EEE 5502 HW #11 Hudanyun Sheng

Question #1

I spent 8 hours.

Question #2

(a) $H_0(z)G_0(z)+H_1(z)G_1(z) = \frac{1}{\sqrt{2}}(1+z^{-1})\frac{1}{\sqrt{2}}(z^{+1}+1) + \frac{1}{\sqrt{2}}(1-z^{-1})\frac{1}{\sqrt{2}}(-z^{+1}+1)$

$= \frac{1}{2}(z+1+1+z^{-1}) + \frac{1}{2}(-z+1+1-z^{-1}) = \frac{1}{2}(z+2+z^{-1}-z+2-z^{-1})$

$= \frac{1}{2}(z-z+4+z^{-1}-z^{-1}) = 2$

$H_0(-z)G_0(z)+H_1(-z)G_1(z) = \frac{1}{\sqrt{2}}(1-z^{-1})\frac{1}{\sqrt{2}}(z^{+1}+1) + \frac{1}{\sqrt{2}}(1+z^{-1})\frac{1}{\sqrt{2}}(-z^{+1}+1)$

$\cancel{= \frac{1}{2}(z+1+z^2+z-z+1-z^2-z) = 1}$

$= \frac{1}{2}(z+1-1-z^{-1}-z+1-1+z^{-1}) = 0$

$\therefore$ The filters satisfy the alias canceling filter bank conditions.

(b) $H_0(z)G_0(z)+H_1(z)G_1(z) = \frac{1}{\sqrt{2}}(1+z^{-1})\frac{1}{\sqrt{2}}(1+z^{-1}) + \frac{1}{\sqrt{2}}(1-z^{-1})\frac{1}{\sqrt{2}}(-1+z^{-1})$

$= \frac{1}{2}(1+z^{-1}+z^{-1}+z^{-2}) + \frac{1}{2}(-1+z^{-1}+z^{-1}-z^{-2})$

$= 2z^{-1} \neq 2$

$H_0(-z)G_0(z)+H_1(-z)G_1(z) = \frac{1}{\sqrt{2}}(1-z^{-1})\frac{1}{\sqrt{2}}(1+z^{-1}) + \frac{1}{\sqrt{2}}(1+z^{-1})\frac{1}{\sqrt{2}}(-1+z^{-1})$

$= \frac{1}{2}(1+z^{-1}-z^{-1}-z^{-2}-1+z^{-1}-z^{-1}+z^{-2})$

$= 0$

$\therefore$ The filters don't satisfy the alias canceling filter bank conditions.

(c) $X_0(z) = X(z)H_0(z) = (1+z^{-1}-z^{-2}-z^{-3})\frac{1}{\sqrt{2}}(1+z^{-1}) = \frac{1}{\sqrt{2}}(1+z^{-1}+z^{-1}+z^{-2}-z^{-2}-z^{-3}-z^{-3}-z^{-4})$

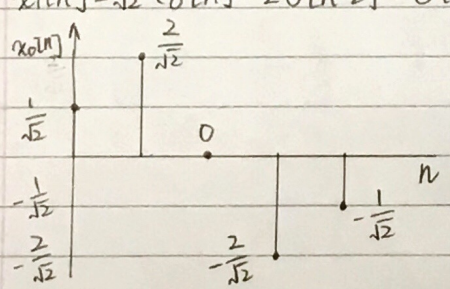
$= \frac{1}{\sqrt{2}}(1+2z^{-1}-2z^{-3}-z^{-4})$

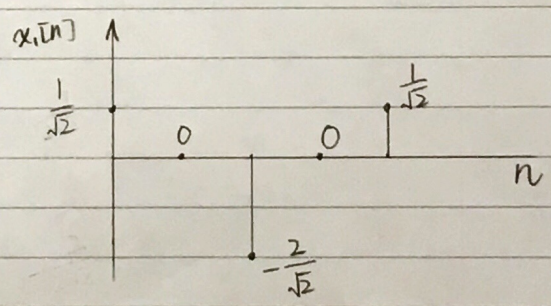
$X_1(z) = X(z)H_1(z) = (1+z^{-1}-z^{-2}-z^{-3})\frac{1}{\sqrt{2}}(1-z^{-1}) = \frac{1}{\sqrt{2}}(1-z^{-1}+z^{-1}-z^{-2}-z^{-2}+z^{-3}-z^{-3}+z^{-4})$

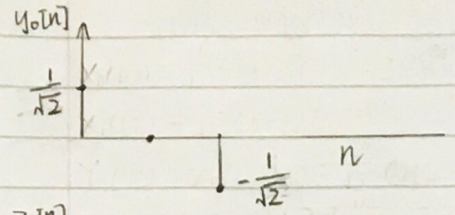
$= \frac{1}{\sqrt{2}}(1-2z^{-2}+z^{-4})$

