Lecture 2: Continuous -Time and Discrete -Time Signals

Foundations of Digital Signal Processing

Outline

- Signal Properties
- Periodicity
- Measures of signal "size"
- Signal Operations
- Special Signals: Impulses and Steps and Exponentials
- System Properties

Notes

- **Homework**
  - To be submitted on Canvas
  - Released on Thursday evenings
  - Due at 11:59 PM on next Thursday
- **MATLAB**
  - Freely available to students via UF Apps
  - https://info.apps.ufl.edu/
  - If ineffective, student licenses are available ($50 or $100)
  - Tutorial? Will try to post something today
- **Slack**
  - Up and operational

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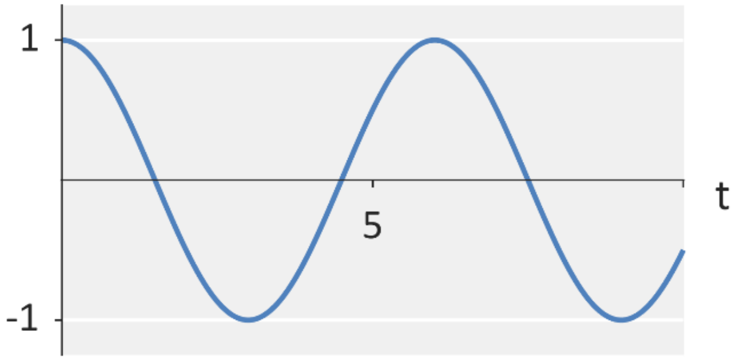
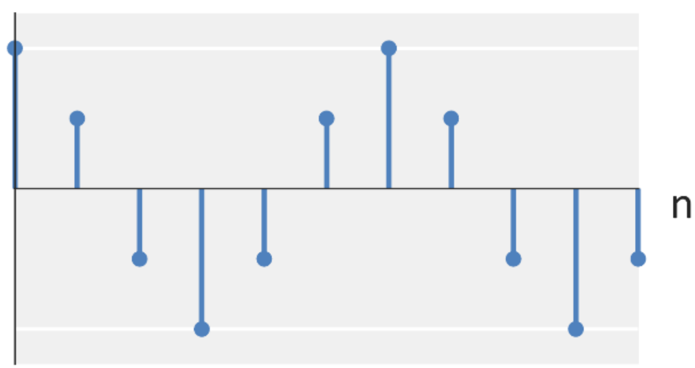
Signal Properties

- **Signals: time signals (1D signals)**
  - Continuous-time signals



$$x(t)$$

  - Discrete-time signals



$$x[n]$$

Signal Properties

- **Problem: Sketch the signal**

$$x[n] = 2^{-|n|}$$

for $-5 < n < 5$

Signal Properties

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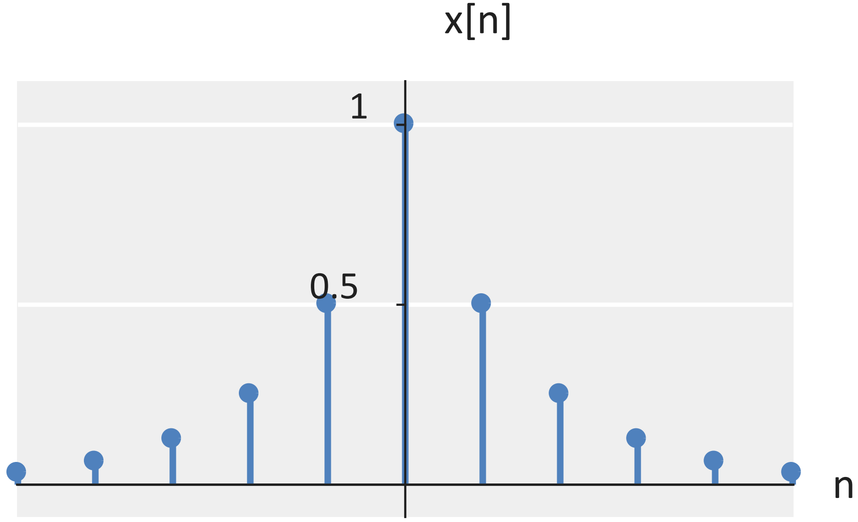
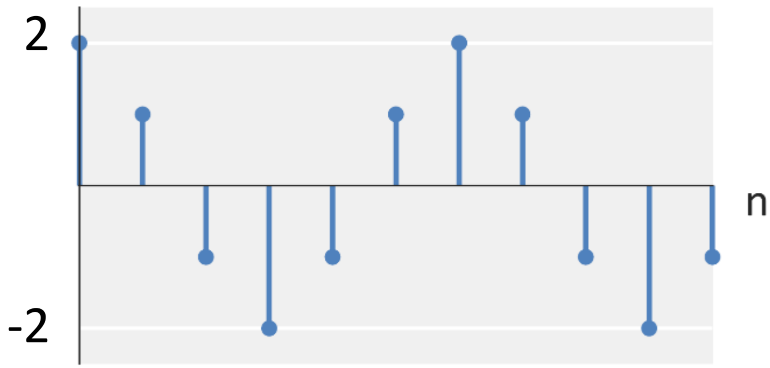
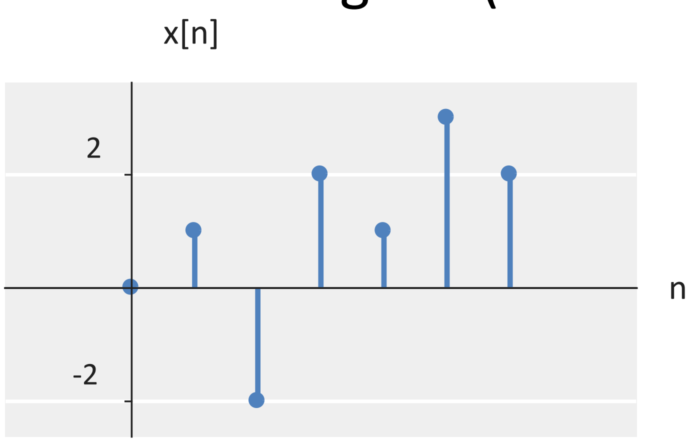


- **Signals: time signals (1D signals)**
  - Discrete-time signals (infinite extent)



$$x[n]$$

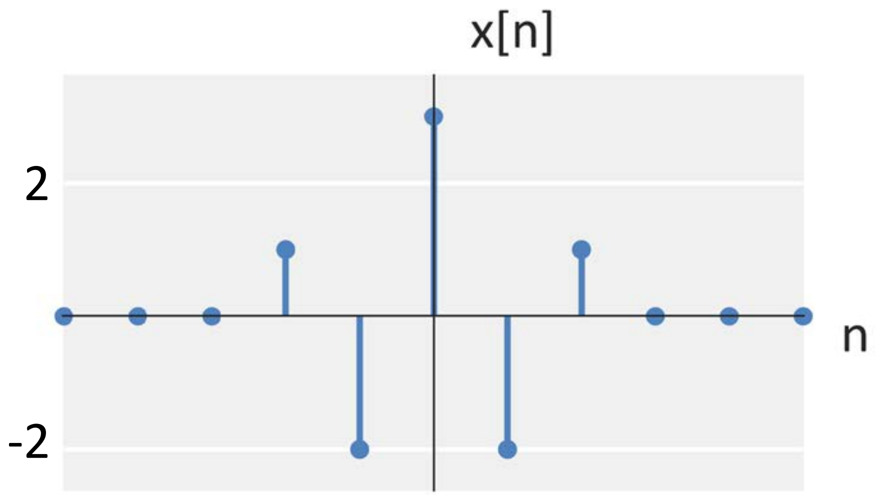
  - Discrete-time signals (finite extent)

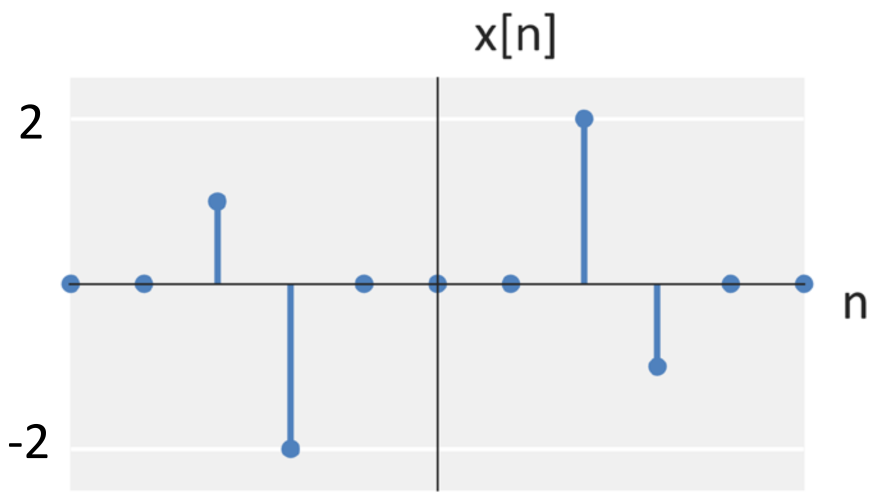


$$x[n] \text{ or } \boldsymbol{x} = \begin{bmatrix} 0 \\ 1 \\ -2 \\ 2 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

Signal Properties

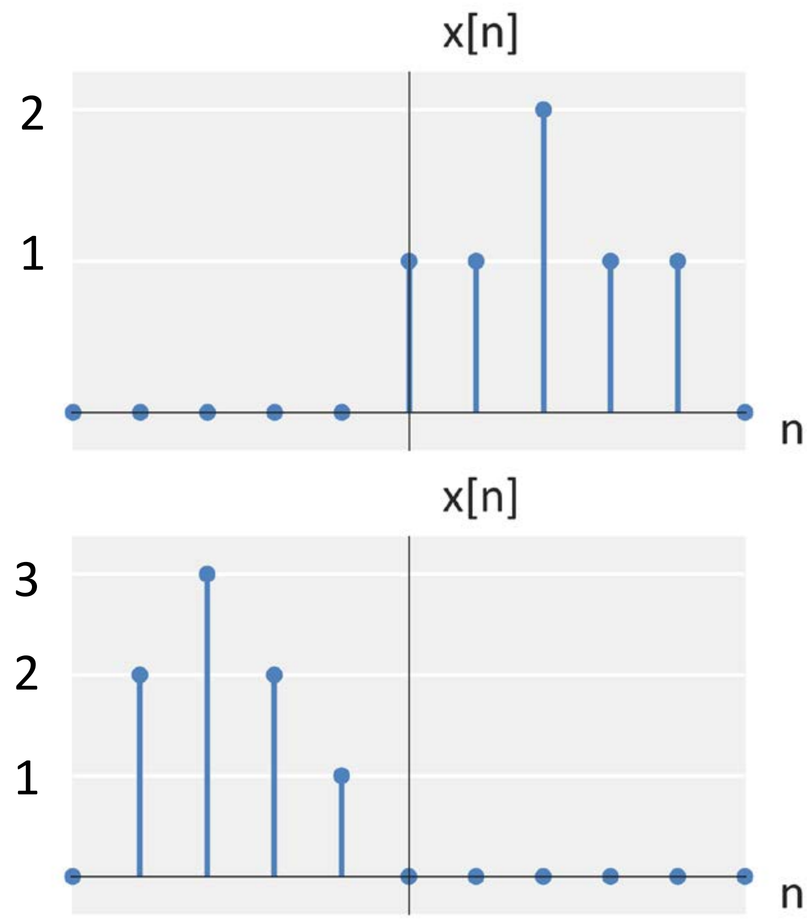
- **Even / Odd**
  - Even signal
    - $x[n] = x[-n]$
  - Odd signal
    - $x[n] = -x[-n]$





