Lecture 3 : Discrete -Time Convolution

Foundations of Digital Signal Processing

Outline

- Homework Questions
- Review of Previous Class
- Discrete-Time Convolution
- The Impulse Response
- Discrete-Time Convolution Again

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Notes

- **Homework Questions**

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Review

- **What did we learn last class?**
  - Signal Properties
  - Periodicity
  - Measures of signal “size”
  - Signal Operations
  - Special Signals: Impulses and Step Functions
  - System Properties

Review

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  - Signal Properties

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  - Special Signals: Impulses and Step Functions

Review

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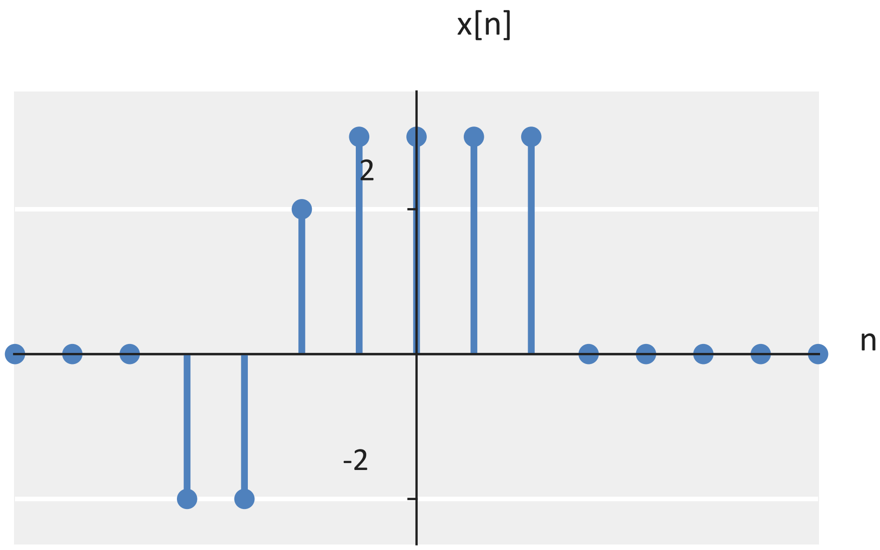
  - System Properties

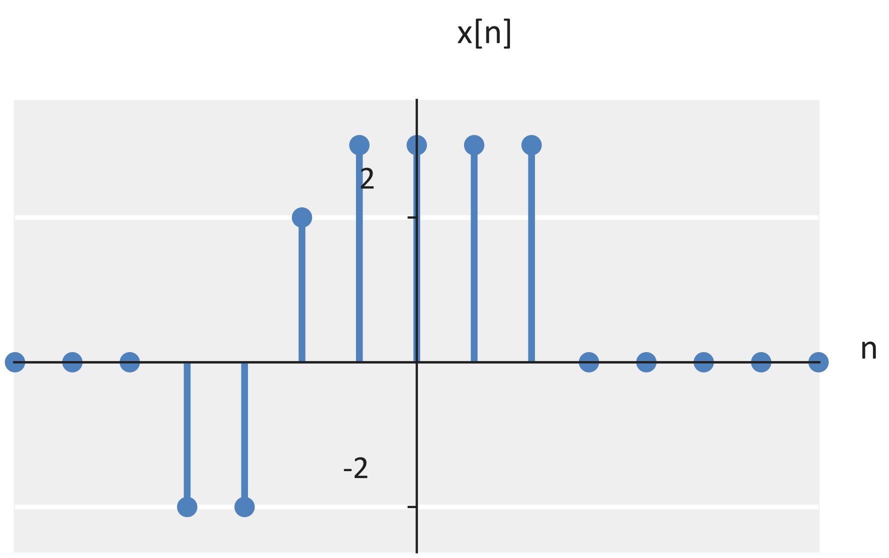
Review

- **Example Problems**
  - Express the following signal as a linear combination of shifted <u>step</u> functions.



Review

- **Example Problems**
  - Express the following signal as a linear combination of shifted <u>impulse</u> functions.



Review

- **Example Problems**
  - Are these systems Linear? Time-Invariant? Memoryless? Causal?
  - $y[n] = n^2 x[n]$
  - $y[n] = x[n] - x[n-1]$
  - $y[n] = x[n] - 3x[n+6]$
  - $y[n] = x[n]x[n-4]$

Review

- **Example Problems**
  - Are these systems Linear? Time-Invariant? Memoryless? Causal?
  - $y[n] = n^2 x[n]$ <- Time Varying
  - $y[n] = x[n] - x[n-1]$ <- Time-Invariant
  - $y[n] = x[n] - 3x[n+6]$ <- Time-Invariant
  - $y[n] = x[n]x[n-4]$ <- Time-Invariant

Review

- **Example Problems**
  - Are these systems Linear? Time-Invariant? Memoryless? Causal?
  - $y[n] = n^2 x[n]$ <- Memoryless
  - $y[n] = x[n] - x[n-1]$ <- Has memory
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Review

- **Example Problems**
  - Are these systems Linear? Time-Invariant? Memoryless? Causal?
  - $y[n] = n^2 x[n]$ <- Causal
  - $y[n] = x[n] - x[n-1]$ <- Causal
  - $y[n] = x[n] - 3x[n+6]$ <- Non-causal
  - $y[n] = x[n]x[n-4]$ <- Causal

Lecture 3: Discrete-Time Convolution

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- The Impulse Response
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Convolution

- **Linear and Time-Invariant (LTI)**
  - If a system is both linear and time-invariant, then...

Linear: $ay_1[n] + by_2[n] = H\{ax_1[n] + bx_2[n]\}$

Time-invariant: $y[n+N] = H\{x[n+N]\}$

Convolution

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  - Recall

$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\delta[n-m]$$

$$H\{x[n]\} = H\left\{\sum_{m=-\infty}^{\infty} x[m]\delta[n-m]\right\}$$

Convolution

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Apply linearity

Convolution

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Apply time-invariance

Convolution

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$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\delta[n-m]$$

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\,h[n-m]$$

Convolution!

Convolution

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  - Recall

$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\delta[n-m]$$

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\,h[n-m] = x[n] * h[n]$$

Convolution

- **Definition of convolution**

$$y[n] = x[n] * h[n] = \underbrace{\sum_{m=-\infty}^{\infty} x[m]\, h[n-m]}_{\text{Inner product of x[m] and h[n-m]}}$$

Convolution

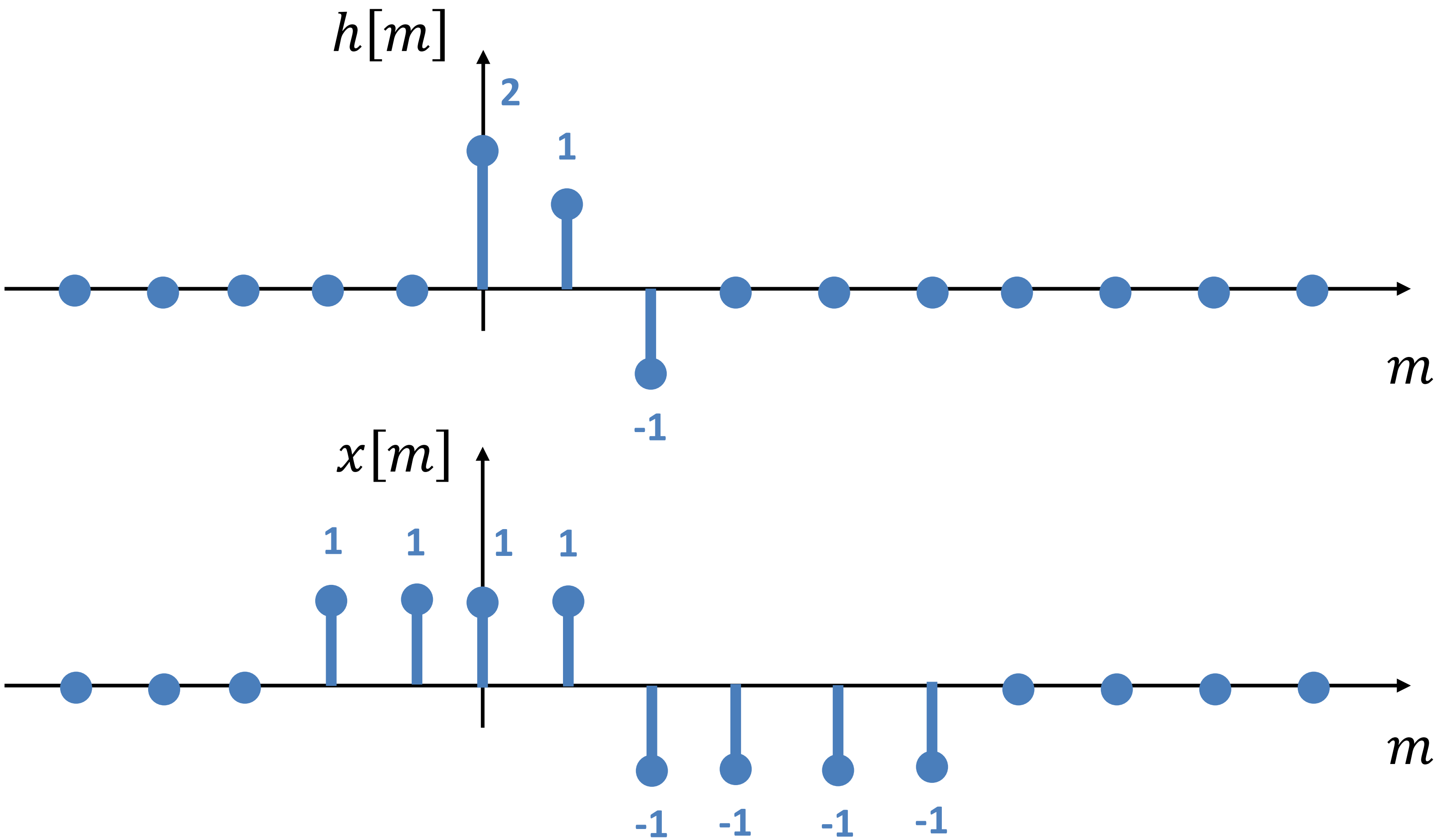
- **Problem: Consider the definition of convolution:**

$$x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

  - Show that:
    - $x[n] * h[n] = h[n] * x[n]$

Convolution

- **Convolution** $y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$



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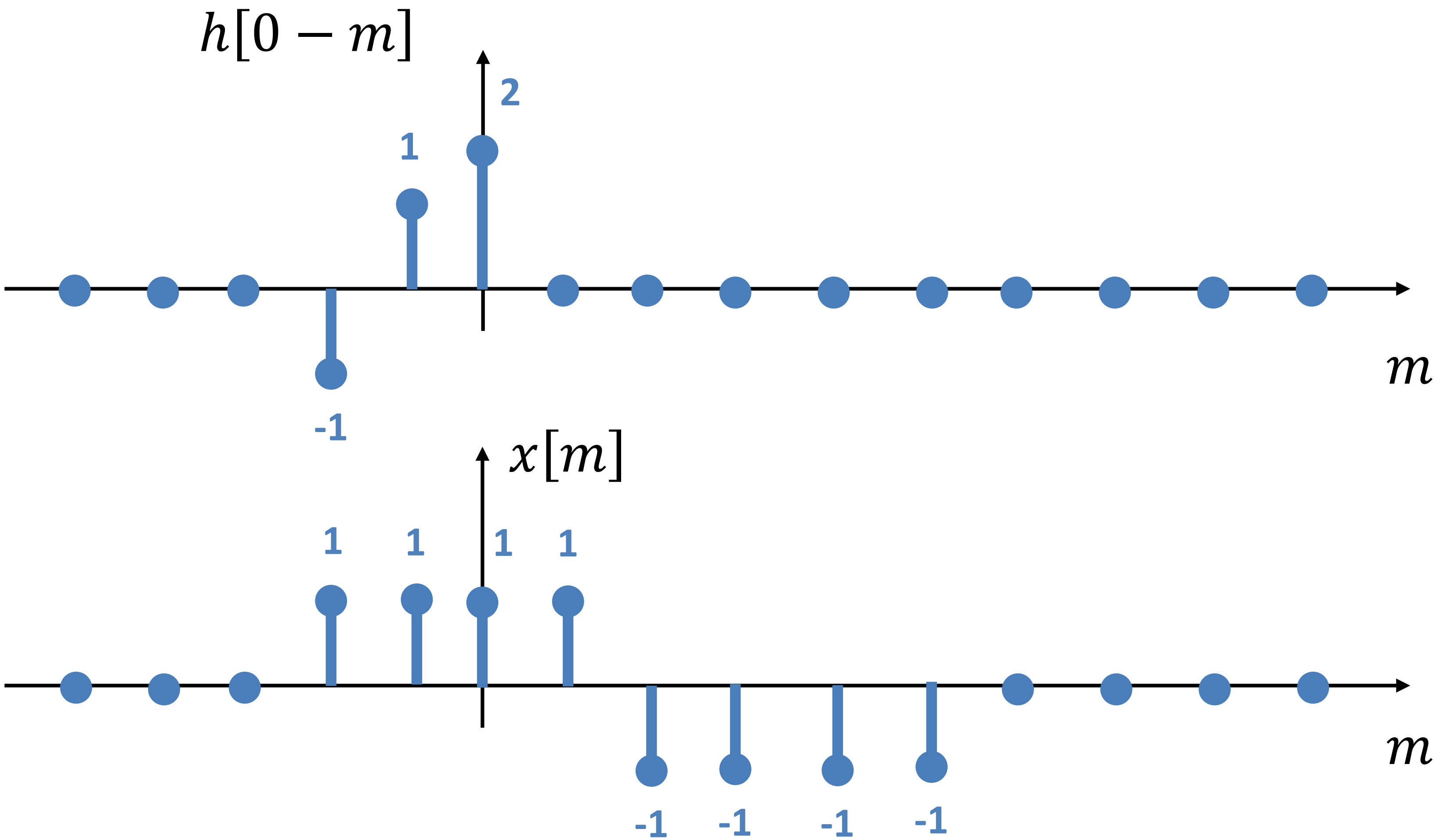

Convolution

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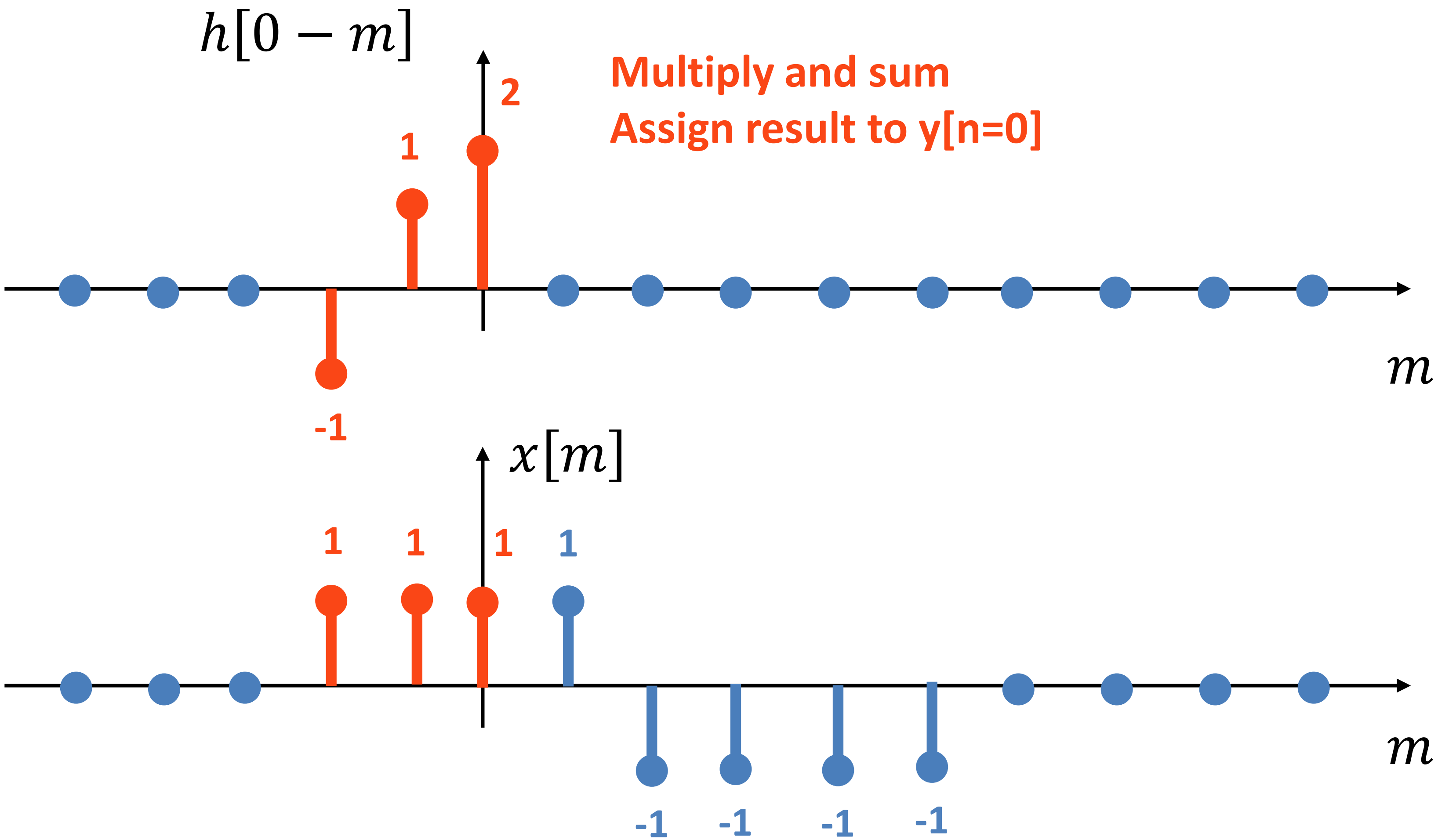

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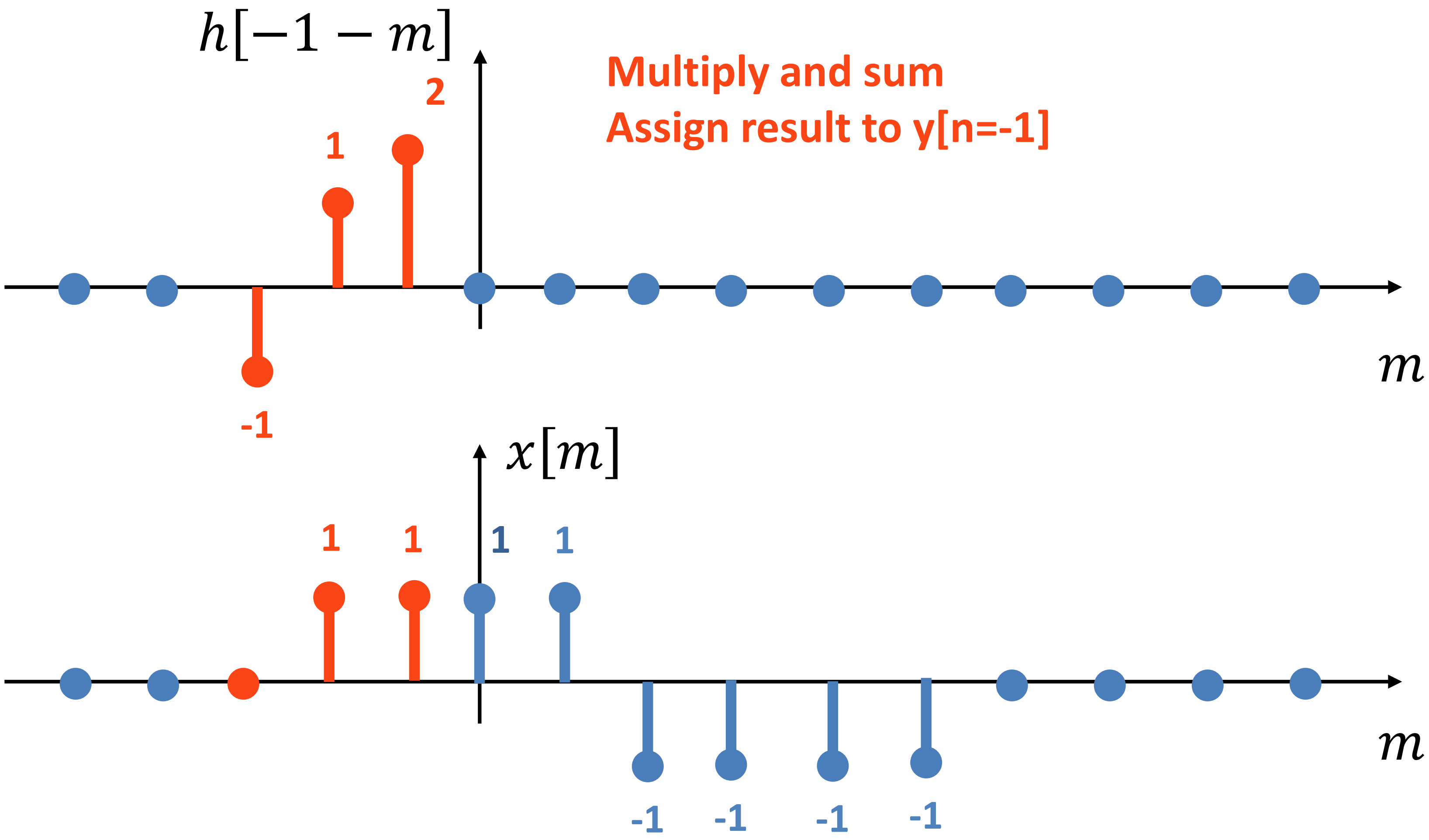

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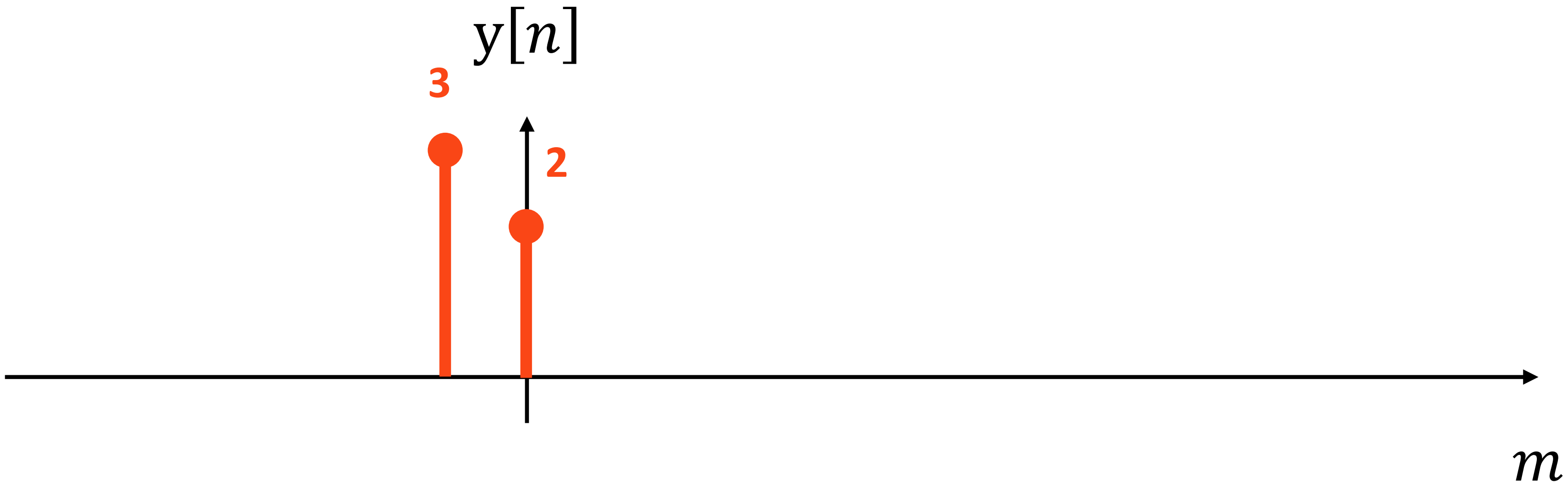

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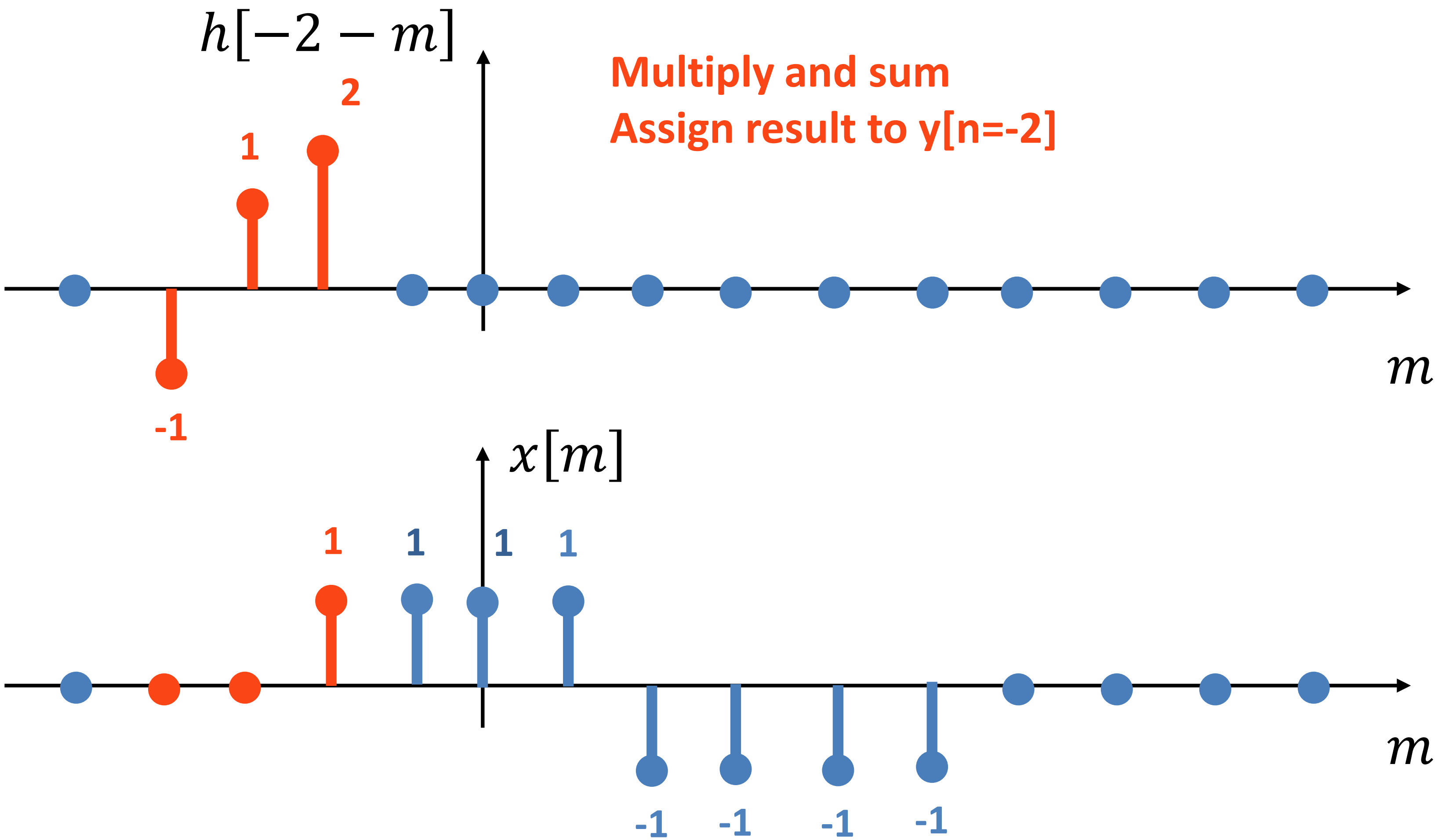
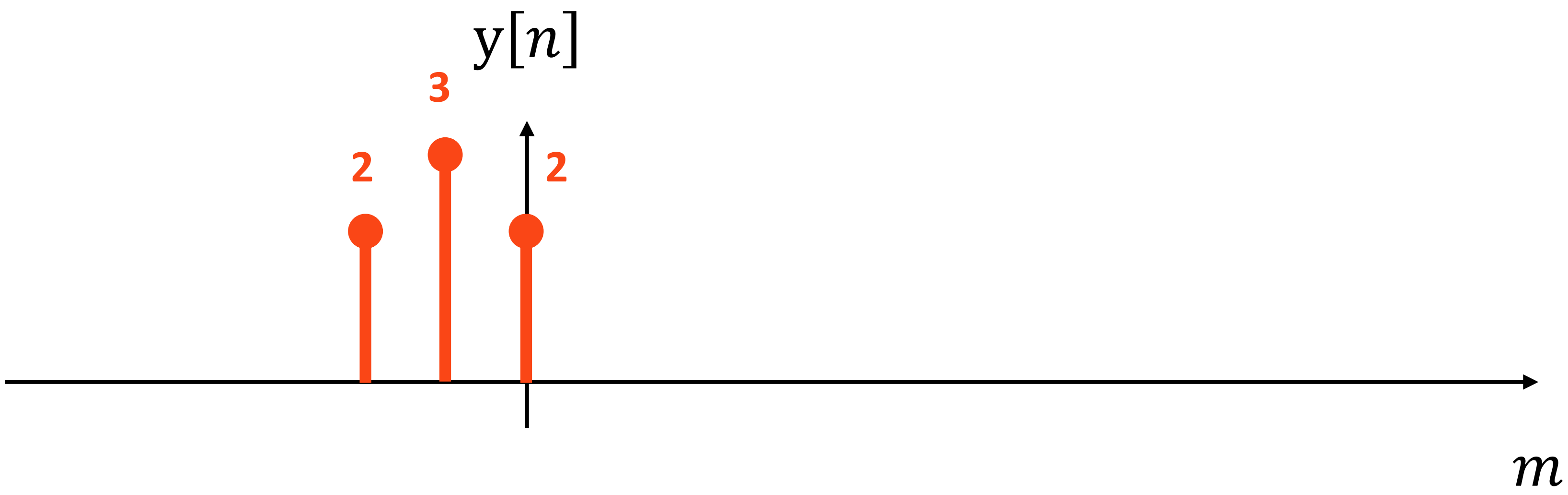


Multiply and sum
Assign result to y[n=-2]

Convolution

- **Convolution** $y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$



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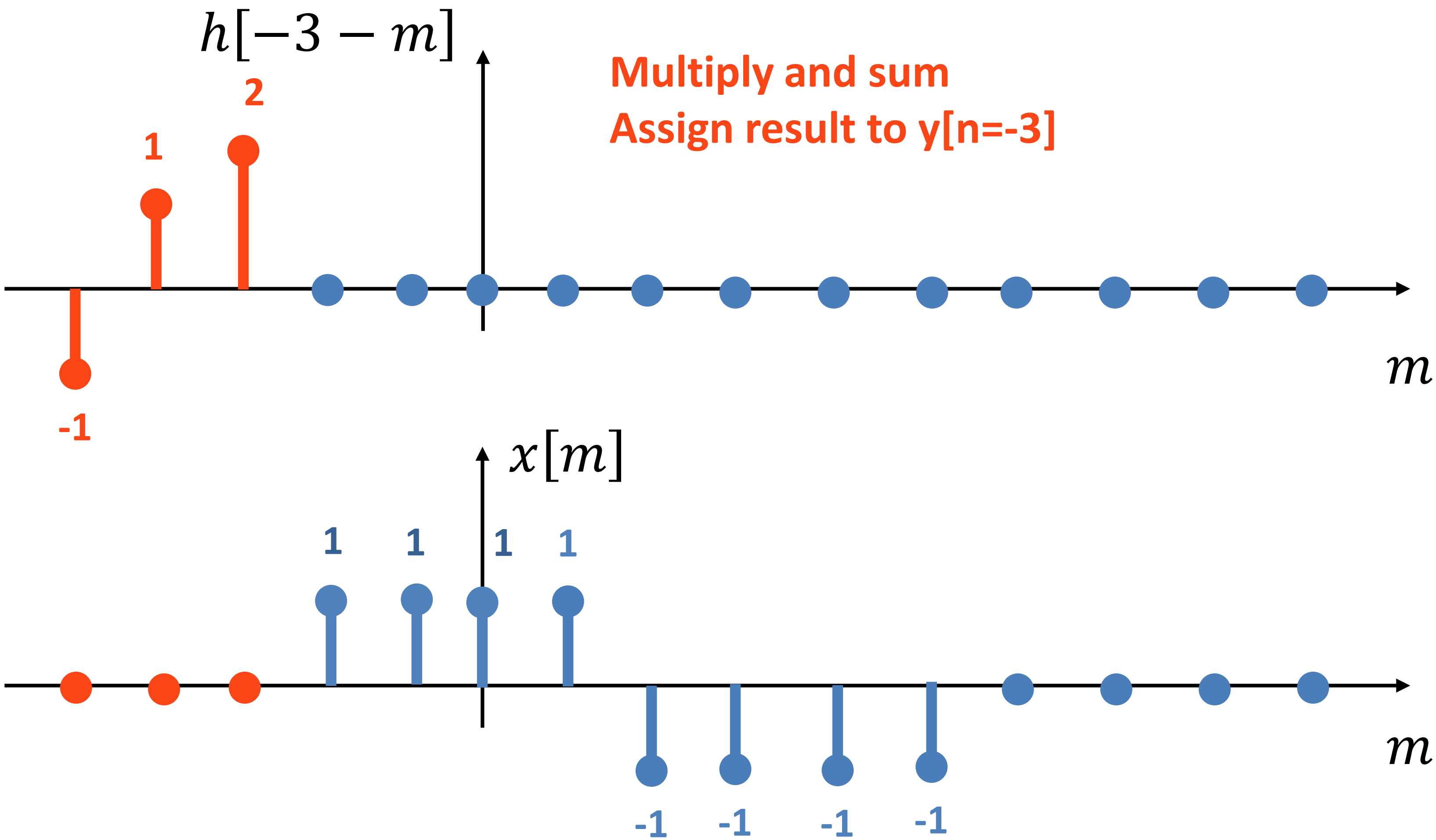

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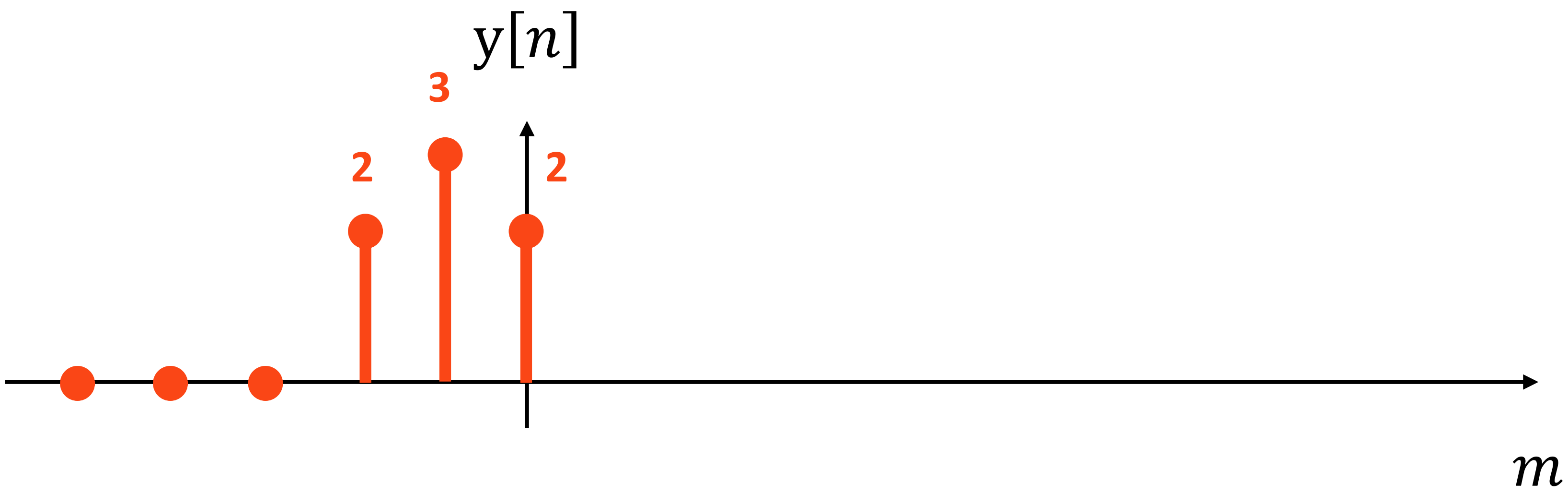

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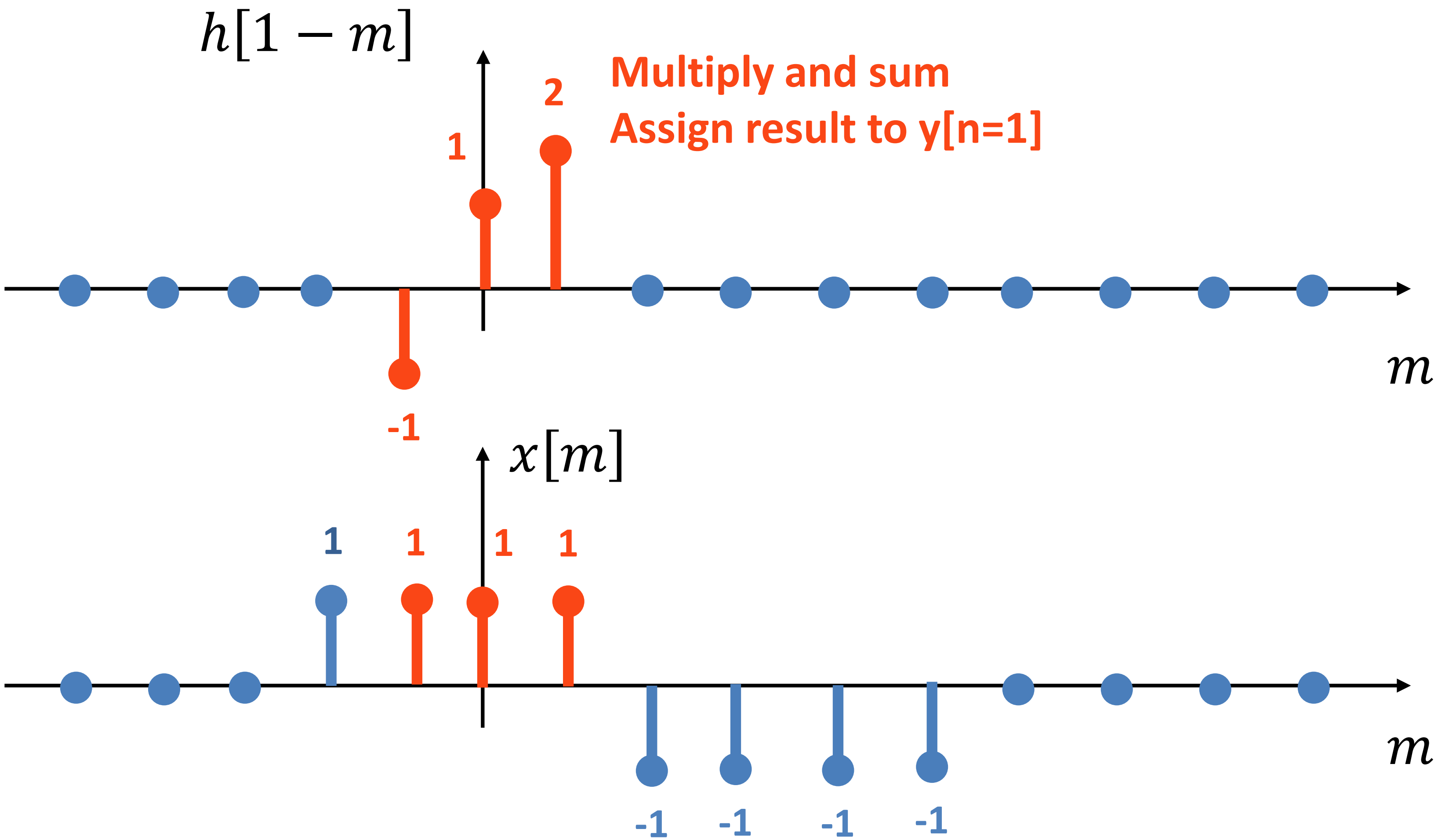

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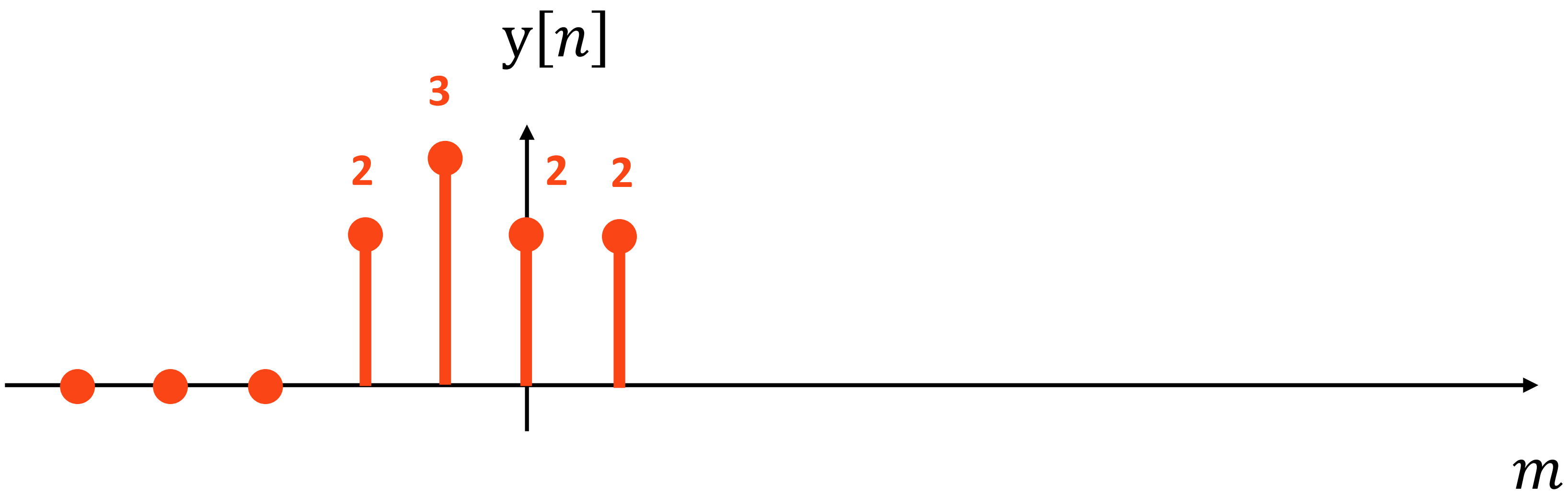

