EEE-6512: Image Processing and Computer Vision

September 29, 2017

Lecture #6: Binary Image Processing

Damon L. Woodard, Ph.D.
Dept. of Electrical and Computer Engineering
dwoodard@ece.ufl.edu

Chapter Outline

- Morphological Operations
- Labeling Regions
- Computing Distance in a Digital Image
- Region Properties
- Skeletonization
- Boundary Representations

Morphological Operations

- **Mathematical morphology:** an entire branch of mathematics that has been developed to process images by considering the shape of the pixel regions.

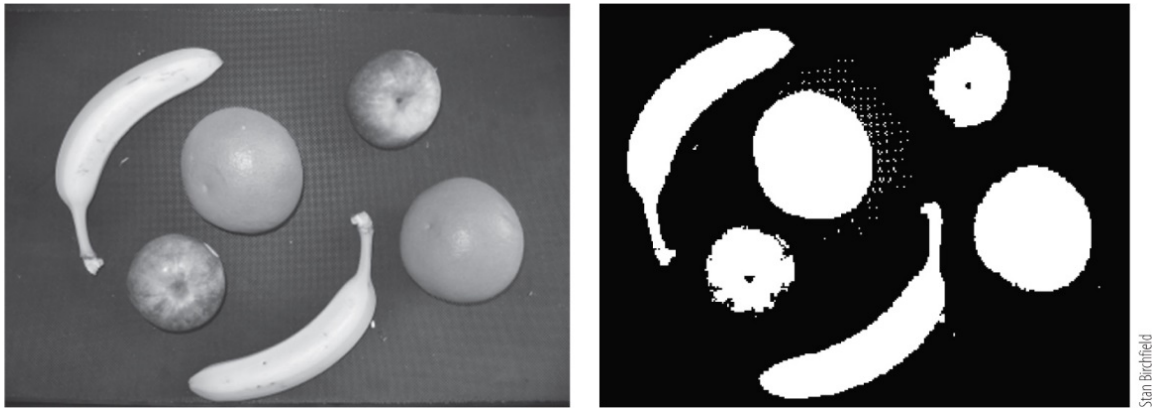


Figure 4.1 Left: A grayscale image of several types of fruit on a dark conveyor belt. Right: A binary image resulting from thresholding. The white pixels are ON and indicate the foreground, while the black pixels are OFF and indicate the background.

Stan Birchfield

Mathematical Morphology (cont.)

- The language of mathematical morphology is **set theory**.
- It is a mathematical tool for investigating **geometric structure** in an image.
- It is an approach for processing a digital image based on its **shape**.

Mathematical Morphology Goals

- To simplify image data, preserve essential shape characteristics, and eliminate noise

- Allows the underlying shape to be identified and reconstructed optimally from its noisy distorted forms.

Mathematical Morphology Uses

- Image pre-processing (noise filtering, shape simplification)
- Enhancing object structures (skeletonization, thinning, convex hull, object marking)
- Segmentation of the object from background
- Quantitative descriptors of objects (area, perimeter, projection, Euler-Poincaré characteristics)

Mathematical Morphology Main Idea

- Morphological operators often take a binary image and a structuring element as input and combine them using a set operator (intersection, union, inclusion, complement).

- The structuring element is shifted over the image and at each pixel of the image its elements are compared with the set of the underlying pixels.

- If the two sets of elements match the condition defined by the set operator (e.g. if set of pixels in the structuring element is a subset of the underlying image pixels), the pixel underneath the origin of the structuring element is set to a pre-defined value (0 or 1 for binary images).

- A morphological operator is therefore defined by its **structuring element** and the **applied set operator.**

Binary Image as a Set

- A **binary image** is an array of values such that $I(x, y)$ returns 1 or 0 for each pixel location (x, y).