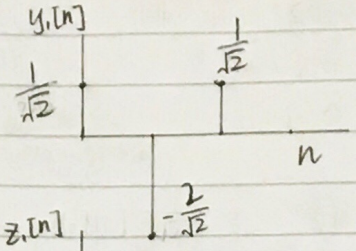
$x_0[n] = \frac{1}{\sqrt{2}}(\delta[n]+2\delta[n-1]-2\delta[n-3]-\delta[n-4])$

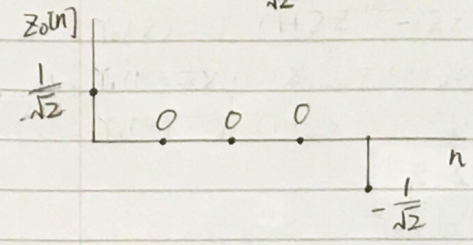
$x_1[n] = \frac{1}{\sqrt{2}}(\delta[n]-2\delta[n-2]+\delta[n-4])$

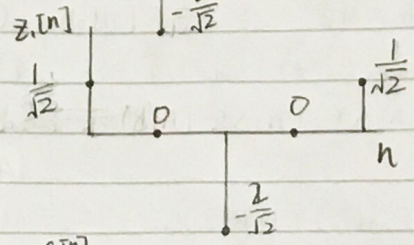


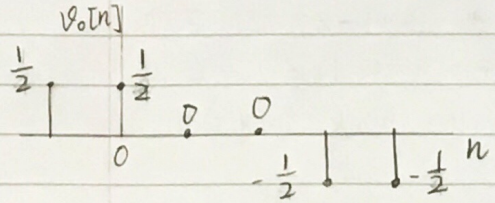


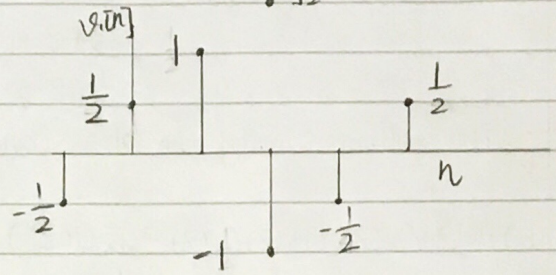


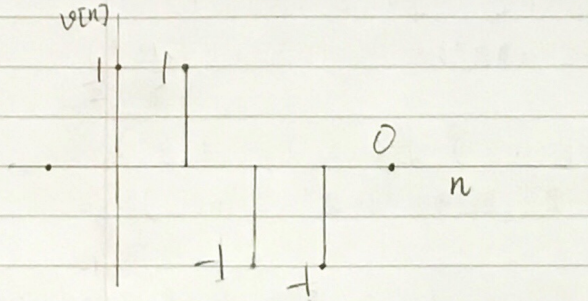












$Z_0(z) = \frac{1}{\sqrt{2}}(1 - z^{-4})$ $\quad V_0(z) = \frac{1}{\sqrt{2}}(1-z^{-4})\frac{1}{\sqrt{2}}(z+1) = \frac{1}{2}(z+1-z^{-3}-z^{-4})$

$\therefore v_0[n] = \frac{1}{2}[\delta[n] - \delta[n-3] - \delta[n-4] + \delta[n+1]]$

$Z_1(z) = \frac{1}{\sqrt{2}}(1-2z^{-2}+z^{-4})$ $\quad V_1(z) = \frac{1}{\sqrt{2}}(1-2z^{-2}+z^{-4})\frac{1}{\sqrt{2}}(-z+1) = \frac{1}{2}(z^{-4}-z^{-3}-2z^{-2}$
$+2z^{-1}+1-z)$

$\therefore v_1[n] = \frac{1}{2}[\delta[n+1] + \delta[n] + 2\delta[n-1] - 2\delta[n-2] - \delta[n-3] + \delta[n-4]]$

