EEE 5502 HW #04 Hudanyun Sheng

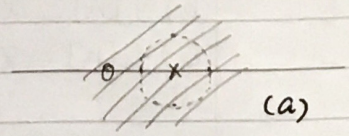
Question #1

I. spent 5 hours

Question #2

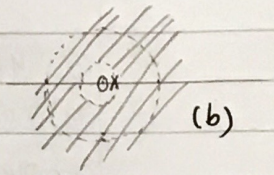
(a) $X(z) = 1 + 2z^{-1}$ zero: $z = -2$. Pole: $z = 0$

ROC: ~~$|z| > R$~~ $|z| > 0$. The system is stable.



(b) $X(z) = \frac{1}{1 - 0.25z^{-1}} = \frac{4z}{4z - 1}$ zero: $z = 0$. pole: $z = \frac{1}{4}$

ROC: $|z| > \frac{1}{4}$. The system is stable.

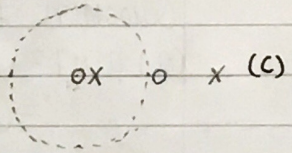


(c) $X(z) = (-1)[-0.2^n u[-n-1]] - 2^n u[n]$

$= -\frac{1}{1 - 0.2z^{-1}} - \frac{1}{1 - 2z^{-1}}$ zero: $z = 0$, $z = \frac{11}{10}$. pole: $z = \frac{1}{5}$, $z = 2$

ROC: $|z| < \frac{1}{5} \cap |z| > 2$ hence, no region of convergence.

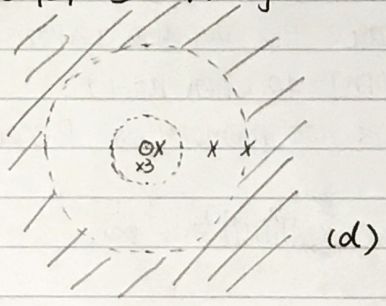
The system is unstable



(d) $X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}} \cdot \frac{1}{1 - \frac{1}{2}z^{-1}} \cdot \frac{1}{1 - 3z^{-1}}$

zeros: $z = 0$, $z = 0$, $z = 0$. poles: $z = \frac{1}{3}$, $z = 2$, $z = 3$

ROC: $|z| > 3$. The system is stable



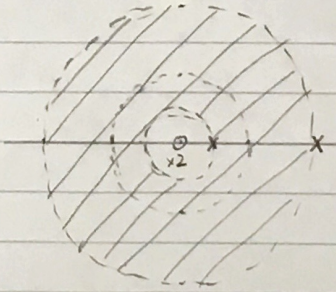
(e) $h[n] = (\frac{1}{2})^n u[n] + (\frac{1}{2})^{-n} u[-n-1] = (\frac{1}{2})^n u[n] - [-2^n u[-n-1])$

$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} - \frac{1}{1 - 2z^{-1}}$

zeros: $z = 0$, $z = 0$. poles: $z = 2$, $z = \frac{1}{2}$

ROC: $\frac{1}{2} < |z| < 2$

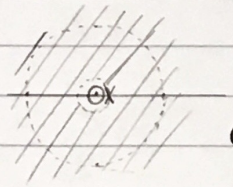
The system is stable.



Question #3

(a) $H(z) = \frac{10z}{10z-1}$ zero: $z=0$. pole: $z=\frac{1}{10}$. ROC: $|z| > \frac{1}{10}$

The system is stable.

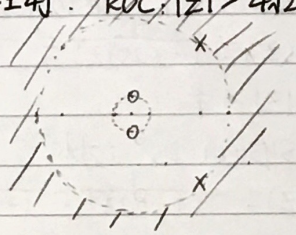


(a)

(b) $H(z) = \frac{z^2+1}{(z-4)^2+16}$ zero: $z=\pm j$. pole: $z=4\pm 4j$. ROC: $|z| > 4\sqrt{2}$

The system is unstable.

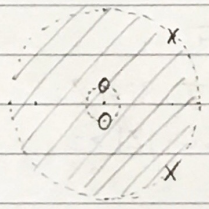
plot for (b) →



(c) zero: $z=\pm j$. pole: $z=4\pm 4j$

ROC: $|z| < 4\sqrt{2}$

The system is stable.

