

4.19 Solution: Poincaré formula: Euler number = # of vertices - # of edges + # of faces

Shape 1: Regions = 1, holes = 1, Euler number = $1 - 1 = 0$

of vertices = 5, # of edges = 5, # of faces = 0

$$\therefore \text{Euler number} = 5 - 5 + 0 = 0$$

Shape 2: # of regions = 1, # of holes = 1, Euler number = $1 - 1 = 0$

of vertices = 11, # of edges = 12, # of faces = 1

$$\therefore \text{Euler number} = 11 - 12 + 1 = 0$$

Shape 3: # of regions = 1, # of holes = 3, Euler number = $1 - 3 = -2$

of vertices = 4, # of edges = 7, # of faces = 1

$$\therefore \text{Euler number} = 4 - 7 + 1 = -2$$

Shape 4: # of regions = 1, # of holes = 3, Euler number = $1 - 3 = -2$

of vertices = 16, # of edges = 19, # of faces = 1

$$\therefore \text{Euler number} = 16 - 19 + 1 = -2$$

4.26 Solution:

Breadth-first search will grow the region

5.44 Solution:

$$I = \begin{bmatrix} 218 & 87 & 246 & 63 & 175 \\ 106 & 161 & 231 & 32 & 207 \\ 16 & 141 & 136 & 140 & 202 \\ 86 & 253 & 55 & 112 & 188 \\ 73 & 85 & 165 & 209 & 99 \end{bmatrix}$$

$$(1) B_s = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad I \oplus B = \begin{bmatrix} 218 & 246 & 246 & 246 & 207 \\ 218 & 246 & 246 & 246 & 207 \\ 253 & 253 & 253 & 231 & 209 \\ 253 & 253 & 253 & 209 & 209 \\ 253 & 253 & 253 & 209 & 209 \end{bmatrix}$$

$$I \ominus B = \begin{bmatrix} 87 & 87 & 32 & 32 & 32 \\ 16 & 16 & 32 & 32 & 32 \\ 16 & 16 & 32 & 32 & 32 \\ 16 & 16 & 55 & 55 & 99 \\ 73 & 55 & 55 & 55 & 99 \end{bmatrix}$$

$$(2) I \cdot B = \begin{bmatrix} 218 & 218 & 246 & 207 & 207 \\ 218 & 218 & 231 & 207 & 207 \\ 218 & 218 & 209 & 207 & 207 \\ 253 & 253 & 209 & 207 & 207 \\ 253 & 253 & 209 & 209 & 209 \end{bmatrix}$$

$$I \circ B = \begin{bmatrix} 87 & 87 & 87 & 32 & 32 \\ 87 & 87 & 87 & 32 & 32 \\ 16 & 55 & 55 & 99 & 99 \\ 73 & 73 & 55 & 99 & 99 \\ 73 & 73 & 55 & 99 & 99 \end{bmatrix}$$

$$(3) I'_{WTH} = I - I \circ B = \begin{bmatrix} 131 & 0 & 159 & 31 & 143 \\ 19 & 74 & 144 & 0 & 175 \\ 0 & 86 & 81 & 41 & 103 \\ 13 & 180 & 0 & 13 & 89 \\ 0 & 12 & 110 & 110 & 0 \end{bmatrix}$$

$$I'_{BTH} = \cancel{I} \cdot B - I$$

$$= \begin{bmatrix} 0 & 131 & 0 & 144 & 32 \\ 112 & 57 & 0 & 175 & 0 \\ 202 & 77 & 73 & 67 & 5 \\ 167 & 0 & 154 & 95 & 19 \\ 180 & 168 & 44 & 0 & 110 \end{bmatrix}$$