Lecture 26: Filter Banks to Wavelets

Foundations of Digital Signal Processing

Outline

- DFT Filter Banks [without downsampling]
- DFT Filter Bank [with downsampling]
- General Two-Channel Filter Banks
- Two-Channel Filter Bank Perfect Reconstruction (aliasing canceling)
- Two-Channel Filter Bank Perfect Reconstruction (orthogonal filter bank)
- Multi-Channel Filter Bank Perfect Reconstruction
- Wavelets

News

- **Schedule / Plan**
  - ~~*Tomorrow:, Nov. 16 Homework #10*~~
  - ~~*Tuesday, Nov. 19: Coding Assignment #6*~~
  - *Next Week:* *No Due Dates (except Monday)*
  - *Thursday, Nov. 29th:* *Homework #11*
  - *Tuesday, Dec. 4th:* *Exam #3*
  - *Wednesday, Dec. 5th:* *Coding Assignment #7 (short)*
  - *Wednesday, Dec. 12th:* *Final Exam*
  - *Friday, Dec. 14th:* *EEE5502 Reports Due*

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- Polyphase Filters
- Multi-Channel Filter Bank Perfect Reconstruction
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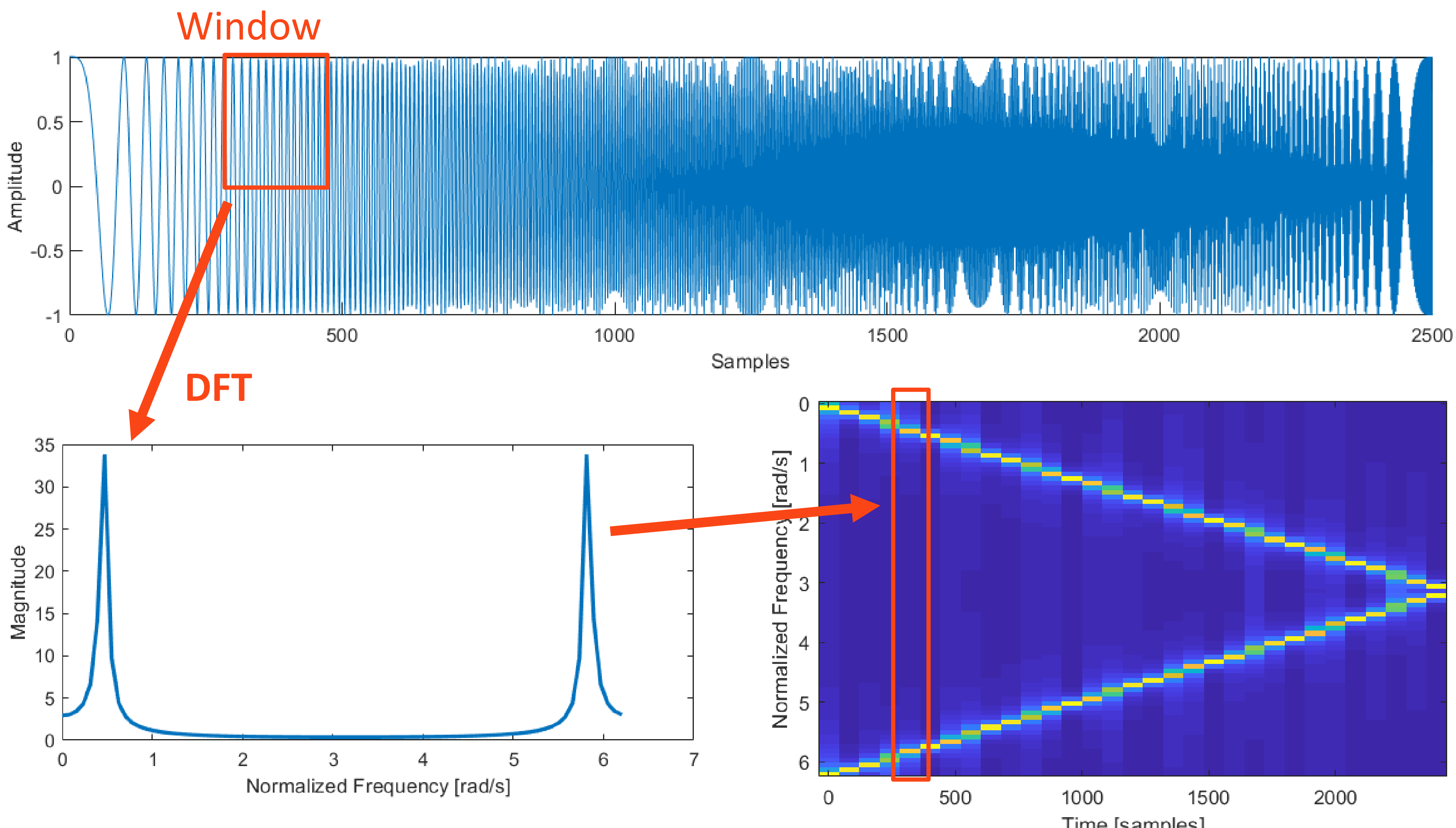
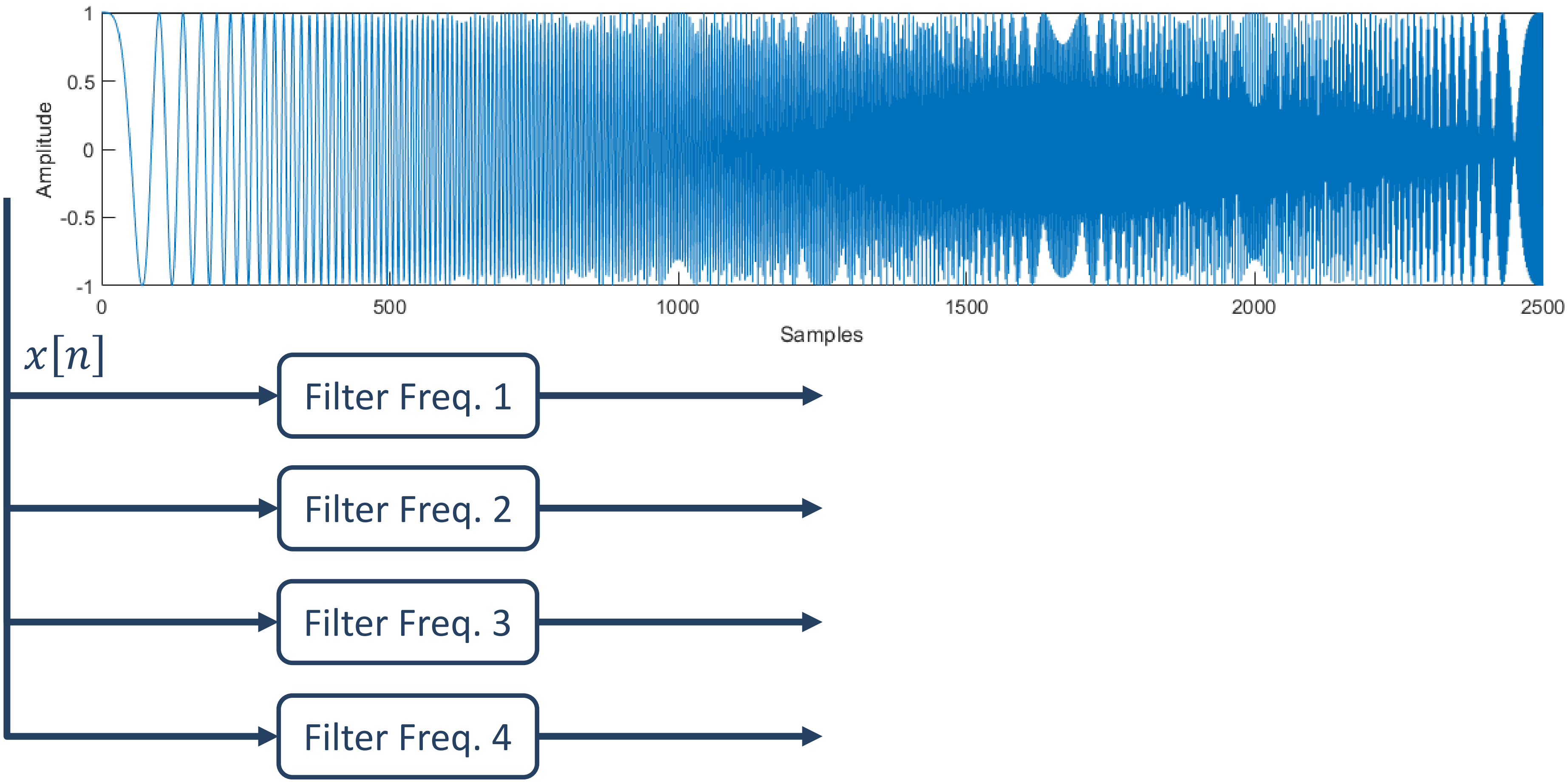
Filter Banks

- **The Short Time Fourier Transform Process**

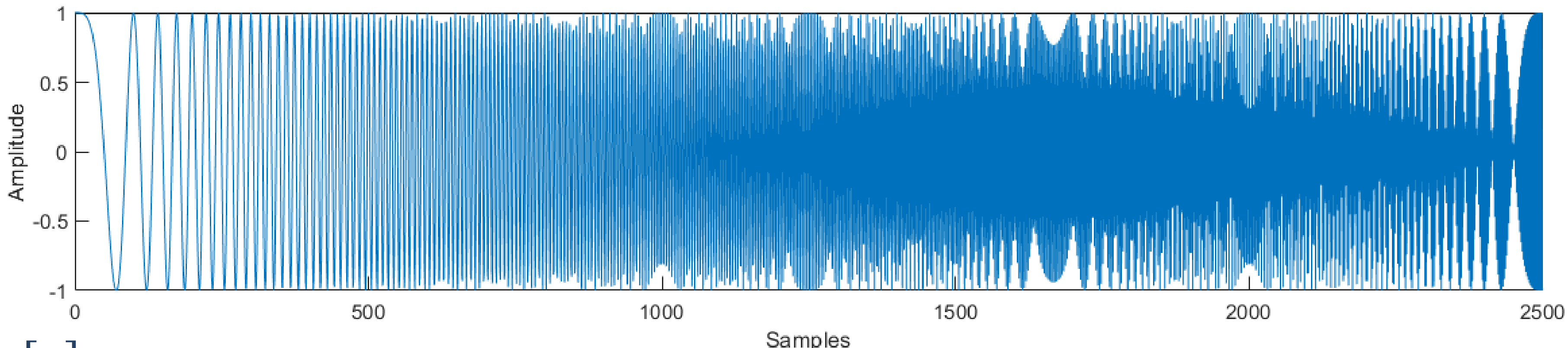


Inefficient Filter Bank Process



Filter Banks

Inefficient Filter Bank Process



$x[n]$

$$h_0[n] = e^{j\frac{2\pi}{4}n(0)}$$

$$h_1[n] = e^{j\frac{2\pi}{4}n(1)}$$

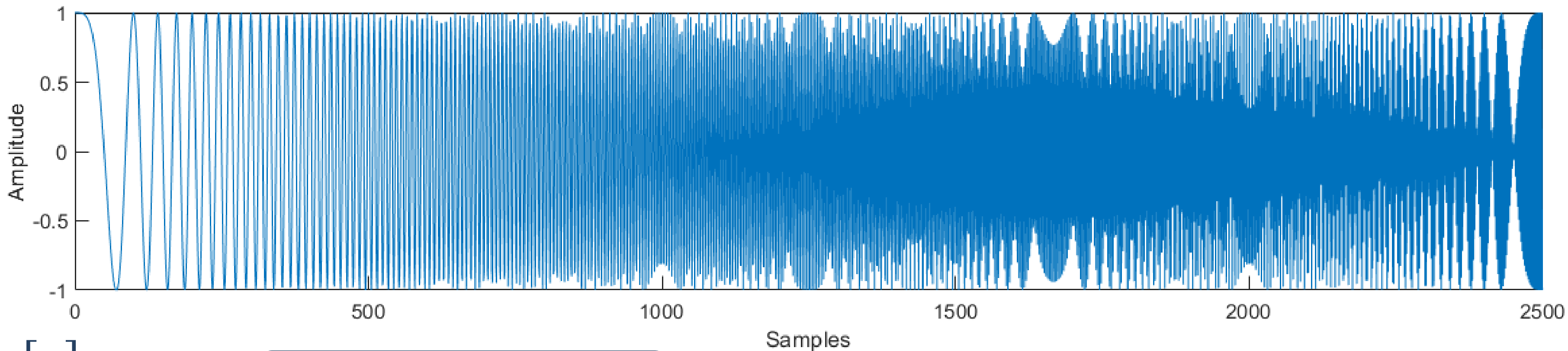
$$h_2[n] = e^{j\frac{2\pi}{4}n(2)}$$

$$h_3[n] = e^{j\frac{2\pi}{4}n(3)}$$

Assume all length-4 filters

Filter Banks

Inefficient Filter Bank Process



$x[n]$

$$h_0[n] = e^{j\frac{2\pi}{4}n(0)} \rightarrow e^{j\frac{2\pi}{N}n(0)} * x[n]$$

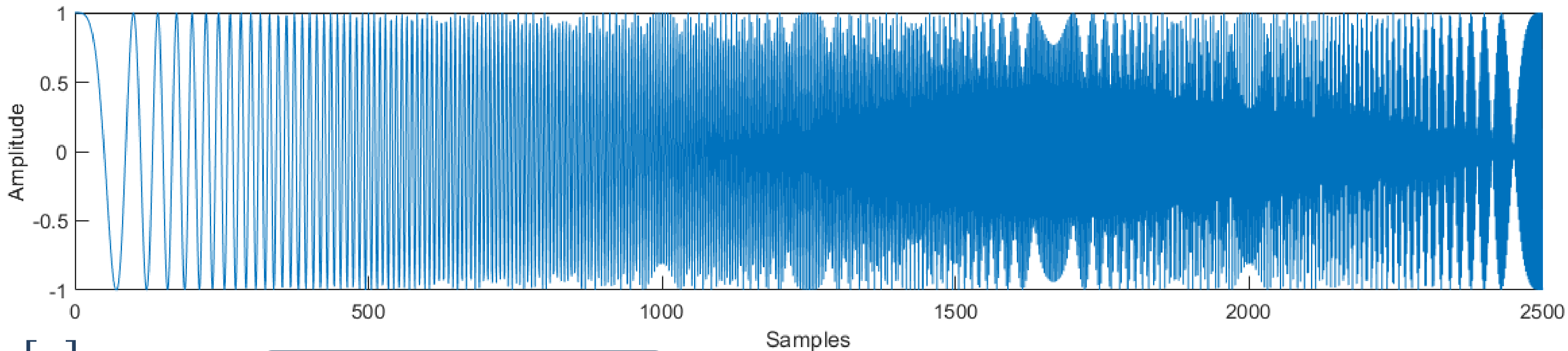
$$h_1[n] = e^{j\frac{2\pi}{4}n(1)} \rightarrow e^{j\frac{2\pi}{N}n(1)} * x[n]$$

$$h_2[n] = e^{j\frac{2\pi}{4}n(2)} \rightarrow e^{j\frac{2\pi}{N}n(2)} * x[n]$$

$$h_3[n] = e^{j\frac{2\pi}{4}n(3)} \rightarrow e^{j\frac{2\pi}{N}n(3)} * x[n]$$

Assume all length-4 filters

Inefficient Filter Bank Process



$x[n]$

$$h_0[n] = e^{j\frac{2\pi}{4}n(0)} \rightarrow \sum_{m=0}^{N-1} e^{j\frac{2\pi}{N}(n-m)(0)}\, x[m]$$

$$h_1[n] = e^{j\frac{2\pi}{4}n(1)} \rightarrow \sum_{m=0}^{N-1} e^{j\frac{2\pi}{N}(n-m)(1)}\, x[m]$$

$$h_2[n] = e^{j\frac{2\pi}{4}n(2)} \rightarrow \sum_{m=0}^{N-1} e^{j\frac{2\pi}{N}(n-m)(2)}\, x[m]$$

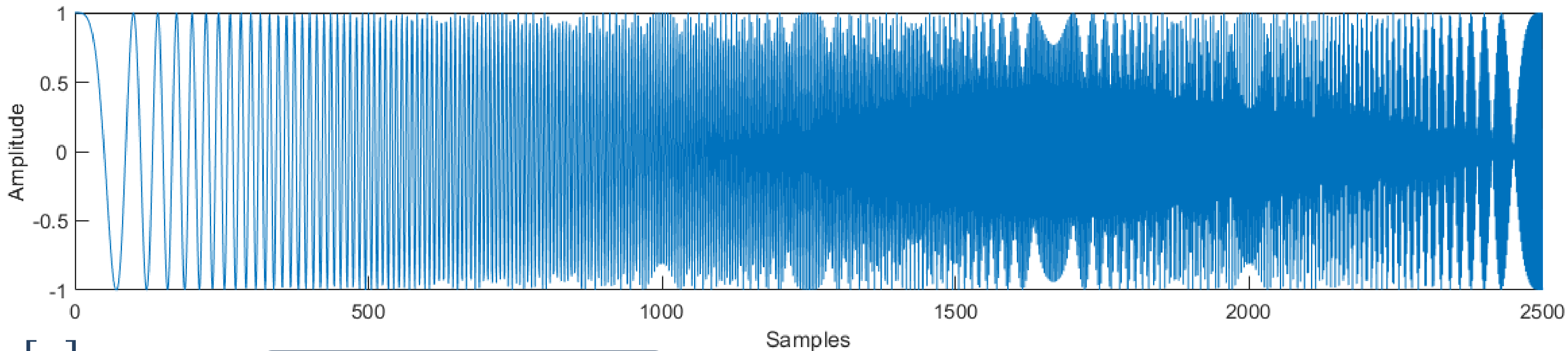
$$h_3[n] = e^{j\frac{2\pi}{4}n(3)} \rightarrow \sum_{m=0}^{N-1} e^{j\frac{2\pi}{N}(n-m)(3)}\, x[m]$$

Assume all length-4 filters

Hard solution: Use math

Filter Banks

Inefficient Filter Bank Process



$x[n]$

$$h_0[n] = e^{j\frac{2\pi}{4}n(0)} \rightarrow e^{j\frac{2\pi}{N}n(0)} \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m(0)}\, x[m]$$

$$h_1[n] = e^{j\frac{2\pi}{4}n(1)} \rightarrow e^{j\frac{2\pi}{N}n(1)} \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m(1)}\, x[m]$$

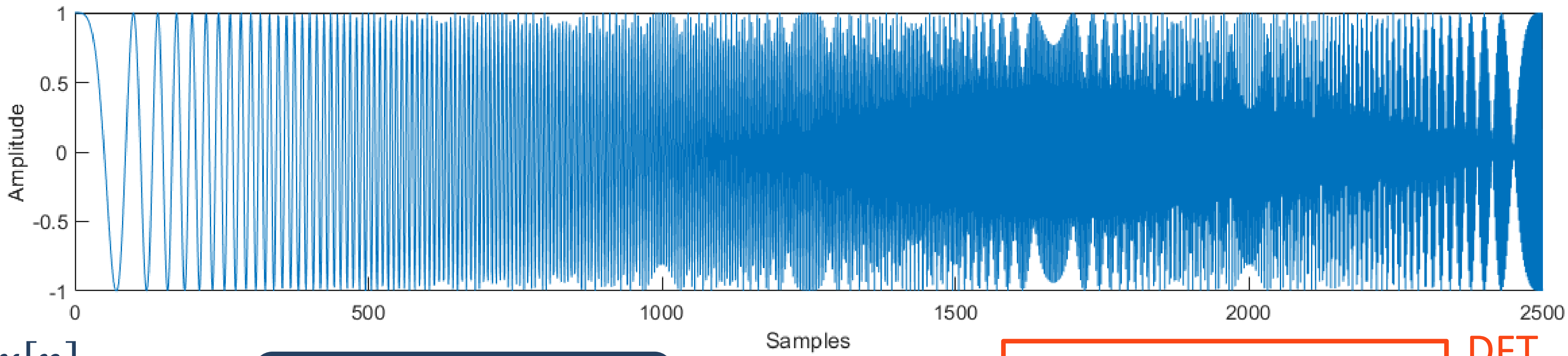
$$h_2[n] = e^{j\frac{2\pi}{4}n(2)} \rightarrow e^{j\frac{2\pi}{N}n(2)} \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m(2)}\, x[m]$$

$$h_3[n] = e^{j\frac{2\pi}{4}n(3)} \rightarrow e^{j\frac{2\pi}{N}n(3)} \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m(3)}\, x[m]$$

Assume all length-4 filters

Filter Banks

Inefficient Filter Bank Process



$x[n]$

$h_0[n] = e^{j\frac{2\pi}{4}n(0)}$ → $e^{j\frac{2\pi}{N}n(0)} \boxed{\sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m(0)}\, x[m]}$ DFT for k = 0

$h_1[n] = e^{j\frac{2\pi}{4}n(1)}$ → $e^{j\frac{2\pi}{N}n(1)} \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m(1)}\, x[m]$

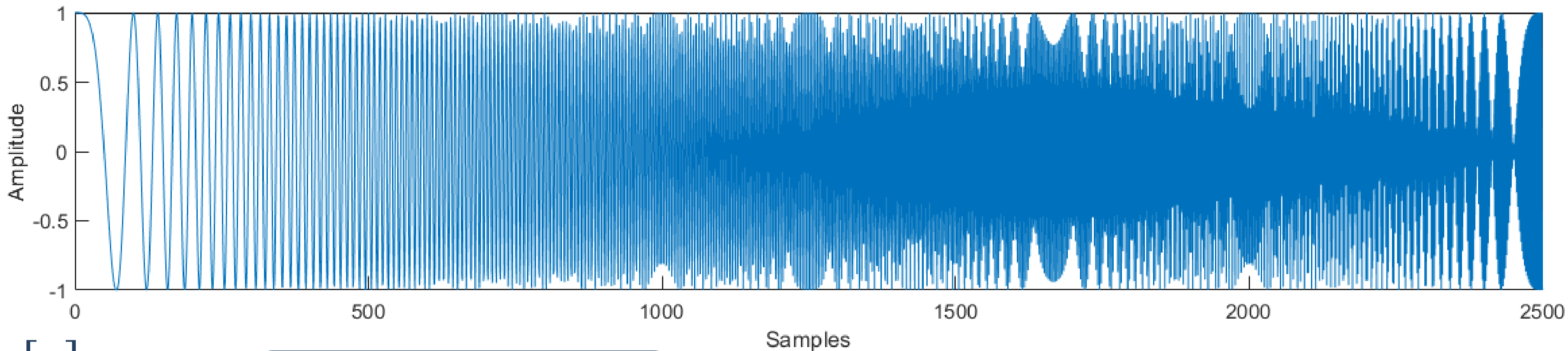
$h_2[n] = e^{j\frac{2\pi}{4}n(2)}$ → $e^{j\frac{2\pi}{N}n(2)} \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m(2)}\, x[m]$

$h_3[n] = e^{j\frac{2\pi}{4}n(3)}$ → $e^{j\frac{2\pi}{N}n(3)} \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m(3)}\, x[m]$

Assume all length-4 filters

Filter Banks

- **Inefficient Filter Bank Process**



$x[n]$

$$h_0[n] = e^{j\frac{2\pi}{4}n(0)} \rightarrow e^{j\frac{2\pi}{N}n(0)} X[n, 0]$$

$$h_1[n] = e^{j\frac{2\pi}{4}n(1)} \rightarrow e^{j\frac{2\pi}{N}n(1)} \boxed{\sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m(1)} x[m]}$$

DFT for k = 1

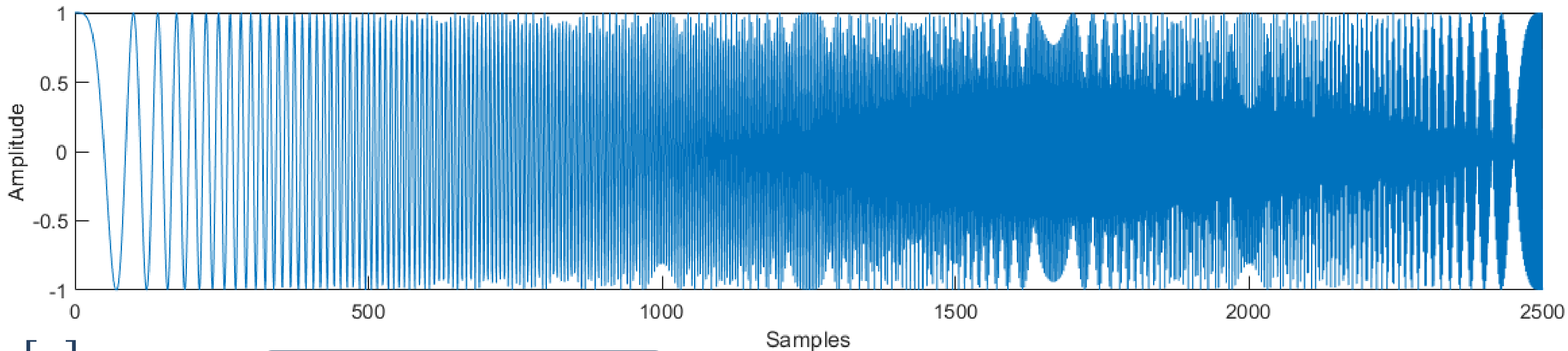
$$h_2[n] = e^{j\frac{2\pi}{4}n(2)} \rightarrow e^{j\frac{2\pi}{N}n(2)} \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m(2)} x[m]$$

$$h_3[n] = e^{j\frac{2\pi}{4}n(3)} \rightarrow e^{j\frac{2\pi}{N}n(3)} \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m(3)} x[m]$$

Assume all length-4 filters

Filter Banks

Inefficient Filter Bank Process



$x[n]$

$$h_0[n] = e^{j\frac{2\pi}{4}n(0)} \rightarrow e^{j\frac{2\pi}{N}n(0)}X[n,0]$$

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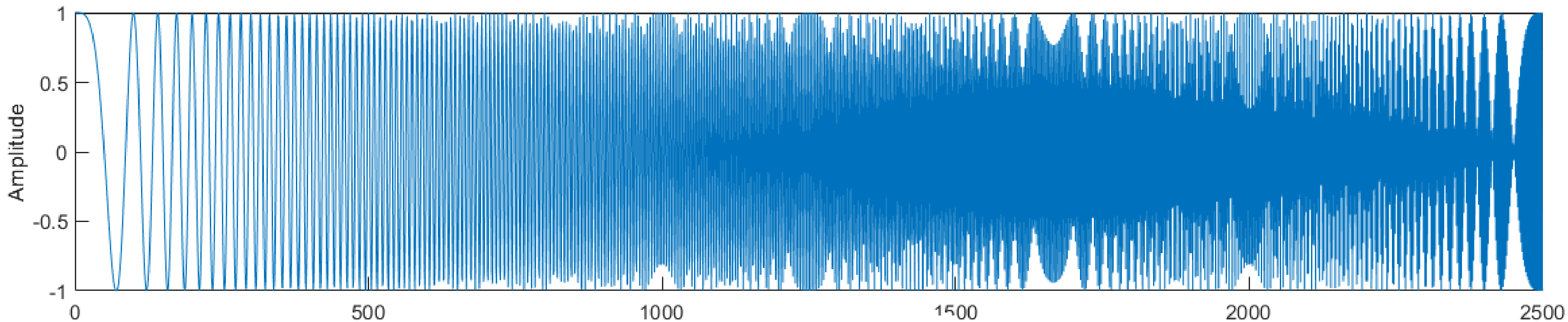
Assume all length-4 filters

Filter Banks

Inefficient Filter Bank Process



$x[n]$

$h_0[n] = e^{j\frac{2\pi}{4}n(0)}$ → $e^{-j\frac{2\pi}{4}n(0)}$ → $X[n,0]$

$h_1[n] = e^{j\frac{2\pi}{4}n(1)}$ → $e^{-j\frac{2\pi}{4}n(1)}$ → $X[n,1]$

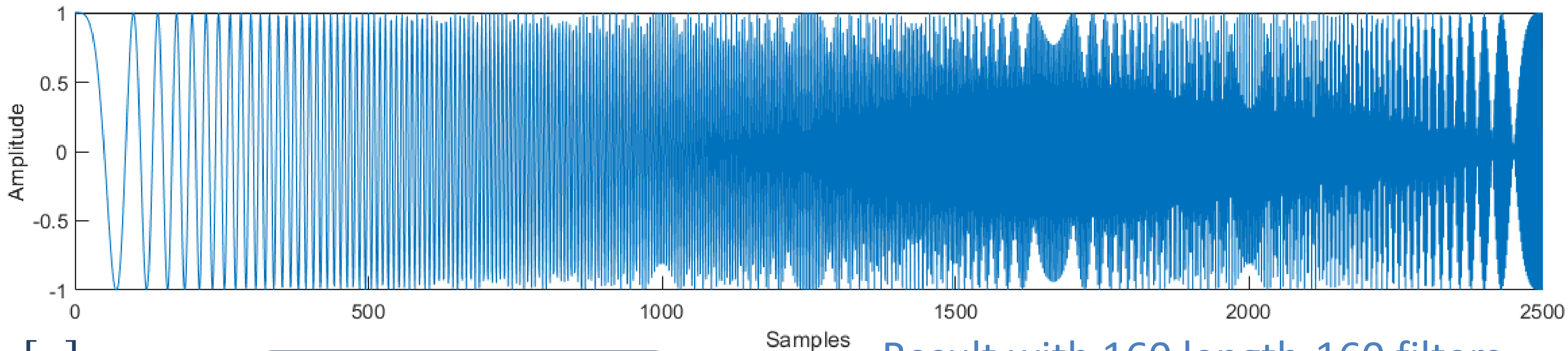
$h_2[n] = e^{j\frac{2\pi}{4}n(2)}$ → $e^{-j\frac{2\pi}{4}n(2)}$ → $X[n,2]$

$h_3[n] = e^{j\frac{2\pi}{4}n(3)}$ → $e^{-j\frac{2\pi}{4}n(3)}$ → $X[n,3]$

Assume all length-4 filters

We have the DFT coefficients!

Inefficient Filter Bank Process



$x[n]$

$$h_0[n] = e^{j\frac{2\pi}{4}n(0)}$$

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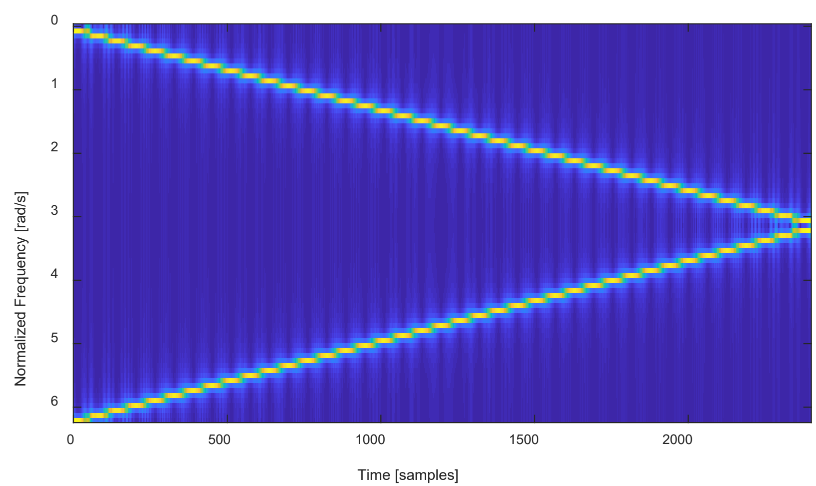
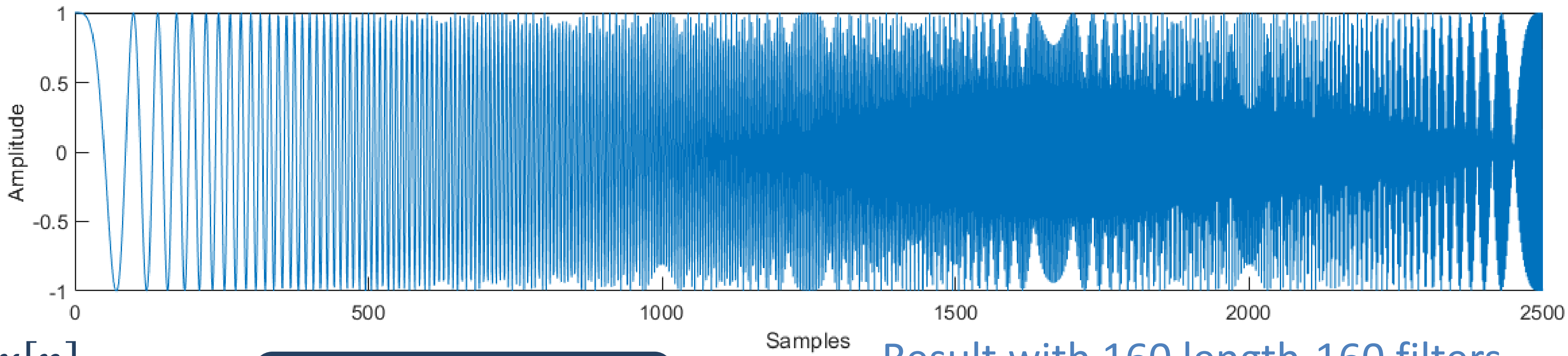
Assume all length-4 filters

Result with 160 length-160 filters



Filter Banks

- **Question: Why is this <u>not</u> a preferred approach?**



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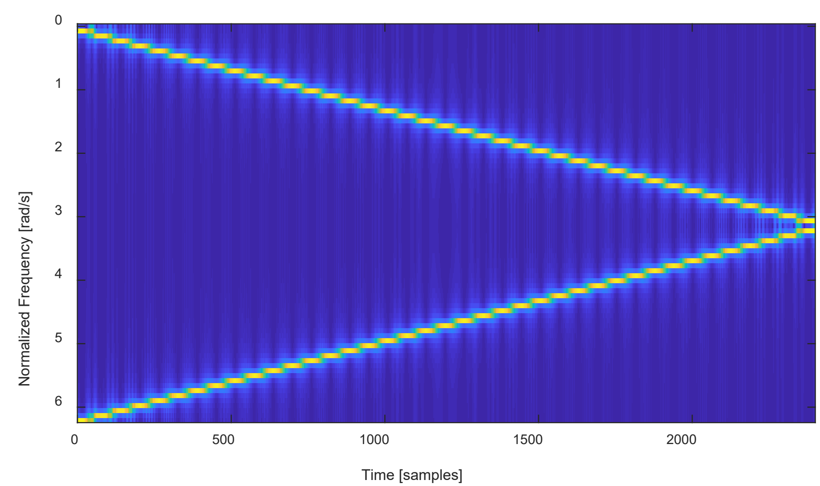
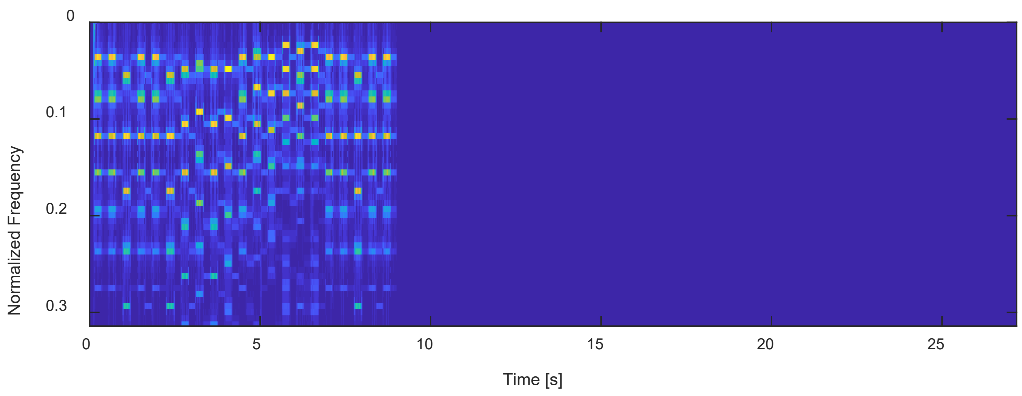
Assume all length-4 filters

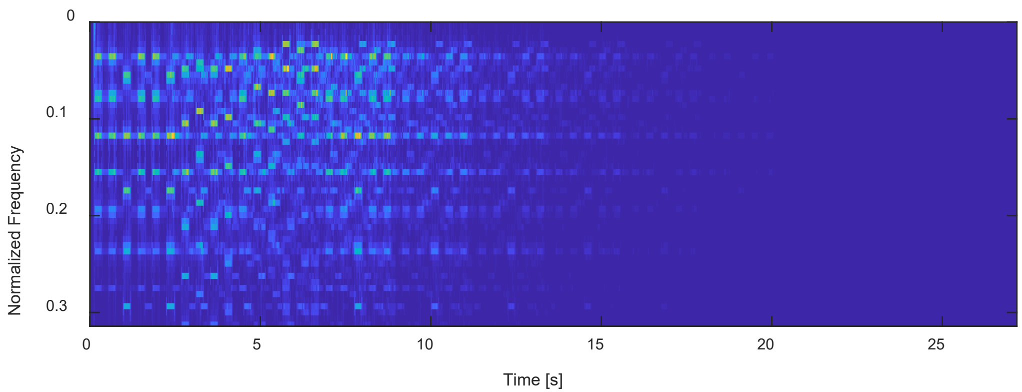
Result with 160 length-160 filters



Filter Banks

- **Example:** **Example from our coding assignment**





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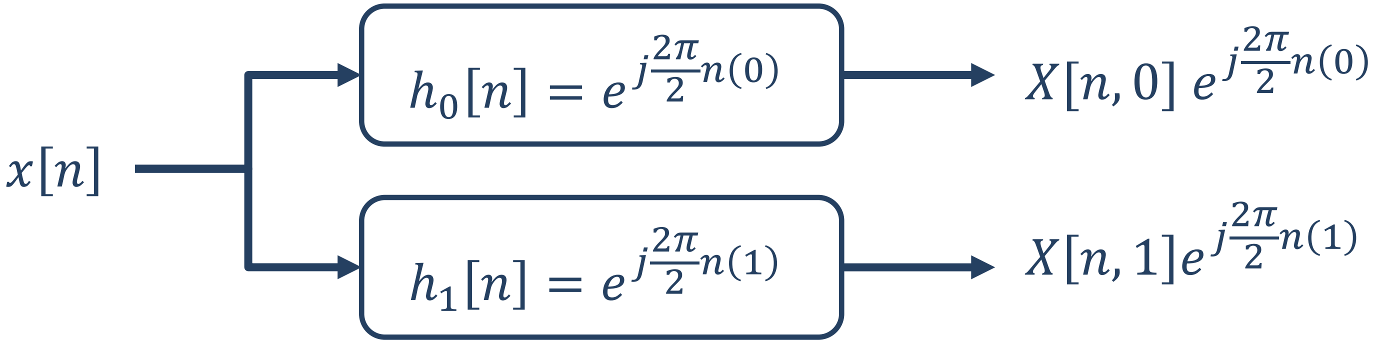
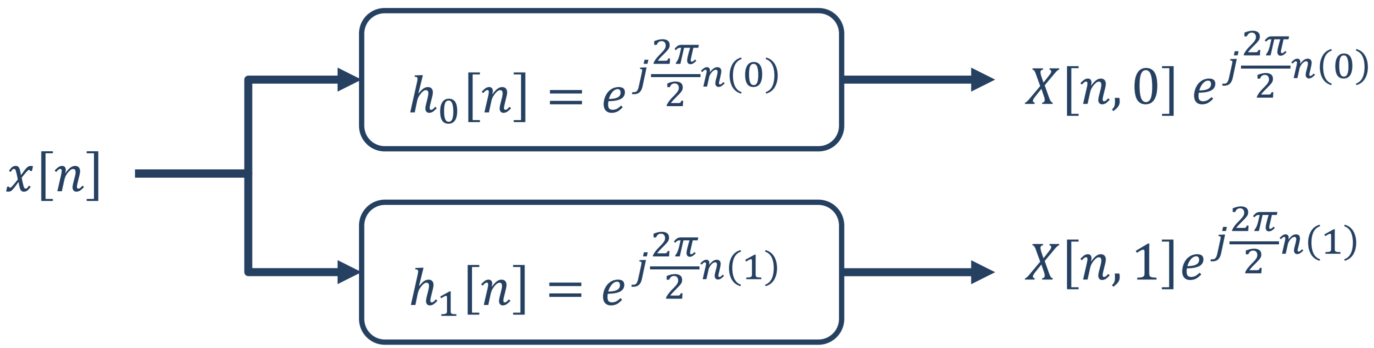
Filter Banks

- **Consider the following filter bank**



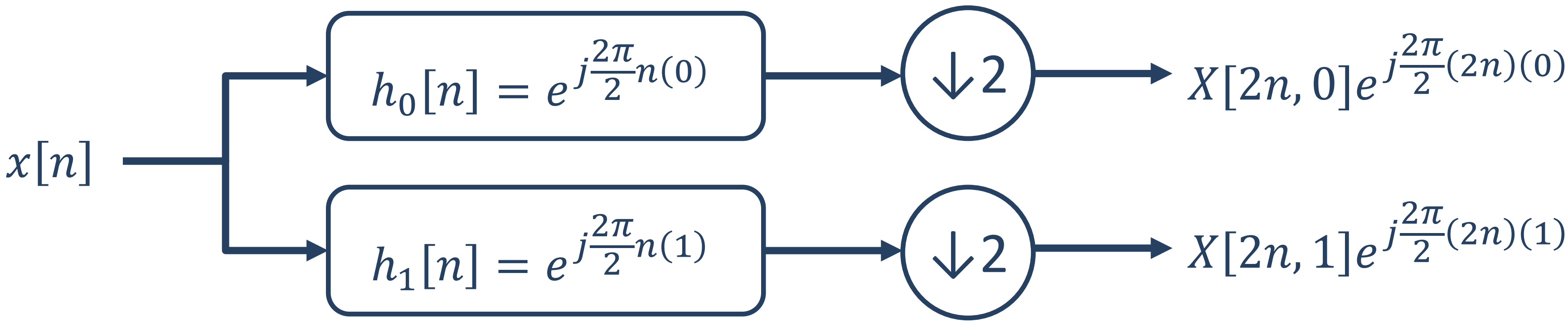
Filter Banks

- **Consider the following filter bank**
  - **Question:** How do I make this like the STFT????