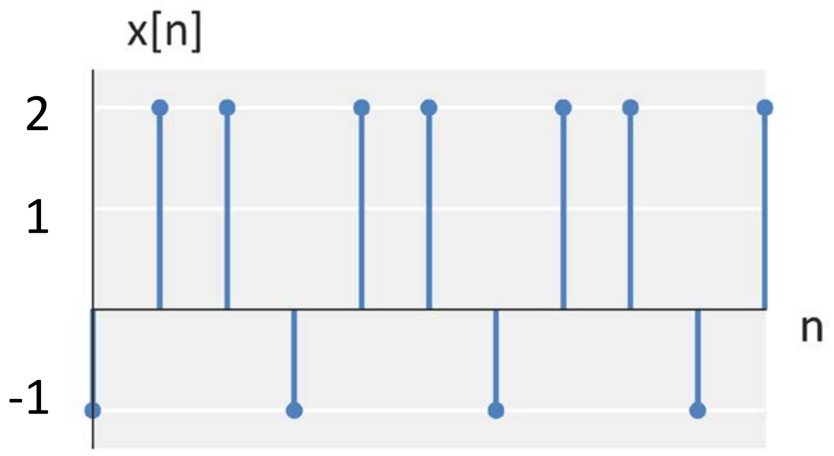
Signal Properties

- **Causal / Acausal**
  - Causal signal
    - ◊ $x[n] = 0, n < 0$
  - Anti-causal signal
    - ◊ $x[n] = 0, n \geq 0$
  - Acausal signal
    - ◊ Has non-zero components before and after $n = 0$



Signal Properties

- **Periodic / Aperiodic**
  - Periodic signal
    - $x[n] = x[n + aN]$ where N and a are integers
    - N is the period of the signal



Signal Properties

- **Problem: Consider the signal**

$$x[n] = 2^{-|n|}$$

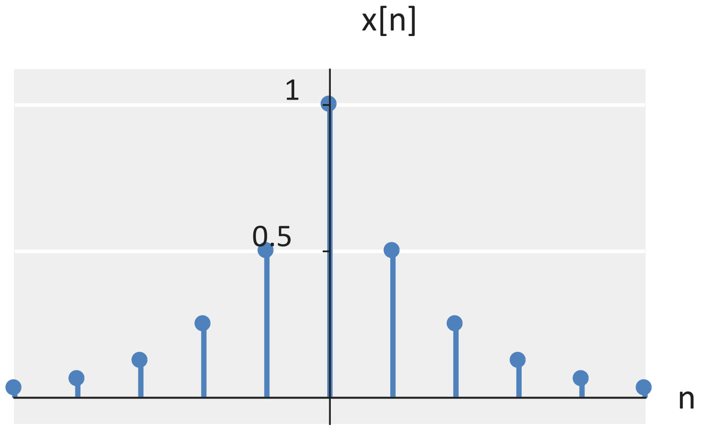
  - Sketch the signal.
  - Is the signal even, odd, or neither?
  - Is the signal causal, anti-causal, or acausal?
  - Is the signal periodic or aperiodic?

Signal Properties

- **Problem: Consider the signal**

$$x[n] = 2^{-|n|}$$

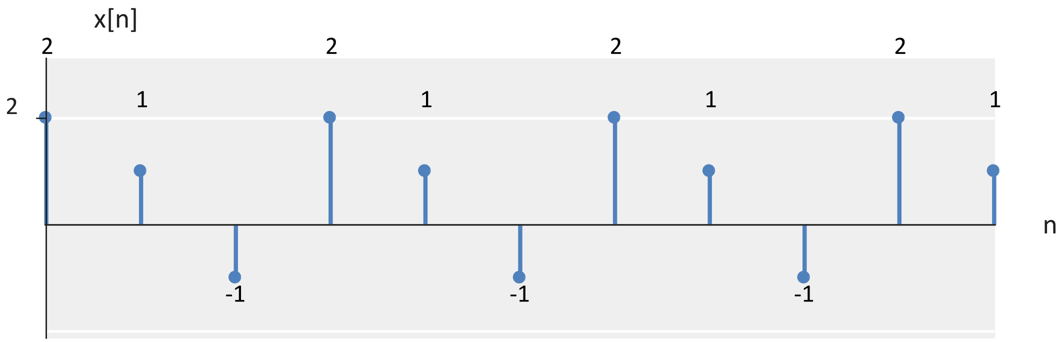
  - Sketch the signal.



  - Is the signal even, odd, or neither? **Even.**
  - Is the signal causal, anti-causal, or acausal? **Acausal.**
  - Is the signal periodic or aperiodic? **Aperiodic.**

Signal Properties

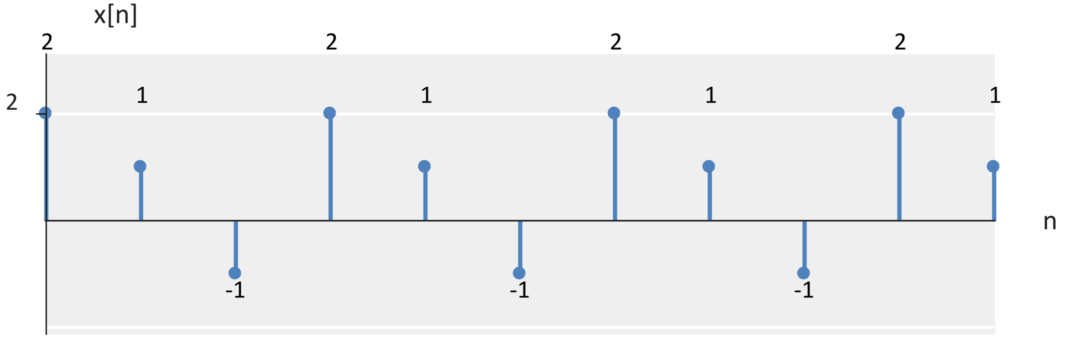
- **Problem: Consider the signal (assume pattern continues for $-\infty < n < \infty$)**



  - Is the signal even, odd, or neither?
  - Is the signal causal, anti-causal, or acausal?
  - Is the signal periodic or aperiodic?

Signal Properties

- **Problem: Consider the signal** **(assume pattern continues for $-\infty < n < \infty$)**



  - Is the signal even, odd, or neither? **Neither.**
  - Is the signal causal, anti-causal, or acausal? **Acausal.**
  - Is the signal periodic or aperiodic? **Periodic.**

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- **Periodicity**
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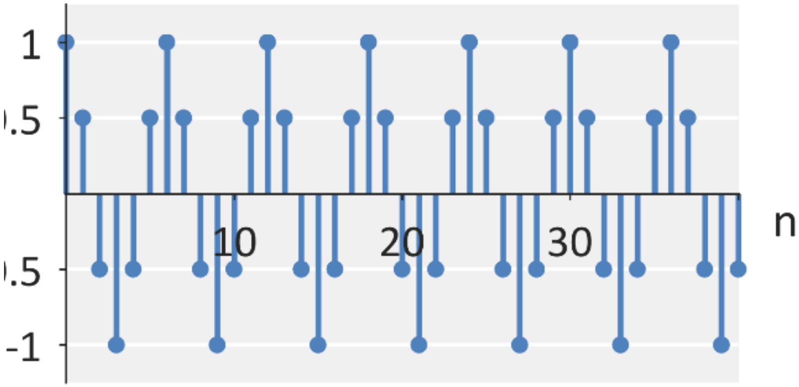
Periodicity

- **Discrete-time Periodicity**
  - A discrete-time signal is periodic if
    - ◊ Its period is an integer N
  - A discrete-time signal is periodic if and only if
    - ◊ Its frequency $f_0 = k/N$ is a rational number
  - Two discrete-time sinusoids are identical if
    - ◊ The angular frequencies are separated by integer multiples of 2π
  - The highest frequency discrete-time sinusoid has
    - ◊ An angular frequency $\omega_0 = \pi$

Periodicity

■ Computing Periodicity

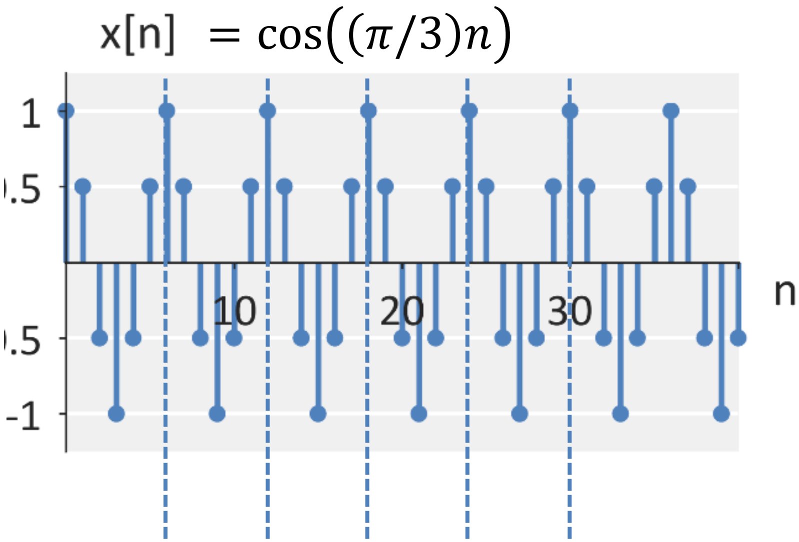
x[n] $= \cos\left((\pi/3)n\right)$



- **Question:** Is this signal periodic?