Question #3

(a) $H_0(z^{-1}) = \frac{1}{2}(1+z-z^2-z^3) = G_0(z)$

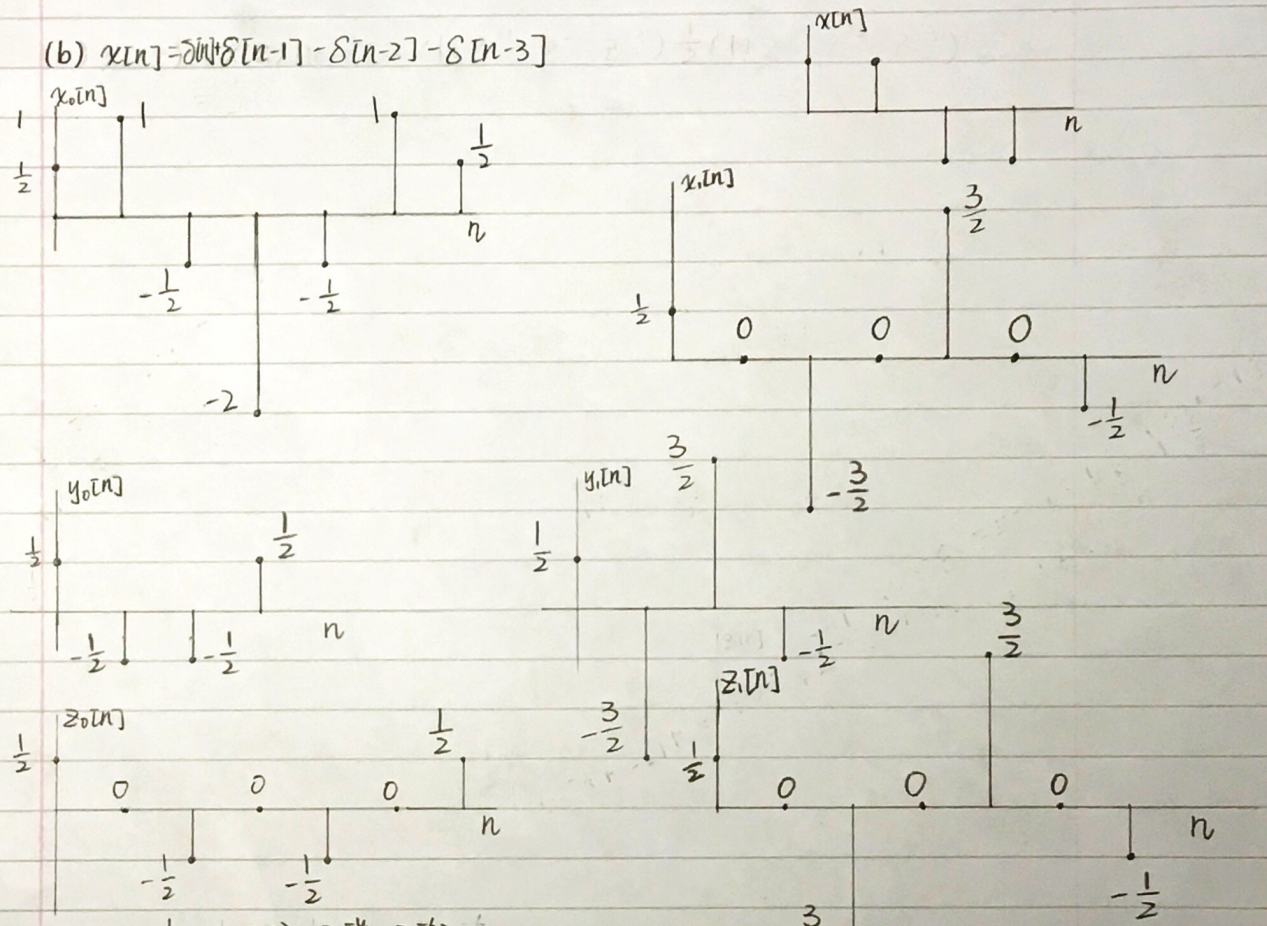
$$H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = \frac{1}{4}(1+z^{-1}-z^{-2}-z^{-3})(1+z-z^2-z^3)$$
$$+\frac{1}{4}(1-z-z^2-z^3)(1-z^{-1}-z^{-2}-z^{-3})$$
$$= 2+\frac{1}{2}z^{-1}+z-\frac{1}{2}z^{-3}-\frac{1}{2}z^{-2} \neq 2$$

$H_1(z^{-1}) = \frac{1}{2}(1-z-z^2+z^3) = G_1(z)$

$$H_1(z)H_1(z^{-1}) + H_1(-z)H_1(-z^{-1}) = \frac{1}{4}(1-z^{-1}-z^{-2}+z^{-3})(1-z-z^2+z^3)$$
$$+\frac{1}{4}(1+z^{-1}-z^{-2}-z^{-3})(1+z-z^2-z^3)$$
$$= 2-z^{-2}-z^2 \neq 2$$

$\therefore$ The filters do not satisfy the orthogonal filter bank conditions.

