EEE 6512 Image Processing and Computer Vision Homework #5

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5.1 Solution:

a) The width of the given kernel is 9. $w=9$.

b) The half-width is 4. $\tilde{w}=\frac{1}{2}(9-1)=4$.

c) Zero-based index of the central element (which is 4) is 4.

5.2 Solution:

$$f_{out}(x)=\frac{1}{4}\begin{bmatrix}1&2&1&0&0&0&0&0\\0&1&2&1&0&0&0&0\\0&0&1&2&1&0&0&0\\0&0&0&1&2&1&0&0\\0&0&0&0&1&2&1&0\\0&0&0&0&0&1&2&1\end{bmatrix}\begin{bmatrix}5\\5\\4\\0\\3\\8\\2\\2\end{bmatrix}=\frac{1}{4}\begin{bmatrix}19\\13\\7\\14\\21\\14\end{bmatrix}=\begin{bmatrix}4.75\\3.25\\1.75\\3.5\\5.25\\3.5\end{bmatrix}$$

When the signal is real, which is our case. ($f=f^*$), the only difference between convolution and cross-correlation is that convolution flips the kernel. In our case, the kernel is symmetric, so the result of cross-correlation would be same with convolution.

Output signal = [4.75 3.25 1.75 3.5 5.25 3.5]

5.3 Solution:

$$I\circledast G=\frac{1}{16}\begin{bmatrix}5&5&4&0&3&3\\5&5&4&0&3&3\\6&6&2&1&8&8\\7&7&9&4&2&2\\7&7&9&4&2&2\end{bmatrix}\circledast\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}$$

$$=\frac{1}{16}\begin{bmatrix}77&50&33&52\\86&64&50&69\\110&98&69&55\end{bmatrix}\approx\begin{bmatrix}4.8&3.1&2.1&3.3\\5.4&4&3.1&4.3\\6.9&6.1&4.3&3.4\end{bmatrix}$$

5.7 Solution:

(a) Smoothing kernel. (b) Differentiating kernel (C) Differentiating (d) Smoothing kerne

5.14 Solution:

$\tilde{w} \approx 2.5\sigma - 0.5 = 2.5 \times 1.8 - 0.5 = 4$

$\therefore w = 9$

$\therefore \text{kernel} = \frac{1}{C} [\exp^{\frac{-4^2}{2(1.8)^2}} \ \exp^{\frac{-3^2}{2(1.8)^2}} \ \exp^{\frac{-2^2}{2(1.8)^2}} \ \exp^{\frac{-1^2}{2(1.8)^2}} \ \exp^{0} \ \exp^{\frac{-1^2}{2(1.8)^2}} \ \exp^{\frac{-2^2}{2(1.8)^2}} \ \exp^{\frac{-3^2}{2(1.8)^2}} \ \exp^{\frac{-4^2}{2(1.8)^2}}]$

C = sum of elements without normalization.

$= \frac{1}{4.461} \cancel{= 0.2216} [0.0847 \ 0.2494 \ 0.5394 \ 0.8570 \ 1.0000 \ 0.8570 \ 0.5394 \ 0.2494 \ 0.0847]$

$= [\cancel{0.0188}^{0.0190} \ \cancel{0.0553}^{0.0559} \ \cancel{0.1196}^{0.1209} \ \cancel{0.1899}^{0.1921} \ \cancel{0.2216}^{0.2242} \ \cancel{0.1899}^{0.1921} \ \cancel{0.1196}^{0.1209} \ \cancel{0.0553} \ \cancel{0.0188} \ 0.0559 \ 0.0190]$

$\sigma^2 = 2(0.0190 \cdot 4^2 + 0.0559 \cdot 3^2 + 0.1209 \cdot 2^2 + 0.1921 \cdot 1^2) + 0.2242 \ 0^2 = 2.97$

standard deviation $= \sqrt{2.97} \approx 1.72$

5.19 Solution

The linear shift-varying system can also be represented as matrix multiplication. When border effects are ignored, and the convolution matrix has every diagonal descending from top left to the bottom right contains a constant value, i.e. it is a Toeplitz matrix, then the system is shift-invariant. Otherwise, it is shift-varying.