Periodicity

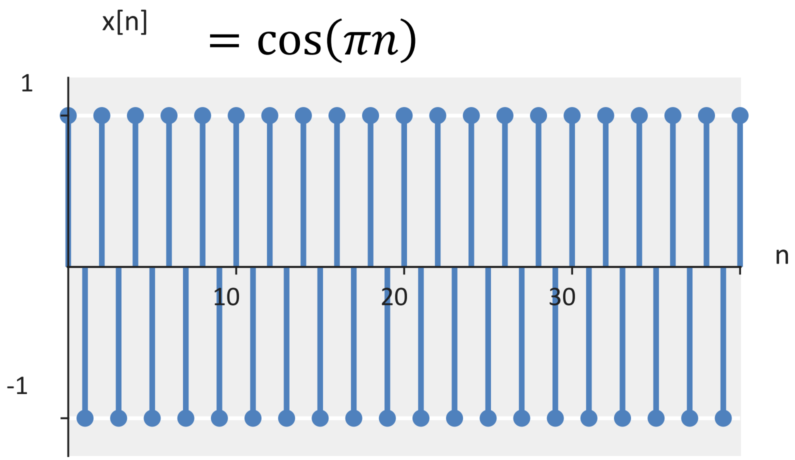
- **Computing Periodicity**



- **Question:** Is this signal periodic? **Yes.**
  - Integer Period : $N = 6$
  - Rational Frequency : $f = 1/6$

Periodicity

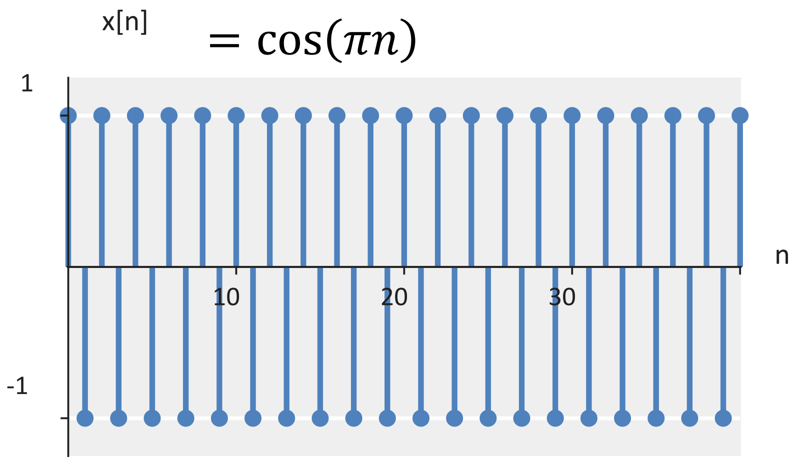
- **Computing Periodicity**



  - **Question:** Is this signal periodic?

Periodicity

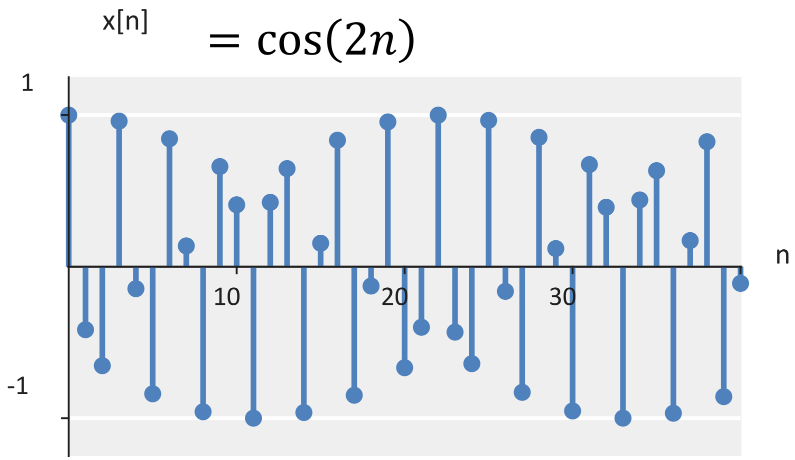
Computing Periodicity



- **Question:** Is this signal periodic? **Yes.**
  - ◊ Integer Period : $N = 1$
  - ◊ Rational Frequency : $f = 1$

Periodicity

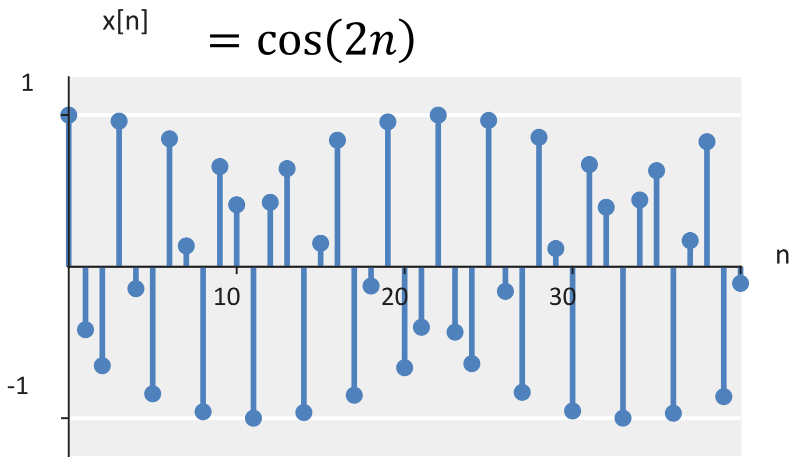
- **Computing Periodicity**



  - **Question:** Is this signal periodic?

Periodicity

Computing Periodicity



- **Question:** Is this signal periodic? **No.**
  - Integer Period : $N =$ does not exist
  - Rational Frequency : $f =$ does not exist

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Signal Size

- **Signal Energy**
  - Signal energy (infinite length signal)

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

  - Signal energy (finite length signal)

$$E_x = \sum_{n=0}^{N-1} |x[n]|^2$$

Signal Size

- **Signal Energy**
  - Signal energy (infinite length signal)

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

  - Signal energy (finite length signal) (ℓ_2 vector norm)

$$E_x = \sum_{n=0}^{N-1} |x[n]|^2 = \|\boldsymbol{x}\|_2$$

Signal Size

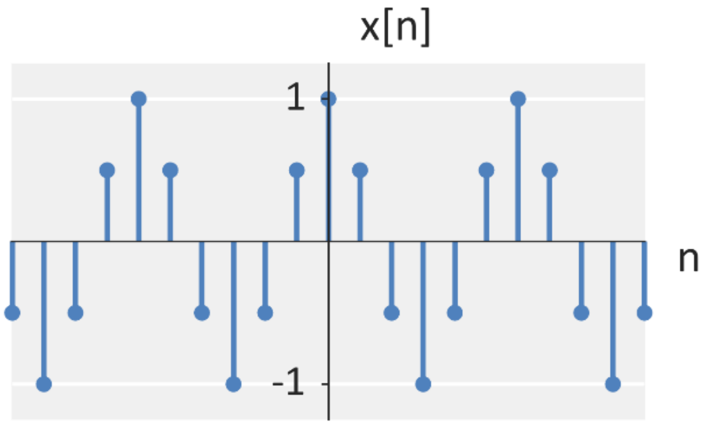
- **Signal Power**
  - Signal power (infinite length signal)

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

  - For periodic signals, this simplifies to:

$$P_x = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

where N is a period of the signal.



Signal Size

- **Types of signals**

	Energy	Power
Energy Signal	Finite $0 < E_x < \infty$	0
Power Signal	∞	Finite $0 < P_x < \infty$

Signal Size

- **Question: Are there other measures of signal "size"?**

Signal Size

- **Question: Are there other measures of signal "size"?**
  - Yes!

Signal Size

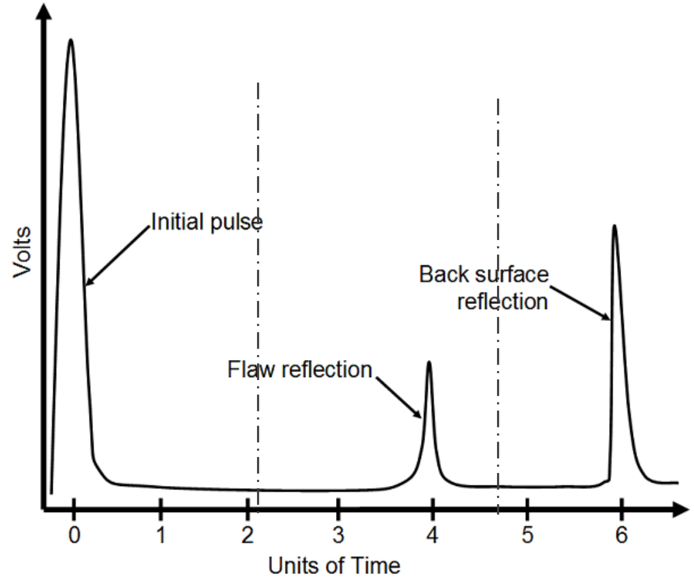
- **Applications**
  - When is energy (or signal size, in general) important?

Signal Size

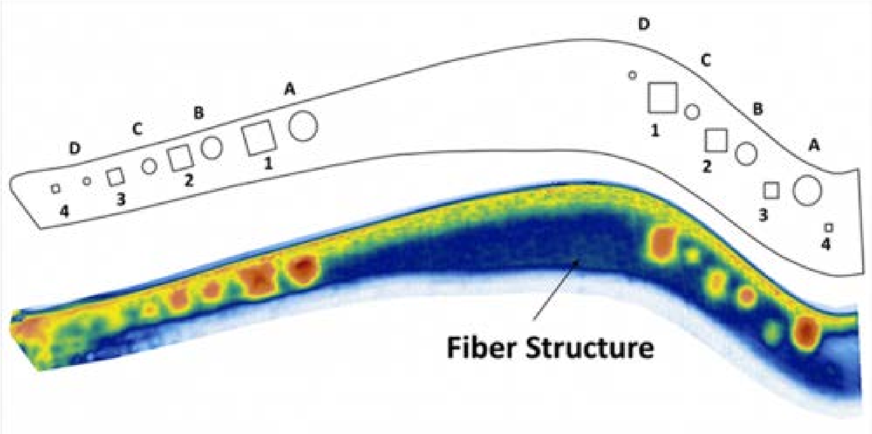
- **Applications**
  - When is energy (or signal size, in general) important?