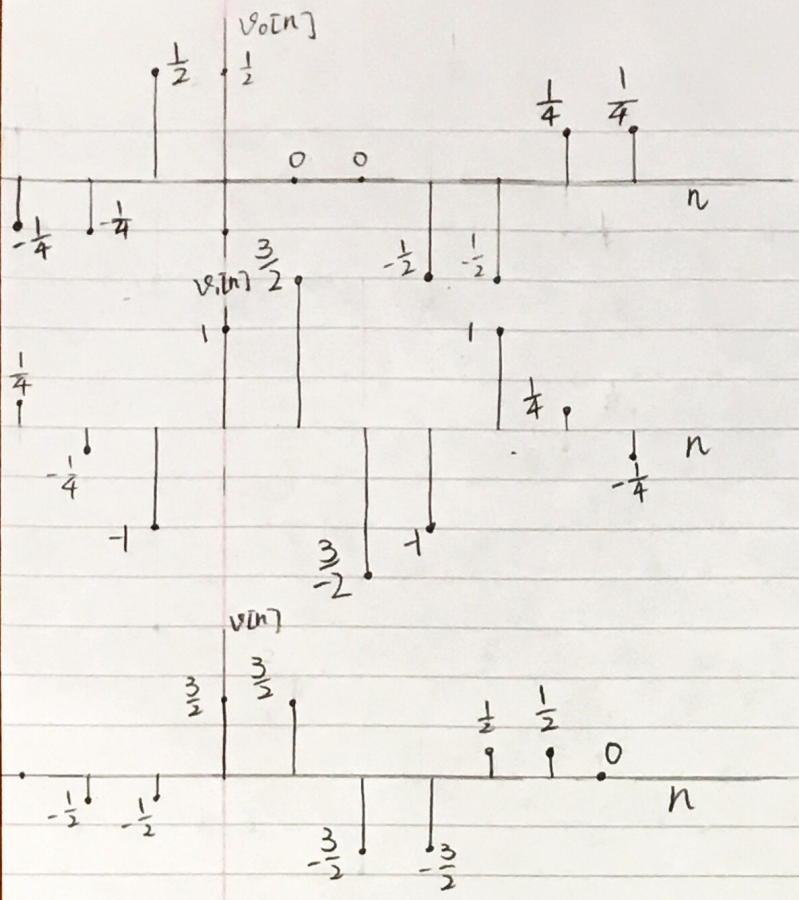
(b) $x[n] = \delta[n]+\delta[n-1]-\delta[n-2]-\delta[n-3]$



$Z_0(z) = \frac{1}{2}(1-z^{-2}-z^{-4}+z^{-6})$

$V_0(z) = G_0(z)Z_0(z) = \frac{1}{4}(z^{-6}+z^{-5}-2z^{-4}-2z^{-3}+2+2z-z^2-z^3)$

$\therefore v_0[n] = \frac{1}{4}(\delta[n-6]+\delta[n-5]-2\delta[n-4]+2\delta[n-3]+2\delta[n]+2\delta[n+1]-\delta[n+2]-\delta[n+3])$



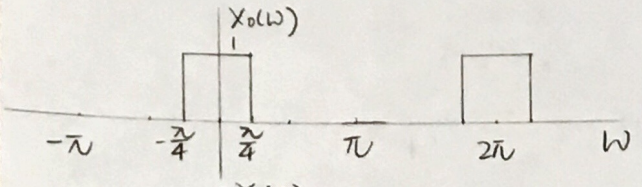
$$Z_1(z) = \frac{1}{2}(1 - 3z^{-2} + 3z^{-4} - z^{-6})$$

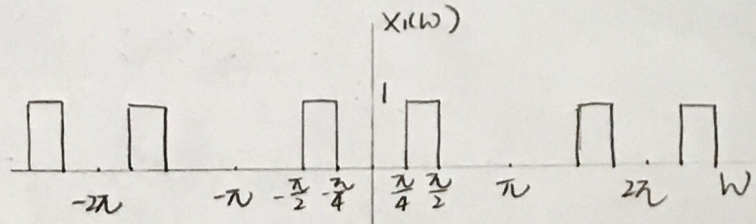
$$V_1(z) = Z_1(z)G_1(z)$$

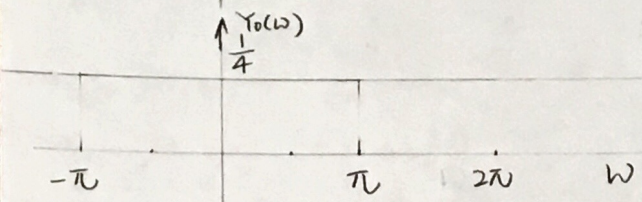
$$= \frac{1}{4}(z^3 - z^2 - 4z + 4 + 6z^{-1} - 6z^{-2} - 4z^{-3} + 4z^{-4} + z^{-5} - z^{-6})$$