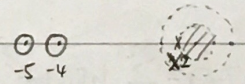


(c)

(d) zero: $z=-4$, $z=-5$

Pole: $z=-\frac{1}{2}$, $z=-\frac{1}{2}$

ROC: $|z| < \frac{1}{2}$. The system is unstable.



(d)

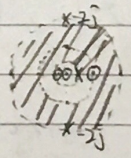
(e) $H(z) = \frac{z(z-1)(z+\frac{1}{4})}{(z^2+4)(z-\frac{1}{2})}$

Zero: $z=0$, $z=1$, $z=-\frac{1}{4}$

Pole: $z=\pm 2j$, $z=\frac{1}{2}$

ROC: $\frac{1}{2} < |z| < 2$

The system is Stable.



(e)

Question #4:

(a) $Y(z) - 1.8\,Y(z)z^{-1} = X(z)$

$\therefore Y(z)(1-1.8z^{-1}) = X(z)$

$\therefore H(z) = Y(z)/X(z) = 1/(1-1.8z^{-1}) \quad \therefore h[n] = 1.8^n u[n]$

(b) $Y(z) - 1.8\,Y(z)z^{-1} = X(z)z^{-1} + 2X(z)z^{-2}$

$\therefore Y(z)(1-1.8z^{-1}) = X(z)(z^{-1} + 2z^{-2})$

$\therefore H(z) = Y(z)/X(z) = (z^{-1}+2z^{-2})/(1-1.8z^{-1})$

$$= \frac{z^{-1}}{1-1.8z^{-1}} + 2\cdot\frac{z^{-2}}{1-1.8z^{-1}}$$

$\therefore h[n] = 1.8^{n-1}u[n-1] + 2(1.8^{n-2})u[n-2]$

(c) $Y(z) - 3Y(z)z^{-1} + 2Y(z)z^{-2} = X(z)$

$\therefore Y(z)(1-3z^{-1}+2z^{-2}) = X(z)$

$\therefore H(z) = Y(z)/X(z) = \frac{1}{1-3z^{-1}+2z^{-2}} = \frac{2}{1-2z^{-1}} - \frac{1}{1-z^{-1}}$

$\therefore h[n] = 2(2^n)u[n] - u[n] = (2^{n+1}-1)u[n]$

(d) $Y(z)z^{-1} - 3Y(z)z^{-2} + 2Y(z)z^{-3} = X(z)$

$\therefore Y(z)(z^{-1} - 3z^{-2} + 2z^{-3}) = X(z)$

$\therefore H(z) = Y(z)/X(z) = 1/(z^{-1}-3z^{-2}+2z^{-3}) = \frac{4}{1-2z^{-1}} - \frac{1}{1-z^{-1}} + \frac{1}{z^{-1}}$

$\therefore h[n] = 4\cdot 2^n u[n] - u[n] + \delta[n+1] = (2^{n+2}-1)u[n] + \delta[n+1]$

Question #5:

(a) $X(\Omega) = \frac{1}{1-0.2e^{-j\Omega}}$

(b) $X(\Omega) = (1/(1-0.2e^{-j\Omega}))\cdot e^{-j\Omega\cdot 12} = e^{-12j\Omega}/(1-0.2e^{-j\Omega})$

(c) $X(\Omega) = 0.2e^{-12j\Omega}/(1-0.2e^{-j\Omega})$

(d) $X(\Omega) = e^{12j\Omega}/(1-0.2e^{j\Omega})$.

(e) $X(\Omega) = (e^{-12j\Omega}/(1-0.2e^{-j\Omega}))\cdot(e^{12j\Omega}/(1-0.2e^{j\Omega}))$

$$= \frac{1}{1.04-0.2(e^{j\Omega}+e^{-j\Omega})}$$

Question #6

(a) $x[n] = \cos(\frac{\pi}{3}n)$

(b) $x[n] = 3 \cdot (-\frac{1}{2})^n u[n]$

(c) $X(\omega) = e^{-j\omega 4}\left(\frac{1}{5-e^{-j\omega}}\right)$

$\therefore x[n] = \delta[n-4] * \frac{0.2^n}{5} u[n] = 0.2^{n-3} u[n-4]$

(d) $X(\omega) = \cos(\omega n) - j\sin(\omega n) = e^{-j\omega n}$

$\therefore x[n] = \delta[n-n_0]$