5.27 Solution:

① $LoG = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $I = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $I \circledast LoG = 1$.

② $LoG = \frac{1}{6}\begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix}$, $I = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $I \circledast LoG = \frac{1}{6} \times (1+4+1) = 1$

③ $LoG = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, $I = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $I \circledast LoG = \frac{1}{3}(1+1+1) = 1$

④ $LoG = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & -4 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $I \circledast LoG = \frac{1}{2}(1+1) = 1$

6.1 Solution:

(a) $f = \frac{8\pi}{2\pi} = 4\,Hz$ $\quad \cos 8\pi t = \frac{1}{2}(e^{j8\pi t} + e^{-j8\pi t})$

$$G(f) = \int_{-\infty}^{+\infty} \frac{1}{2}(e^{j8\pi t} + e^{-j8\pi t}) e^{-j2\pi f t} dt = \frac{1}{2}\int_{-\infty}^{\infty} (e^{j2\pi(4-f)t} + e^{-j2\pi(4+f)t}) dt$$

$$= \frac{1}{2}[\delta(f-4) + \delta(f+4)]$$

(b) $f = \frac{16}{2\pi} = \frac{8}{\pi} \approx 2.55\,Hz$ $\quad \cos(16t+8) = \cos(16(t+0.5)) = \frac{1}{2}(e^{j(16t+8)} + e^{-j(16t+8)})$

$$\cancel{G(f) = \int_{-\infty}^{+\infty} \frac{1}{2}[e^{j(16t+8)} + e^{-j(16t+8)}] e^{-j2\pi f t} dt}$$

$$\cancel{= \frac{1}{2}\int_{-\infty}^{\infty} e^{-j2\pi(f-2.55)t} e^{8j} dt + \frac{1}{2}\int_{-\infty}^{\infty} e^{-j2\pi(f+2.55)t} e^{-8j} dt}$$

Fourier transform of cos(16t):

$$G(\cos(16t)) = \int_{-\infty}^{\infty} \frac{1}{2}[e^{-j\cdot 16t} + e^{j\cdot 16t}] \cdot e^{-j2\pi f t} dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} e^{-j2\pi(2.55+f)t} dt + \frac{1}{2}\int_{-\infty}^{\infty} e^{-j2\pi(f-2.55)t} dt$$

$$= \cancel{\delta(f} \; \frac{1}{2}[\delta(f+2.55) + \delta(f-2.55)]$$

$$\therefore G(\cos(16t+8)) = \frac{1}{2}[\delta(f+2.55) + \delta(f-2.55)] \cdot e^{-j2\pi f(-0.5)}$$

$$= \frac{1}{2}e^{-j\pi f}[\delta(f+2.55) + \delta(f-2.55)]$$

(c) $f = \frac{44}{2\pi} = \frac{22}{\pi} \approx 7\,Hz$ $\quad \sin 44t = \sin(2\pi \cdot 7t) = (e^{j2\pi \cdot 7t} - e^{-j2\pi \cdot 7t})\frac{j}{2}$

$$G(\sin 44t) = \frac{j}{2}\int_{-\infty}^{\infty}(e^{j2\pi \cdot 7t} - e^{-j2\pi \cdot 7t})e^{-j2\pi ft}dt = \frac{j}{2}\int_{-\infty}^{\infty} e^{-j2\pi(f-7)t}dt - \frac{j}{2}\int_{-\infty}^{\infty} e^{-j2\pi(f+7)t}dt$$

$$= \frac{j}{2}[\delta(f-7) - \delta(f+7)]$$

6.4 Solution:

Nyquist rate: a property of the signal, which is twice the highest frequency of the signal. Minimum sampling rate necessary to preserve the signal.