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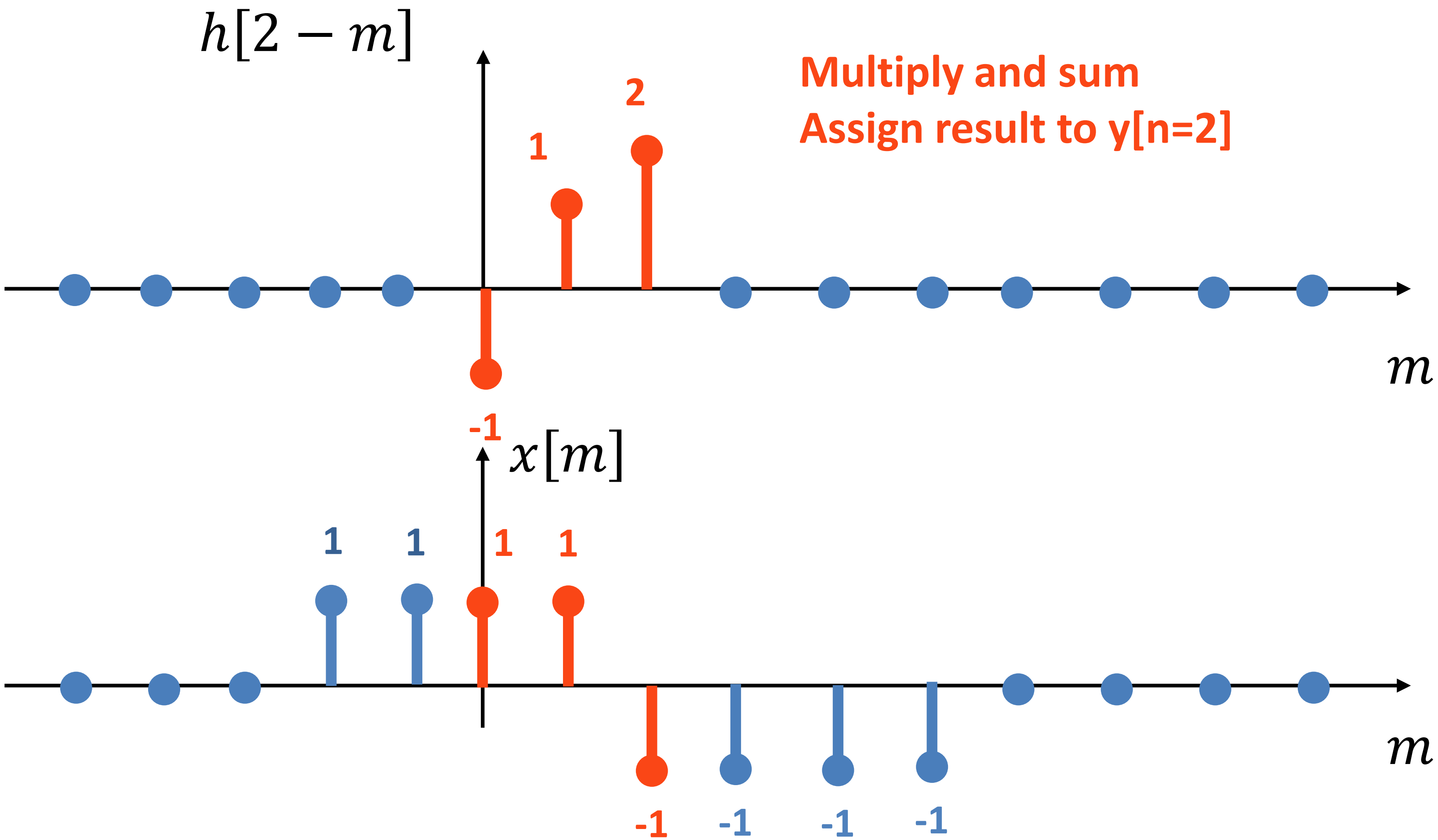

Multiply and sum
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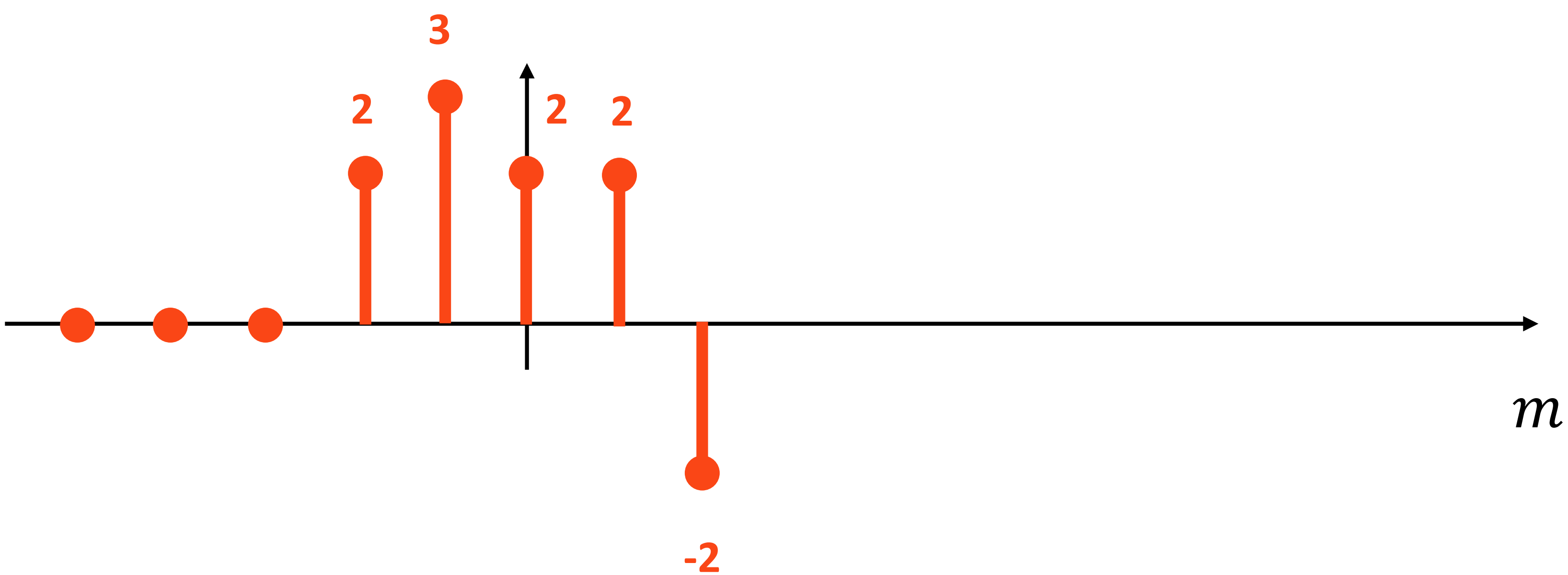

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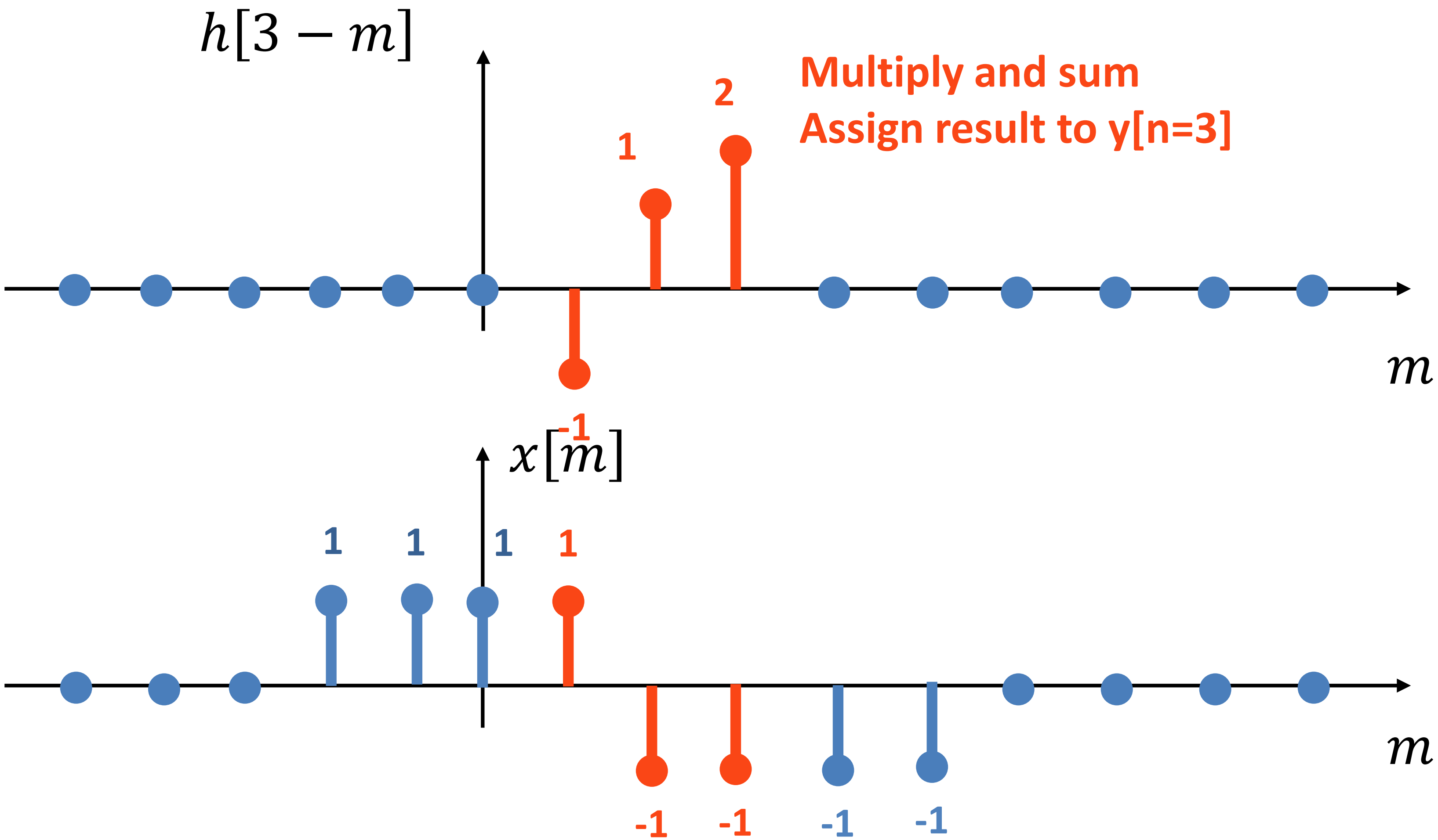
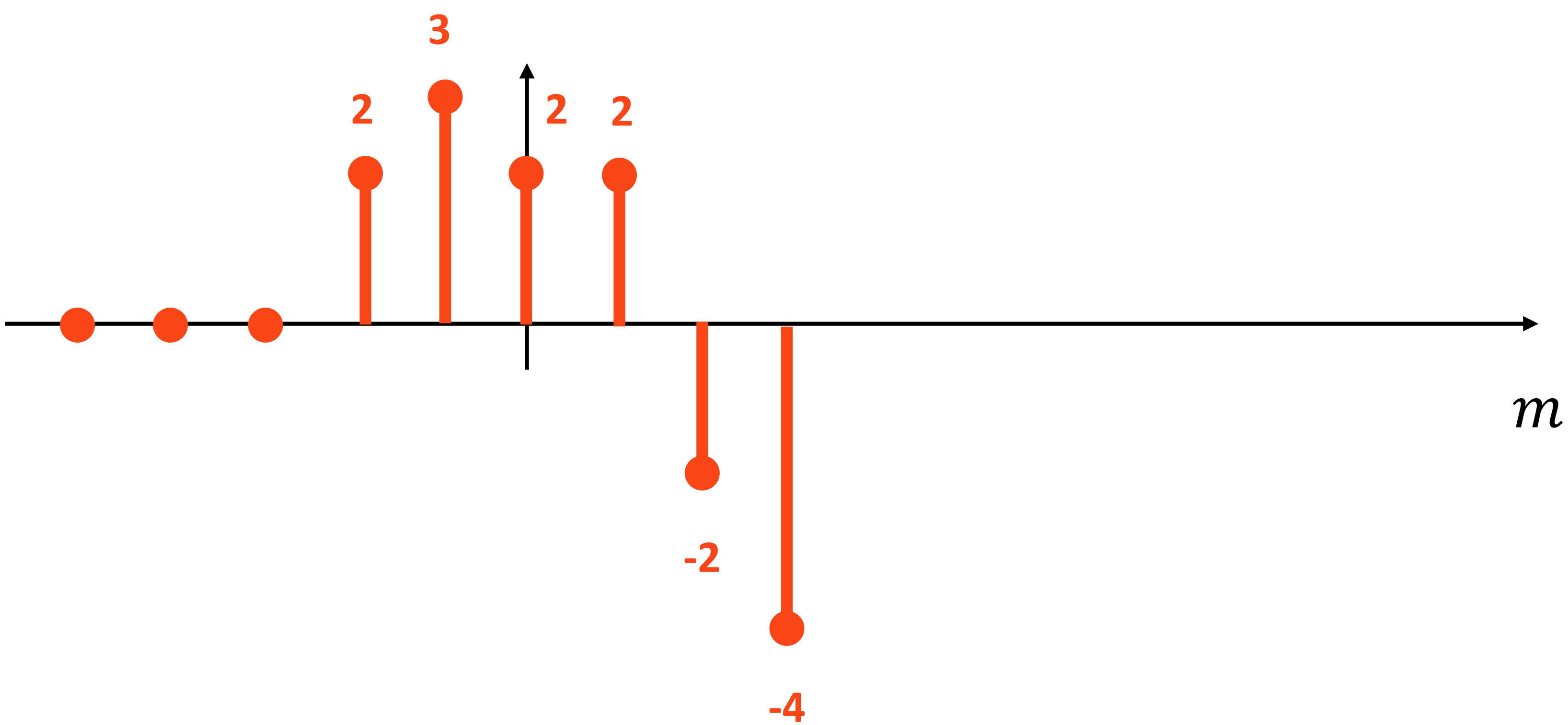


Multiply and sum
Assign result to y[n=3]

Convolution

- **Convolution** $y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$



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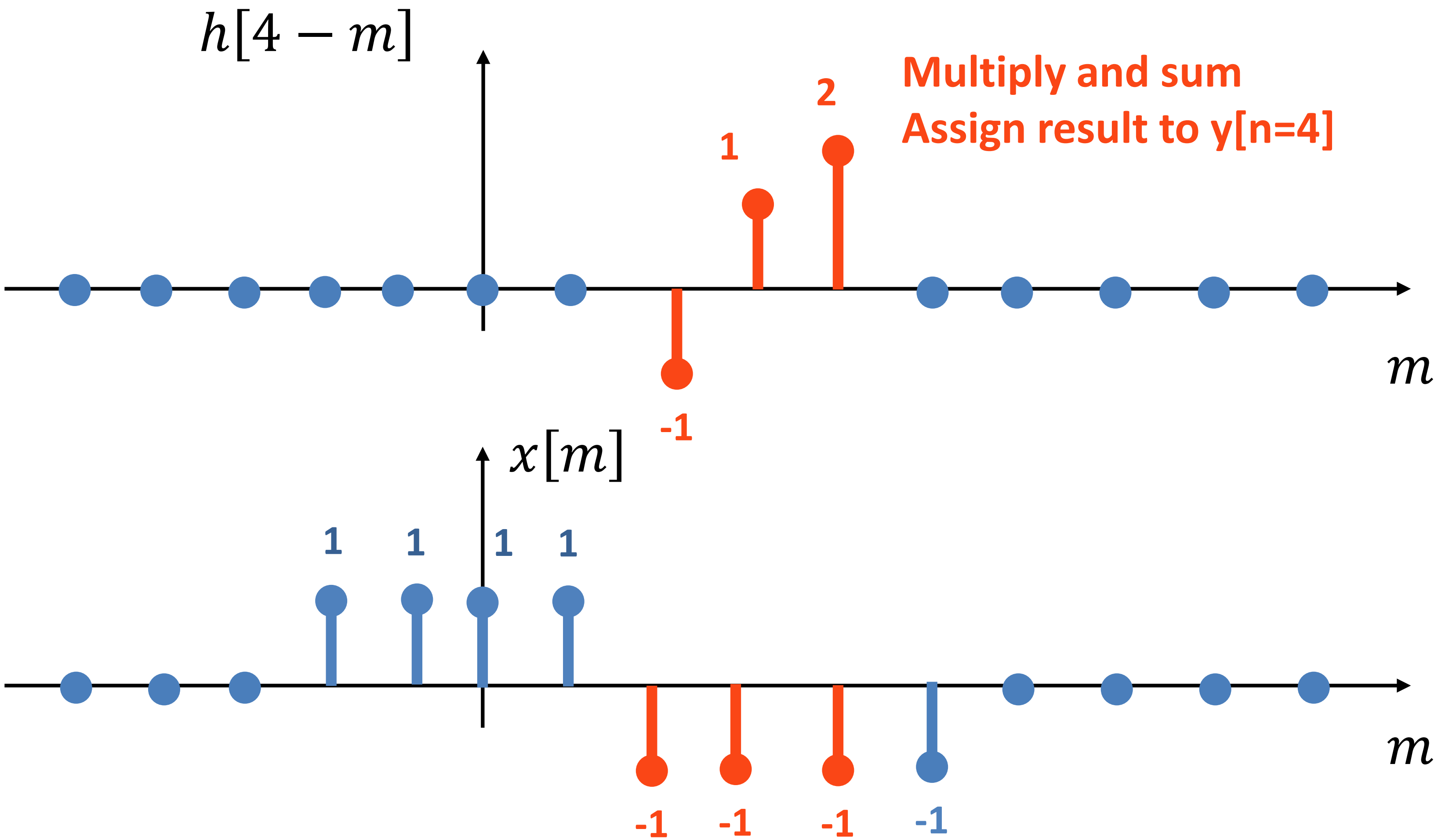

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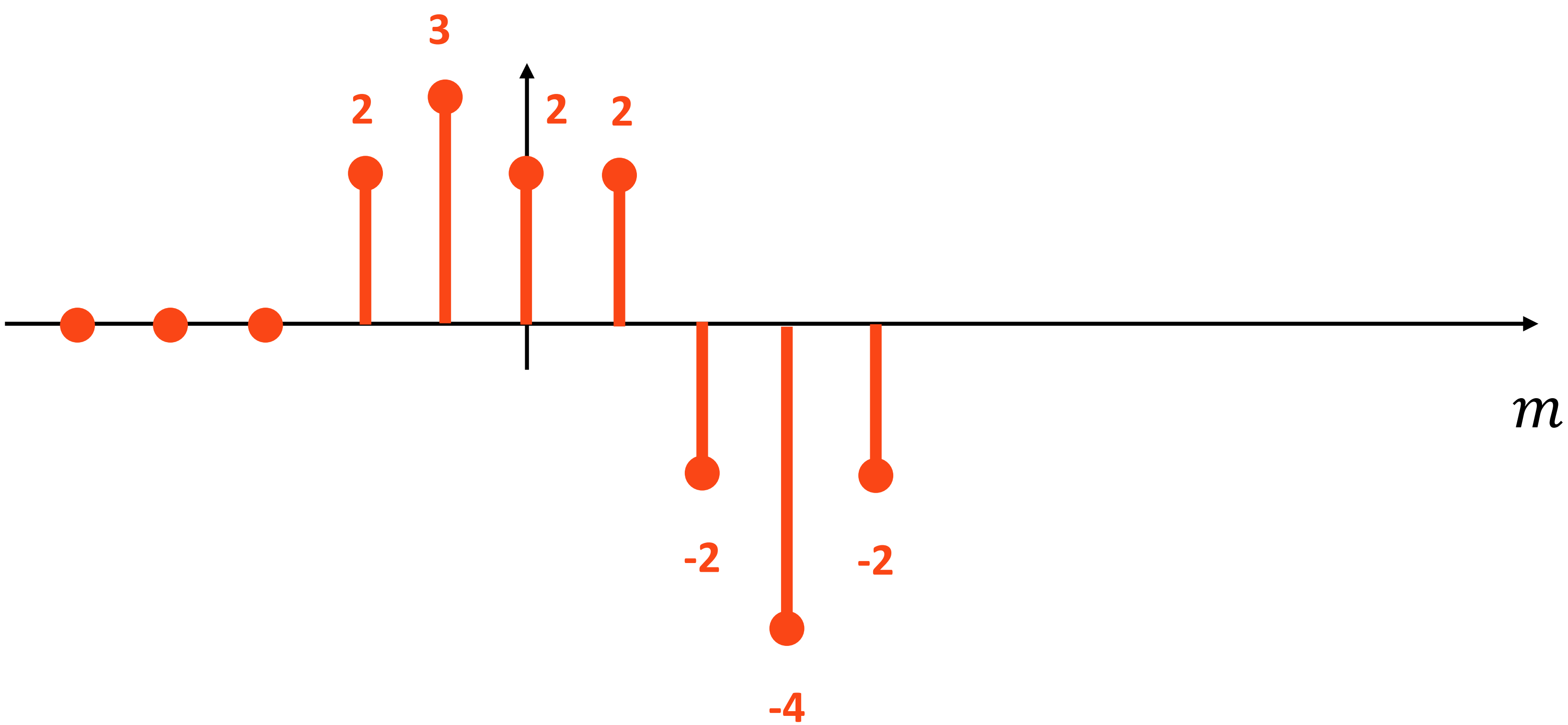

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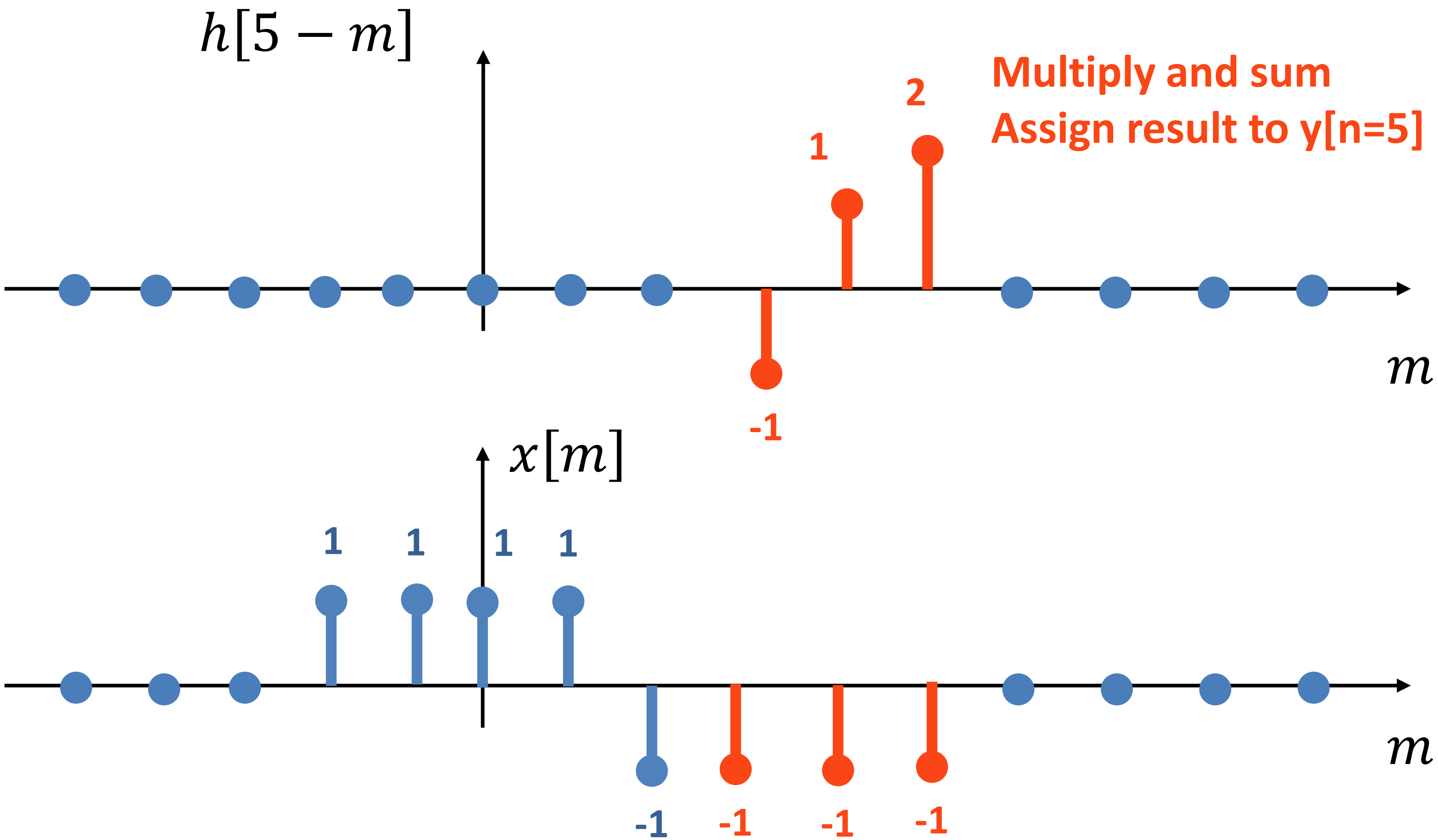

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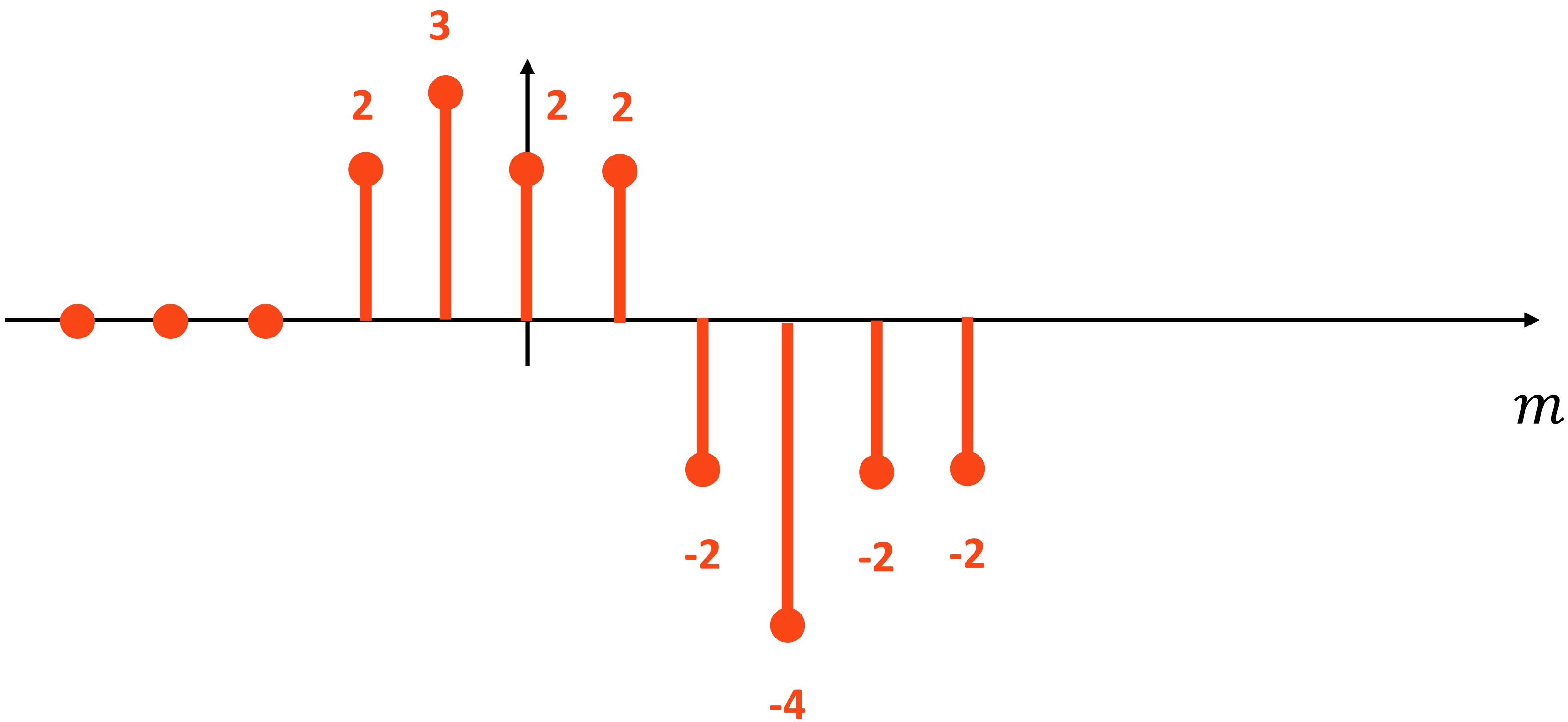

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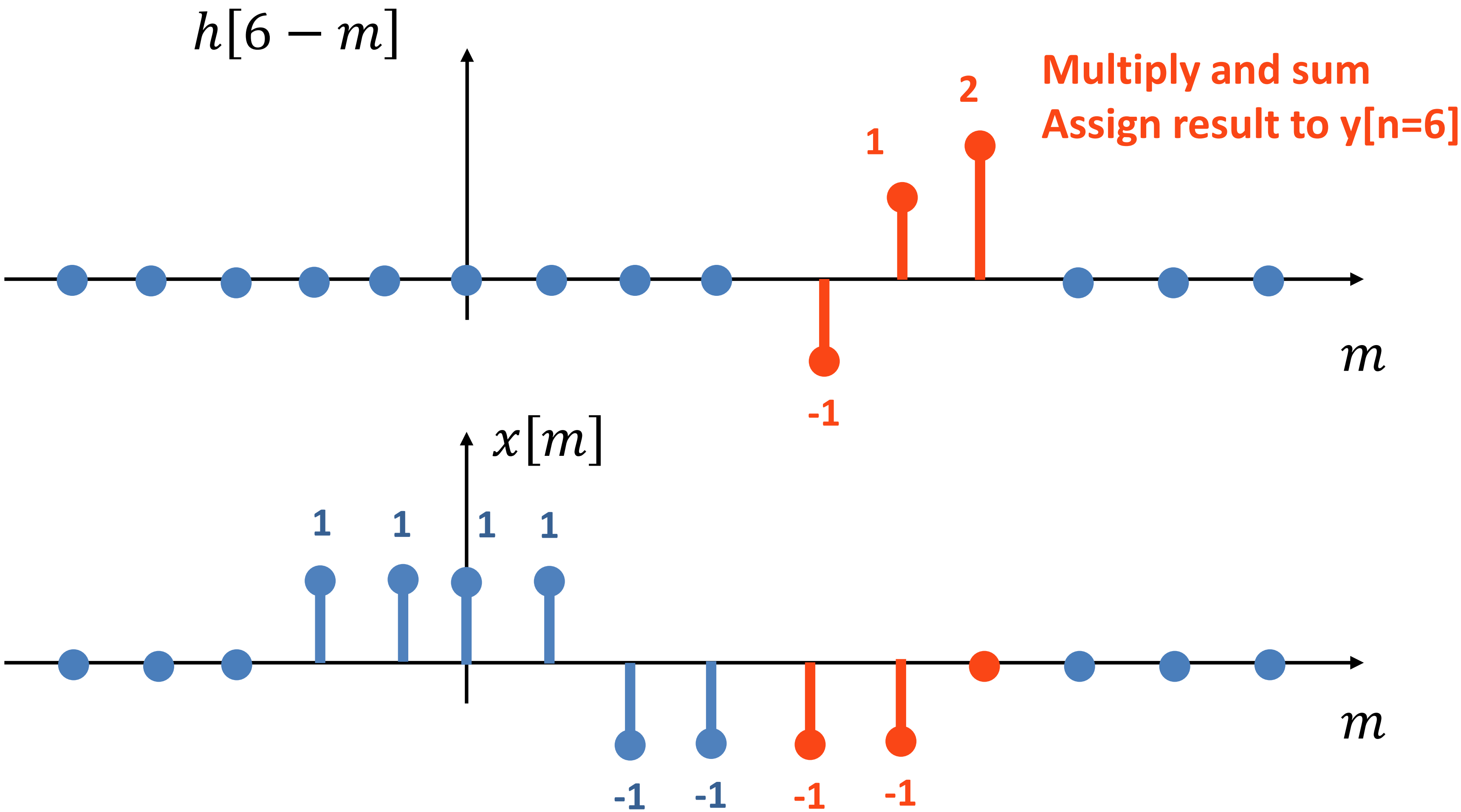

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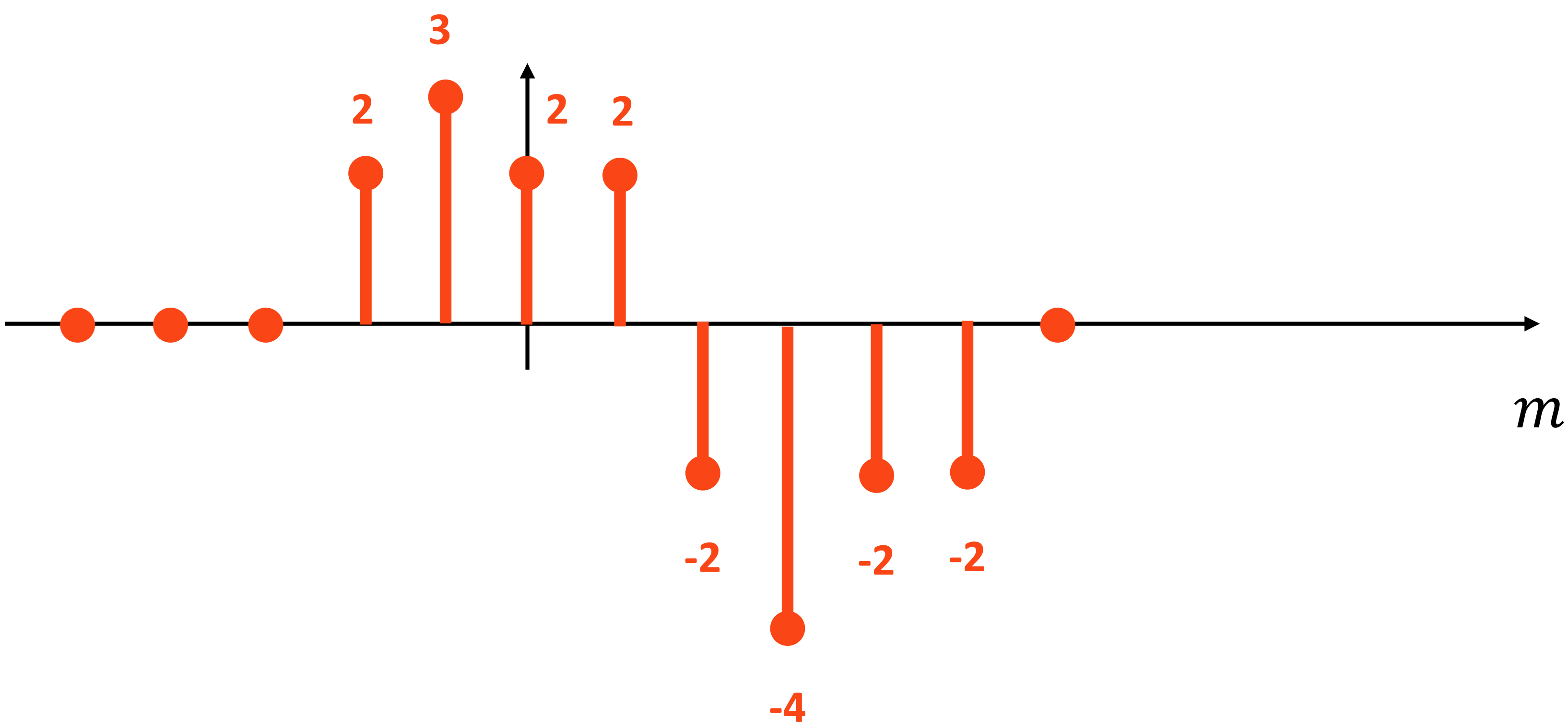

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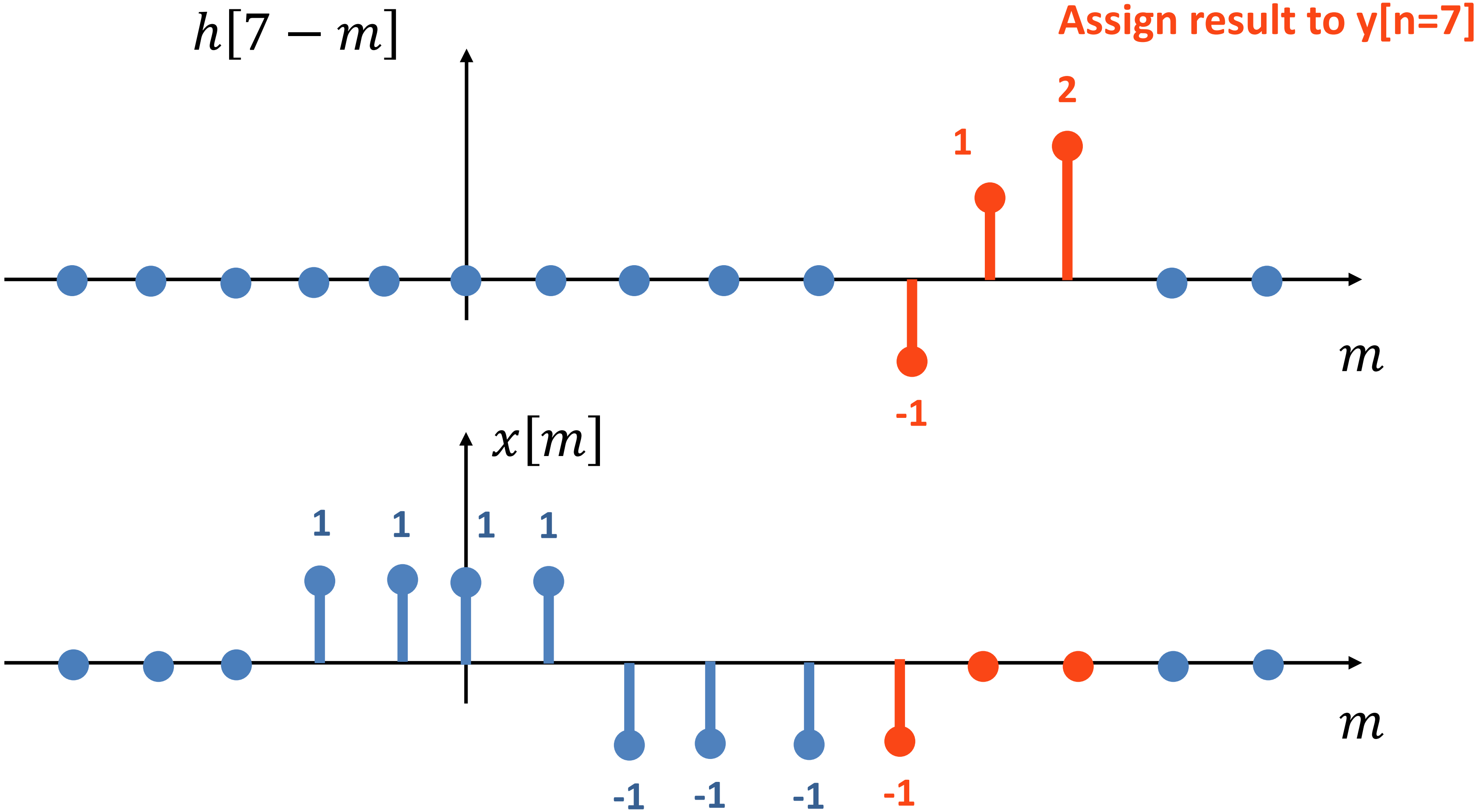

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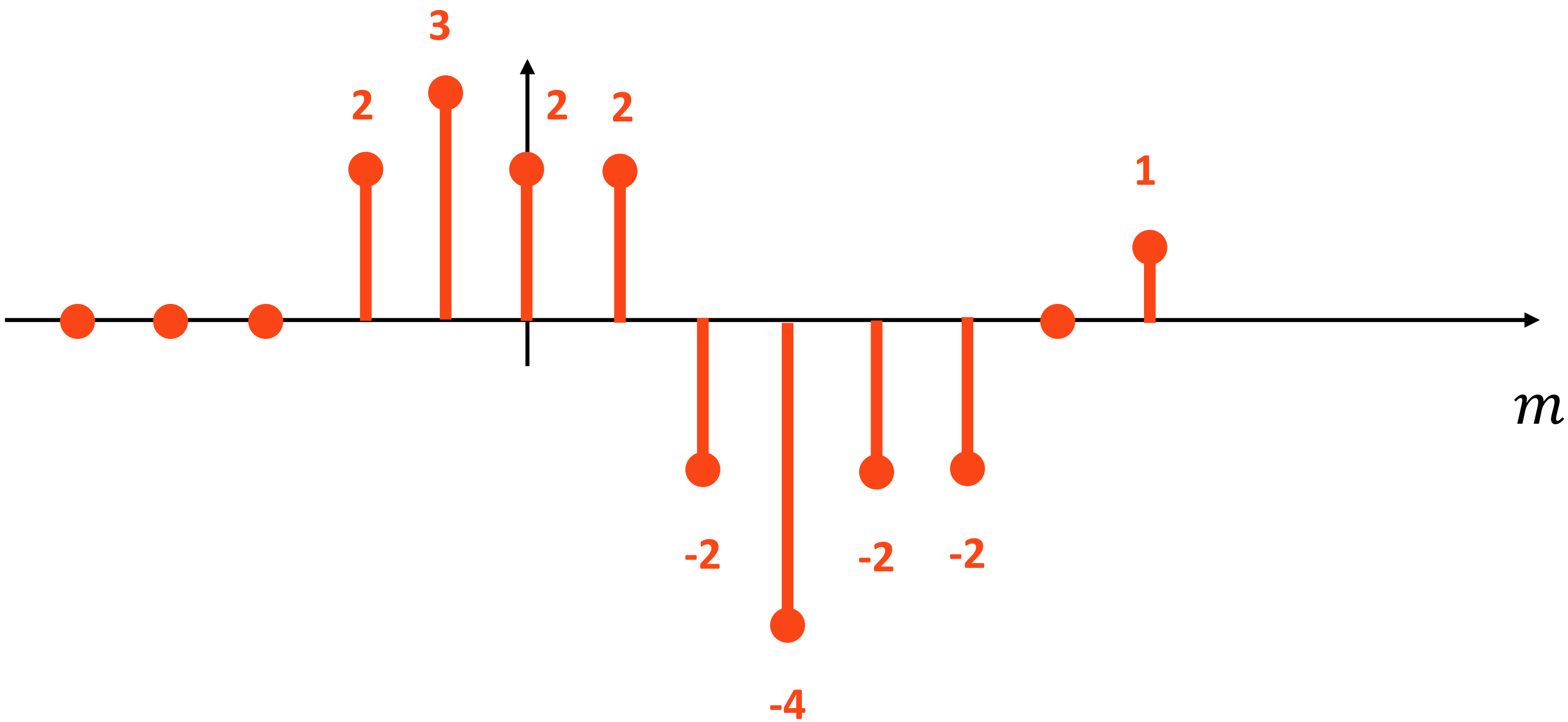