  - **Now recall:** The short-time Fourier Transform gave us
    - $X[Mn, 0]$ <- M = shift amount (often window length)

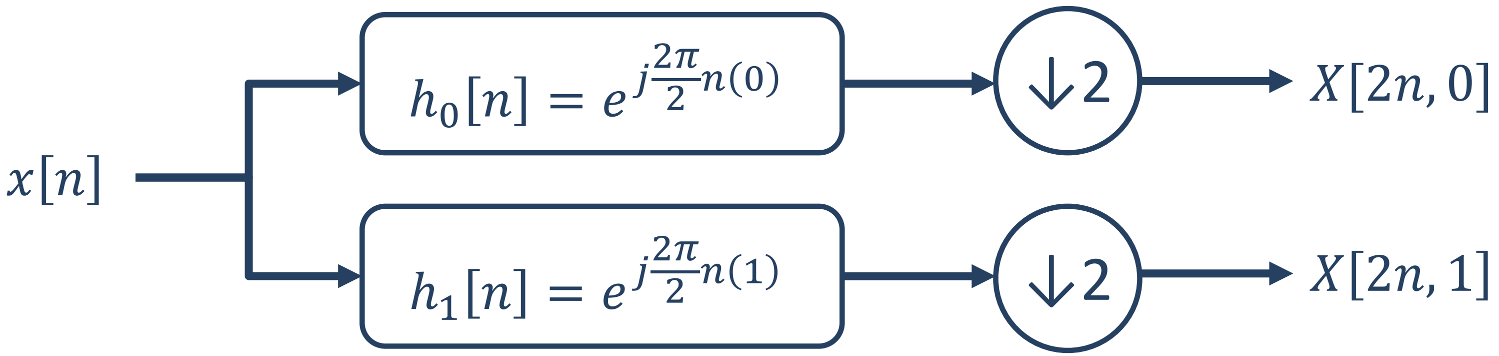
- **Consider the following filter bank**
  - **Question:** How do I make this like the STFT????



  - **Now recall:** The short-time Fourier Transform gave us
    - ◊ $X[Mn, 0]$ <- M = shift amount (often window length)

Filter Banks

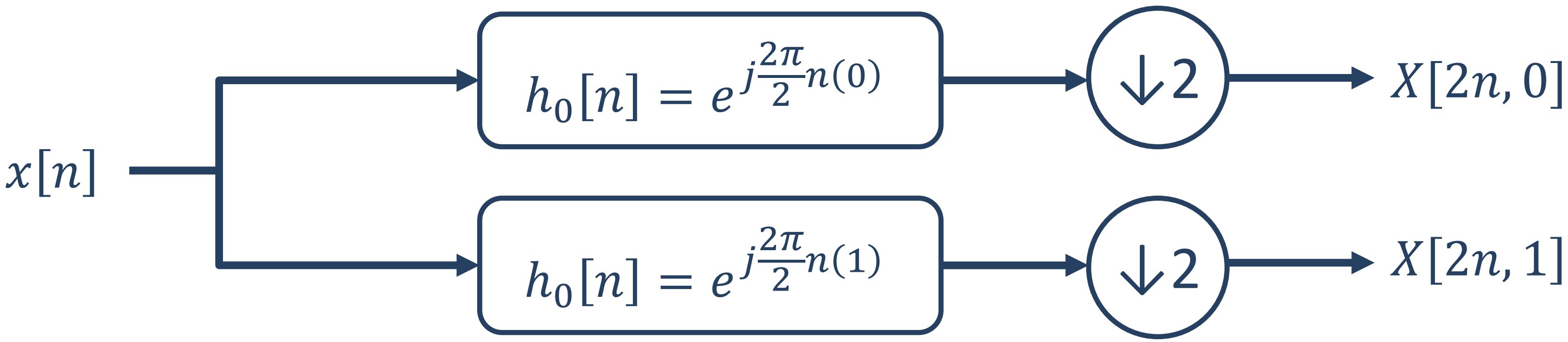
- **Consider the following filter bank**
  - **Question:** How do I make this like the STFT????



  - **Now recall:** The short-time Fourier Transform gave us
    - $X[Mn, 0]$ <- M = shift amount (often window length)

Filter Banks

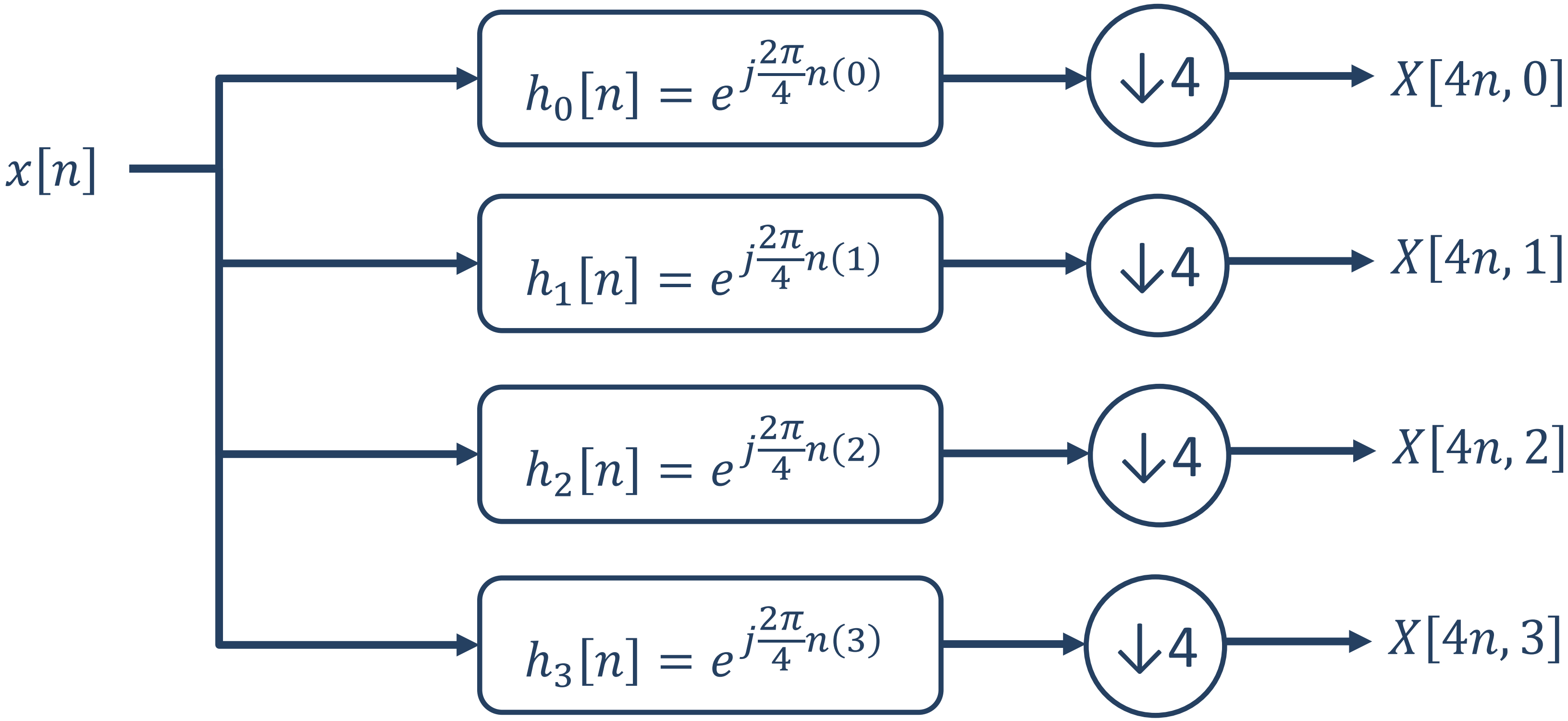
- **Consider the following filter bank**



  - **Result:** It is now exactly the same as the STFT with a window of length 2 and shift of 2 between windows
  - **But,** we do not need to buffer $x[n]$

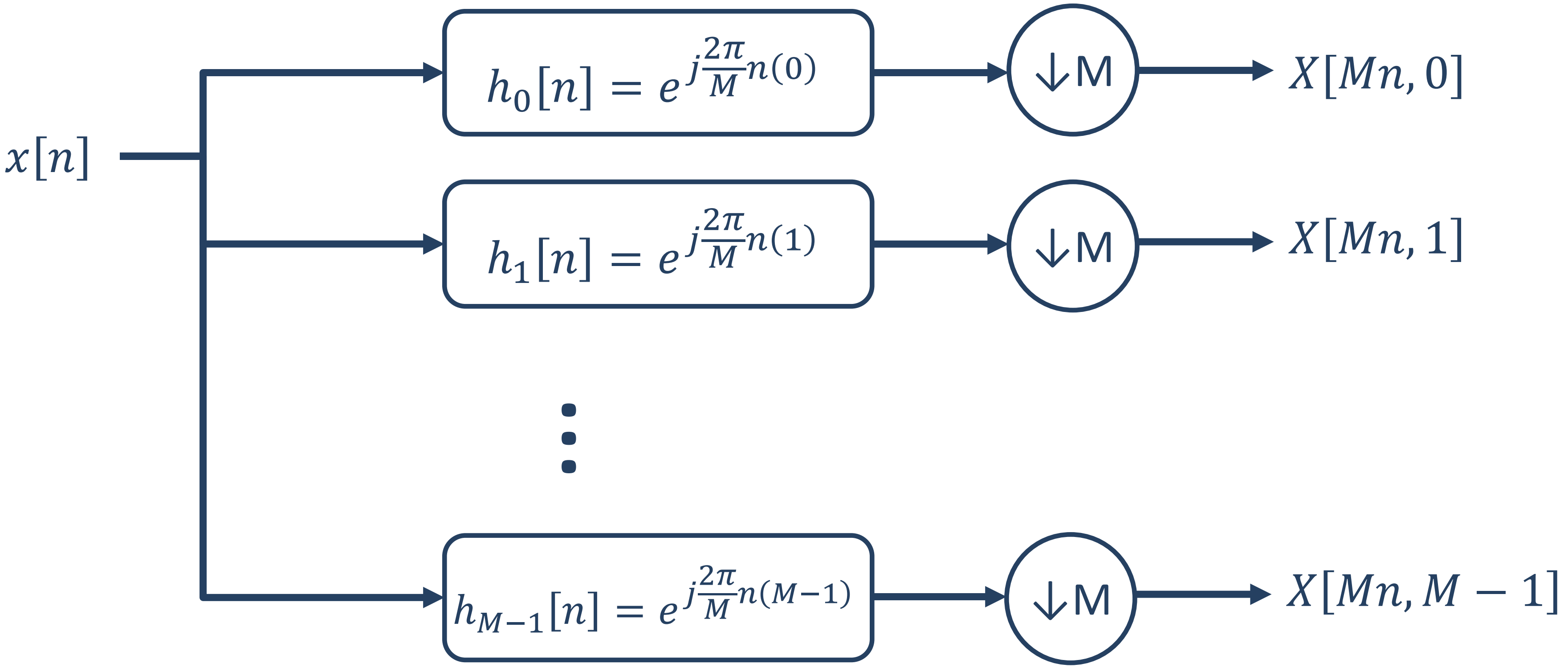
Filter Banks

- **Consider the following filter bank**



Filter Banks

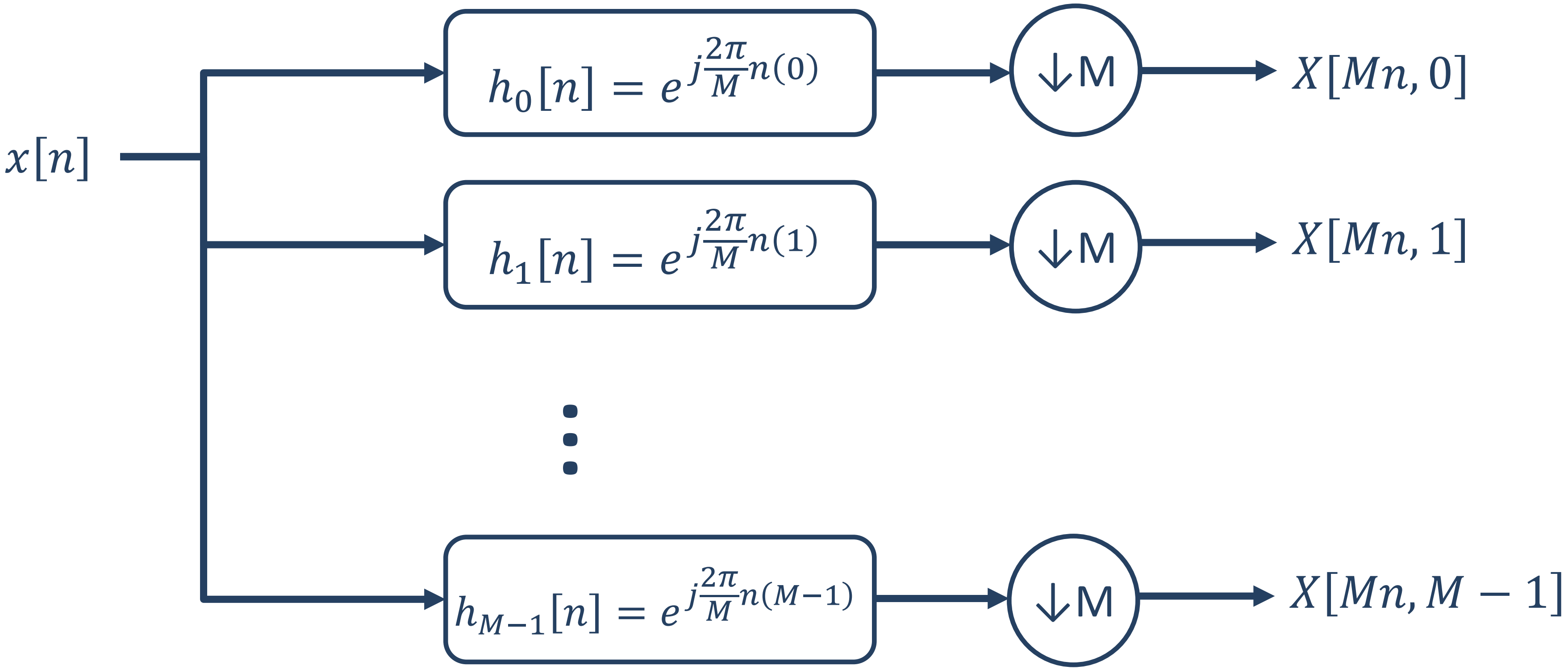
- **Hence, this is an M-point DFT**



- **So, I can implement the STFT as a filter bank…**
  - Can I do more?

Filter Banks

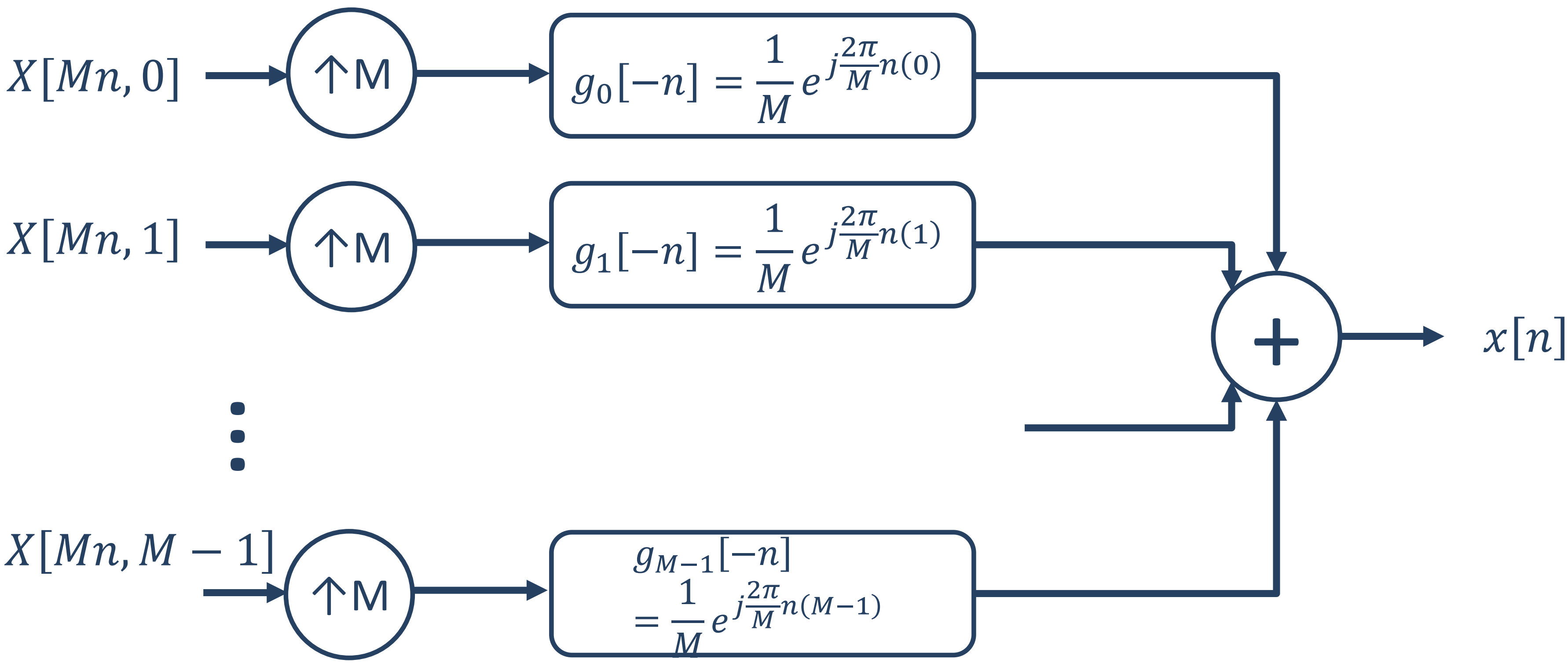
- **Hence, this is an M-point DFT**



- **So, I can implement the STFT as a filter bank…**
  - **Question:** How do I get back into the time domain?

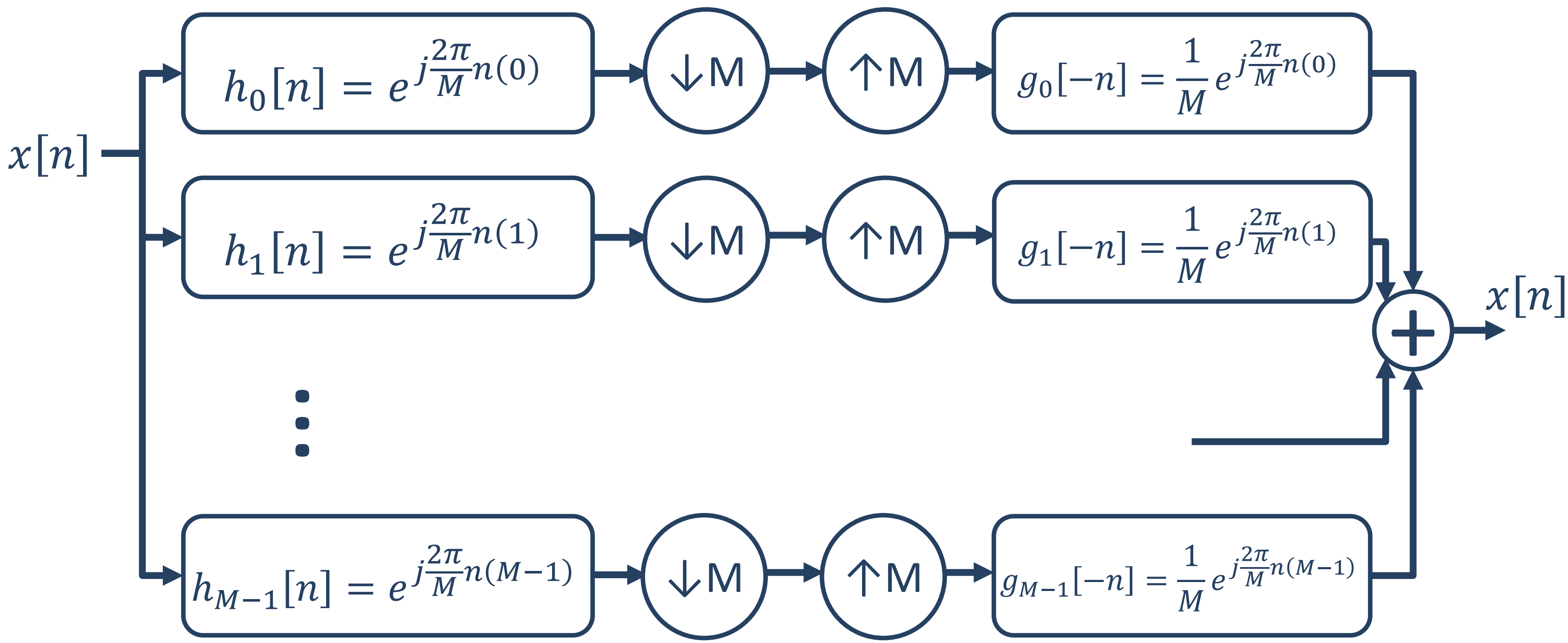
Filter Banks

- **Hence, this is an M-point IDFT**



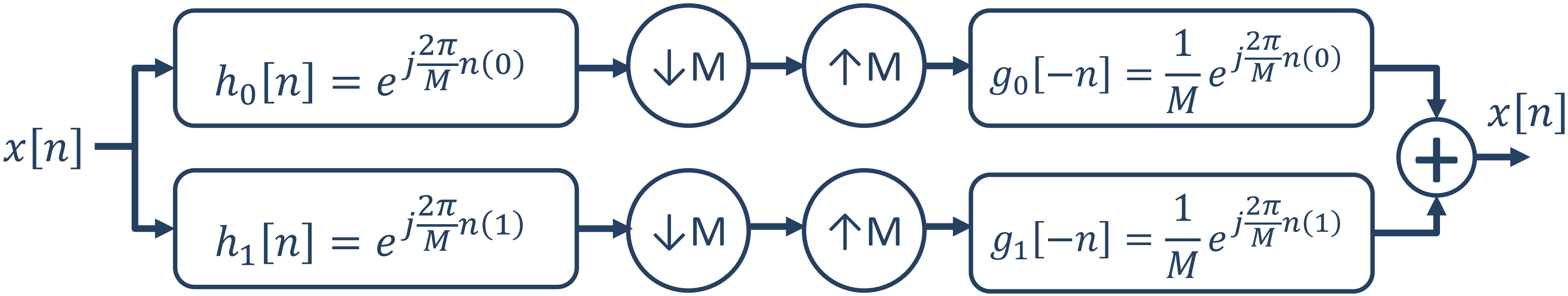
Filter Banks

- **Hence, this is an M-point DFT and IDFT**



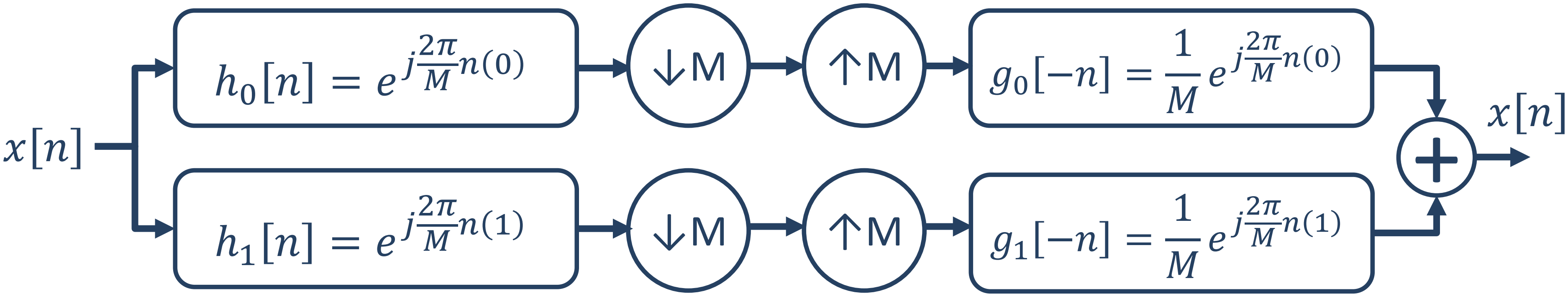
Filter Banks

- **Example: The M=2 point DFT and IDFT**



Filter Banks

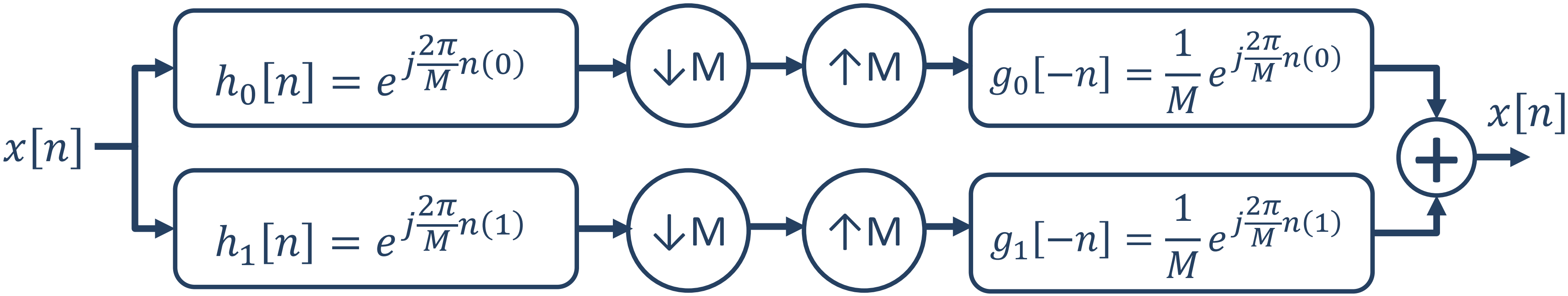
- **Example: The M=2 point DFT and IDFT**



  - What are the filter coefficients for $h_0[n]$ and $h_1[n]$?

Filter Banks

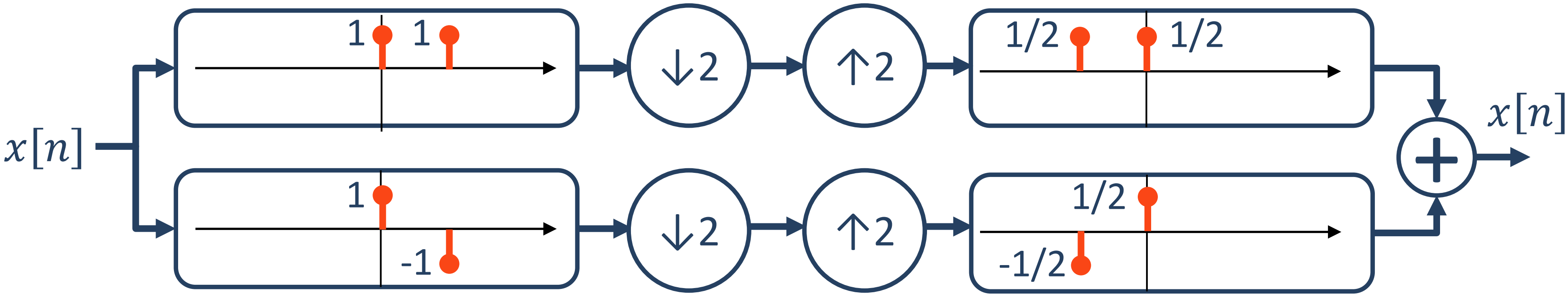
- **Example: The M=2 point DFT and IDFT**



- $h_0[n] = e^{j\frac{2\pi}{2}n(0)} = 1$ for $0 \leq n \leq 1$
- $h_1[n] = e^{j\frac{2\pi}{2}n(1)} = e^{\pi n}$ for $0 \leq n \leq 1$
- $g_0[n] = \frac{1}{M}e^{-j\frac{2\pi}{2}n(0)} = \frac{1}{M}$ for $-1 \leq n \leq 0$
- $g_1[n] = \frac{1}{M}e^{-j\frac{2\pi}{2}n(1)} = \frac{1}{M}e^{\pi n}$ for $-1 \leq n \leq 0$

Filter Banks

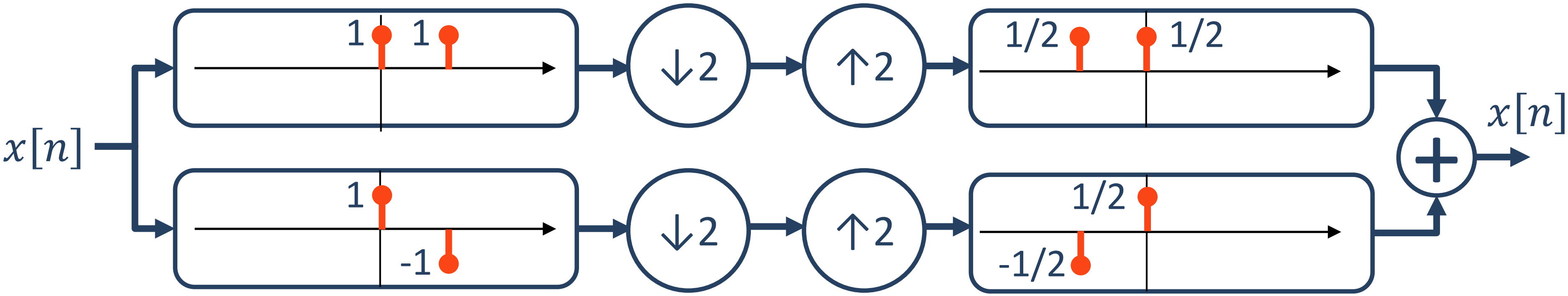
- **Example: The M=2 point DFT and IDFT**



- $h_0[n] = \delta[n] + \delta[n-1]$
- $h_1[n] = \delta[n] - \delta[n-1]$
- $g_0[n] = \frac{1}{2}[\delta[n+1] + \delta[n]]$
- $g_1[n] = \frac{1}{2}[-\delta[n+1] + \delta[n]]$

Filter Banks

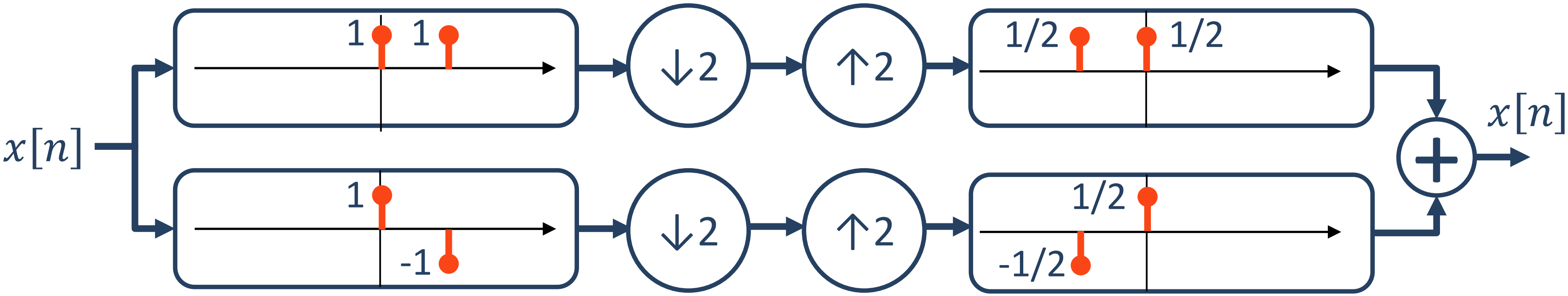
- **Example: The M=2 point DFT and IDFT**



  - $h_0[n] = \delta[n] + \delta[n-1]$
    - ◊ $H(\omega) = e^{-j\frac{2\pi}{2}\omega(0)} + e^{-j\frac{2\pi}{2}\omega(1)}$
  - $h_1[n] = \delta[n] - \delta[n-1]$
    - ◊ $H(\omega) = e^{-j\frac{2\pi}{2}\omega(0)} - e^{-j\frac{2\pi}{2}\omega(1)}$

Filter Banks

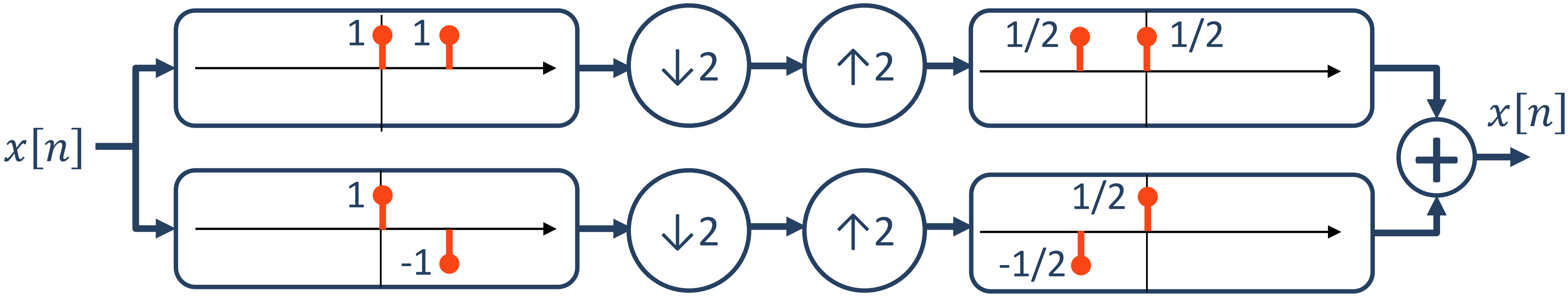
- **Example: The M=2 point DFT and IDFT**



  - $h_0[n] = \delta[n] + \delta[n-1]$
    - ◊ $H_0(\omega) = 1 + e^{-j\pi\omega} = 2\cos(\omega/2)\, e^{-j\frac{\pi}{2}\omega}$
  - $h_1[n] = \delta[n] - \delta[n-1]$
    - ◊ $H_1(\omega) = 1 - e^{-j\pi\omega} = 2j\sin(\omega/2)\, e^{-j\frac{\pi}{2}\omega}$

Filter Banks

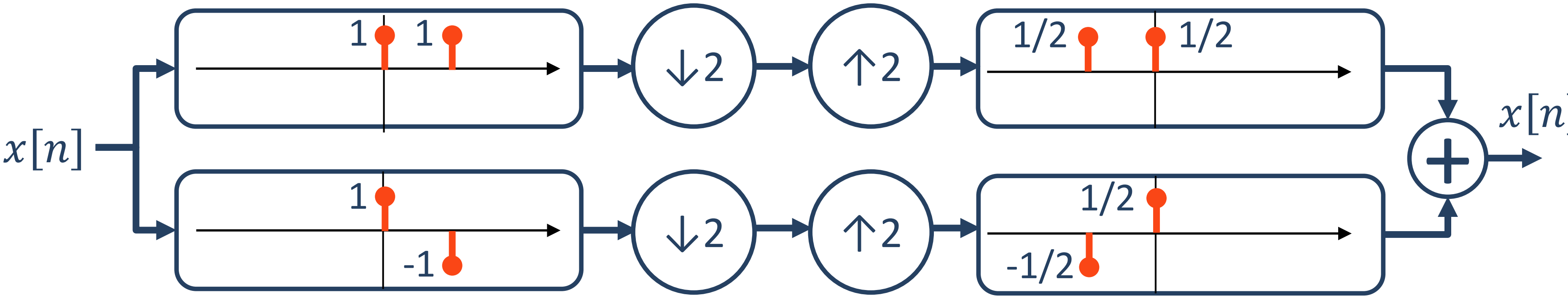
- **Example: The M=2 point DFT and IDFT**



- $h_0[n] = \delta[n] + \delta[n-1]$
  - $|H_0(\omega)| = 2|\cos(\omega/2)|$
- $h_1[n] = \delta[n] - \delta[n-1]$
  - $|H_1(\omega)| = 2|\sin(\omega/2)|$

Filter Banks

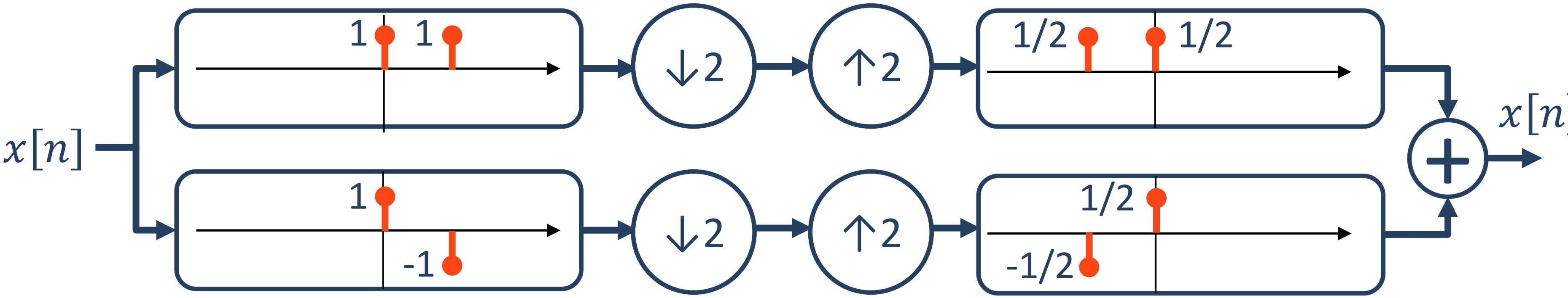
- **Example: The M=2 point DFT and IDFT**