Nyquist frequency: a property of the sampling system, which is half the sampling rate. Highest frequency that is preserved by sampling.

6.6 Solution:

(a) No aliasing would occur since $300\,Hz < \frac{1000}{2}Hz$.

(b) Aliasing will occur since $600\,Hz > \frac{1000}{2}Hz$. Aliasing frequency $= |1000 \cdot ROUND(600/1000) - 600|$
$= |1000 - 600| = 400\,Hz$

(c) Aliasing will occur since $1200\,Hz > \frac{1000}{2}Hz$

Aliasing frequency $= |1000 \cdot ROUND(1200/1000) - 1200| = 200\,Hz$.

6.7 Solution:

When the DFT of all these 5 samples are nonzero, the discrete frequencies captured by the DFT is $0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$.

Or ~~that~~ the frequencies $0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ are all the possible frequencies can be captured by the DFT.

6.11 Solution:

$$G(k)=\sum_{x=1}^{W-1} g(x)\, e^{-j2\pi kx/W}$$

$\because g(x)=\begin{cases}1, & x=0\\ 0, & \text{otherwise}\end{cases}$ $\quad \therefore G(k)=e^{-j2\pi\cdot 0/W}+0=1$

$\therefore \mathrm{DFT}([\underline{1}\ \ 0 \cdots 0]) = [1\ \ 1 \cdots 1]$ (all "1")

6.12 Solution:

(a) $g(0)=1,\ g(1)=\frac{\sqrt{2}}{2},\ g(2)=0,\ g(3)=-\frac{\sqrt{2}}{2},\ g(4)=-1,\ g(5)=-\frac{\sqrt{2}}{2},\ g(6)=0,\ g(7)=\frac{\sqrt{2}}{2}$

$$G(k)=\sum_{x=0}^{7} g(x)e^{-j2\pi kx/W}=\sum_{x=0}^{7} g(x)[\cos(2\pi kx/W)-j\sin(2\pi kx/W)]$$

$\therefore G_{even}(k)=\sum_{x=0}^{7} g(x)\cos(2\pi kx/8),\ G_{odd}(k)=-\sum_{x=0}^{7}\sin(2\pi kx/8).\ G(k)=G_{even}(k)+jG_{odd}(k).$

$G_{even}(0)=\sum_{x=0}^{7} g(x)=0,\ G_{odd}(0)=0,\ G(0)=0$

$G_{even}(1)=\sum_{x=0}^{7} g(x)\cos(\frac{\pi x}{4})=1(1)+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{2})+0-\frac{\sqrt{2}}{2}(-\frac{\sqrt{2}}{2})-1(-1)-\frac{\sqrt{2}}{2}(-\frac{\sqrt{2}}{2})+0+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{2})=4$

$G_{odd}(1)=-\sum_{x=0}^{7} g(x)\sin(\frac{\pi x}{4})=-(1(0)+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{2})+0(1)-\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{2})-0-\frac{\sqrt{2}}{2}(-\frac{\sqrt{2}}{2})+0+\frac{\sqrt{2}}{2}(-\frac{\sqrt{2}}{2}))=0 \quad \therefore G(1)=4$

$G_{even}(2)=\sum_{x=0}^{7} g(x)\cos(\frac{\pi x}{2})=1(1)+\frac{\sqrt{2}}{2}(0)+0-\frac{\sqrt{2}}{2}(-1)-1-\frac{\sqrt{2}}{2}(\overset{1}{\cancel{0}})+\overset{(-1)}{0}+\frac{\sqrt{2}}{2}(0)=0$

$\therefore G(2)=0$

$G_{odd}(2)=-\sum_{x=0}^{7} g(x)\sin(\frac{\pi x}{2})=-(1(0)+\frac{\sqrt{2}}{2}(1)+0-\frac{\sqrt{2}}{2}(-1)-0-\frac{\sqrt{2}}{2}(1)+0+\frac{\sqrt{2}}{2}(-1))=0$