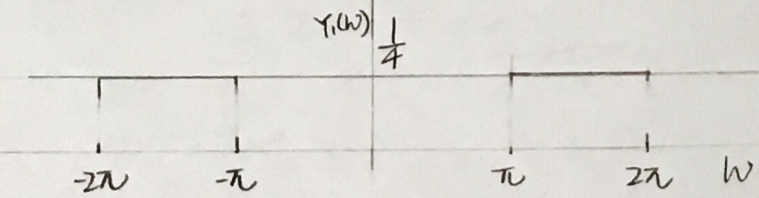
Question #4

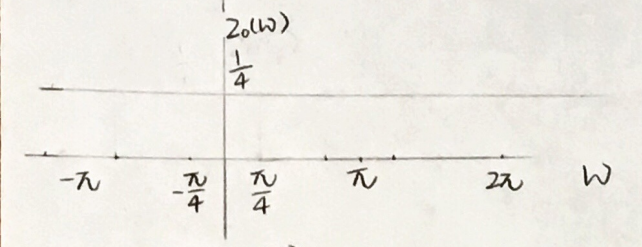
$X(z)=1 \Rightarrow x[n]=\delta[n] \Rightarrow X(\omega)=1$

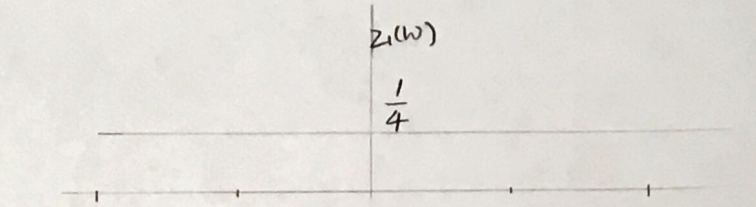


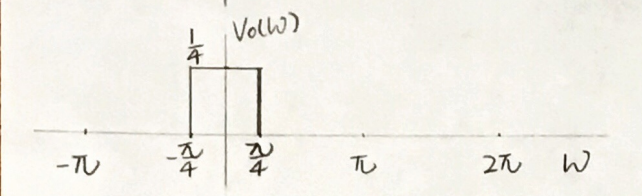


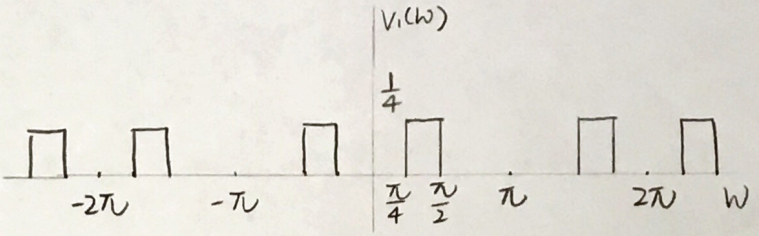


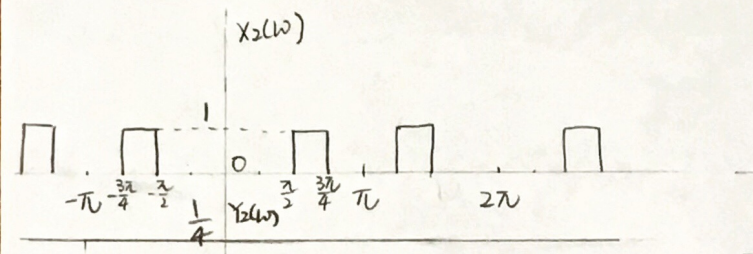


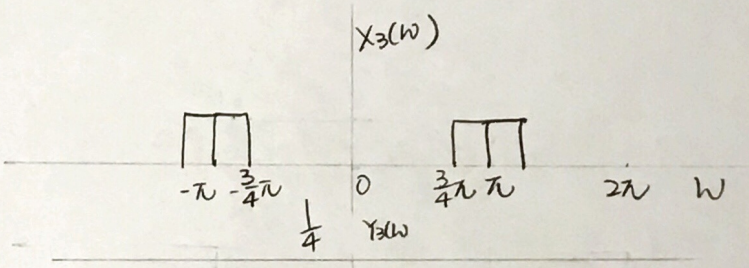


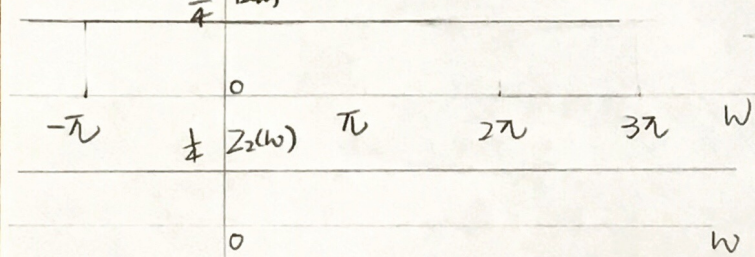


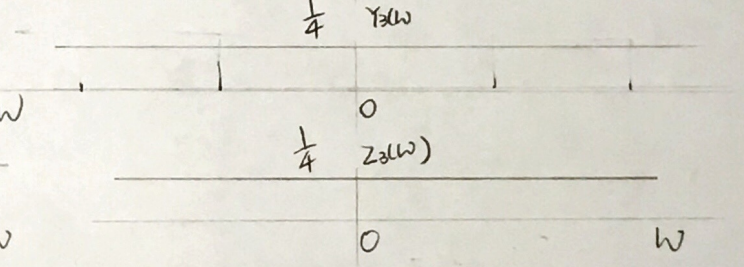


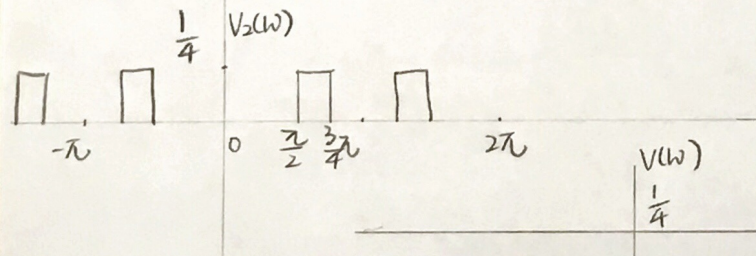


