EEE 5502 HW #08 Hudanyun Sheng

Question #1

I spent 10 hours.

Question #2

(a) $h[n]$ has length of 11 (for non-zeros) and is symmetric at $n=5$

Assume $h'[n] = h[n+5]$, where $h'[n]$ is symmetric at $n=0$, i.e. $h'[n] = h'[-n]$

Then $h[n] = h'[n-5]$ $\quad \therefore H(\omega) = |H(\omega)| e^{-j\omega 5}$

$\therefore \angle H(\omega) = -5\omega$

(b) Group delay $= \frac{d\angle H(\omega)}{d\omega} = -5$

(c) $\angle -H(\omega) = \angle(-1) + \angle H(\omega) = \pi - 5\omega$

(d) $H(\omega)(1+e^{-j\omega})$

$\angle[H(\omega)(1+e^{-j\omega})] = \angle H(\omega) + \angle(1+e^{-j\omega}) = -5\omega + \arctan \frac{-\sin\omega}{1+\cos\omega}$

$= -5\omega + \arctan \frac{-2\sin\frac{\omega}{2}\cos\frac{\omega}{2}}{2\cos^2\frac{\omega}{2}} = -5\omega + \arctan(-\tan\frac{\omega}{2}) = -5\omega - \frac{\omega}{2} = -\frac{11}{2}\omega$

Question #3

(a) Zeros: $z = \pm 0.5j$, $z = \pm 2j$. Poles: $z = 0$ (x4)

$X(z) = \frac{(z+0.5j)(z-0.5j)(z+2j)(z-2j)}{z^4} = \frac{(z^2+0.25)(z^2+4)}{z^4} = (1+0.25z^{-2})(1+4z^{-2})$

$X(\omega) = (1+0.25e^{-2j\omega})(1+4e^{-2j\omega}) = 1+4.25e^{-2j\omega} + e^{-4j\omega}$

$\angle X(\omega) = \mathrm{atan}[(-4.25\sin 2\omega - \sin 4\omega)/(1+4.25\cos 2\omega + \cos 4\omega)]$

$= \mathrm{atan} \frac{-4.25\sin 2\omega - 2\sin 2\omega \cos 2\omega}{4.25\cos 2\omega + 2\cos^2 2\omega} = \mathrm{atan} \frac{-\sin 2\omega(4.25+2\cos 2\omega)}{\cos 2\omega(4.25+2\cos 2\omega)} = \mathrm{atan}(-\tan 2\omega)$

$= -2\omega$

(b) group delay $= -2$

(c) $\frac{1}{2}[X(\omega-\frac{\pi}{2}) + X(\omega-\frac{\pi}{2})] = X(\omega-\frac{\pi}{2}) = 1+4.25e^{-2j(\omega-\frac{\pi}{2})} + e^{-4j(\omega-\frac{\pi}{2})} = 1-4.25e^{-2j\omega} + e^{-4j\omega}$

$\angle X(\omega-\frac{\pi}{2}) = \mathrm{atan}[(4.25\sin 2\omega - \sin 4\omega)/(1-4.25\cos 2\omega + \cos 4\omega)]$

$= \mathrm{atan} \frac{4.25\sin 2\omega - 2\sin 2\omega\cos 2\omega}{2\cos^2 2\omega - 4.25\cos 2\omega} = \mathrm{atan} \frac{\sin 2\omega(4.25-2\cos 2\omega)}{\cos 2\omega(2\cos 2\omega - 4.25)} = -2\omega$

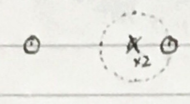
Question #4

(a) $y[n] = x[n] + 2x[n-1] - 3x[n-2]$

(b) $Y(z) = X(z) + 2X(z)z^{-1} - 3X(z)z^{-2}$. $H(z) = \cancel{z}\, 1 + 2z^{-1} - 3z^{-2}$

(c) $Y(z) = X(z)(1 + 2z^{-1} - 3z^{-2})$

$H(z) = 1 + 2z^{-1} - 3z^{-2}$ Zero: $z = 1$ or -3. Pole: $z = 0$ (x2)



(d) $y[n] = a_3(a_1x[n] + a_2x[n-1]) + a_4(a_1x[n-1] + a_2x[n-2])$

$= a_1a_3x[n] + (a_2a_3 + a_1a_4)x[n-1] + a_2a_4x[n-2]$

$a_1a_3 = 1$, $a_2a_3 + a_1a_4 = 2$, $a_2a_4 = -3$

$a_1a_2a_3 + a_1^2a_4 = 2a_1$ $\therefore a_2 + a_1^2a_4 = 2a_1$ $\therefore a_2a_4 + a_1^2a_4^2 = 2a_1a_4$ $\therefore \cancel{a_1a_4 =}$

$\therefore a_1^2a_4^2 - 2a_1a_4 - 3 = 0$ $\therefore a_1a_4 = 3$ or -1

If $a_1a_4 = 3$, then $a_2a_3 = -1$, $a_1 = -1, a_2 = -1, a_3 = -1, a_4 = -3$ or $a_1 = 1, a_2 = -1, a_3 = 1, a_4 = 3$

