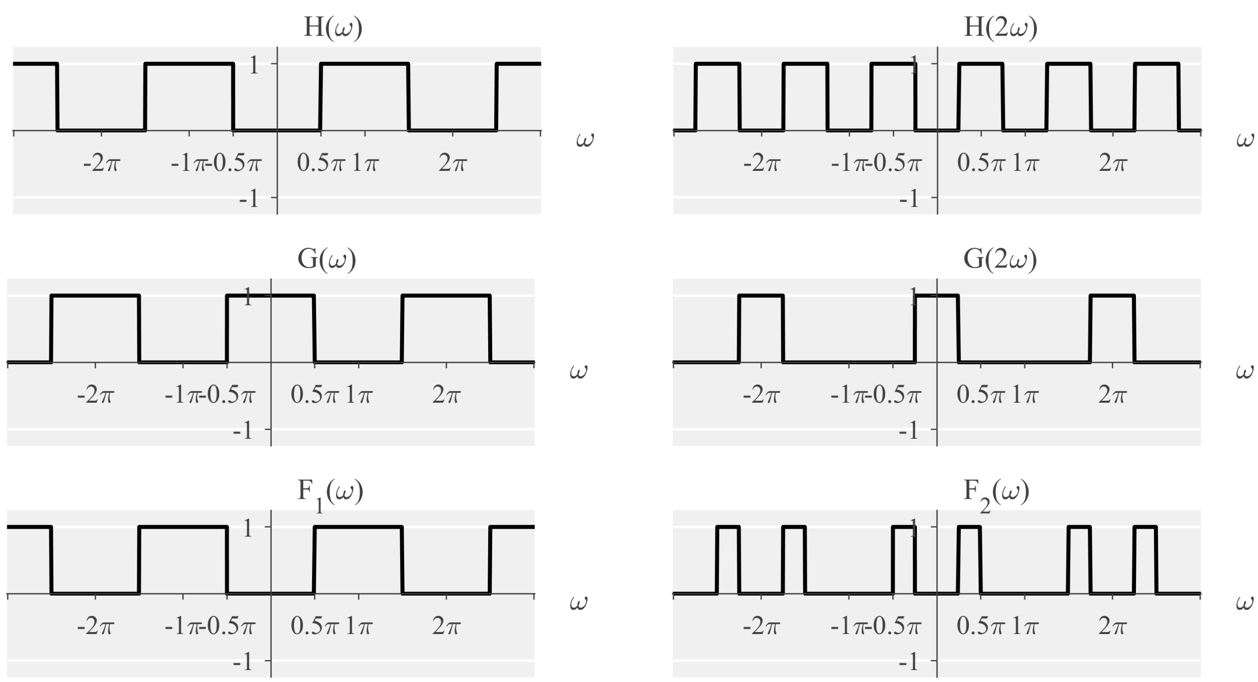
Let the high pass filter $H(z)$ and low pass filter $G(z)$ be defined by frequency responses:

$$G(\omega) = \sum_{k=-\infty}^{\infty} u(\omega + \pi/2 - 2\pi k) - u(\omega - \pi/2 - 2\pi k)$$
$$H(\omega) = \sum_{k=-\infty}^{\infty} u(\omega + \pi/2 - \pi - 2\pi k) - u(\omega - \pi/2 - \pi - 2\pi k)$$

Use the Noble identities to simplify the wavelet bank (left) diagram and represent it as a filter bank (right). Determine M_1, M_2, and M_3. Sketch $|F_1(\omega)|$, $|F_2(\omega)|$, and $|F_3(\omega)|$.

Solution: $M_1 = 2,\ M_2 = 4,\ M_3 = 4.$

$$F_1(\omega) = H(z)$$
$$F_2(\omega) = G(z)H(z^2)$$
$$F_3(\omega) = G(z)G(z^2)$$



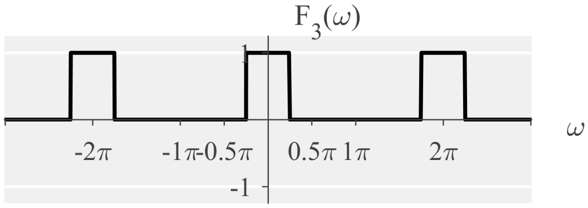

$F_3(\omega)$
1
-1
-2π
-1π
-0.5π
0.5π
1π
2π
ω

Table of Discrete-Time Fourier Transform Pairs:

$$\text{Discrete-Time Fourier Transform} \quad : \quad X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$\text{Inverse Discrete-Time Fourier Transform} \quad : \quad x[n] = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega t}\, d\omega\ .$$