$G_{even}(3)=\sum_{x=0}^{7} g(x)\cos(\frac{3\pi x}{4})=1(1)+\frac{\sqrt{2}}{2}(-\frac{\sqrt{2}}{2})+0-\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{2})-(-1)-\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{2})+0+\frac{\sqrt{2}}{2}(-\frac{\sqrt{2}}{2})=0$

$G_{odd}(3)=-\sum_{x=0}^{7} g(x)\sin(\frac{3\pi x}{4})=-(1(0)+\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{2})+0(-1)-\frac{\sqrt{2}}{2}(\frac{\sqrt{2}}{2})-0-\frac{\sqrt{2}}{2}(-\frac{\sqrt{2}}{2})+0+\frac{\sqrt{2}}{2}(-\frac{\sqrt{2}}{2}))=0 \quad \therefore G(3)=0$

$G_{even}(4)=\sum_{x=0}^{7} g(x)\cos(\pi x)=1(1)+\frac{\sqrt{2}}{2}(-1)+0-\frac{\sqrt{2}}{2}(-1)-1-\frac{\sqrt{2}}{2}(-1)+0+\frac{\sqrt{2}}{2}(-1)=0$

$G_{odd}(4)=-\sum_{x=0}^{7} g(x)\sin(\pi x)=0 \quad \therefore G(4)=0$

$G_{even}(5)=\sum_{x=0}^{7} g(x)\cos(\frac{5\pi x}{4})=1-\frac{1}{2}+0-\frac{1}{2}+1-\frac{1}{2}+0-\frac{1}{2}=0$

$G_{odd}(5)=-\sum_{x=0}^{7} g(x)\sin(\frac{5\pi x}{4})=0-\frac{1}{2}+0+\frac{1}{2}-0-\frac{1}{2}+0+\frac{1}{2}=0 \quad \therefore G(5)=0$

$G_{even}(6)=\sum_{x=0}^{7} g(x)\cos(\frac{3\pi}{2}x)=1-0-0-0-1+0+0-0=0$

$G_{odd}(6)=\sum_{x=0}^{7} g(x)\sin(\frac{3\pi x}{2})=0-\frac{\sqrt{2}}{2}+0-\frac{\sqrt{2}}{2}+0+\frac{\sqrt{2}}{2}-0+\frac{\sqrt{2}}{2}=0 \quad \therefore G(6)=0$

$G_{even}(7)=\sum_{x=0}^{7} g(x)\cos(\frac{7\pi x}{4})=1+\frac{1}{2}-0+\frac{1}{2}+1+\frac{1}{2}+0+\frac{1}{2}=4$

$G_{odd}(7)=\sum_{x=0}^{7} g(x)\sin(\frac{7\pi x}{4})=0-\frac{1}{2}-0+\frac{1}{2}-0-\frac{1}{2}-0+\frac{1}{2}=0 \quad \therefore G(7)=4$

$\therefore \mathrm{DFT}=[0\ 4\ 0\ 0\ 0\ 0\ 0\ 4]$. The frequencies captured by the DFT is $\frac{1}{8}$ sample^{-1} and $\frac{7}{8}$ samples^{-1} (or $-\frac{1}{8}$ samples^{-1} equivalently).

