EEL5840 Fundamental Machine Learning Homework 4

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Question 1

It is attached in the other file.

Question 2

It is attached in the other file.

Question 3

The code is attached at the end of this file.

Suppose the true mean of the Gaussian distribution is 2 (μ = 2). And we are drawing random samples from this Gaussian distribution, with a Gaussian prior distribution on the mean parameters. Assume the variance for the Gaussian likelihood and Gaussian prior is fixed and known. And our goal is to estimate the mean. Since we are to explore how do the initial values of the prior mean, prior variance and likelihood variance interact to affect the final estimate of mean, below shows my experiment results, for each experiment I draw 10 samples.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Number of draws | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| mu\_0 = 4 | sigma = 0.5, sigma\_0 = 0.5 | sample | 1.8725 | 2.0822 | 2.3739 | 1.8635 | 2.7882 | 1.7595 | 2.1638 | 2.3324 | 2.0426 | 2.4405 |
| true probability | 0.3993 | 0.5653 | 0.7727 | 0.3924 | 0.9425 | 0.3153 | 0.6284 | 0.7469 | 0.5339 | 0.8108 |
| ML probability | 0.5000 | 0.5831 | 0.7015 | 0.3560 | 0.8818 | 0.2335 | 0.5277 | 0.6390 | 0.4212 | 0.7044 |
| MAP probability | 0.0167 | 0.2269 | 0.5524 | 0.2420 | 0.8770 | 0.1975 | 0.4939 | 0.6290 | 0.4051 | 0.7096 |
| sigma = 2, sigma\_0 = 0.5 | sample | 2.6464 | 0.4317 | -1.6107 | 5.7172 | 0.7909 | 2.2067 | 3.1263 | 2.2272 | 0.1905 | 1.0646 |
| true probability | 0.6267 | 0.2165 | 0.0355 | 0.9685 | 0.2727 | 0.5412 | 0.7133 | 0.5452 | 0.1828 | 0.3200 |
| ML probability | 0.5000 | 0.2899 | 0.1469 | 0.9750 | 0.3438 | 0.6006 | 0.7299 | 0.5567 | 0.2182 | 0.3793 |
| MAP probability | 0.2621 | 0.0527 | 0.0077 | 0.9117 | 0.1593 | 0.4203 | 0.6236 | 0.4634 | 0.1443 | 0.2811 |
| sigma = 0.5, sigma\_0 = 2 | sample | 2.2471 | 1.2584 | 1.4899 | 1.7765 | 2.0548 | 2.5644 | 1.8550 | 2.6308 | 2.2377 | 2.5871 |
| true probability | 0.6894 | 0.0690 | 0.1538 | 0.3274 | 0.5437 | 0.8705 | 0.3859 | 0.8964 | 0.6828 | 0.8798 |
| ML probability | 0.5000 | 0.1614 | 0.3630 | 0.5663 | 0.7187 | 0.9085 | 0.4703 | 0.9019 | 0.6736 | 0.8494 |
| MAP probability | 0.4183 | 0.0803 | 0.2583 | 0.5084 | 0.7184 | 0.9359 | 0.5234 | 0.9382 | 0.7558 | 0.9077 |
| sigma = 2, sigma\_0 = 2 | sample | 1.3771 | 0.8600 | -0.0515 | 0.1825 | 1.5802 | -1.3977 | 3.2152 | 1.7644 | 3.3983 | 2.5393 |
| true probability | 0.3777 | 0.2843 | 0.1525 | 0.1817 | 0.4169 | 0.0447 | 0.7283 | 0.4531 | 0.7578 | 0.6063 |
| ML probability | 0.5000 | 0.4486 | 0.3483 | 0.4189 | 0.6537 | 0.1810 | 0.8841 | 0.6597 | 0.8626 | 0.7245 |
| MAP probability | 0.2560 | 0.3009 | 0.2340 | 0.3221 | 0.6127 | 0.1307 | 0.8823 | 0.6737 | 0.8912 | 0.7783 |
| mu\_0 = 2 | sigma = 0.5, sigma\_0 = 0.5 | sample | 1.8699 | 2.3001 | 2.2970 | 0.9070 | 1.3365 | 1.2795 | 2.2009 | 2.7351 | 1.8366 | 2.4062 |
| true probability | 0.3973 | 0.7258 | 0.7237 | 0.0144 | 0.0922 | 0.0748 | 0.6561 | 0.9292 | 0.3719 | 0.7917 |
| ML probability | 0.5000 | 0.6665 | 0.6113 | 0.0305 | 0.2086 | 0.2204 | 0.8209 | 0.9590 | 0.4793 | 0.8361 |
| MAP probability | 0.4482 | 0.7191 | 0.6733 | 0.0152 | 0.1249 | 0.1292 | 0.7771 | 0.9649 | 0.5004 | 0.8668 |
| sigma = 2, sigma\_0 = 0.5 | sample | 3.0911 | -0.1033 | 2.7949 | 0.4962 | 5.0325 | 1.9349 | 5.2720 | 1.1499 | 3.1789 | 1.8744 |
| true probability | 0.7073 | 0.1465 | 0.6545 | 0.2261 | 0.9353 | 0.4870 | 0.9491 | 0.3354 | 0.7222 | 0.4750 |
| ML probability | 0.5000 | 0.2123 | 0.6677 | 0.2957 | 0.9170 | 0.4457 | 0.9055 | 0.2565 | 0.6256 | 0.3825 |
| MAP probability | 0.6962 | 0.1460 | 0.6556 | 0.2370 | 0.9364 | 0.4832 | 0.9426 | 0.3049 | 0.6833 | 0.4236 |
| sigma = 0.5, sigma\_0 = 2 | sample | 2.4077 | 2.3994 | 2.0601 | 2.2856 | 2.2064 | 1.5065 | 2.3798 | 1.6714 | 1.6980 | 2.0885 |
| true probability | 0.7926 | 0.7878 | 0.5478 | 0.7161 | 0.6601 | 0.1618 | 0.7762 | 0.2555 | 0.2729 | 0.5702 |
| ML probability | 0.5000 | 0.4967 | 0.3235 | 0.4979 | 0.4479 | 0.1011 | 0.6568 | 0.1877 | 0.2295 | 0.5145 |
| MAP probability | 0.5191 | 0.5022 | 0.2855 | 0.4714 | 0.4218 | 0.0661 | 0.6109 | 0.1401 | 0.1661 | 0.4412 |
| sigma = 2, sigma\_0 = 2 | sample | 5.6177 | -0.1597 | 2.3984 | -1.0421 | 0.5527 | 0.8135 | 2.8027 | 3.8843 | 2.6010 | 1.2539 |
| true probability | 0.9648 | 0.1401 | 0.5789 | 0.0641 | 0.2346 | 0.2765 | 0.6559 | 0.8269 | 0.6181 | 0.3545 |
| ML probability | 0.5000 | 0.0743 | 0.4561 | 0.0849 | 0.3226 | 0.3917 | 0.7313 | 0.8444 | 0.6293 | 0.3786 |
| MAP probability | 0.8171 | 0.0432 | 0.3836 | 0.0374 | 0.2058 | 0.2819 | 0.6792 | 0.8432 | 0.6402 | 0.3769 |