- **Example: Ultrasonic C-Scan Inspection**

Ultrasonic signal from one location



http://www.ni.com/white-paper/3368/en/



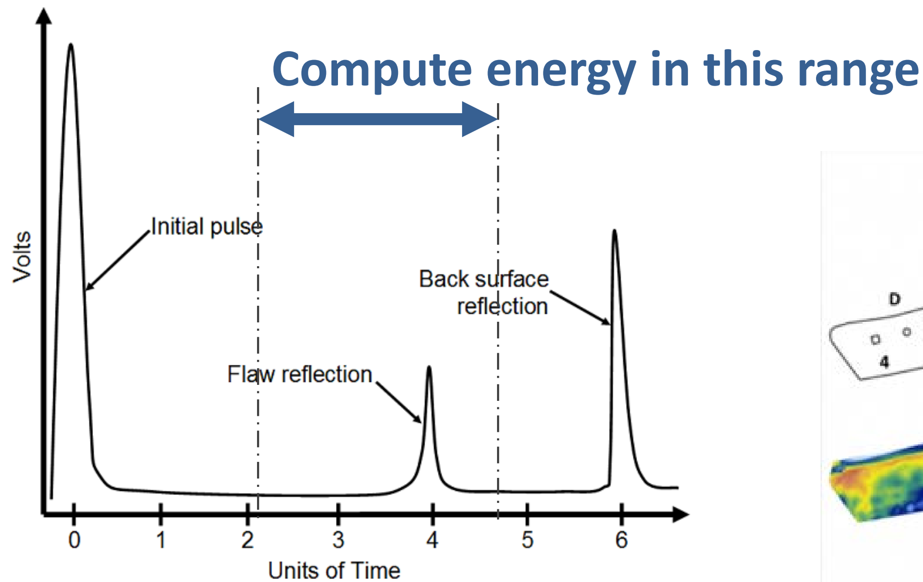
Source: https://www.tecscan.ca/products/ultrasonic-immersion-scanners/scan3d-high-precision-immersion-scanners/

Signal Size

- **Applications**
  - When is energy (or signal size, in general) important?

- **Example: Ultrasonic C-Scan Inspection**

Ultrasonic signal from one location



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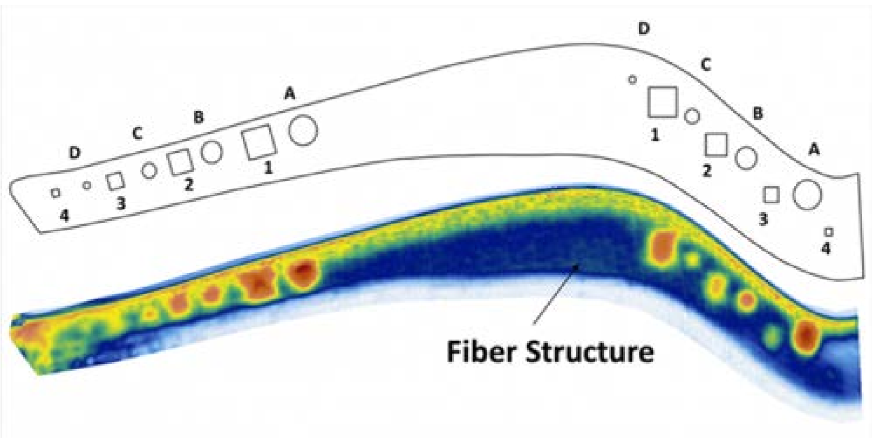

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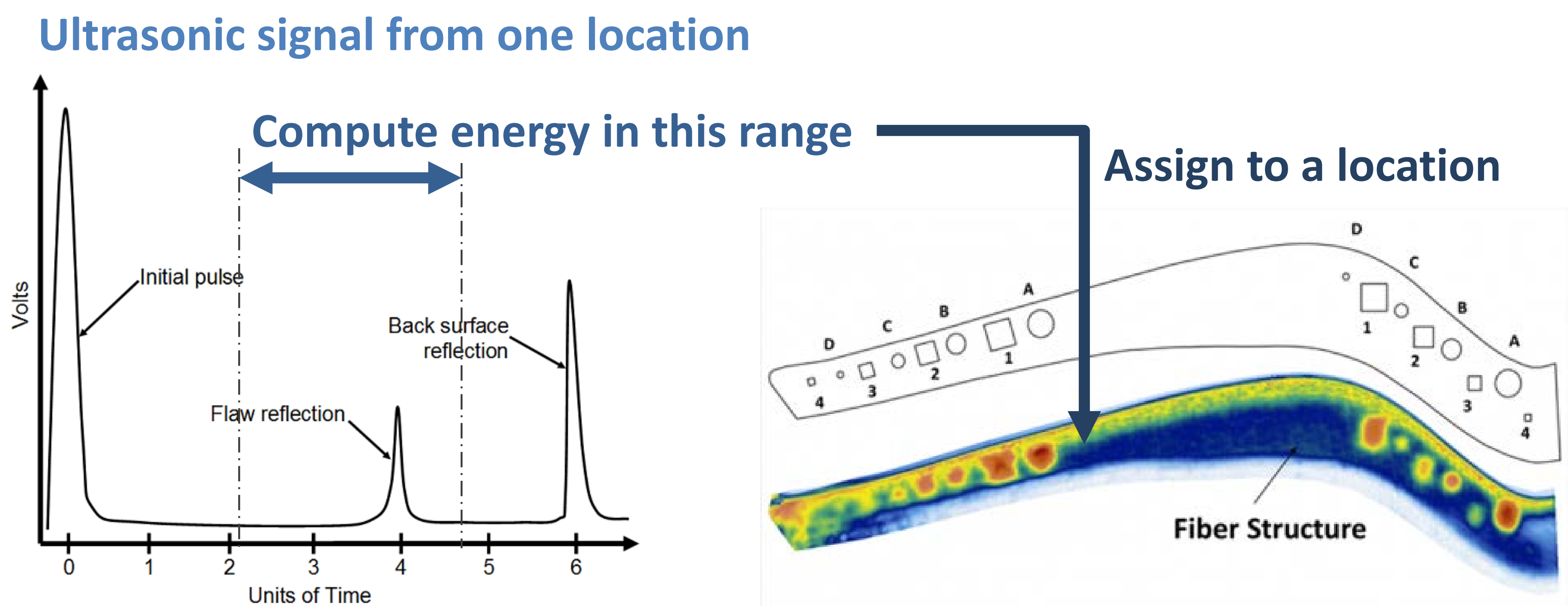
Signal Size

- **Applications**
  - When is energy (or signal size, in general) important?

- **Example: Ultrasonic C-Scan Inspection**



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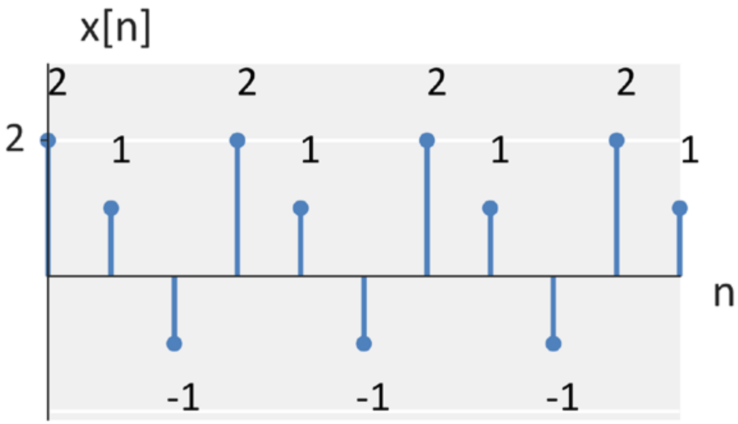
Signal Size

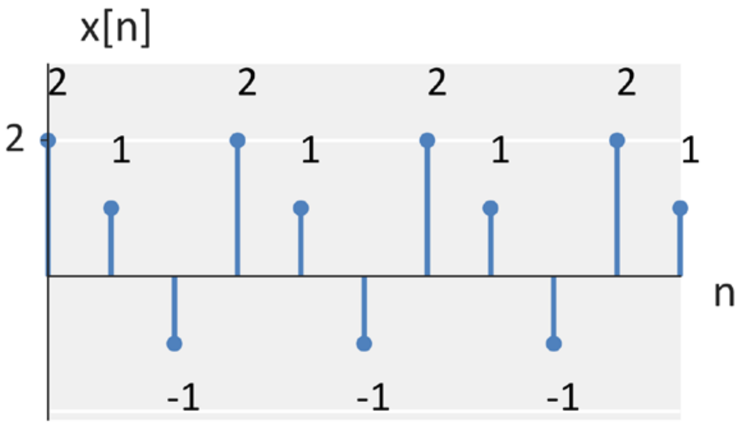
- **Consider the signal**



  - Compute the energy of the signal.
  - Compute the power of the signal.

Signal Size

- **Consider the signal**



- Compute the energy of the signal. $E_x = \infty$
- Compute the power of the signal.

$$P_x = \frac{1}{3}[2^2 + 1^1 + (-1)^2] = 2$$

Signal Size

- **Consider the signal**

$$x[n] = 2^{-|n|}$$

  - Compute the energy of the signal.
  - Compute the power of the signal.

Signal Size

- **Consider the signal**

$$x[n] = 2^{-|n|}$$

  - Compute the energy of the signal.
  - Compute the power of the signal.

- **Solution:**

$$E_x = \sum_{n=-\infty}^{\infty} 2^{-|n|} = 2\sum_{n=1}^{\infty} 2^{-n} + \sum_{n=0}^{0} 2^{-n}$$

$$= 2\sum_{n=1}^{\infty} 2^{-n} + 1 = 2\sum_{n=1}^{\infty}(1/2)^{n} + 1 =$$

$$= 2\sum_{n=1}^{\infty}(1/2)^{n} + 1 = 2\left[\frac{1}{1-1/2} - 1\right] + 1 = 3$$

Geometric series

Signal Size

- **Consider the signal**

$$x[n] = 2^{-|n|}$$

  - Compute the energy of the signal.
  - Compute the power of the signal.

- **Solution:**

$$P_x = 0$$

Signal Size

- **Consider the signal**
  - Assume
    - E_x is the energy of $x[n]$
    - E_y is the energy of y$[n]$
  - Show that the energy of $x[n] + y[n]$ is not $E_x + E_y$.

Signal Size

- **Consider the signal**
  - Assume
    - E_x is the energy of $x[n]$
    - E_y is the energy of y$[n]$
  - Show that the energy of $x[n] + y[n]$ is not $E_x + E_y$.

