EEE 5502 HW #10 Hudanyun Sheng

Question #1

I spent 15 hours.

Question #2

(a) $H_d(\omega) = \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{5\pi}{6} + 2\pi k) + \delta(\omega + \frac{5\pi}{6} + 2\pi k)$

$h_d[n] = \frac{1}{\pi}\cos(\frac{5\pi}{6}n)$ shift $\frac{N-1}{2} = 2.5 \Rightarrow h_d[n] = \frac{1}{\pi}\cos[\frac{5\pi}{6}(n-\frac{5}{2})]$

$w[n] = \frac{1}{2}[1-\cos(\frac{2\pi n}{N-1})] = \frac{1}{2}[1-\cos(\frac{2\pi n}{5})]$

$h[n] = h_d[n]\,w[n] = \begin{cases} \frac{1}{2\pi}\cos[\frac{5\pi}{6}(n-\frac{5}{2})](1-\cos(\frac{2\pi n}{5})), & \text{for } 0 \le n \le 6 \\ 0, & \text{otherwise.} \end{cases}$

(b) $\cancel{|H(\omega)| = \frac{1}{2\pi}\cdot\frac{1}{2\pi}\cdot\pi^2[\sum_{k=-\infty}^{\infty}\{\delta(\omega-\frac{5\pi}{6}-2\pi k)+\delta(\omega+\frac{5}{6}\pi-2\pi k)\}e^{j\frac{5}{2}n}}$

$H(\omega) = \frac{1}{2\pi}\mathcal{F}\{\cos\frac{5\pi}{6}(n-\frac{5}{2})]\circledast \mathcal{F}\{1-\cos(\frac{2\pi n}{5})\}$

$= \frac{1}{4\pi^2}\cdot\pi\sum_{k=-\infty}^{\infty}\{\delta(\omega-\frac{5}{6}\pi-2\pi k)+\delta(\omega+\frac{5}{6}\pi-2\pi k)\}e^{j\frac{5}{2}n}$

$\circledast\, 2\pi\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k) - \pi\sum_{k=-\infty}^{\infty}\{\delta(\omega-\frac{2}{5}\pi-2\pi k)+\delta(\omega+\frac{2}{5}\pi-2\pi k)\}$

$|H(\omega)| = \frac{1}{4}[2\sum_{k=-\infty}^{\infty}\delta(\omega-\frac{5}{6}\pi-2\pi k) - \sum_{k=-\infty}^{\infty}(\omega-\frac{37}{30}\pi-2\pi k) - \sum_{k=-\infty}^{\infty}\delta(\omega-\frac{13}{30}\pi-2\pi k)$

$+2\sum_{k=-\infty}^{\infty}\delta(\omega+\frac{5}{6}\pi-2\pi k) - \sum_{k=-\infty}^{\infty}\delta(\omega-\frac{13}{30}\pi-2\pi k) - \sum_{k=-\infty}^{\infty}\delta(\omega-\frac{37}{30}\pi-2\pi k)$

* The plot is attached in a separate PDF file using MATLAB.

(c) $G = [0\ 0\ 0\ 0\ 0\ 1\ 0]$

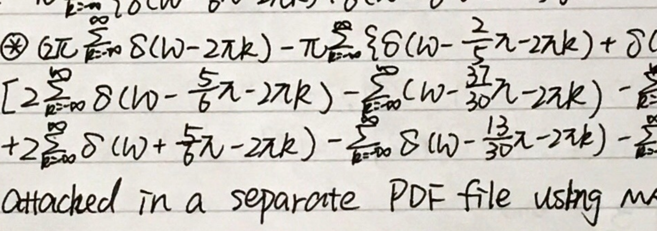
$g[n] = H[0] + 2\sum_{k=1}^{6} G[k]\cos(\frac{2\pi}{12}(n-5.5)k)$

$= 2\cdot\cos(\frac{5\pi}{6}(n-5.5))$

(d) $G(\omega) = 2\pi\sum_{k=-\infty}^{\infty}\{\delta(\omega-\frac{5}{6}\pi-2k\pi)+\delta(\omega+\frac{5}{6}\pi-2k\pi)\}e^{-j\omega 5.5}$

$|G(\omega)| = 2\pi|\sum_{k=-\infty}^{\infty}\delta(\omega-\frac{5}{6}\pi-2k\pi)+\delta(\omega+\frac{5}{6}\pi-2k\pi)|$

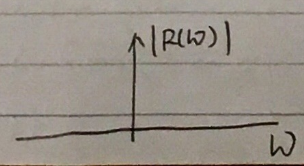
~~The plot is attached in the separate file.~~



(e) $R[k] = [0\ 0\ 0\ 0]$

$r[n] = 0$

(f) $|R(\omega)| = 0$



Question #3

(a) $H_d(z) = \frac{1}{1-(1/2)z^{-2}} = \frac{1}{2}\left(\frac{1}{1-\frac{1}{\sqrt{2}}z^{-1}} + \frac{1}{1+\frac{1}{\sqrt{2}}z^{-1}}\right)$