(b) $g(0)=0,\ g(1)=\frac{\sqrt{2}}{2},\ g(2)=1,\ g(3)=\frac{\sqrt{2}}{2},\ g(4)=0,\ g(5)=-\frac{\sqrt{2}}{2},\ g(6)=-1,\ g(7)=-\frac{\sqrt{2}}{2}$

$G(k)=\sum_{x=0}^{7} g(x)\left[\cos(2\pi kx/8)-j\sin(2\pi kx/8)\right]$

$G_{even}(k)=\sum_{x=0}^{7} g(x)\cos\left(\frac{\pi kx}{4}\right),\ G_{odd}(k)=-\sum_{x=0}^{7} g(x)\sin\left(\frac{\pi kx}{4}\right).\ G(k)=G_{even}(k)+j\sin G_{odd}(k)$

$G_{even}(0)=\sum_{x=0}^{7} g(x)\cos(0)=0+\frac{\sqrt{2}}{2}+1+\frac{\sqrt{2}}{2}+0-\frac{\sqrt{2}}{2}-1-\frac{\sqrt{2}}{2}=0$

$G_{odd}(0)=-\sum_{x=0}^{7} g(x)\sin(0)=0 \quad \therefore G(0)=0$

$G_{even}(1)=\sum_{x=0}^{7} g(x)\cos\left(\frac{\pi x}{4}\right)=0+\frac{1}{2}+0-\frac{1}{2}-0+\frac{1}{2}+0-\frac{1}{2}=0.$

$G_{odd}(1)=-\sum_{x=0}^{7} g(x)\sin\left(\frac{\pi x}{4}\right)=-(0+\frac{1}{2}+1+\frac{1}{2}+0+\frac{1}{2}+1+\frac{1}{2})=-4 \quad \therefore G(1)=-4j$

$G_{even}(2)=\sum_{x=0}^{7} g(x)\cos\left(\frac{2\pi x}{4}\right)=0+0-1-0+0-0+1+0=0$

$G_{odd}(2)=\sum_{x=0}^{7} g(x)\sin\left(\frac{2\pi x}{4}\right)=-(0+\frac{\sqrt{2}}{2}+0-\frac{\sqrt{2}}{2}-0-\frac{\sqrt{2}}{2}-0+\frac{\sqrt{2}}{2})=0. \quad \therefore G(2)=0$

$G_{even}(3)=\sum_{x=0}^{7} g(x)\cos\left(\frac{3\pi x}{4}\right)=0-\frac{1}{2}-0+\frac{1}{2}-0-\frac{1}{2}-0+\frac{1}{2}=0$

$G_{odd}(3)=-\sum_{x=0}^{7} g(x)\sin\left(\frac{3\pi x}{4}\right)=-(0+\frac{1}{2}-1+\frac{1}{2}+0+\frac{1}{2}-1+\frac{1}{2})=0 \quad \therefore G(3)=0$

$G_{even}(4)=\sum_{x=0}^{7} g(x)\cos\left(\frac{4\pi x}{4}\right)=0-\frac{\sqrt{2}}{2}+1-\frac{\sqrt{2}}{2}+0+\frac{\sqrt{2}}{2}-1+\frac{\sqrt{2}}{2}=0$

$G_{odd}(4)=-\sum_{x=0}^{7} g(x)\sin\left(\frac{4\pi x}{4}\right)=0 \quad \therefore G(4)=0$

$G_{even}(5)=\sum_{x=0}^{7} g(x)\cos\left(\frac{5\pi x}{4}\right)=0-\frac{1}{2}+0+\frac{1}{2}-0-\frac{1}{2}+0+\frac{1}{2}=0$

$G_{odd}(5)=\sum_{x=0}^{7} g(x)\sin\left(\frac{5\pi x}{4}\right)=-(0-\frac{1}{2}+1-\frac{1}{2}+0-\frac{1}{2}+1-\frac{1}{2})=0 \quad \therefore G(5)=0$

$G_{even}(6)=\sum_{x=0}^{7} g(x)\cos\left(\frac{6\pi x}{4}\right)=0-0-1+0+0+0+1+0=0$

$G_{odd}(6)=-\sum_{x=0}^{7} g(x)\sin\left(\frac{6\pi x}{4}\right)=-(0-\frac{\sqrt{2}}{2}+0+\frac{\sqrt{2}}{2}-0+\frac{\sqrt{2}}{2}-0-\frac{\sqrt{2}}{2})=0 \quad \therefore G(6)=0$