- **Solution**

The energy of $x[n] + y[n]$ is

$$\sum_{n=-\infty}^{\infty} |x[n] + y[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2 + |y[n]|^2 + 2x[n]y[n]$$

$$\neq E_x + E_y = \sum_{n=-\infty}^{\infty} |x[n]|^2 + |y[n]|^2$$

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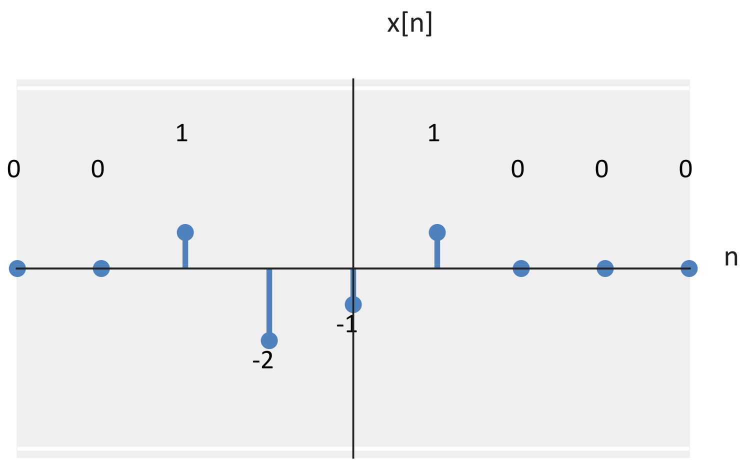
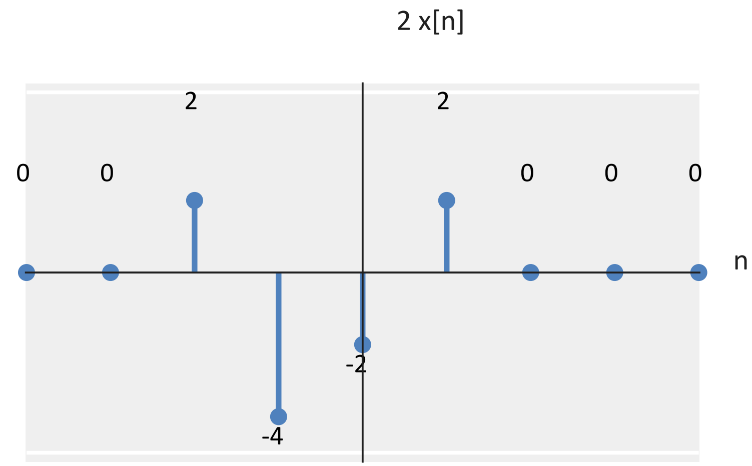
Signal Operations

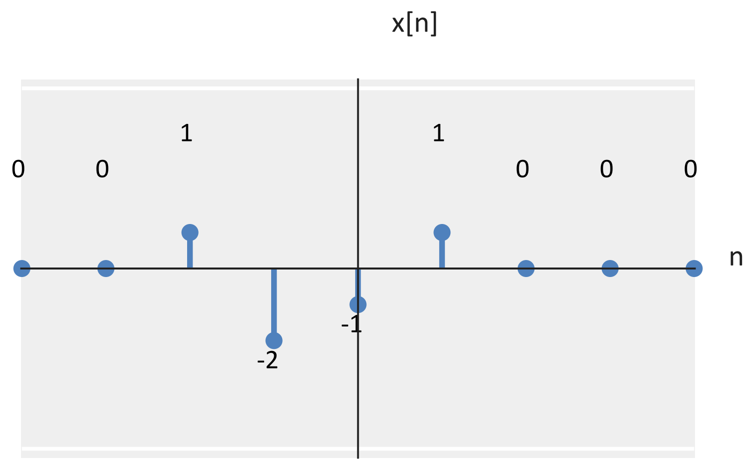
- **Amplitude Change**
  - Amplify signal by A
    - $A\,x[n]$

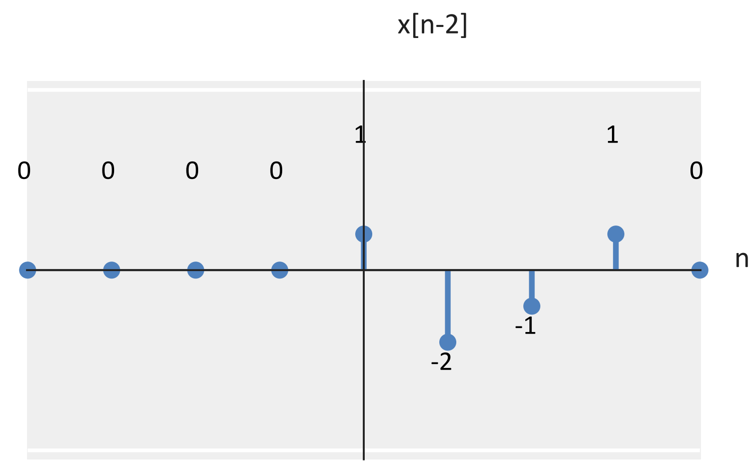




Signal Operations

- **Time Shifting**
  - Shift to the right by N_0
    - $x[n - N_0]$





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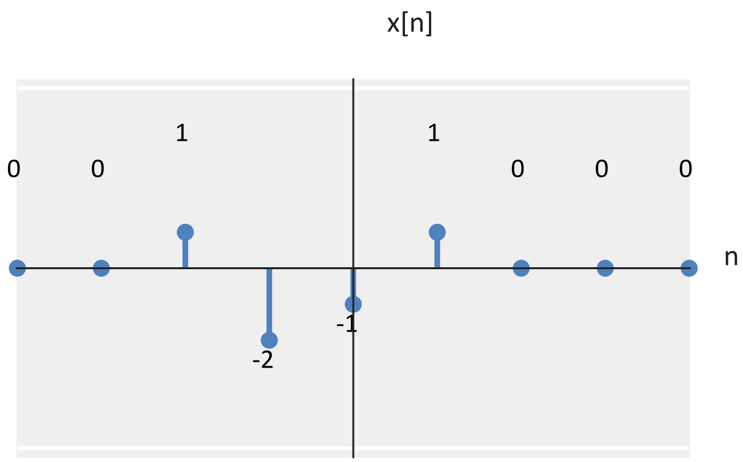
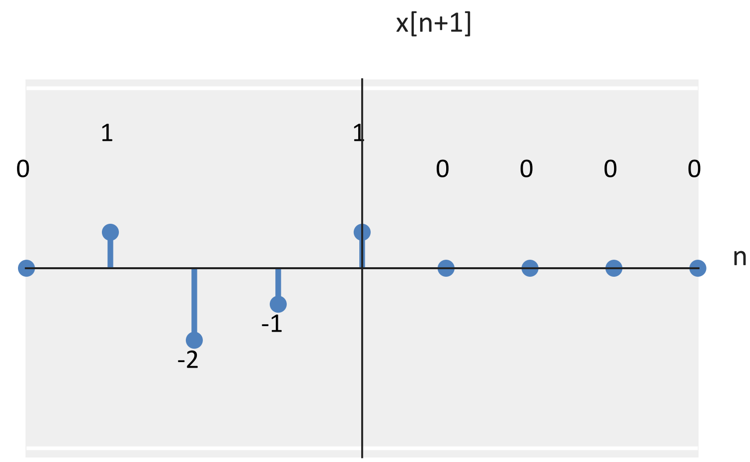
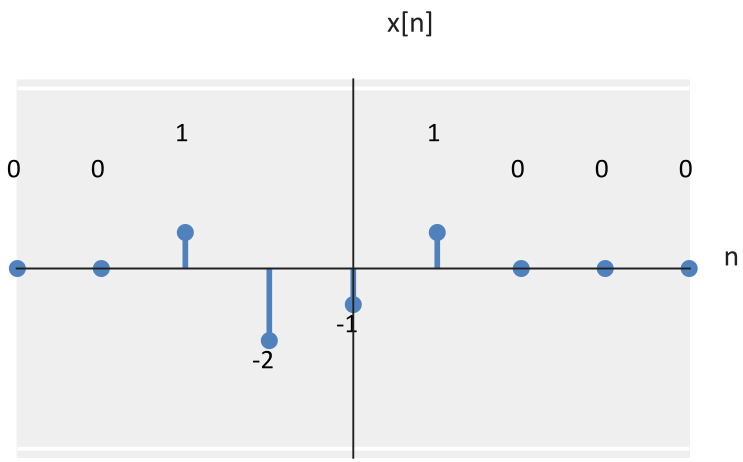
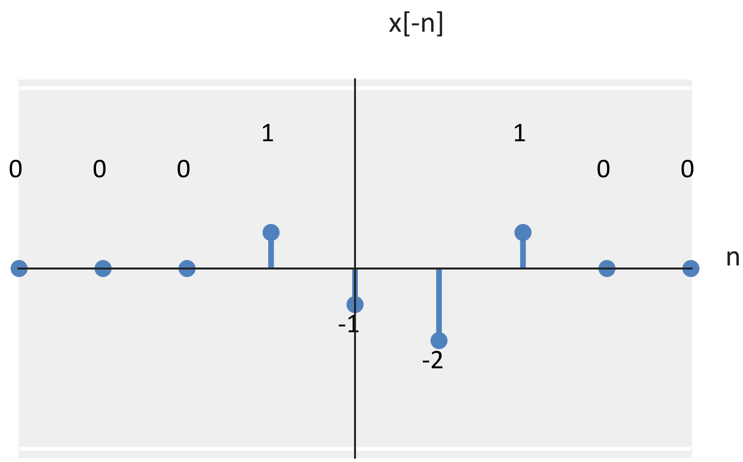




Signal Operations

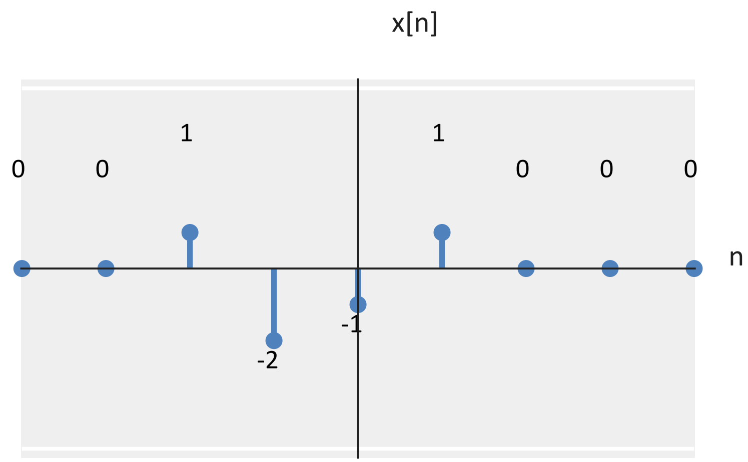
- **Time Reversal**
  - Flip the signal around n = 0
    - $x[-n]$