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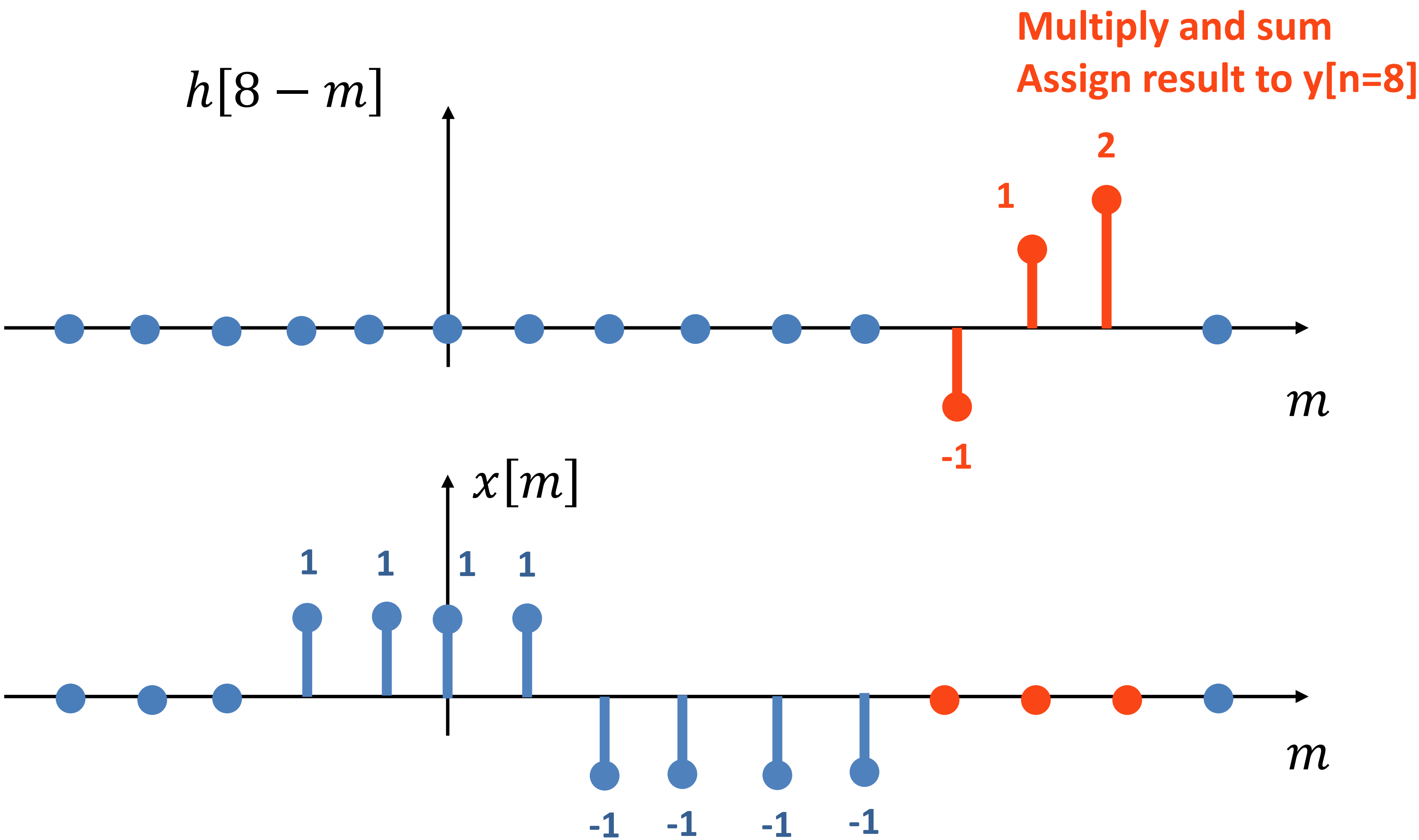

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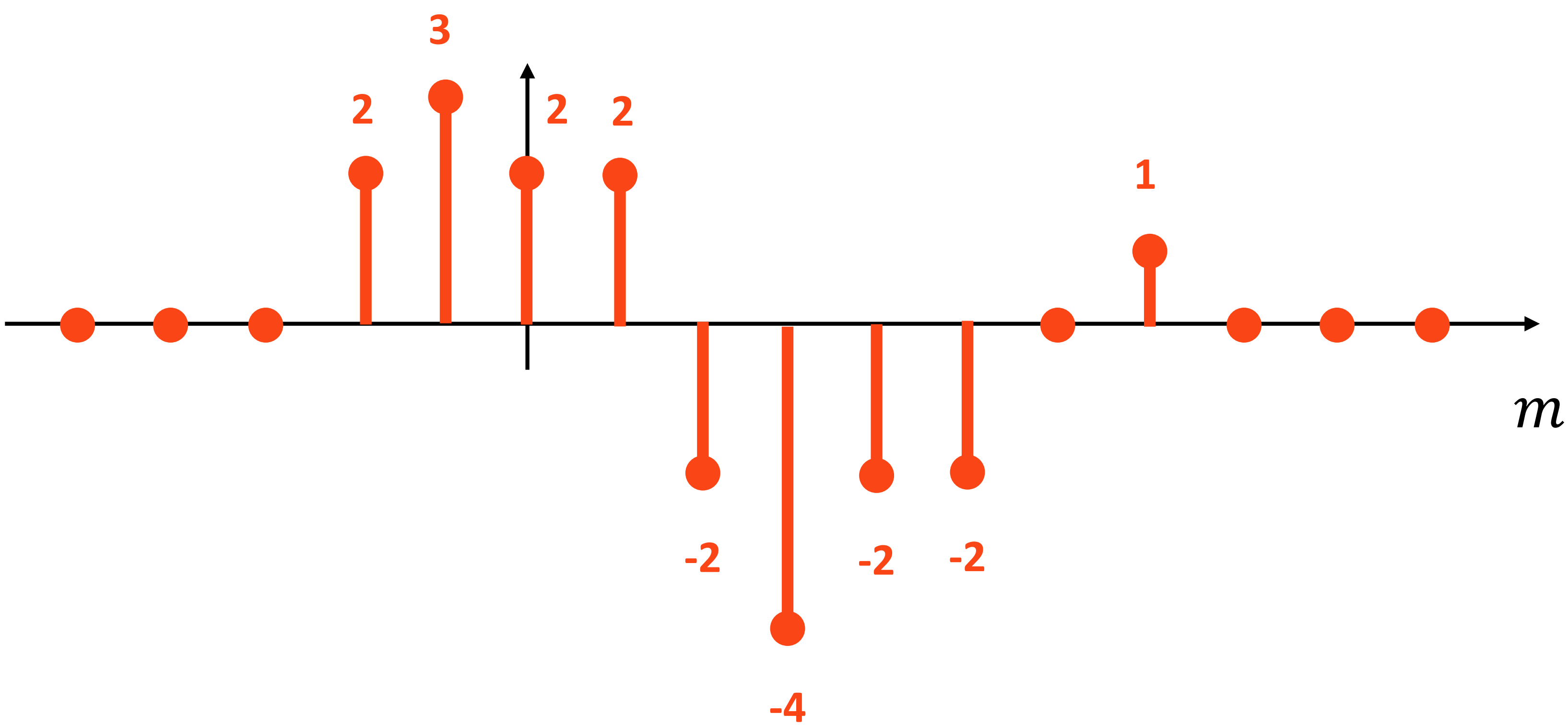
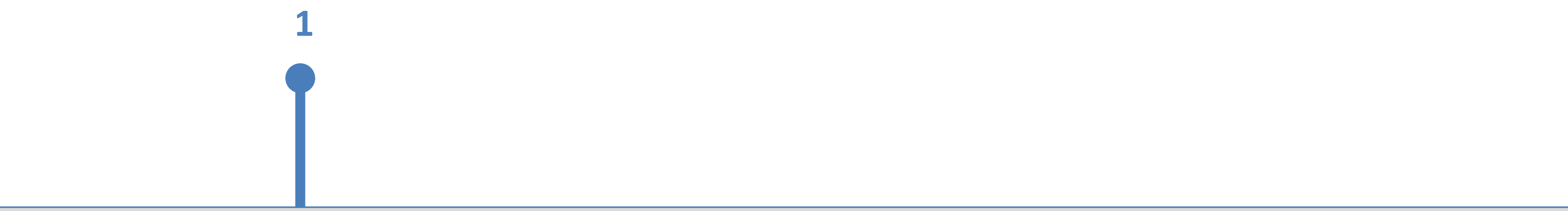


A slightly different perspective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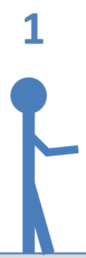
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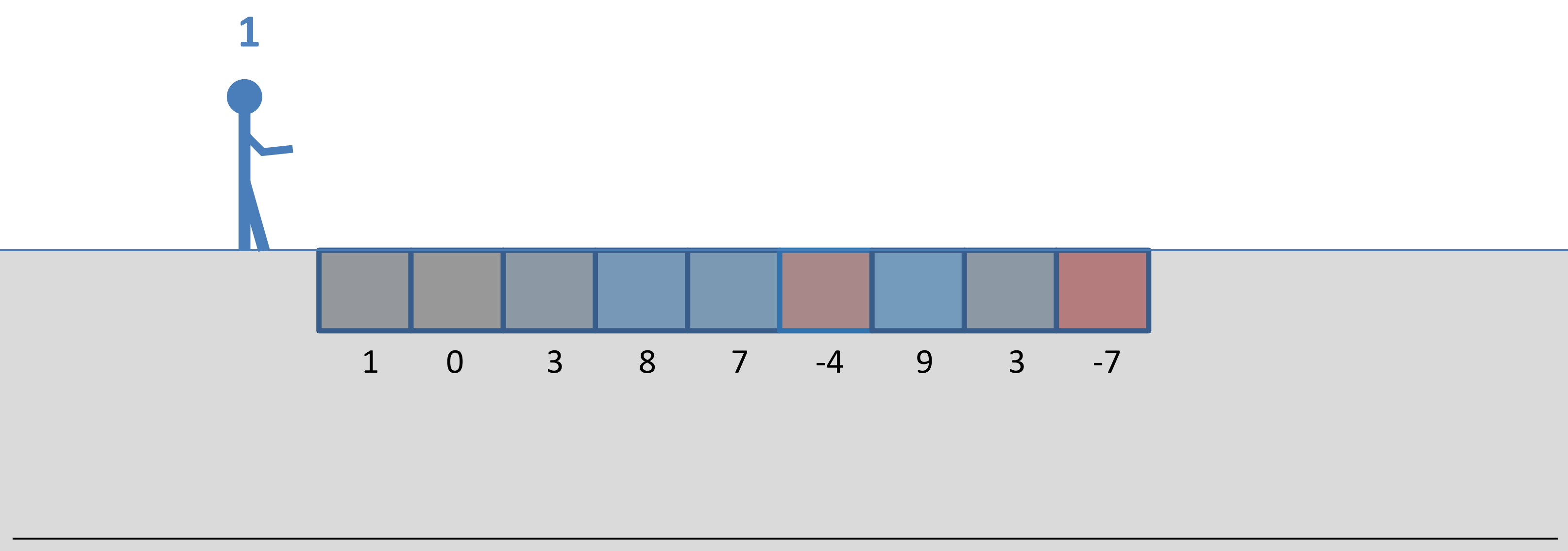
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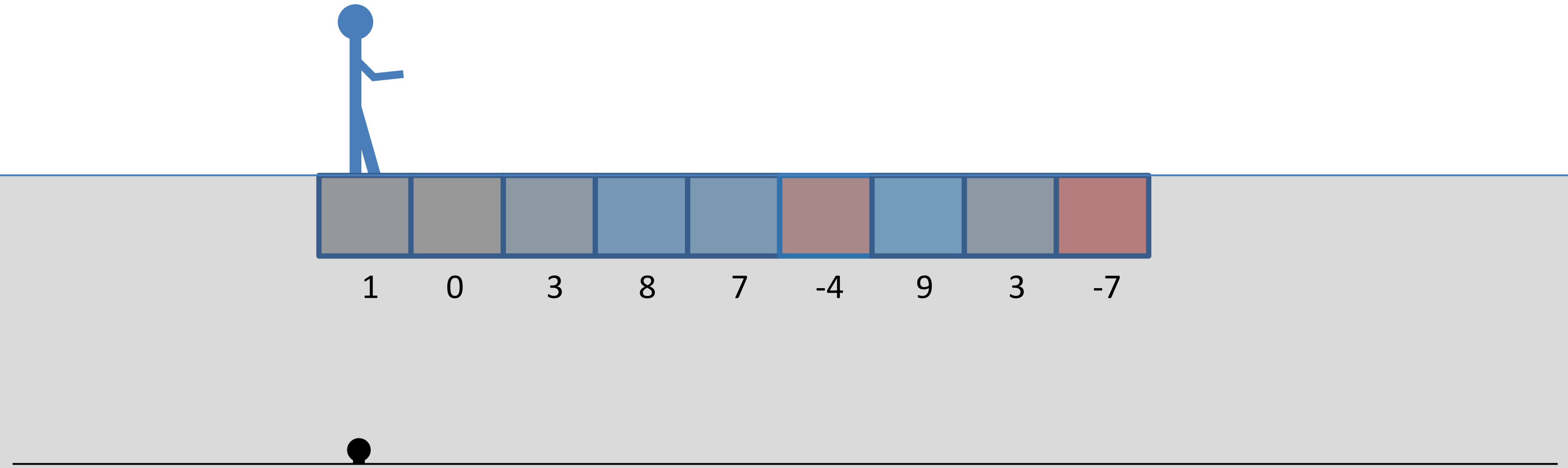
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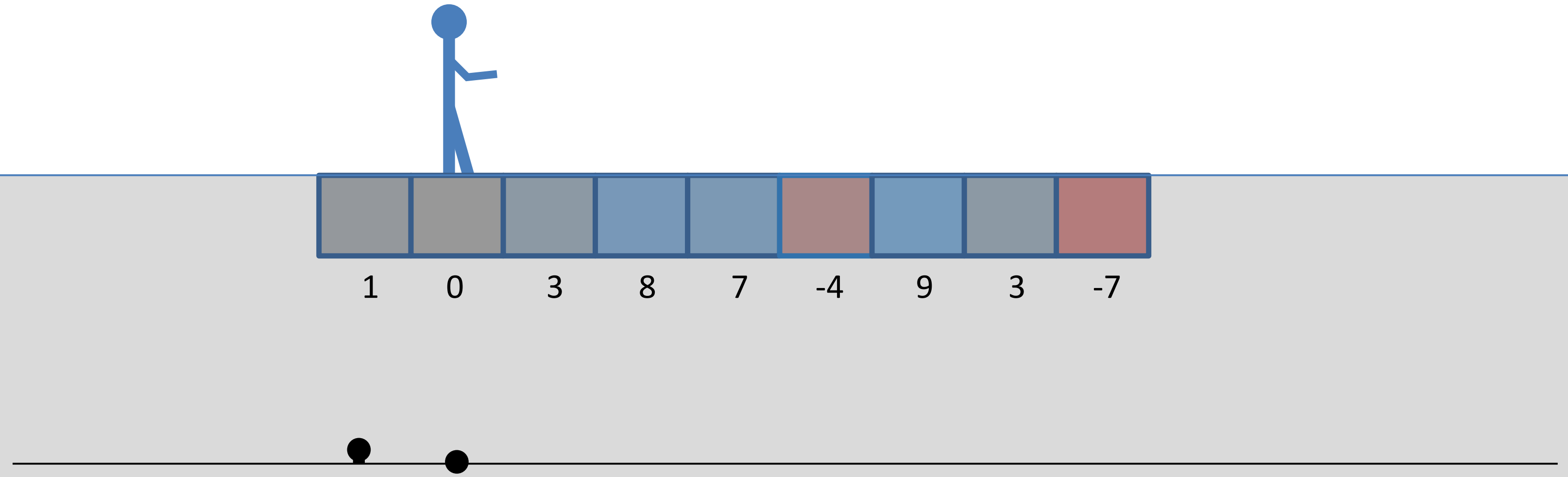
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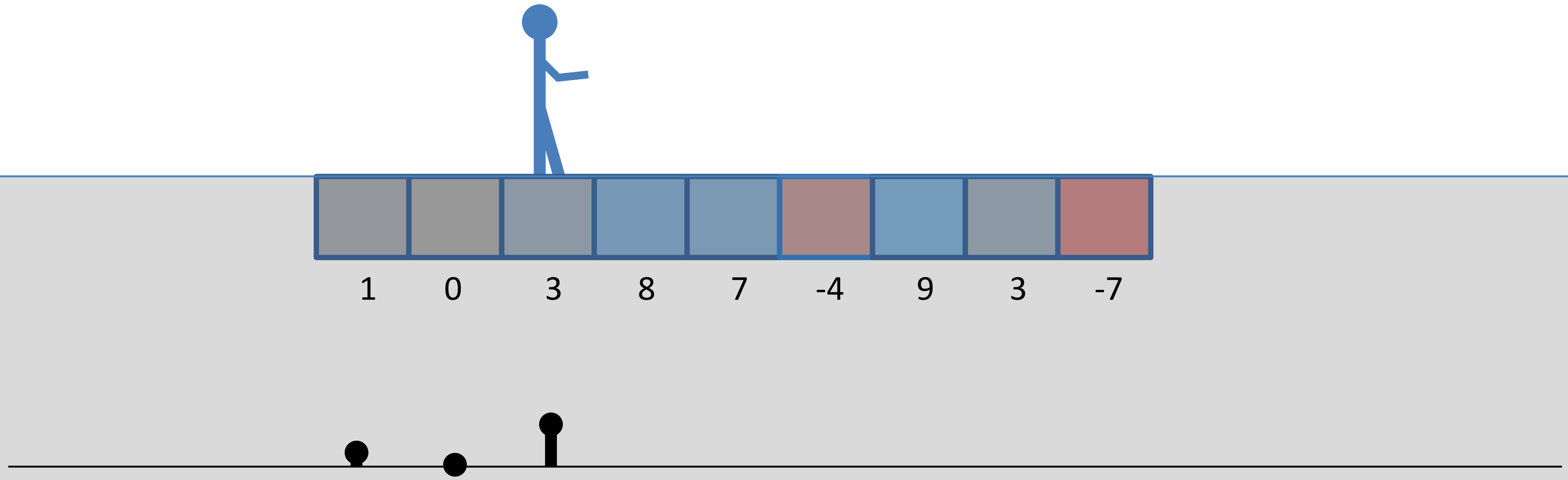
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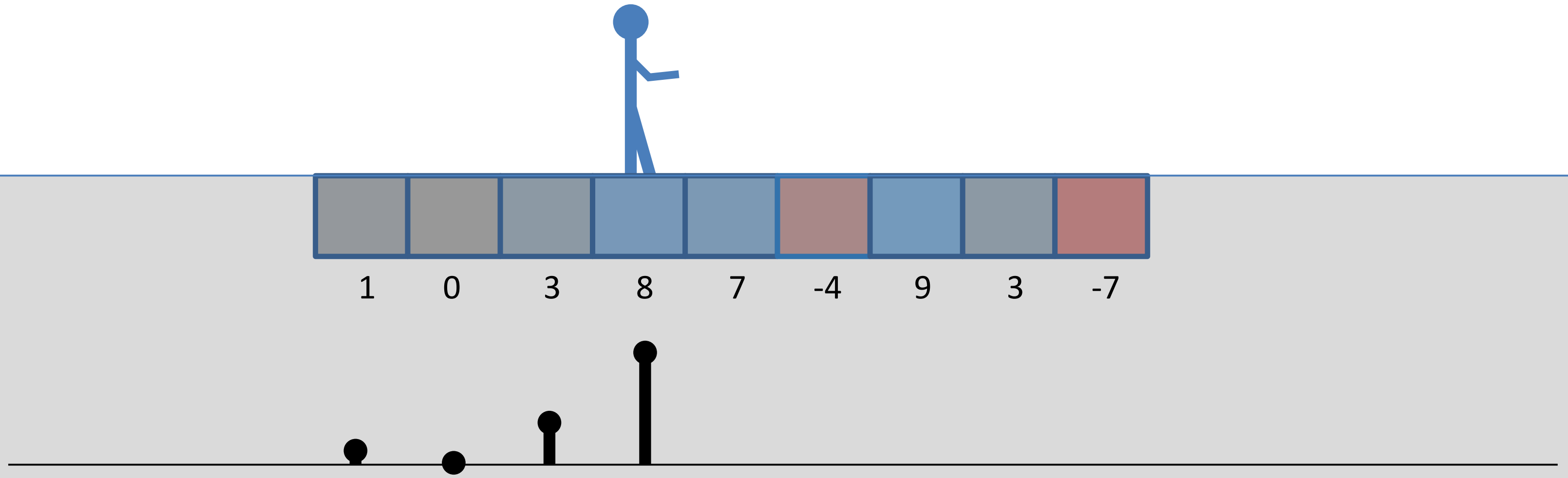
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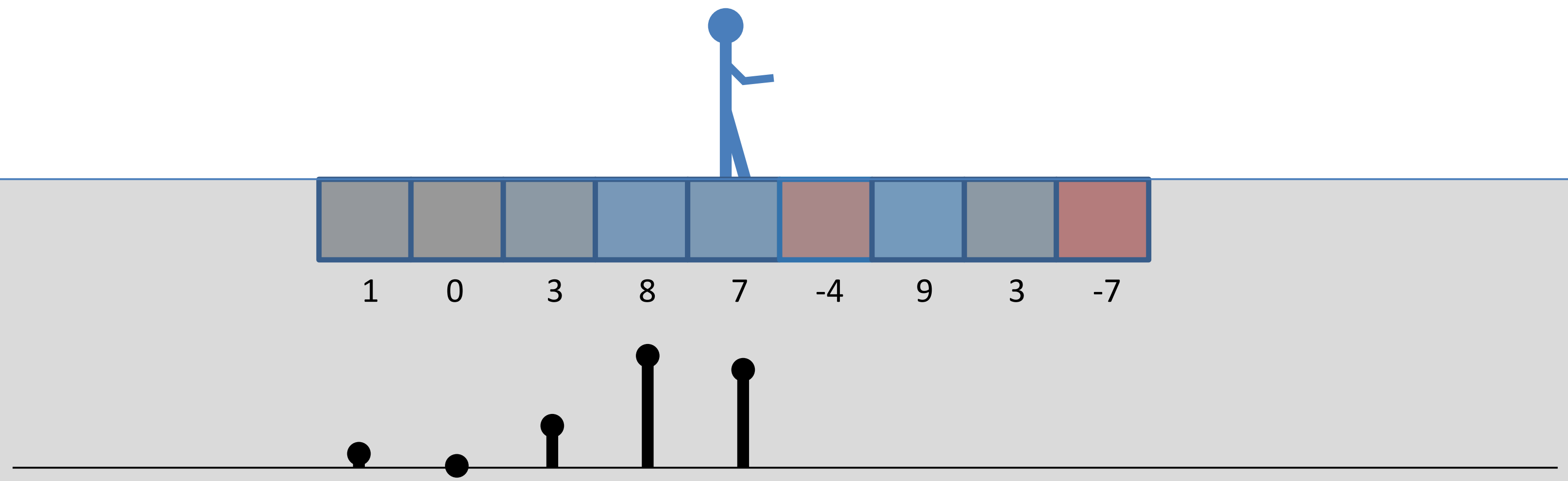
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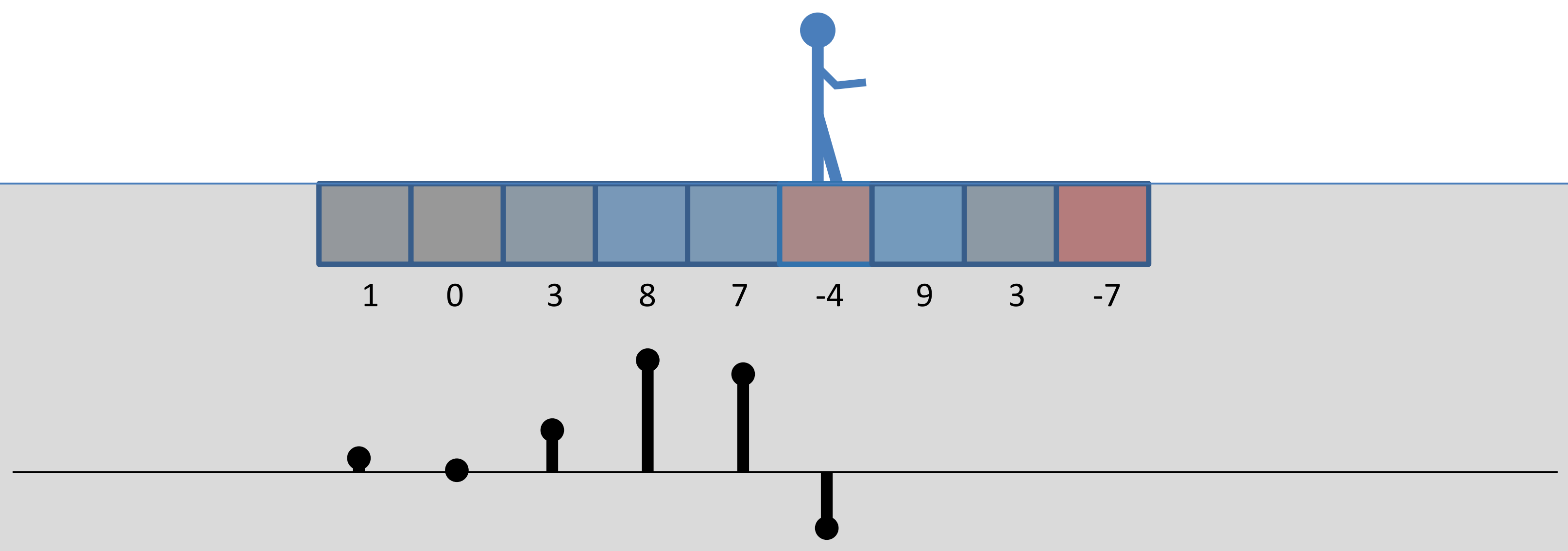
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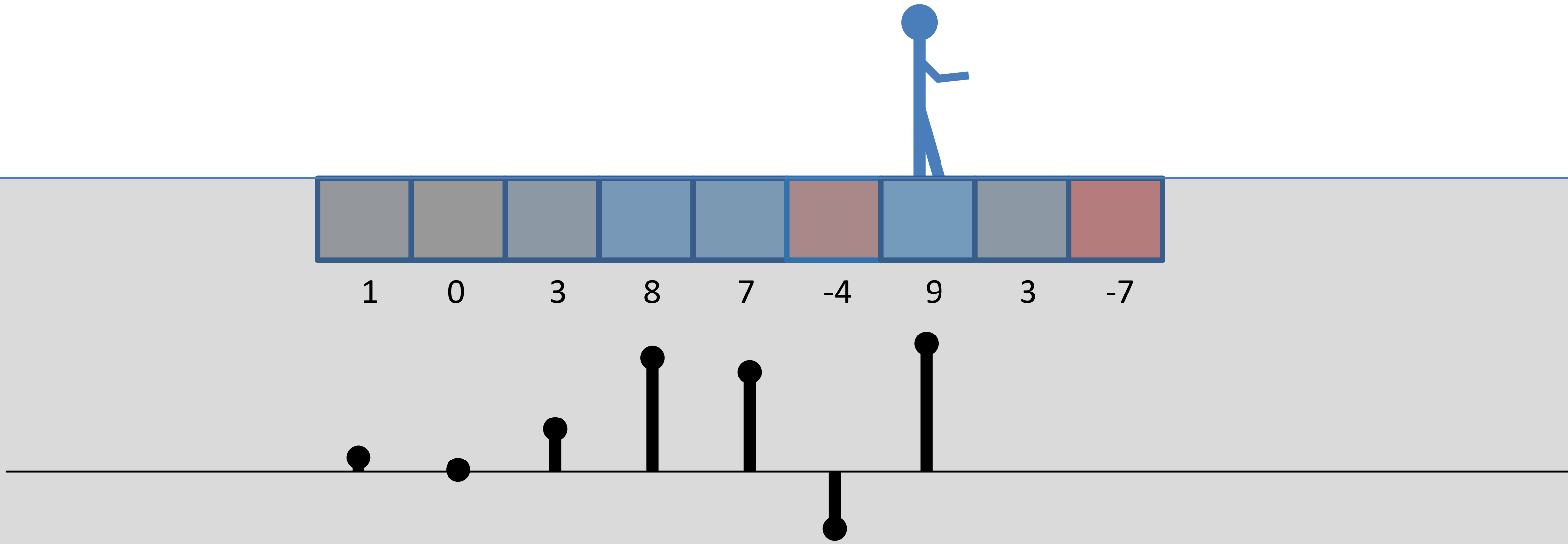
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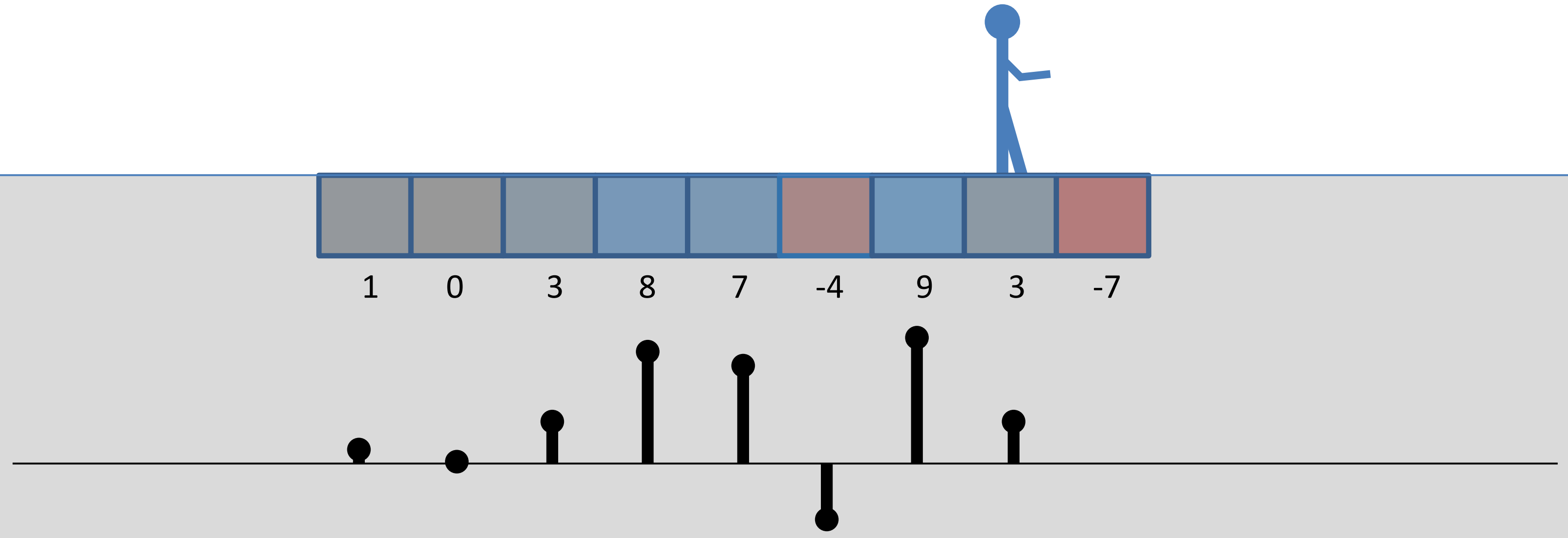
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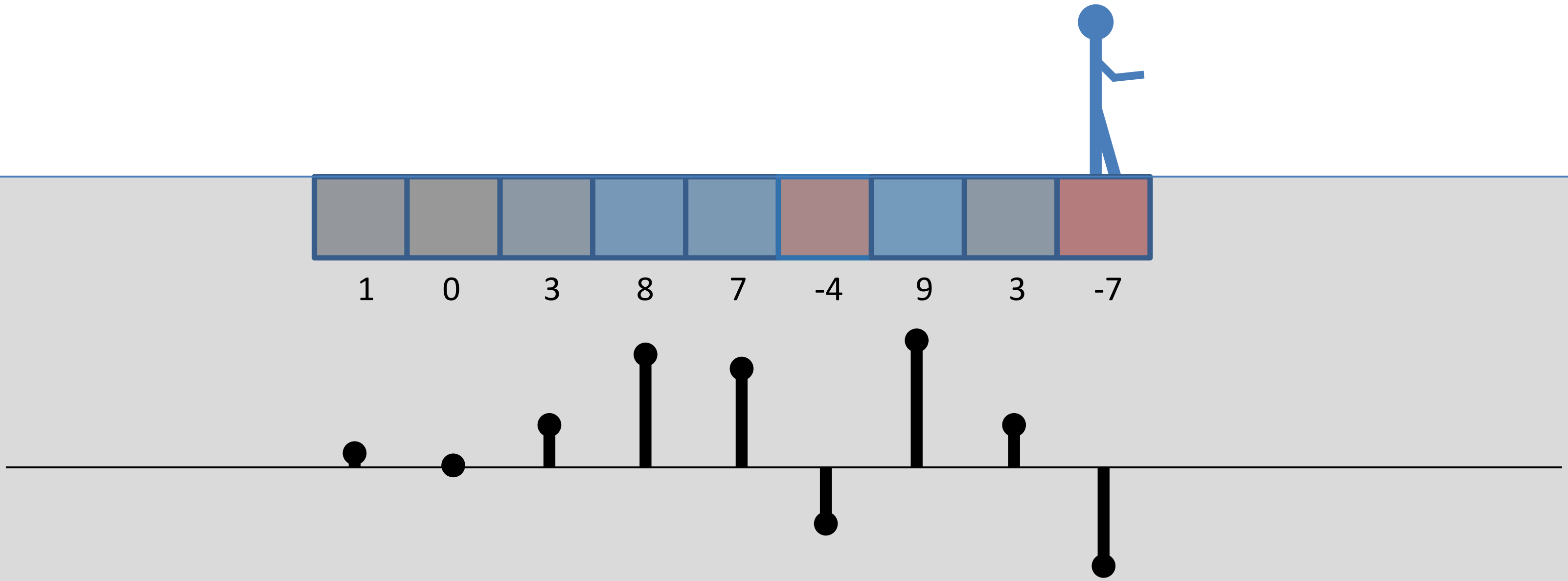

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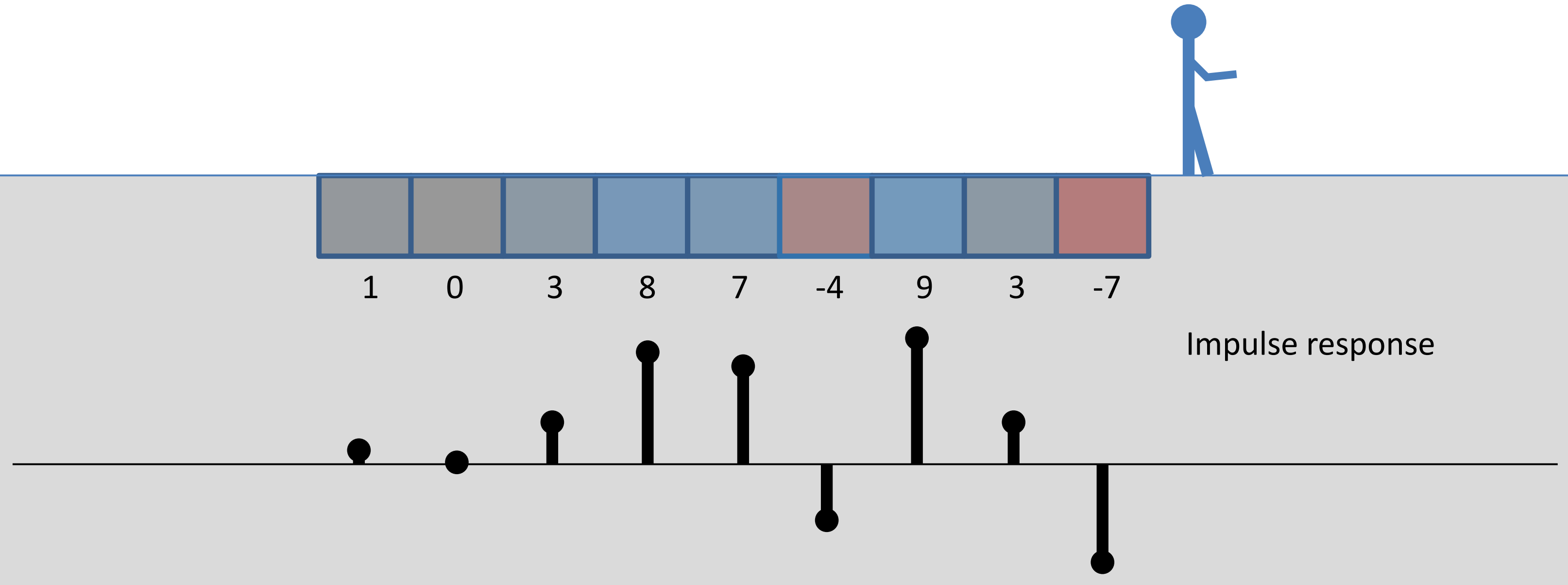
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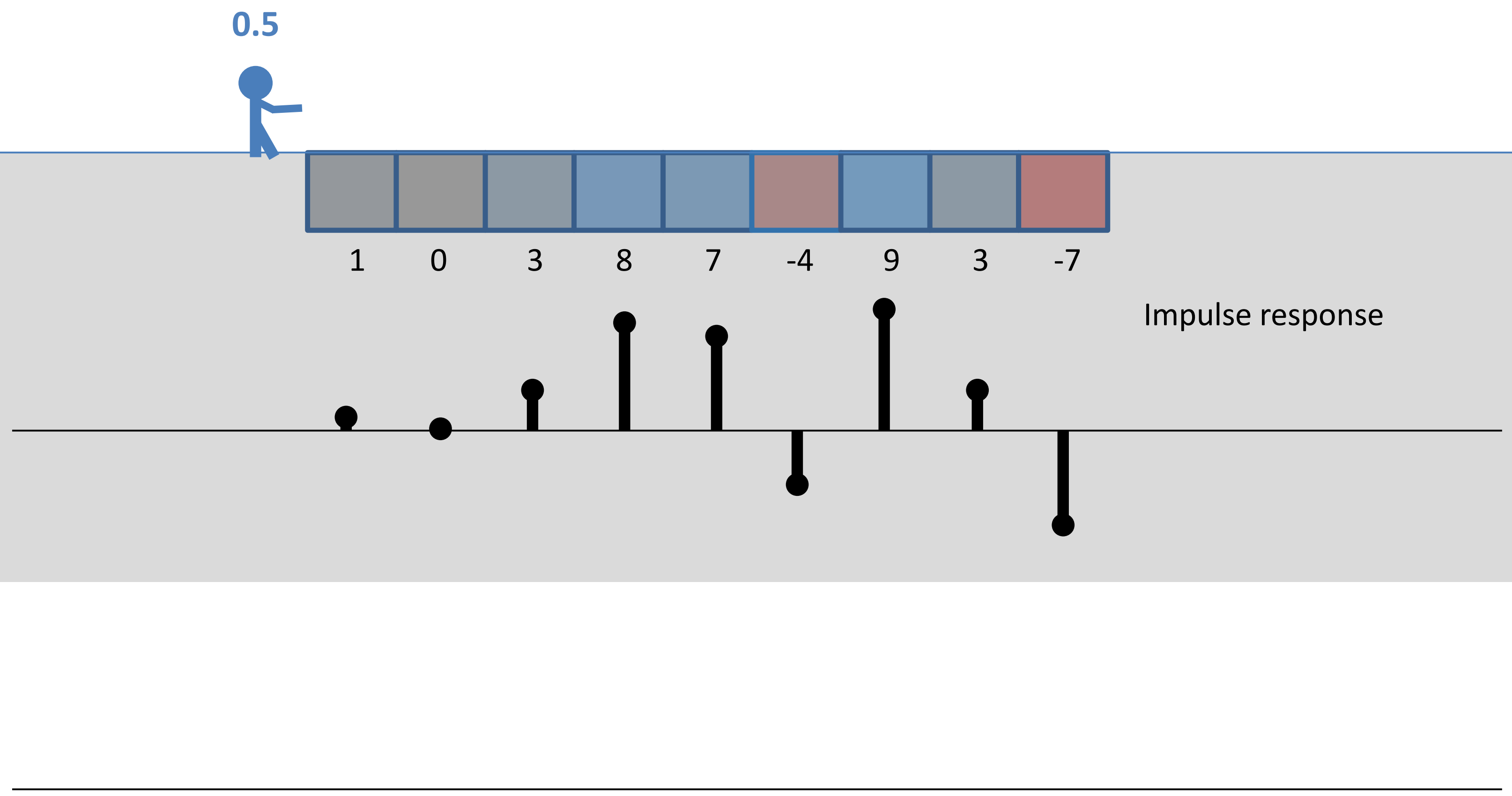
Impulse response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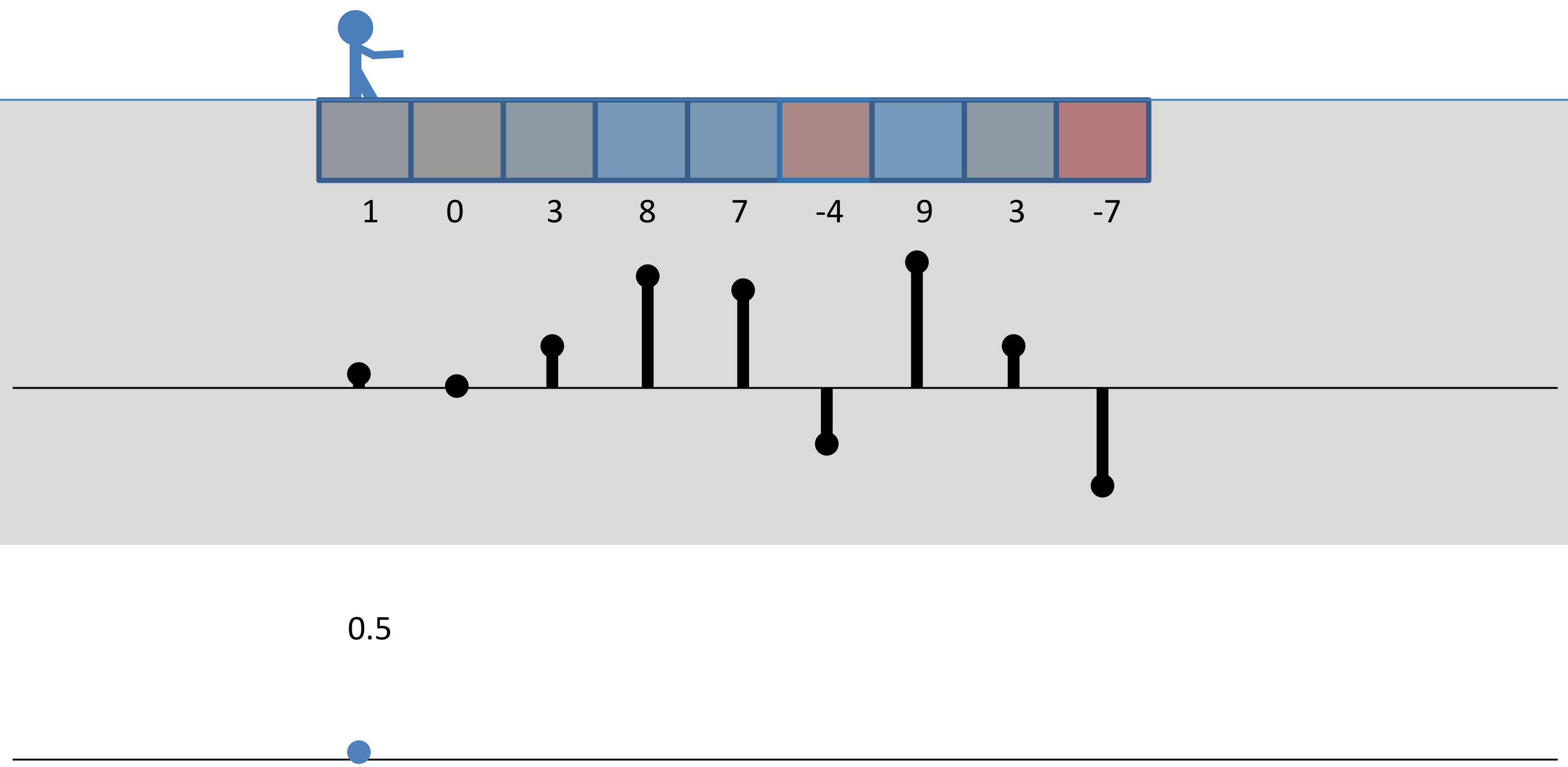
Impulse response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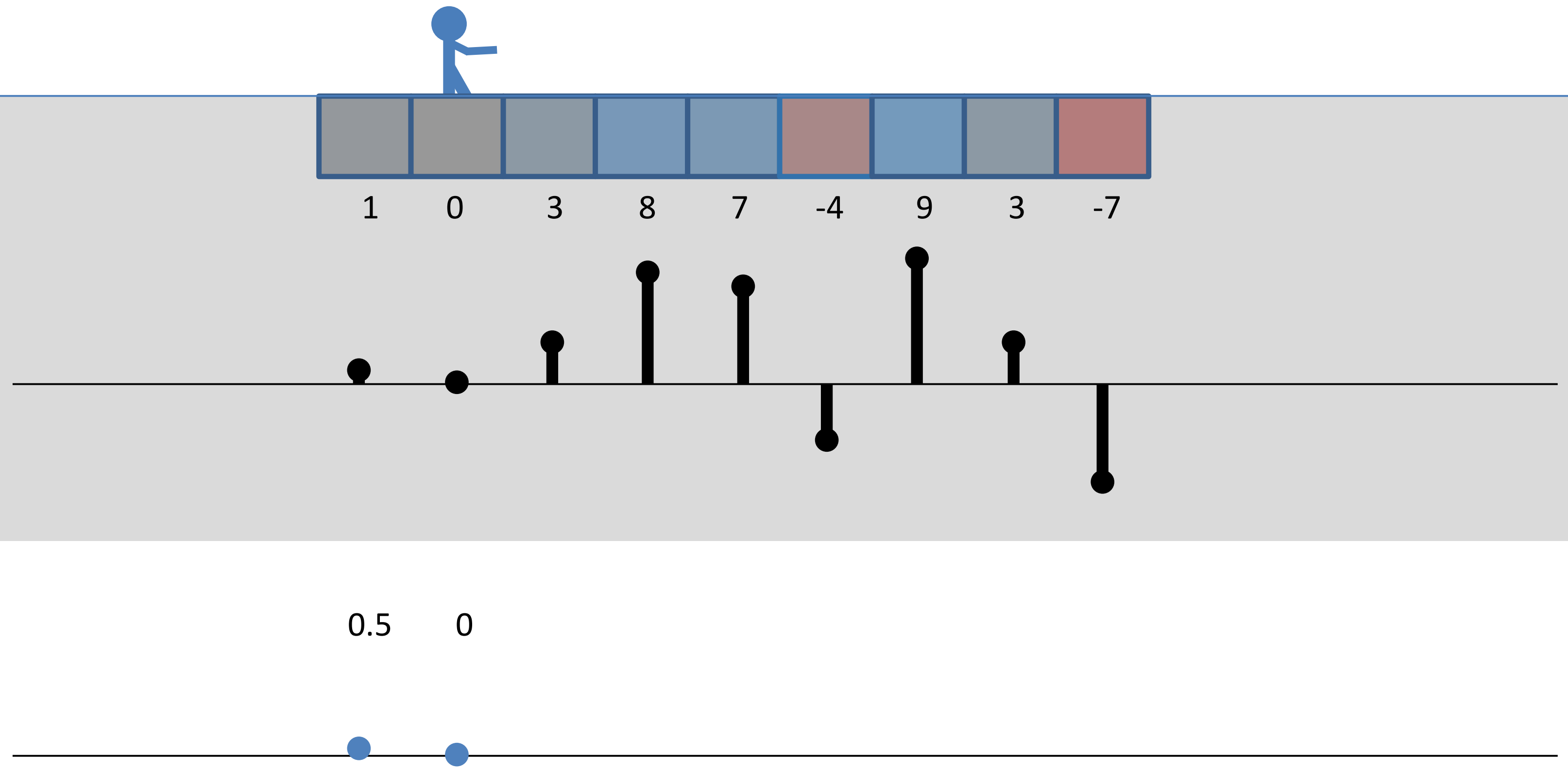