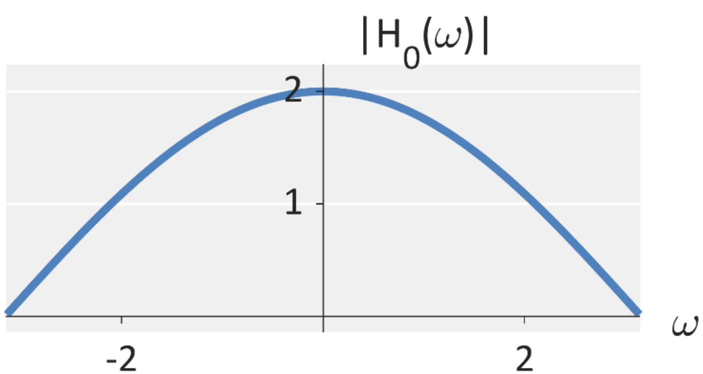


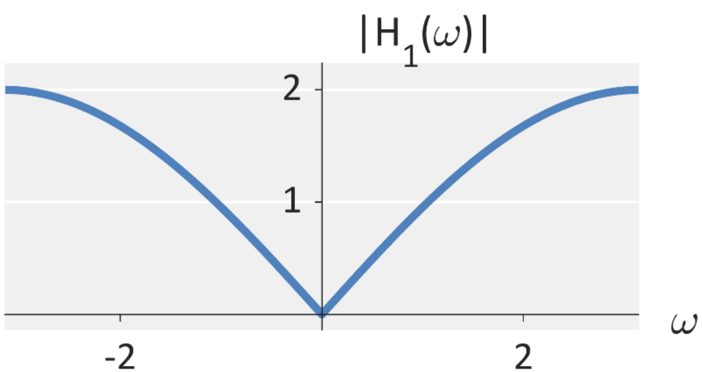
- $h_0[n] = \delta[n] + \delta[n-1]$
  - $|H_0(\omega)| = 2|\cos(\omega/2)|$
  - $|G_0(\omega)| = |\cos(\omega/2)|$
- $h_1[n] = \delta[n] - \delta[n-1]$
  - $|H_1(\omega)| = 2|\sin(\omega/2)|$
  - $|G_1(\omega)| = |\sin(\omega/2)|$

Filter Banks

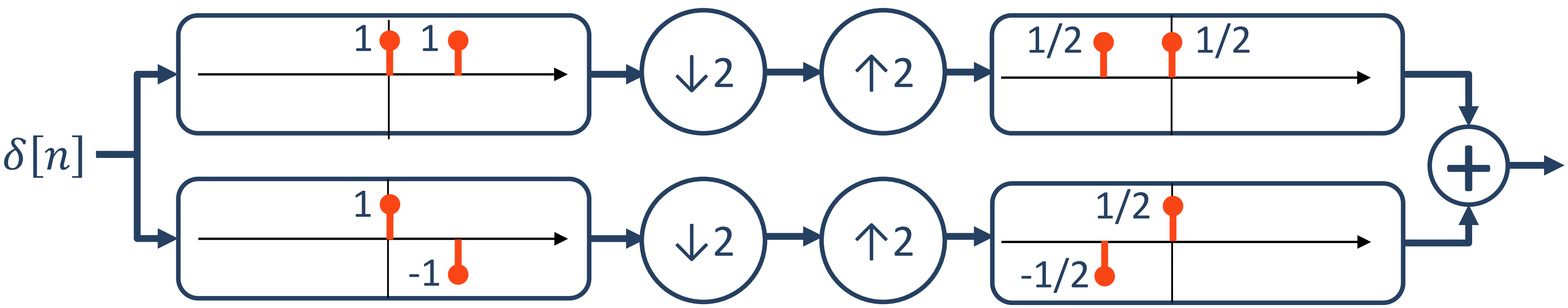
- **Example: The M=2 point DFT and IDFT**





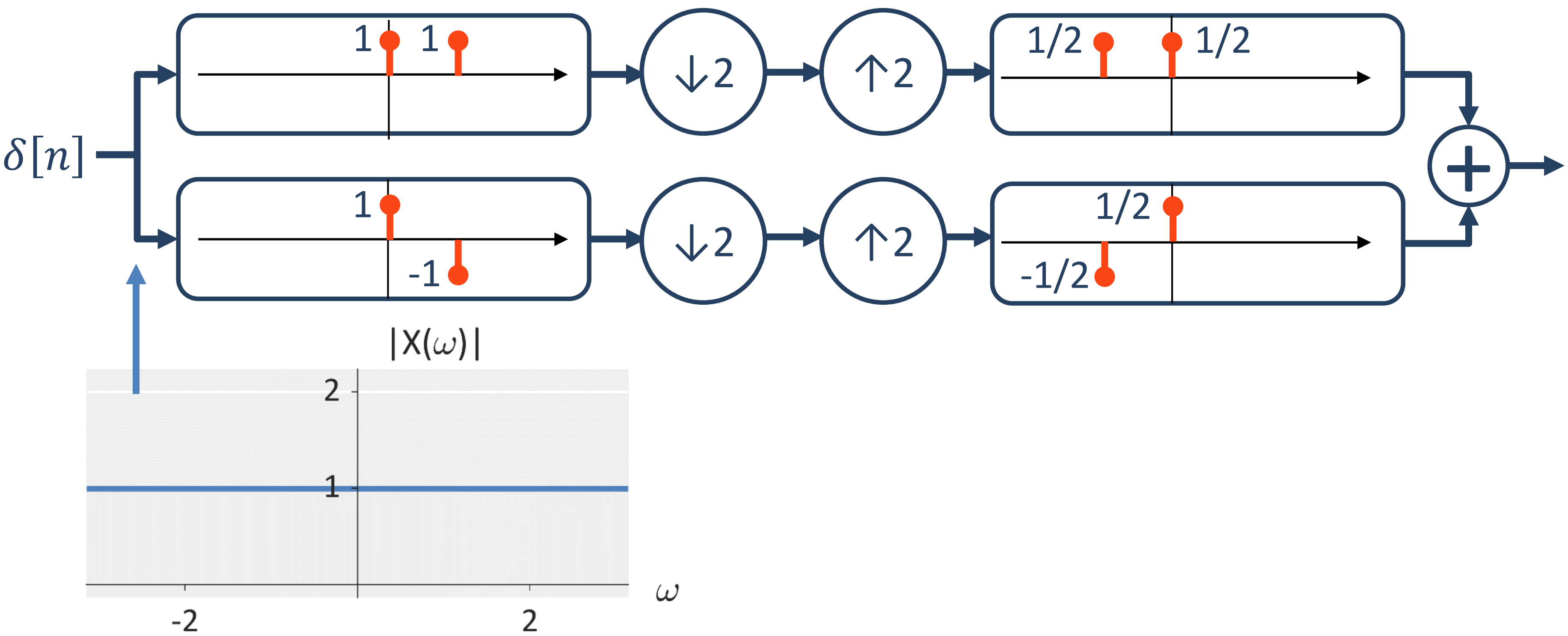


- **Example:** **Plot of the intermediate frequency magnitudes**



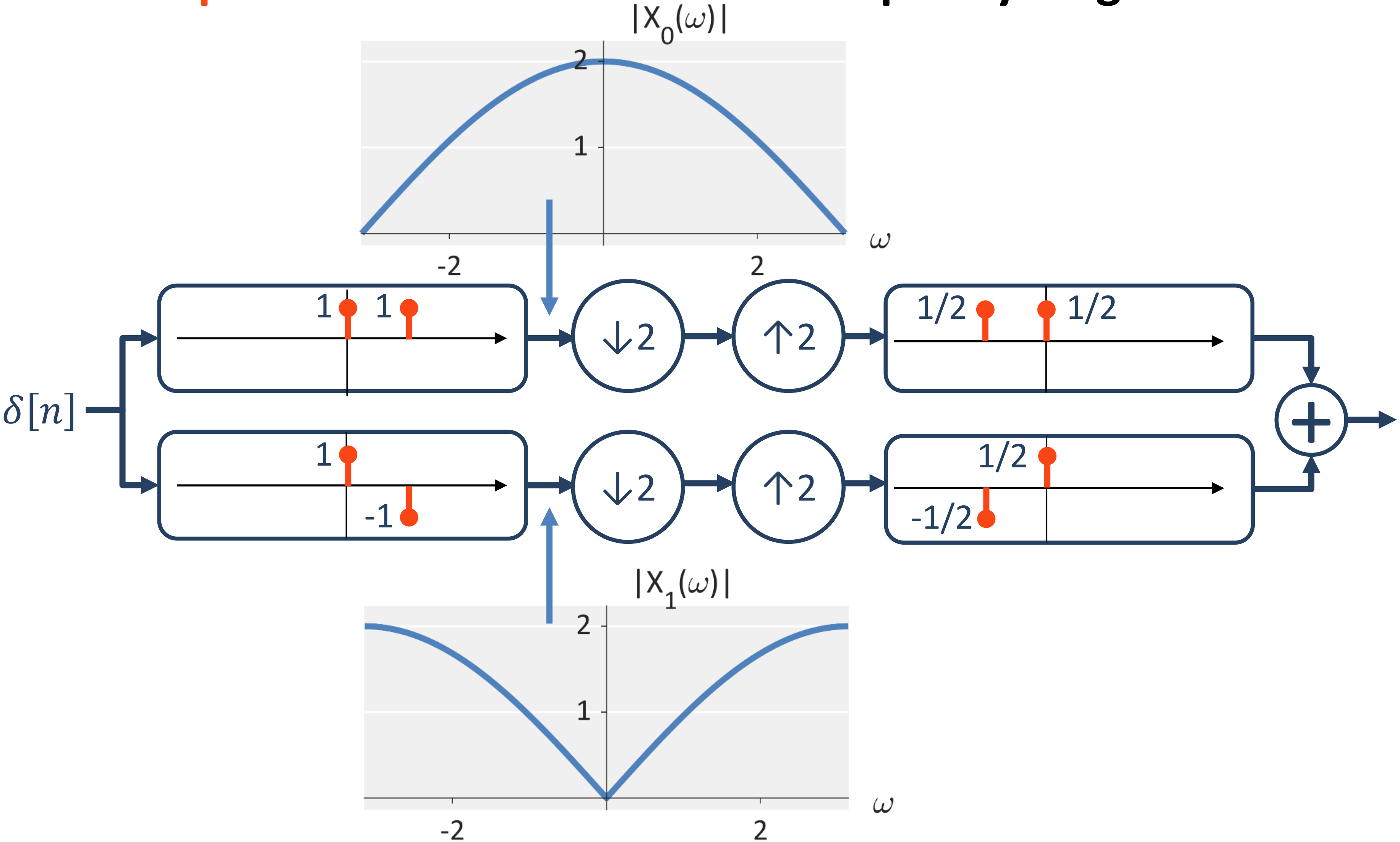
Filter Banks

- **Example: Plot of the intermediate frequency magnitudes**



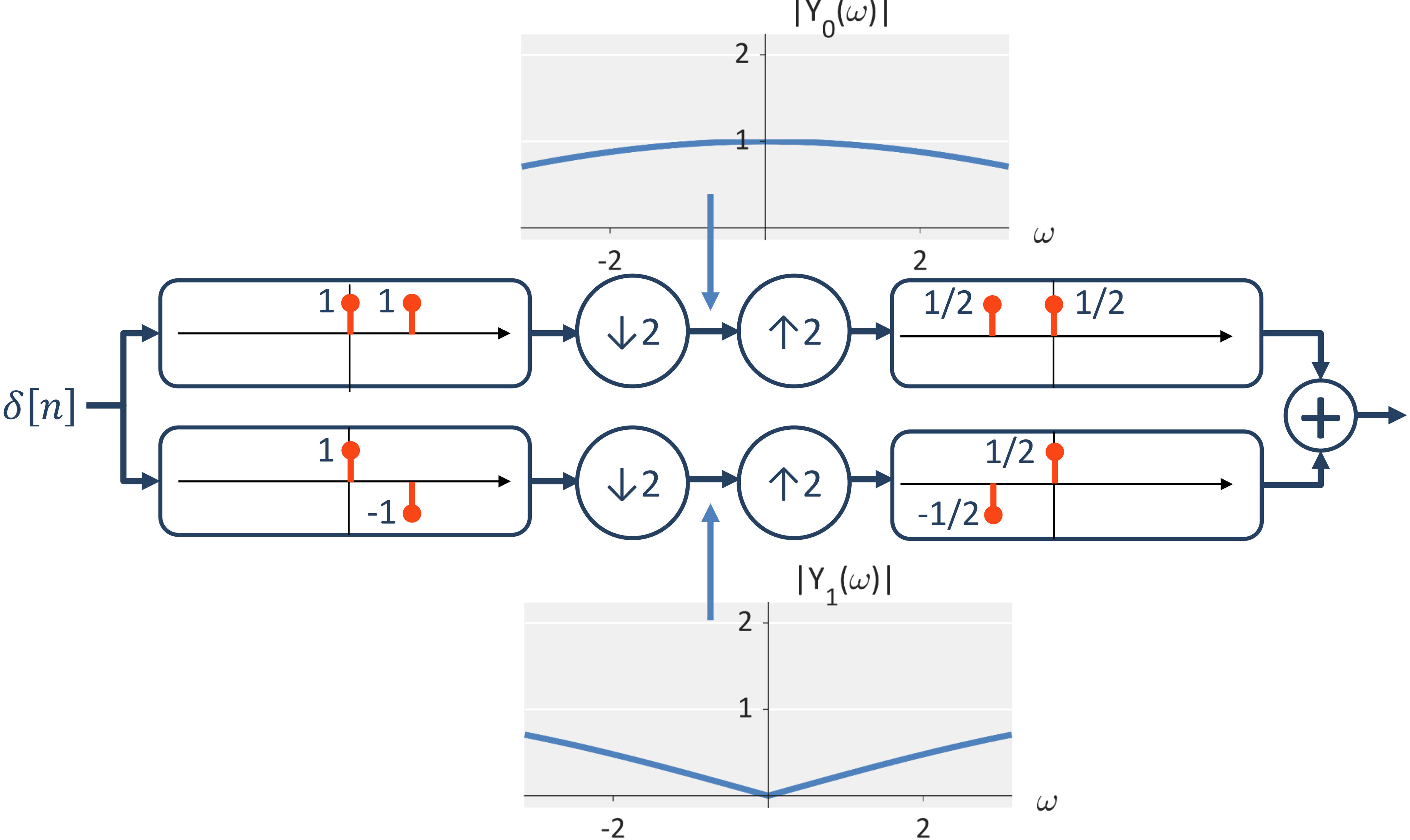
Filter Banks

■ **Example: Plot of the intermediate frequency magnitudes**



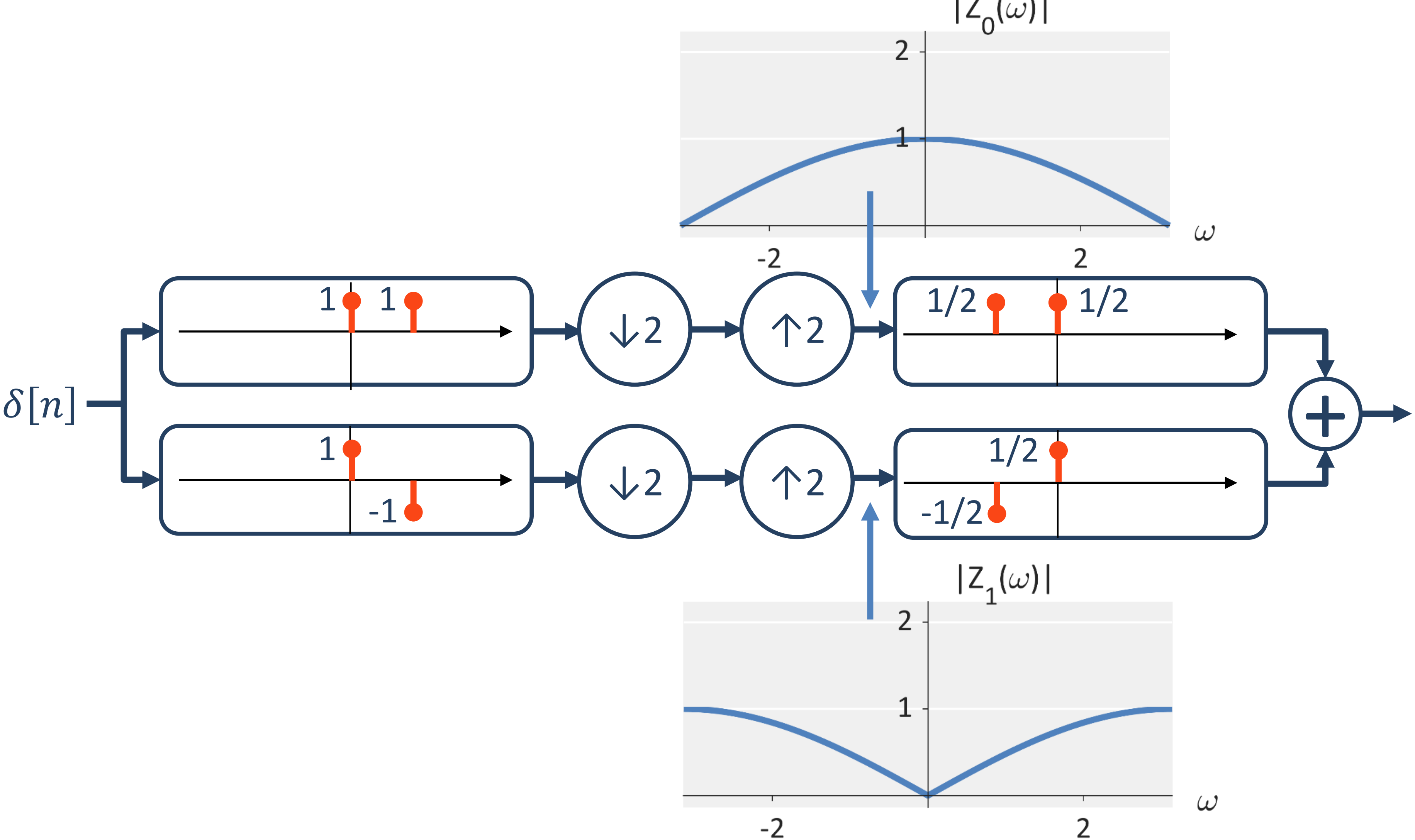
Filter Banks

- **Example:** **Plot of the intermediate frequency magnitudes**



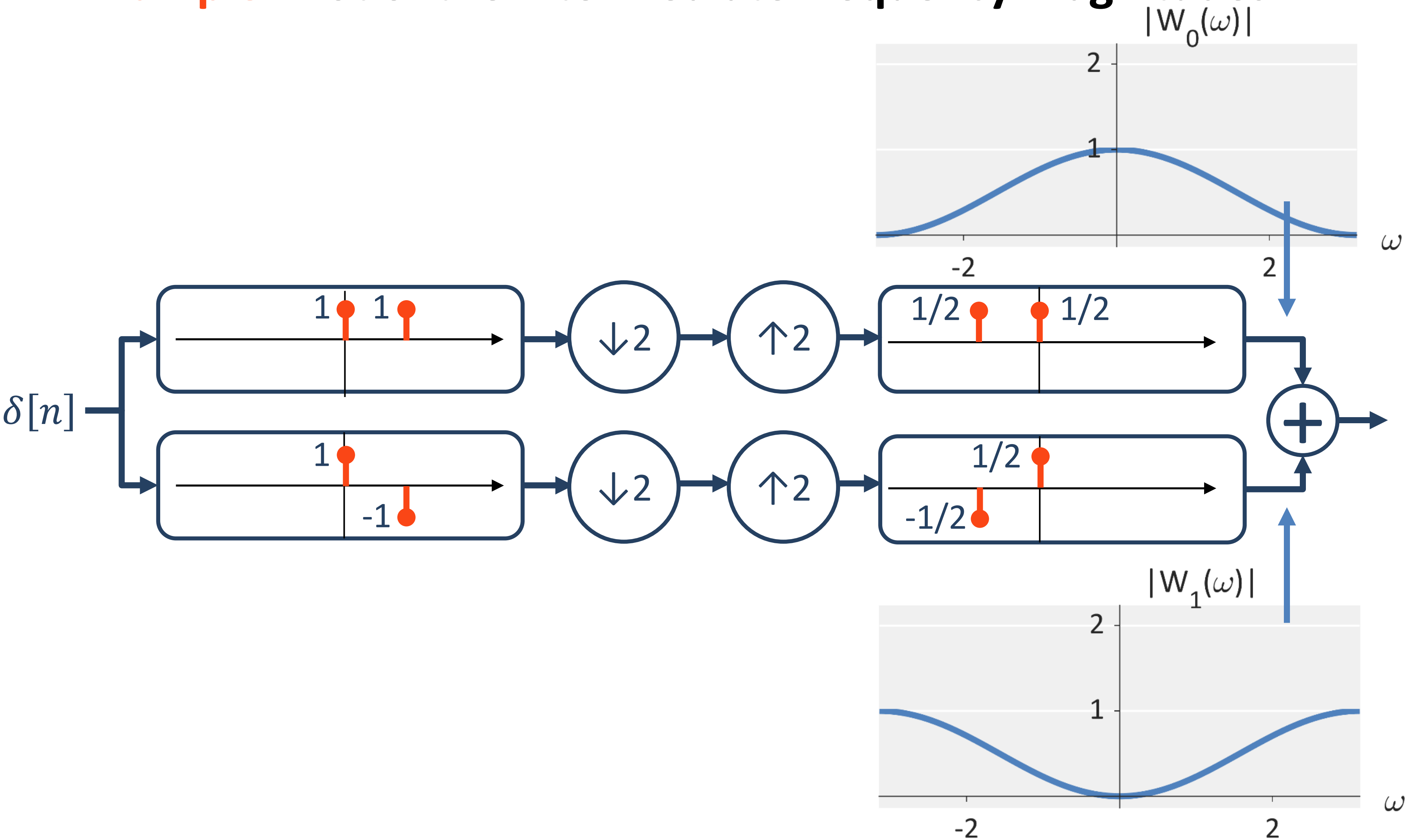
Filter Banks

- **Example: Plot of the intermediate frequency magnitudes**



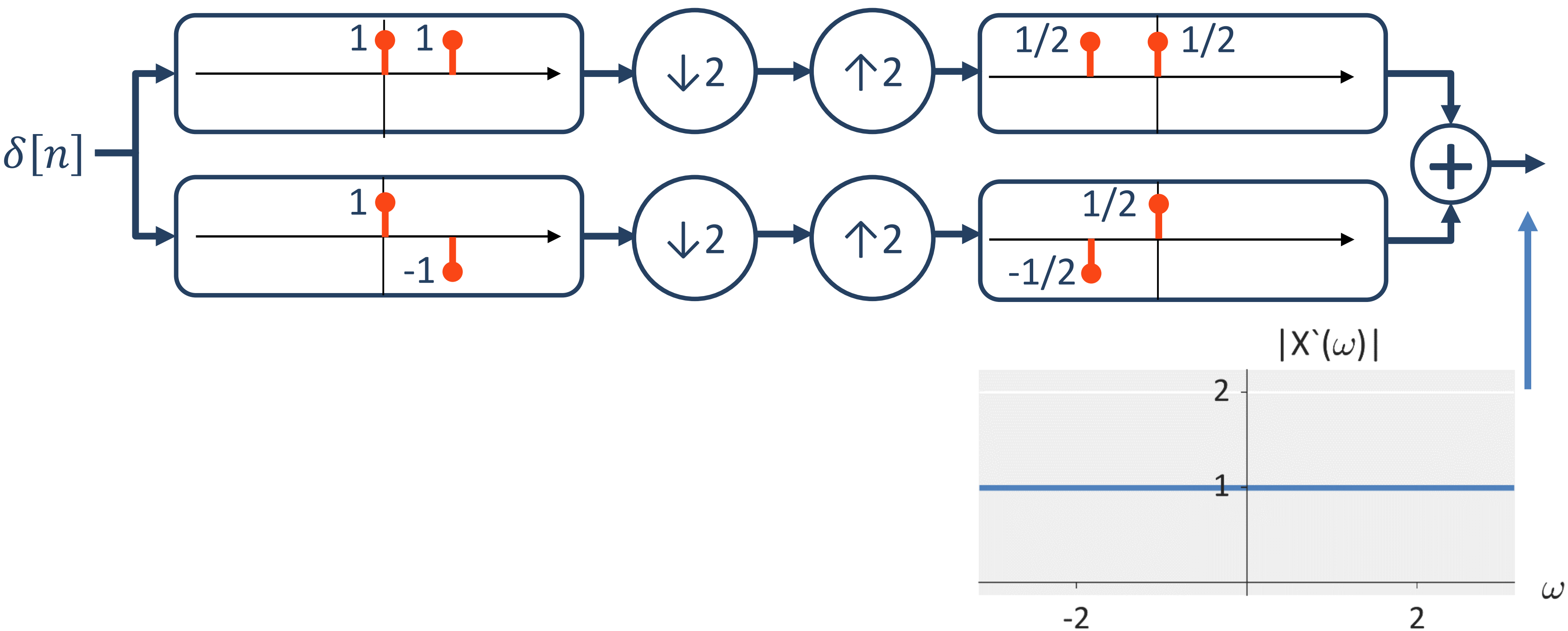
Filter Banks

- **Example: Plot of the intermediate frequency magnitudes**



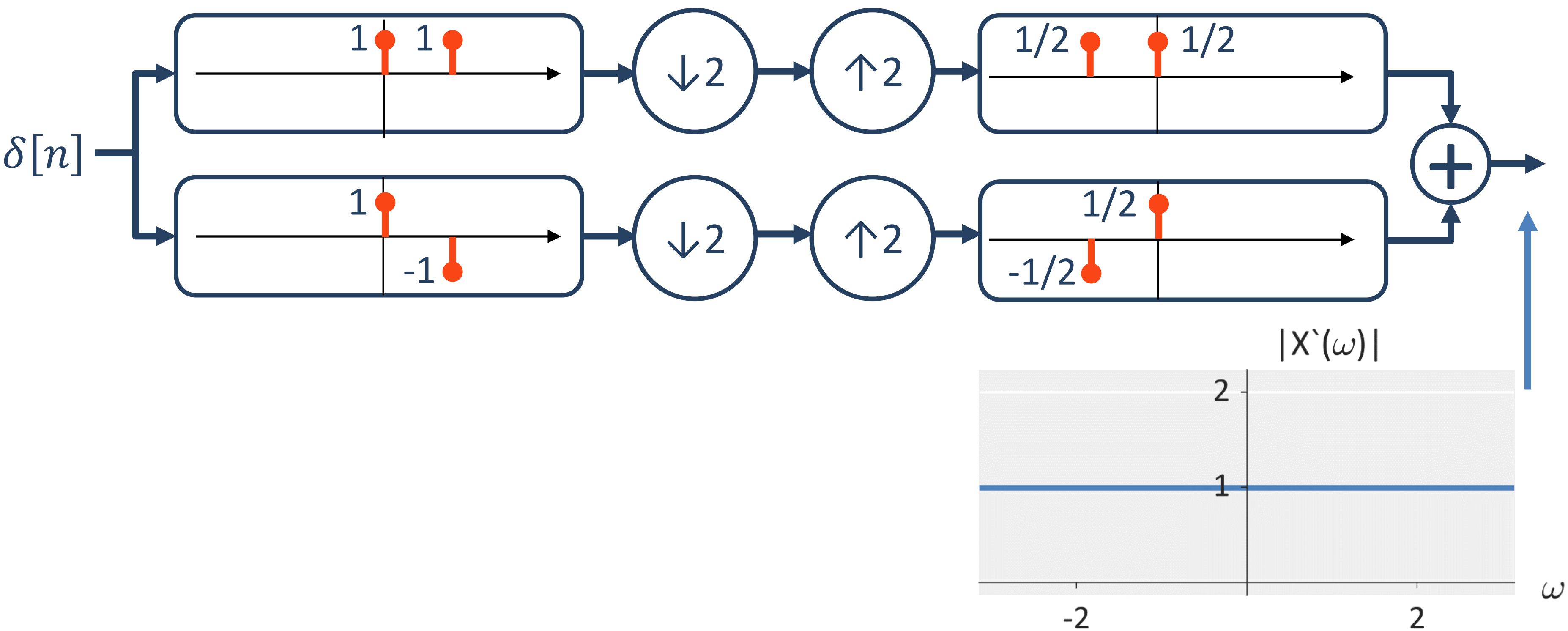
Filter Banks

- **Example: Plot of the intermediate frequency magnitudes**



Filter Banks

- **Question: Can we generalize this?**



Lecture 26: Filter Banks to Wavelets

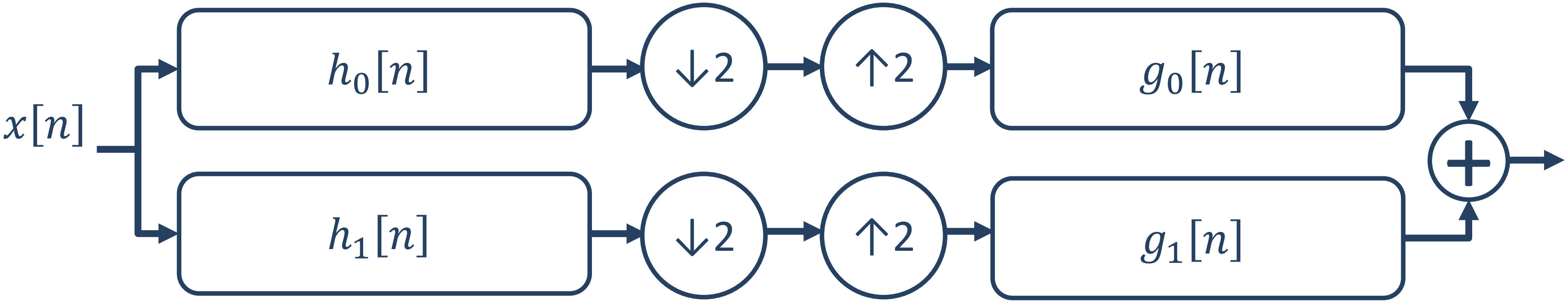
Foundations of Digital Signal Processing

Outline

- DFT Filter Banks [without downsampling]
- DFT Filter Bank [with downsampling]
- **General Two-Channel Filter Banks**
- Two-Channel Filter Bank Perfect Reconstruction (aliasing canceling)
- Two-Channel Filter Bank Perfect Reconstruction (orthogonal filter bank)
- Polyphase Filters
- Multi-Channel Filter Bank Perfect Reconstruction
- Wavelets

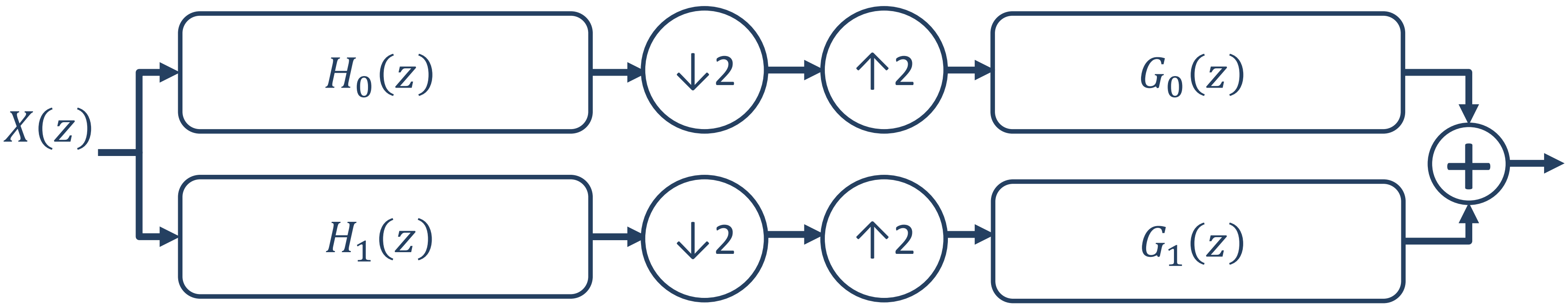
Filter Banks

- **Question: Can we generalize perfect reconstruction?**



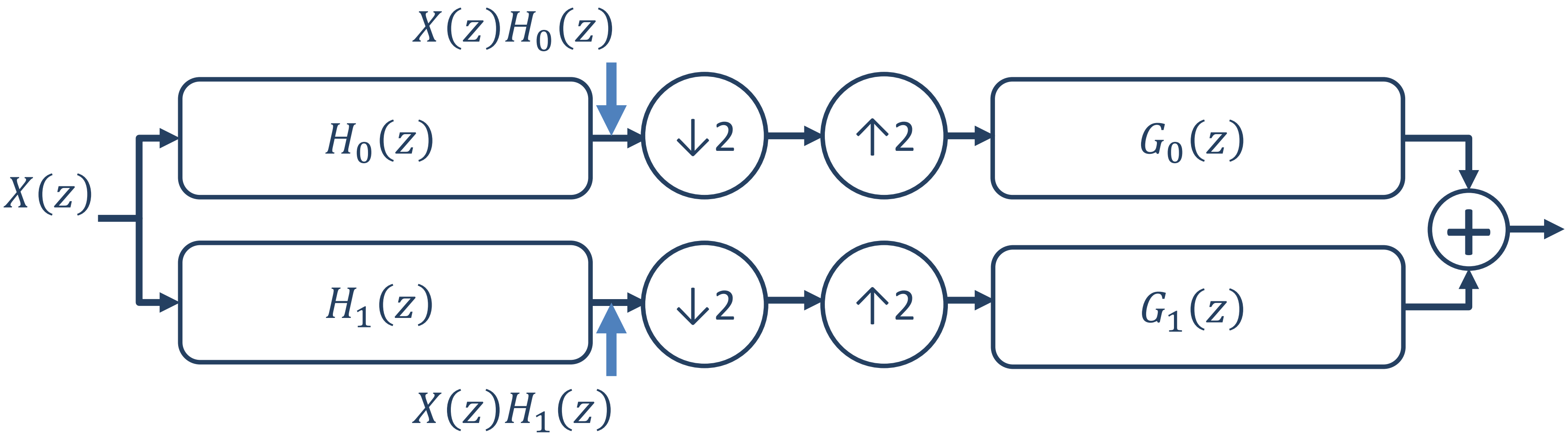
Filter Banks

- **Question: Can we generalize perfect reconstruction?**



Filter Banks

- **Question: Can we generalize perfect reconstruction?**



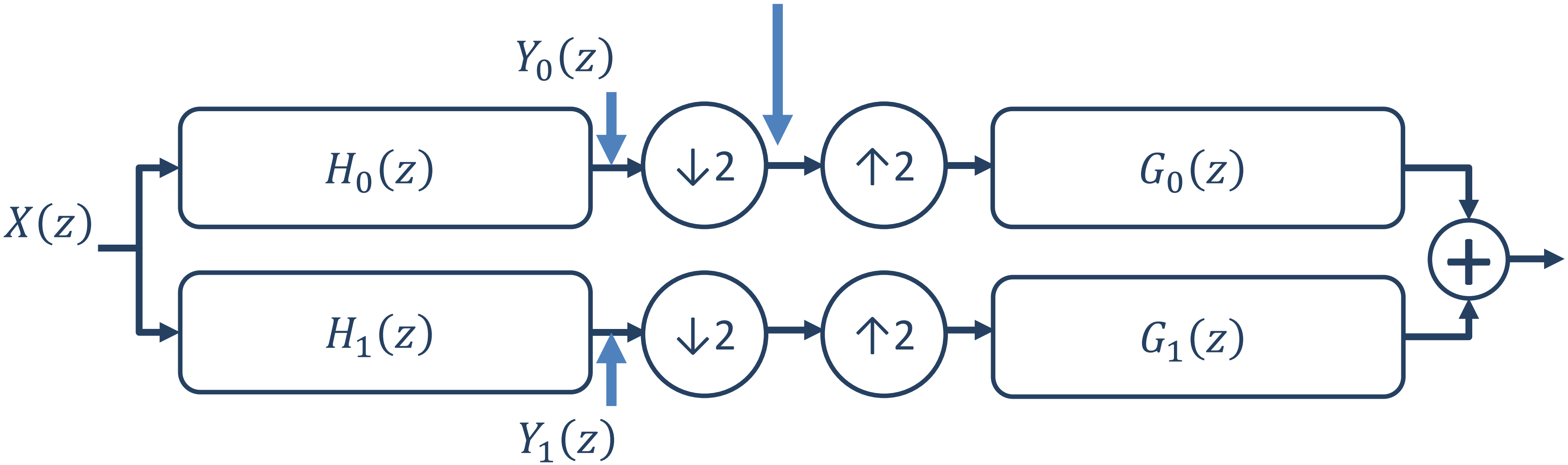
Filter Banks

- **Question: Can we generalize perfect reconstruction?**

Stretch by 2 and than shift by 2π

DTFT: $Y_0(\omega/2) + Y_0\big((\omega - 2\pi)/2\big)$

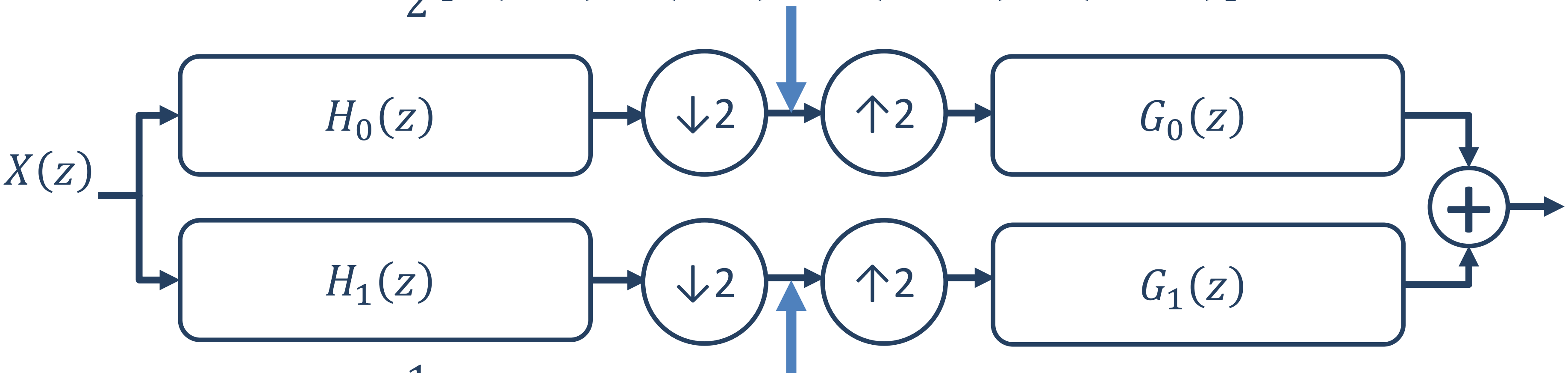
Z-Domain: $Y_0\big(z^{1/2}\big) + Y_0\big(z^{1/2}e^{-j\pi}\big) = Y_0\big(z^{1/2}\big) + Y_0\big(-z^{1/2}\big)$