$G_{even}(7)=\sum_{x=0}^{7} g(x)\cos\left(\frac{7\pi x}{4}\right)=0+\frac{1}{2}-0-\frac{1}{2}-0+\frac{1}{2}+0-\frac{1}{2}=0$

$G_{odd}(7)=\sum_{x=0}^{7} g(x)\sin\left(\frac{7\pi x}{4}\right)=-(0-\frac{1}{2}-1-\frac{1}{2}+0-\frac{1}{2}-1-\frac{1}{2})=4 \quad \therefore G(7)=4j$

$\therefore$ DFT $=[0\ \ -4j\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 4j]$. The frequencies captured by DFT is $\frac{1}{8}$ samples^{-1} and $\frac{7}{8}$ (or $-\frac{1}{8}$) samples^{-1}.

(c) $g(x)=[1\ \ \frac{\sqrt{2}}{2}\ \ 0\ \ -\frac{\sqrt{2}}{2}\ \ -1\ \ -\frac{\sqrt{2}}{2}\ \ 0\ \ \frac{\sqrt{2}}{2}\ \ 1\ \ \frac{\sqrt{2}}{2}\ \ 0\ \ -\frac{\sqrt{2}}{2}\ \ -1\ \ -\frac{\sqrt{2}}{2}\ \ 0\ \ \frac{\sqrt{2}}{2}]$

$G(k)=\sum_{x=0}^{15} g(x)\left[\cos(2\pi kx/16)-j\sin(2\pi kx/16)\right]$

$G_{even}(k)=\sum_{x=0}^{15} g(x)\cos\left(\frac{\pi kx}{8}\right),\ G_{odd}(k)=-\sum_{x=0}^{15} g(x)\sin\left(\frac{\pi kx}{8}\right).\ G(k)=G_{even}(k)+jG_{odd}(k).$

$G_{even}(0)=\sum_{x=0}^{15} g(x)\cos(0)=0,\ G_{odd}(0)=-\sum_{x=0}^{15} g(x)\sin(0)=0 \quad \therefore G(0)=0$

$G_{even}(1)=\sum_{x=0}^{15} g(x)\cos\left(\frac{\pi x}{8}\right)=\cancel{8}0\ \ G_{odd}(1)=-\sum_{x=0}^{15} g(x)\sin\left(\frac{\pi x}{8}\right)=0 \quad \therefore G(1)=0$

$G_{even}(2)=\sum_{x=0}^{15} g(x)\cos\left(\frac{2\pi x}{8}\right)=8,\ G_{odd}(2)=-\sum_{x=1}^{15} g(x)\sin\left(\frac{2\pi x}{8}\right)=0 \quad \therefore G(2)=8$

$G_{even}(3)=\sum_{x=0}^{15} g(x)\cos\left(\frac{3\pi x}{8}\right)=0,\ G_{odd}(3)=0. \quad \therefore G(3)=0$

$G_{even}(4)=\sum_{x=0}^{15} g(x)\cos(\frac{4\pi x}{8})=0$, $G_{odd}(4)=-\sum_{x=0}^{15} g(x)\sin(\frac{4\pi x}{8})=0$. $\therefore G(4)=0$

$G_{even}(5)=\sum_{x=0}^{15} g(x)\cos(\frac{5\pi x}{8})=0$. $G_{odd}(5)=-\sum_{x=0}^{15} g(x)\sin(\frac{5\pi x}{8})=0$. $\therefore G(5)=0$

$G_{even}(6)=0$, $G_{odd}(6)=0$ ~~$\therefore G_{odd}(6)=0$. G_{odd}~~ $\therefore G(6)=0$

$G_{even}(7)=0$, $G_{odd}(7)=0$ $\therefore G(7)=0$. $G_{even}(9)=0$, $G_{odd}(9)=0$ $\therefore G(9)=0$