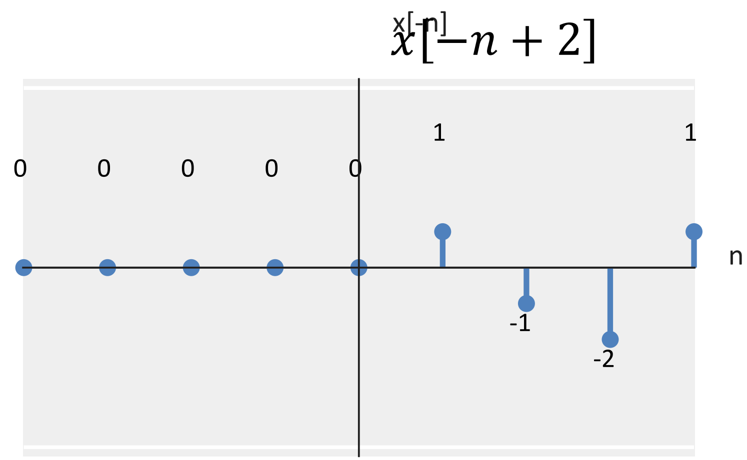




Signal Operations

- **Shift & Time Reversal**
  - Shift first, then flip around the axis
    - $x[-n+2]$





Signal Operations

- **Operations on two signals**
  - Sum

$$z[n] = x[n] + y[n]$$

$$z[n] = \sum_{k=0}^{K-1} x_k[n]$$

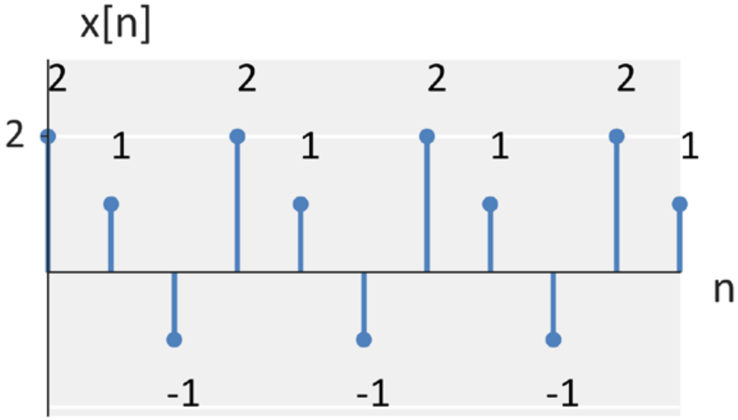
  - Multiplication

$$z[n] = x[n]y[n]$$

$$z[n] = \prod_{k=0}^{K} x_k[n]$$

Signal Operations

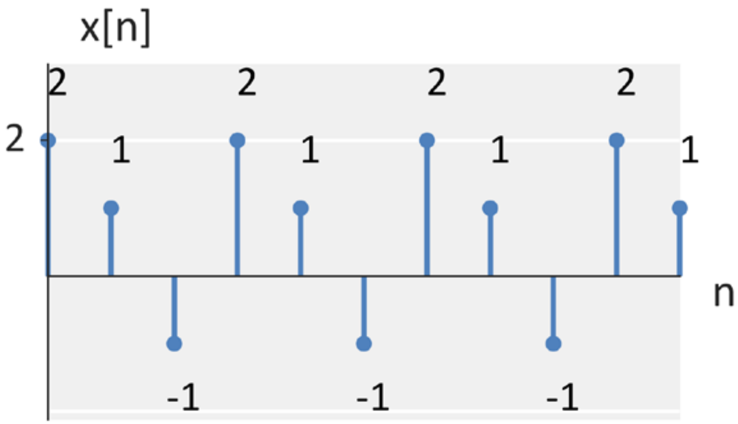
- **Problem: Consider the signal** **(assume the pattern continues for $-\infty < n < \infty$)**



  - Sketch $x[n + 4]$

Signal Operations

- **Problem: Consider the signal** **(assume the pattern continues for $-\infty < n < \infty$)**



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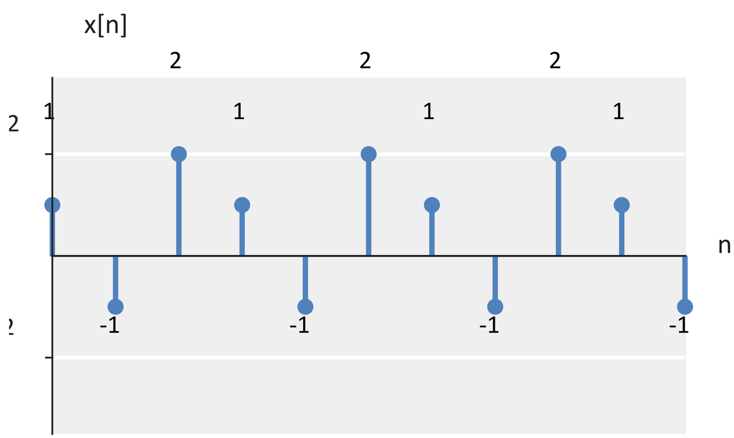

Signal Operations

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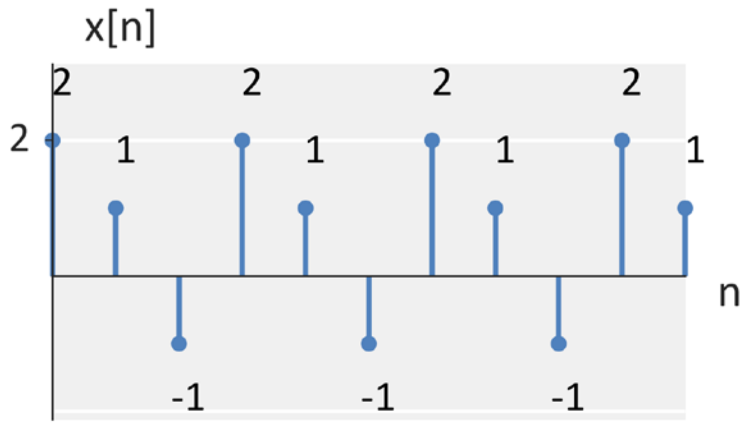
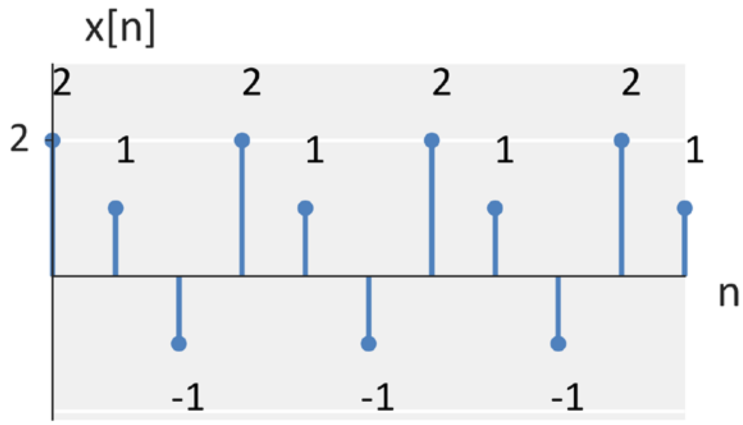
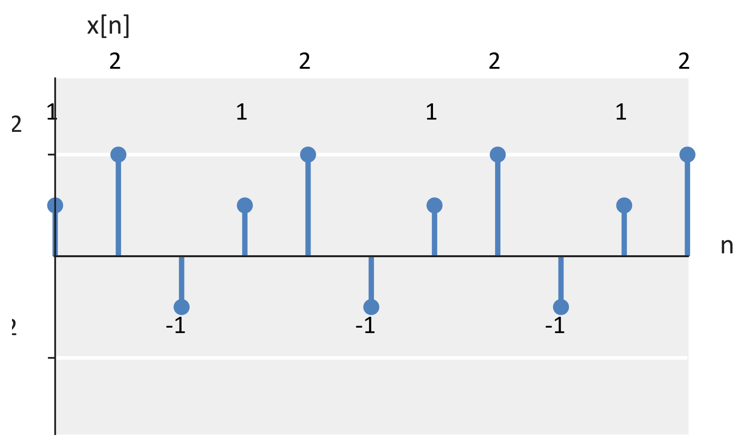


  - Sketch $x[4 - n]$

- **Problem: Consider the signal** **(assume the pattern continues for $-\infty < n < \infty$)**



  - Sketch $x[4-n]$



Signal Operations

- **Example Problems**
  - Assume $x[n]$ has an energy of $E_x = 10$.
  - Compute the energy of $Ax[n - N_1]$

Signal Operations

- **Example Problems**
  - Assume $x[n]$ has an energy of $E_x = 10$.
  - Compute the energy of $Ax[n - N_1]$

- **Solution:**

$$\mathrm{E}'_x = \sum_{n=-\infty}^{\infty} |\mathrm{A}x[n - N_1]|^2$$

$$= \sum_{n=-\infty}^{\infty} |\mathrm{A}|^2 |x[n - N_1]|^2$$

$$= |\mathrm{A}|^2 \sum_{n=-\infty}^{\infty} |x[n - N_1]|^2$$

$$= |\mathrm{A}|^2 \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= 10|\mathrm{A}|^2$$

Lecture 2: Continuous -Time and Discrete -Time Signals

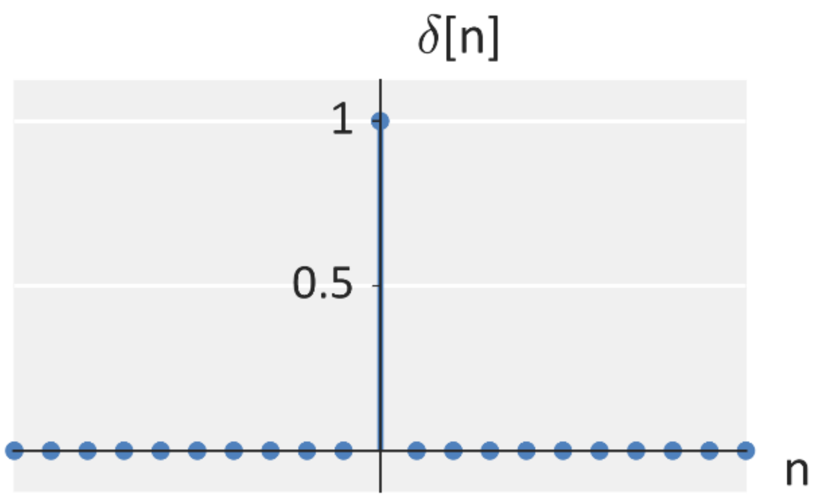
Foundations of Digital Signal Processing

Outline

- Signal Properties
- Periodicity
- Measures of signal “size”
- Signal Operations
- **Special Signals: Impulses and Steps and Exponentials**
- System Properties