$h_d[n] = \left(\frac{1}{\sqrt{2}}\right)^{n+2} u[n] + \left(-\frac{1}{\sqrt{2}}\right)^{n+2} u[n]$

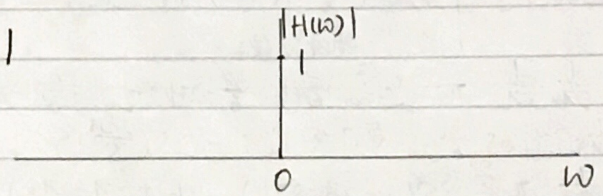
$w[n] = [1\ 1] = \delta[n] + \delta[n-1]$

$\cancel{h[n] = h_d[n]w[n] = \left(\frac{1}{\sqrt{2}}\right)^{n+2}\delta[n] + \left(\frac{1}{\sqrt{2}}\right)^{n+2}\delta[n-1] - \left(-\frac{1}{\sqrt{2}}\right)^{n+2}\delta[n] - \left(-\frac{1}{\sqrt{2}}\right)^{n+2}\delta[n-1]}.$

$\cancel{= \left(\left(\frac{1}{\sqrt{2}}\right)^{n+2} - \left(-\frac{1}{\sqrt{2}}\right)^{n+2}\right)\delta[n] + \left(\left(\frac{1}{\sqrt{2}}\right)^{n+2} - \left(-\frac{1}{\sqrt{2}}\right)^{n+2}\right)\delta[n-1]}$

$h[n] = h_d[n]w[n] = \left(\frac{1}{2} + \frac{1}{2}\right)\delta[n] \cancel{+ \left(\frac{1}{\sqrt{2}}\right)^3 - \left(\frac{1}{\sqrt{2}}\right)^3 \delta[} = \delta[n]$

(b) $|H(\omega)| = \cancel{\delta[n]}\ 1$



(c) $w[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$

$v[n] = \delta[n] + \left(\frac{1}{4} + \frac{1}{4}\right)\delta[n-2] = \delta[n] + \frac{1}{2}\delta[n-2]$.

(d) $V(\omega) = 1 + \frac{1}{2}e^{-2j\omega}$

$|V(\omega)| = \sqrt{\left(1+\frac{1}{2}\cos(-2\omega)\right)^2 + \left(\frac{1}{2}\sin(-2\omega)\right)^2}$

$= \sqrt{1 + \frac{1}{4} + \cos(2\omega)}$ The plot of $|V(\omega)|$ is shown in separate file.

(e) $H_d(\omega) = \frac{1}{1-(1/2)e^{-2j\omega}}$ $G[k] = [2\ \ 2]$

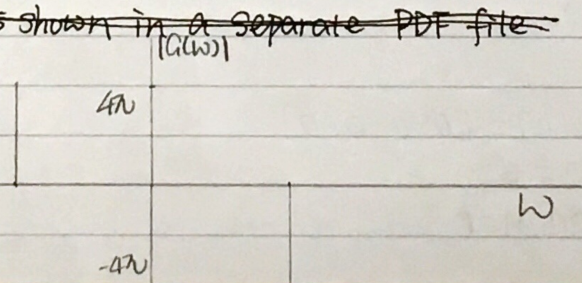
$g[n] = G[0] + 2\sum_{k=1}^{1} G[k]\cos\left(\frac{2\pi}{2}(n-0.5)k\right)]$

$= 2 + 2\cdot 2\cos(\pi(n-0.5)) = 2 + 4\sin(\pi n)$

(f) $G(\omega) = 2\cdot 2\pi \sum_{k=-\infty}^{\infty}\delta(\omega - 2\pi k) - 4\cdot\frac{\pi}{j}\sum_{k=-\infty}^{\infty}\{\delta(\omega-\pi) - \delta(\omega+\pi)\}]$

$= 4\pi\left[\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k) - \frac{1}{j}\sum_{k=-\infty}^{\infty}\{\delta(\omega-\pi-2\pi k) - \delta(\omega+\pi-2\pi k)\}\right]$

~~The plot of $|G(\omega)|$ is shown in a separate PDF file.~~



~~(e) $H_d(w) = \frac{1}{1-\frac{1}{2}e^{-2jw}}$~~ ~~$G[k] = [2 \quad 2]$~~

~~$g[n] = G[0] + 2\sum_{k=1}^{1} G[k]\cos(\frac{2\pi}{2}(n-0.5)k)$~~

~~$=2$~~

~~(f) $G(w) = 2\pi\sum_{k=-\infty}^{\infty}\delta(w-2\pi k)$~~

Question #4

(a) $H(S) = \frac{1}{S^2+\pi}$ $\quad S = \frac{1}{T}(1-z^{-1}) = 1-z^{-1}$

$\therefore H(z) = \frac{1}{1+z^{-2}-2z^{-1}+\pi}$

* (b) $H(w) = 1/(1+e^{-2jw}-2e^{-jw}+\pi)$

The plot of $|H(w)|$ is shown in the attached ~~sp~~ separate file.