Filter Banks

- **Question: Can we generalize perfect reconstruction?**

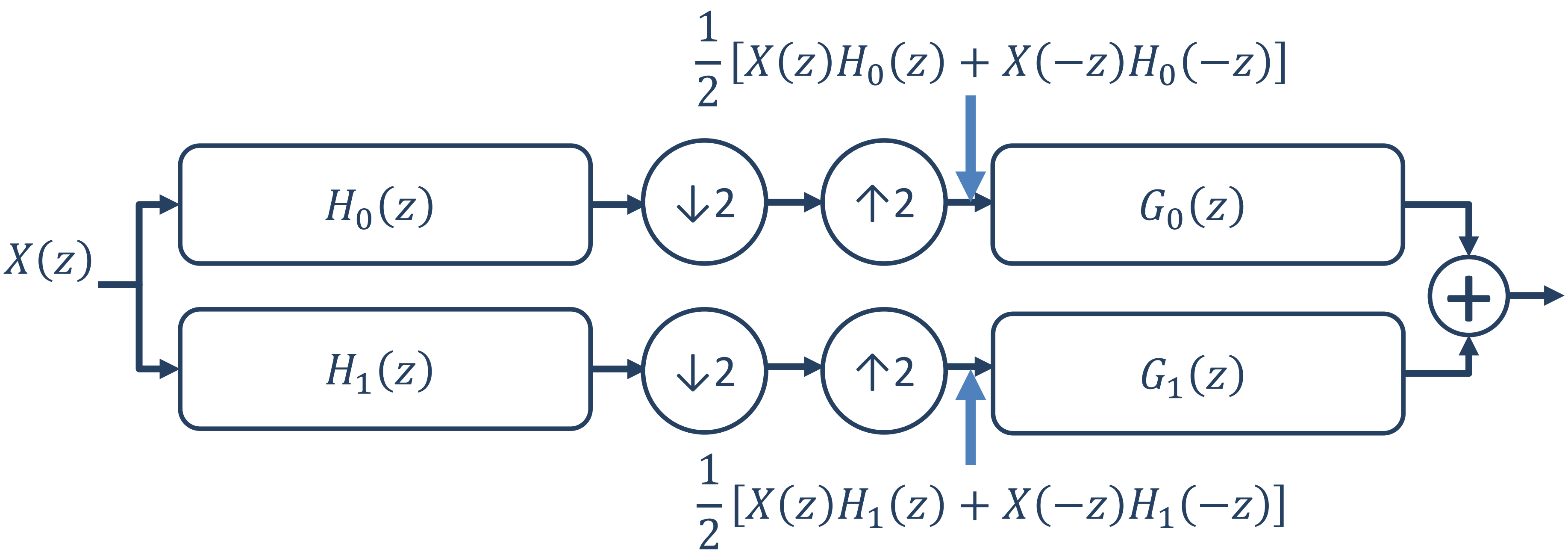
$$\frac{1}{2}\left[X\left(z^{1/2}\right)H_0\left(z^{1/2}\right)+X\left(-z^{1/2}\right)H_0\left(-z^{1/2}\right)\right]$$



$$\frac{1}{2}\left[X\left(z^{1/2}\right)H_1\left(z^{1/2}\right)+X\left(-z^{1/2}\right)H_1\left(-z^{1/2}\right)\right]$$

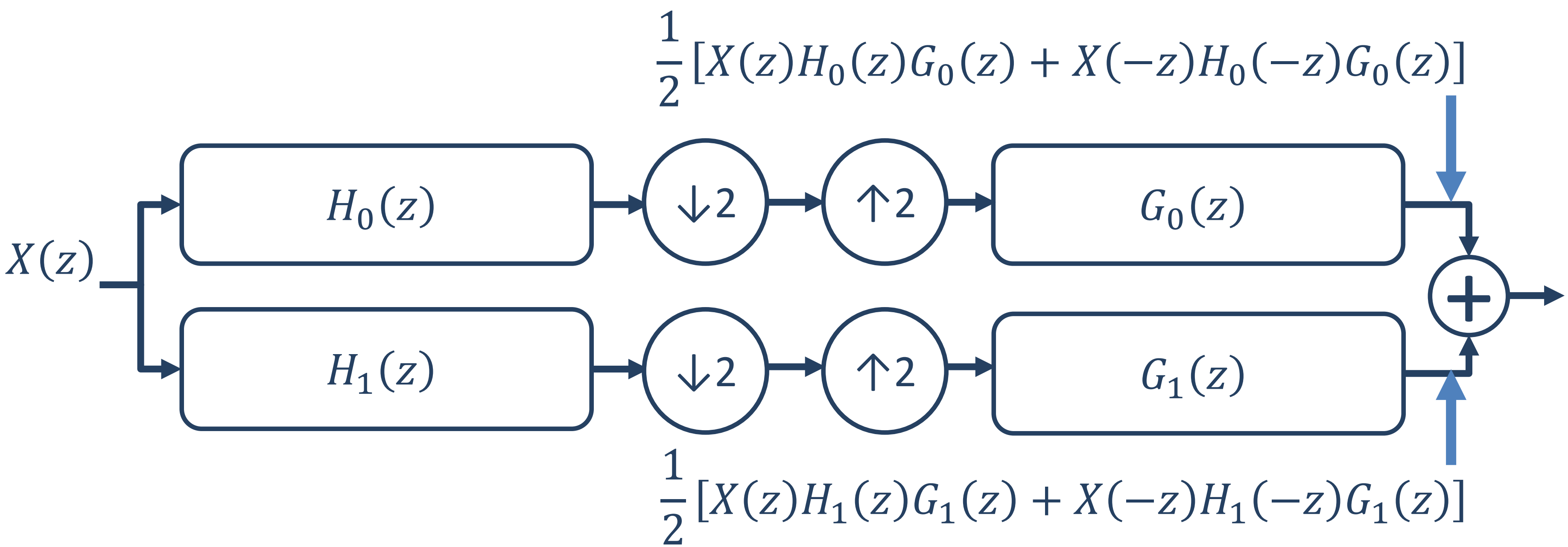
Filter Banks

- **Question: Can we generalize perfect reconstruction?**

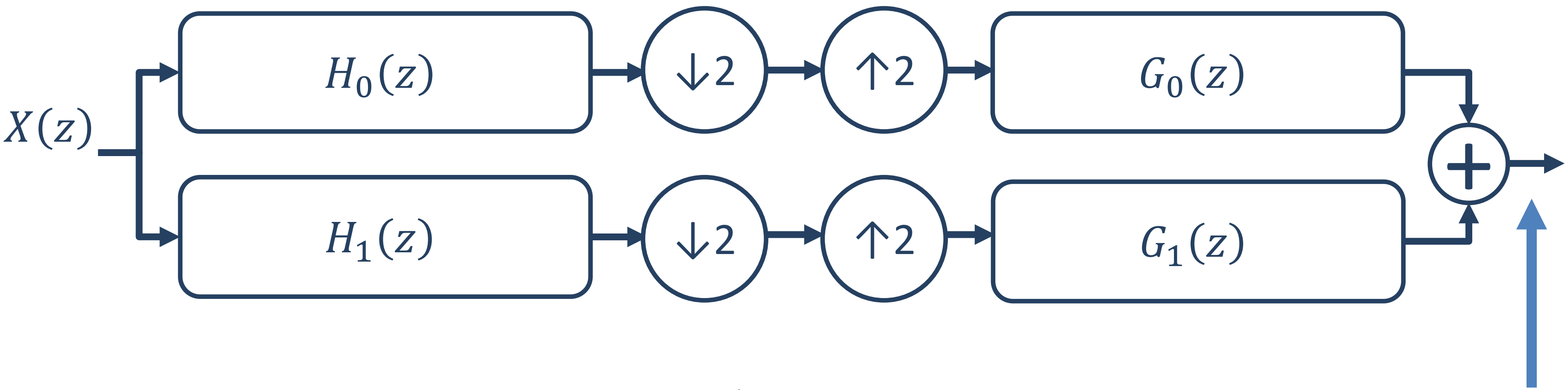


Filter Banks

- **Question: Can we generalize perfect reconstruction?**



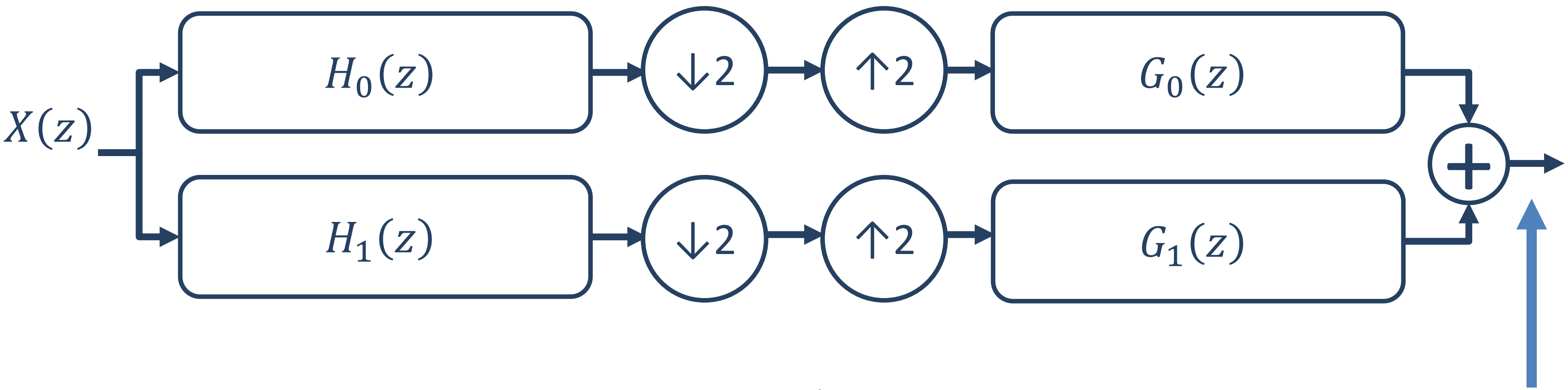
- **Question: Can we generalize perfect reconstruction?**



$$\frac{1}{2}[X(z)H_0(z)G_0(z) + X(-z)H_0(-z)G_0(z)]$$
$$+\frac{1}{2}[X(z)H_1(z)G_1(z) + X(-z)H_1(-z)G_1(z)]$$

Filter Banks

- **Question: Can we generalize perfect reconstruction?**



No aliasing term -> $$\frac{1}{2}X(z)[H_0(z)G_0(z) + H_1(z)G_1(z)]$$

Repeat (might cause aliasing)-> $$+\frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

Lecture 26: Filter Banks to Wavelets

Foundations of Digital Signal Processing

Outline

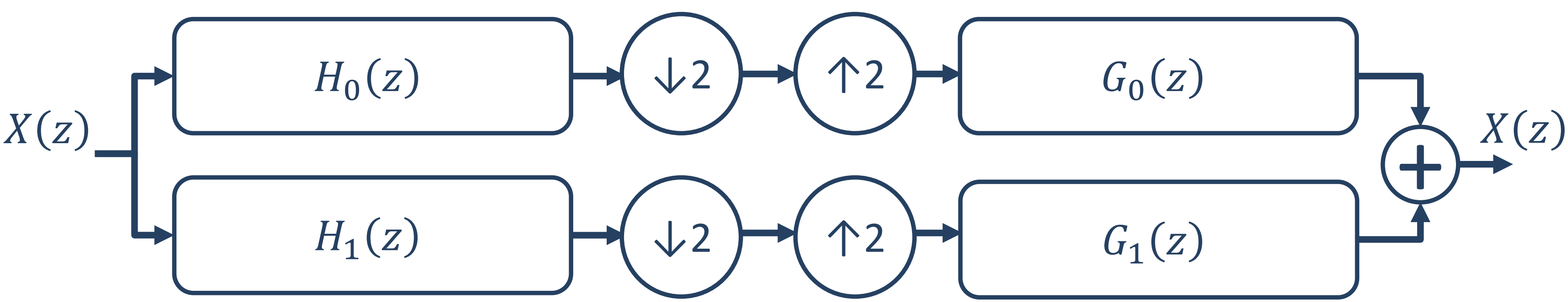
- DFT Filter Banks [without downsampling]
- DFT Filter Bank [with downsampling]
- General Two-Channel Filter Banks
- **Two-Channel Filter Bank Perfect Reconstruction (aliasing canceling)**
- Two-Channel Filter Bank Perfect Reconstruction (orthogonal filter bank)
- Polyphase Filters
- Multi-Channel Filter Bank Perfect Reconstruction
- Wavelets

Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ [H_0(z)G_0(z) + H_1(z)G_1(z)] + \frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

  - **Option 1:** Alias canceling

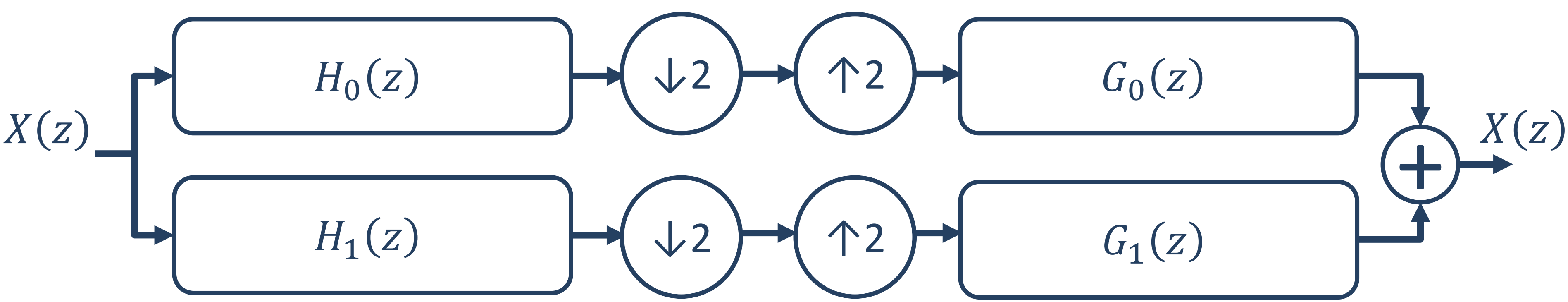


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ \ [H_0(z)G_0(z) \quad + H_1(z)G_1(z)]$$
$$+\frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

  - **Option 1:** Alias canceling
    - ◊ $H_0(z)G_0(z) \quad + H_1(z)G_1(z) \quad = 2$
    - ◊ $H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0$

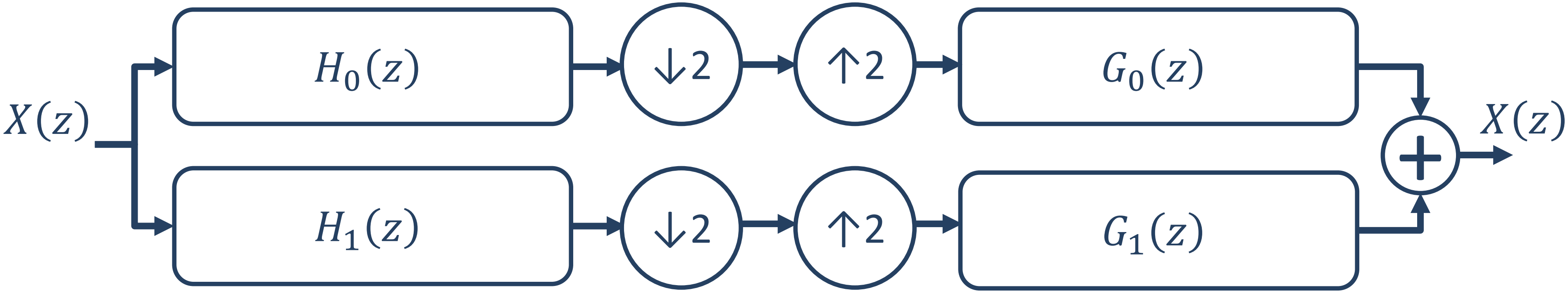


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ \ [H_0(z)G_0(z) \quad + H_1(z)G_1(z)]$$
$$+\frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

- **Option 1:** Alias canceling (**what is this in frequency?**)
  - $H_0(z)G_0(z) \quad + H_1(z)G_1(z) \quad = 2$
  - $H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0$

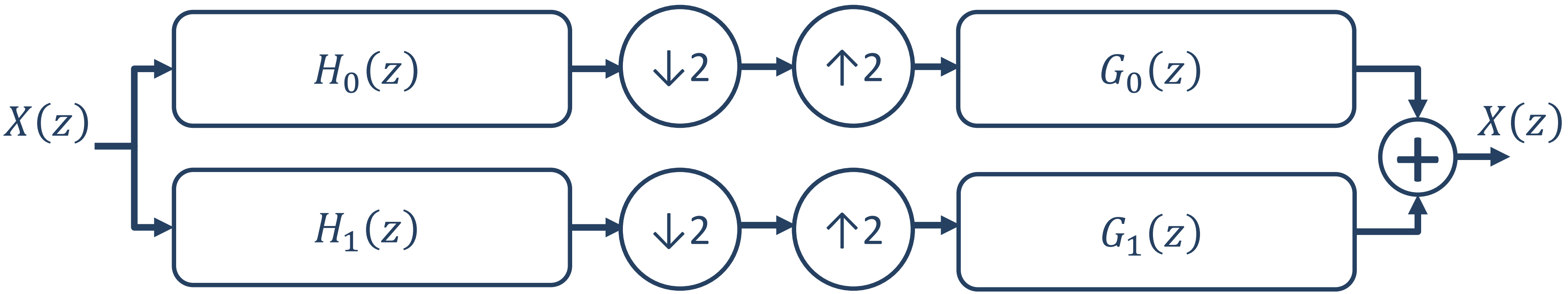


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ [H_0(z)G_0(z) \quad + H_1(z)G_1(z)]$$
$$+\frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

  - **Option 1:** Alias canceling (**what is this in frequency?**)
    - ◊ $H_0(\omega)G_0(\omega) \qquad + H_1(\omega)G_1(\omega) \qquad = 2$
    - ◊ $H_0(\omega-\pi)G_0(\omega) + H_1(\omega-\pi)G_1(\omega) = 0$

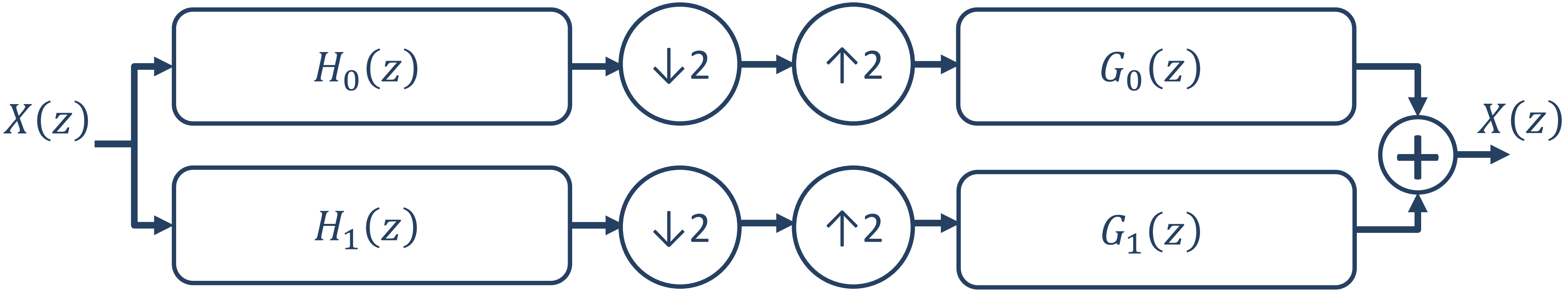


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ [H_0(z)G_0(z) + H_1(z)G_1(z)] + \frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

  - **Option 1:** Alias canceling (**what is this in time?**)
    - ◊ $H_0(\omega)G_0(\omega) + H_1(\omega)G_1(\omega) = 2$
    - ◊ $H_0(\omega-\pi)G_0(\omega) + H_1(\omega-\pi)G_1(\omega) = 0$

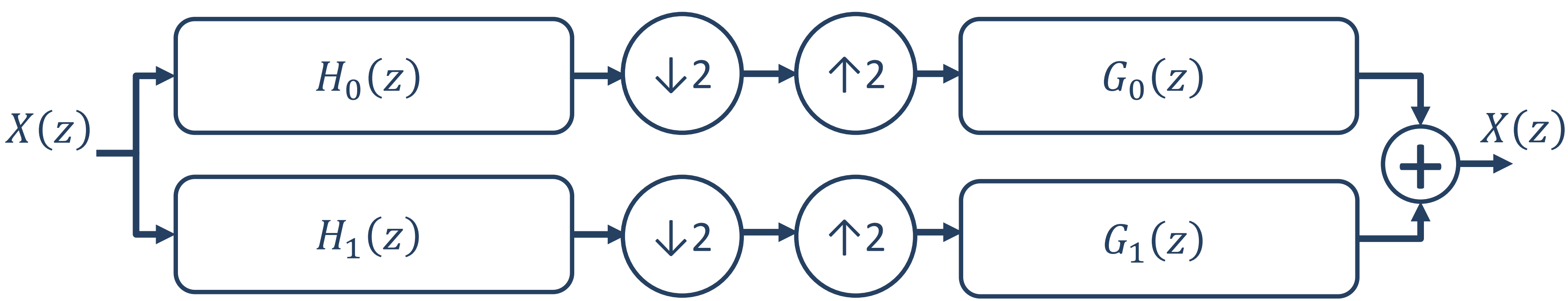


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ \ [H_0(z)G_0(z) \quad + H_1(z)G_1(z)]$$
$$+\frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

  - **Option 1:** Alias canceling (**what is this in time?**)
    - ◊ $h_0[n] * g_0[n] + h_1[n] * g_1[n] = 2\delta[n]$
    - ◊ $\left[(-1)^n h_0[n]\right] * g_0[n] + \left[(-1)^n h_1[n]\right] * g_1[n] = 0$

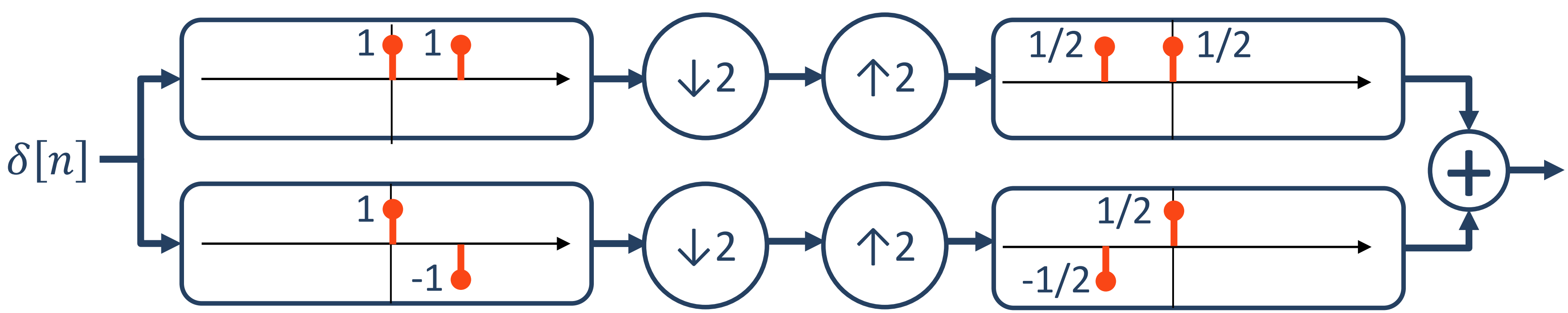


Filter Banks

- **Example: Alias Canceling**

$$X(z) = \frac{1}{2}X(z)\ \ [H_0(z)G_0(z) \quad + H_1(z)G_1(z)] + \frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

  - **Do the following filters satisfy alias canceling?**
    - ◊ $H_0(z)G_0(z) \quad + H_1(z)G_1(z) \quad = 2$
    - ◊ $H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0$

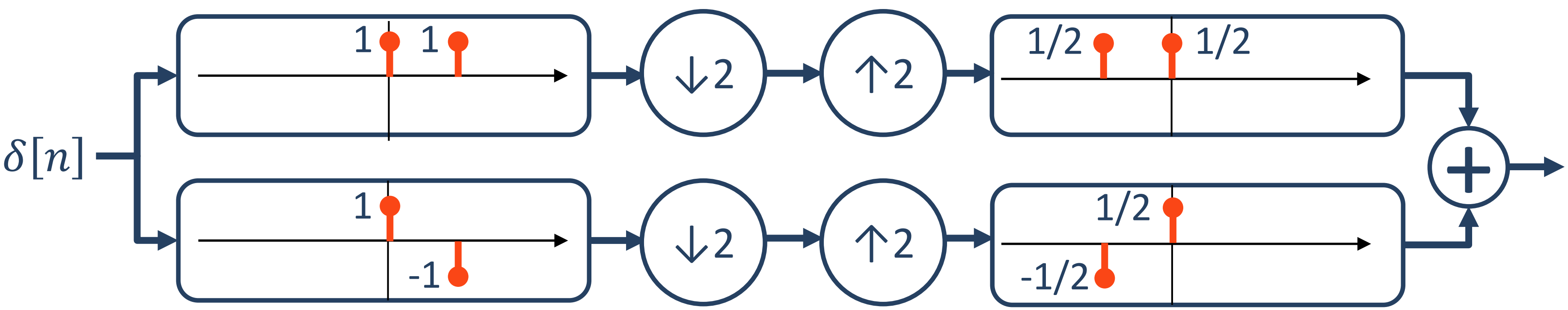


Filter Banks

Example: Alias Canceling

$$X(z) = \frac{1}{2}X(z)\ \ [H_0(z)G_0(z) \quad + H_1(z)G_1(z)]$$
$$+\frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

- **Do the following filters satisfy alias canceling?**
  - ◊ $\frac{1}{2}(1+z^{-1})(z^{+1}+1) + \frac{1}{2}(1-z^{-1})(-z^{+1}+1) = 2$
  - ◊ $\frac{1}{2}(1-z^{-1})(z^{+1}+1) + \frac{1}{2}(1+z^{-1})(-z^{+1}+1) = 0$

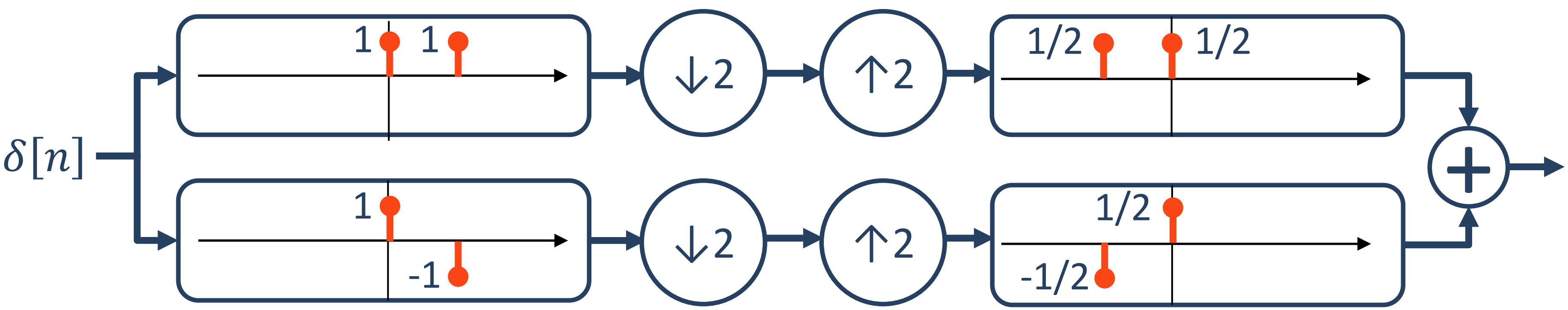


Filter Banks

- **Example: Alias Canceling**

$$\hat{X}(z) = \frac{1}{2}X(z)\ \ [H_0(z)G_0(z) \quad + H_1(z)G_1(z)] \\ +\frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

- **Do the following filters satisfy alias canceling?**
  - ◊ $\frac{1}{2}(2 + z^{+1} + z^{-1}) + \frac{1}{2}(2 - z^{+1} - z^{-1}) = 2$
  - ◊ $\frac{1}{2}(z^{+1} - z^{-1}) + \frac{1}{2}(-z^{+1} + z^{-1}) = 0$

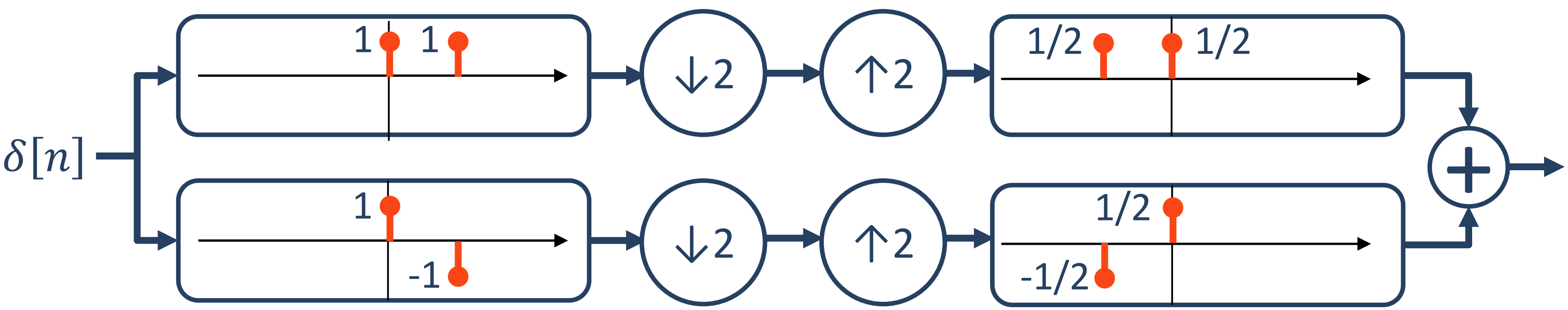


Filter Banks

Example: Alias Canceling

$$X(z) = \frac{1}{2}X(z)\ \ [H_0(z)G_0(z) \quad + H_1(z)G_1(z)]$$
$$+\frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

- **Do the following filters satisfy alias canceling?**
  - $2 = 2$
  - $0 = 0$

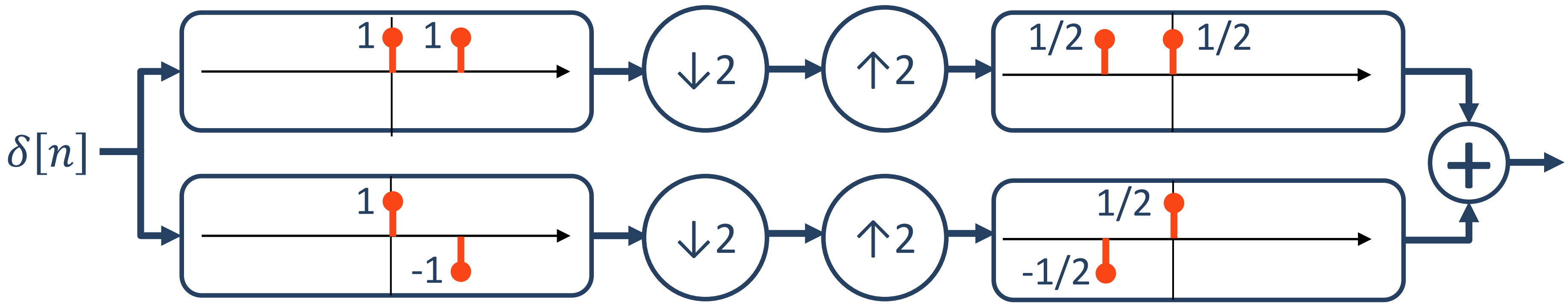


Filter Banks

■ Example: Alias Canceling

$$X(z) = \frac{1}{2}X(z)\ \ [H_0(z)G_0(z) \quad + H_1(z)G_1(z)]$$
$$+\frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

- **Do the following filters satisfy alias canceling?**
  - ◊ $h_0[n] * g_0[n] + h_1[n] * g_1[n] = 2\delta[n]$
  - ◊ $\left[(-1)^n h_0[n]\right] * g_0[n] + \left[(-1)^n h_1[n]\right] * g_1[n] = 0$



Lecture 26: Filter Banks to Wavelets

Foundations of Digital Signal Processing

Outline

- DFT Filter Banks [without downsampling]
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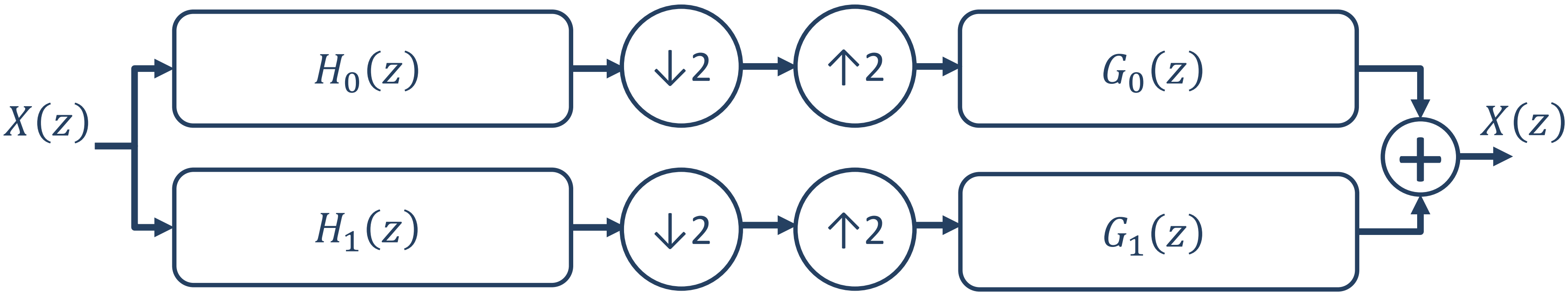
- **Two-Channel Filter Bank Perfect Reconstruction (orthogonal filter bank)**
- Polyphase Filters
- Multi-Channel Filter Bank Perfect Reconstruction
- Wavelets

Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ \ [H_0(z)G_0(z) \quad + H_1(z)G_1(z)]$$
$$+\frac{1}{2}X(-z)[H_0(-z)G_0(z) + H_1(-z)G_1(z)]$$

  - **Option 2:** Orthogonal Filter Bank

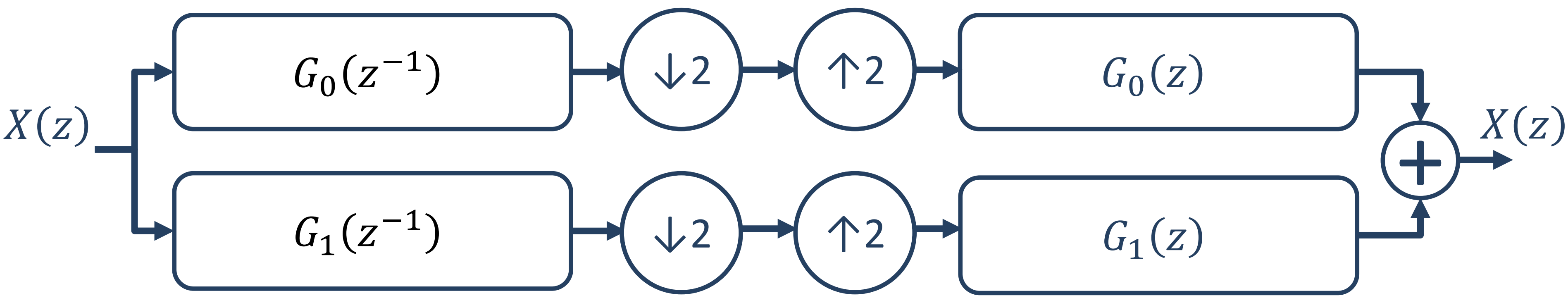


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ [G_0(z^{-1})G_0(z) + G_1(z^{-1})G_1(z)]$$
$$+\frac{1}{2}X(-z)[G_0(-z^{-1})G_0(z) + G_1(-z^{-1})G_1(z)]$$

- **Option 2:** Orthogonal Filter Bank
  - ◊ Let $H_0(z) = G_0(z^{-1})$
  - ◊ Let $H_1(z) = G_1(z^{-1})$

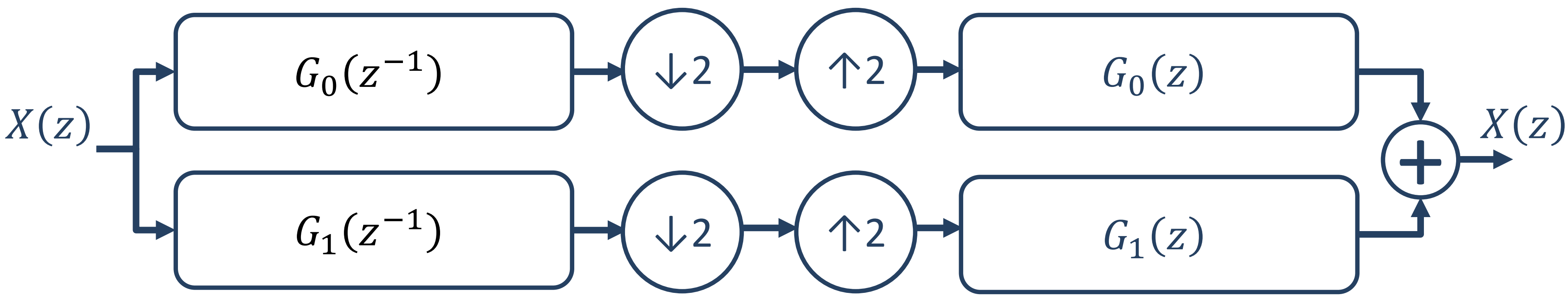


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ \ [G_0(z^{-1})G_0(z) \quad + G_1(z^{-1})G_1(z)]$$
$$+\frac{1}{2}X(-z)[G_0(-z^{-1})G_0(z) + G_1(-z^{-1})G_1(z)]$$

  - **Option 2:** Orthogonal Filter Bank
    - ◊ Let $H_0(z) = G_0(z^{-1})$
    - ◊ Let $H_1(z) = G_1(z^{-1})$
    - ◊ Assume $X(z) = \alpha G_0(z) + \beta G_1(z)$



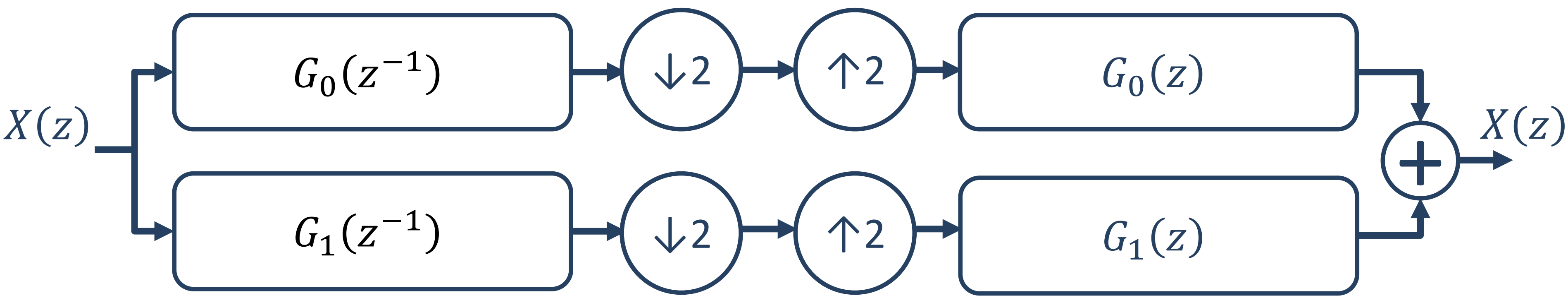
Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}[\alpha G_0(z) + \beta G_1(z)][G_0(z^{-1})G_0(z) + G_1(z^{-1})G_1(z)]$$
$$+\frac{1}{2}[\alpha G_0(-z) + \beta G_1(-z)][G_0(-z^{-1})G_0(z) + G_1(-z^{-1})G_1(z)]$$