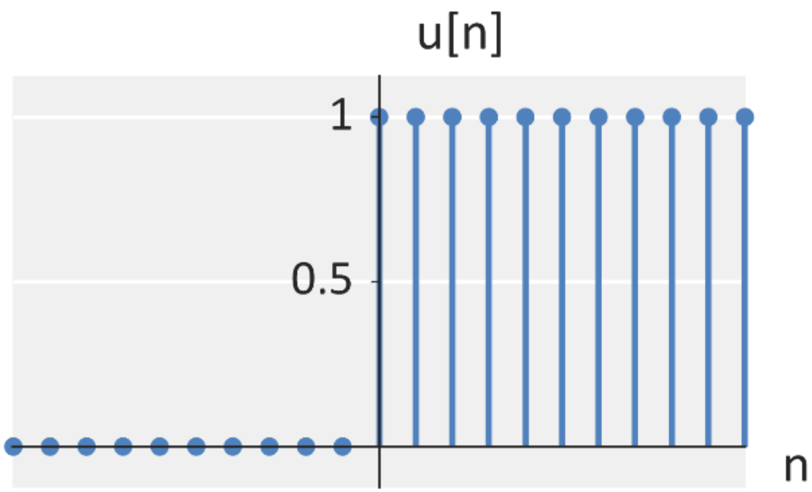
Special Signals

- **Impulse Signals**
  - $\delta[n] = \begin{cases} 1 & \text{when} \quad n = 0 \\ 0 & \text{when} \quad n \neq 0 \end{cases}$



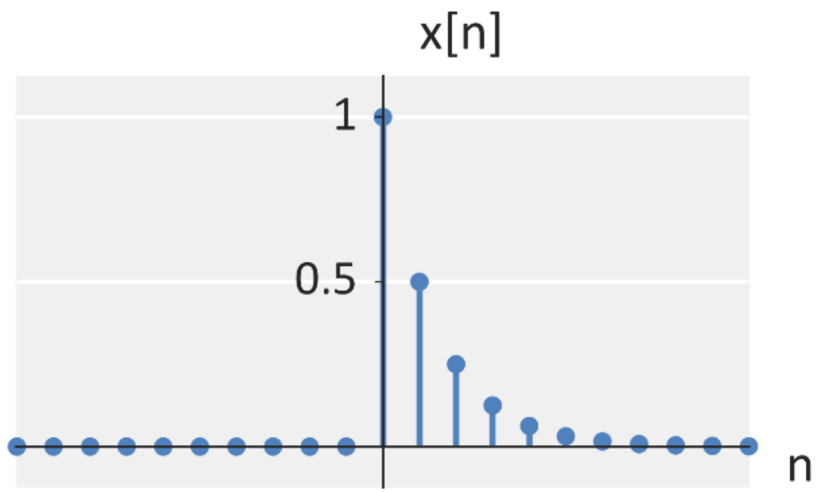
Special Signals

- **Heaviside step functions**
  - $u[n] = \begin{cases} 1 & \text{when} \quad n \geq 0 \\ 0 & \text{when} \quad n < 0 \end{cases}$



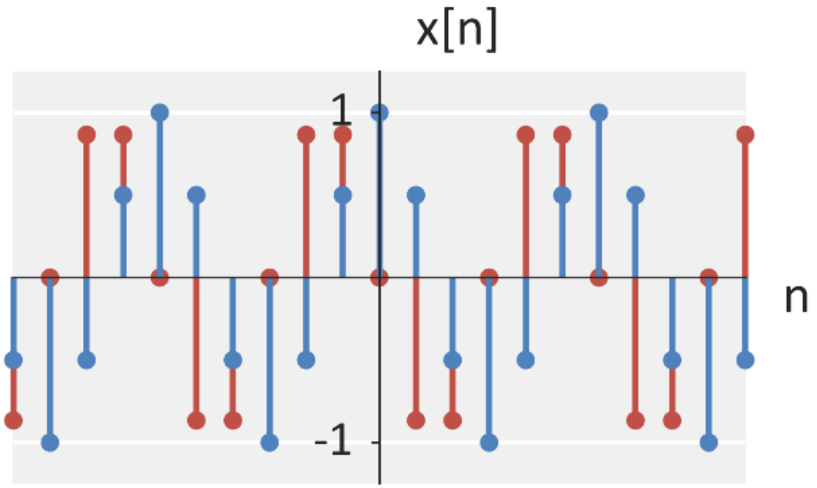
Special Signals

- **Stepped Exponentials**
  - $x[n] = e^{-an}u[n]$
  - $x[n] = r^{-n}u[n], \qquad r > 1$



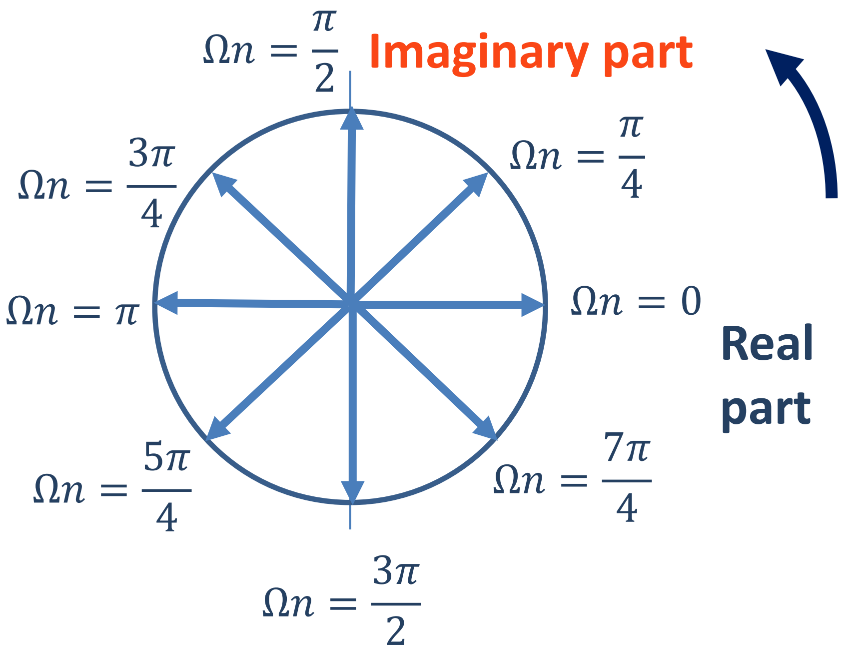
Special Signals

- **Complex Exponentials**
  - $x[n] = e^{-(\mathrm{j}\Omega+\sigma)n}$
  - $x[n] = (r + j\gamma)^{-n}$



Special Signals

- **Complex Exponentials**
  - $x[n] = e^{-j\Omega n}$
  - $= \cos(\Omega n) + j\sin(\Omega n)$
  - $=$ real $+ j$ imag



Special Signals

- **Example Problem**

$$\text{Assume } x[n] = \cos\big((\pi/3)n\big)$$

$$\text{Let } y[n] = x[n]\delta[n]$$

  - Simplify $y[n]$

Special Signals

Example Problem

$$\text{Assume } x[n] = \cos\big((\pi/3)n\big)$$

$$\text{Let } y[n] = x[n]\delta[n]$$

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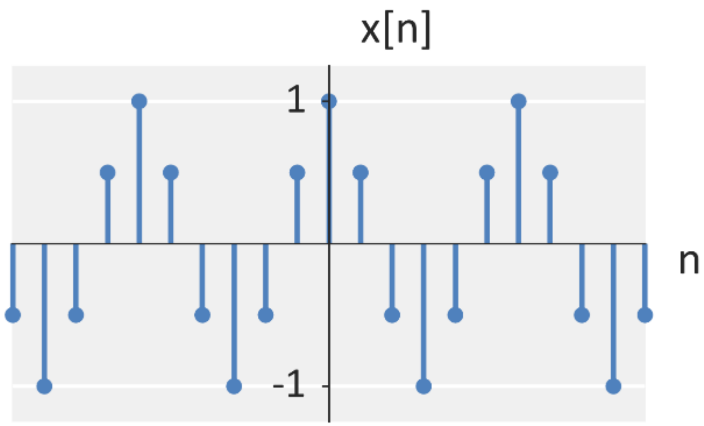

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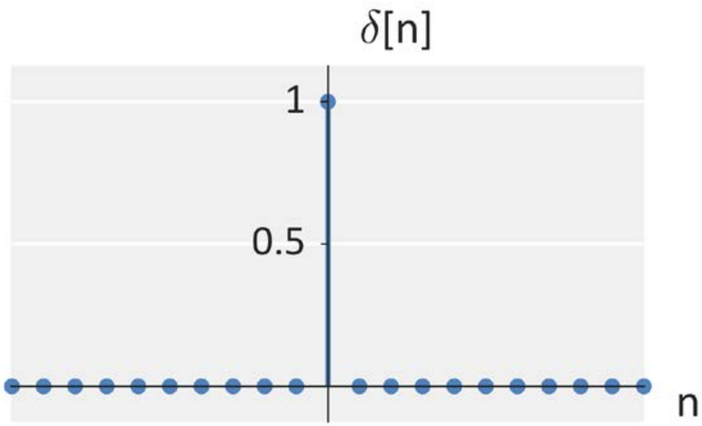

Special Signals

- **Example Problem**

$$\text{Assume } x[n] = \cos\big((\pi/3)n\big)$$

$$\text{Let } y[n] = x[n]\delta[n]$$

  - Simplify $y[n]$
  - $y[n] = x[n]\delta[n]$
  - $y[n] = x[0]\delta[n] = \cos\big((\pi/3)0\big) = \delta[n]$

Special Signals

- **Example Problem**

$$\text{Assume } x[n] = \cos\big((\pi/3)n\big)$$

$$\text{Let } y[n] = x[n]\delta[n-3]$$

  - Simplify $y[n]$

Special Signals

Example Problem

$$\text{Assume } x[n] = \cos\big((\pi/3)n\big)$$

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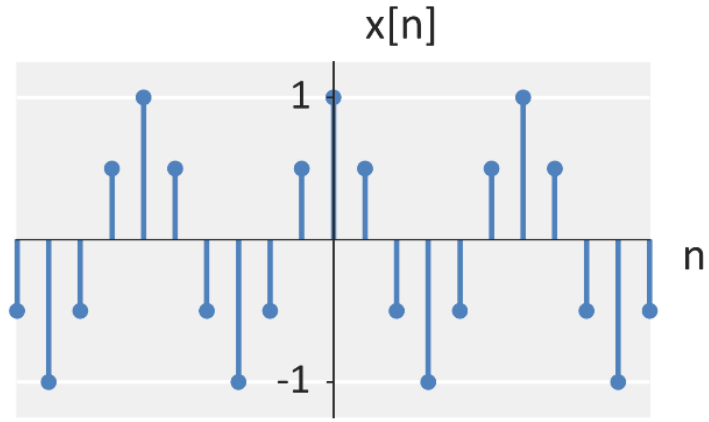