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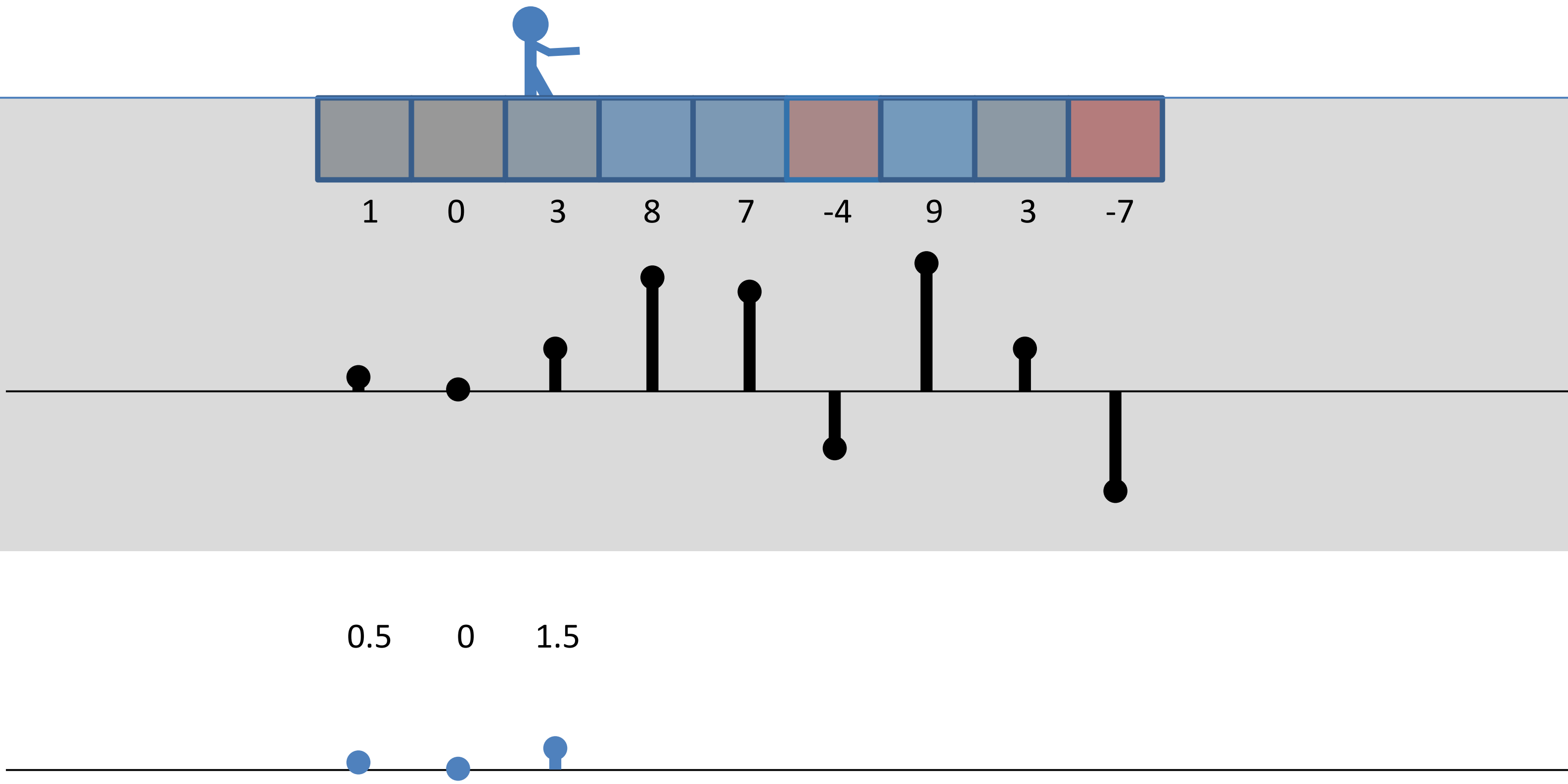
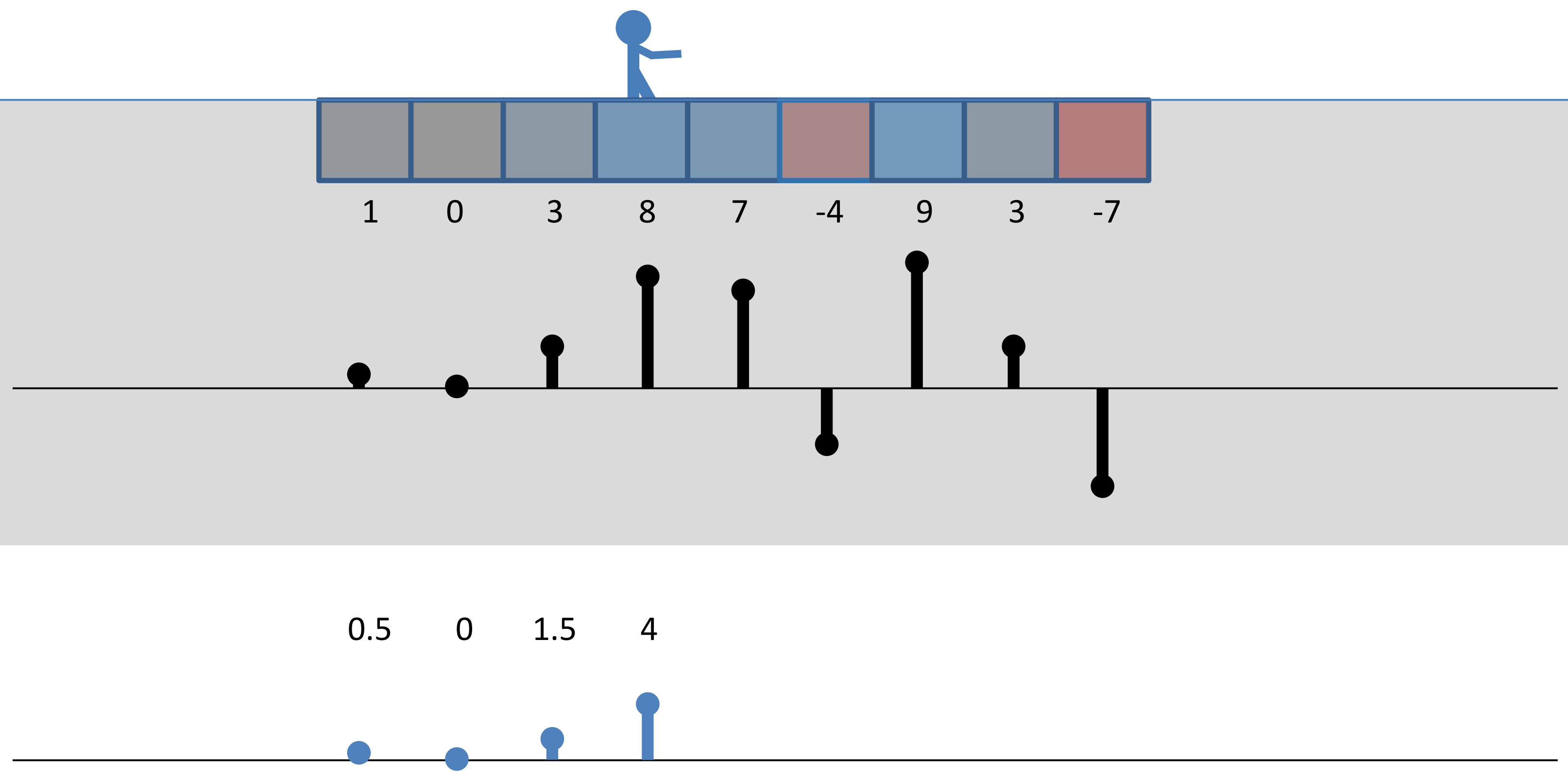
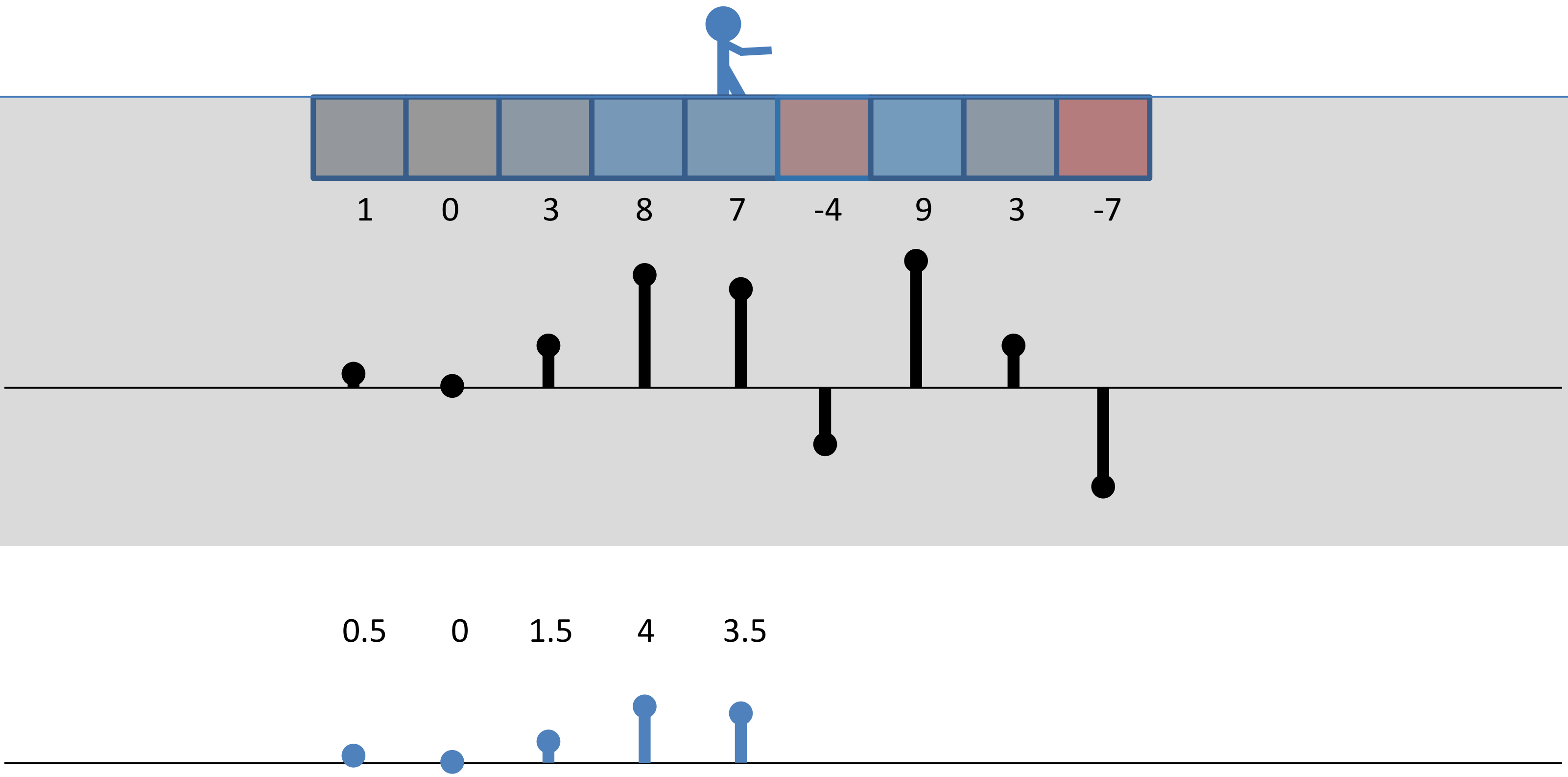
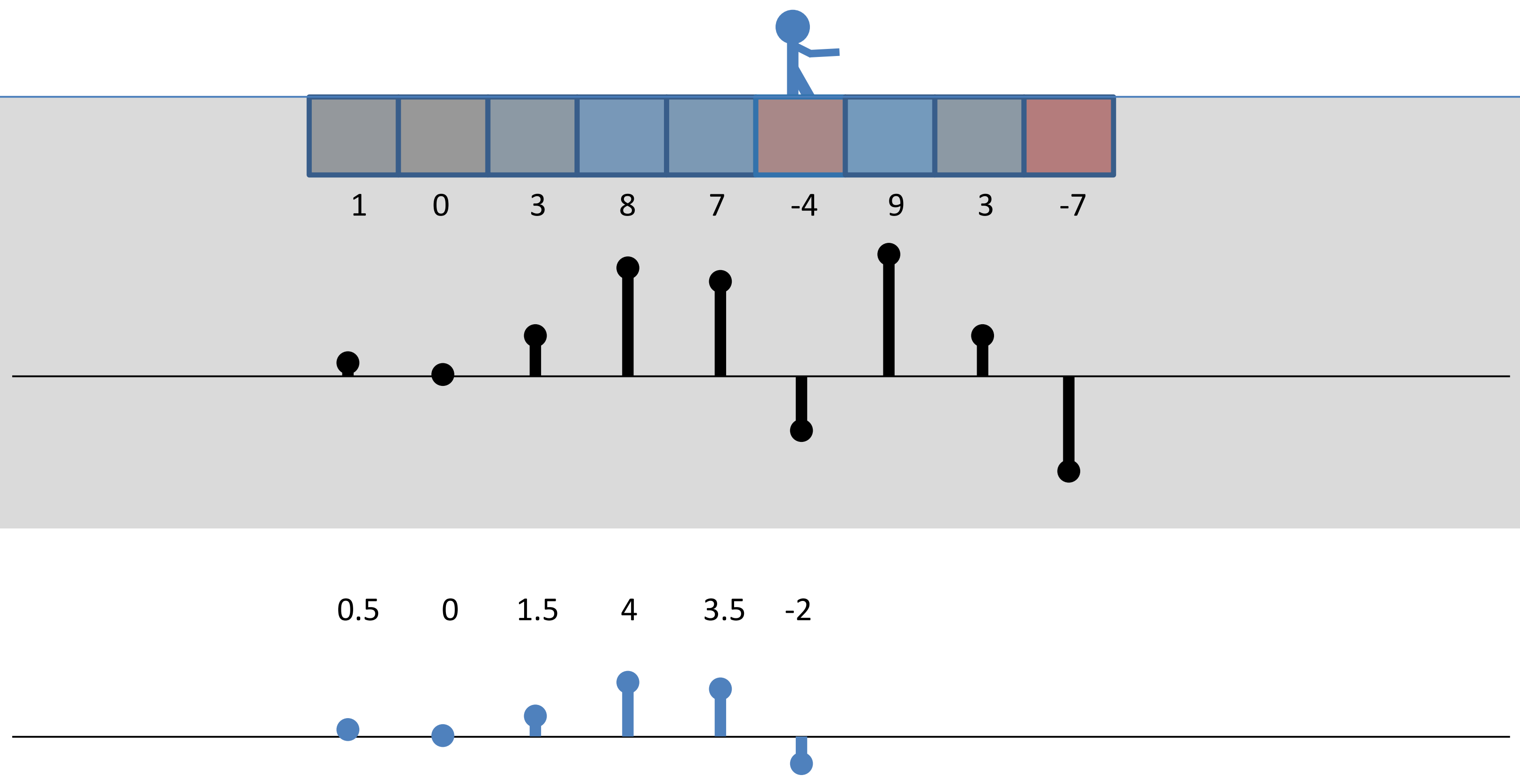