- **Option 2:** Orthogonal Filter Bank
  - ◊ Let $H_0(z) = G_0(z^{-1})$
  - ◊ Let $H_1(z) = G_1(z^{-1})$
  - ◊ Assume $X(z) = \alpha G_0(z) + \beta G_1(z)$

i.e., $G_0(z)$ and $G_1(z)$ are bases of $X(z)$

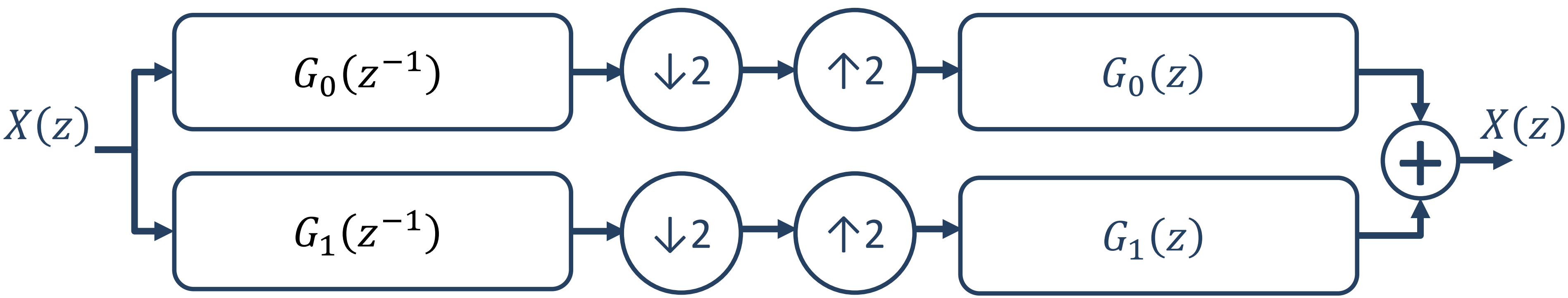


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}\alpha\left[G_0(z)[G_0(z)G_0(z^{-1}) + G_0(-z)G_0(-z^{-1})] + G_1(z)[G_0(z)G_1(z^{-1}) + G_0(-z)G_1(-z^{-1})]\right]$$
$$+ \frac{1}{2}\beta\left[G_0(z)[G_1(z)G_0(z^{-1}) + G_1(-z)G_0(-z^{-1})] + G_1(z)[G_1(z)G_1(z^{-1}) + G_1(-z)G_1(-z^{-1})]\right]$$

  - **Option 2:** Orthogonal Filter Bank
    - Let $H_0(z) = G_0(z^{-1})$
    - Let $H_1(z) = G_1(z^{-1})$
    - Assume $X(z) = \alpha G_0(z) + \beta G_1(z)$

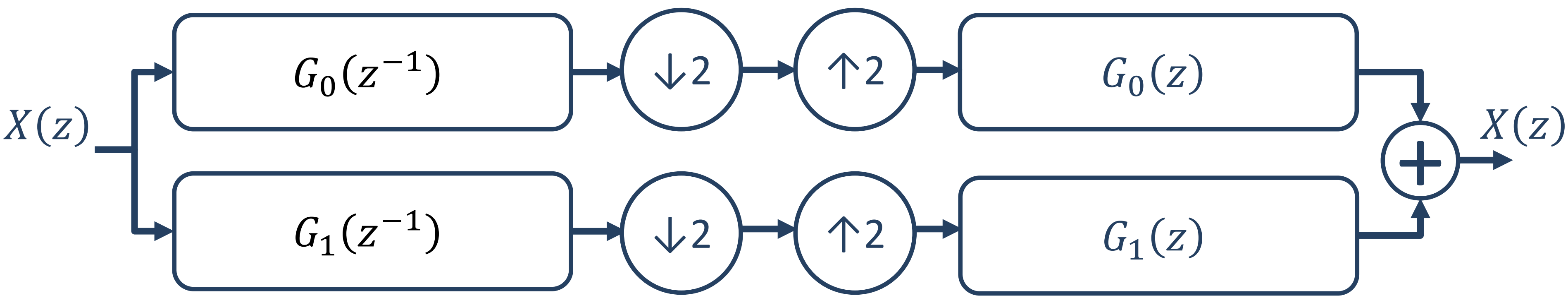


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$
\begin{aligned}
X(z) \\
&= \frac{1}{2}\alpha\left[G_0(z)\left[\cancelto{2}{G_0(z)G_0(z^{-1}) + G_0(-z)G_0(-z^{-1})}\right] + G_1(z)\left[\cancelto{0}{G_0(z)G_1(z^{-1}) + G_0(-z)G_1(-z^{-1})}\right]\right] \\
&+ \frac{1}{2}\beta\left[G_0(z)\left[\cancelto{0}{G_1(z)G_0(z^{-1}) + G_1(-z)G_0(-z^{-1})}\right] + G_1(z)\left[\cancelto{2}{G_1(z)G_1(z^{-1}) + G_1(-z)G_1(-z^{-1})}\right]\right]
\end{aligned}
$$

- **Option 2:** Orthogonal Filter Bank
  - $G_0(z)G_0(z^{-1}) + G_0(-z)G_0(-z^{-1}) = 2$
  - $G_1(z)G_1(z^{-1}) + G_1(-z)G_1(-z^{-1}) = 2$
  - $G_0(z)G_1(z^{-1}) + G_0(-z)G_1(-z^{-1}) = 0$
  - $G_1(z)G_0(z^{-1}) + G_1(-z)G_0(-z^{-1}) = 0$



Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \alpha G_0(z) + \beta G_1(z)$$

- **Option 2:** Orthogonal Filter Bank
  - $G_0(z)G_0(z^{-1}) + G_0(-z)G_0(-z^{-1}) = 2$
  - $G_1(z)G_1(z^{-1}) + G_1(-z)G_1(-z^{-1}) = 2$
  - $G_0(z)G_1(z^{-1}) + G_0(-z)G_1(-z^{-1}) = 0$
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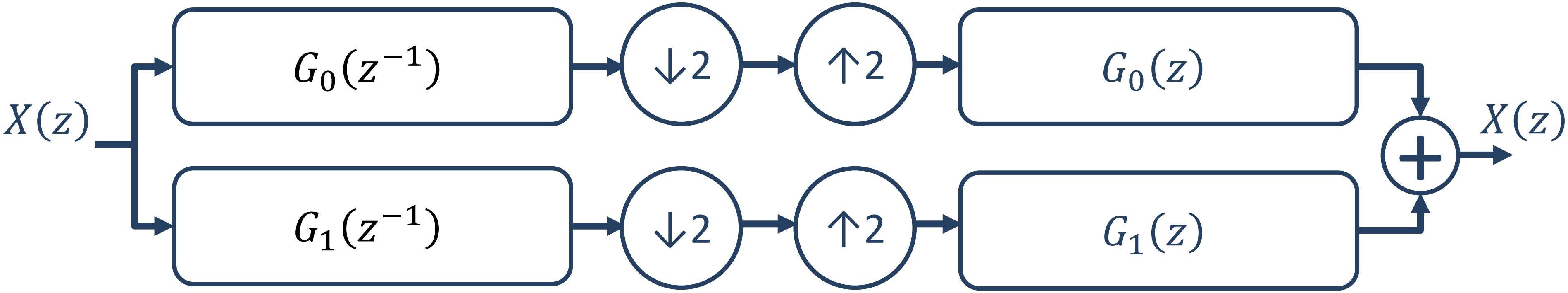


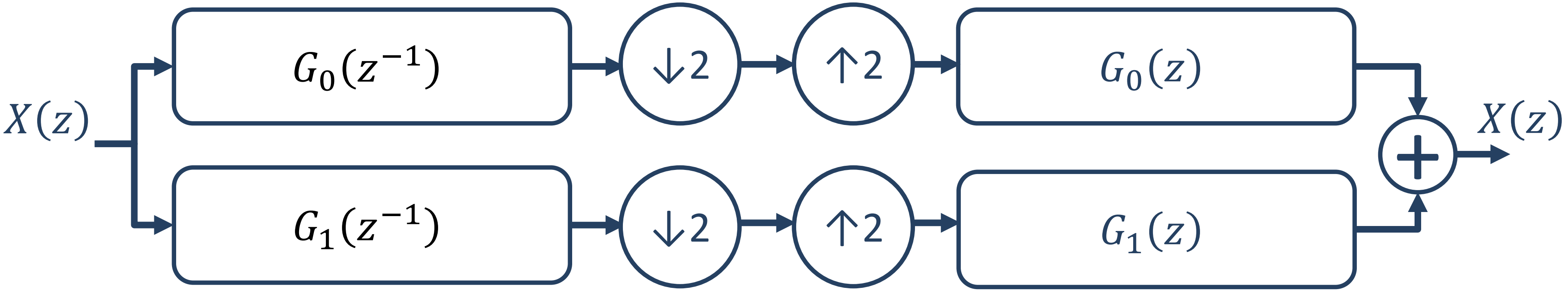
Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \alpha G_0(z) + \beta G_1(z)$$

- **Option 2:** Orthogonal Filter Bank
  - ◊ $G_0(z)G_0(z^{-1}) + G_0(-z)G_0(-z^{-1}) = 2$
  - ◊ $G_1(z)G_1(z^{-1}) + G_1(-z)G_1(-z^{-1}) = 2$
  - ◊ $G_0(z)G_1(z^{-1}) + G_0(-z)G_1(-z^{-1}) = 0$
  - ◊ $G_1(z)G_0(z^{-1}) + G_1(-z)G_0(-z^{-1}) = 0$

Equations are same if:
$z \leftarrow z^{-1}$
So we only need one.

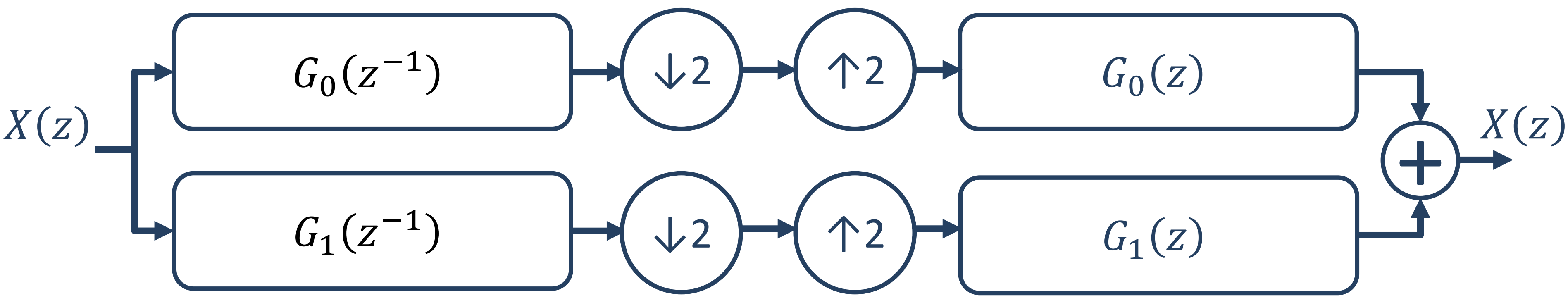


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \alpha G_0(z) + \beta G_1(z)$$

- **Option 2:** Orthogonal Filter Bank
  - ◊ $G_0(z)G_0(z^{-1}) + G_0(-z)G_0(-z^{-1}) = 2$
  - ◊ $G_1(z)G_1(z^{-1}) + G_1(-z)G_1(-z^{-1}) = 2$
  - ◊ $G_0(z)G_1(z^{-1}) + G_0(-z)G_1(-z^{-1}) = 0$

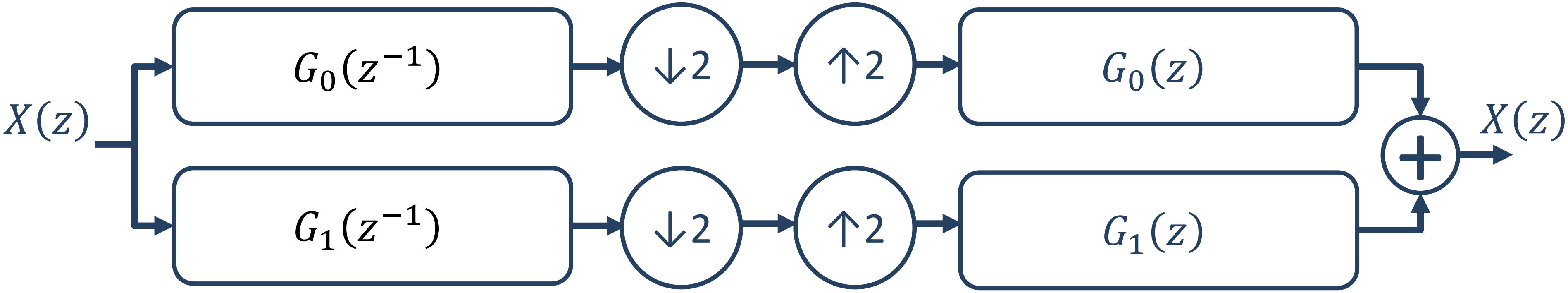


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \alpha G_0(z) + \beta G_1(z)$$

  - **Option 2:** Orthogonal Filter Bank (**what is this in frequency?**)
    - ◊ $G_0(\omega)G_0(-\omega) + G_0(\omega-\pi)G_0(-\omega-\pi) = 2$
    - ◊ $G_1(\omega)G_1(-\omega) + G_1(\omega-\pi)G_1(-\omega-\pi) = 2$
    - ◊ $G_0(\omega)G_1(-\omega) + G_0(\omega-\pi)G_1(-\omega-\pi) = 0$



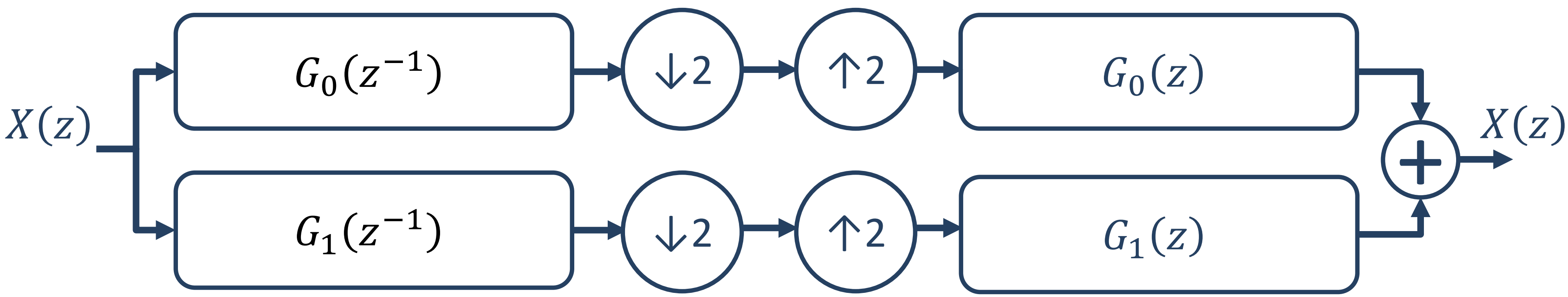
Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \alpha G_0(z) + \beta G_1(z)$$

- **Option 2:** Orthogonal Filter Bank (**what is this in frequency?**)
  - $G_0(\omega)G_0^*(\omega) + G_0(\omega-\pi)G_0^*(\omega-\pi) = 2$
  - $G_1(\omega)G_1^*(\omega) + G_1(\omega-\pi)G_1^*(\omega-\pi) = 2$
  - $G_0(\omega)G_1^*(\omega) + G_0(\omega-\pi)G_1^*(\omega-\pi) = 0$

Assuming real filter coefficients



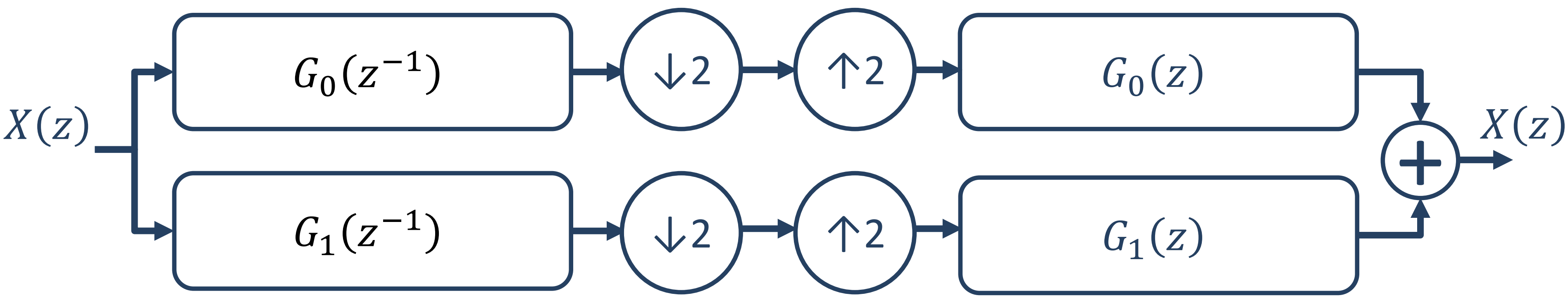
Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \alpha G_0(z) + \beta G_1(z)$$

- **Option 2:** Orthogonal Filter Bank (**what is this in frequency?**)
  - ◊ $|G_0(\omega)|^2 + |G_0(\omega - \pi)|^2 = 2$
  - ◊ $|G_1(\omega)|^2 + |G_1(\omega - \pi)|^2 = 2$
  - ◊ $G_0(\omega)G_1^*(\omega) + G_0(\omega - \pi)G_1^*(\omega - \pi) = 0$

Assuming real filter coefficients



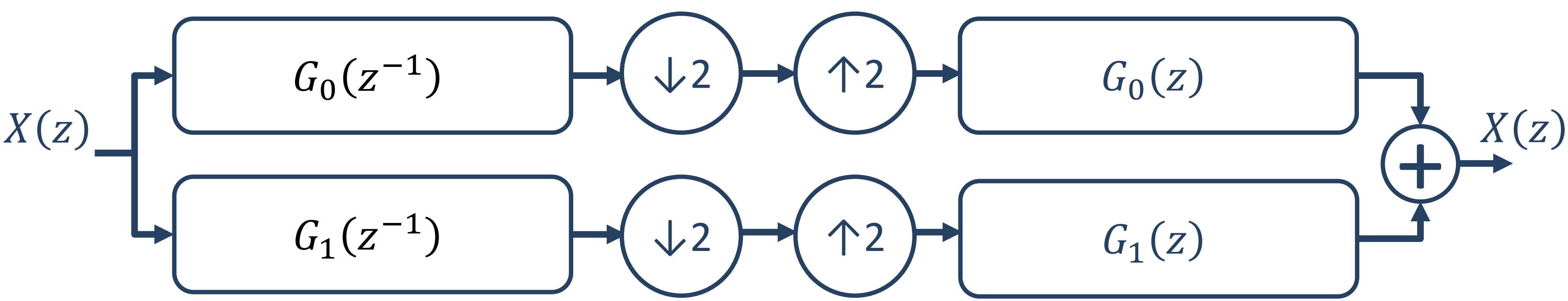
Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \alpha G_0(z) + \beta G_1(z)$$

- **Option 2:** Orthogonal Filter Bank (**what is this in time?**)
  - $|G_0(\omega)|^2 + |G_0(\omega - \pi)|^2 = 2$
  - $|G_1(\omega)|^2 + |G_1(\omega - \pi)|^2 = 2$
  - $G_0(\omega)G_1^*(\omega) + G_0(\omega - \pi)G_1^*(\omega - \pi) = 0$

Assuming real filter coefficients

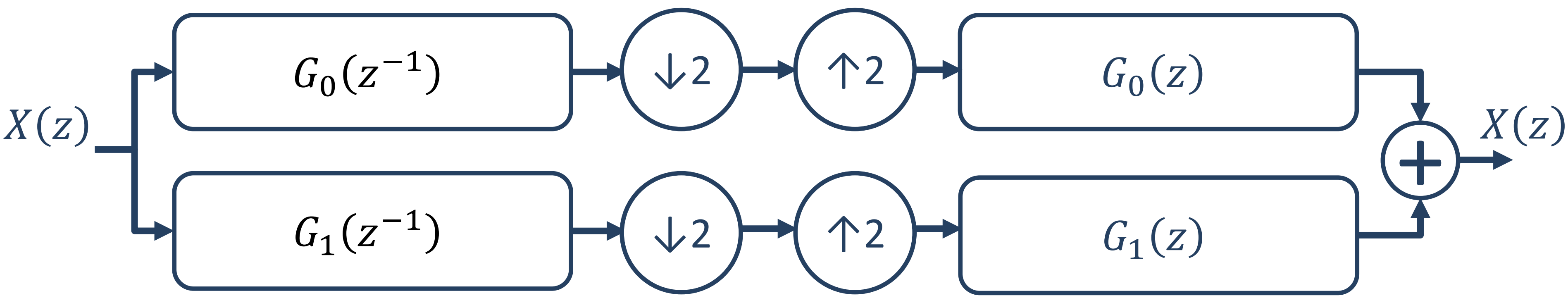


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

  $X(z) = \alpha G_0(z) + \beta G_1(z)$

  - **Option 2:** Orthogonal Filter Bank (**what is this in time?**)
    - $g_0[n] * g_0[-n] + [(-1)^n g_0[n]] * [(-1)^n g_0[-n]] = 2\delta[n]$
    - $g_1[n] * g_1[-n] + [(-1)^n g_1[n]] * [(-1)^n g_1[-n]] = 2\delta[n]$
    - $g_0[n] * g_1[-n] + [(-1)^n g_0[n]] * [(-1)^n g_1[-n]] = 0$

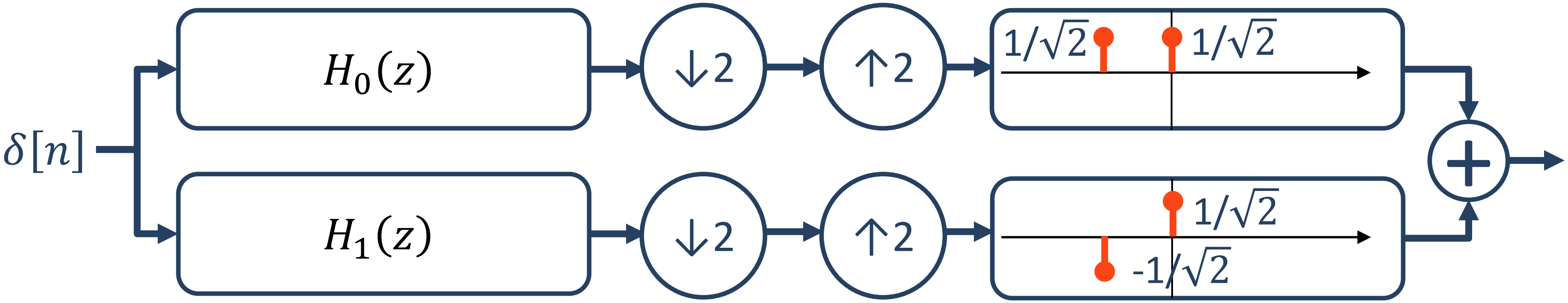


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ \left[G_0(z^{-1})G_0(z) \quad + G_1(z^{-1})G_1(z)\right]$$
$$+\frac{1}{2}X(-z)\left[G_0(-z^{-1})G_0(z) + G_1(-z^{-1})G_1(z)\right]$$

- **Do the following filters satisfy an orthogonal filter bank?**
  - Let $H_0(z) = G_0(z^{-1})$
  - Let $H_1(z) = G_1(z^{-1})$

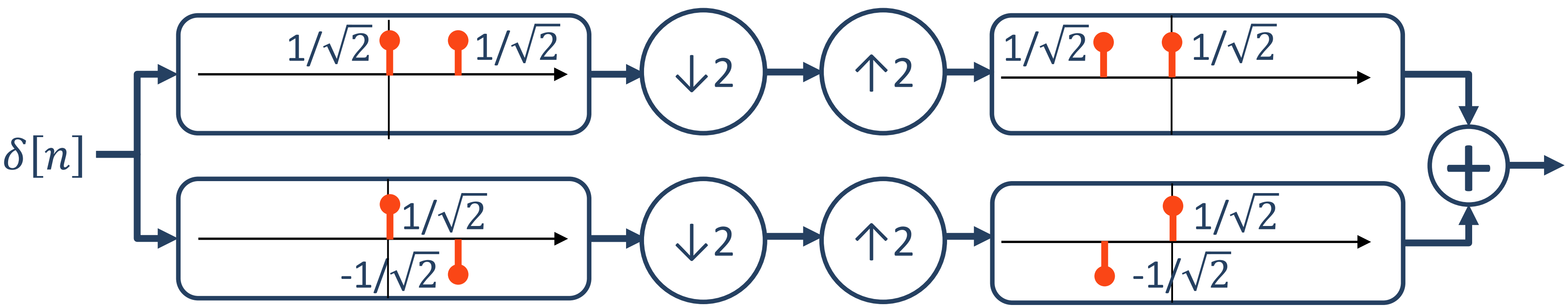


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ \left[G_0(z^{-1})G_0(z) \quad + G_1(z^{-1})G_1(z)\right]$$
$$+\frac{1}{2}X(-z)\left[G_0(-z^{-1})G_0(z) + G_1(-z^{-1})G_1(z)\right]$$

- **Do the following filters satisfy an orthogonal filter bank?**
  - Let $H_0(z) = G_0(z^{-1})$
  - Let $H_1(z) = G_1(z^{-1})$

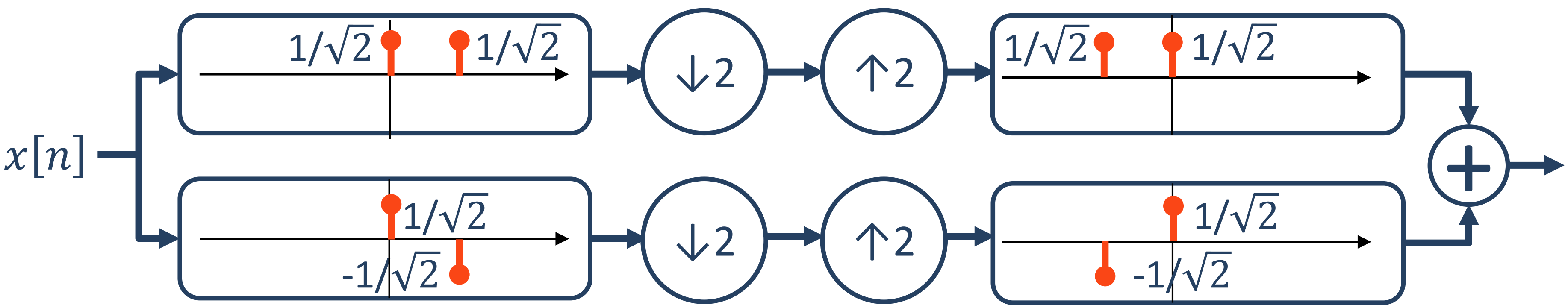


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ \left[G_0(z^{-1})G_0(z) \quad + G_1(z^{-1})G_1(z)\right]$$
$$+\frac{1}{2}X(-z)\left[G_0(-z^{-1})G_0(z) + G_1(-z^{-1})G_1(z)\right]$$

  - **Do the following filters satisfy an orthogonal filter bank?**
    - $G_0(z)G_0(z^{-1}) + G_0(-z)G_0(-z^{-1}) = 2$
    - $G_1(z)G_1(z^{-1}) + G_1(-z)G_1(-z^{-1}) = 2$
    - $G_0(z)G_1(z^{-1}) + G_0(-z)G_1(-z^{-1}) = 0$

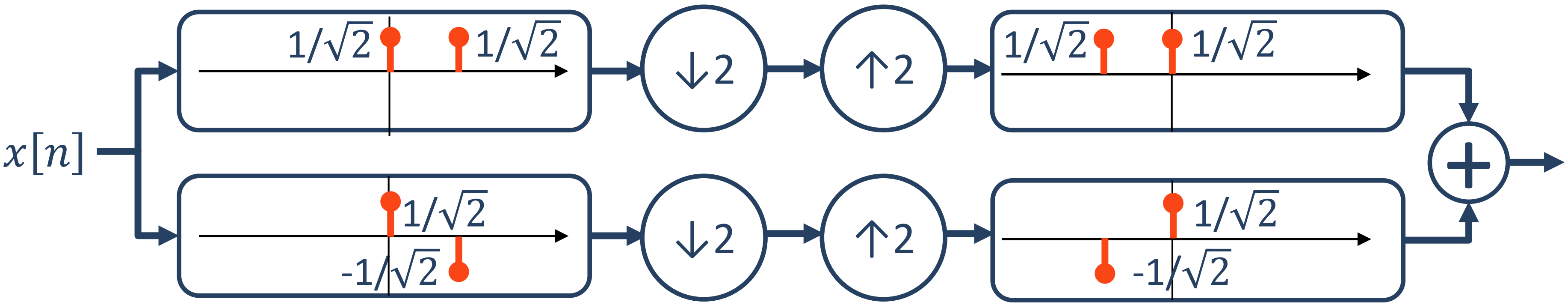


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ \ [G_0(z^{-1})G_0(z) \quad + G_1(z^{-1})G_1(z)]$$
$$+\frac{1}{2}X(-z)[G_0(-z^{-1})G_0(z) + G_1(-z^{-1})G_1(z)]$$

  - **Do the following filters satisfy an orthogonal filter bank?**
    - $\frac{1}{2}(z^{+1}+1)(z^{-1}+1)+\frac{1}{2}(-z^{+1}+1)(-z^{-1}+1) = 2$
    - $\frac{1}{2}(-z^{+1}+1)(z^{-1}+1)+\frac{1}{2}(z^{+1}+1)(-z^{-1}+1) = 2$
    - $\frac{1}{2}(z^{+1}+1)(-z^{-1}+1)+\frac{1}{2}(-z^{+1}+1)(z^{-1}+1) = 0$

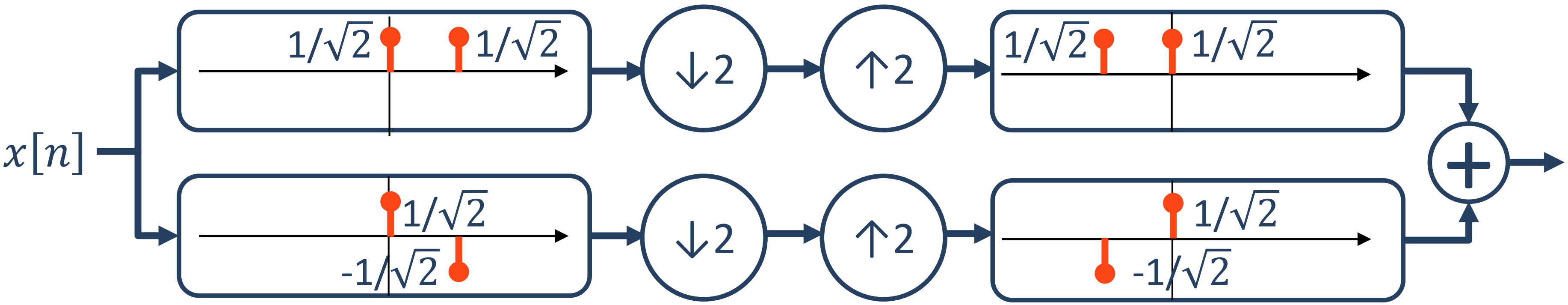


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ \ [G_0(z^{-1})G_0(z) \quad + G_1(z^{-1})G_1(z)]$$
$$+\frac{1}{2}X(-z)[G_0(-z^{-1})G_0(z) + G_1(-z^{-1})G_1(z)]$$

  - **Do the following filters satisfy an orthogonal filter bank?**
    - $\frac{1}{2}(2 + z^{+1} + z^{-1}) + \frac{1}{2}(2 - z^{+1} - z^{-1}) = 2$
    - $\frac{1}{2}(2 - z^{-1} - z^{+1}) + \frac{1}{2}(2 + z^{-1} + z^{+1}) = 2$
    - $\frac{1}{2}(z^{+1} - z^{-1}) + \frac{1}{2}(-z^{+1} + z^{-1}) = 0$

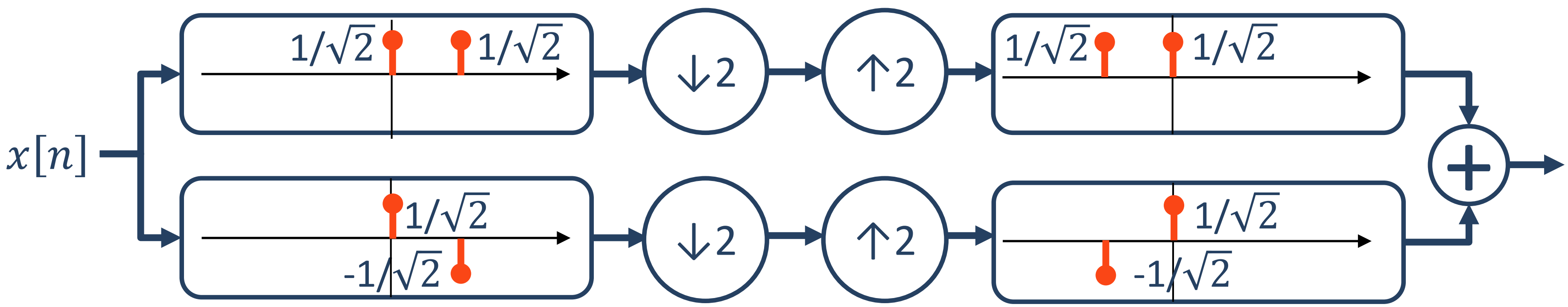


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ [G_0(z^{-1})G_0(z) \quad + G_1(z^{-1})G_1(z)]$$
$$+\frac{1}{2}X(-z)[G_0(-z^{-1})G_0(z) + G_1(-z^{-1})G_1(z)]$$

  - **Do the following filters satisfy an orthogonal filter bank?**
    - ◊ $2 = 2$
    - ◊ $2 = 2$
    - ◊ $0 = 0$

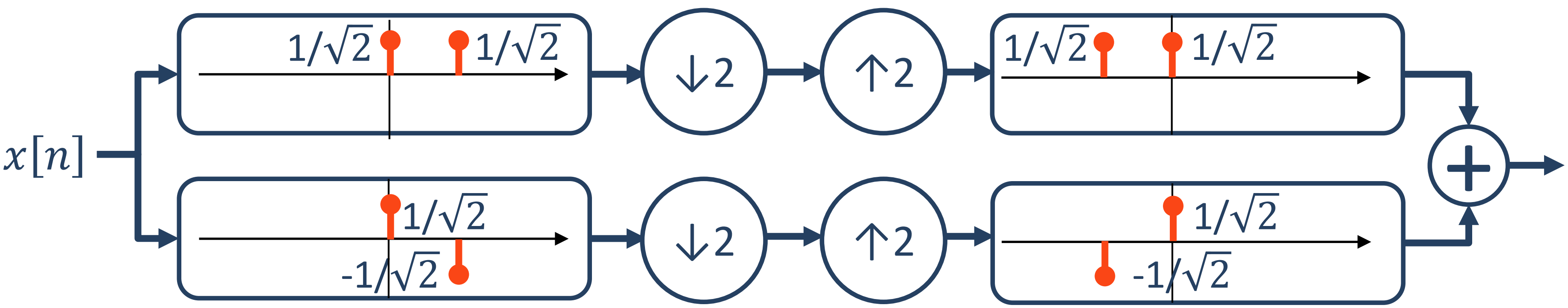


Filter Banks

- **Question: Can we generalize perfect reconstruction?**

$$X(z) = \frac{1}{2}X(z)\ \ [G_0(z^{-1})G_0(z)\quad + G_1(z^{-1})G_1(z)]$$
$$+\frac{1}{2}X(-z)[G_0(-z^{-1})G_0(z) + G_1(-z^{-1})G_1(z)]$$

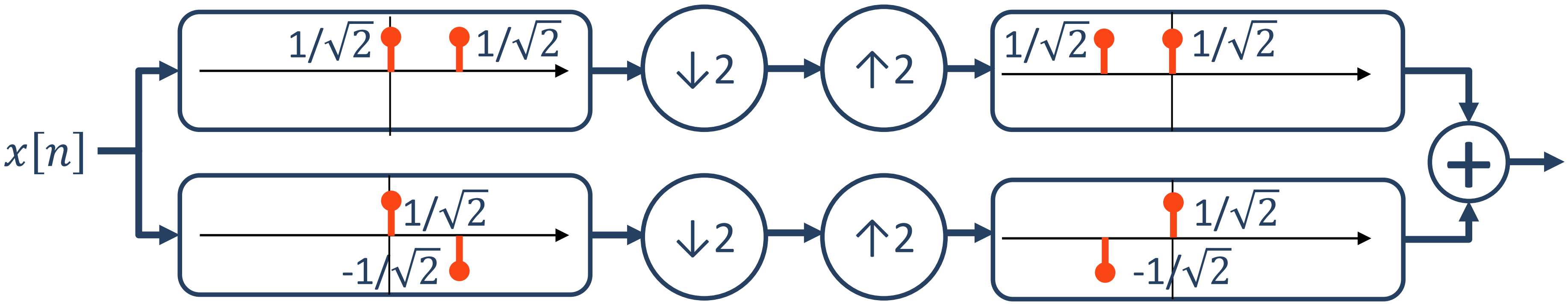
  - **Do the following filters satisfy an orthogonal filter bank?**
    - $g_0[n] * g_0[-n] + [(-1)^n g_0[n]] * [(-1)^n g_0[-n]] = 2\delta[n]$
    - $g_1[n] * g_1[-n] + [(-1)^n g_1[n]] * [(-1)^n g_1[-n]] = 2\delta[n]$
    - $g_0[n] * g_1[-n] + [(-1)^n g_0[n]] * [(-1)^n g_1[-n]] = 0$



Filter Banks

- **Question:**
  - What are the benefits of this approach over alias canceling?