- Two fundamental set operators are *union* and *intersection*:

$$\mathcal{A} \cup \mathcal{B} \equiv \{\mathbf{z} : \mathbf{z} \in \mathcal{A} \text{ or } \mathbf{z} \in \mathcal{B}\} \qquad \text{(union)}$$

$$\mathcal{A} \cap \mathcal{B} \equiv \{\mathbf{z} : \mathbf{z} \in \mathcal{A} \text{ and } \mathbf{z} \in \mathcal{B}\} \qquad \text{(intersection)}$$

Binary Image as a Set (cont'd)

- Additional operators:

$$\mathcal{A}_{\mathbf{b}} \equiv \{\mathbf{z} : \mathbf{z} = \mathbf{a} + \mathbf{b}, \mathbf{a} \in \mathcal{A}\} \quad \text{(translation)}$$

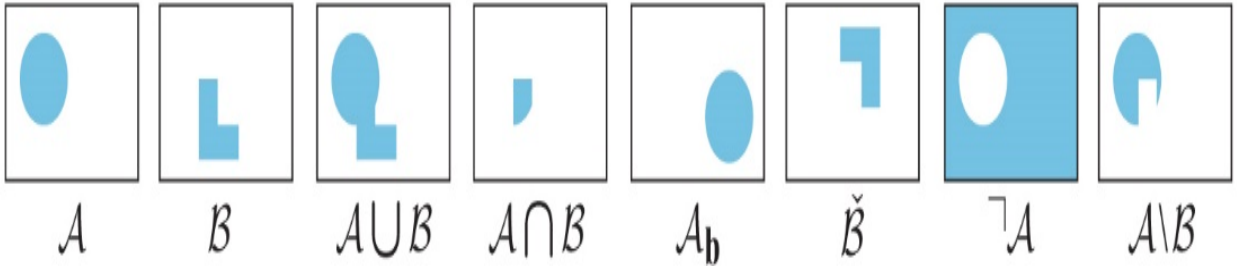
$$\check{\mathcal{B}} \equiv \{\mathbf{z} : \mathbf{z} = -\mathbf{b}, \mathbf{b} \in \mathcal{B}\} \quad \text{(reflection)}$$

$$\neg\mathcal{A} \equiv \{\mathbf{z} : \mathbf{z} \notin \mathcal{A}\} \quad \text{(complement)}$$

$$\mathcal{A} \setminus \mathcal{B} \equiv \{\mathbf{z} : \mathbf{z} \in \mathcal{A}, \mathbf{z} \notin \mathcal{B}\} = \mathbf{A} \bigcap \neg\mathcal{B} \quad \text{(difference)}$$

Set Operators

Figure 4.2 Set operators. The first two columns show two sets $\mathcal{A}$ and $\mathcal{B}$ in blue. Then, from left to right, shown are the union, intersection, shift of $\mathcal{A}$ by some amount **b** (not related to $\mathcal{B}$), reflection about the origin (assumed to be at the center), complement, and set difference.



Minkowski Addition and Subtraction

- The **Minkowski addition** of two sets *A* and *B* is defined as the set of points resulting from all possible vector additions of elements of the two sets:

$$\mathcal{A} \oplus \mathcal{B} \equiv \{\mathbf{z} : \mathbf{z} = \mathbf{a} + \mathbf{b}, \mathbf{a} \in \mathcal{A}, \mathbf{b} \in \mathcal{B}\}$$

$$= \bigcup_{\mathbf{b} \in \mathcal{B}} \{\mathbf{a} + \mathbf{b} : \mathbf{a} \in \mathcal{A}\} = \bigcup_{\mathbf{b} \in \mathcal{B}} \mathcal{A}_{\mathbf{b}}$$

- **Minkowski subtraction** of two sets:

$$\mathcal{A} \ominus \mathcal{B} \equiv \{\mathbf{z} : \mathbf{z} - \mathbf{b} \in \mathcal{A}, \forall \mathbf{b} \in \mathcal{B}\}$$

$$= \bigcap_{\mathbf{b} \in \mathcal{B}} \{\mathbf{a} + \mathbf{b} : \mathbf{a} \in \mathcal{A}\} = \bigcap_{\mathbf{b} \in \mathcal{B}} \mathcal{A}_{\mathbf{b}}$$