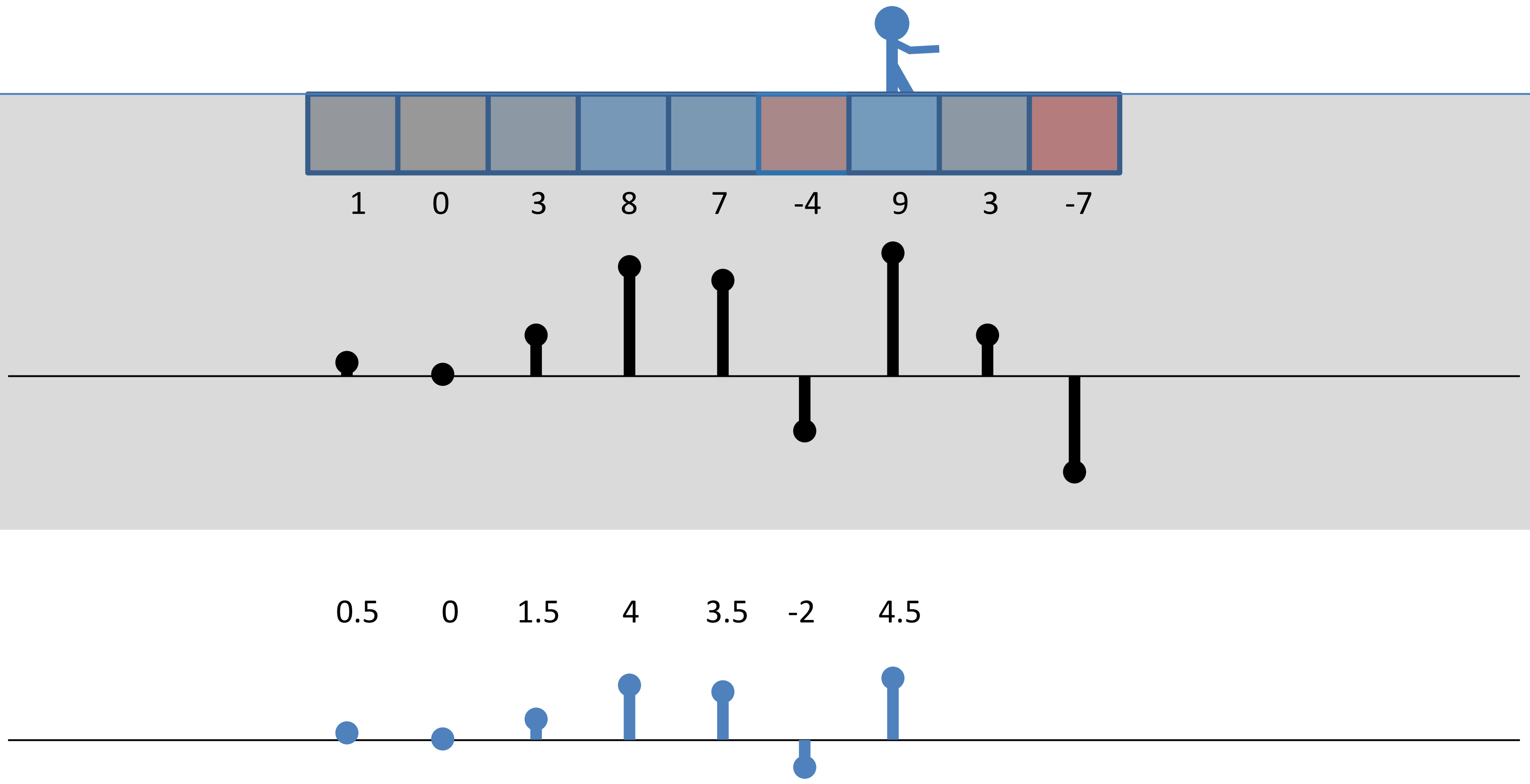
1 0 3 8 7 -4 9 3 -7
0.5 0 1.5

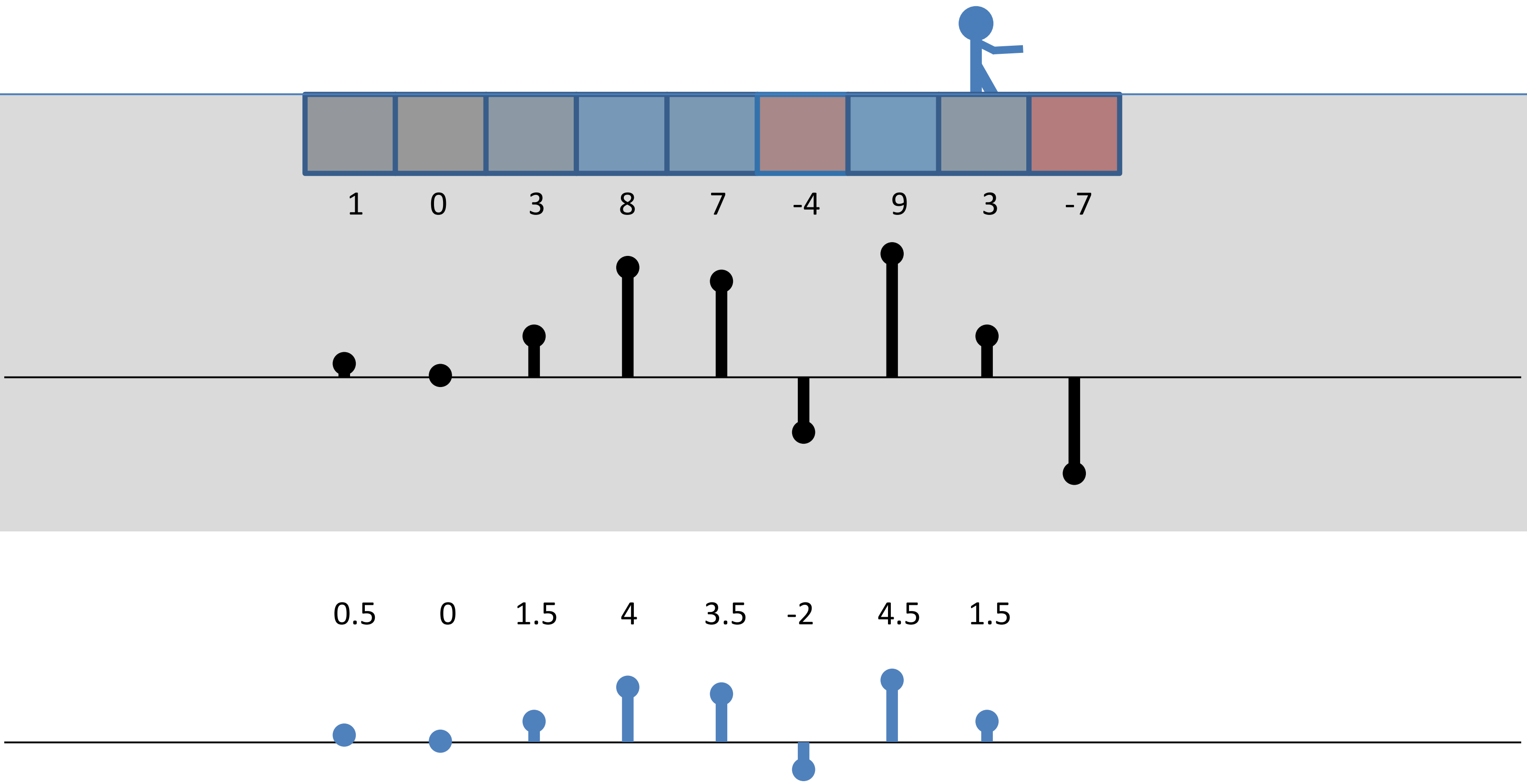

1
0
3
8
7
-4
9
3
-7
0.5
0
1.5
4

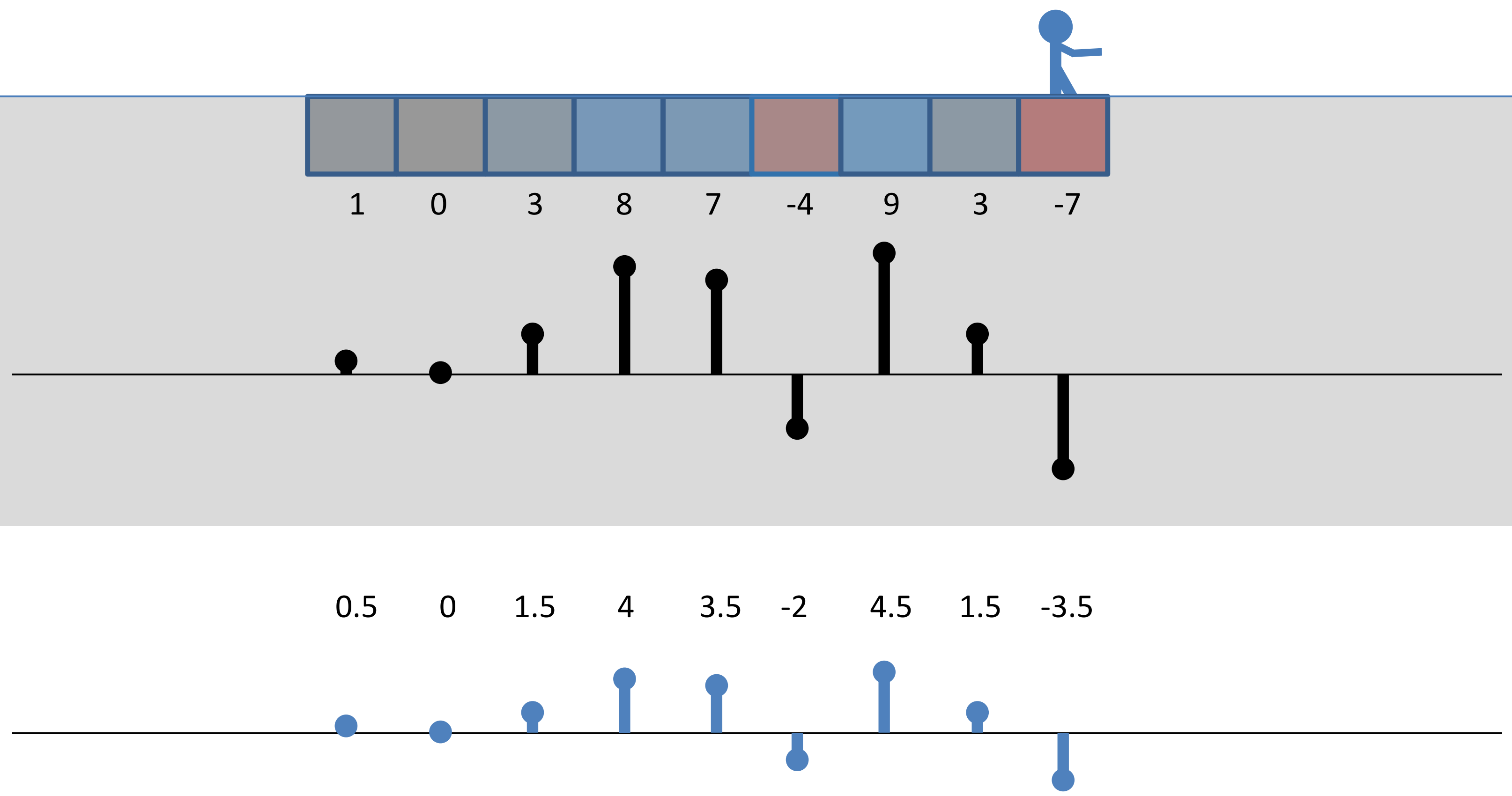

1 0 3 8 7 -4 9 3 -7
0.5 0 1.5 4 3.5

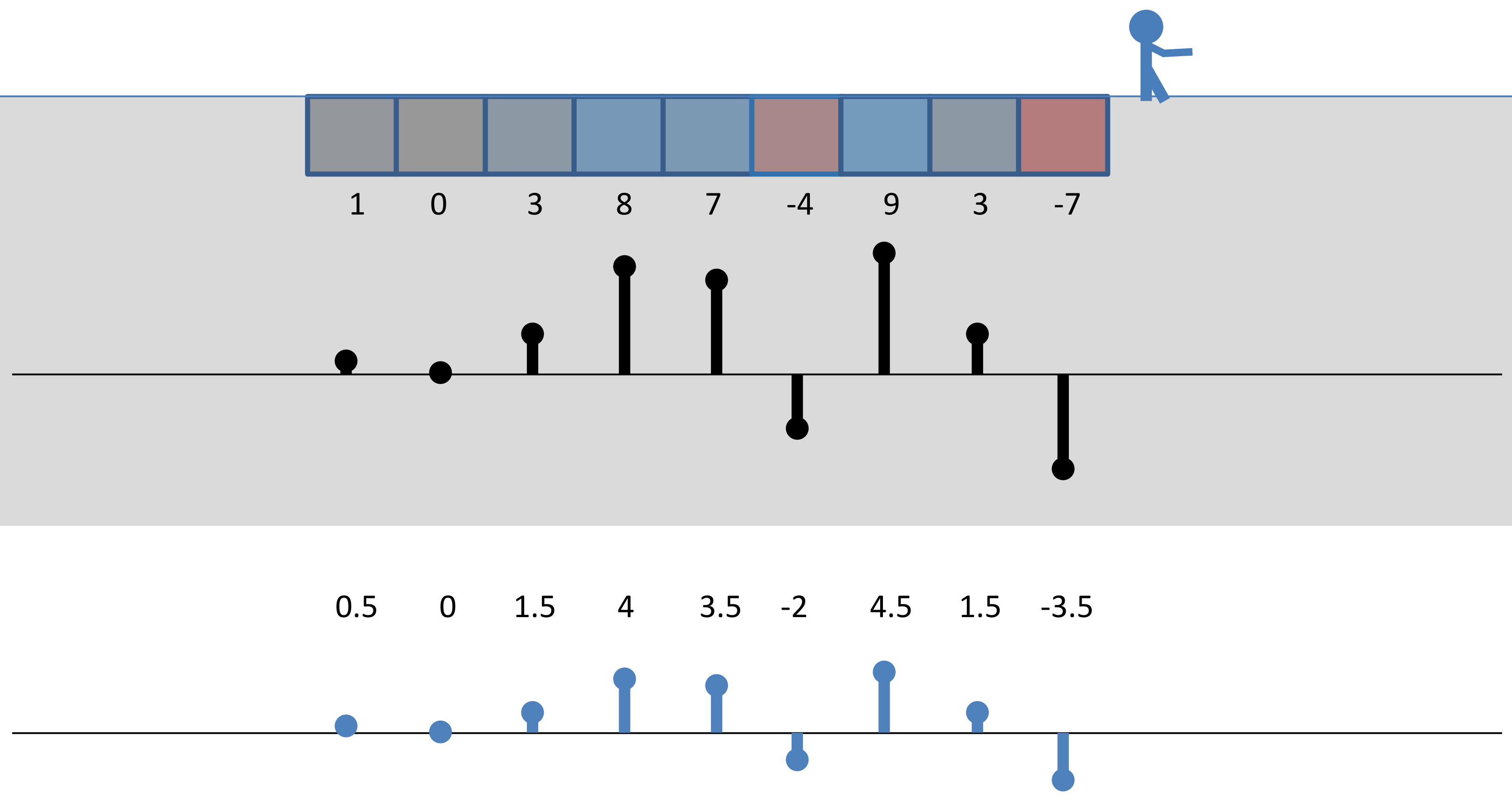


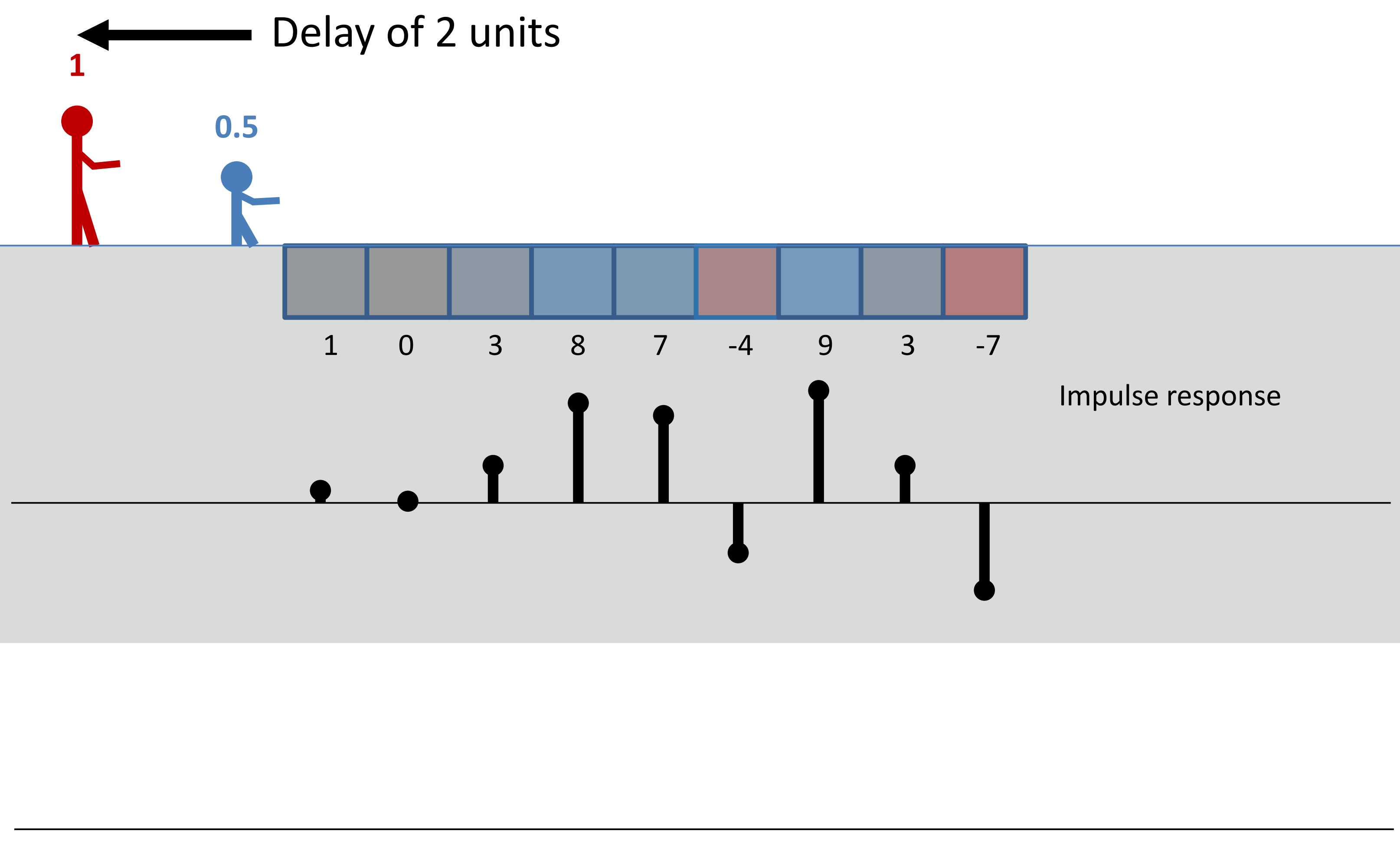
1
0
3
8
7
-4
9
3
-7
0.5
0
1.5
4
3.5
-2

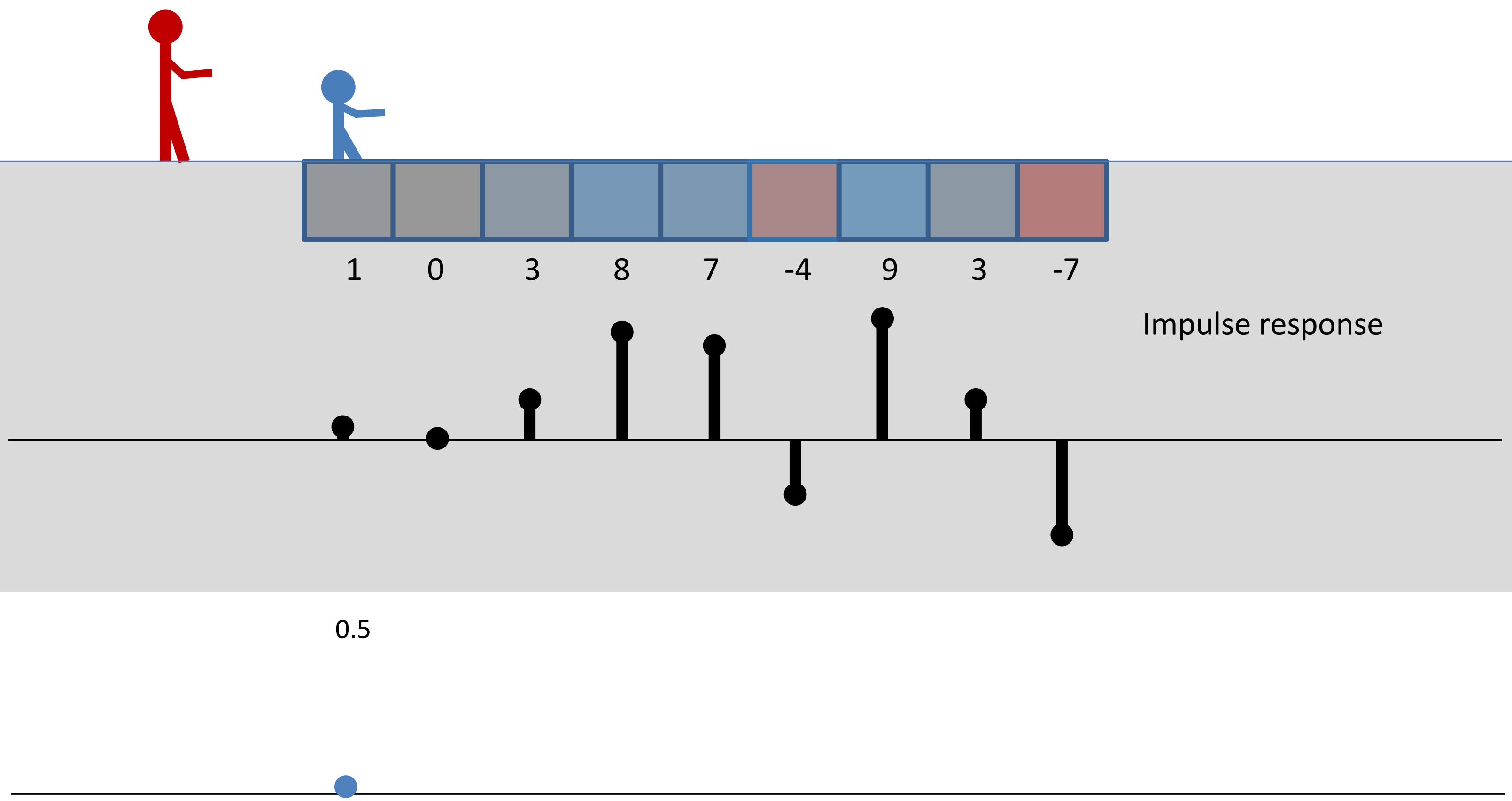

1
0
3
8
7
-4
9
3
-7
0.5
0
1.5
4
3.5
-2
4.5

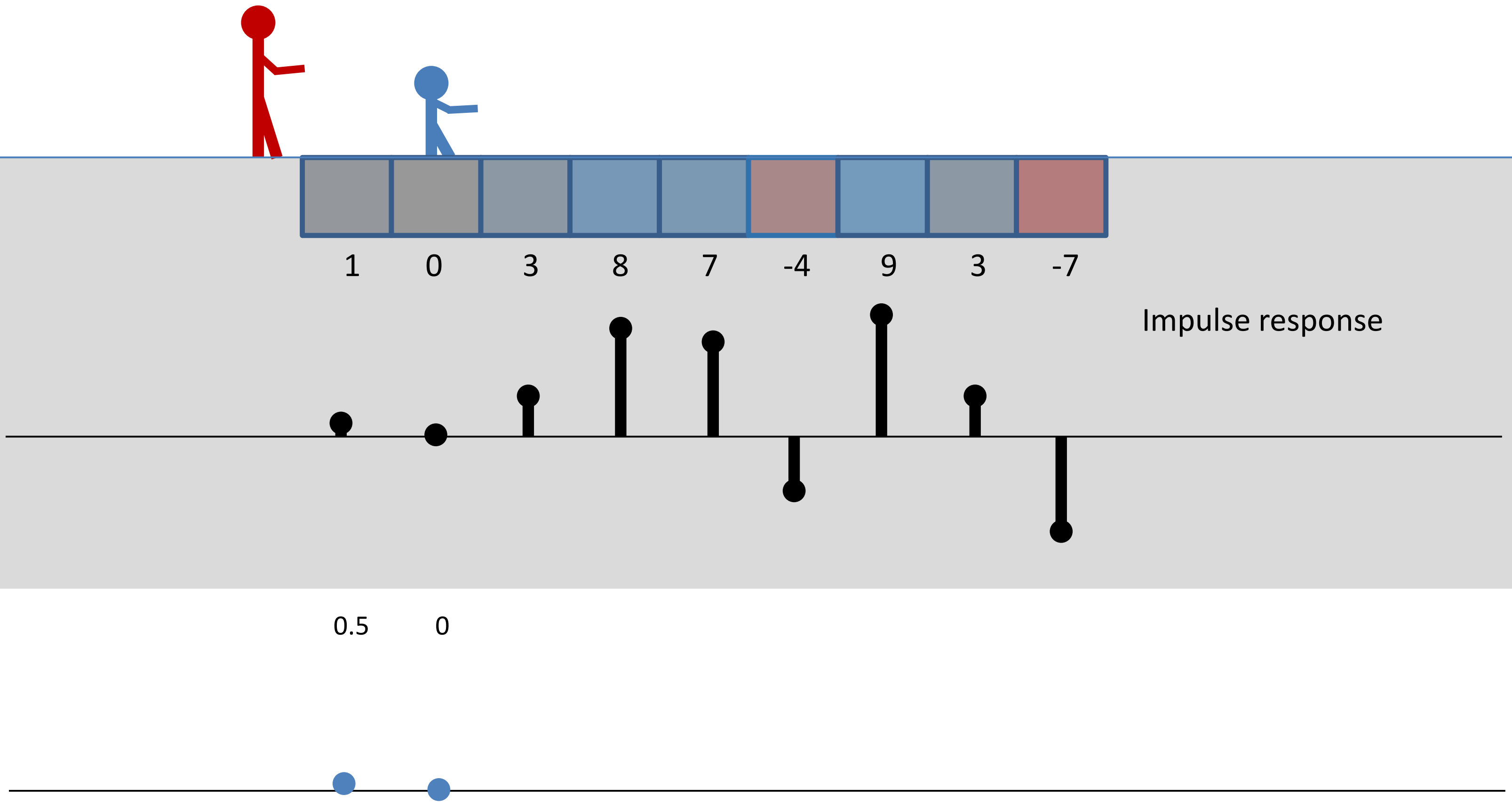

1 0 3 8 7 -4 9 3 -7
0.5 0 1.5 4 3.5 -2 4.5 1.5

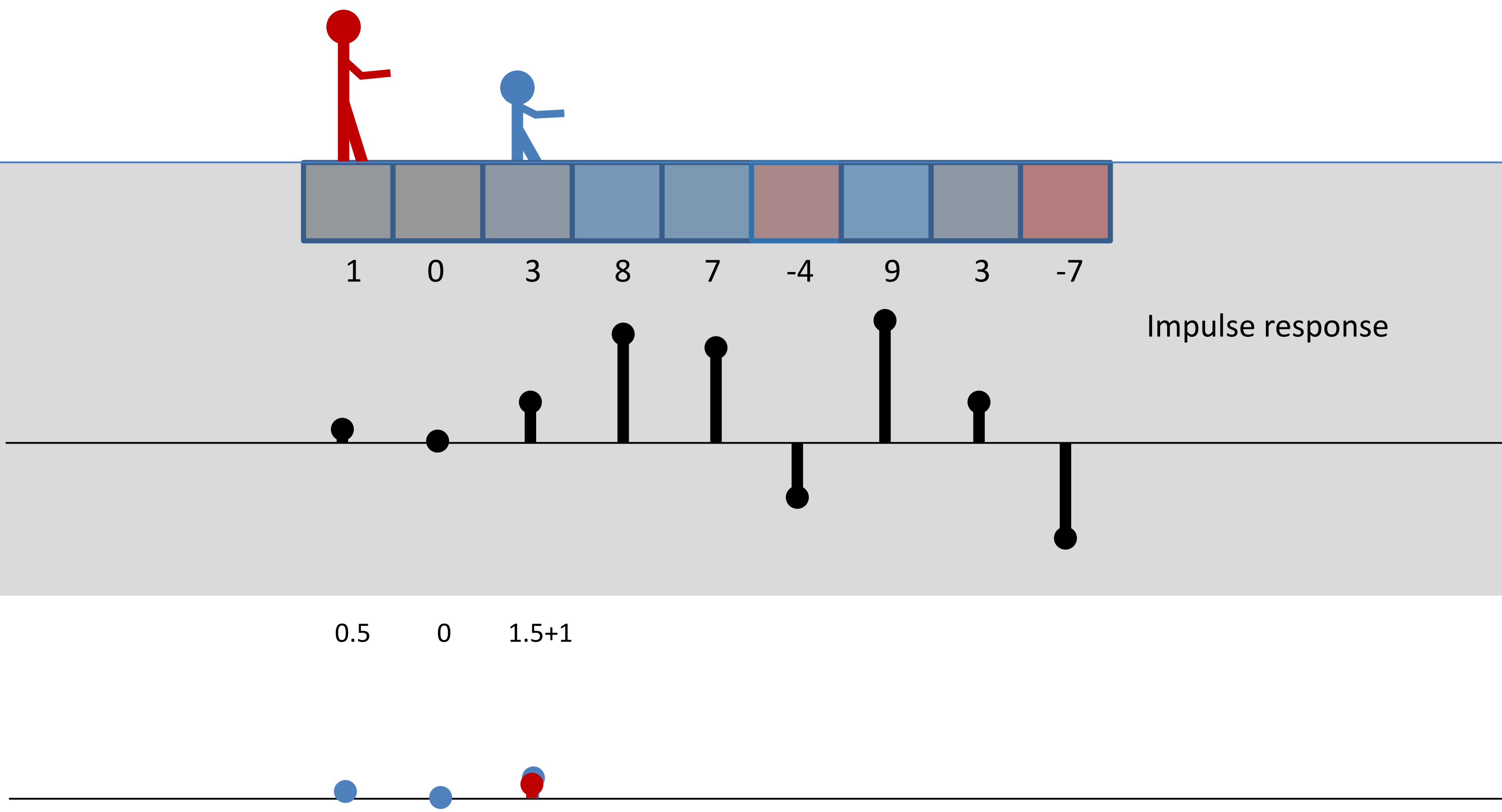

1 0 3 8 7 -4 9 3 -7
0.5 0 1.5 4 3.5 -2 4.5 1.5 -3.5

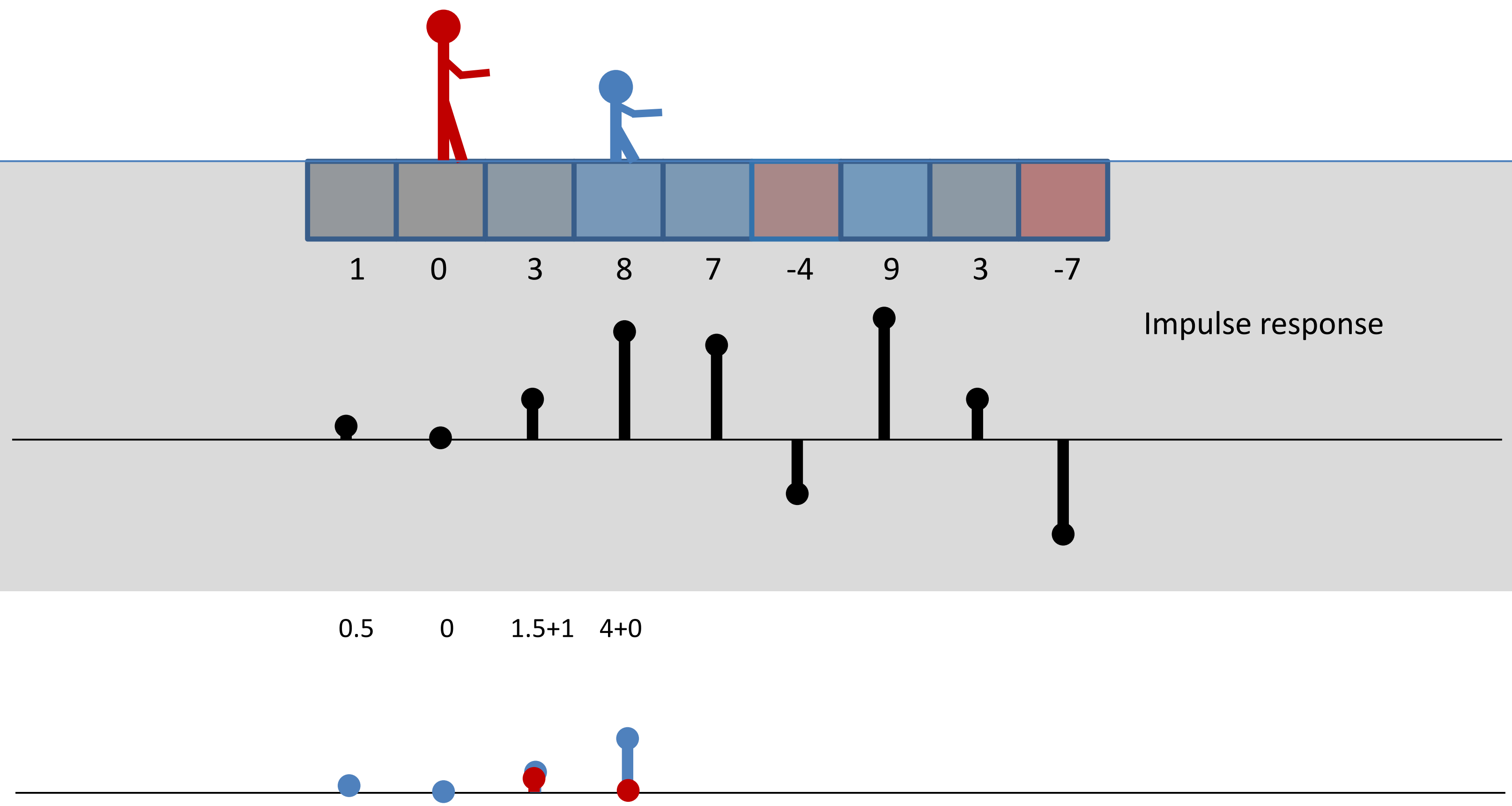

1 0 3 8 7 -4 9 3 -7
0.5 0 1.5 4 3.5 -2 4.5 1.5 -3.5

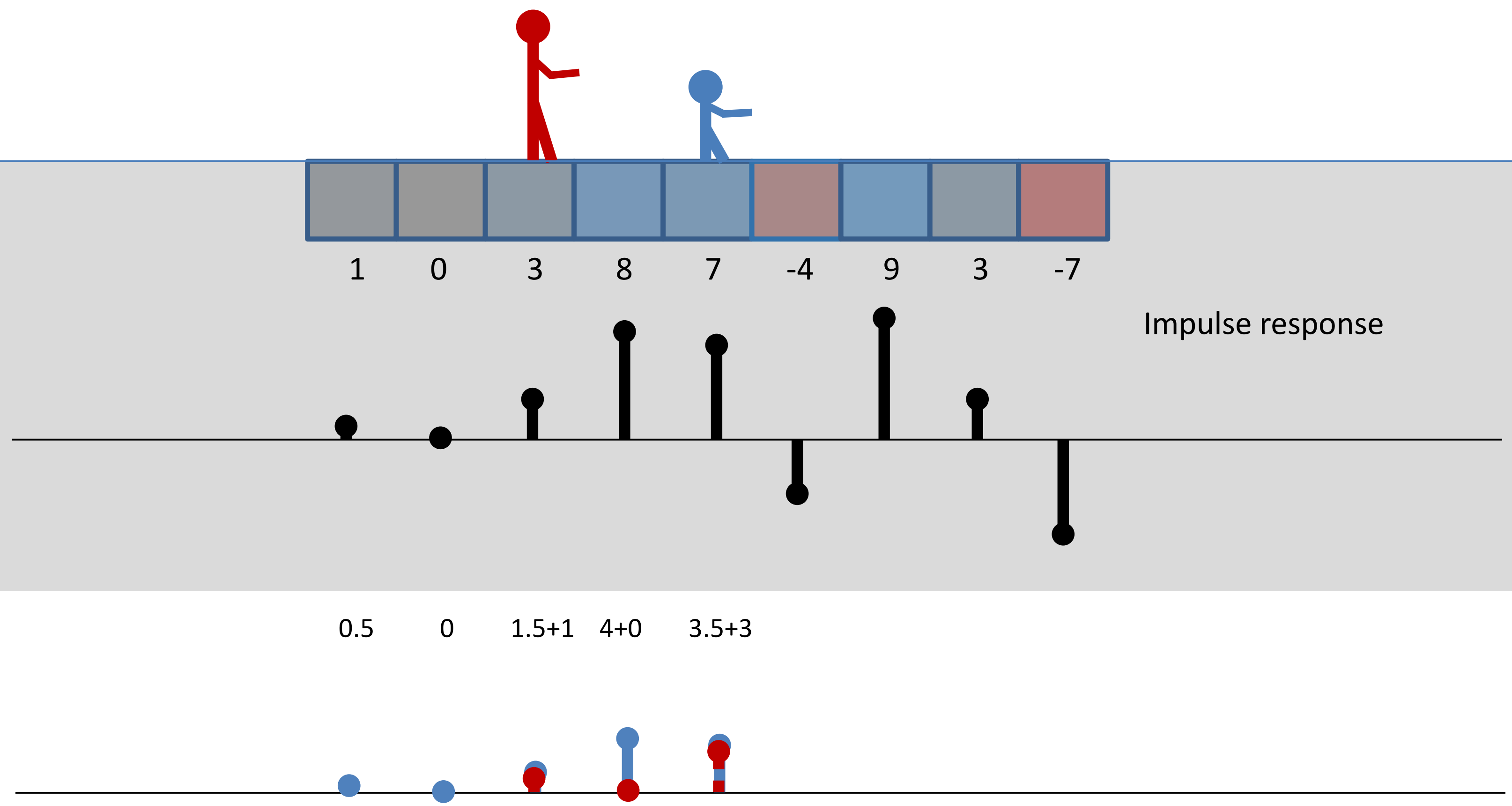

Delay of 2 units
1
0.5
1 0 3 8 7 -4 9 3 -7
Impulse response

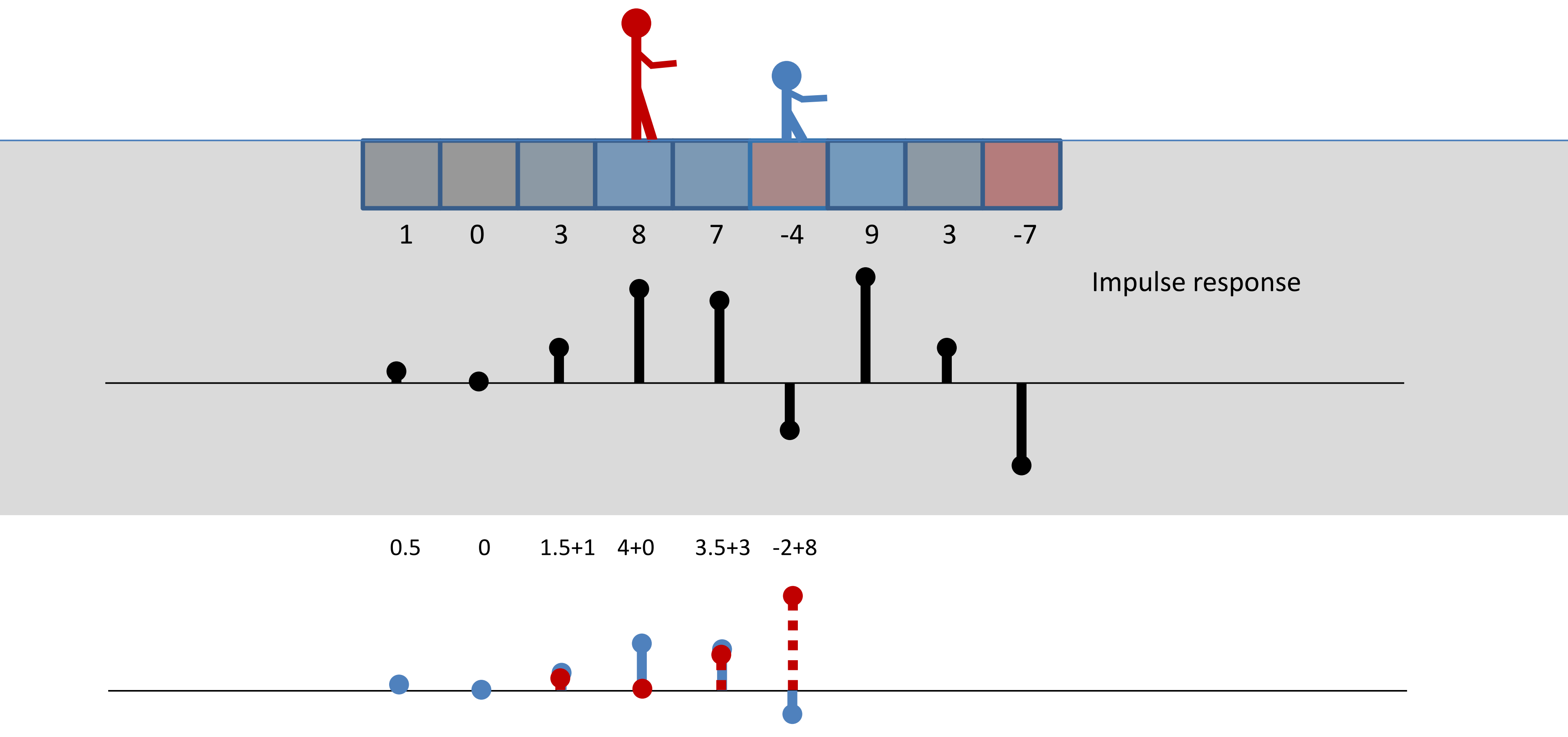

1
0
3
8
7
-4
9
3
-7
Impulse response
0.5

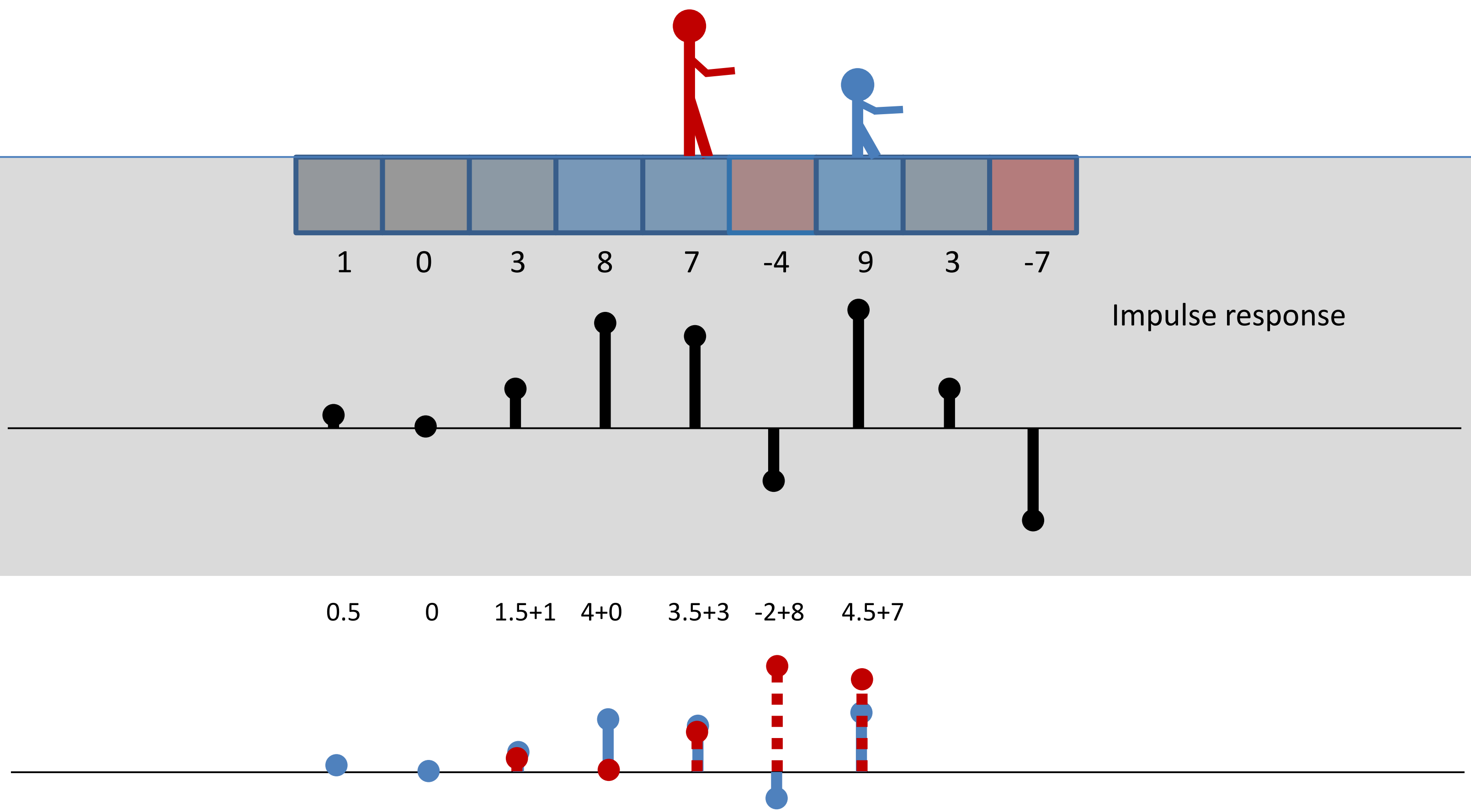

1
0
3
8
7
-4
9
3
-7
Impulse response
0.5
0

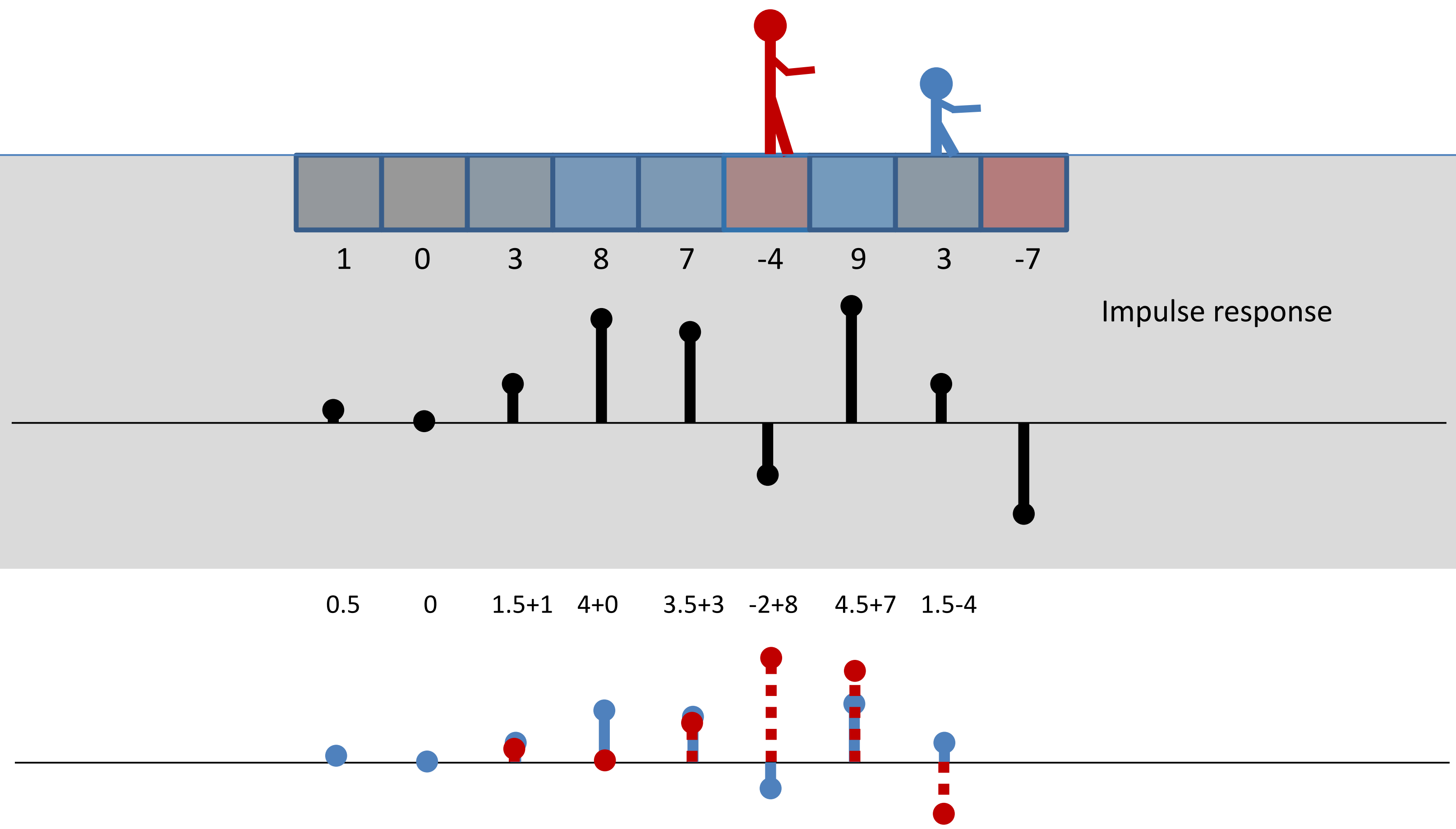

1
0
3
8
7
-4
9
3
-7
Impulse response
0.5
0
1.5+1

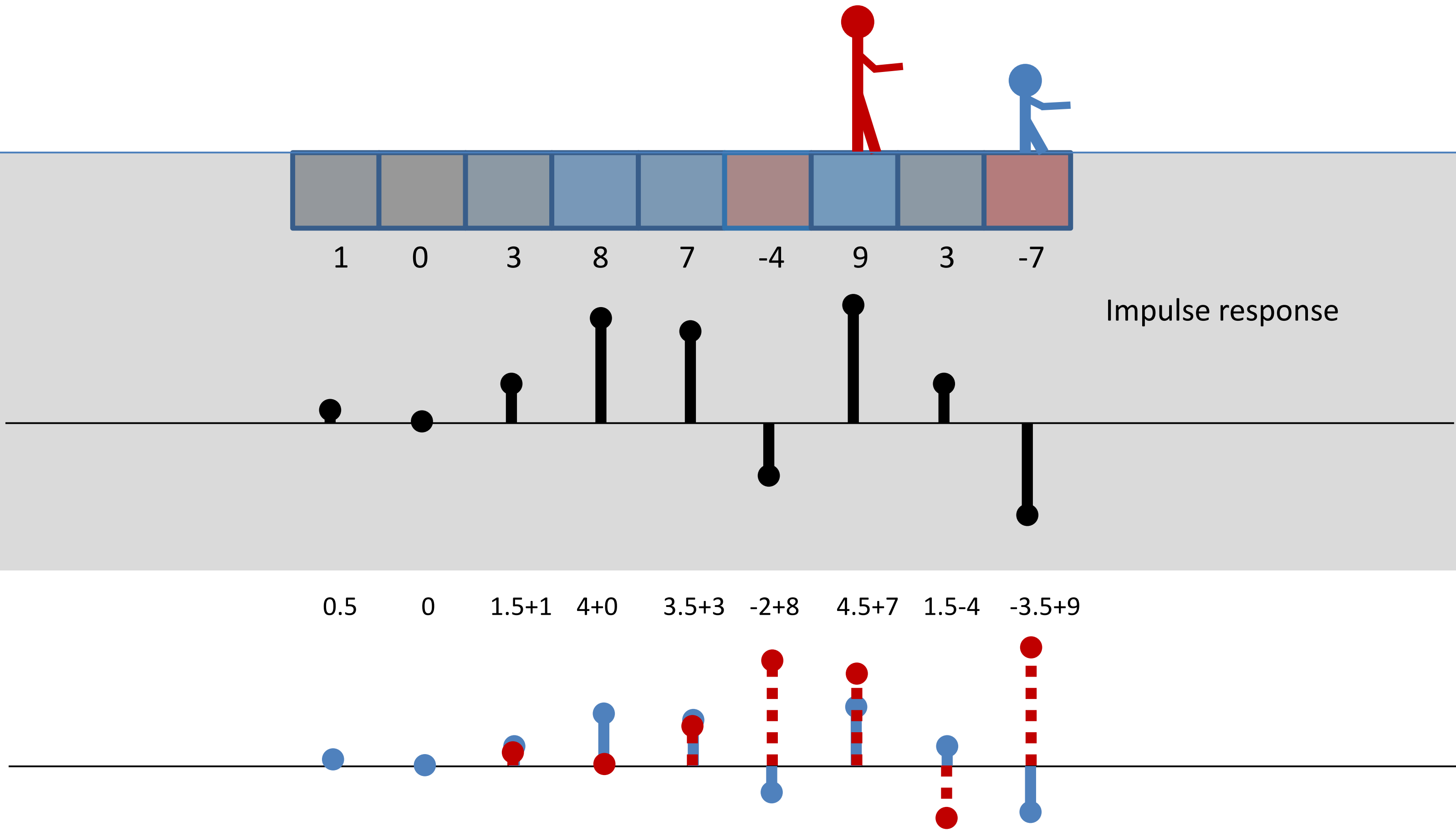

1 0 3 8 7 -4 9 3 -7
Impulse response
0.5 0 1.5+1 4+0

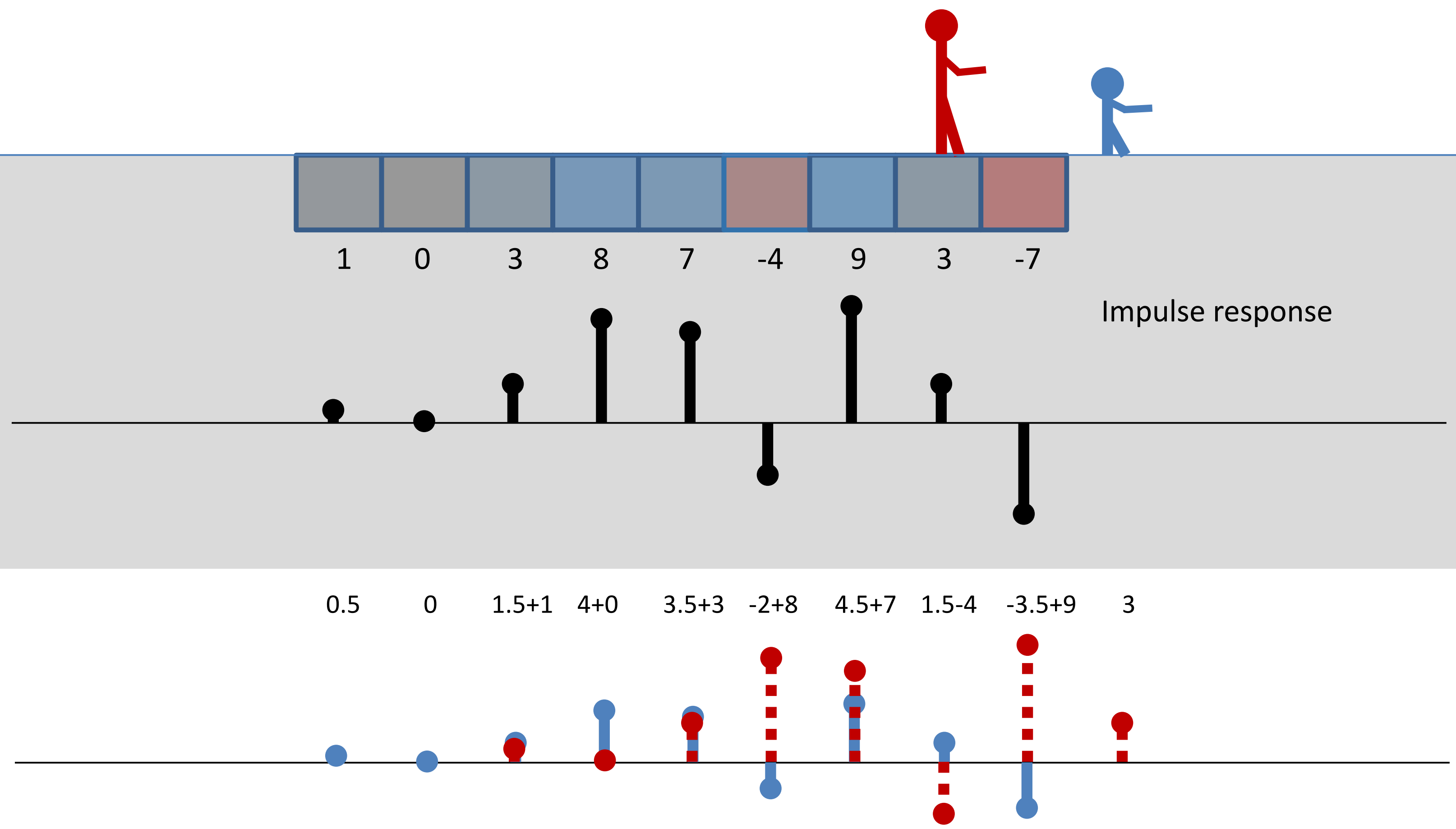

1 0 3 8 7 -4 9 3 -7
Impulse response
0.5
0
1.5+1
4+0
3.5+3

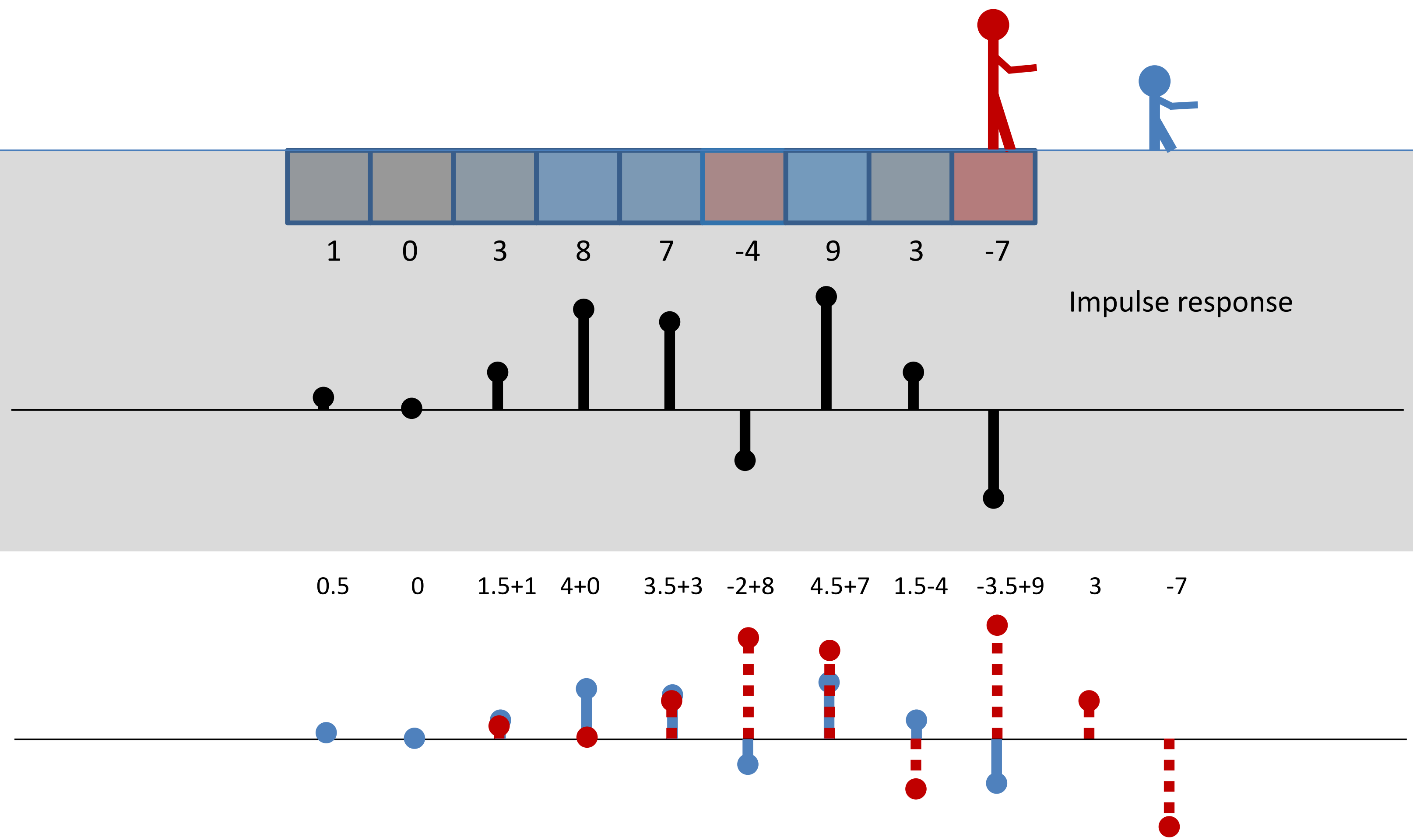

1
0
3
8
7
-4
9
3
-7
Impulse response
0.5
0
1.5+1
4+0
3.5+3
-2+8

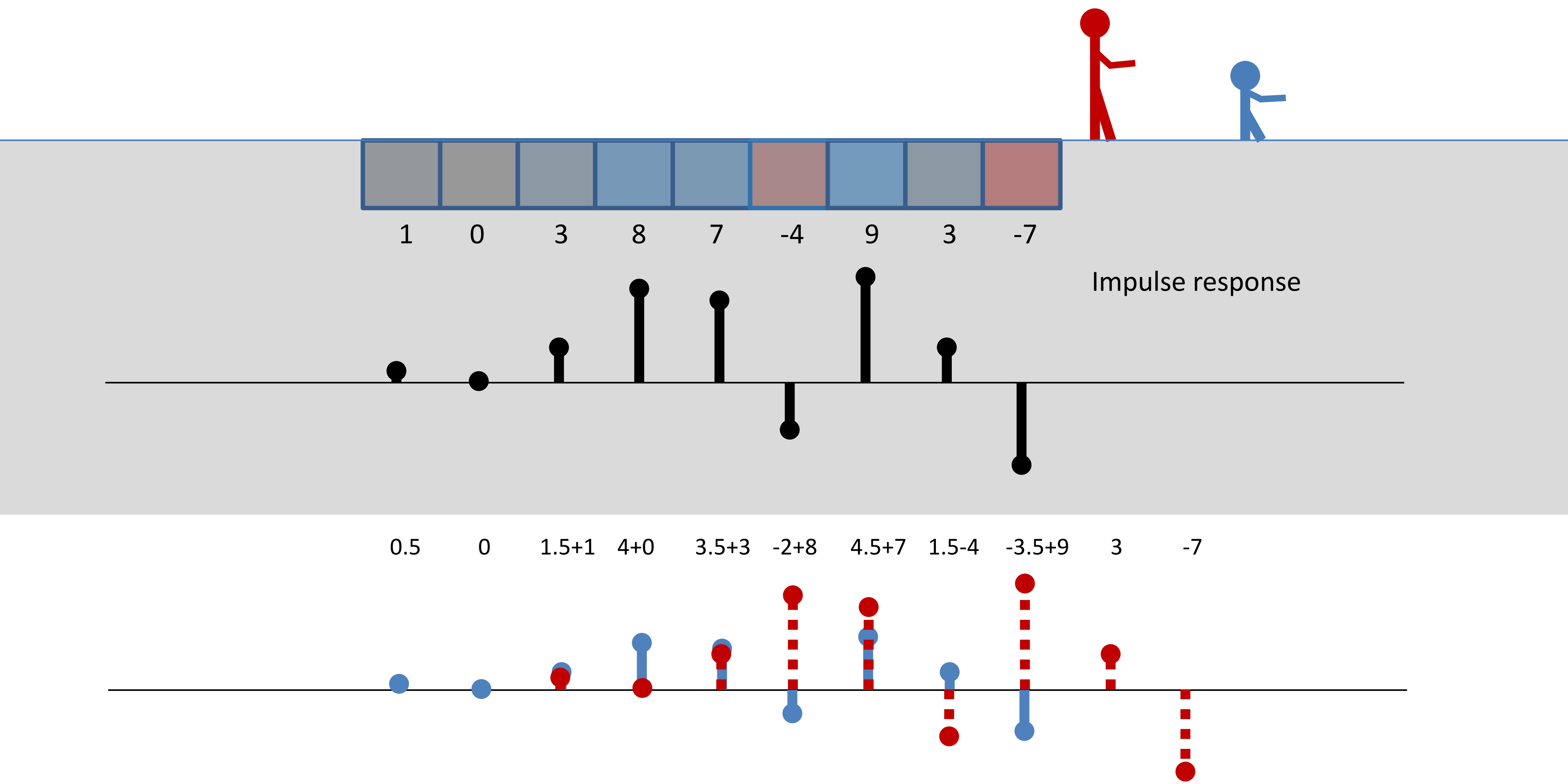

1
0
3
8
7
-4
9
3
-7
Impulse response
0.5
0
1.5+1
4+0
3.5+3
-2+8
4.5+7


1
0
3
8
7
-4
9
3
-7
Impulse response
0.5
0
1.5+1
4+0
3.5+3
-2+8
4.5+7
1.5-4

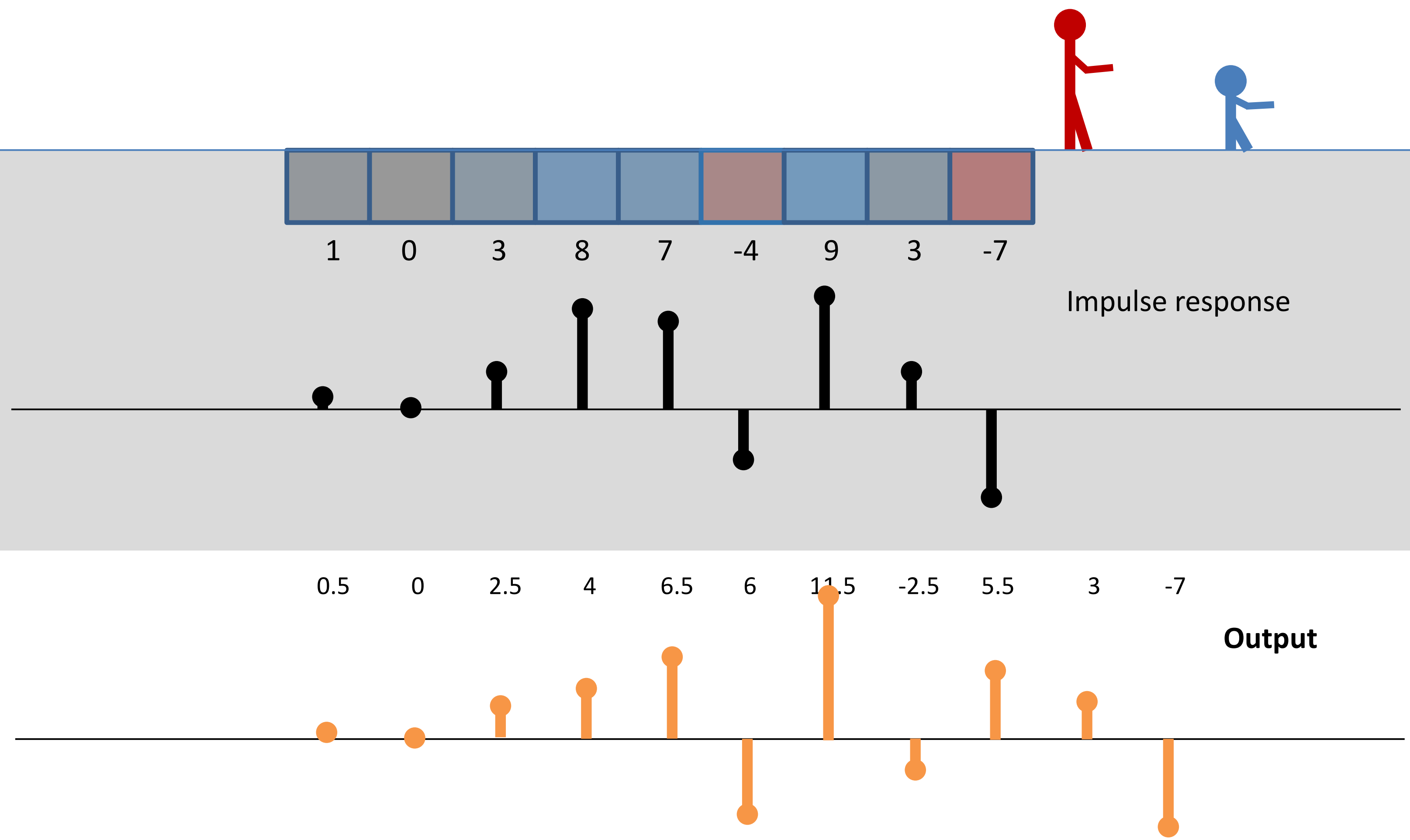

1
0
3
8
7
-4
9
3
-7
Impulse response
0.5
0
1.5+1
4+0
3.5+3
-2+8
4.5+7
1.5-4
-3.5+9