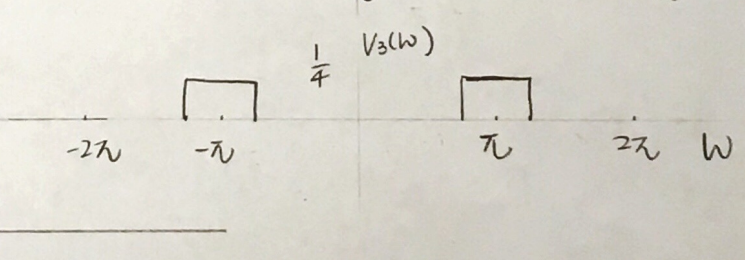


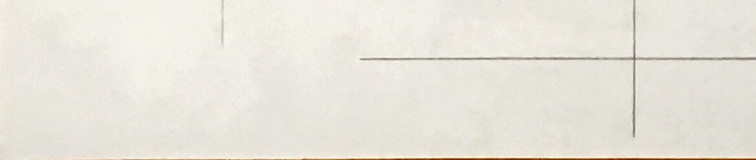


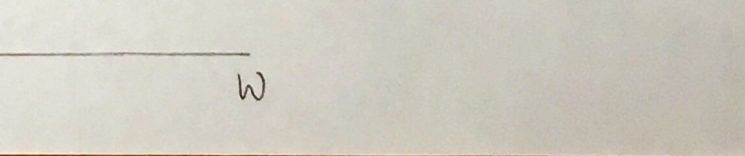


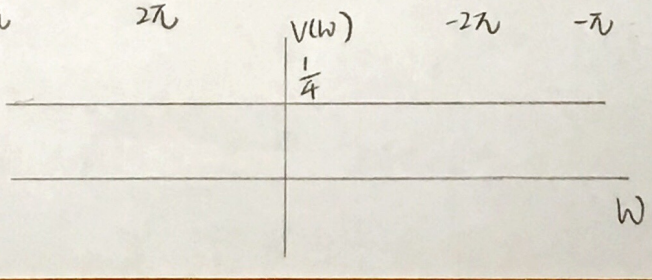








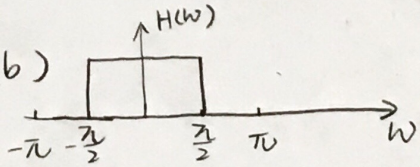


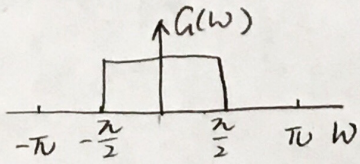


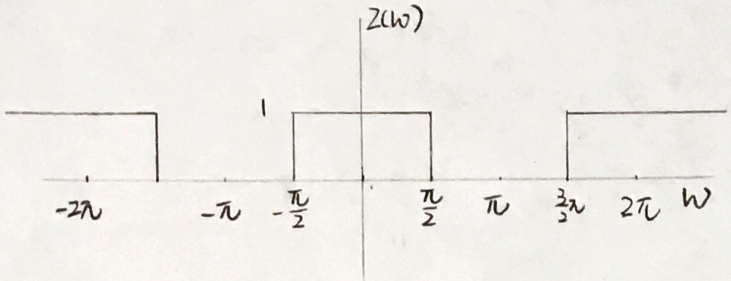
Question # 5

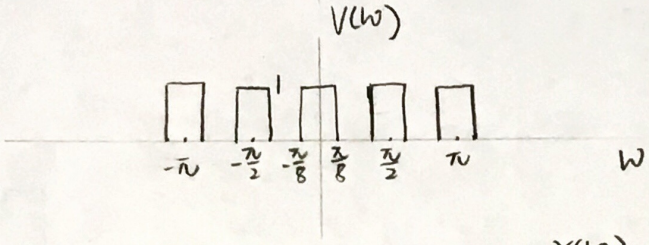
(a) $X(z) \rightarrow \boxed{H(z)G(z)} \xrightarrow{Z(z)} \uparrow 4 \xrightarrow{V(z)} \downarrow 3 \rightarrow Y(z)$