Minkowski Addition and Subtraction

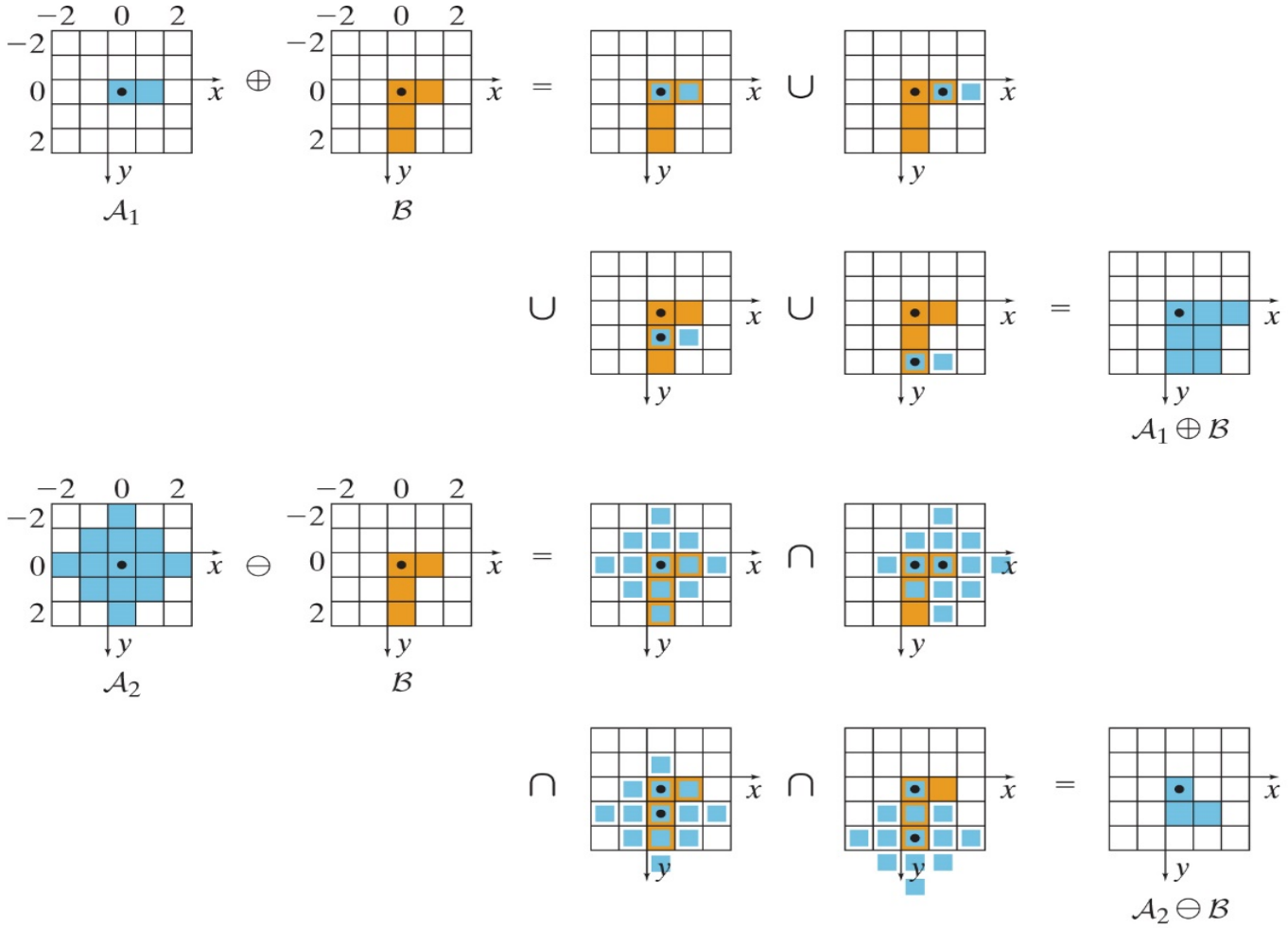


Figure 4.3 Minkowski addition and subtraction, from Equations (4.10) and (4.12) , respectively. For both operations, the first set ($\mathcal{A}_1$ or $\mathcal{A}_2$) is translated so that its origin is placed at each element of the second set ($\mathcal{B}$). The union of the blue cells then yields $\mathcal{A}_1 \oplus \mathcal{B}$, while their intersection yields $\mathcal{A}_2 \ominus \mathcal{B}$. In other words, each ON (blue) cell in $\mathcal{A}_1 \oplus \mathcal{B}$ is ON (blue) in *at least one* of the intermediate results, while each ON (blue) cell in $\mathcal{A}_2 \ominus \mathcal{B}$ is ON (blue) in *all* of the intermediate results. Colored cells are ON (value 1); white cells are OFF (value 0); and all pixels outside the 5 × 5 image are assumed to be OFF. The small black dots indicate the origins of the coordinate systems. (As explained later, this is the "center-in" approach.)

Properties of the Minkowski operators

$$\mathcal{A} \oplus \mathcal{B} = \mathcal{B} \oplus \mathcal{A} \qquad \text{(commutativity)}$$

$$\mathcal{A} \ominus \mathcal{B} = \neg\mathcal{B} \ominus \neg\mathcal{A} \qquad \text{(non-commutativity)}$$

$$(\mathcal{A} \oplus \mathcal{B}) \oplus \mathcal{C} = \mathcal{A} \oplus (\mathcal{B} \oplus \mathcal{C}) \qquad \text{(associativity, separability)}$$

$$(\mathcal{A} \ominus \mathcal{B}) \ominus \mathcal{C} = \mathcal{A} \ominus (\mathcal{B} \oplus \mathcal{C}) \qquad \text{(separability)}$$

$$(\mathcal{A} \oplus \mathcal{B}) \oplus \mathcal{C} = (\mathcal{A} \oplus \mathcal{C}) \oplus \mathcal{B} \qquad \text{(order does not matter)}$$