$G_{even}(8)=0$, $G_{odd}(8)=0$ $\therefore G(8)=0$ $G_{even}(10)=0$, $G_{odd}(10)=0$ $\therefore G(10)=0$

$G_{even}(11)=0$, $G_{odd}(11)=0$. $\therefore G(11)=0$. $G_{even}(12)=0$, $G_{odd}(12)=0$. $\therefore G(12)=0$

$G_{even}(13)=0$. $G_{odd}(13)=0$ $\therefore G(13)=0$. $G_{even}(14)=8$, $G_{odd}(14)=0$. $\therefore G(14)=8$

$G_{even}(15)=G_{odd}(15)=0$ $\therefore G(15)=0$. ~~$G_{even}(16)=G_{odd}(16)=0$. $\therefore G(16)=0$~~

$\therefore DFT=[0\ 0\ 8\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 8\ 0]$ The frequencies captured by DFT is $\frac{2}{16}$ and $\frac{14}{16}$ (or $-\frac{2}{16}$ equilvalently) $samples^{-1}$, or $\frac{1}{8}$ $samples^{-1}$ and $-\frac{1}{8}$ $samples^{-1}$.

(d) $g(x)=\cos(\frac{2\pi}{16}x)$ $x=0,1,\cdots,15$. $W=16$.

$G(k)=\sum_{x=0}^{15} g(x)[\cos(2\pi kx/16)-j\sin(2\pi kx/16)]$. $G_{even}(k)=\sum_{x=0}^{15} g(x)\cos(\frac{\pi kx}{8})$

$G_{odd}(k)=-\sum_{x=0}^{15} g(x)\sin(\frac{\pi kx}{8})$

$G_{even}(0)=G_{odd}(0)=0$. $\therefore G(0)=0$.

$G_{even}(1)=8$, $G_{odd}(1)=0$ $\therefore G(1)=8$.

$G_{even}(2)=G_{odd}(2)=0$ $\therefore G(2)=0$. $G_{even}(3)=G_{odd}(3)=0$. $\therefore G(3)=0$

$G_{even}(4)=G_{odd}(4)=0$ $\therefore G(4)=0$. $G_{even}(5)=G_{odd}(5)=0$. $\therefore G(5)=0$.

$G_{even}(6)=G_{odd}(6)=0$ $\therefore G(6)=0$. $G_{even}(7)=G_{odd}(7)=0$. $\therefore G(7)=0$.

$G_{even}(8)=G_{odd}(8)$ $\therefore G(8)=0$.

$G_{even}(9)=G_{even}(10)=G_{even}(11)=G_{even}(12)=G_{even}(13)=G_{even}(14)=0$

$G_{odd}(9)=G_{odd}(10)=G_{odd}(11)=G_{odd}(12)=G_{odd}(13)=G_{odd}(14)=G_{odd}(15)=0$

~~$\therefore G(15)=$~~

$G_{even}(15)=8$ $\therefore G(15)=8$

$\therefore DFT=[0\ 8\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 8]$

The frequencies captured by DFT is $\frac{1}{16}$ $samples^{-1}$ and $\frac{15}{16}$ (or $-\frac{1}{16}$) $samples^{-1}$.

6.15 Solution:

(a) $1-\frac{7\pi}{32}=\frac{25\pi}{32}$ $\therefore$ The negative frequency is $-\frac{25\pi}{32}$.

(b) $1-\frac{15\pi}{32}=\frac{17\pi}{32}$ $\therefore$ The negative frequency is $-\frac{17\pi}{32}$.

(c) $1-\frac{19\pi}{32}=\frac{13\pi}{32}$ $\therefore$ The negative frequency is $-\frac{13\pi}{32}$.

6.20 Solution:

The 2D DFT is also rotated clockwise by 30 degrees.