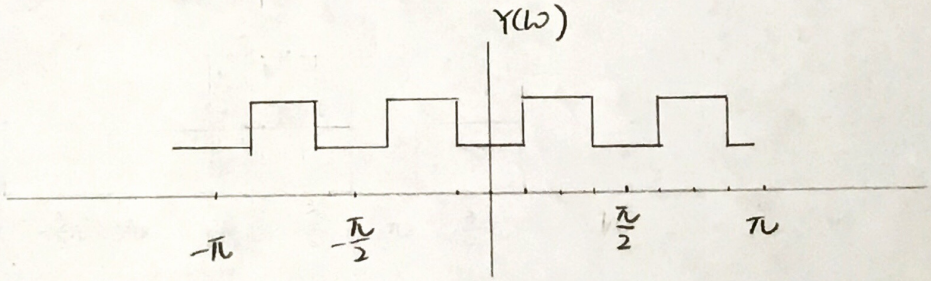
(b)



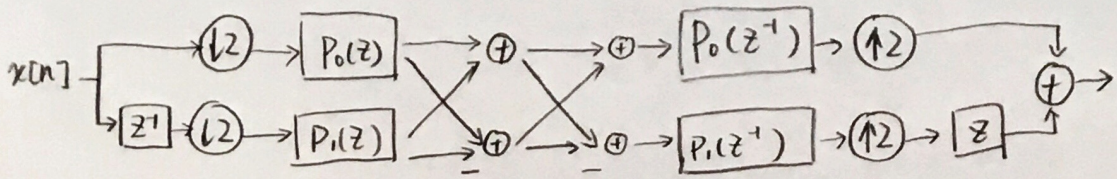








Question #6



$H_0(z) = P_0(z^2) + z^{-1}P_1(z^2)$

$H_1(z) = P_0(z^2) - z^{-1}P_1(z^2)$

$P_0(z^2) = \frac{1}{2}[H_0(z) + H_1(z)] = \frac{1}{2}\frac{1}{2}(2 - 2z^{-2})$

$= \frac{1}{2}(1 - z^{-2})$

$\therefore P_0(z) = \frac{1}{2}(1 - z^{-1})$

$H_0(z) - H_1(z) = 2z^{-1}P_1(z^2) = \frac{1}{2}(2z^{-1} - 2z^{-3})$

$\therefore P_1(z^2) = \frac{1}{2}(z^{-2} - z^{-4})$

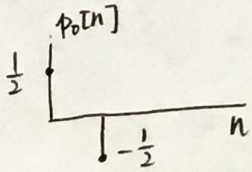
$\therefore P_1(z) = \frac{1}{2}(z^{-1} - z^{-2})$

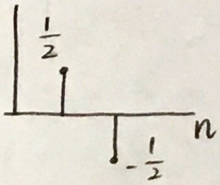
$p_0[n] = \frac{1}{2}(\delta[n] - \delta[n-1])$

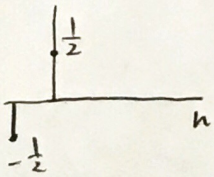
$p_1[n] = \frac{1}{2}(\delta[n-1] - \delta[n-2])$

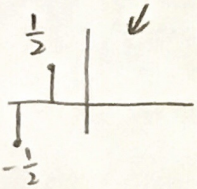
$P_0(z^{-1}) = \frac{1}{2}(1 - z) \rightarrow$