1
0
3
8
7
-4
9
3
-7
Impulse response
0.5
0
1.5+1
4+0
3.5+3
-2+8
4.5+7
1.5-4
-3.5+9
3

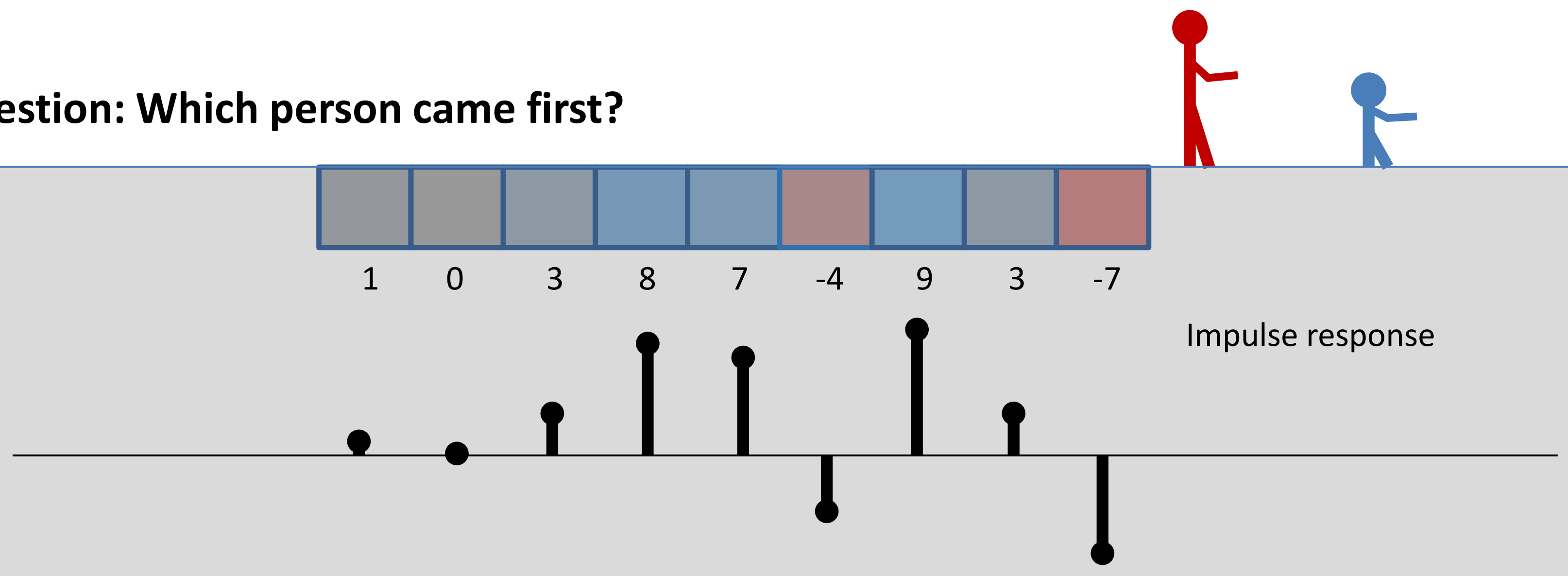

1 0 3 8 7 -4 9 3 -7
Impulse response
0.5 0 1.5+1 4+0 3.5+3 -2+8 4.5+7 1.5-4 -3.5+9 3 -7

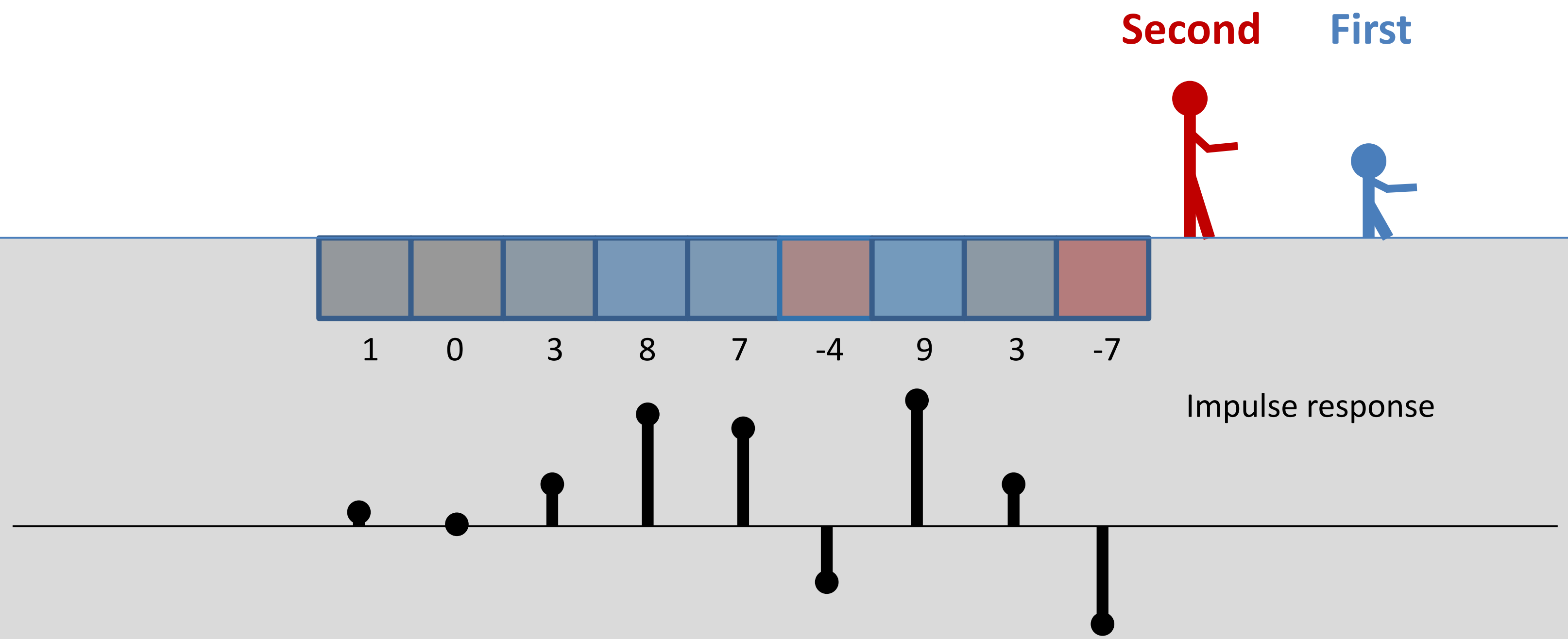


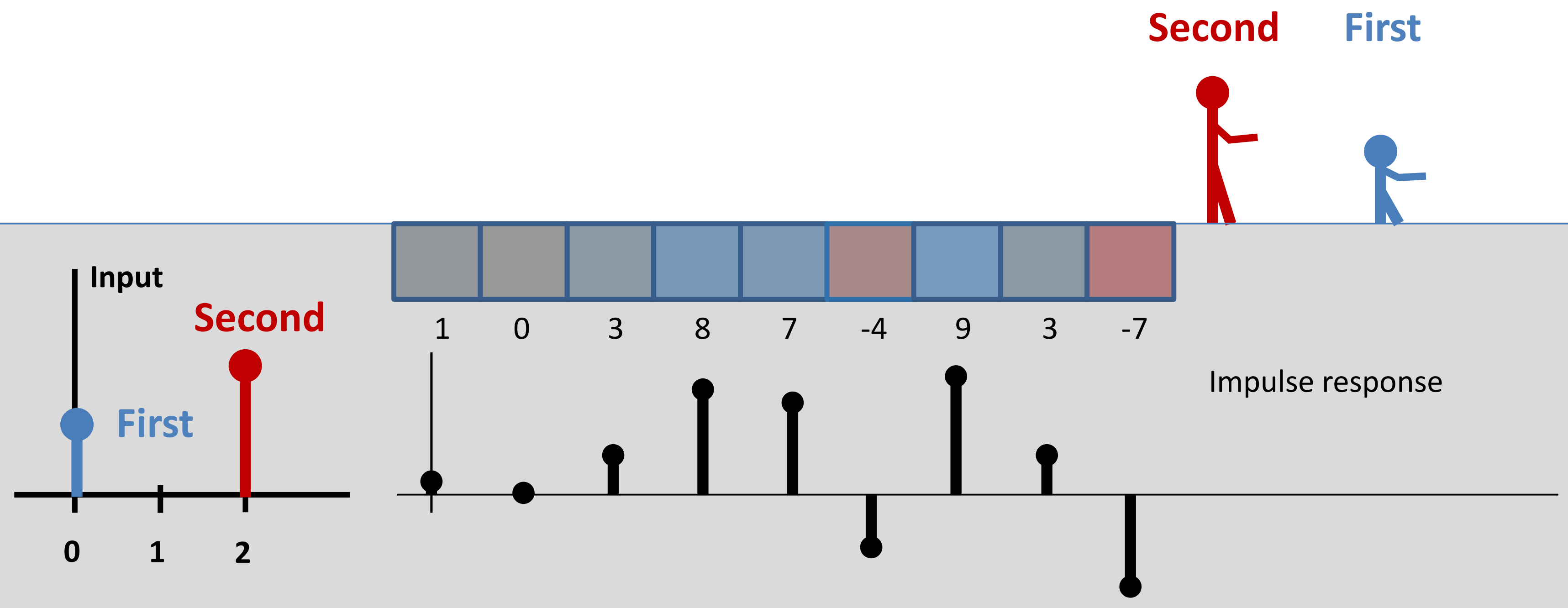
1
0
3
8
7
-4
9
3
-7
Impulse response
0.5
0
1.5+1
4+0
3.5+3
-2+8
4.5+7
1.5-4
-3.5+9
3
-7

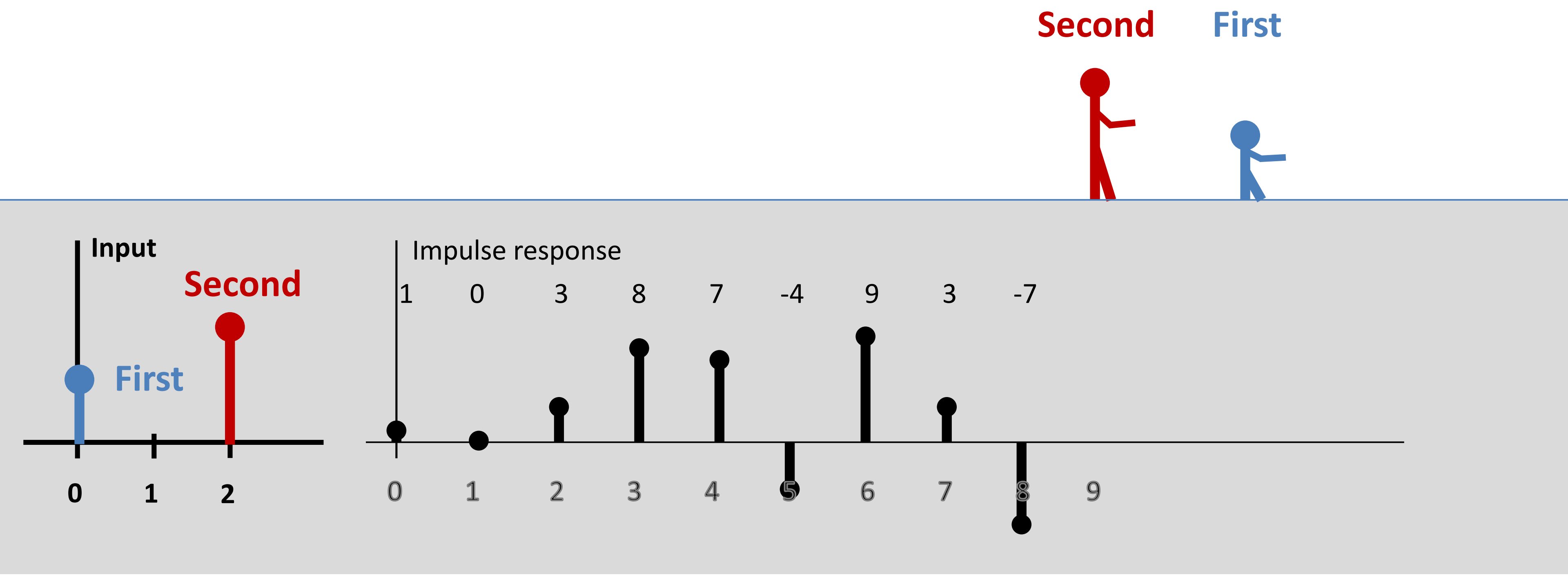

1
0
3
8
7
-4
9
3
-7
Impulse response
0.5
0
2.5
4
6.5
6
11.5
-2.5
5.5
3
-7
Output

Question: Which person came first?

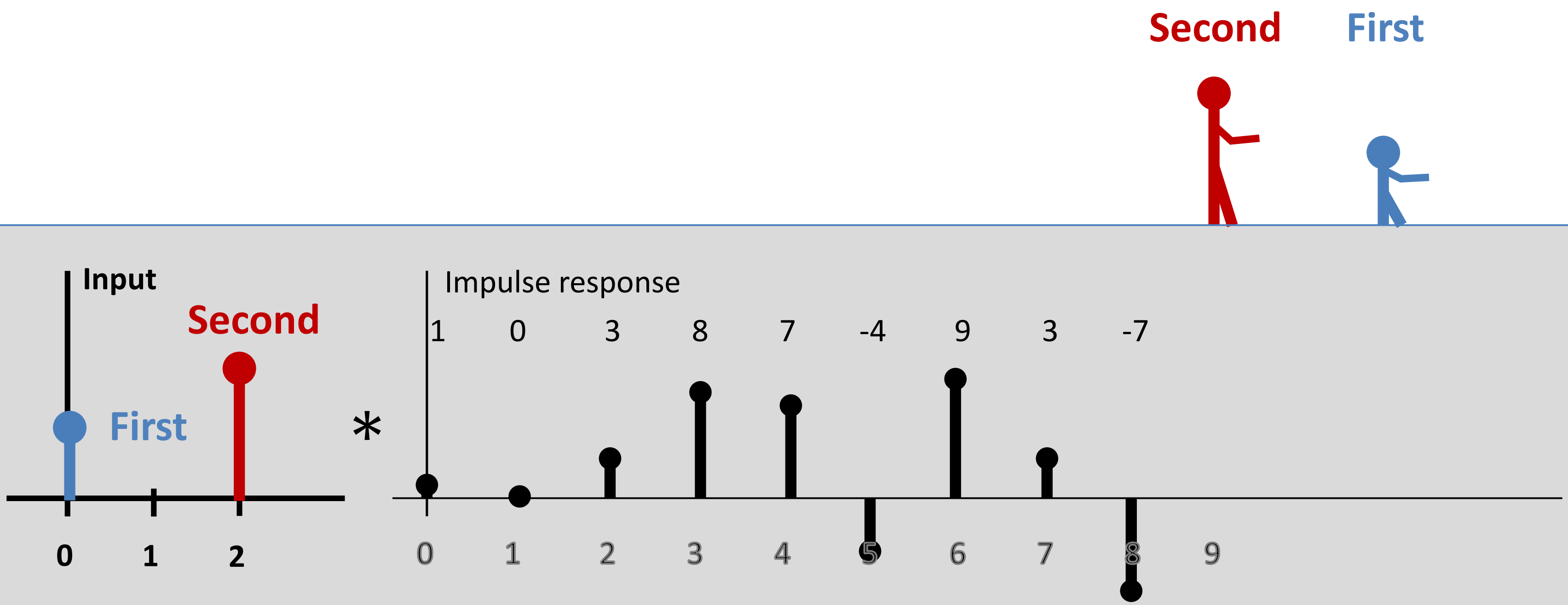


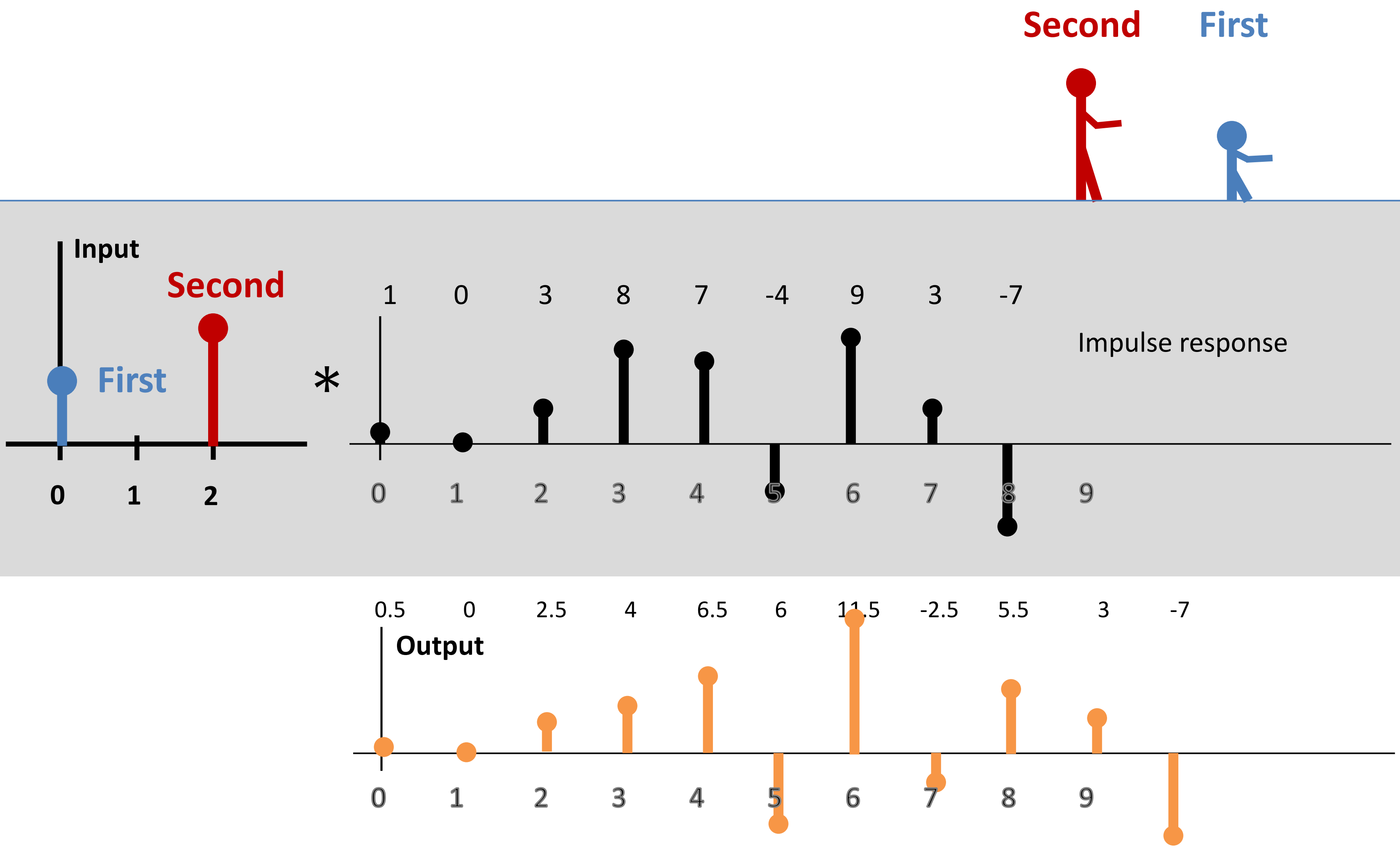

Second
First
1
0
3
8
7
-4
9
3
-7
Impulse response


Second
First
Input
Second
First
0
1
2
1
0
3
8
7
-4
9
3
-7
Impulse response



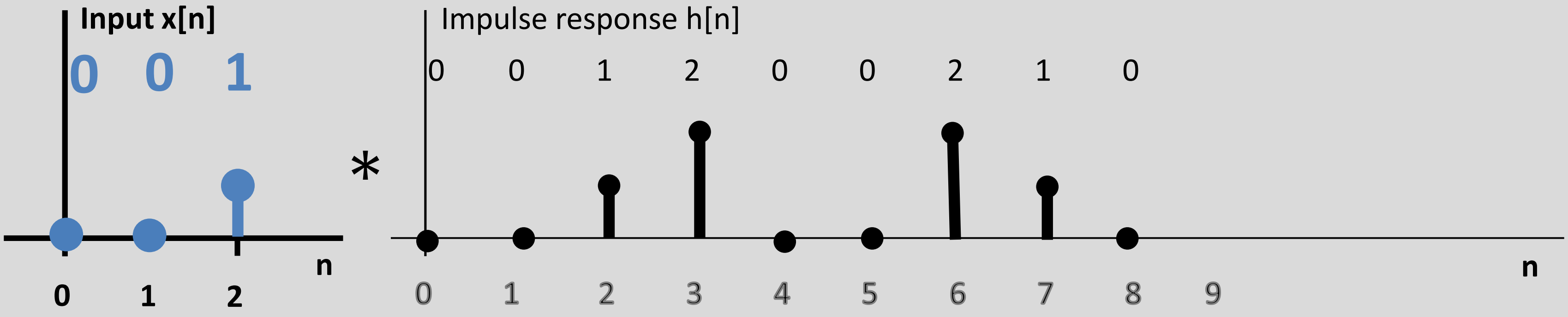
Second
First
Input
Second
First
0
1
2
Impulse response
1
0
3
8
7
-4
9
3
-7
0
1
2
3
4
5
6
7
8
9


Second
First
Input
Second
First
0
1
2
*
Impulse response
1 0 3 8 7 -4 9 3 -7
0 1 2 3 4 5 6 7 8 9

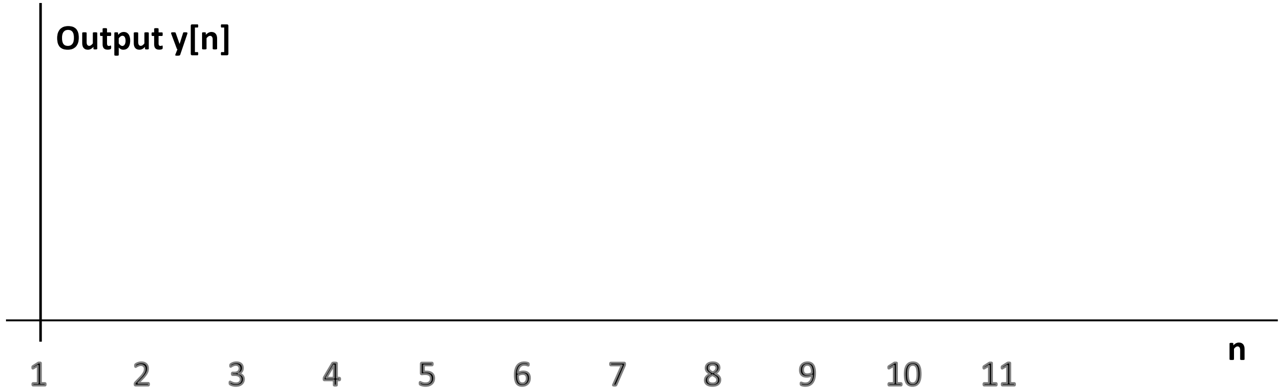


Second
First
Input
Second
First
0
1
2
*
1 0 3 8 7 -4 9 3 -7
Impulse response
0 1 2 3 4 5 6 7 8 9
0.5 0 2.5 4 6.5 6 11.5 -2.5 5.5 3 -7
Output
0 1 2 3 4 5 6 7 8 9

Discrete-time Convolution Example:
Shifted Impulse

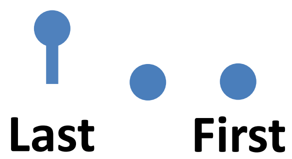


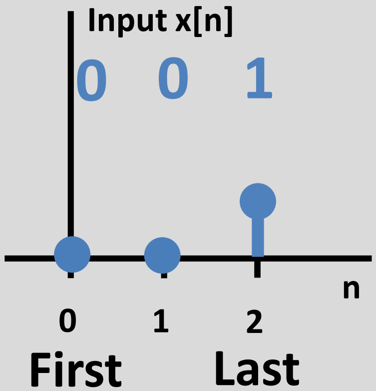
Input x[n]
0 0 1
0 1 2
n
*
Impulse response h[n]
0 0 1 2 0 0 2 1 0
0 1 2 3 4 5 6 7 8 9
n



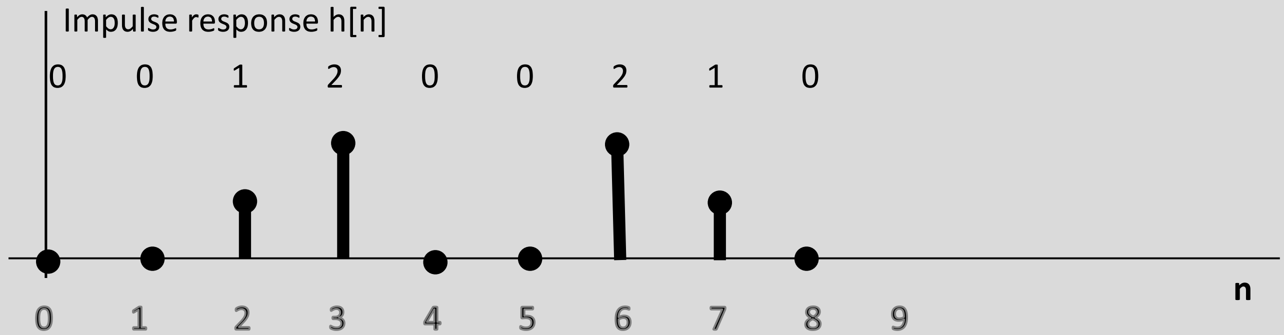
Output y[n]
1 2 3 4 5 6 7 8 9 10 11
n

Step 1: Time reverse

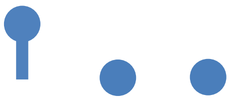




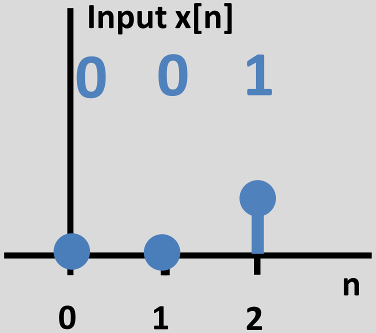
*



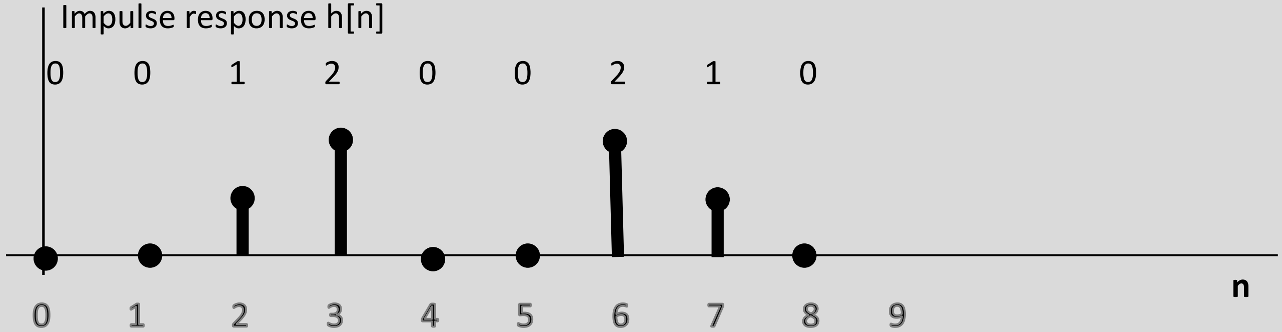
Step 2: Shift, multiply, and sum (note: all values not shown are zeros)