  - **Option 2:** Orthogonal Filter Bank
    - ◊ $G_0(z)G_0(z^{-1}) + G_0(-z)G_0(-z^{-1}) = 2$
    - ◊ $G_1(z)G_1(z^{-1}) + G_1(-z)G_1(-z^{-1}) = 2$
    - ◊ $G_0(z)G_1(z^{-1}) + G_0(-z)G_1(-z^{-1}) = 0$



Lecture 26: Filter Banks to Wavelets

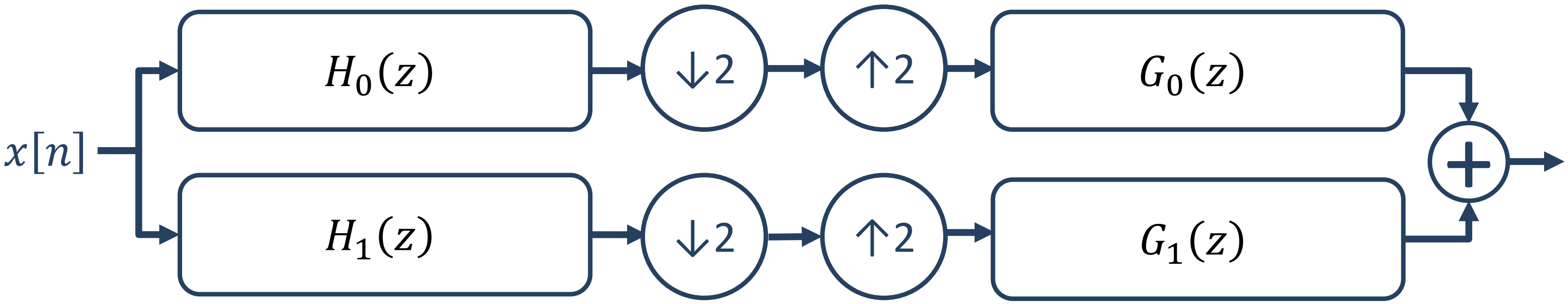
Foundations of Digital Signal Processing

Outline

- DFT Filter Banks [without downsampling]
- DFT Filter Bank [with downsampling]
- General Two-Channel Filter Banks
- Two-Channel Filter Bank Perfect Reconstruction (aliasing canceling)
- Two-Channel Filter Bank Perfect Reconstruction (orthogonal filter bank)
- **Polyphase Filters**
- Multi-Channel Filter Bank Perfect Reconstruction
- Wavelets

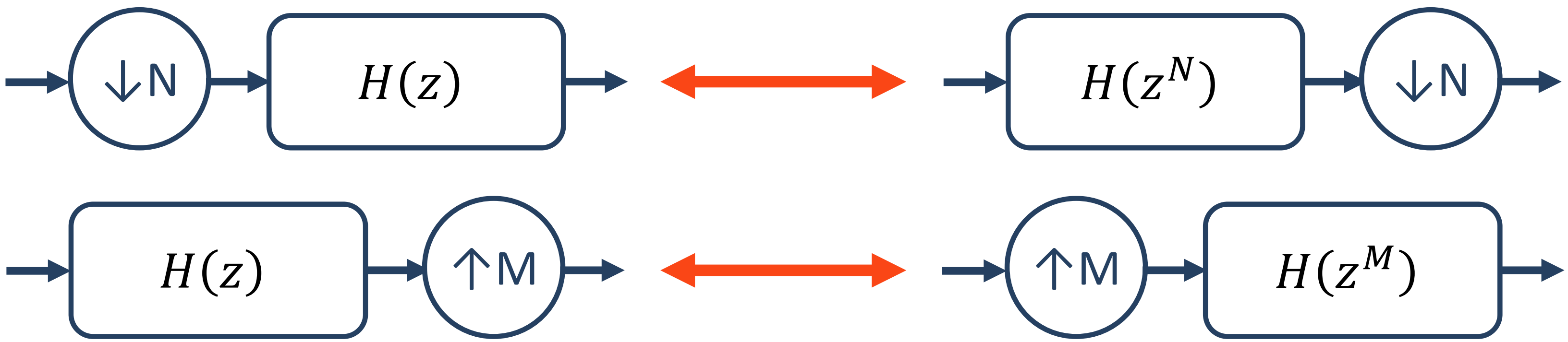
Polyphase Filters

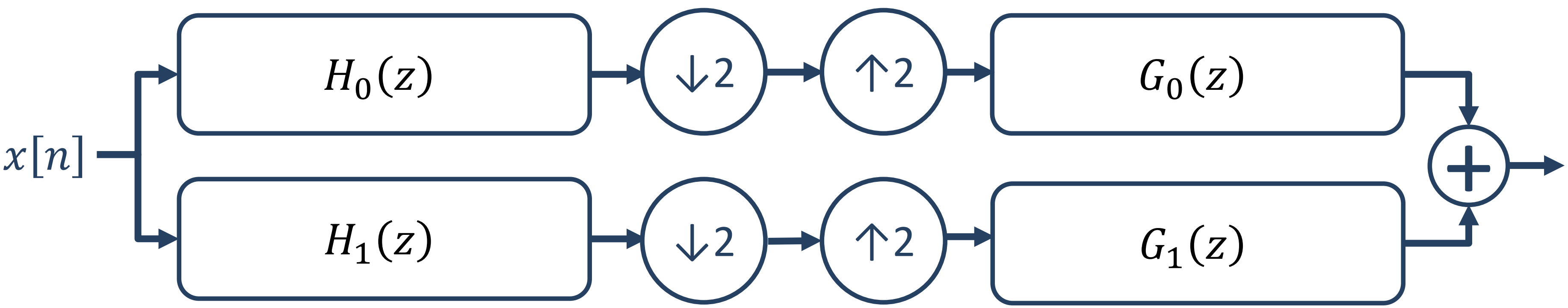
- **Question: Can we make this more efficient?**



Polyphase Filters

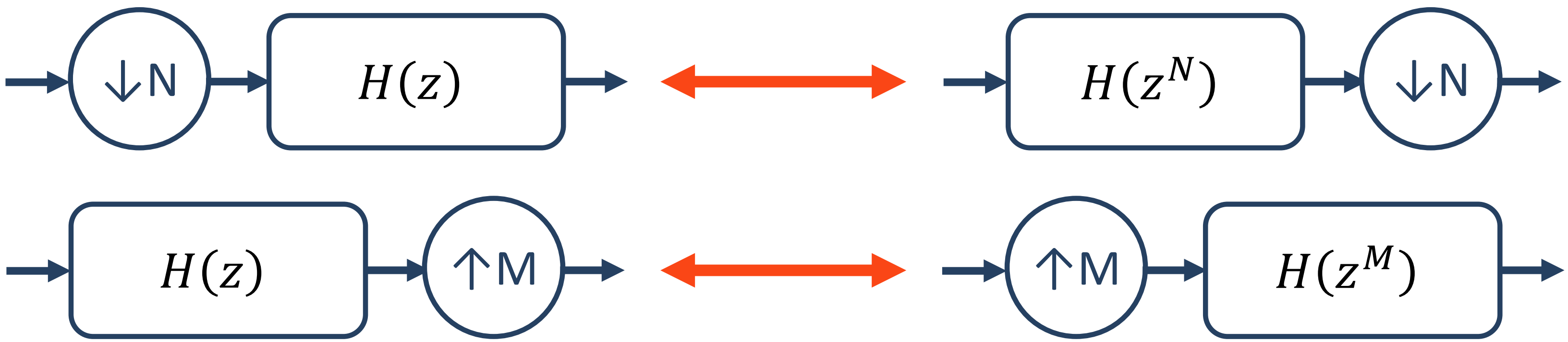
- **Noble Properties**



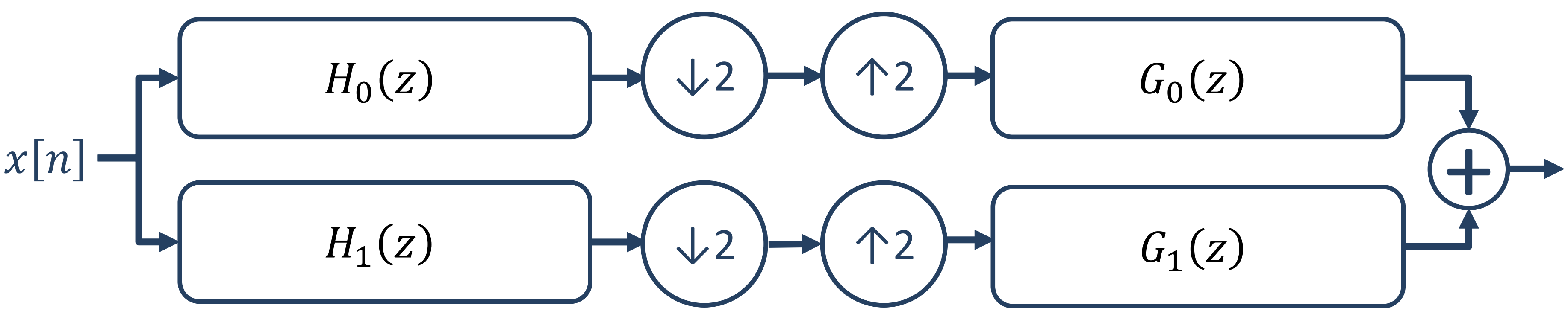


Polyphase Filters

- **Noble Properties**

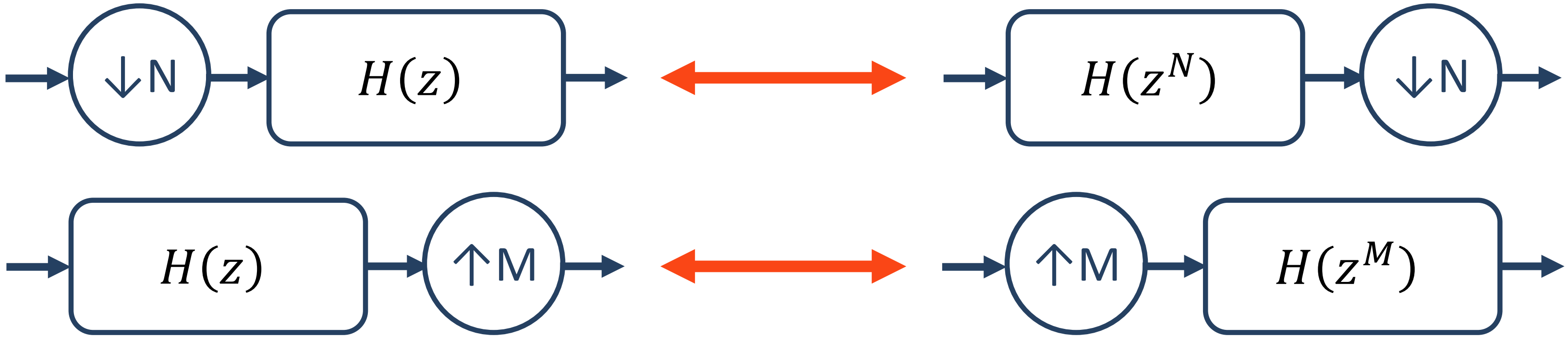


- **Question: How do we apply these properties?**

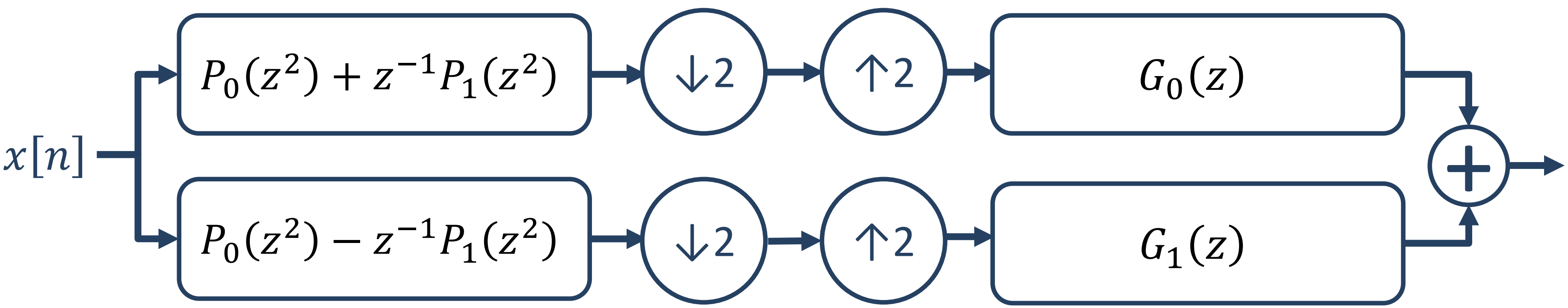


Polyphase Filters

- **Noble Properties**

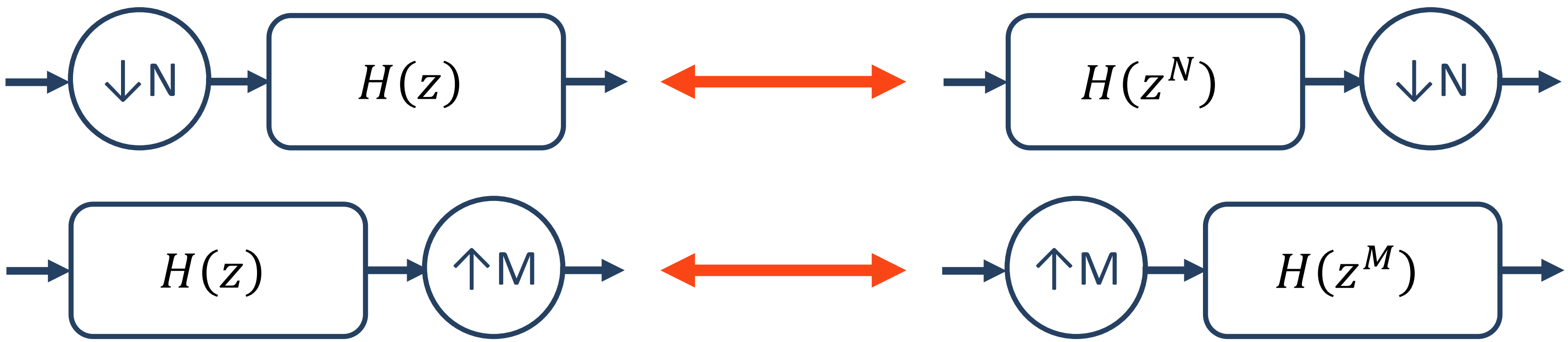


- **Question: How do we apply these properties?**

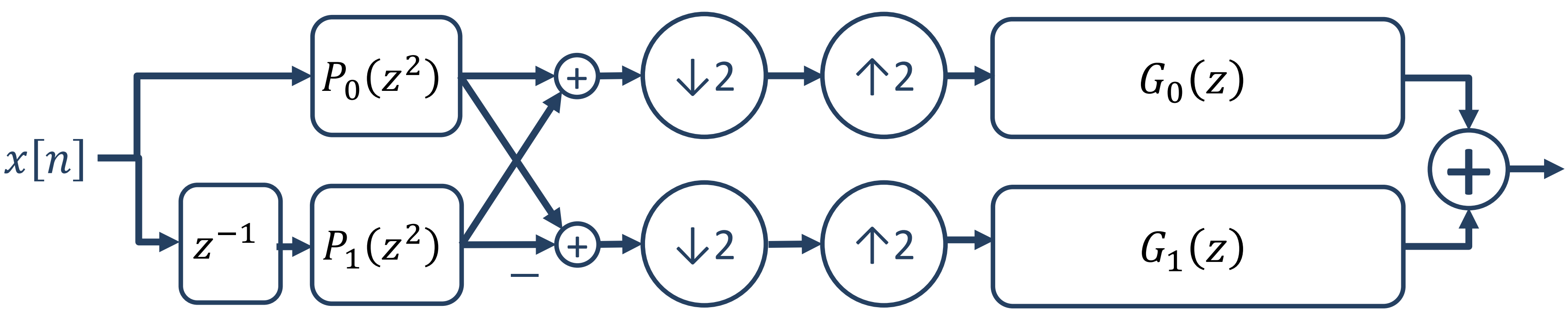


Polyphase Filters

- **Noble Properties**

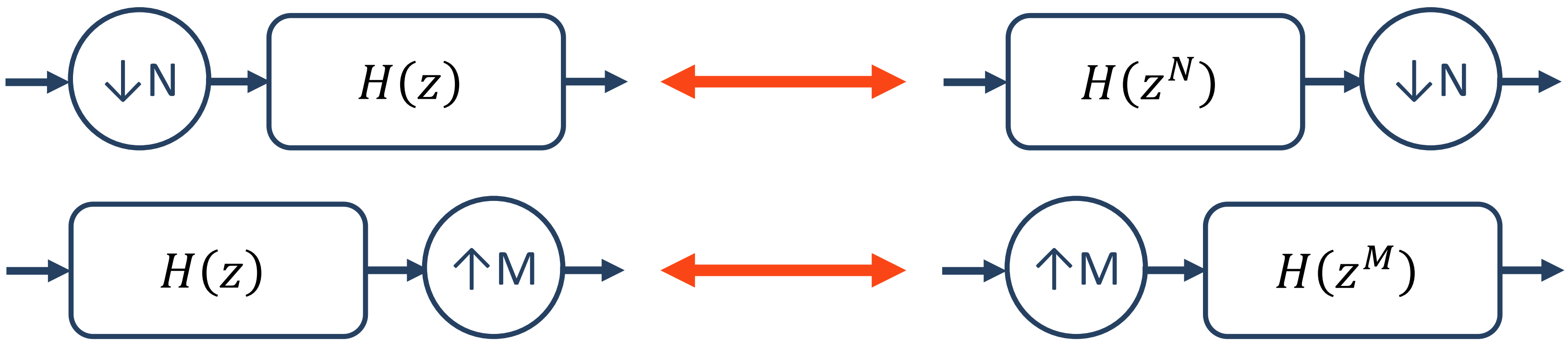


- **Question: How do we apply these properties?**

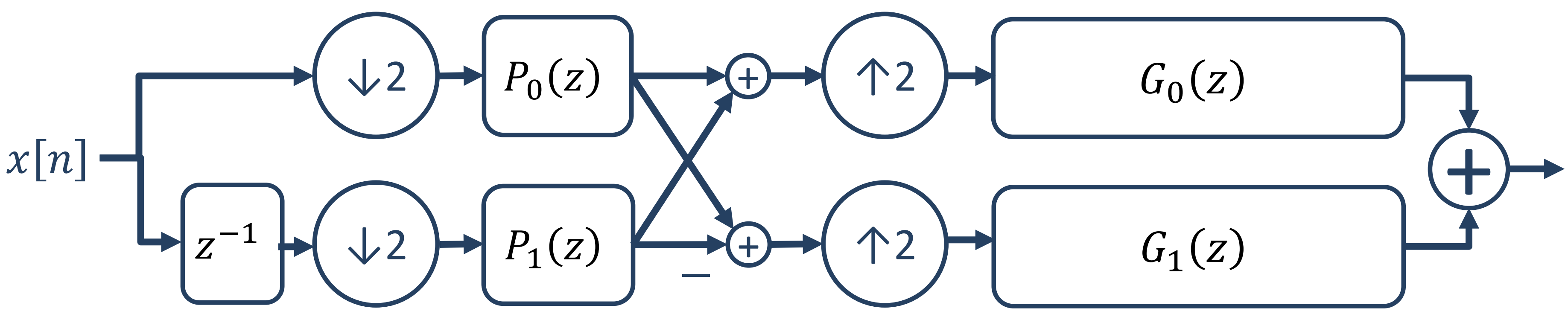


Polyphase Filters

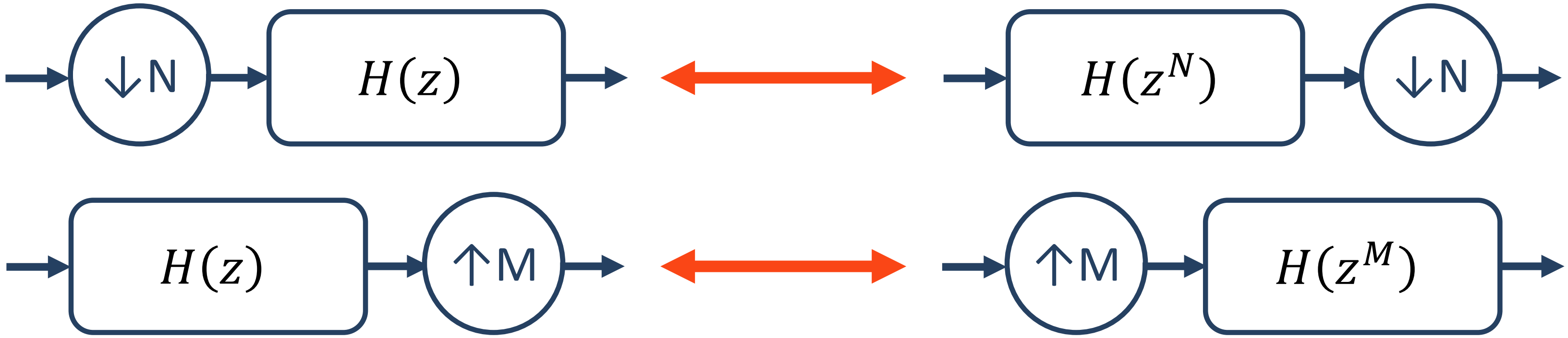
- **Noble Properties**



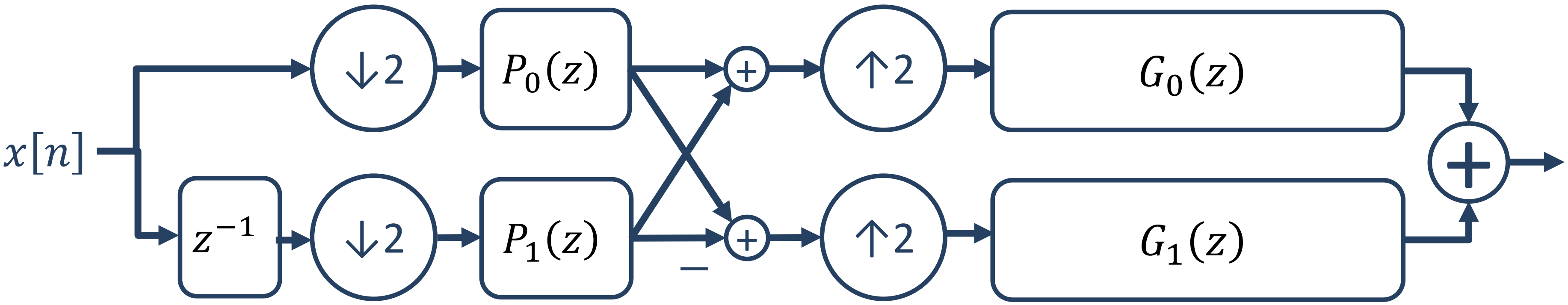
- **Question: How do we apply these properties?**



- **Noble Properties**

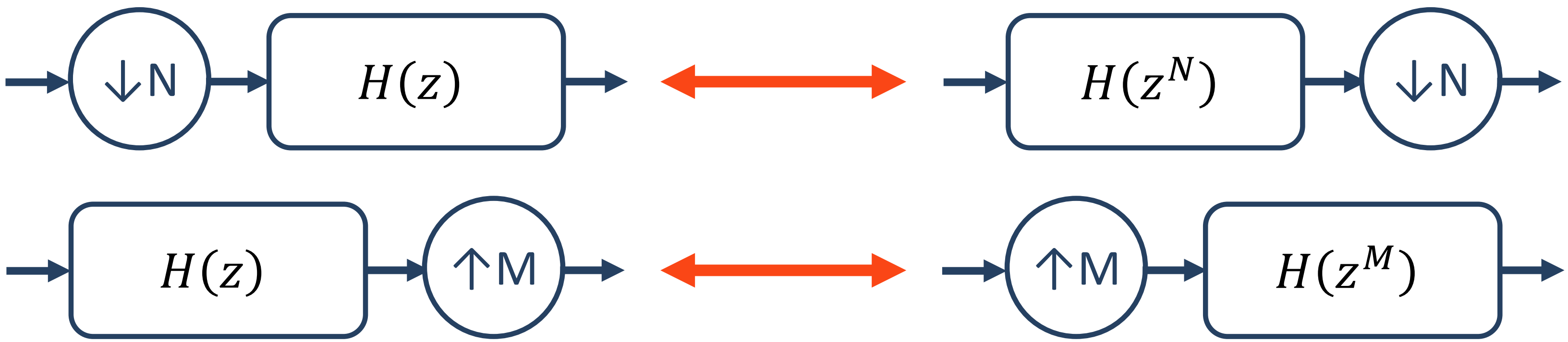


- **Question: How do we apply these properties?**

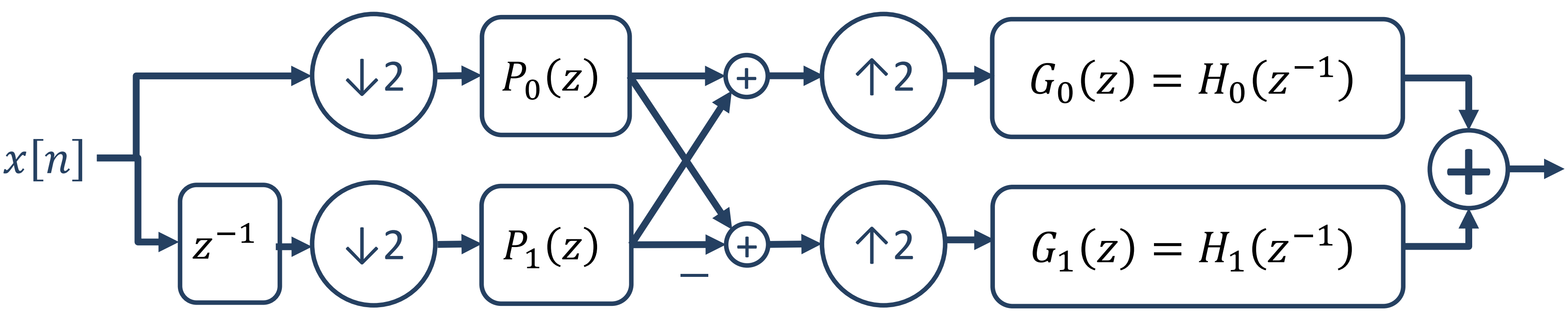


Polyphase Filters

Noble Properties

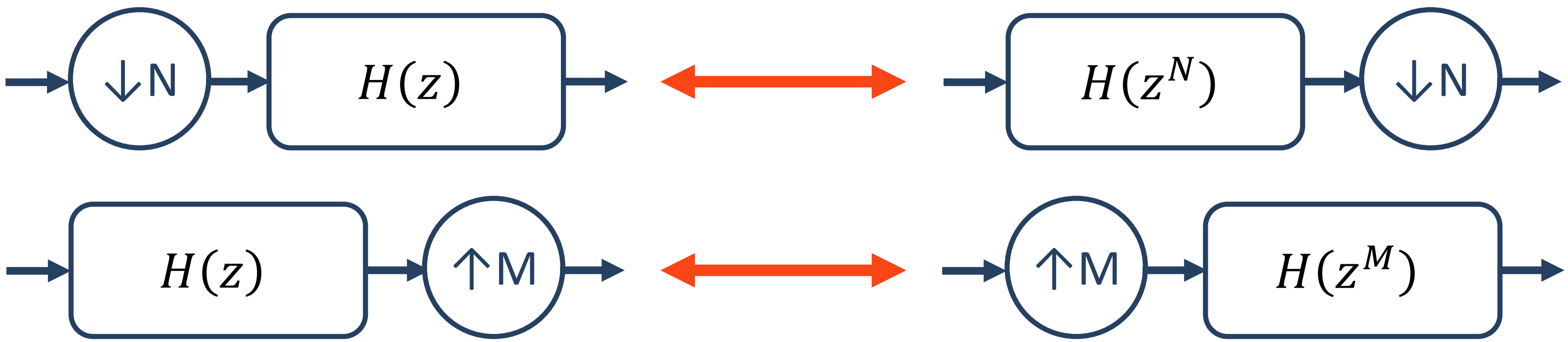


Question: How do we apply these properties?

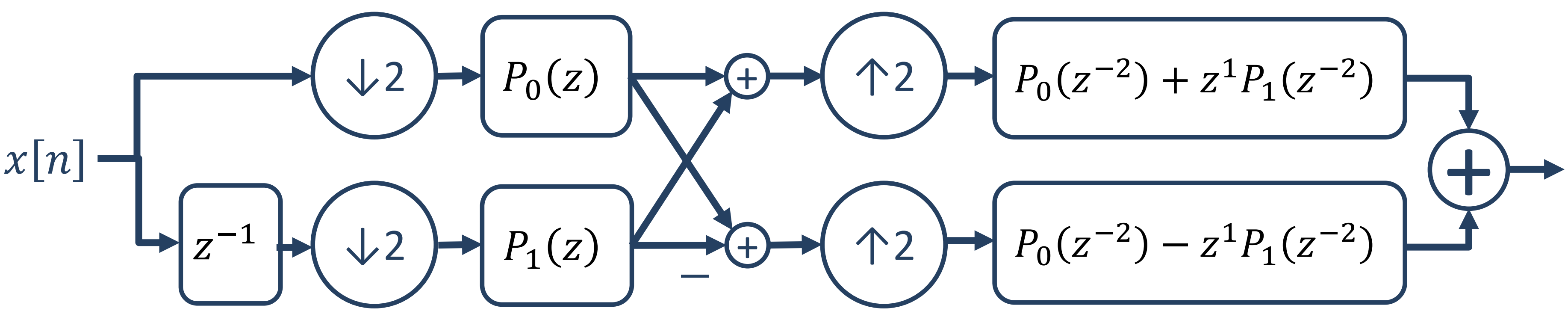


Polyphase Filters

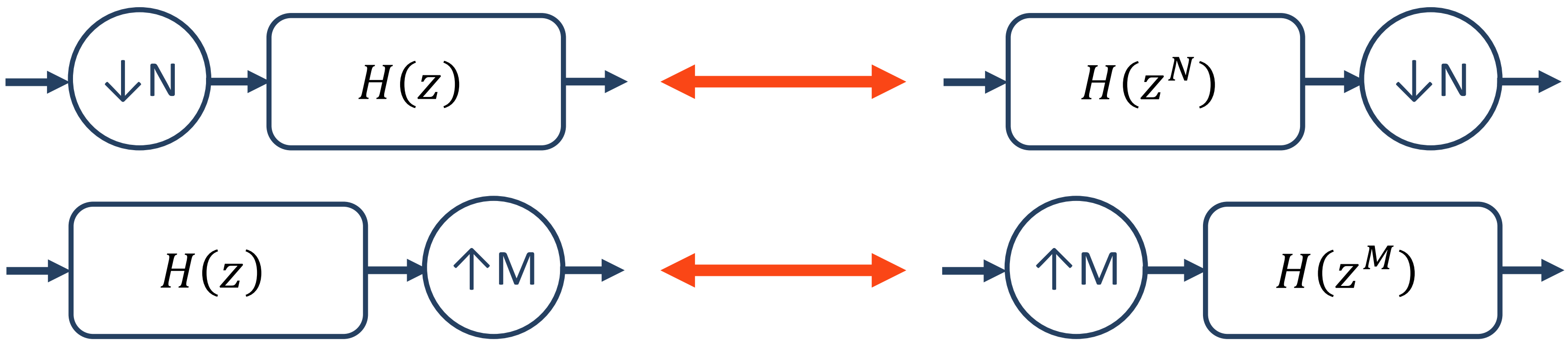
- **Noble Properties**



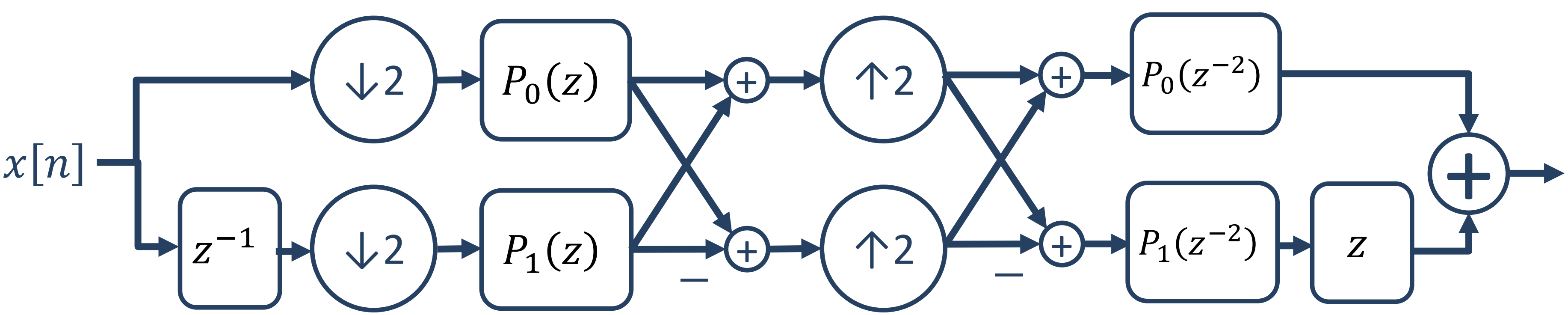
- **Question: How do we apply these properties?**



- **Noble Properties**

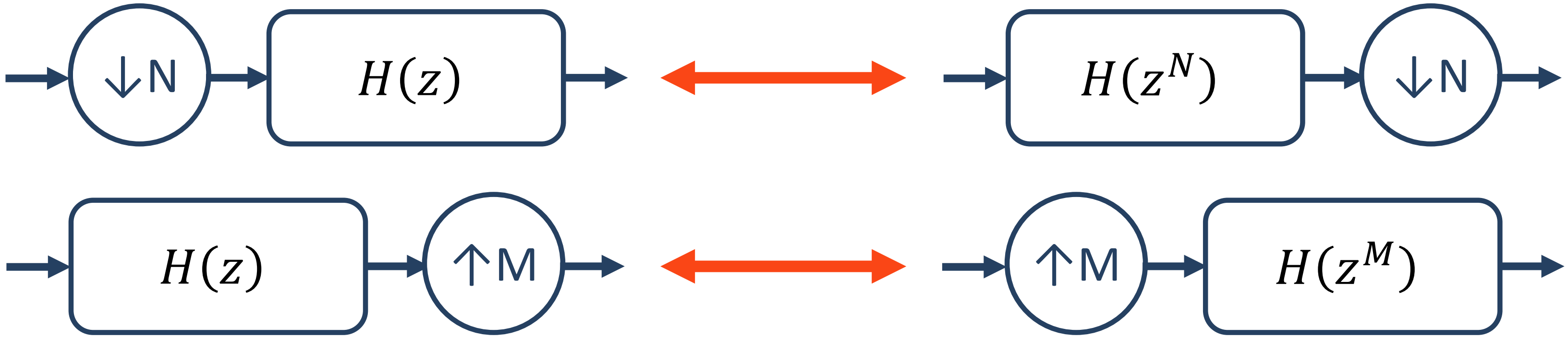


- **Question: How do we apply these properties?**

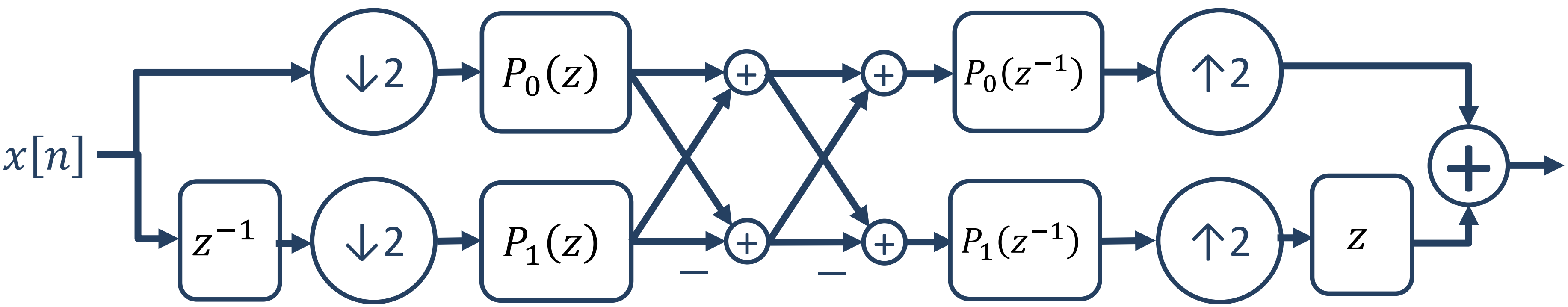


Polyphase Filters

- **Noble Properties**

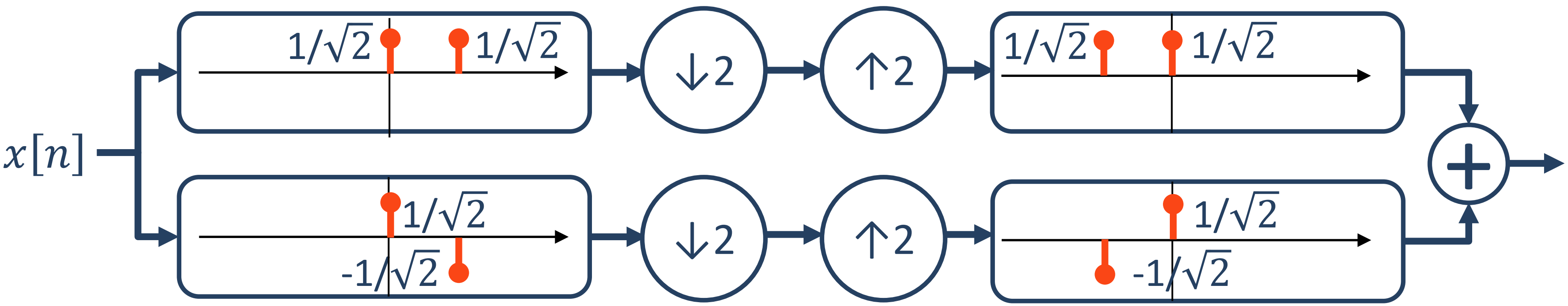


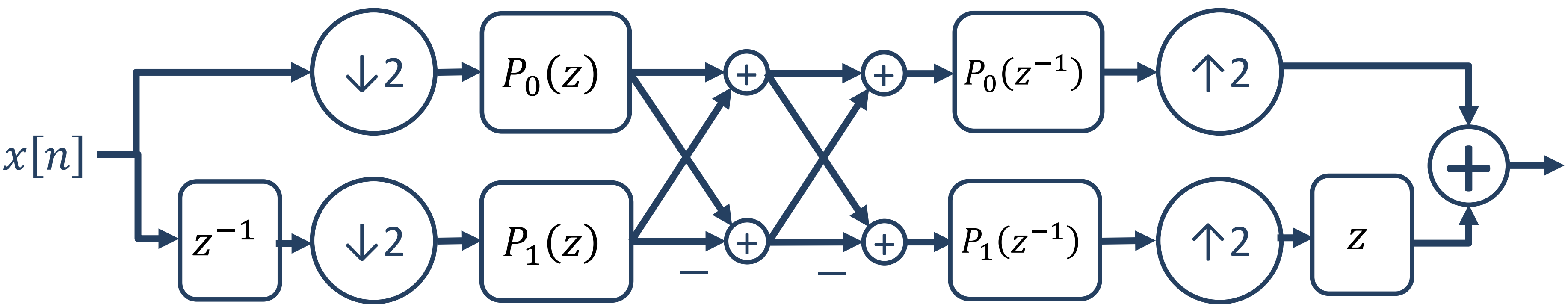
- **Question: How do we apply these properties?**



Polyphase Filters

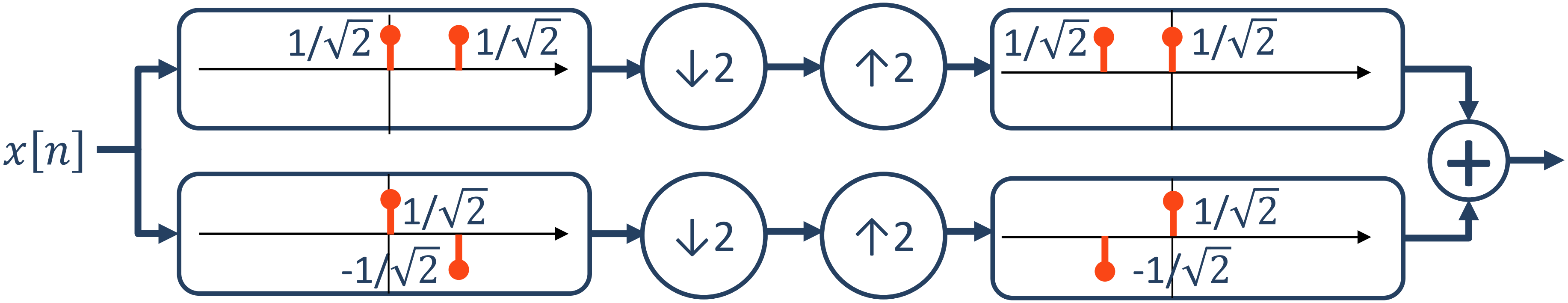
- **Example:** **What is $P_0(z)$ and $P_1(z)$ for the filters below?**