$x[n]$	$X(\omega)$	condition
$a^n u[n]$	$\dfrac{1}{1-ae^{-j\omega}}$	$\|a\|<1$
$(n+1)a^n u[n]$	$\dfrac{1}{(1-ae^{-j\omega})^2}$	$\|a\|<1$
$\dfrac{(n+r-1)!}{n!(r-1)!}a^n u[n]$	$\dfrac{1}{(1-ae^{-j\omega})^r}$	$\|a\|<1$
$\delta[n]$	1	
$\delta[n-n_0]$	$e^{-j\omega n_0}$	
$x[n]=1$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega-2\pi k)$	
$u[n]$	$\dfrac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega-2\pi k)$	
$e^{j\omega_0 n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega-\omega_0-2\pi k)$	
$\cos(\omega_0 n)$	$\pi \sum_{k=-\infty}^{\infty} \{\delta(\omega-\omega_0-2\pi k)+\delta(\omega+\omega_0-2\pi k)\}$	
$\sin(\omega_0 n)$	$\dfrac{\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\omega-\omega_0-2\pi k)-\delta(\omega+\omega_0-2\pi k)\}$	
$\sum_{k=-\infty}^{\infty} \delta[n-kN]$	$\dfrac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega-\dfrac{2\pi k}{N}\right)$	
$x[n]=\begin{cases} 1 & , \quad \|n\| \le N \\ 0 & , \quad \|n\| > N \end{cases}$	$\dfrac{\sin(\omega(N+1/2))}{\sin(\omega/2)}$	
$\dfrac{\sin(Wn)}{\pi n} = \dfrac{W}{\pi}\text{sinc}\left(\dfrac{Wn}{\pi}\right)$	$X(\omega)=\begin{cases} 1 & , \quad 0 \le \|\omega\| \le W \\ 0 & , \quad W < \|\omega\| \le \pi \end{cases}$	

$X(\omega)$ is periodic with period 2π

Table of Discrete-Time Fourier Transform Properties: For each property, assume

$$x[n] \stackrel{DTFT}{\longleftrightarrow} X(\omega) \quad \text{and} \quad y[n] \stackrel{DTFT}{\longleftrightarrow} Y(\omega)$$

Property	Time domain	DTFT domain
Linearity	$Ax[n] + By[n]$	$AX(\omega) + BY(\omega)$
Time Shifting	$x[n - n_0]$	$X(\omega)e^{-j\omega n_0}$
Frequency Shifting	$x[n]e^{j\omega_0 n}$	$X(\omega - \omega_0)$
Conjugation	$x^*[n]$	$X^*(-\omega)$
Time Reversal	$x[-n]$	$X(-\omega)$
Convolution	$x[n] * y[n]$	$X(\omega)Y(\omega)$
Multiplication	$x[n]y[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{2\pi} X(\theta)Y(\omega - \theta)d\theta$
Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(\omega)$
Accumulation	$\sum_{k=-\infty}^{\infty} x[k]$	$\frac{1}{1-e^{-j\omega}} + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
Frequency Differentiation	$nx[n]$	$j\dfrac{dX(\omega)}{d\omega}$
Parseval's Relation for Aperiodic Signals	$\sum_{k=-\infty}^{\infty} \lvert x[k]\rvert^2$	$\frac{1}{2\pi} \int_{2\pi} \lvert X(\omega)\rvert^2 d\omega$

Table of Z-Transform Pairs:

$$\text{Z-Transform} \quad : \quad X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$\text{Inverse Z-Transform} \quad : \quad x[n] = \frac{1}{2\pi j} \oint_{\mathcal{C}} X(z) z^{n-1} \, dz \, .$$

$x[n]$	$X(\omega)$	ROC
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{1}{1 - az^{-1}}$	$\|z\| < \|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$\|z\| < \|a\|$
$\delta[n]$	1	All z
$\delta[n - n_0]$	z^{-n_0}	All z
$u[n]$	$\frac{1}{1 - z^{-1}}$	$\|z\| > 1$
$\cos(\omega_0 n) u[n]$	$\frac{1 - z^{-1}\cos(\omega_0)}{1 - 2z^{-1}\cos(\omega_0) + z^{-2}}$	$\|z\| > 1$
$\sin(\omega_0 n) u[n]$	$\frac{z^{-1}\sin(\omega_0)}{1 - 2z^{-1}\cos(\omega_0) + z^{-2}}$	$\|z\| > 1$
$a^n \cos(\omega_0 n) u[n]$	$\frac{1 - az^{-1}\cos(\omega_0)}{1 - 2az^{-1}\cos(\omega_0) + a^2 z^{-2}}$	$\|z\| > \|a\|$
$a^n \sin(\omega_0 n) u[n]$	$\frac{az^{-1}\sin(\omega_0)}{1 - a2z^{-1}\cos(\omega_0) + a^2 z^{-2}}$	$\|z\| > \|a\|$

Table of Z-Transform Properties: For each property, assume

$$x[n] \overset{Z}{\longleftrightarrow} X(z) \quad \text{and} \quad y[n] \overset{Z}{\longleftrightarrow} Y(z)$$

Property	Time domain	Z-domain
Linearity	$Ax[n] + By[n]$	$AX(z) + BY(z)$
Time Shifting	$x[n - n_0]$	$X(z)z^{-n_0}$
Z-scaling	$a^n x[n]$	$X(a^{-1}z)$
Conjugation	$x^*[n]$	$X^*(z^*)$
Time Reversal	$x[-n]$	$X(z^{-1})$
Convolution	$x[n] * y[n]$	$X(z)Y(z)$
Differentiation in z-domain	$nx[n]$	$-z\frac{dX(z)}{dz}$
Initial Value Theorem	$x[n]$ is causal	$x(0) = \lim_{z\to\infty} X(z)$