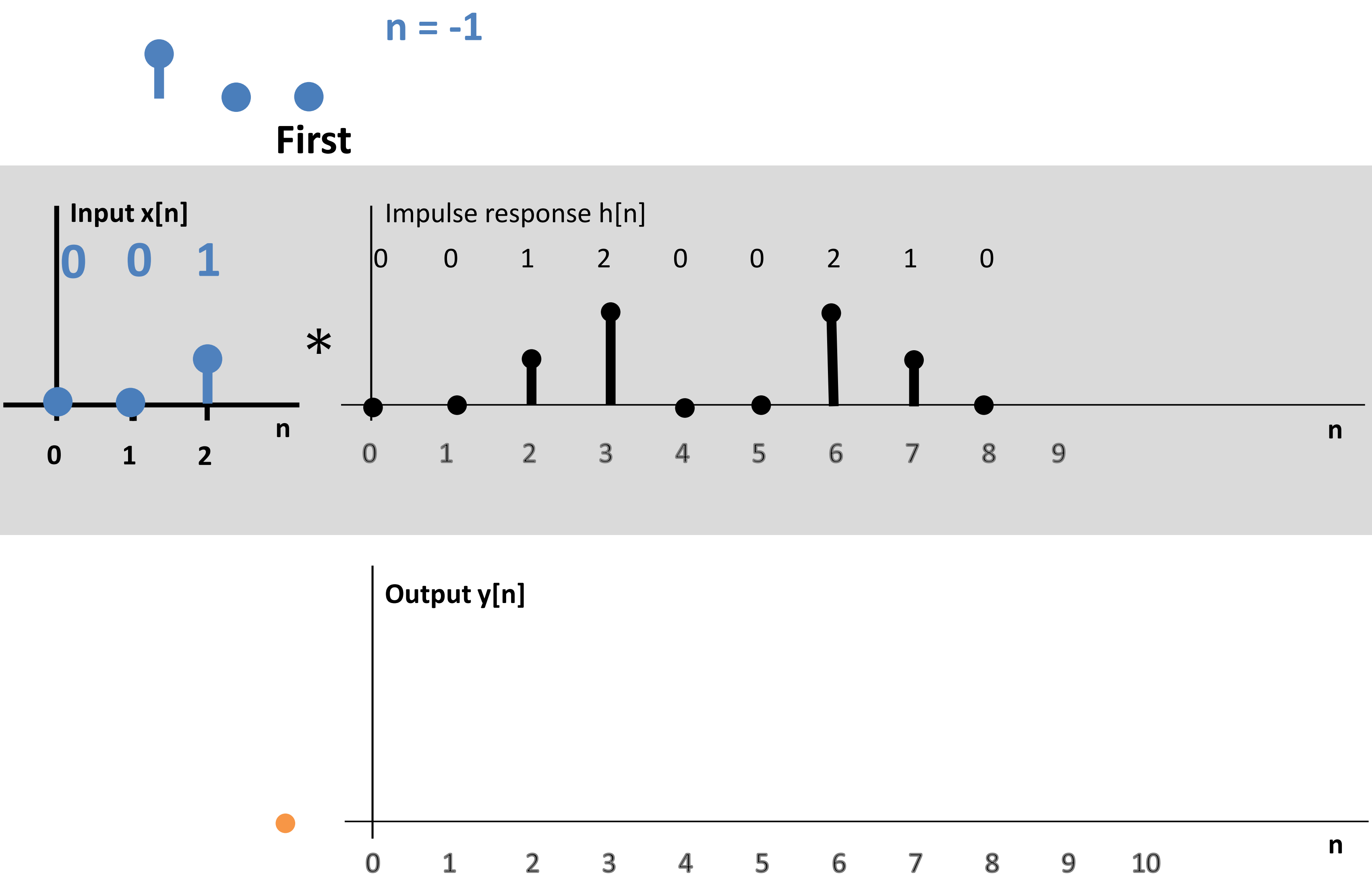
First



*

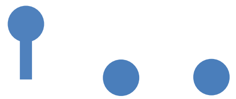


Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

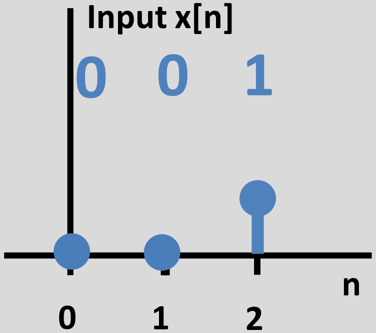


Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

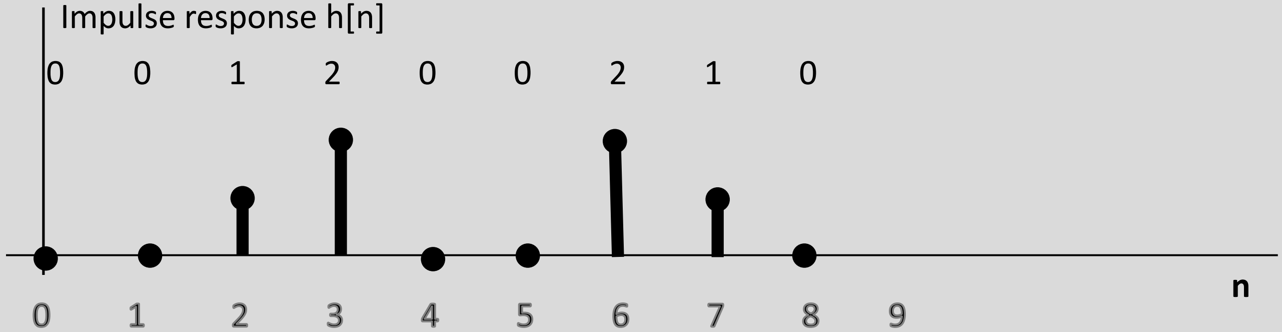
n = 0



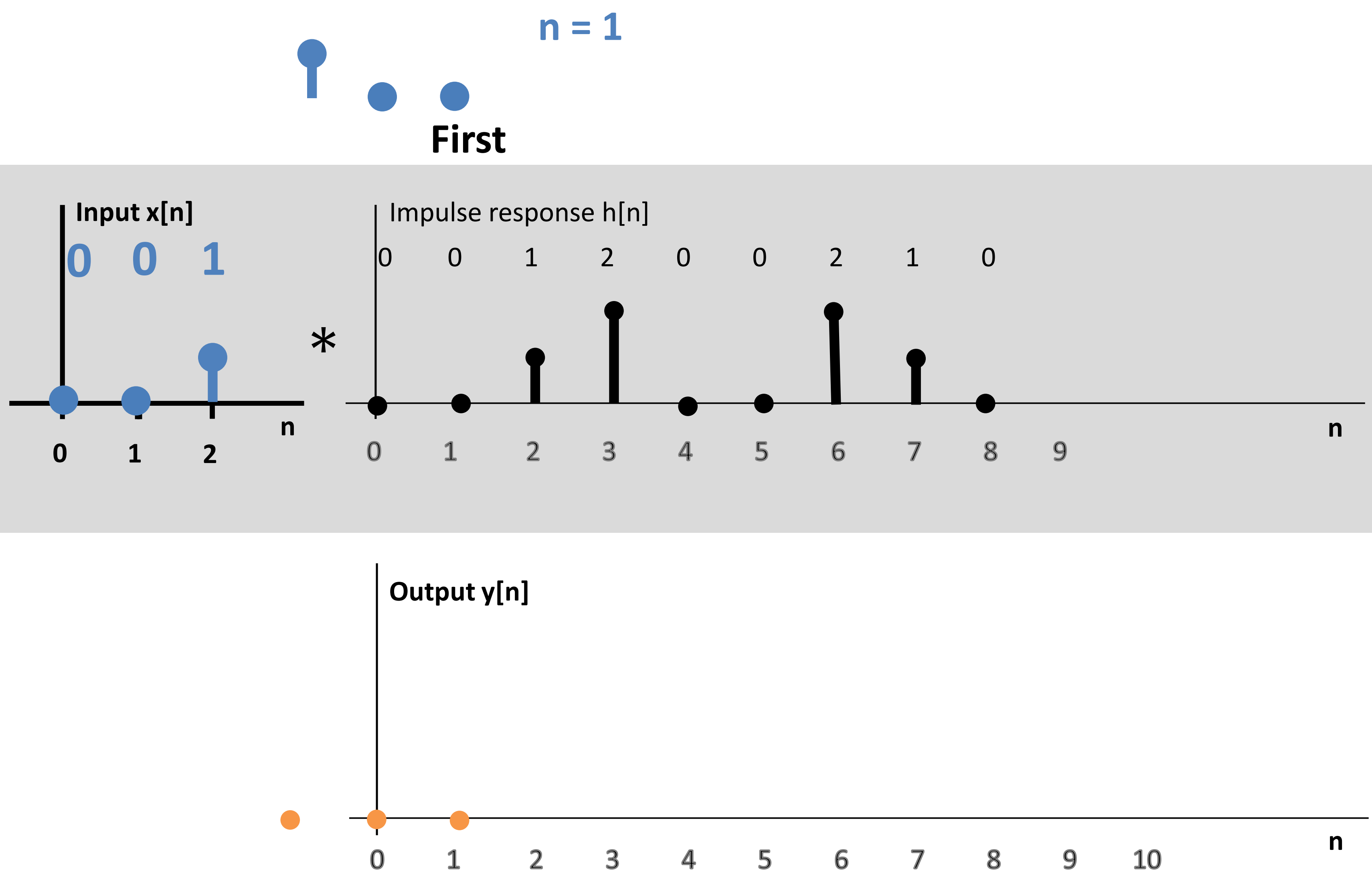
First



*

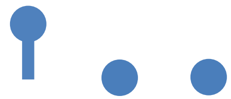


Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

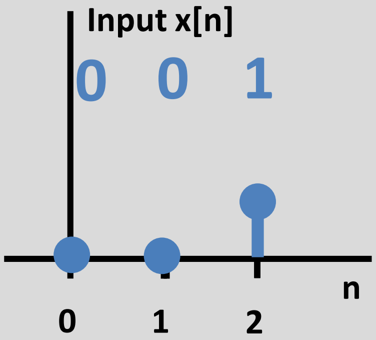


Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

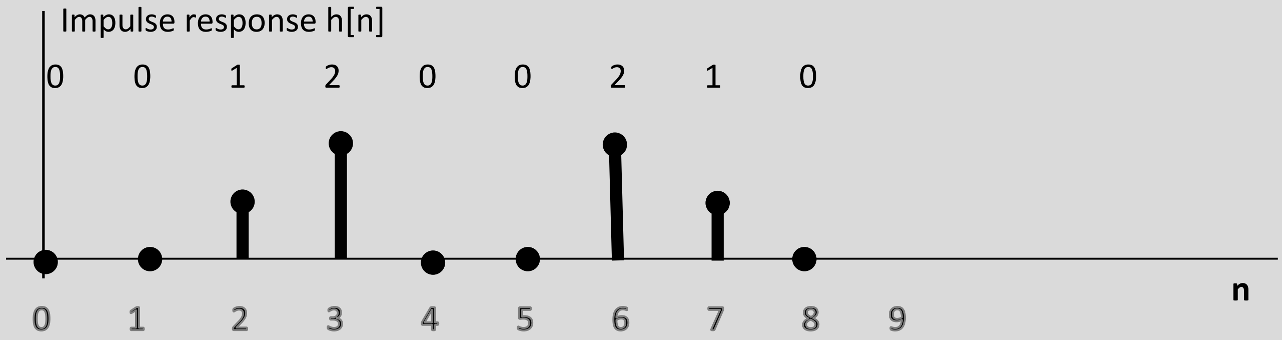
n = 2

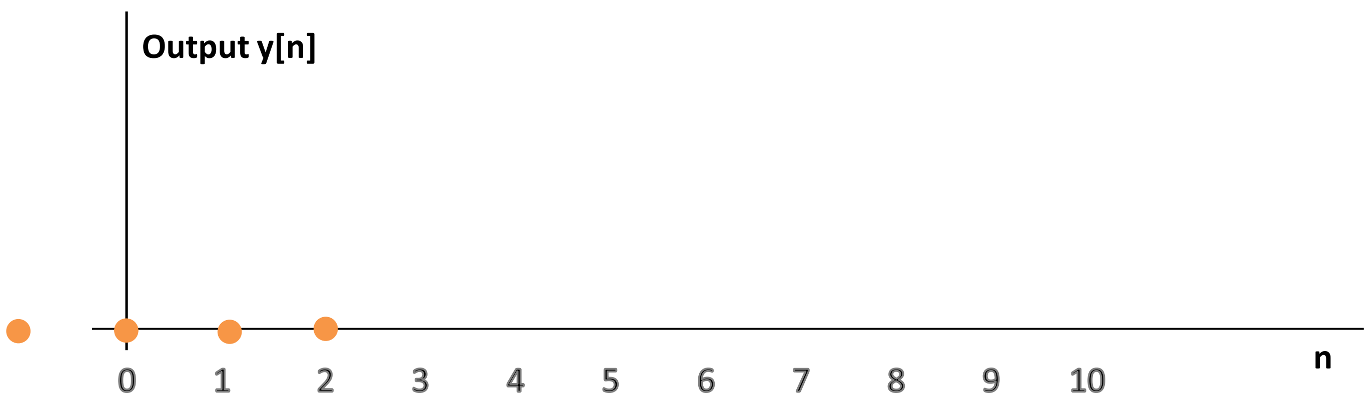


First



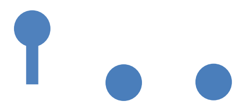
*



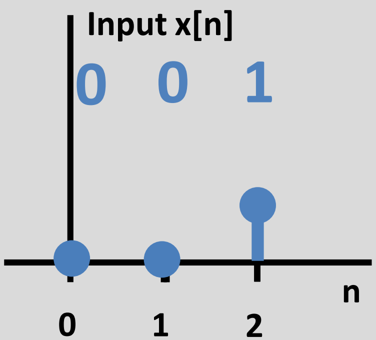


Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

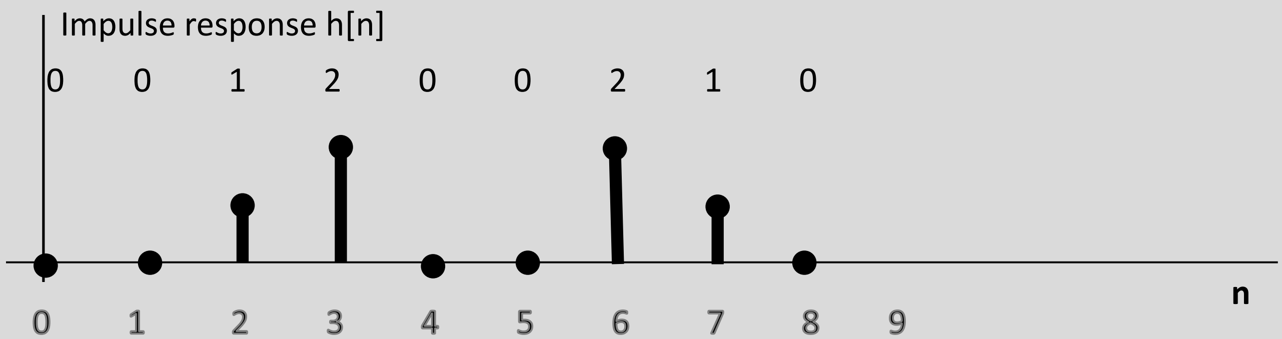
n = 3

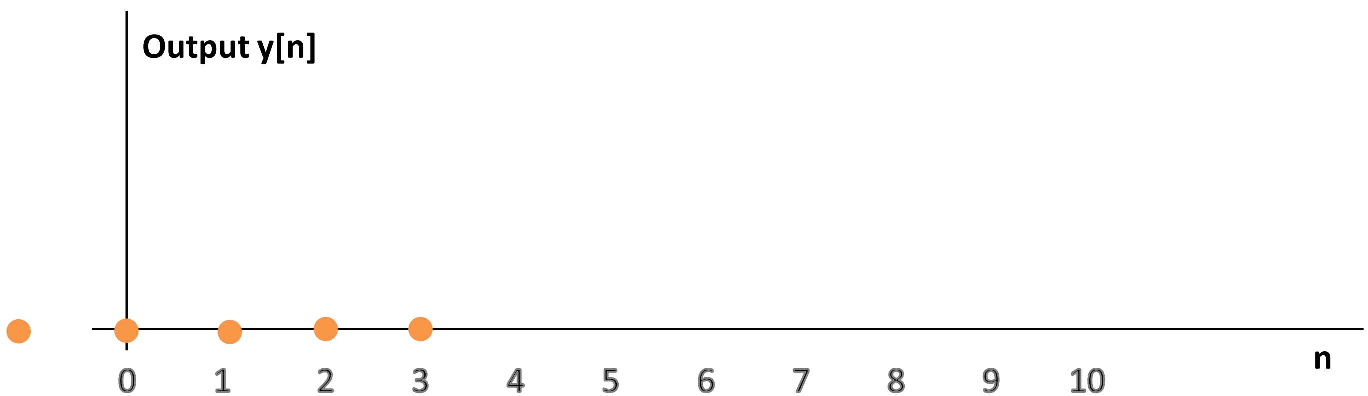


First

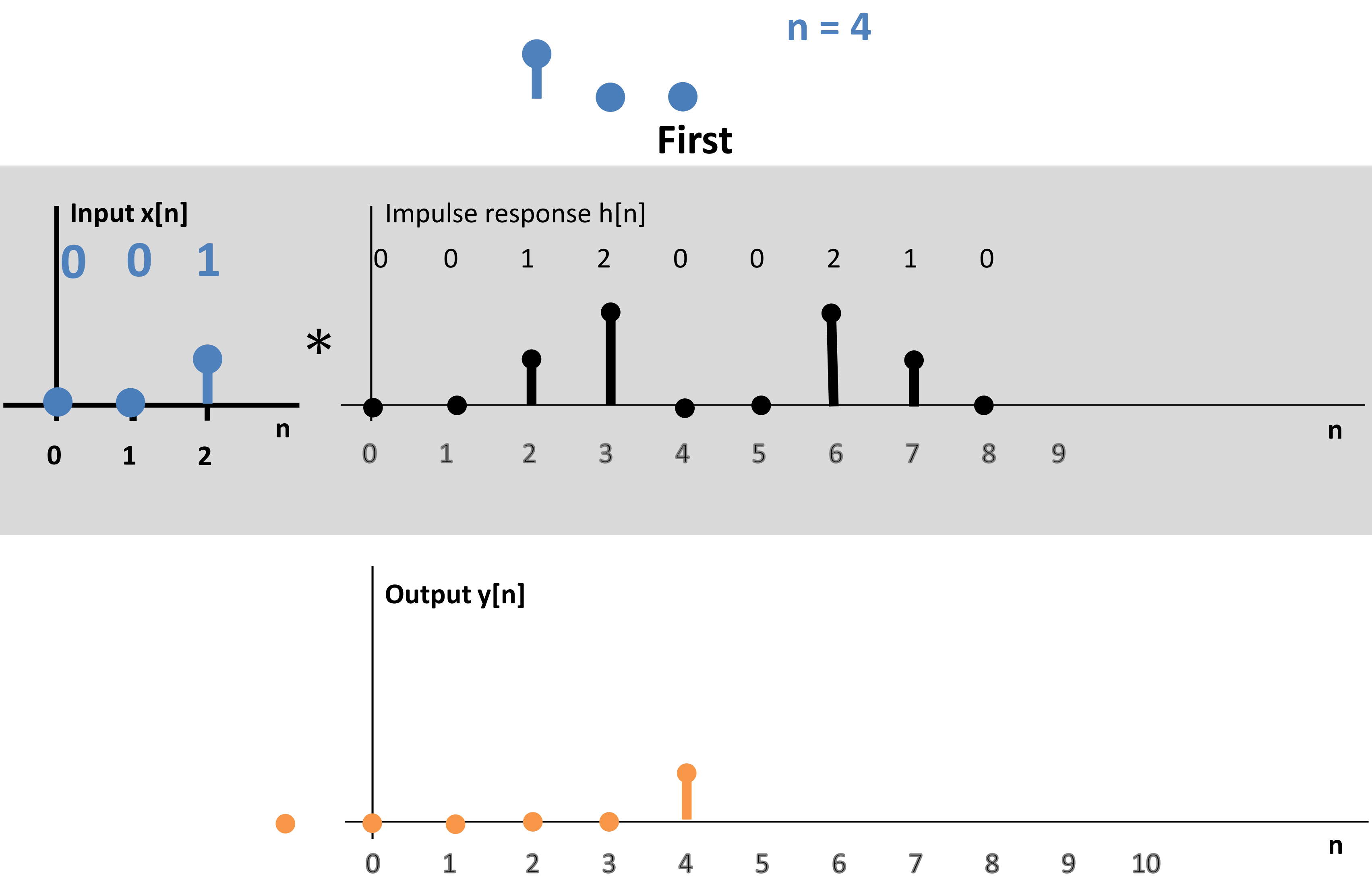


*

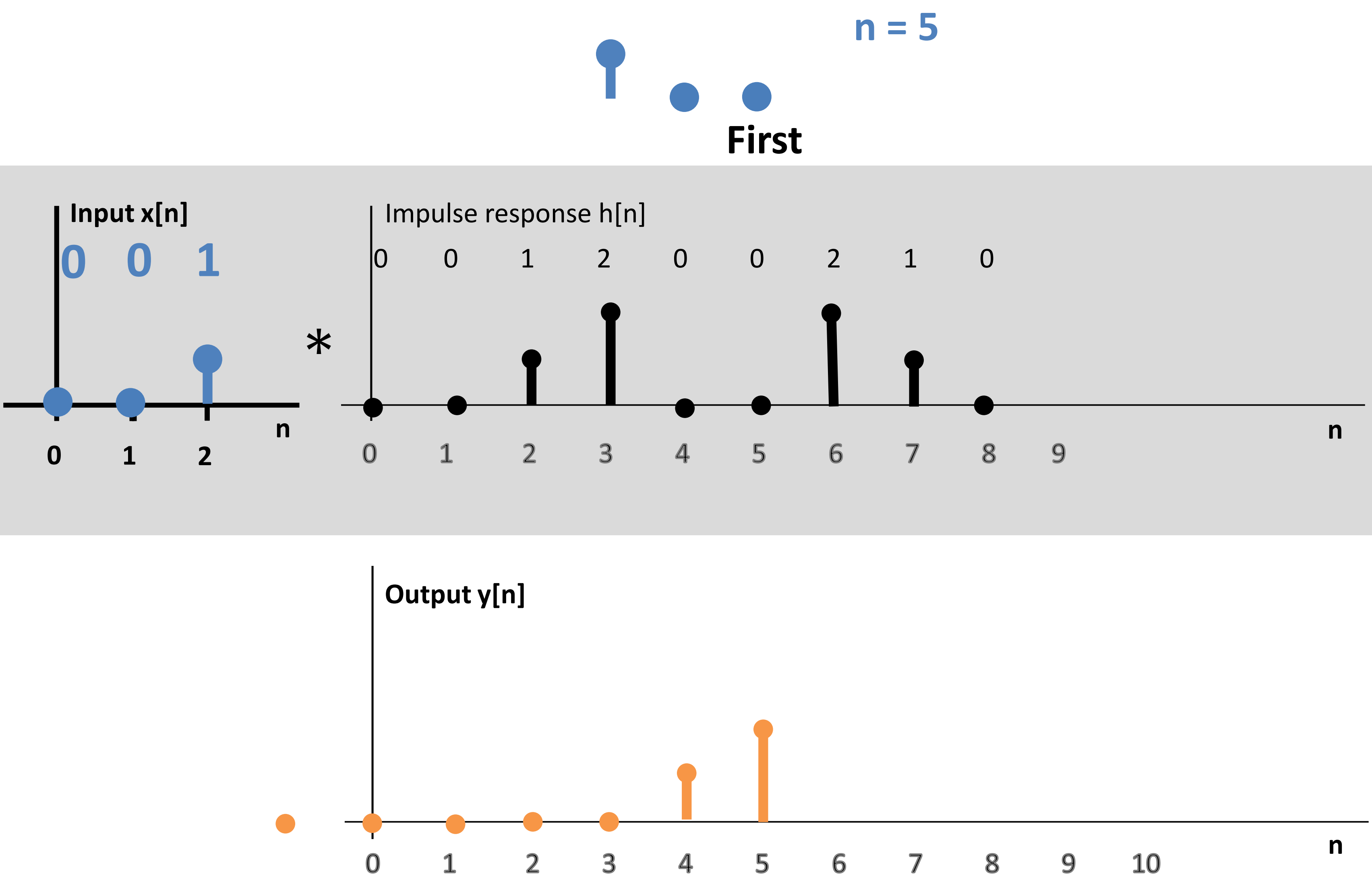




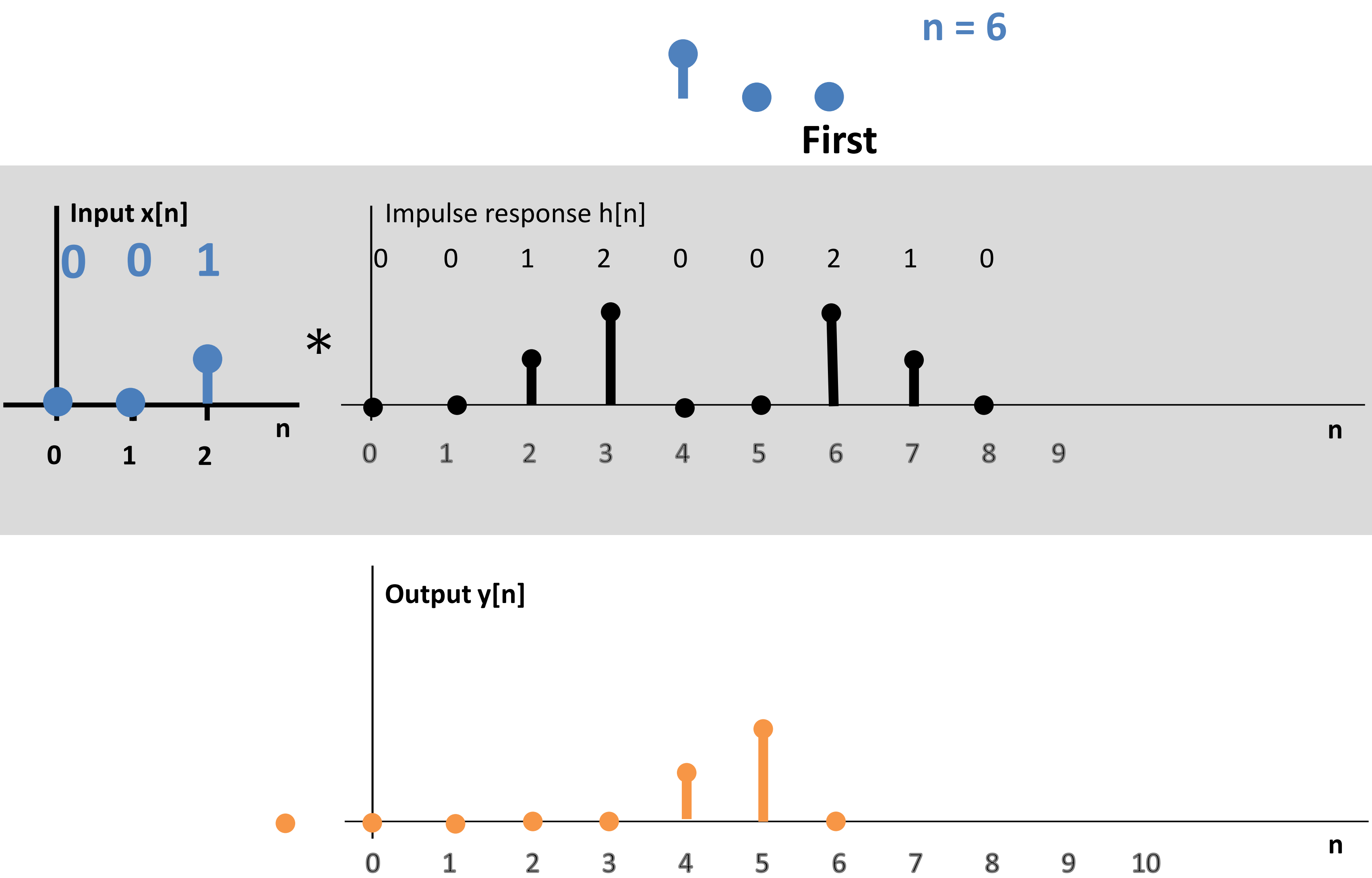
Step 2: Shift, multiply, and sum (note: all values not shown are zeros)



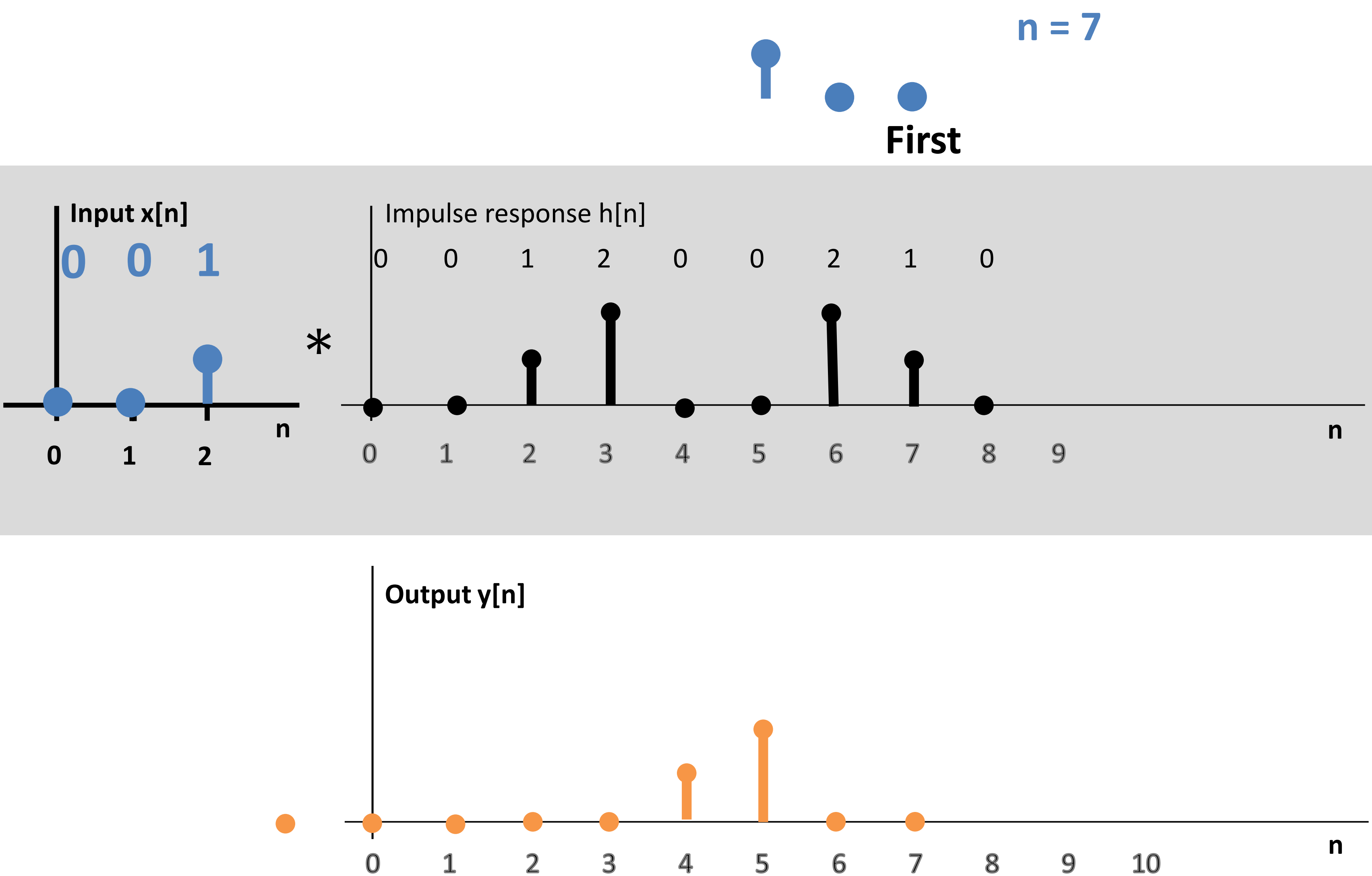
Step 2: Shift, multiply, and sum (note: all values not shown are zeros)



Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

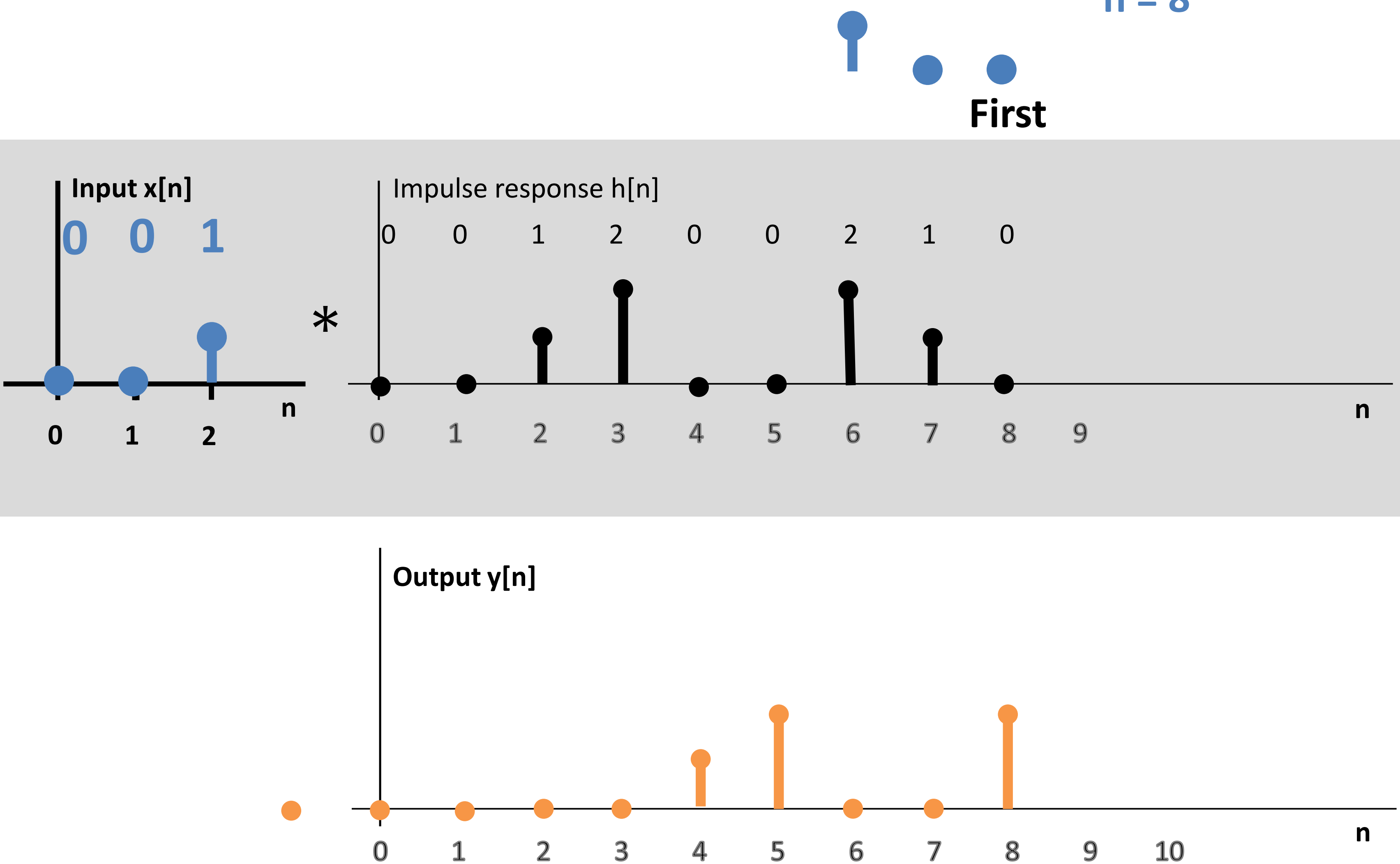


Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

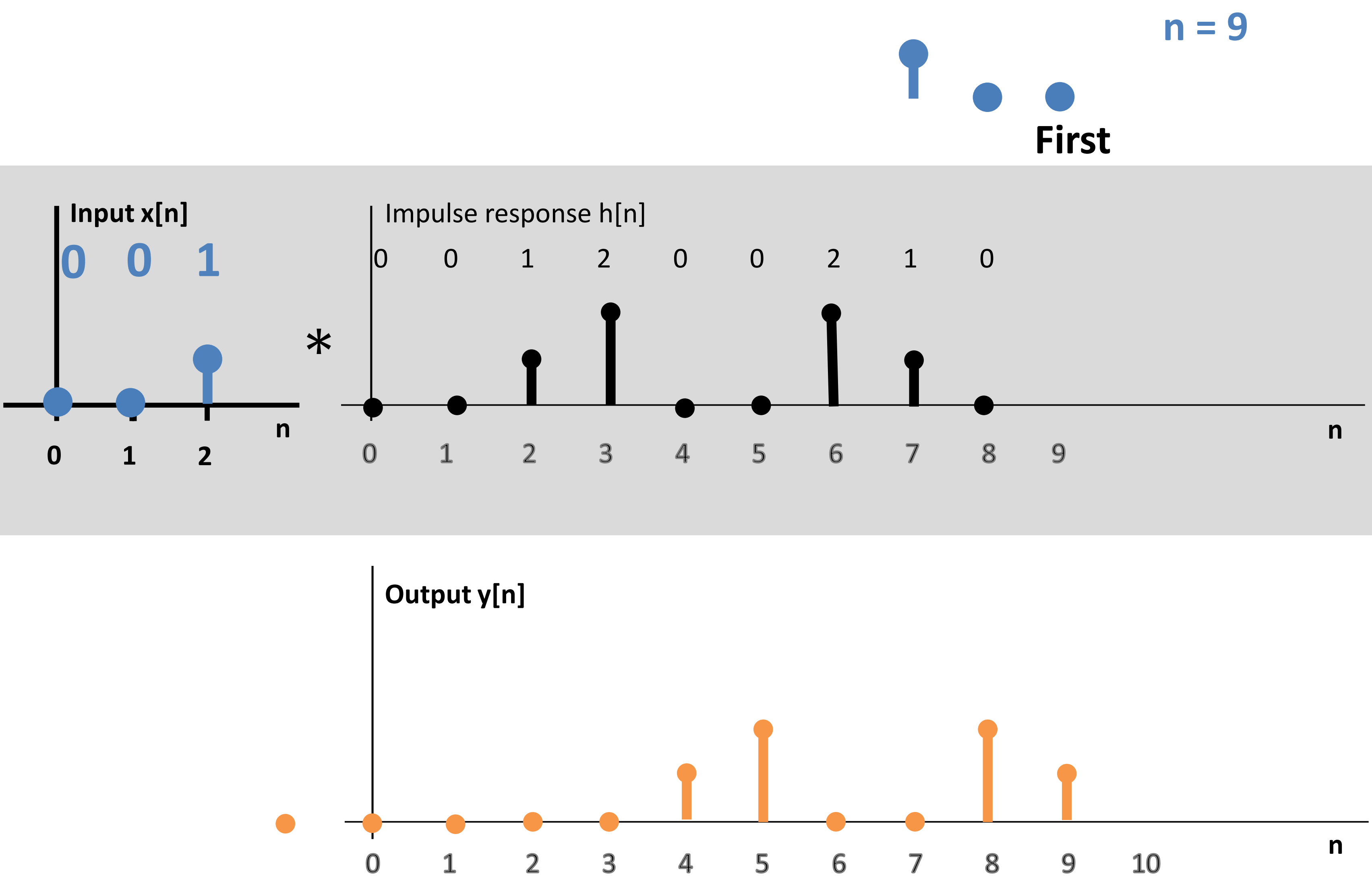


Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

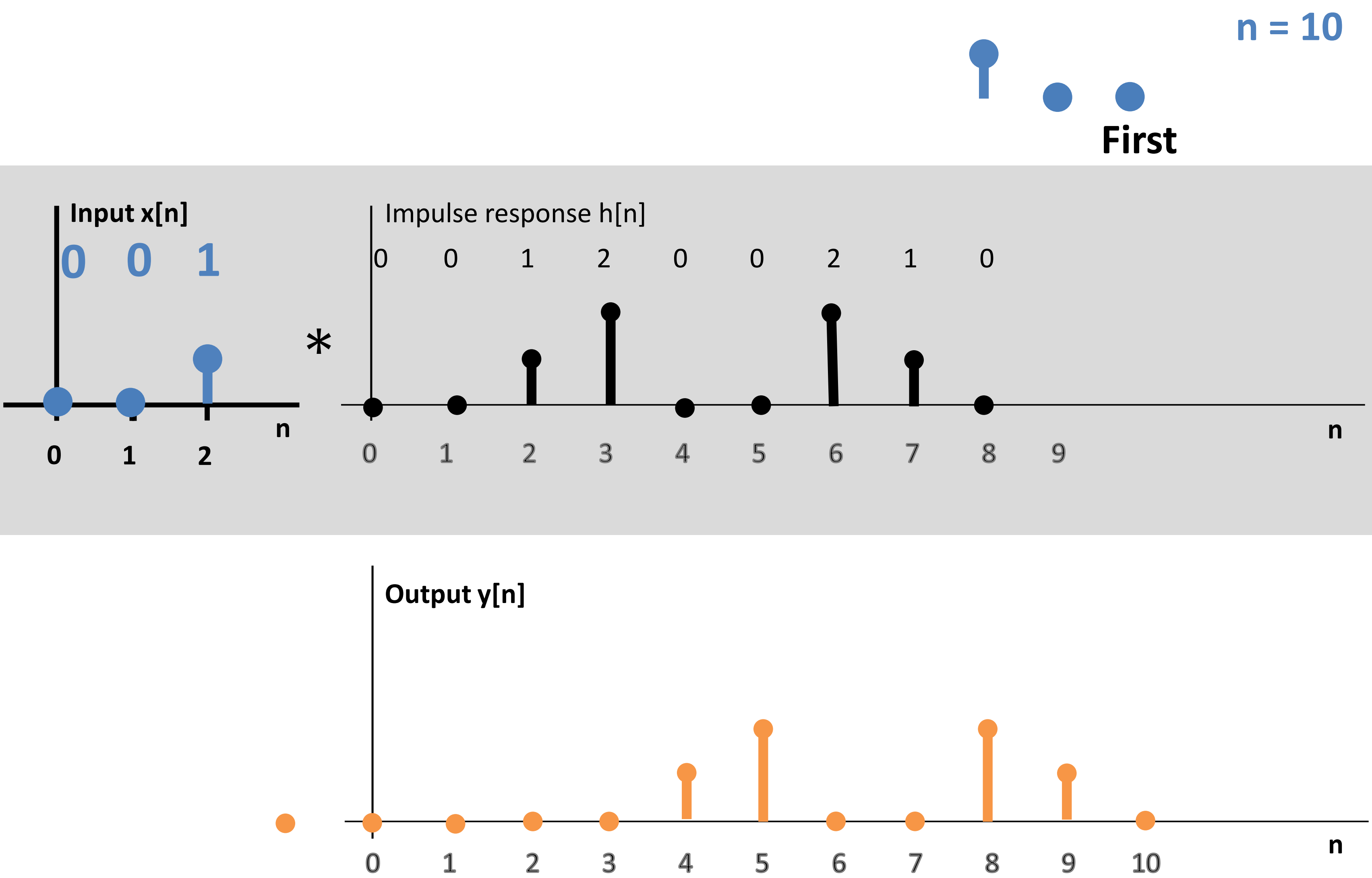
n = 8



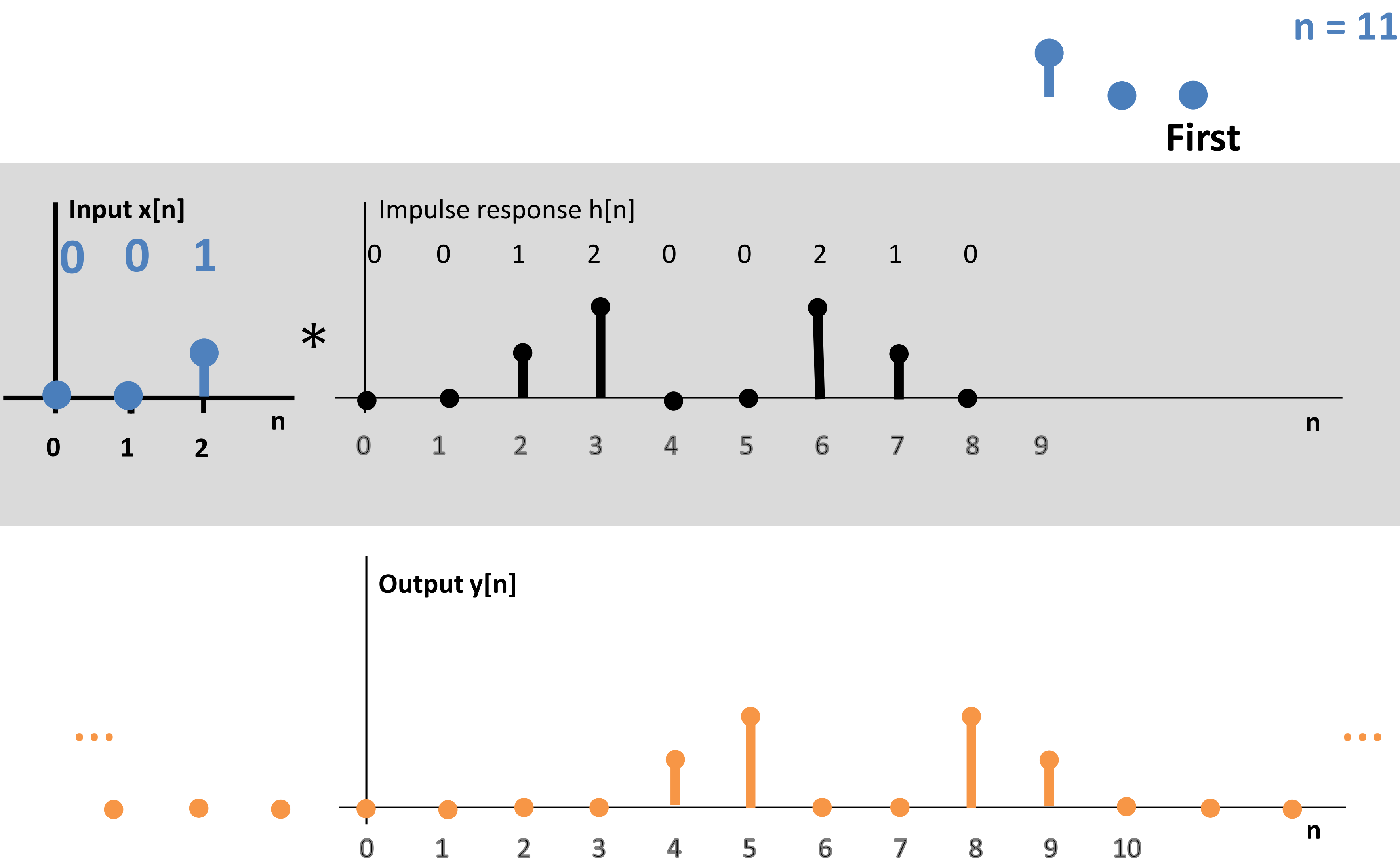
Step 2: Shift, multiply, and sum (note: all values not shown are zeros)



Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

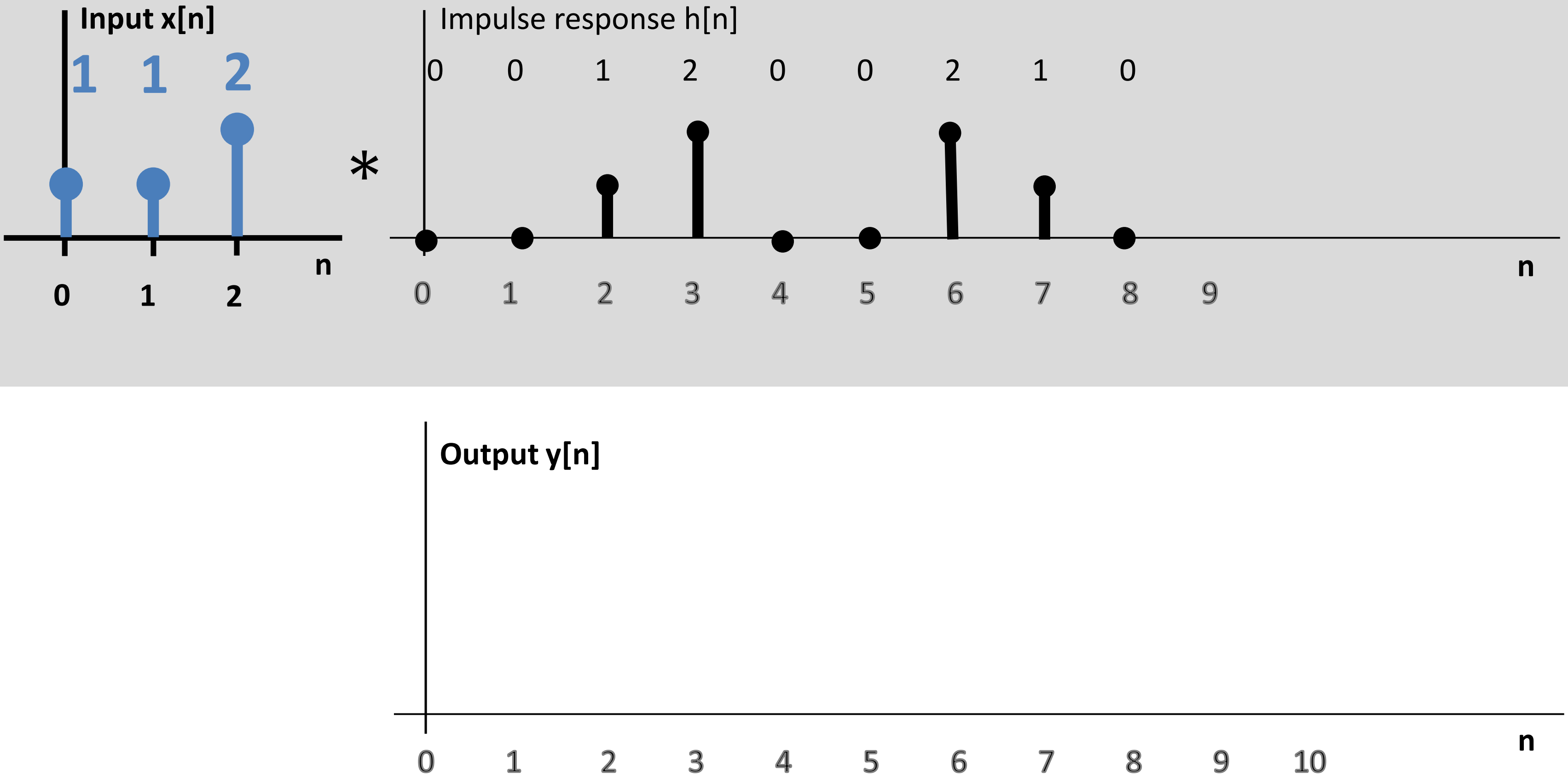


Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

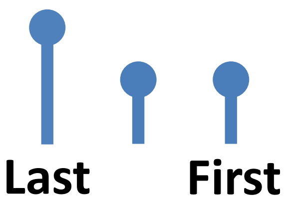


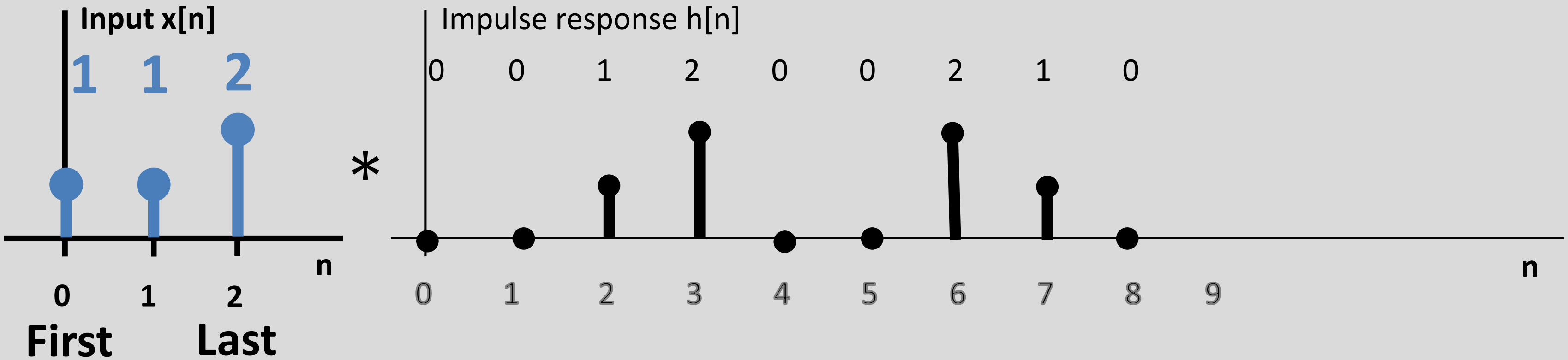
Discrete-time Convolution Example:

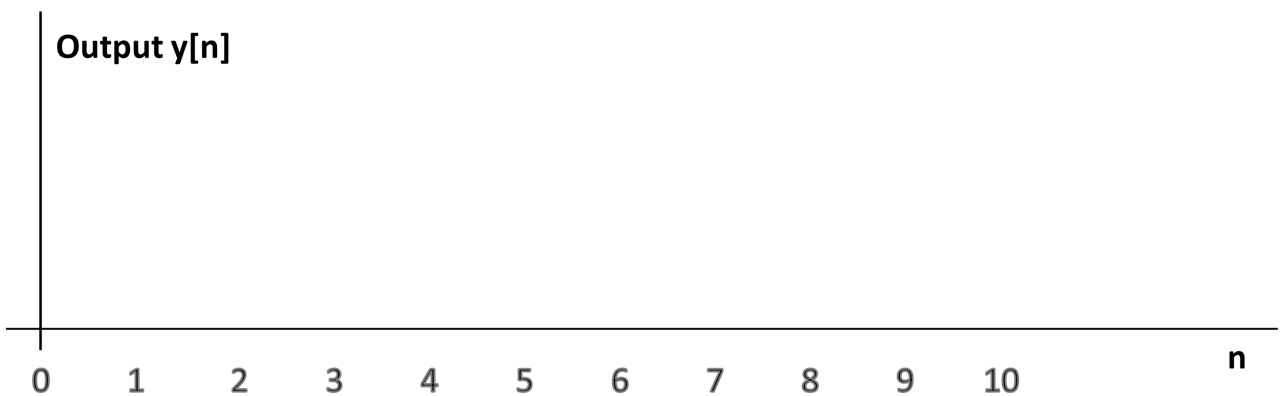
Three Impulses


Input x[n]
1 1 2
0 1 2
n
*
Impulse response h[n]
0 0 1 2 0 0 2 1 0
0 1 2 3 4 5 6 7 8 9
n
Output y[n]
0 1 2 3 4 5 6 7 8 9 10
n