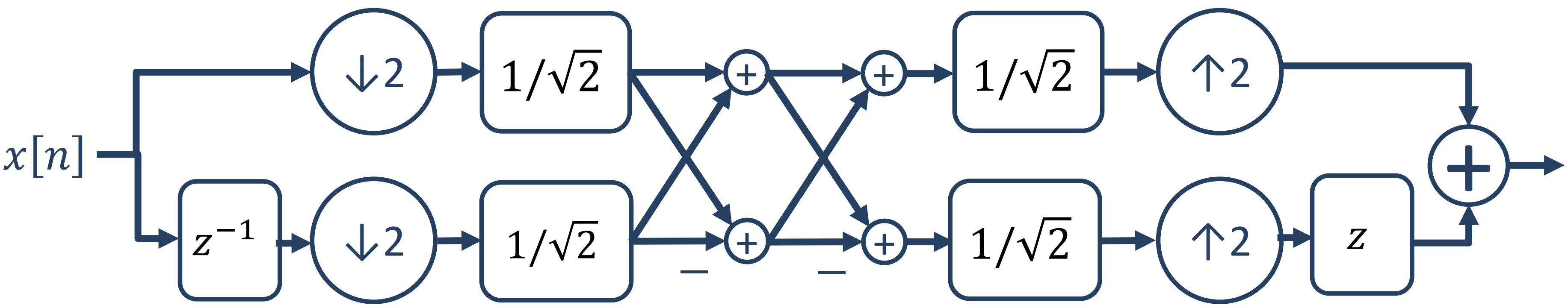




Polyphase Filters

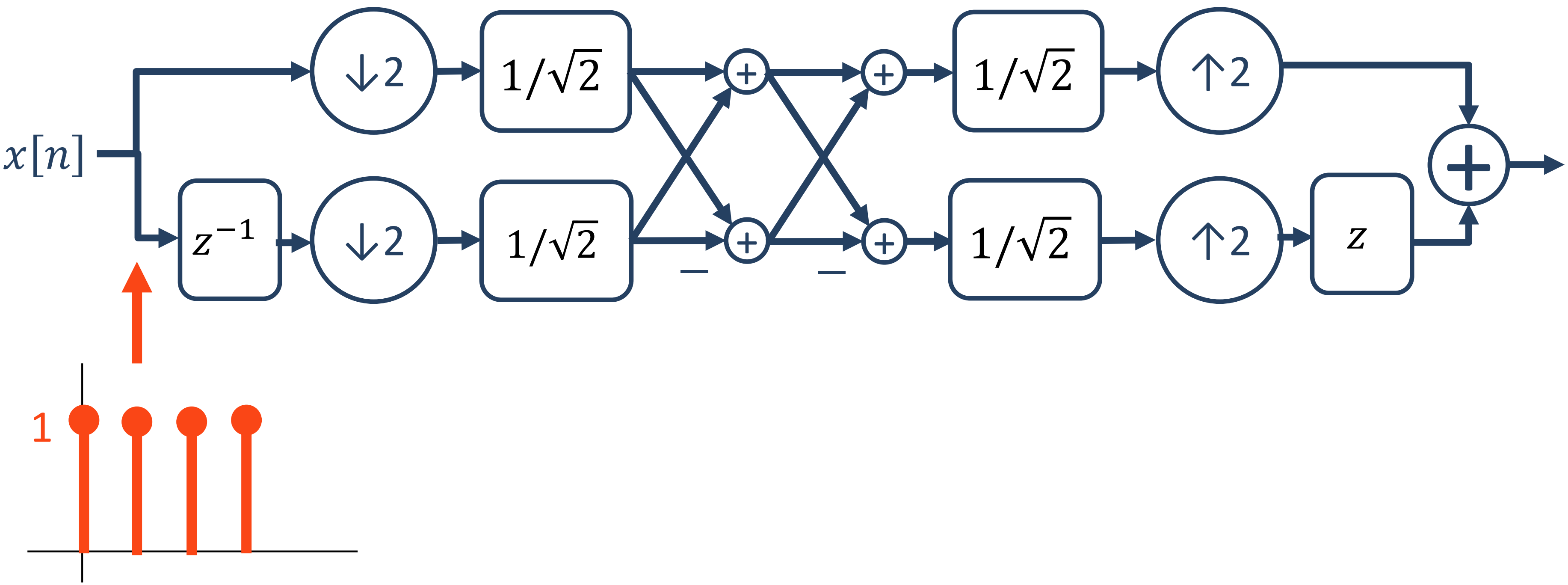
- **Example: What is $P_0(z)$ and $P_1(z)$ for the filters below?**
  - $P_0(z) = 1/\sqrt{2}$
  - $P_1(z) = 1/\sqrt{2}$





Polyphase Filters

- **Example:** **What is $P_0(z)$ and $P_1(z)$ for the filters below?**



Polyphase Filters

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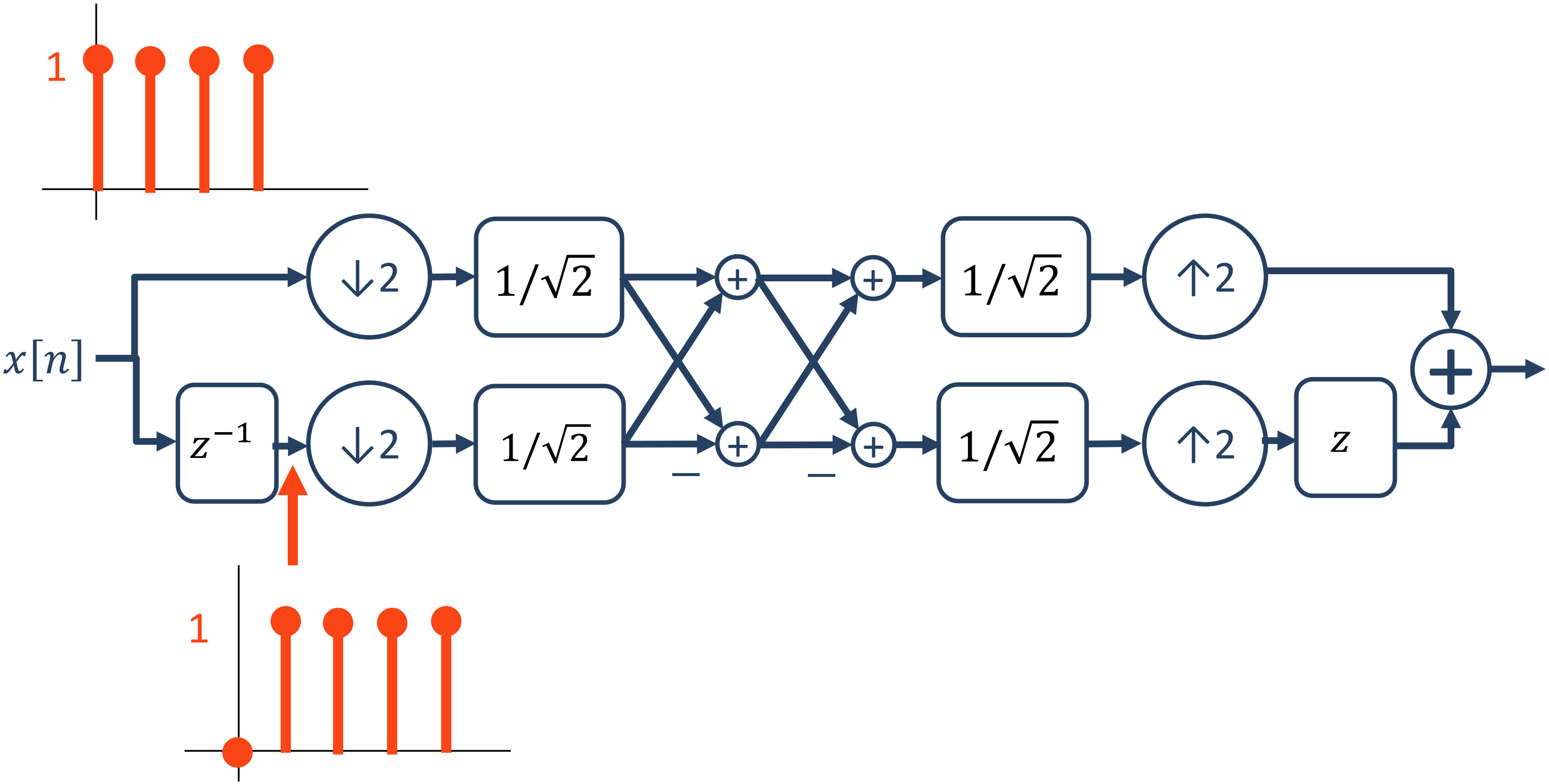

Polyphase Filters

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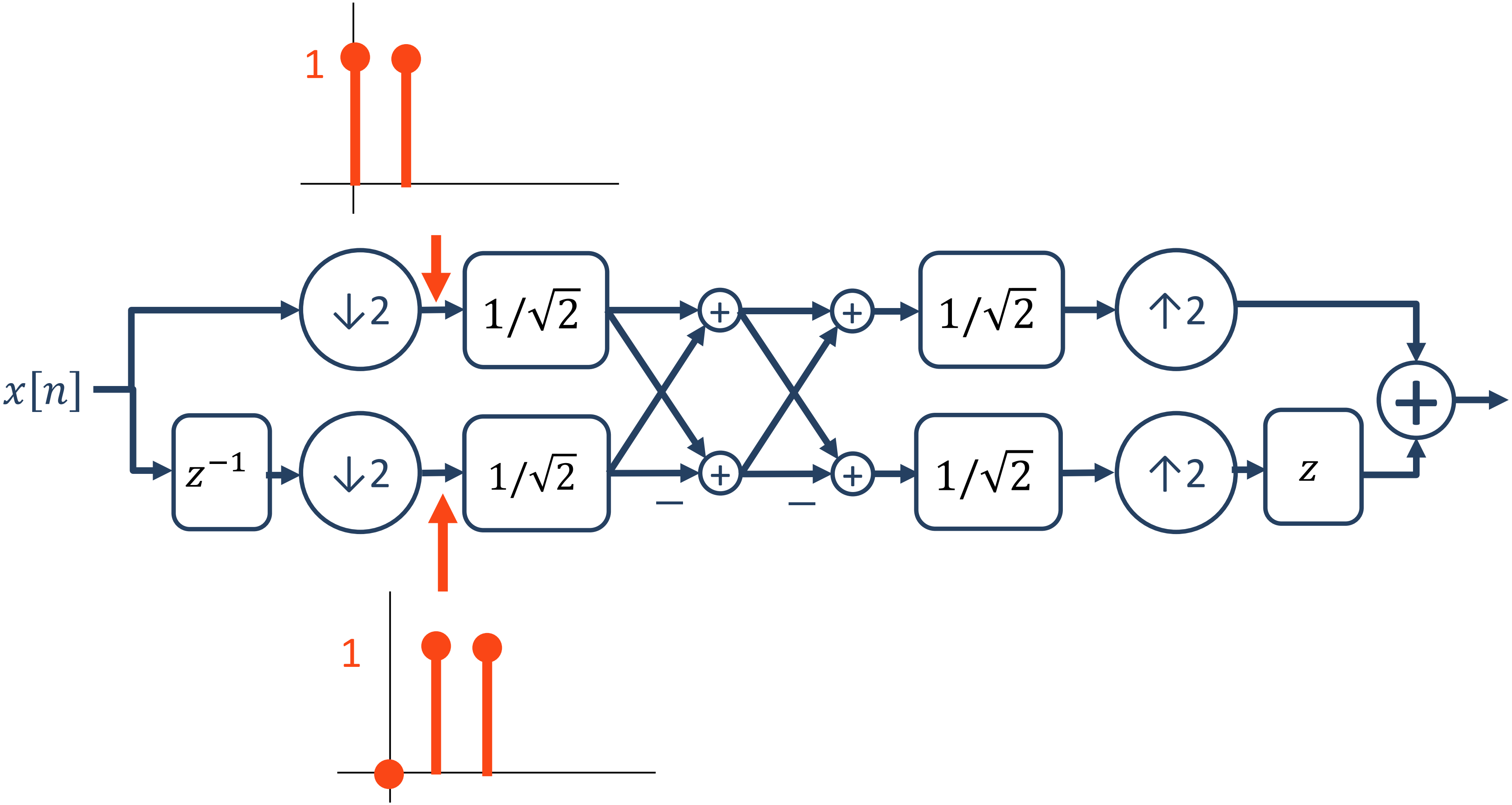

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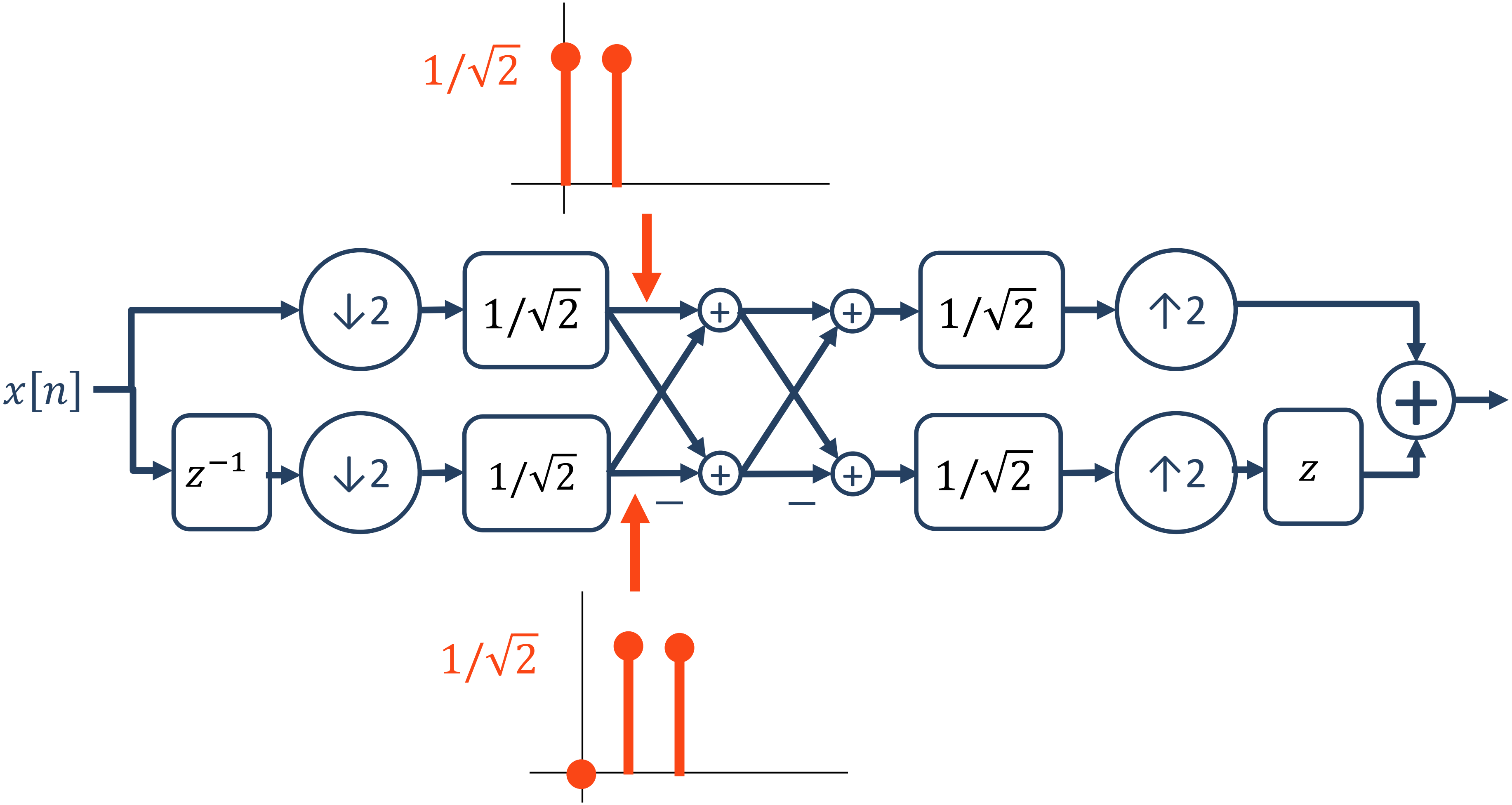

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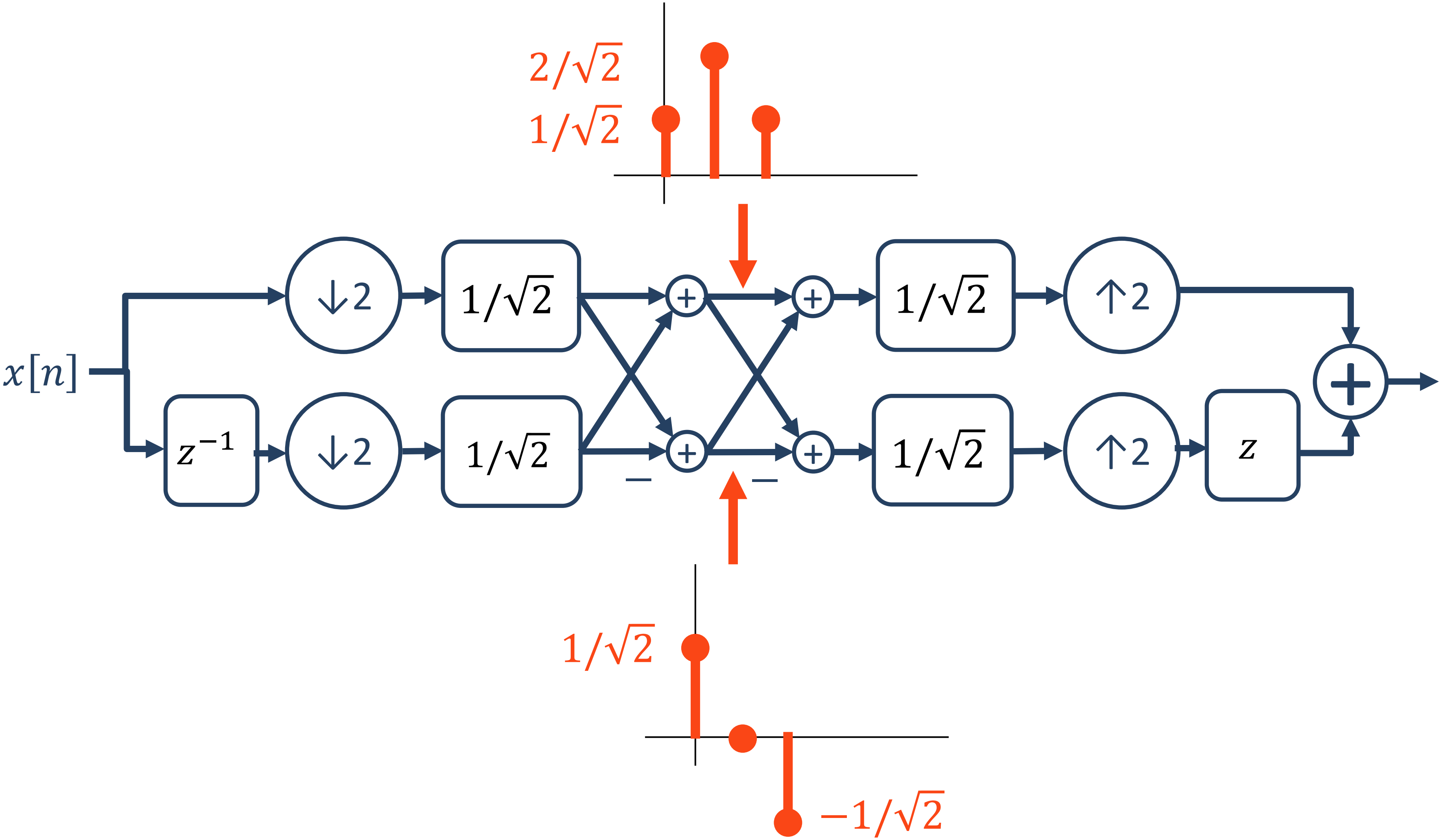

Polyphase Filters

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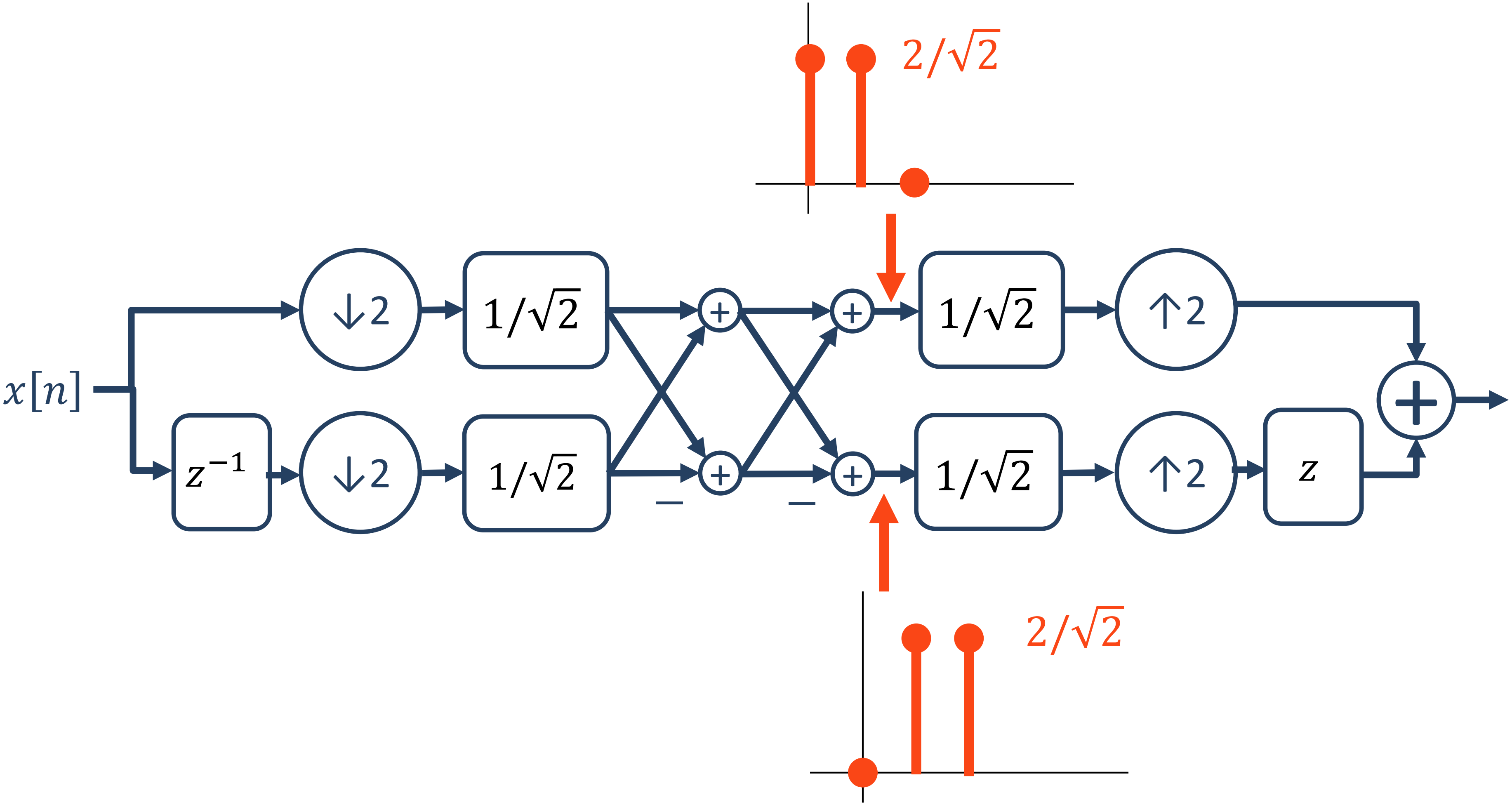

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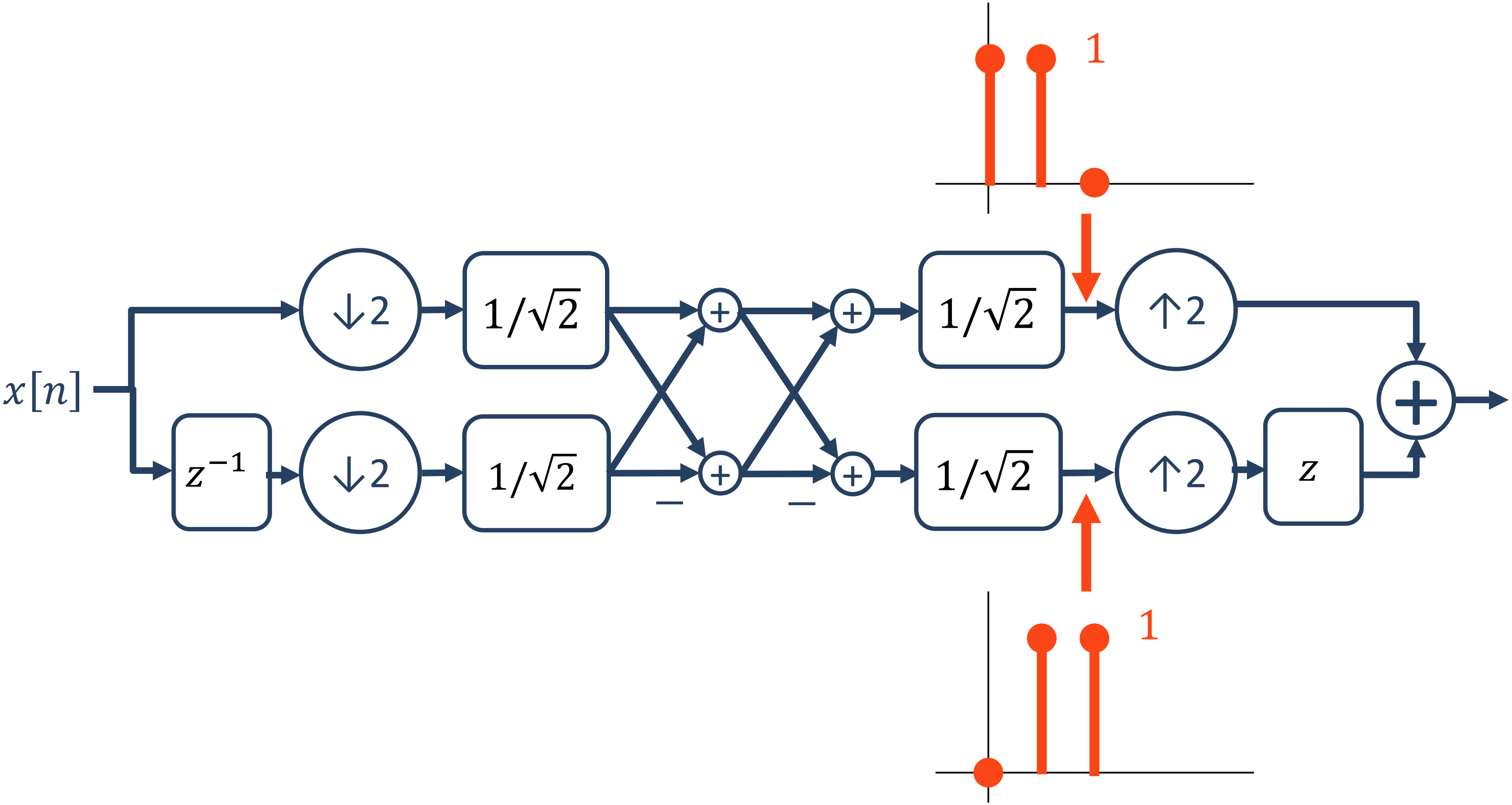

Polyphase Filters

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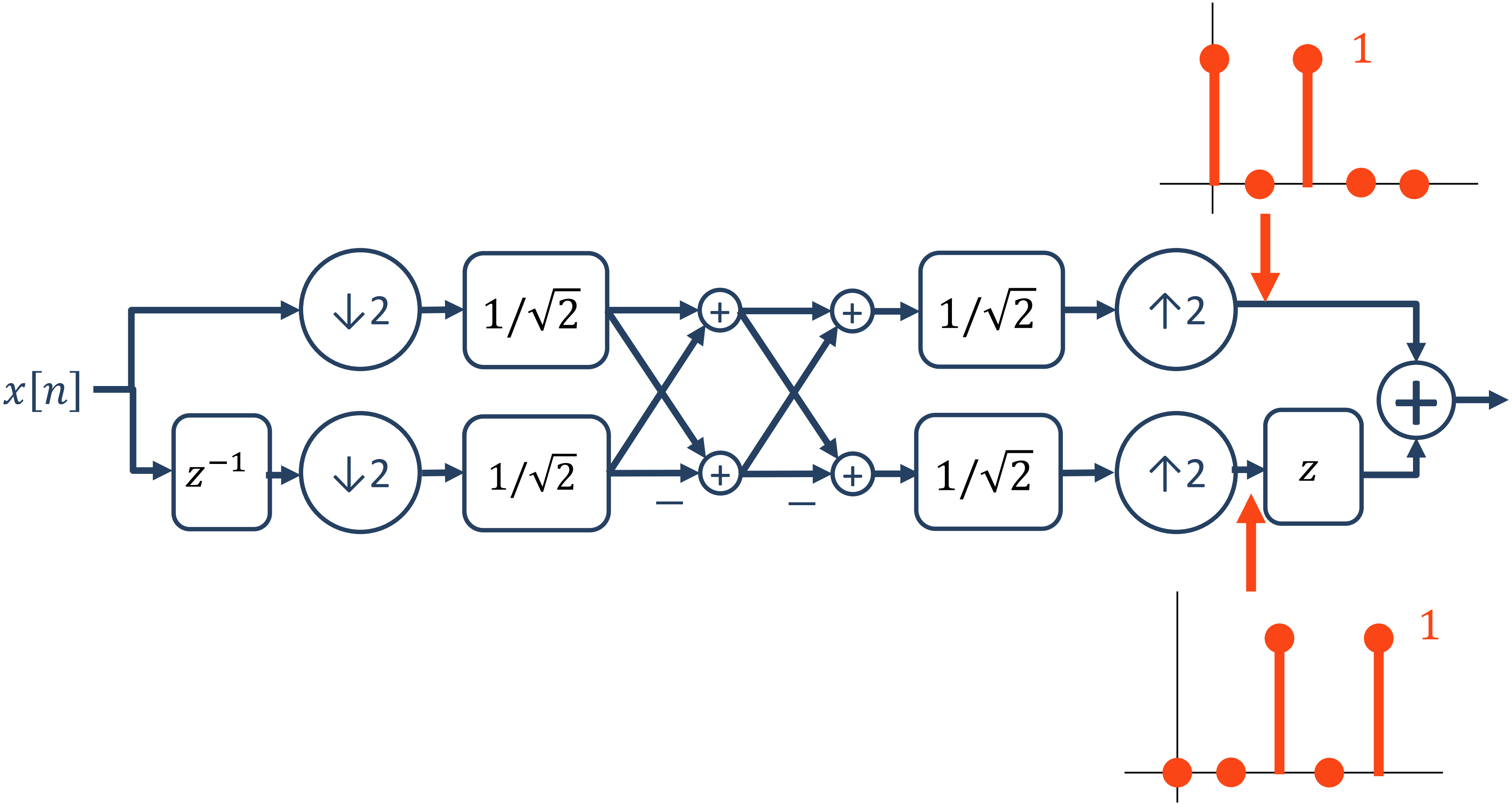

Polyphase Filters

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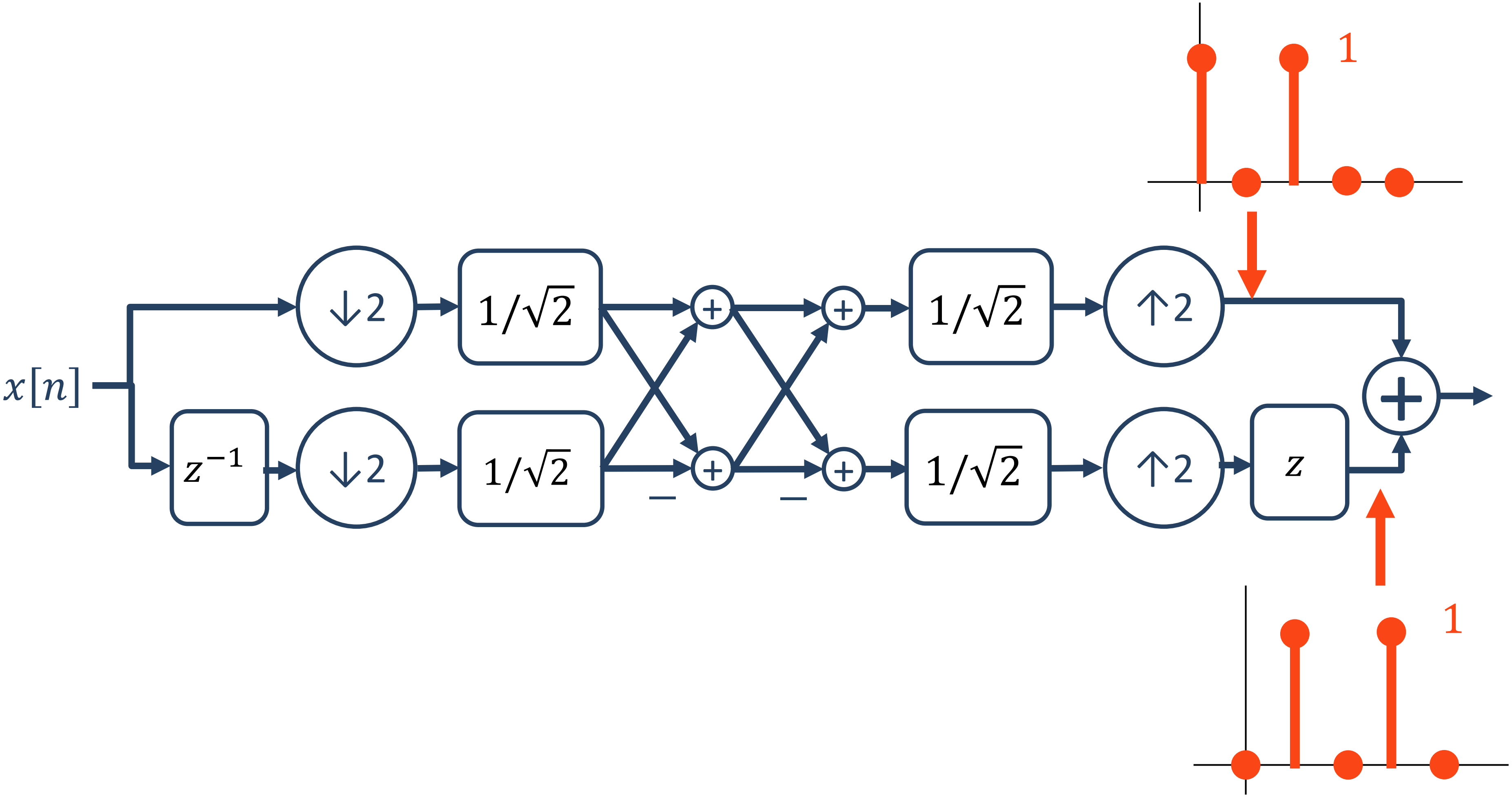

Polyphase Filters

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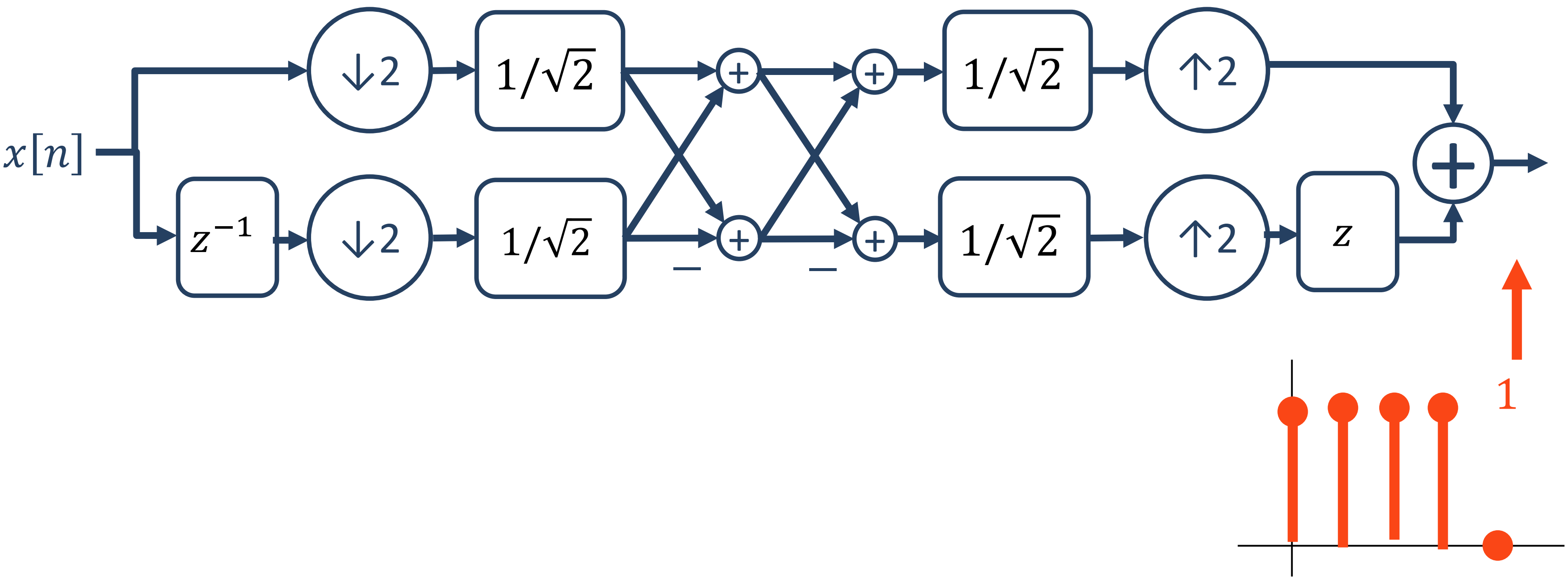


Lecture 26: Filter Banks to Wavelets

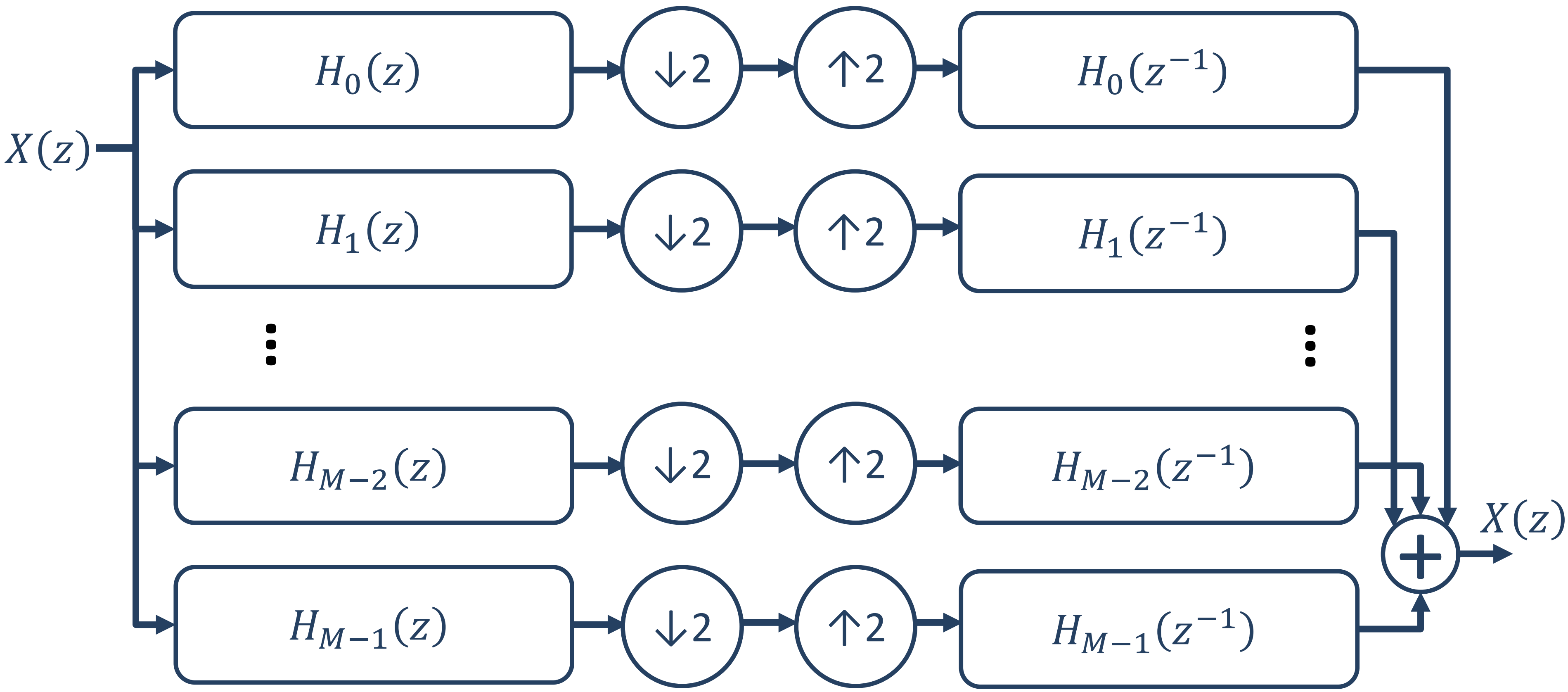
Foundations of Digital Signal Processing

Outline

- DFT Filter Banks [without downsampling]
- DFT Filter Bank [with downsampling]
- General Two-Channel Filter Banks
- Two-Channel Filter Bank Perfect Reconstruction (aliasing canceling)
- Two-Channel Filter Bank Perfect Reconstruction (orthogonal filter bank)
- Polyphase Filters
- **Multi-Channel Filter Bank Perfect Reconstruction**
- Wavelets