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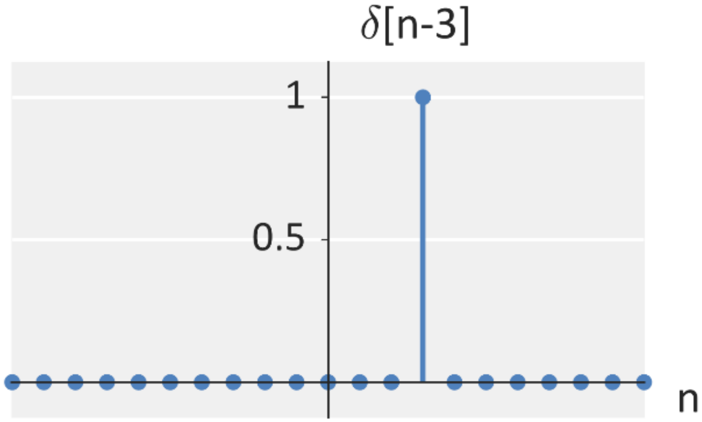

Special Signals

- **Example Problem**

$$\text{Assume } x[n] = \cos\big((\pi/3)n\big)$$

$$\text{Let } y[n] = x[n]\delta[n-3]$$

  - Simplify $y[n]$ (i.e., remove $\delta[n]$).
  - $y[n] = x[n]\delta[n]$
  - $y[n] = x[5] = \cos\big((\pi/3)3\big) = -\delta[n-3]$

Special Signals

- **Example Problem**
  - Show that

$$x[n] = \sum_{m=-\infty}^{\infty} x[n]\delta[n-m]$$

Special Signals

- **Example Problem**
  - Show that

$$x[n] = \sum_{m=-\infty}^{\infty} x[n]\delta[n-m]$$

- **Solutions**

$$x[n] = \sum_{m=-\infty}^{\infty} x[n]\delta[n-m]$$

$$= \sum_{m=-\infty}^{\infty} x[m]\delta[n-m] = x[m] \sum_{m=-\infty}^{\infty} \delta[n-m]$$

$$= x[n]$$

Lecture 2: Continuous -Time and Discrete -Time Signals

Foundations of Digital Signal Processing

Outline

- Signal Properties
- Periodicity
- Measures of signal "size"
- Signal Operations
- Special Signals: Impulses and Steps and Exponentials
- **System Properties**

System Properties

- **Input-output system model**
  - A generic system is define by

$$y[n] = H\{x[n]\}$$

Output

System

Input

System Properties

- **Linear / Nonlinear**
  - A system is linear if

$$y_1[n] = H\{x_1[n]\} \quad , \quad y_2[n] = H\{x_2[n]\}$$

then

$$ay_1[n] + by_2[n] = H\{ax_1[n] + bx_2[n]\}$$

System Properties

- **Linear / Nonlinear**
  - A system is linear if

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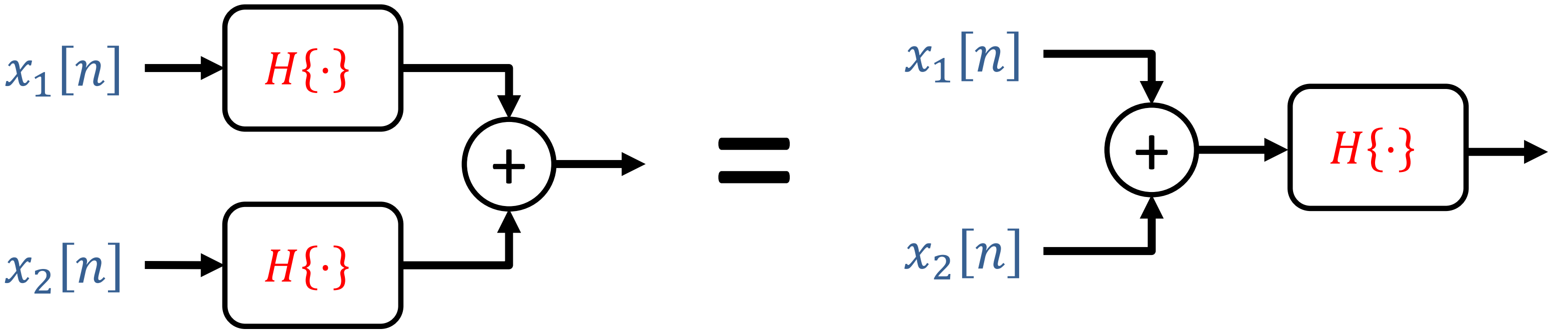

System Properties

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  - **Question:** Why do I care about linearity?

System Properties

- **Linear / Nonlinear**
  - A system is linear if

$$y_1[n] = H\{x_1[n]\} \quad , \quad y_2[n] = H\{x_2[n]\}$$

then

$$ay_1[n] + by_2[n] = H\{ax_1[n] + bx_2[n]\}$$

  - **Question:** What is an example of a non-linear system?

System Properties

- **Time invariant / time varying**
  - A system is time-invariant if

$$y[n] = H\{x[n]\}$$

then

$$y[n+N] = H\{x[n+N]\}$$

System Properties

- **Time invariant / time varying**
  - A system is time-invariant if

$$y[n] = H\{x[n]\}$$

then

$$y[n+N] = H\{x[n+N]\}$$

  - **Question:** Why do I care about time-invariance?

System Properties

- **Time invariant / time varying**
  - A system is time-invariant if

$$y[n] = H\{x[n]\}$$

then

$$y[n+N] = H\{x[n+N]\}$$

  - **Question:** What is an example of a time varying system?

System Properties

- **Memoryless / with memory**
  - A system $H\{\cdot\}$ is memoryless if

$y[n]$ is a function of $x[n]$ at **only** time n

$y[n]$ is **not** a function of $x[n-N]$ for any $N>0$ (past values of $x[n-N]$)

$y[n]$ is **not** a function of $x[n+N]$ for any $N>0$ (future values of $x[n-N]$)

System Properties

- **Memoryless / with memory**
  - A system $H\{\cdot\}$ is memoryless if

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  - **Question:** What is an example of a system without memory?

System Properties

- **Memoryless / with memory**
  - A system $H\{\cdot\}$ has memory if

$y[n]$ is a function of $x[n]$ at **only** time n

$y[n]$ is **not** a function of $x[n-N]$ for any $N>0$ (past values of $x[n-N]$)

$y[n]$ is **not** a function of $x[n+N]$ for any $N>0$ (future values of $x[n-N]$)

  - **Question:** What is an example of a system with memory?

System Properties

- **Causal / non-causal**
  - A system $H\{\cdot\}$ is causal if

$y[n]$ **is only** a function of $x[n-N]$ for any $N \geq 0$
(past and present values of $x[n-N]$)

System Properties

- **Causal / non-causal**
  - A system $H\{\cdot\}$ is causal if

$y[n]$ **is only** a function of $x[n-N]$ for any $N \geq 0$
(past and present values of $x[n-N]$)

  - **Question:** What is an example of a non-causal system?

System Properties

- **Bounded-input, bounded-output (BIBO) stable / unstable**
  - A system $H\{\cdot\}$ is BIBO stable if

$$x[n] < \infty \rightarrow H\{x[n]\} < \infty$$

System Properties

- **Consider**
  - $y[n] = H\{x[n]\} = 5x[n] + 1$

  - Is this system linear?
  - Is this system time-invariant?
  - Is this system memoryless?
  - Is this system causal?

System Properties

- **Consider**
  - $y[n] = H\{x[n]\} = 5x[n] + 1$
  - Is this system linear?
    - ◊ $H\{ax_1[n] + bx_2[n]\} = 5(ax_1[n] + bx_2[n]) + 1$
    - ◊ $H\{ax_1[n]\} + H\{bx_2[n]\} = [5(ax_1[n]) + 1] + [5(ax_2[n]) + 1]$
    - ◊ **Non-linear (non-equal)**
  - Is this system time-invariant?
  - Is this system memoryless?
  - Is this system causal?

System Properties

- **Consider**
  - $y[n] = H\{x[n]\} = 5x[n] + 1$
  - Is this system linear?
  - Is this system time-invariant?
    - $H\{x[n+N]\} = 5(x[n+N]) + 1$
    - $y[n] = 5(ax[n+N]) + 1$
    - **Time-invariant (Equal!)**
  - Is this system memoryless?
  - Is this system causal?

System Properties

- **Consider**
  - $y[n] = H\{x[n]\} = 5x[n] + 1$

  - Is this system linear?
  - Is this system time-invariant?
  - Is this system memoryless?
    - ◊ $y[n]$ only depends on $x[n]$ at the current time
    - ◊ **Memoryless**
  - Is this system causal?

System Properties

- **Consider**
  - $y[n] = H\{x[n]\} = 5x[n] + 1$
  - Is this system linear?
  - Is this system time-invariant?
  - Is this system memoryless?
  - Is this system causal?
    - ◊ **Causal because we are memoryless**

Lecture 2: Continuous -Time and Discrete -Time Signals

Foundations of Digital Signal Processing

Outline

- Signal Properties
- Periodicity
- Measures of signal “size”
- Signal Operations
- Special Signals: Impulses and Steps and Exponentials
- System Properties
- **Convolution**

Convolution

- **Linear and Time-Invariant (LTI)**
  - If a system is both linear and time-invariant, then…

Linear: $ay_1[n] + by_2[n] = H\{ax_1[n] + bx_2[n]\}$

Time-invariant: $y[n + N] = H\{x[n + N]\}$

Convolution

- **Linear and Time-Invariant (LTI)**
  - If a system is both linear and time-invariant, then…

Linear: $ay_1[n] + by_2[n] = H\{ax_1[n] + bx_2[n]\}$

Time-invariant: $y[n+N] = H\{x[n+N]\}$

  - Recall

$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\delta[n-m]$$

$$H\{x[n]\} = H\left\{\sum_{m=-\infty}^{\infty} x[m]\delta[n-m]\right\}$$

Convolution

- **Linear and Time-Invariant (LTI)**
  - If a system is both linear and time-invariant, then…

Linear: $ay_1[n] + by_2[n] = H\{ax_1[n] + bx_2[n]\}$

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  - Recall

$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\delta[n-m]$$

$$H\{x[n]\} = \sum_{m=-\infty}^{\infty} x[m]\, H\{\delta[n-m]\}$$

Apply linearity

Convolution

- **Linear and Time-Invariant (LTI)**
  - If a system is both linear and time-invariant, then...

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  - Recall

$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\delta[n-m]$$

$$H\{x[n]\} = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

Apply time-invariance

Convolution

- **Linear and Time-Invariant (LTI)**
  - If a system is both linear and time-invariant, then…

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Time-invariant: $y[n+N] = H\{x[n+N]\}$

  - Recall

$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\delta[n-m]$$

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\,h[n-m]$$

Convolution!