$$(\mathcal{A} \ominus \mathcal{B}) \ominus \mathcal{C} = (\mathcal{A} \ominus \mathcal{C}) \ominus \mathcal{B} \qquad \text{(order does not matter)}$$

$$\mathcal{A} \oplus \mathcal{B} \subseteq \mathcal{A} \ \text{ if} (0,0) \in \mathcal{B} \qquad \text{(extensivity)}$$

$$\mathcal{A} \ominus \mathcal{B} \subseteq \mathcal{A} \ \text{ if} (0,0) \in \mathcal{B} \qquad \text{(anti-extensivity)}$$

$$\mathcal{A}_1 \oplus \mathcal{B} \subseteq \mathcal{A}_2 \oplus \mathcal{B} \ \text{ if } \mathcal{A}_1 \subseteq \mathcal{A}_2 \qquad \text{(increasing)}$$

$$\mathcal{A}_1 \ominus \mathcal{B} \subseteq \mathcal{A}_2 \ominus \mathcal{B} \ \text{ if } \mathcal{A}_1 \subseteq \mathcal{A}_2 \qquad \text{(increasing)}$$

Properties of the Minkowski operators (cont'd)

$$\mathcal{A} \oplus (\mathcal{B} \bigcup \mathcal{C}) = (\mathcal{A} \oplus \mathcal{B}) \bigcup (\mathcal{A} \oplus \mathcal{C}) \qquad \text{(parallelism)}$$

$$\mathcal{A} \ominus (\mathcal{B} \bigcup \mathcal{C}) = (\mathcal{A} \ominus \mathcal{B}) \bigcap (\mathcal{A} \ominus \mathcal{C}) \qquad \text{(parallelism)}$$

$$(\mathcal{A} \bigcap \mathcal{B}) \ominus \mathcal{C} = (\mathcal{A} \ominus \mathcal{C}) \bigcap (\mathcal{B} \ominus \mathcal{C})$$

$$(\mathcal{A} \bigcup \mathcal{B}) \oplus \mathcal{C} = (\mathcal{A} \oplus \mathcal{C}) \bigcup (\mathcal{B} \oplus \mathcal{C})$$

$$\neg(\mathcal{A} \oplus \mathcal{B}) = \neg\mathcal{A} \ominus \mathcal{B} \qquad \text{(duality)}$$

$$\neg(\mathcal{A} \ominus \mathcal{B}) = \neg\mathcal{A} \oplus \mathcal{B} \qquad \text{(duality)}$$