Step 1: Time reverse

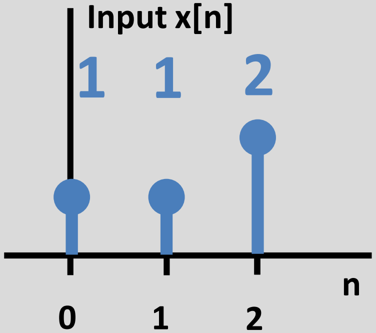






Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

First



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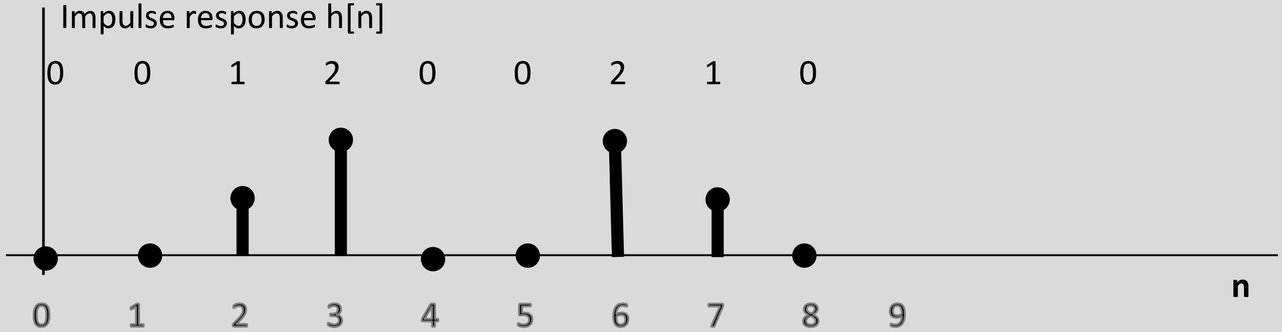
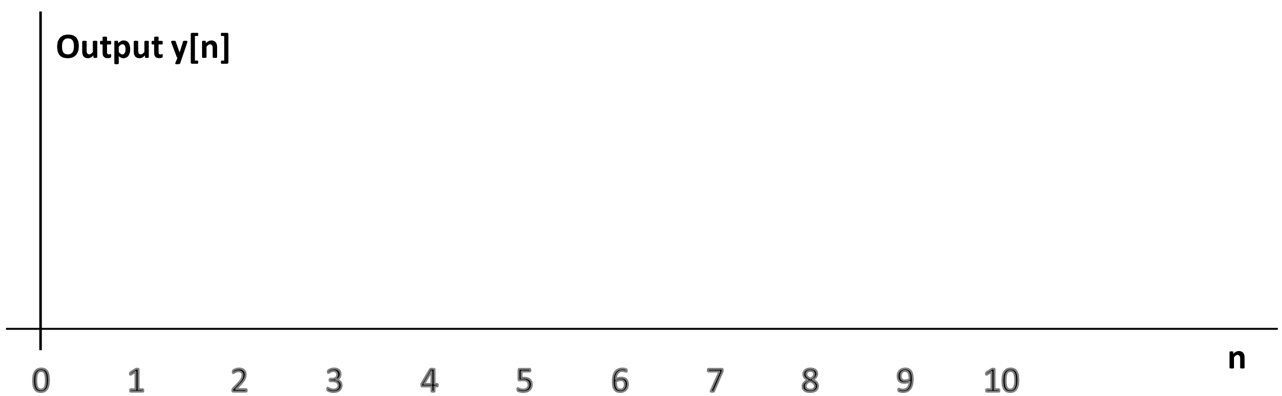
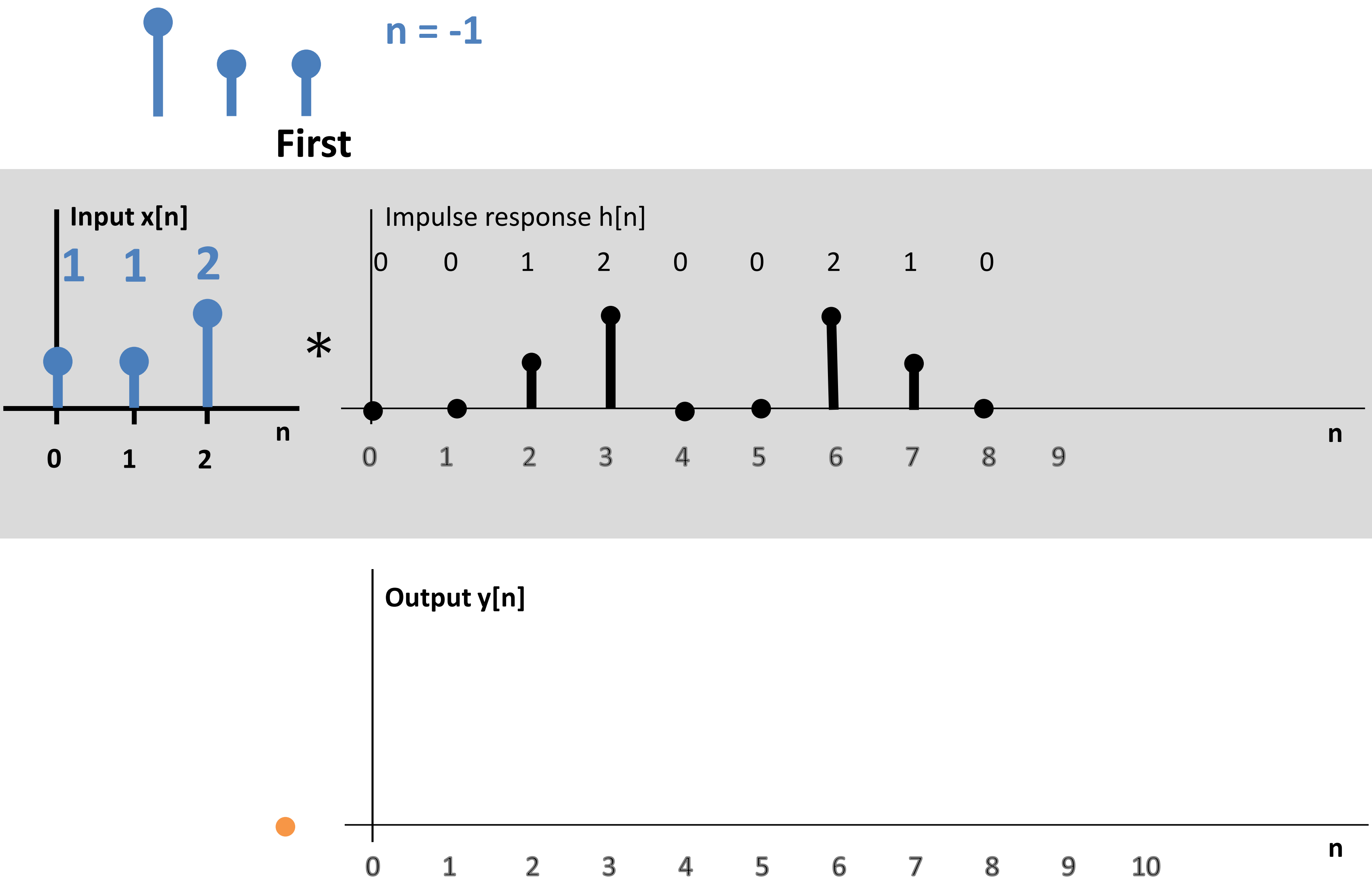




Step 2: Shift, multiply, and sum (note: all values not shown are zeros)



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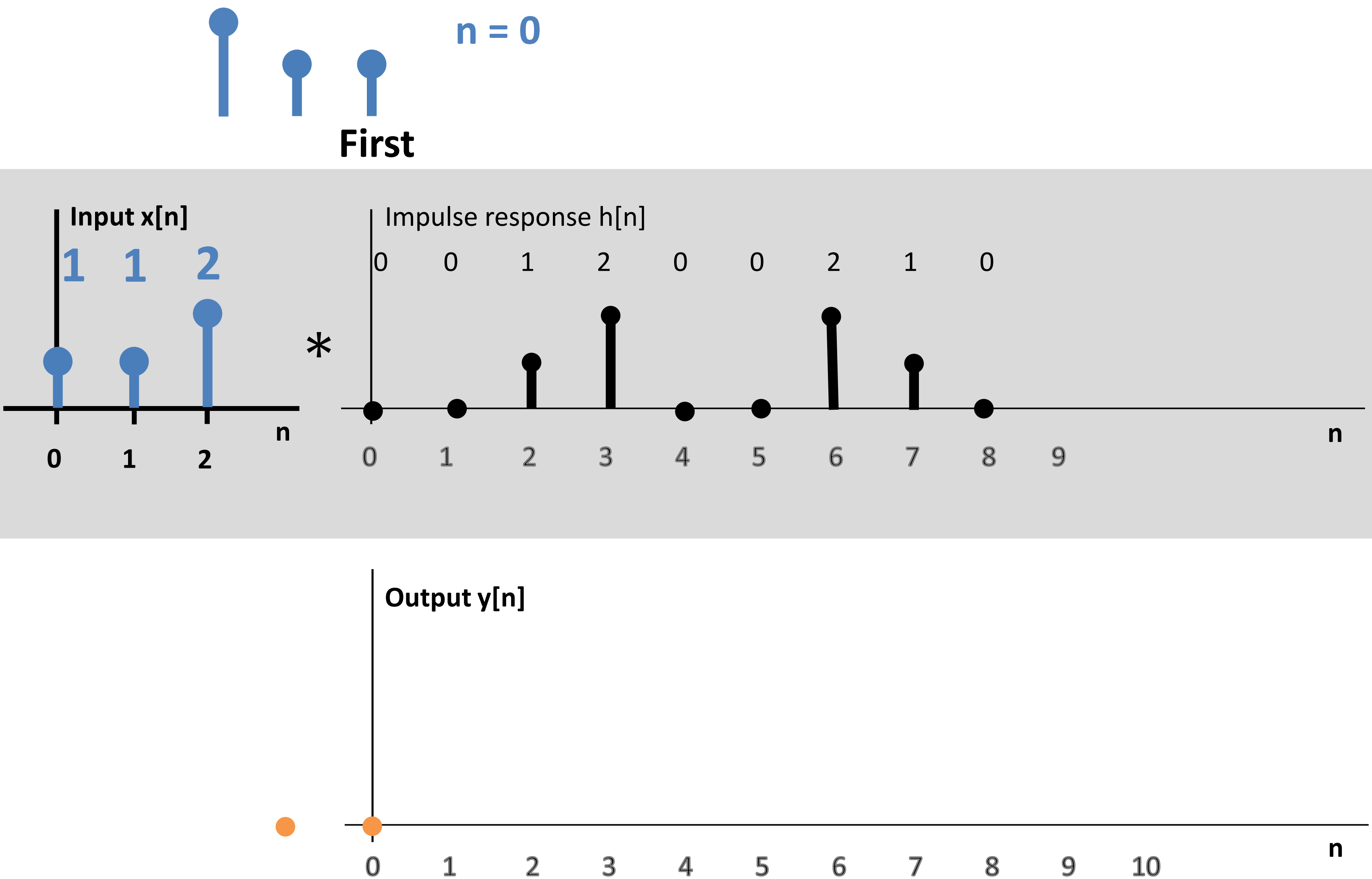
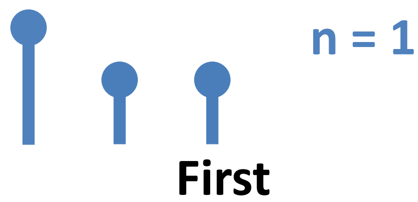
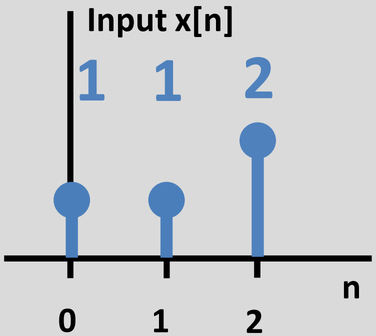


Step 2: Shift, multiply, and sum (note: all values not shown are zeros)





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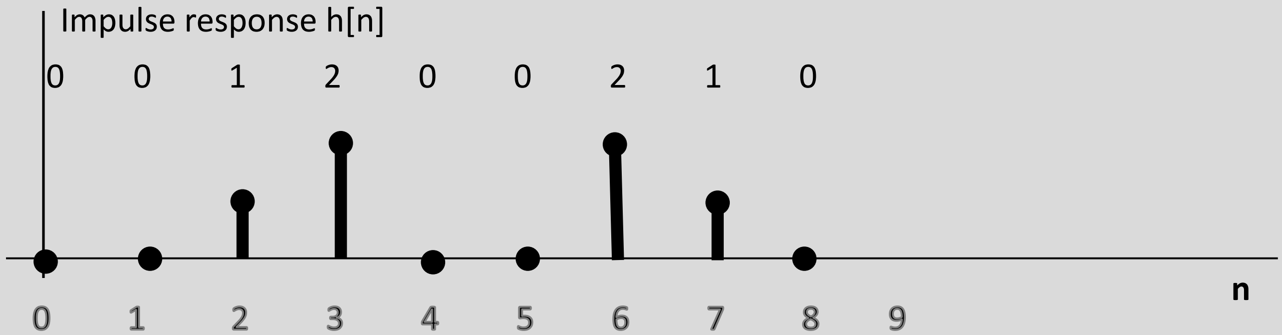
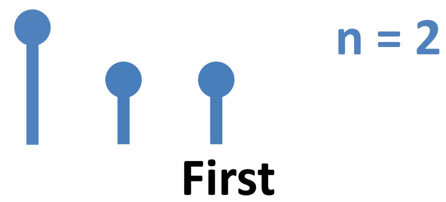
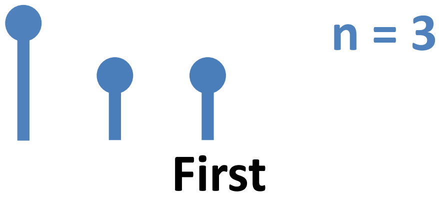


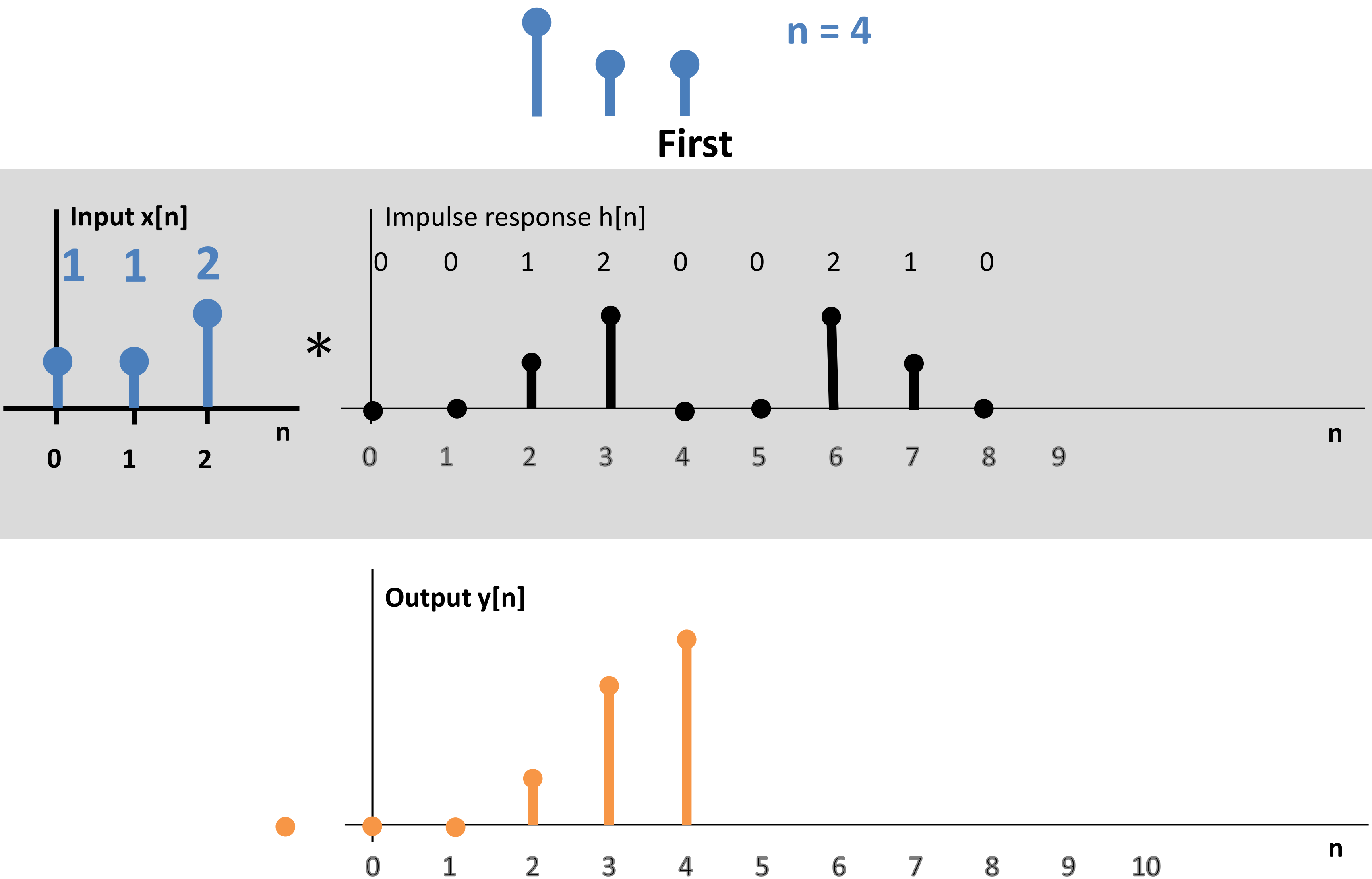
Step 2: Shift, multiply, and sum (note: all values not shown are zeros)



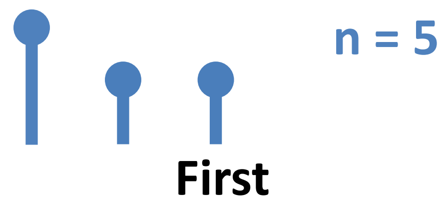
Step 2: Shift, multiply, and sum (note: all values not shown are zeros)

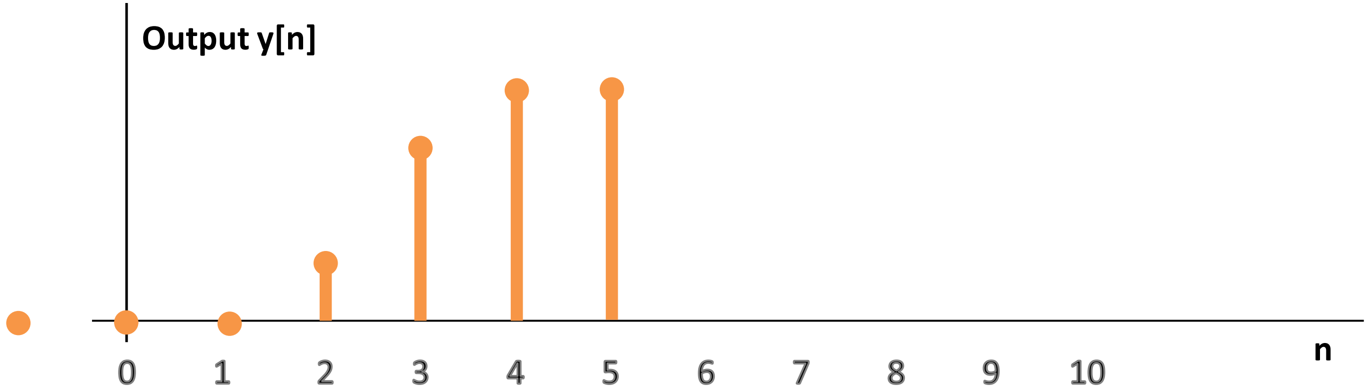


Step 2: Shift, multiply, and sum (note: all values not shown are zeros)



Step 2: Shift, multiply, and sum (note: all values not shown are zeros)





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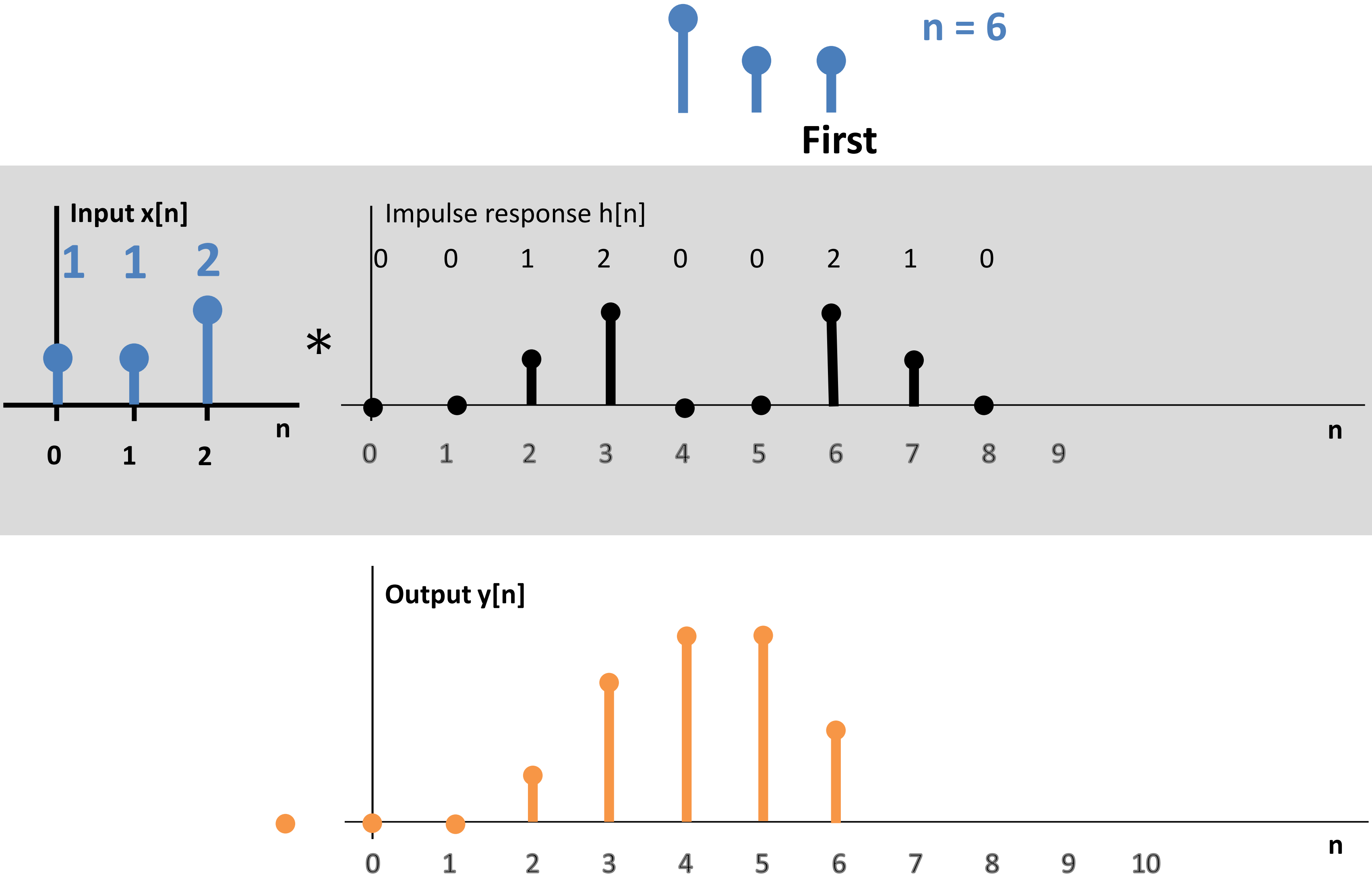
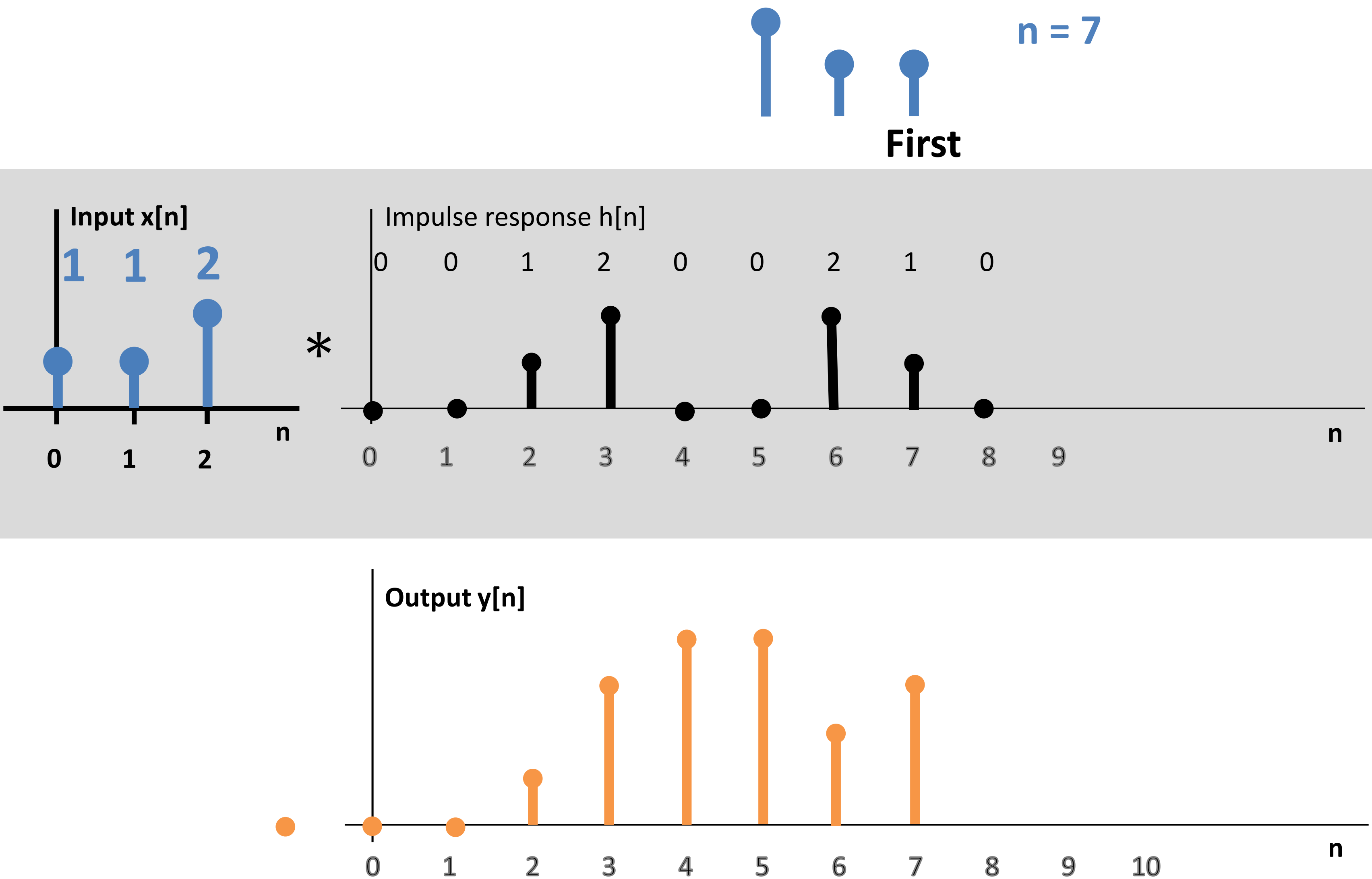


Step 2: Shift, multiply, and sum (note: all values not shown are zeros)



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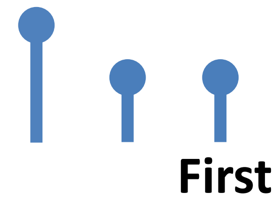
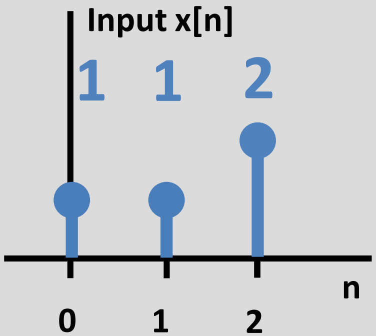
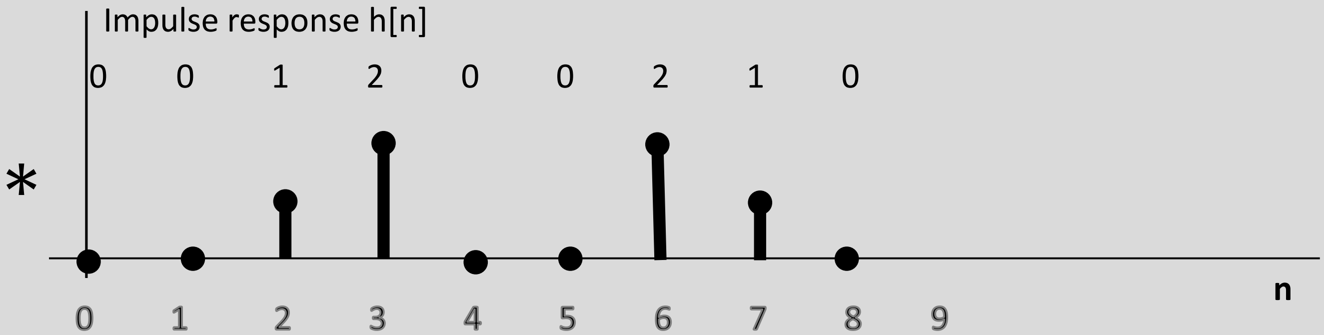
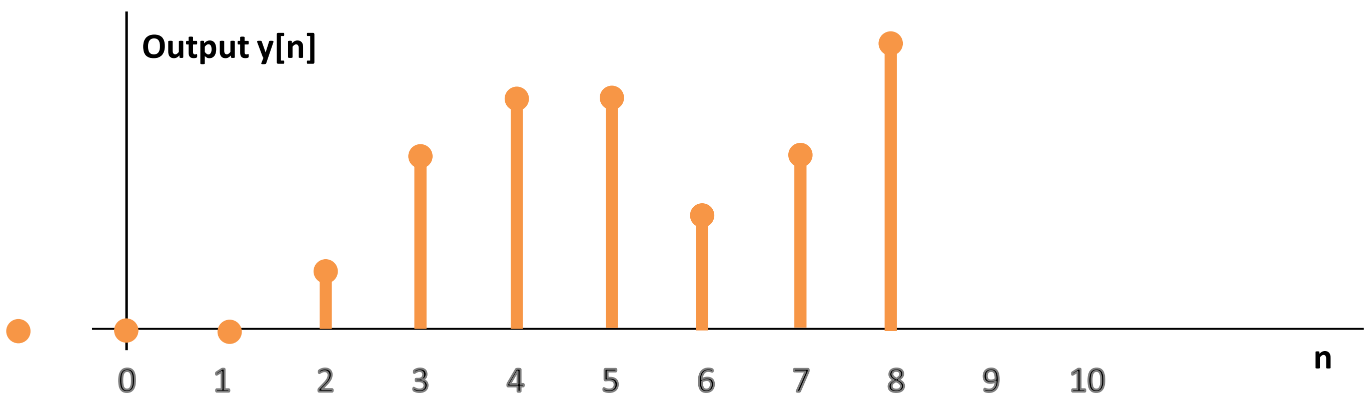
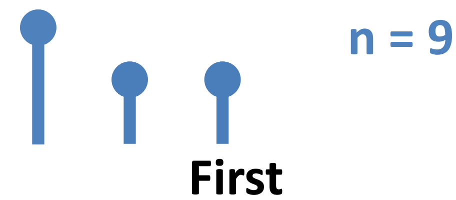


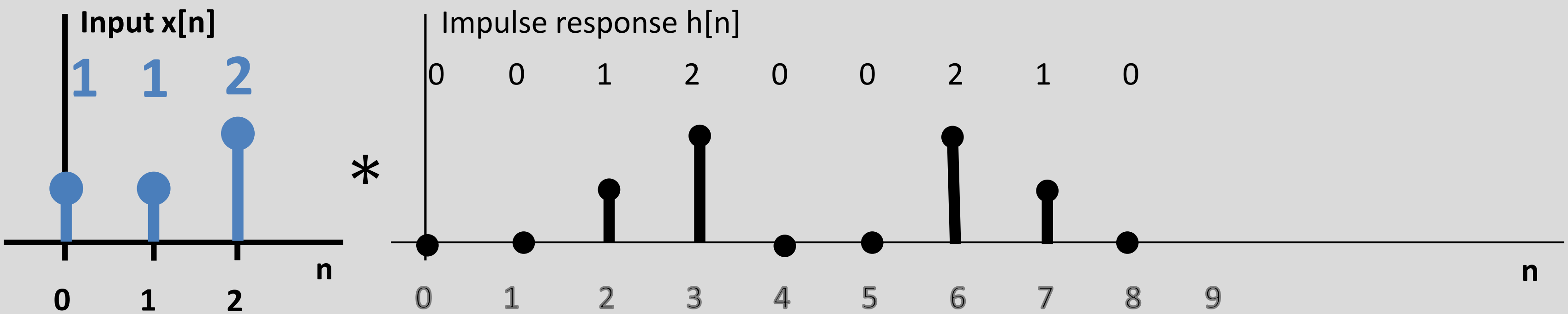


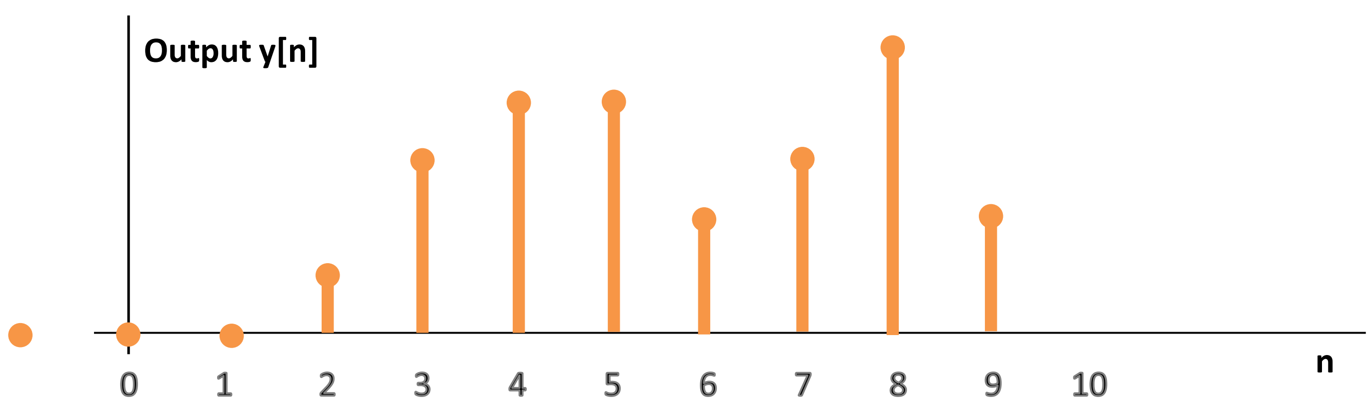




Step 2: Shift, multiply, and sum (note: all values not shown are zeros)







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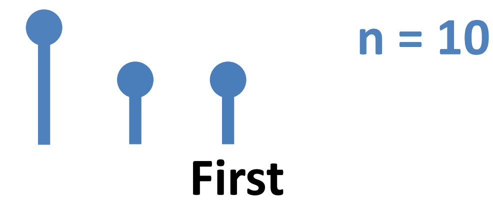

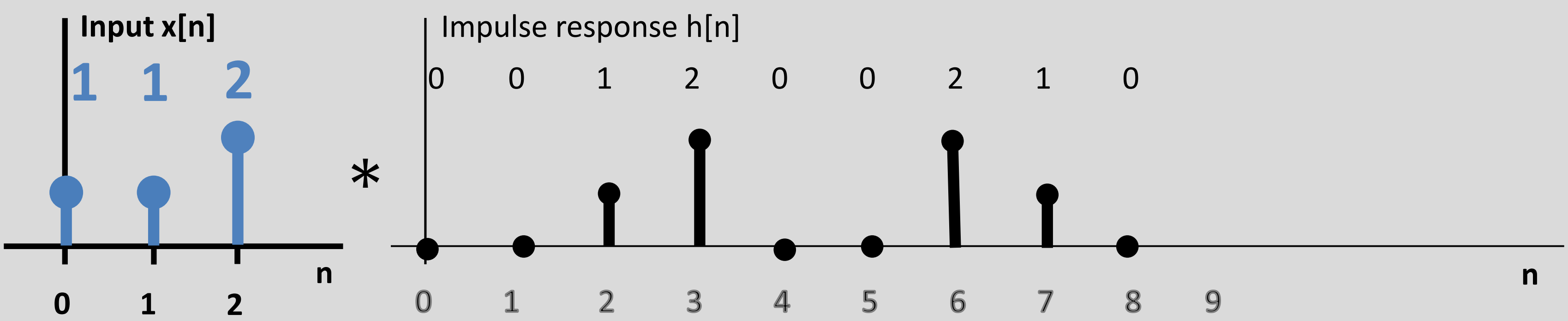

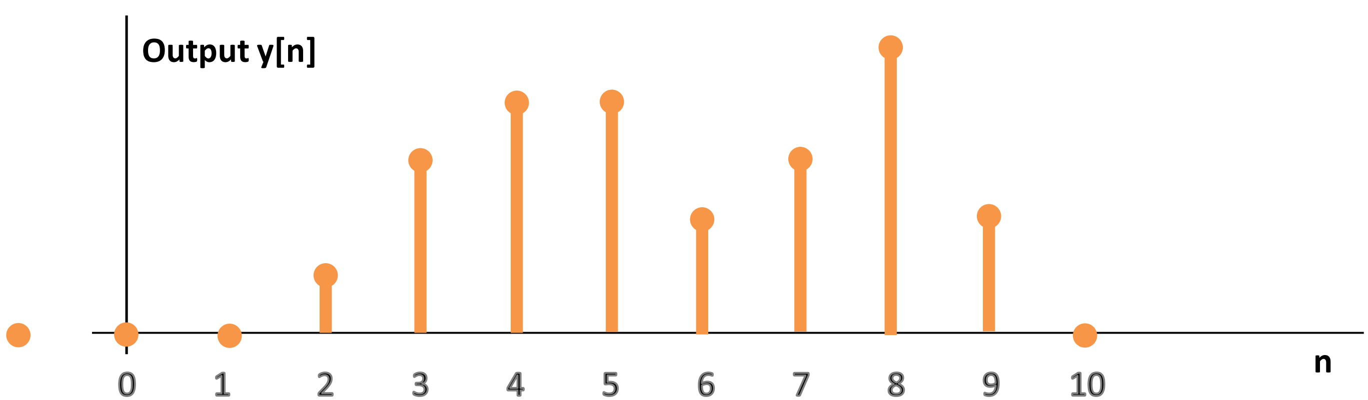

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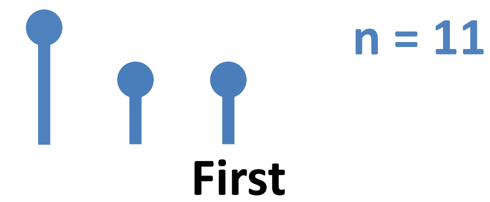

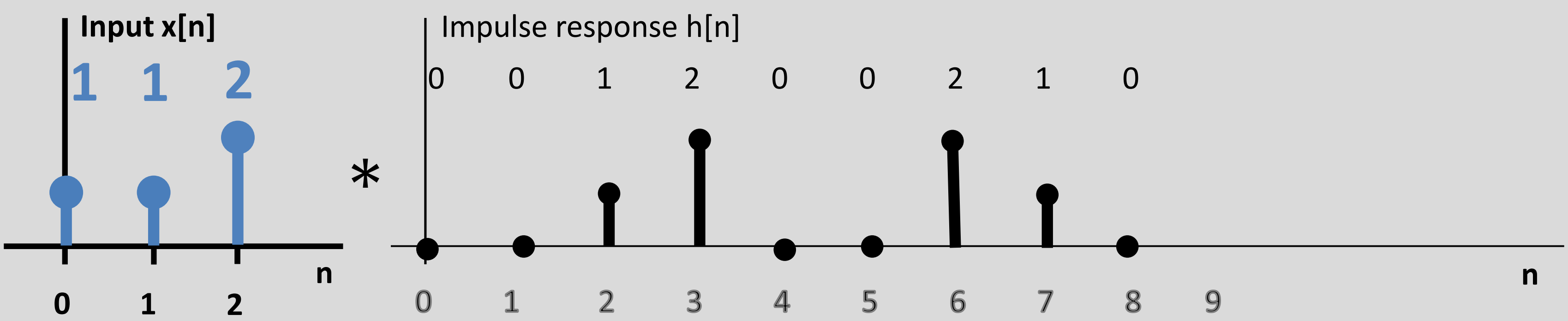

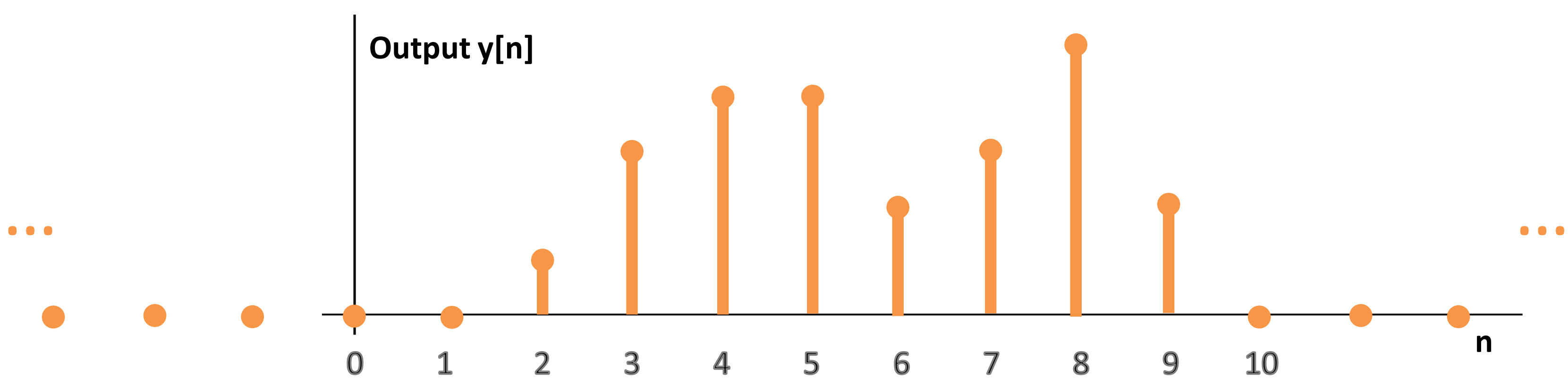

Lecture 3: Discrete-Time Convolution

Foundations of Digital Signal Processing

Outline

- Homework Questions
- Review of Previous Class
- Discrete-Time Convolution
- **The Impulse Response**
- Discrete-Time Convolution Again

Convolution

- **Linear and Time-Invariant (LTI) System**

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

Convolution!

  - Show that if $h[n] = A\delta[n]$, then the system is memoryless.

Convolution

- **Linear and Time-Invariant (LTI) System**

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

Convolution!

  - Show that if $h[n]$ is causal, then the system is causal

Convolution

- **Linear and Time-Invariant (LTI) System**

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

Convolution!

  - Show that if $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$, then the system is BIBO stable

Lecture 3: Discrete -Time Convolution

Foundations of Digital Signal Processing

Outline

- Homework Questions
- Review of Previous Class
- Discrete-Time Convolution
- The Impulse Response
- **Discrete-Time Convolution Again**

Discrete-Time Convolution

- **Go to the notes on the course website!**
  - http://smartdata.ece.ufl.edu/eee5502/lecture.html?lecture=03

Lecture 3: Discrete -Time Convolution

Foundations of Digital Signal Processing

Outline

- Homework Questions
- Review of Previous Class
- Discrete-Time Convolution
- The Impulse Response
- Discrete-Time Convolution Again
- **Correlation**

Convolution

- **Definition of Correlation**

$$y[n] = x[n] * h[n] = \underbrace{\sum_{m=-\infty}^{\infty} x[m]\,h[n+m]}_{\text{Inner product of x[m] and h[n+m]}}$$