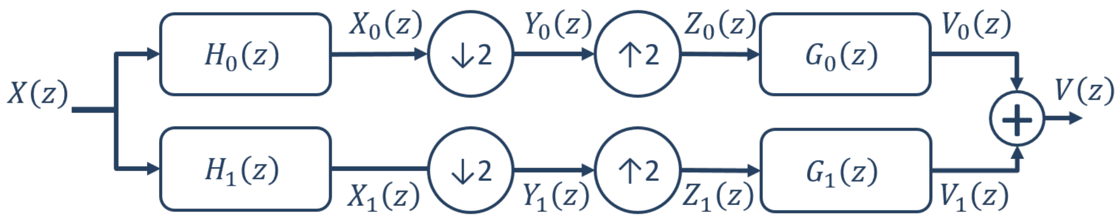
Full Name: ______________________________

EEL 4750 / EEE 5502 (Fall 2018) – HW #11 **Due Date: Dec. 9, 2018**

Question #1: *(2 pts)* How many hours did you spend on this homework?

Question #2: *(12 pts)* Consider the following 2-channel filter bank shown below.



Let the filters be defined by

$$H_0(z) = \frac{1}{\sqrt{2}}\left(1 + z^{-1}\right) \qquad G_0(z) = \frac{1}{\sqrt{2}}\left(z^{+1} + 1\right)$$

$$H_1(z) = \frac{1}{\sqrt{2}}\left(1 - z^{-1}\right) \qquad G_1(z) = \frac{1}{\sqrt{2}}\left(-z^{+1} + 1\right)$$

(a) Do the filters satisfy the alias canceling filter bank conditions? Show that they do or do not.

(b) Change $G_0(z)$ and $G_1(z)$ to be causal: $G_0(z) = \frac{1}{\sqrt{2}}\left(1 + z^{-1}\right)$ and $G_1(z) = \frac{1}{\sqrt{2}}\left(-1 + z^{-1}\right)$. How does the right-hand side of the alias canceling filter bank conditions change under this condition?

(c) Sketch all of the intermediate signals ($X_0(z)$, $X_(z)$, $Y_0(z)$, $Y_1(z)$, $Z_0(z)$, $Z_1(z)$, $V_0(z)$, $V_1(z)$, $V(z)$) in the time domain for excitation $X(z) = 1 + z^{-1} - z^{-2} - z^{-3}$.

Question #3: *(8 pts)* Consider a 2-channel filter bank shown in Question #2.
Let the filters be defined by

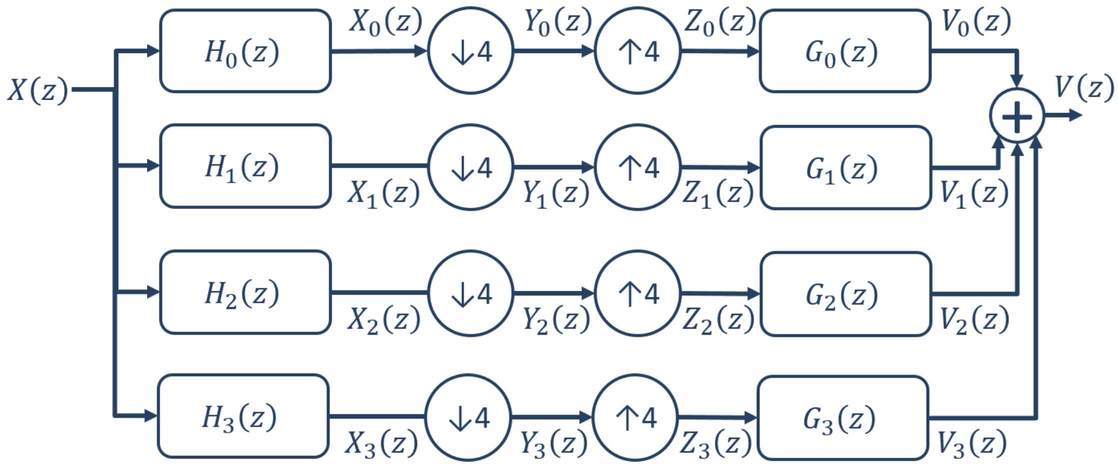
$$H_0(z) = \frac{1}{2}\left(1 + z^{-1} - z^{-2} - z^{-3}\right) \qquad G_0(z) = \frac{1}{2}\left(-z^{+3} - z^{+2} + z^{+1} + 1\right)$$

$$H_1(z) = \frac{1}{2}\left(1 - z^{-1} - z^{-2} + z^{-3}\right) \qquad G_1(z) = \frac{1}{2}\left(z^{+3} - z^{+2} - z^{+1} + 1\right)$$

(a) Do the filters satisfy the orthogonal filter bank conditions? Show that they do or do not.

(b) Sketch all of the intermediate signals ($X_0(z)$, $X_1(z)$, $Y_0(z)$, $Y_1(z)$, $Z_0(z)$, $Z_1(z)$, $V_0(z)$, $V_1(z)$, $V(z)$) in the time domain for excitation $X(z) = 1 + z^{-1} - z^{-2} - z^{-3}$.

Question #4: *(8 pts)* Consider the following 4-channel filter bank shown below.