Dilation and Erosion

- Minkowski addition grows (or “dilates”) the region by increasing its size.
- Minkowski subtraction shrinks (or “erodes”) the region by decreasing its size.

$$\mathcal{A} \oplus \mathcal{B} \equiv A \oplus \mathcal{B} = \{\mathbf{z} : \mathbf{z} = \mathbf{a} + \mathbf{b}, \mathbf{a} \in \mathcal{A}, \mathbf{b} \in \mathcal{B}\} \quad \text{(dilation)}$$

$$\mathcal{A} \check{\ominus} \mathcal{B} \equiv A \ominus \check{\mathcal{B}} = \{\mathbf{z} : \mathbf{z} + \mathbf{b} \in \mathcal{A}, \forall \mathbf{b} \in \mathcal{B}\} \quad \text{(erosion)}$$

- The most common use of dilation and erosion is to remove noise.

Dilation and Erosion

Dilations make objects thicker, can be used to fill in holes, multiple applications can result in image which looks different from input

Erosions remove pixels from object boundaries, used to simplify structure of object. Objects with width one will disappear.

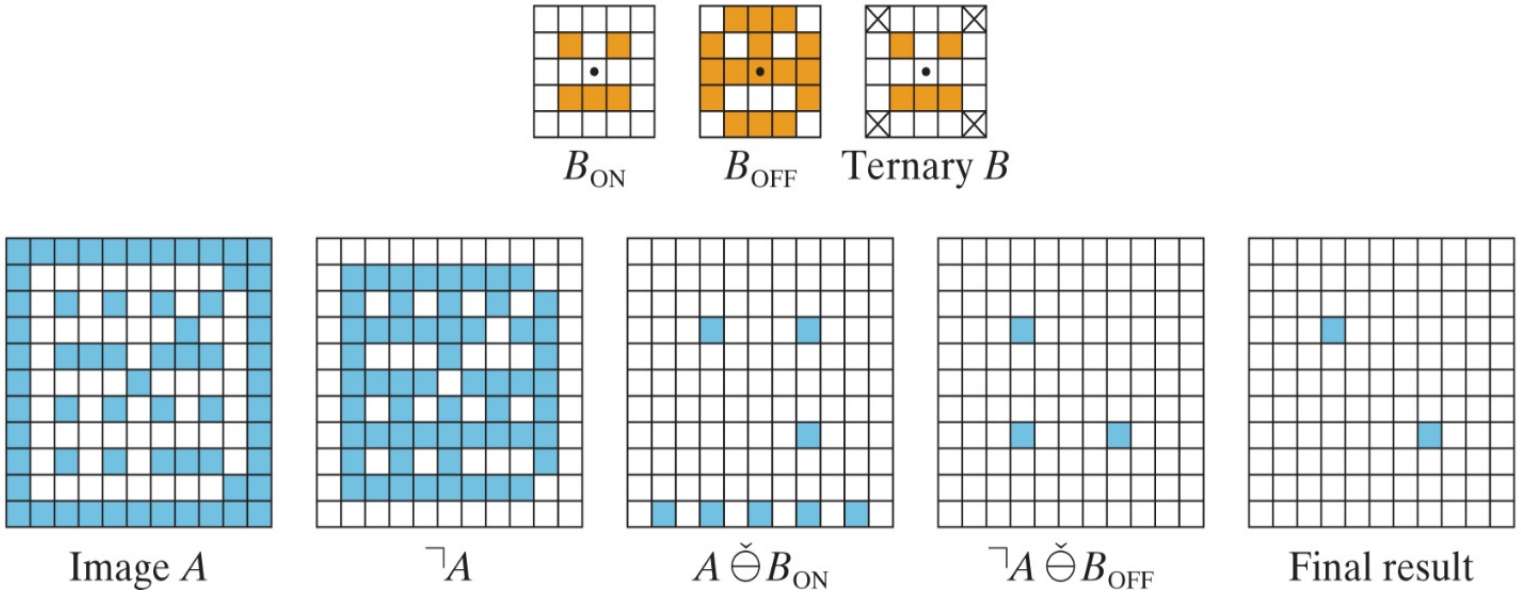
Hit-Miss Operator

- To detect the shape in the image, the **hit-miss operator** uses erosion to find all the places in the image where B_{ON} matches the foreground and B_{OFF} matches the background:

$$A \circledast (B_{ON}, B_{OFF}) \equiv (A \mathbin{\check{\ominus}} B_{ON}) \cap (\urcorner A \mathbin{\check{\ominus}} B_{OFF}) \qquad \text{(hit-miss operator)}$$

Hit or Miss Operator

Figure 4.12 Hit-miss operator. TOP: The foreground pattern B_{ON}, the background pattern B_{OFF}, and the ternary representation B with X indicating DONT-CARE. BOTTOM: From left-to-right: The image A within which to search for the pattern, the negation $\neg A$ of the image, the erosion of the image with the foreground pattern, the erosion of the negated image with the background pattern, and the final result, which shows two successful detections of the smiley face.



Hit-Miss Operator (cont'd)

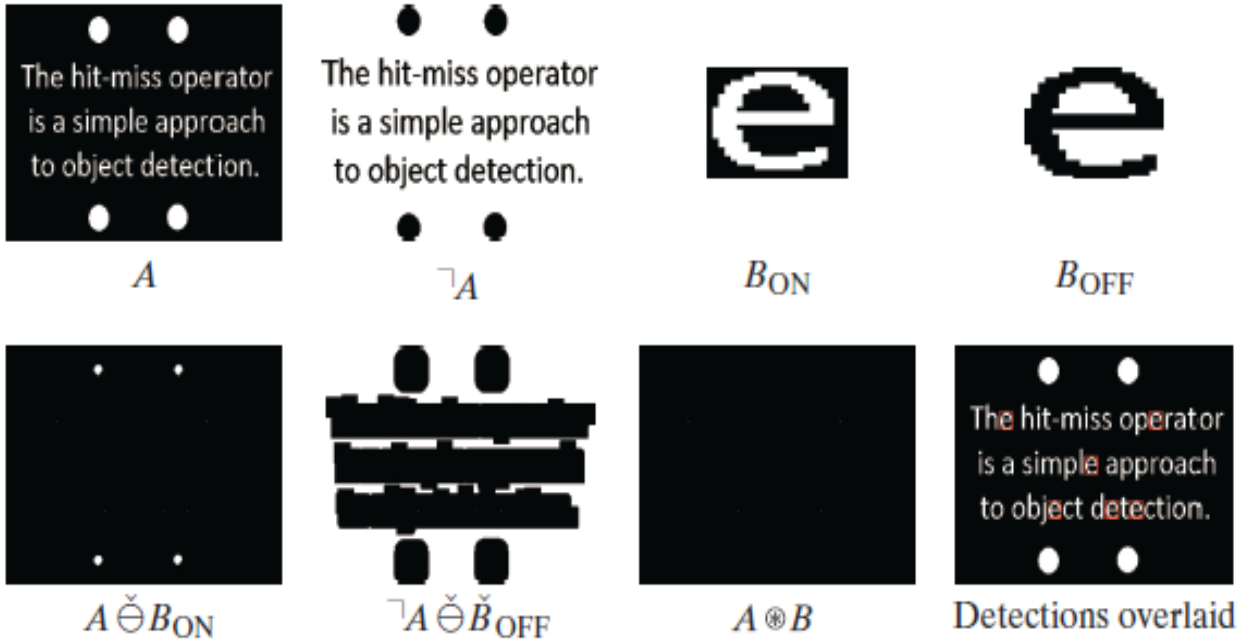


Figure 4.13 The hit-miss operator applied to a binary image, with white indicating ON and black indicating OFF. TOP: From left-to-right: The image, the inverted image, the foreground pattern, and the background pattern. BOTTOM: From left-to-right: The erosion of the image with the foreground pattern, the erosion of the inverted image with the background pattern, the result of hit-miss, and the outlines of the detections overlaid on the original image.