

**Homework learning objectives:** By the end of this homework, you should be able to:

- Compute the DFT of a signal
- Solve for the magnitude and phase of a transfer function
- Obtain the output for a system

**Question #1:** (1 pts) How many hours did you spend on this homework?

**Question #2:** (16 pts) Consider the length-4 signal  $x[n]$  with values

$$[1 \quad 1 \quad 0 \quad 0]$$

Also, consider the length-4 signal  $y[n]$  with values

$$[1 \quad 0 \quad 0 \quad 1]$$

- Sketch the length-4 signals  $x[n]$  and  $y[n]$ .
- Sketch  $x[n] \otimes y[n]$ .
- Compute the length-4 discrete Fourier transform (DFT) of  $x[n]$  to get  $X[k]$ .
- Sketch the length-4 magnitude of the DFT  $|X[k]|$ .
- Sketch the length-4 phase of the DFT  $\angle X[k]$ .
- Compute the length-4 discrete Fourier transform (DFT) of  $y[n]$  to get  $Y[k]$ .
- Sketch the length-4 magnitude of the DFT  $|Y[k]|$ .
- Sketch the length-4 phase of the DFT  $\angle Y[k]$ .
- Explain the similarities and difference between  $|X[k]|$ ,  $|Y[k]|$ ,  $\angle X[k]$ , and  $\angle Y[k]$ .

**Question #3:** (18 pts) Consider the length-4 signal  $x[n]$  with values

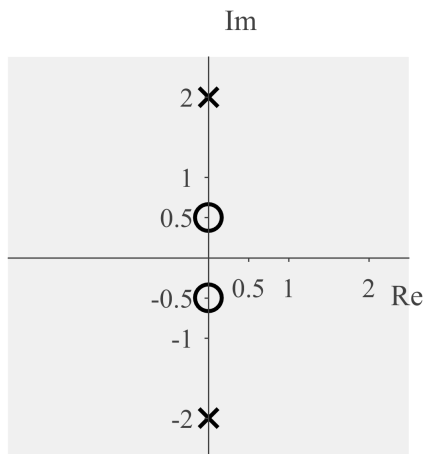
$$[1 \quad -1 \quad 1 \quad -1]$$

- Compute the length-4 discrete Fourier transform (DFT) of  $x[n]$  to get  $X[k]$ .
- Compute the length-2 DFT of the even values of  $x[n]$  (i.e.,  $x[2n]$ ) to get  $X_e[k]$ .
- Compute the length-2 DFT of the odd values of  $x[n]$  (i.e.,  $x[2n+1]$ ) to get  $X_o[k]$ .
- Show that  $X[k]$  is periodic with a fundamental period of 4.
- Show that  $X_e[k]$  and  $X_o[k]$  are periodic with a fundamental periods of 2.
- Show that for  $X_e[k]$  and  $X_o[k]$  above that  $X[k] = X_e[k] + e^{-j(2\pi/4)k} X_o[k]$  and is now periodic with a fundamental period of 4. This is the basic building block of the fast Fourier transform.

**Question #4:** (20 pts) Consider  $y[n] = x[n] - x[n - 1]$ , representing a causal LTI system.

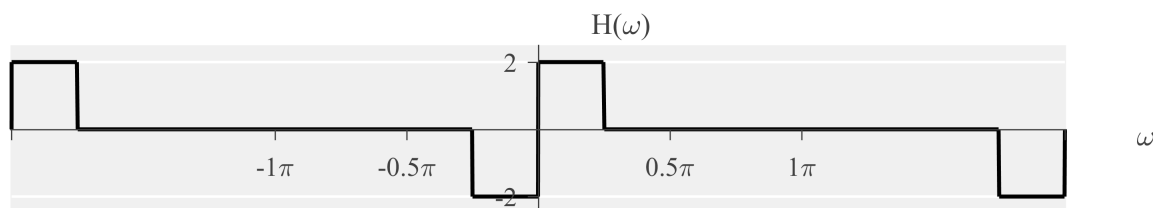
- Determine the systems poles and zeros. Sketch the pole-zero plot for the system.
- Compute the z-transform of the system impulse response  $H(z)$ .
- Sketch the [frequency] magnitude response  $|H(\omega)|$  for the system for  $4\pi \leq \omega \leq 4\pi$ .
- Compute the [frequency] [phase] response  $\angle H(\omega)$  for the system.
- Sketch the [frequency] phase response for the system for  $4\pi \leq \omega \leq 4\pi$ .
- Would you describe this filter as a low pass, bandpass, high pass, or all pass filter?
- Is this system invertible (i.e., is the inverse stable?)?
- Is this a minimum phase system?
- Compute the output of  $x[n] = 10$ .
- Compute the output of  $x[n] = 10 + 5 \cos(\pi/3n) + 13 \cos(\pi)$ .

**Question #5:** (14 pts) Consider the following pole-zero plot, representing a causal LTI system.



- Compute the DTFT of the system impulse response  $H(\omega)$ .
- Sketch the [frequency] magnitude response  $|H(\omega)|$  for the system for  $4\pi \leq \omega \leq 4\pi$ .
- Would you describe this filter as a low pass, bandpass, high pass, or all pass filter?
- Is this system invertible (i.e., is the inverse stable?)?
- Is this a minimum phase system?
- Compute the output of  $x[n] = 10$ .
- Compute the output of  $x[n] = 5 \cos(\pi/3n + \pi/2)$ .

**Question #6:** (16 pts) Consider the following DTFT frequency response  $H(\omega)$ , representing a causal LTI system.



- Sketch the [frequency] magnitude response for the system for  $2\pi \leq \omega \leq 2s\pi$ .
- Sketch the [frequency] phase response for the system for  $2\pi \leq \omega \leq 2\pi$ .
- Would you describe this filter as a low pass, bandpass, high pass, or all pass filter?
- Compute the output of  $x[n] = 5 \cos(\pi/3n + \pi/2)$ .
- Compute the output of  $x[n] = 10 + 5 \cos(\pi/3n + \pi/2) + 13 \cos(\pi)$ . Assume  $H(0) = 2$ .
- Is this system invertible (i.e., is the inverse stable?)?
- Sketch the magnitude response for  $g[n] = h[n] \cos(\pi n)$ . Would you describe this filter as a low pass, bandpass, high pass, or all pass filter?
- Sketch the magnitude response for  $g[n] = h[n] \cos((\pi/2)n)$ . Would you describe this filter as a low pass, bandpass, high pass, or all pass filter?

**Question #7:** (5 pts) *Project (EEE 5502 only)*

- For students in EEE 5502 whom are pursuing advanced degrees, I want to ensure you experience and learn about some modern signal processing trends and techniques. Therefore, your final project report will discuss a “hot” topic in signal processing and its relationship to our class. To identify a “hot” topic, identify one paper from the IEEE Signal Processing Magazine’s top 50 articles (<http://ieeexplore.ieee.org/xpl/topAccessedArticles.jsp?punumber=79>). This paper will serve as the foundation for your report. The IEEE Signal Processing Magazine publishes papers on hot signal processing topics that are geared toward an audience with a good, but not necessarily expert, knowledge of digital signal processing. As a result, it is a good place to start to come up with ideas for your report.

In the comments to your homework submission on Canvas, provide a link to your chosen paper from the IEEE Signal Processing Magazine’s top 50 articles.