$P_1(z^{-1}) = \frac{1}{2}(z - z^2)$

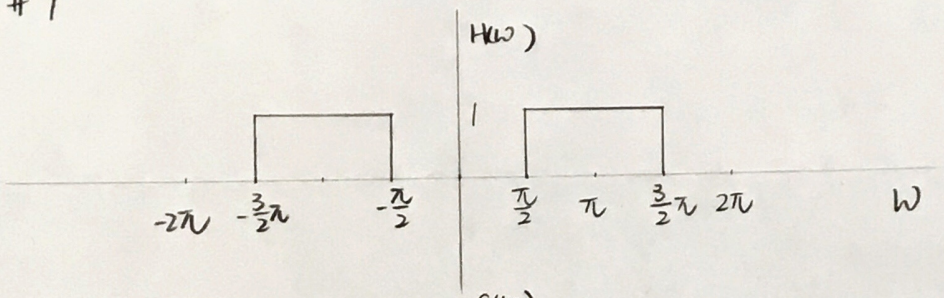


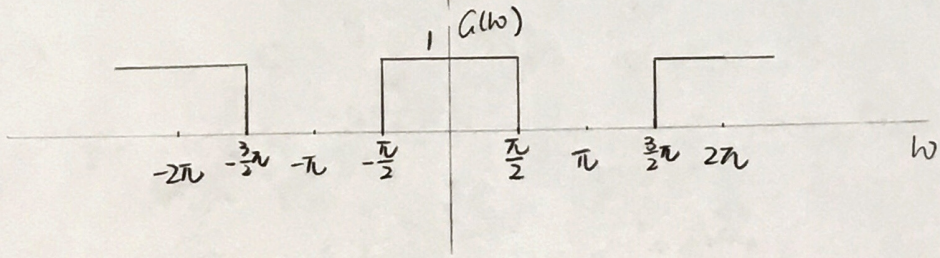


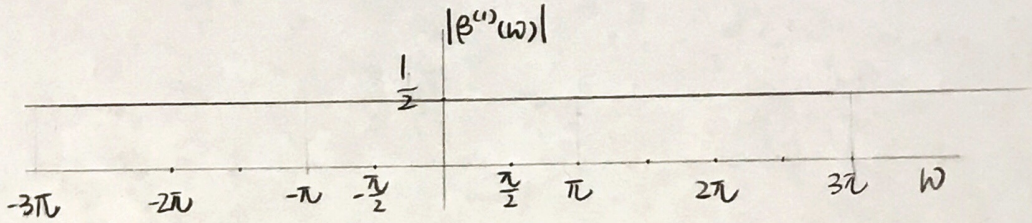


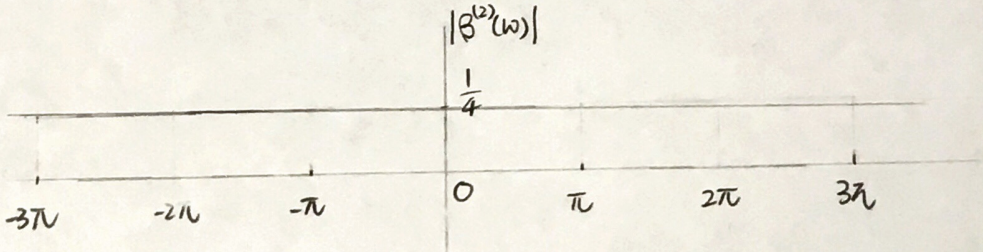


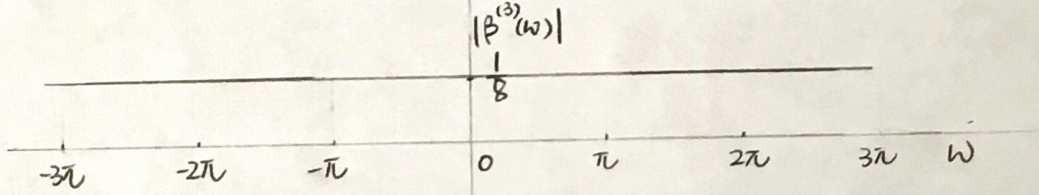
Question #7

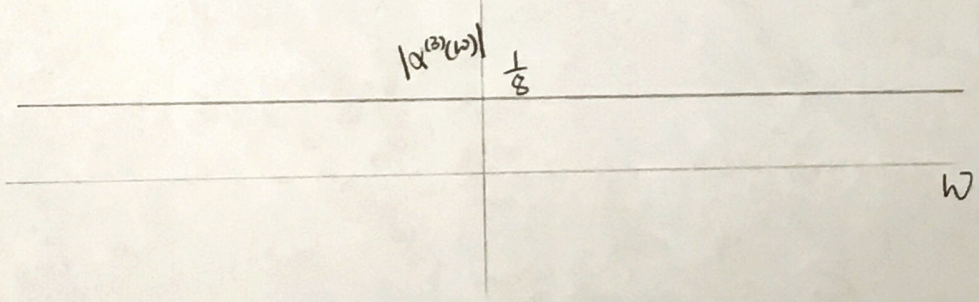

H(ω)
1
-2π
-3/2π
-π/2
π/2
π
3/2π
2π
ω


G(ω)
1
-2π
-3/2π
-π
-π/2
π/2
π
3/2π
2π
ω


|β(1)(ω)|
1/2
-3π
-2π
-π
-π/2
π/2
π
2π
3π
ω


|β(2)(ω)|
1/4
-3π
-2π
-π
0
π
2π
3π


|β(3)(ω)|
1/8
-3π
-2π
-π
0
π
2π
3π
ω


|α(3)(ω)|
1/8
ω