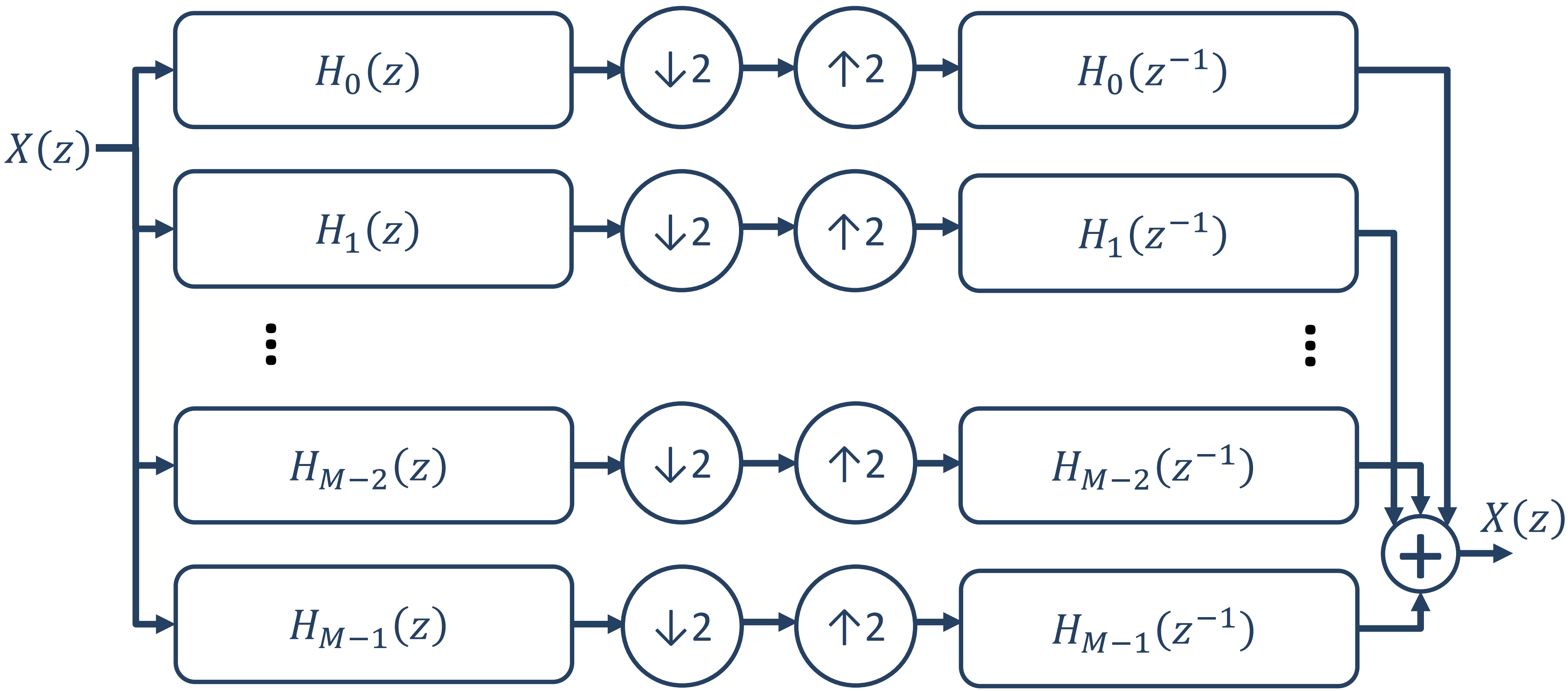
Multi-Channel Filter Banks

- **Question: Can we generalize perfect reconstruction?**



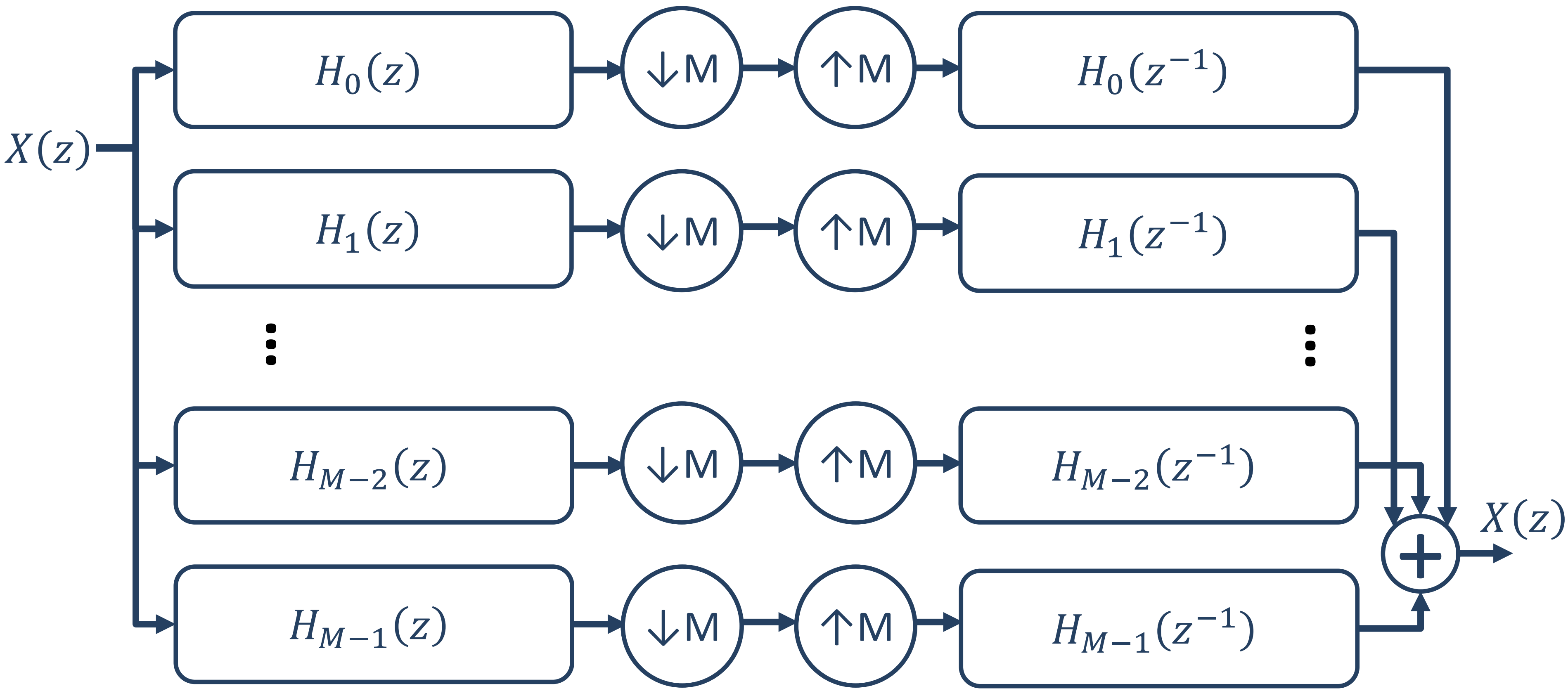
Multi-Channel Filter Banks

- **Question: Can we generalize perfect reconstruction?**
  - $G_m(z)G_m(z^{-1}) + G_m(-z)G_m(-z^{-1}) = 2$ for all m
  - $G_m(z)G_k(z^{-1}) + G_m(-z)G_k(-z^{-1}) = 0$ for all m, k



Multi-Channel Filter Banks

- **Question: Can we generalize perfect reconstruction?**
  - $\sum_{k=0}^{M-1} G_m\left(e^{j\frac{2\pi}{M}k}\, z\right) G_m\left(e^{j\frac{2\pi}{M}k}\, z^{-1}\right) = M$ for all m
  - $\sum_{k=0}^{M-1} G_m\left(e^{j\frac{2\pi}{M}k}\, z\right) G_k\left(e^{j\frac{2\pi}{M}k}\, z^{-1}\right) = 0$ for all m, k



Lecture 26: Filter Banks to Wavelets

Foundations of Digital Signal Processing

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- General Two-Channel Filter Banks
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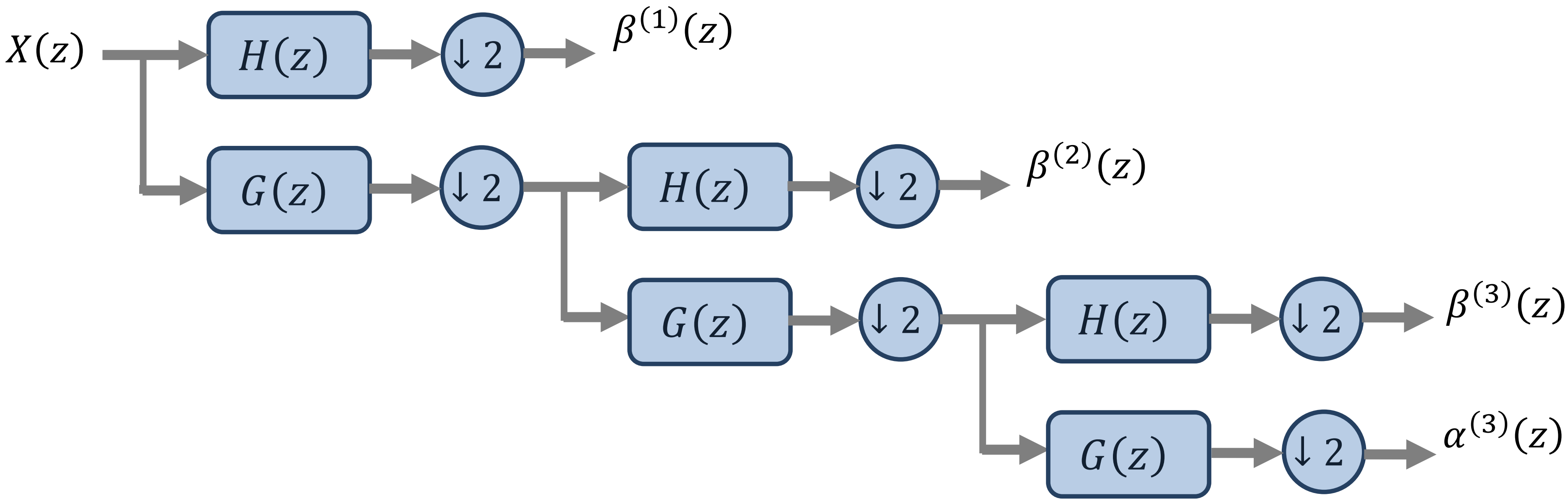
- Polyphase Filters
- Multi-Channel Filter Bank Perfect Reconstruction
- **Wavelets**

Wavelets / Sub-band coding

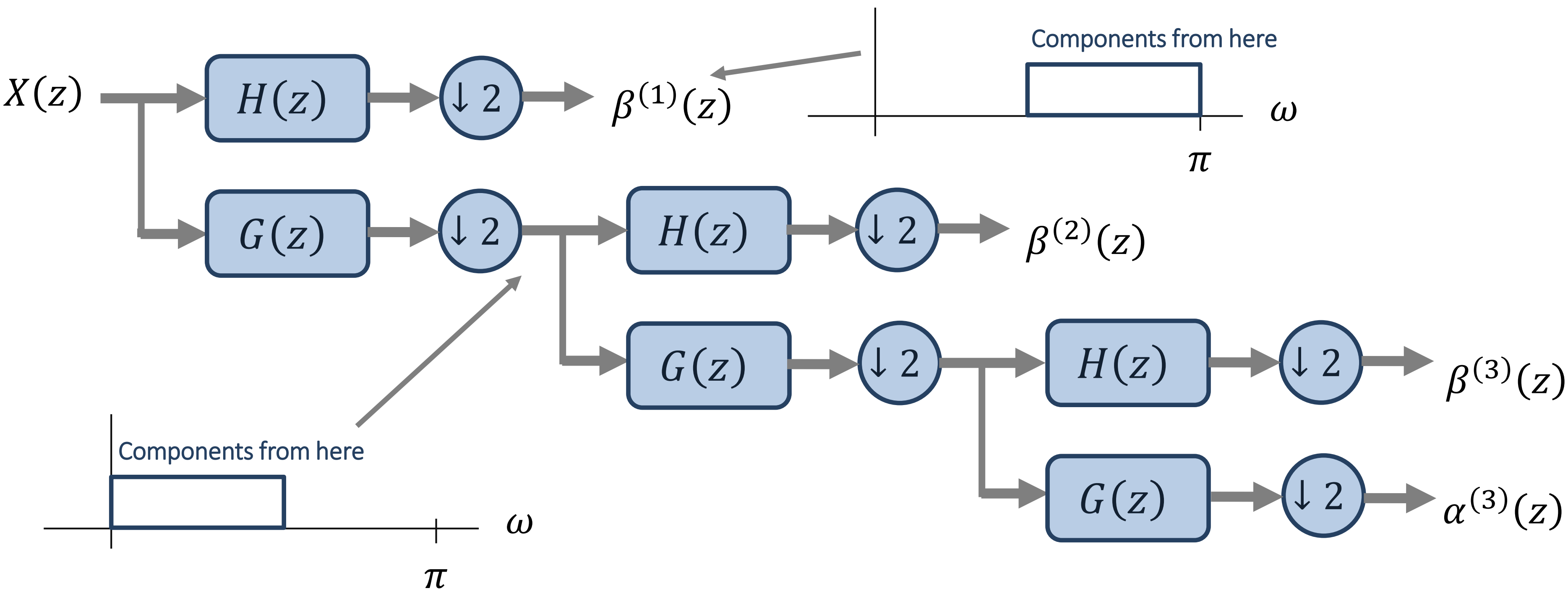
- **Consider this new [analysis] filter bank:** What is going on here?



- **Assume H^* is a half-band high pass filter**
- **Assume G^* is a half-band low pass filter**

Wavelets / Sub-band coding

- **Consider this new [analysis] filter bank: What is going on here?**



Wavelets / Sub-band coding

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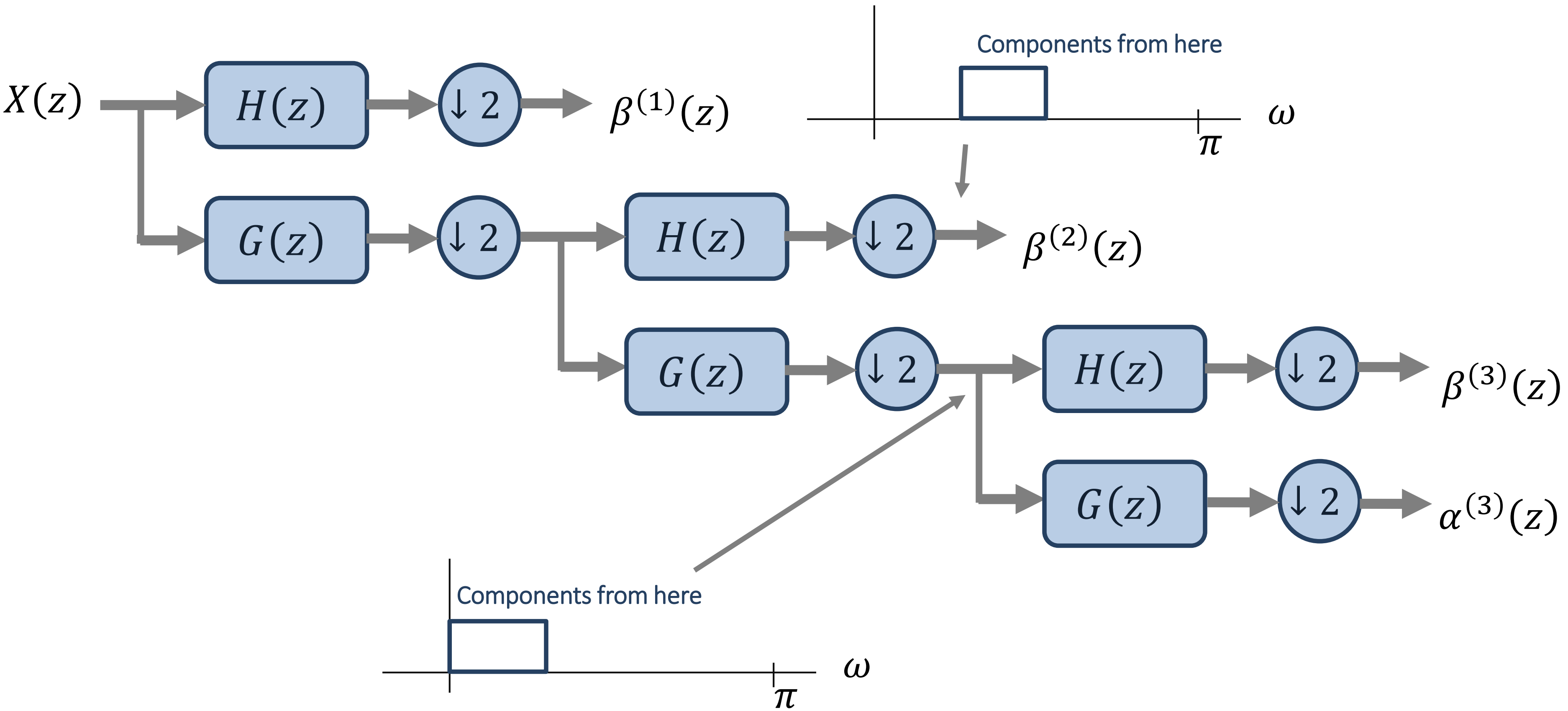

Wavelets / Sub-band coding

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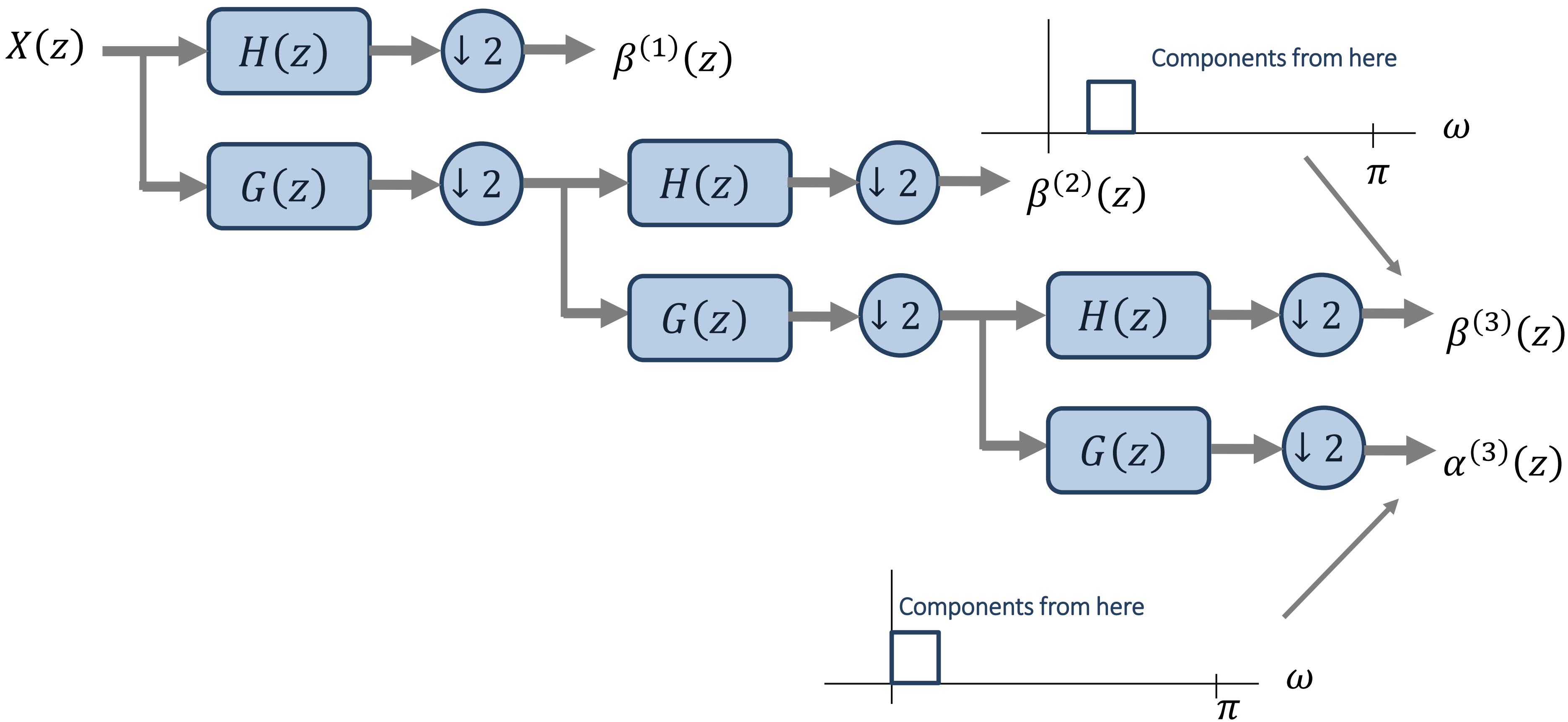

Wavelets / Sub-band coding

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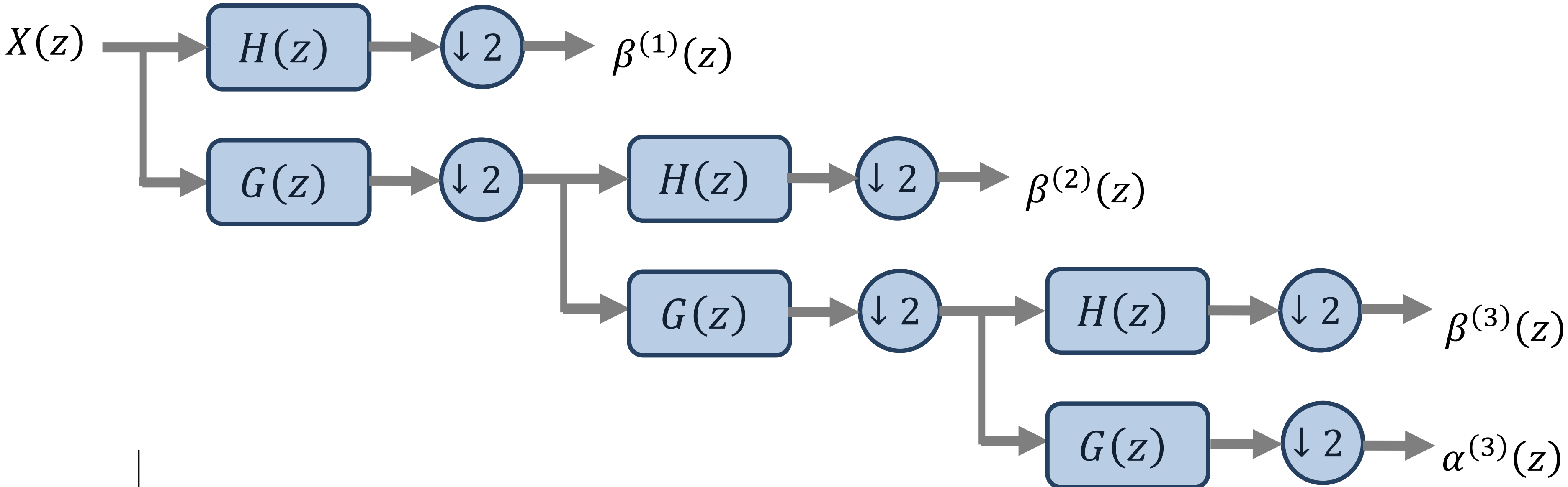
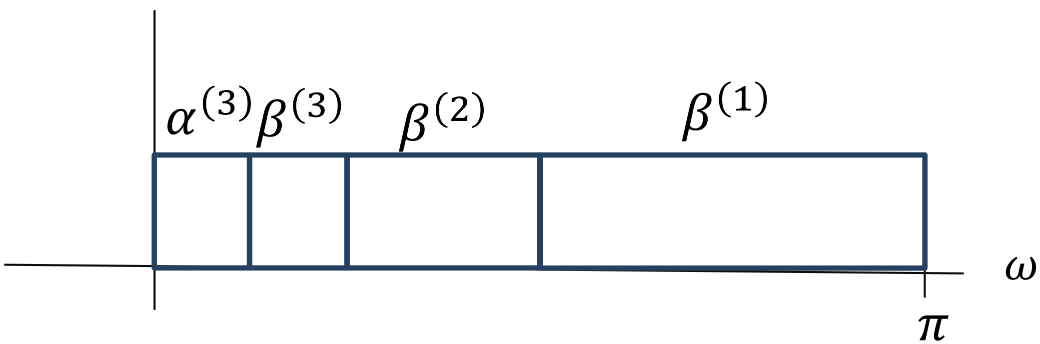
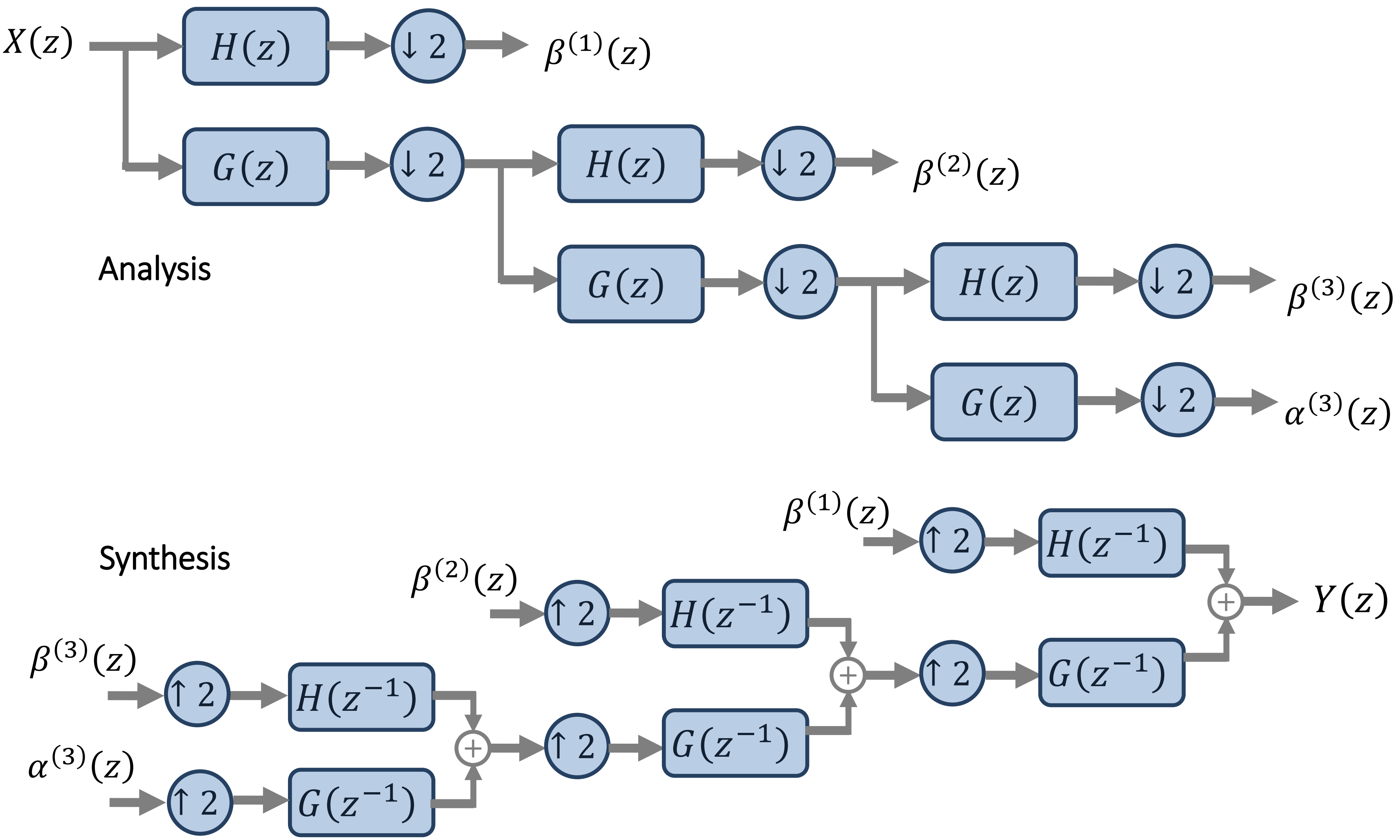


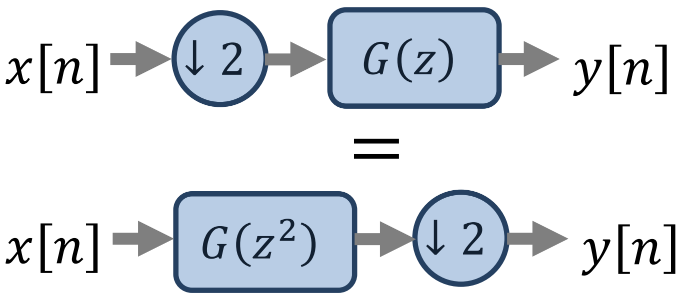


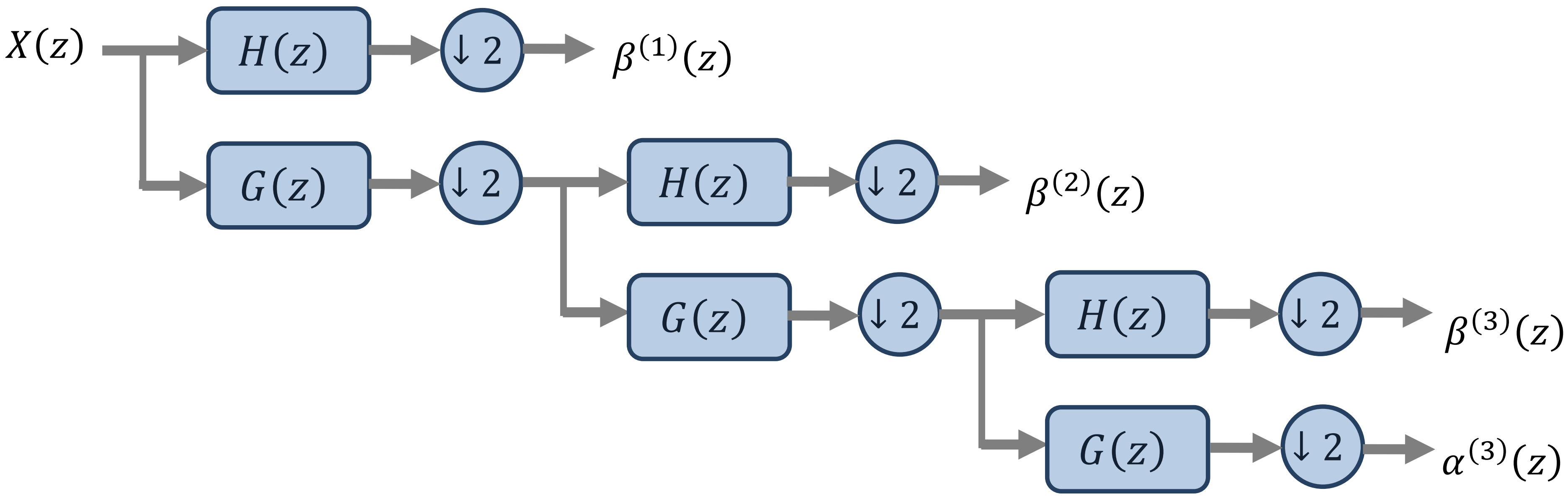
Wavelets / Sub-band coding



Wavelets / Sub-band coding

- **How do you analyze this?**





Wavelets / Sub-band coding

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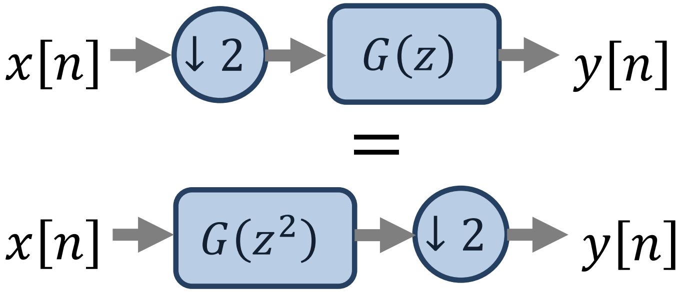

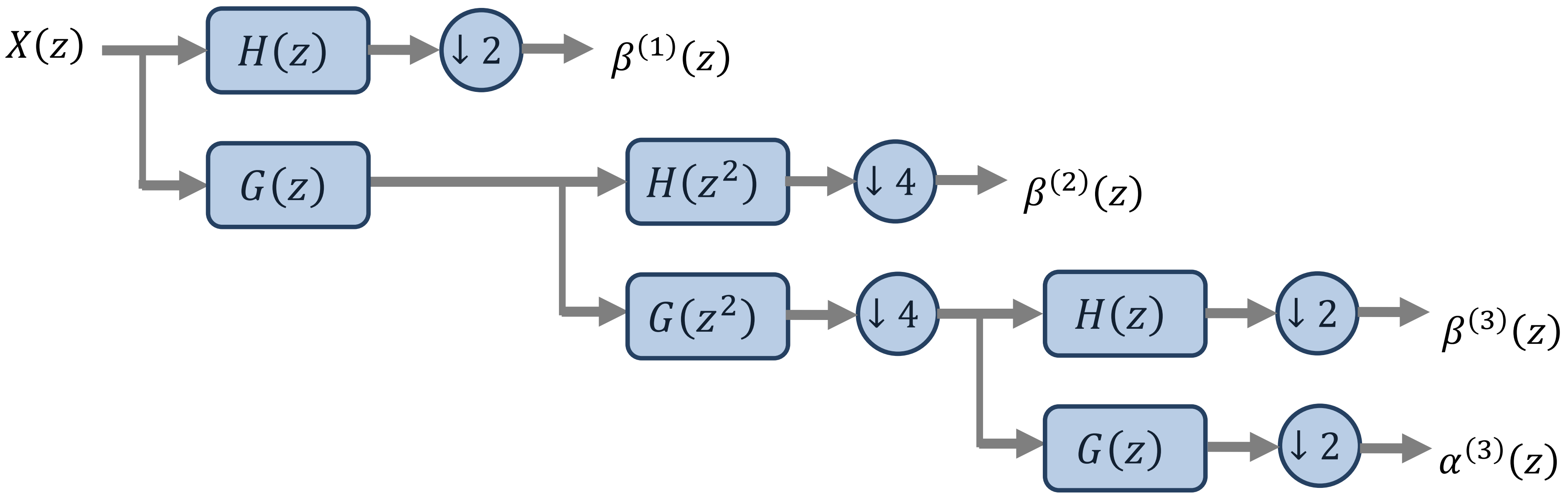

Wavelets / Sub-band coding

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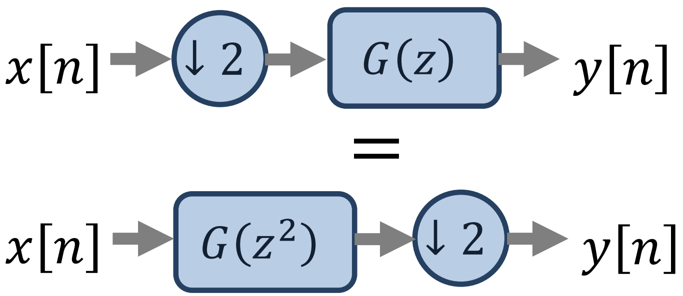

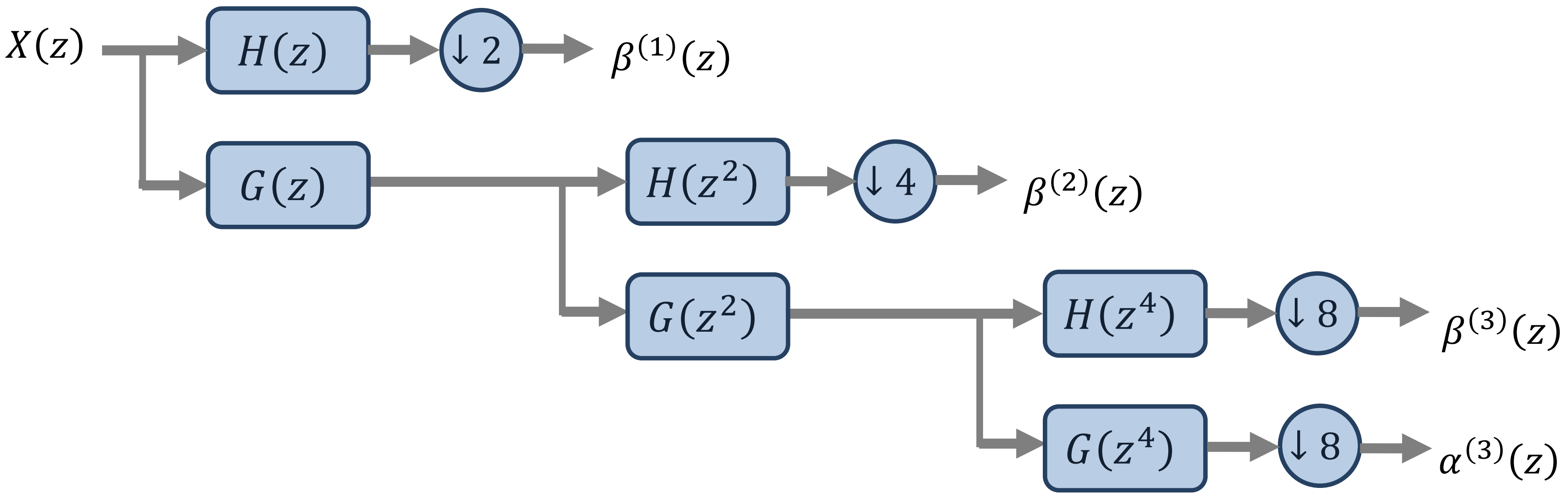

Wavelets / Sub-band coding

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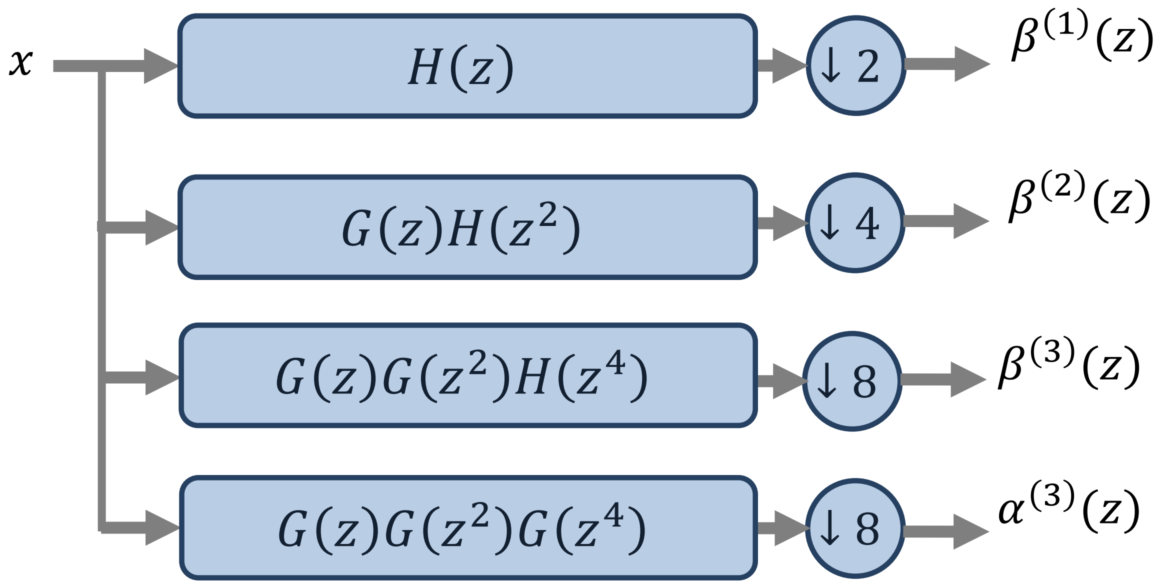

It's a filter bank!

Wavelets / Sub-band coding

- **How do you analyze this?**