(c) poles: $S^2+\pi=0$, $S=\pm\sqrt{\pi}j$

$H(S) = \frac{1}{S^2+\pi} = \cancel{\frac{1}{2j\sqrt{\pi}\,S}\frac{1}{S+\pi}}\ \frac{1}{2j\sqrt{\pi}}\left(\frac{1}{S-\sqrt{\pi}j} - \frac{1}{S+\sqrt{\pi}j}\right)$

$H(z) = \frac{1}{2j\sqrt{\pi}}\left(\frac{1}{1-e^{(\sqrt{\pi}j)}z^{-1}} + \frac{1}{1-e^{(-\sqrt{\pi}j)}z^{-1}}\right)$

$G(z) = \frac{1}{2j\sqrt{\pi}}\left(\frac{1}{1-e^{(j\sqrt{\pi})}z^{-1}} + \frac{1}{1-e^{(-j\sqrt{\pi})}z^{-1}}\right)$

* (d) $G(w) = \frac{1}{2j\sqrt{\pi}}\left(\frac{1}{1-e^{(j\sqrt{\pi})}e^{-jw}} + \frac{1}{1-e^{(-j\sqrt{\pi})}e^{-jw}}\right)$

The plot of $|G(w)|$ is show in the attached file.

(e) $S = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} = 2\sqrt{\pi}\,\frac{1-z^{-1}}{1+z^{-1}}$

$R(z) = \frac{1}{4\pi\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2+\pi} = \frac{z^{-2}+2z^{-1}+1}{\pi(9z^{-2}-6z^{-1}+5)}$

* (f) $R(w) = \frac{e^{-2jw}+2e^{-jw}+1}{\pi(9e^{-2jw}-6e^{-jw}+5)}$

Question #5

(a) $s = j\omega$, $H(\omega) = j\omega$

$|H(\omega)| = |\omega|$ It is a high pass filter.

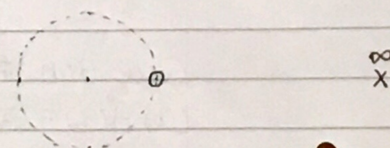
The problem for this filter response regarding discrete-time differentiation is ~~the~~ with ~~discrete-tim~~ taking derivative of absolute values of a function.

(b) $s = \frac{1}{T}(1-z^{-1}) = 1-z^{-1}$

$H(z) = \cancel{j\frac{\ln z}{j} = \ln z}\ 1-z^{-1}$

* (c) $\cancel{|H(\omega)| = |\omega|}$ $H(\omega) = 1-e^{-j\omega}$. Plot shown in attached file.

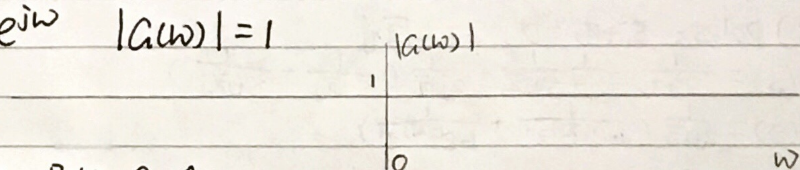
(d) $H(z) = \frac{1-z^{-1}}{\cancel{\ln z}}$. Pole: $z = 0$, ~~zero~~ zero, $z=1$.



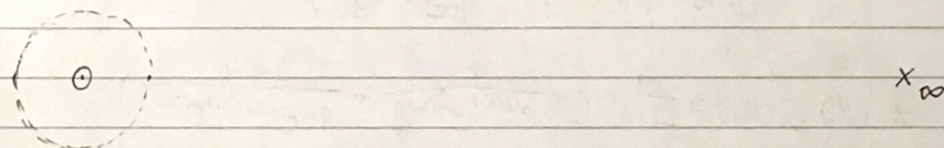
(e) $H(s) = s$. Pole: $s = \infty$. zero: $s = 0$

$H(z) = z$. $G(z) = z$

(f) $G(\omega) = e^{j\omega}$ $|G(\omega)| = 1$



(g) zero: $s=0$. Pole: $s=\infty$



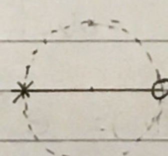
(h) $s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} = 2\frac{1-z^{-1}}{1+z^{-1}}$

$R(z) = 2\frac{1-z^{-1}}{1+z^{-1}}$

* (i) $R(\omega) = 2\frac{1-e^{-j\omega}}{1+e^{-j\omega}}$ $|R(\omega)| = 2\sqrt{\frac{(1-\cos\omega)^2+\sin^2\omega}{(1+\cos\omega)^2+\sin^2\omega}} = 2\sqrt{\frac{1-\cos\omega}{1+\cos\omega}}$

Plot shown in attached / the separate file.

(j) Pole: $z=-1$. zero: $z=1$



(k) I think the bilinear filter is the best differentiator.