If $a_1a_4 = -1$, then $a_2a_3 = 3$ $a_1 = 1$ $a_2 = 3$, $a_3 = 1$, $a_4 = -1$ or $a_1 = -1, a_2 = 3, a_3 = -1, a_4 = 1$

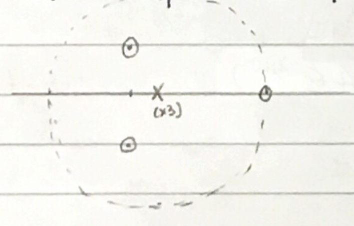
Question #5

(a) $y[n] = -4x[n] + 2x[n-1] + x[n-2] + x[n-3]$

$Y(z) = -4X(z) + 2X(z)z^{-1} + X(z)z^{-2} + X(z)z^{-3}$

(b) $H(z) = -4 + 2z^{-1} + z^{-2} + z^{-3}$ $y[n] = -4x[n] + 2x[n-1] + x[n-2] + x[n-3]$

(c) Zeros: $z = 1$, $z = -\frac{1}{4} + \frac{\sqrt{3}}{4}j$, $z = -\frac{1}{4} - \frac{\sqrt{3}}{4}j$. Poles: $z = 0$ (x3)



(d) The system is stable.

(e) $y[n] = b_1x[n] + b_2x[n-1] + b_3x[n-2] + b_4x[n-3]$

$b_1 = -4$, $b_2 = 2$, $b_3 = 1$, $b_4 = 1$

Question #6

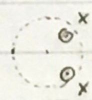
(a) $y[n] = x[n] - x[n-1] + 0.5\,x[n-2] + 2y[n-1] - 2y[n-2]$

(b) $Y(z) = X(z) - X(z)z^{-1} + 0.5X(z)z^{-2} + 2Y(z)z^{-1} - 2Y(z)z^{-2}$

$Y(z)(1 - 2z^{-1} + 2z^{-2}) = X(z)(1 - z^{-1} + 0.5z^{-2})$

$H(z) = \dfrac{1 - z^{-1} + 0.5z^{-2}}{1 - 2z^{-1} + 2z^{-2}}$

(c) Poles: $z = 1 \pm j$. Zeros: $\frac{1}{2} \pm \frac{1}{2}j$



(d) It is a all pass filter

(e) Introduce a new signal at top center node: $t[n]$

$y[n] = a_3 t[n] + a_4 t[n-1] + a_5 t[n-2]$

$t[n] = x[n] + a_1 t[n-1] + a_2 t[n-2]$

$Y(z) = a_3 T(z) + a_4 T(z) z^{-1} + a_5 T(z) z^{-2}$

$= (a_3 + a_4 z^{-1} + a_5 z^{-2}) T(z)$

$T(z) = X(z) + a_1 T(z) z^{-1} + a_2 T(z) z^{-2}$

$X(z) = (1 - a_1 z^{-1} - a_2 z^{-2}) T(z)$

$\dfrac{Y(z)}{X(z)} = \dfrac{a_3 + a_4 z^{-1} + a_5}{1 - a_1 z^{-1} - a_2 z^{-2}}$

Compared with the result in part (c)

$a_1 = 2,\ a_2 = -2,\ a_3 = 1,\ a_4 = -1,\ a_5 = 0.5$

Question #7

(a) $y[n] = \frac{1}{2} v_1[n] + u_1[n-1]$ $\qquad Y(z) = \frac{1}{2} V_1(z) + U_1(z) z^{-1}$

$v_1[n] = x[n] - \frac{1}{2} u_1[n-1]$ $\qquad V_1(z) = X(z) - \frac{1}{2} U_1(z) z^{-1} \Rightarrow X(z) = V_1(z) + \frac{1}{2} U_1(z) z^{-1}$

$v_2[n] = v_1[n] - 2 v_2[n-1]$ $\qquad V_2(z) = V_1(z) - 2V_2(z) z^{-1} \Rightarrow V_1(z) = V_2(z) + 2V_2(z) z^{-1}$

$u_1[n] = 2v_2[n] + v_2[n-1]$ $\qquad U_1(z) = 2V_2(z) + V_2(z) z^{-1}$

$\therefore Y(z) = \frac{1}{2} V_2(z) + V_2(z) z^{-1} + 2V_2(z) z^{-1} + V_2(z) z^{-2} = V_2(z)(\frac{1}{2} + 3z^{-1} + z^{-2})$

$X(z) = V_2(z) + 2V_2(z) z^{-1} + V_2(z) z^{-1} + \frac{1}{2} V_2(z) z^{-2} = V_2(z)(1 + 3z^{-1} + \frac{1}{2} z^{-2})$

$\therefore Y(z)/X(z) = (\frac{1}{2} + 3z^{-1} + z^{-2})/(1 + 3z^{-1} + \frac{1}{2} z^{-2})$

(b) $\cancel{V_2(z) = V_1(z) - 2V_2(z) z^{-1}}$

$X(z) = V_2(z)(1 + 3z^{-1} + \frac{1}{2} z^{-2})$

$V_2(z)/X(z) = 1/(1 + 3z^{-1} + \frac{1}{2} z^{-2})$

(c) $Y(z) = V_2(z)(\frac{1}{2} + 3z^{-1} + z^{-2})$

$Y(z)/V_2(z) = \frac{1}{2}(1 + 6z^{-1} + 2z^{-2})$