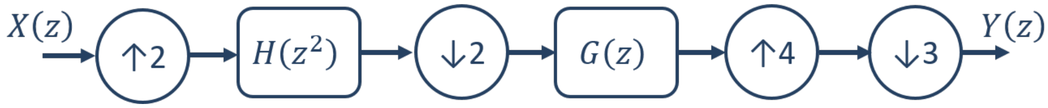
Let the filters be defined by

$$H_0(\omega) = G_0(\omega) = \sum_{k=-\infty}^{\infty} u(\omega + \pi/4 - 2\pi k) - u(\omega - \pi/4 - 2\pi k)$$
$$H_1(\omega) = G_1(\omega) = \sum_{k=-\infty}^{\infty} u(\omega+\pi/2-2\pi k)-u(\omega+\pi/4-2\pi k)+u(\omega-\pi/4-2\pi k)-u(\omega-\pi/2-2\pi k)$$
$$H_2(\omega) = G_2(\omega) = \sum_{k=-\infty}^{\infty} u(\omega+3\pi/4-2\pi k)-u(\omega+\pi/2-2\pi k)+u(\omega-\pi/2-2\pi k)-u(\omega-3\pi/4-2\pi k)$$
$$H_3(\omega) = G_3(\omega) = \sum_{k=-\infty}^{\infty} u(\omega+\pi-2\pi k)-u(\omega+3\pi/4-2\pi k)+u(\omega-3\pi/4-2\pi k)-u(\omega-\pi-2\pi k)$$

Sketch all of the intermediate signals ($X_m(z)$, $Y_m(z)$, $Z_m(z)$, $V_m(z)$, $V(z)$ for $0 \le m \le 3$) in the frequency domain for excitation $X(z) = 1$.

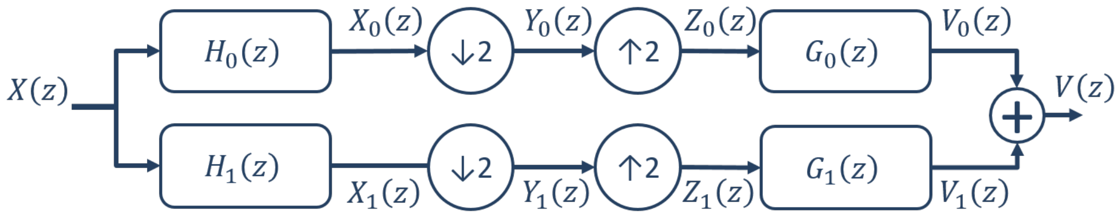
Question #5: *(8 pts)* *Noble Properties*
One of the useful aspects of Noble properties is that they can help simplify complex expressions with downsampling and upsmapling.

(a) Use the Noble properties for upsampling and downsampling to simplify the following block digram. Represent the results as a block diagram. It should only have one downsampling operation, one upsampling operation, and one filter operation



(b) Let $H(\omega) = G(\omega) = \sum_{k=-\infty}^{\infty} u(\omega + \pi/2 - 2\pi k) - u(\omega - \pi/2 - 2\pi k)$. Plot the magnitude response of the resulting filter in your block diagram.

Question #6: *(8 pts)* Consider the following 2-channel bank.



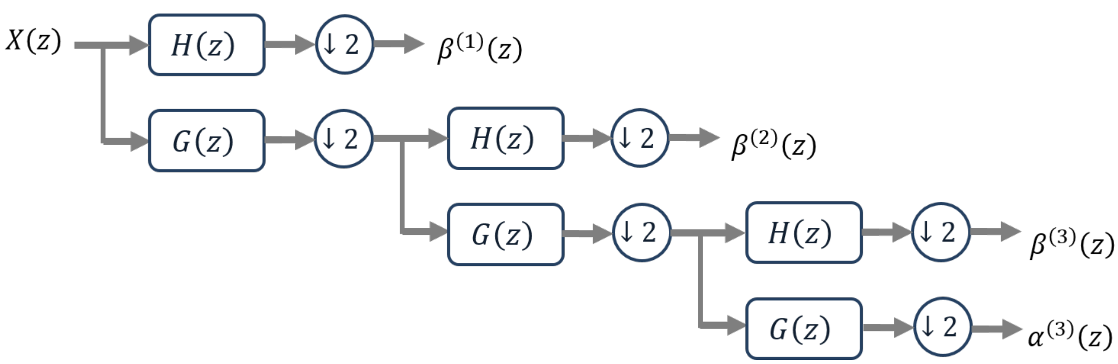
Let the filters be defined by

$$H_0(z) = \frac{1}{2}\left(1 + z^{-1} - z^{-2} - z^{-3}\right) \qquad H_1(z) = \frac{1}{2}\left(-z^{+1} - z^{+2} + z^{+1} + 1\right)$$

$$G_0(z) = \frac{1}{2}\left(1 - z^{-1} - z^{-2} + z^{-3}\right) \qquad G_1(z) = \frac{1}{2}\left(z^{+3} - z^{+2} - z^{+1} + 1\right)$$

Determine the equivalent polyphase filter implementation for this filter bank. Stretch the impulse response of the filter coefficients for each polyphase filter.

Question #7: *(12 pts)* Consider the following wavelet bank.



Let the high pass filter $H(z)$ and low pass filter $G(z)$ be defined by:

$$G(\omega) = \sum_{k=-\infty}^{\infty} u(\omega + \pi/2 - 2\pi k) - u(\omega - \pi/2 - 2\pi k)$$

$$H(\omega) = \sum_{k=-\infty}^{\infty} u(\omega + \pi/2 - \pi - 2\pi k) - u(\omega - \pi/2 - \pi - 2\pi k)$$

Sketch the magnitude responses of $|\beta^{(1)}(\omega)|$,$|\beta^{(2)}(\omega)|$,$|\beta^{(3)}(\omega)|$,$|\alpha^{(3)}(\omega)|$ when $X(\omega) = 1$