Coplanar Force Systems

""You are asking the wrong questions.
If you want to make the world a better place, tell funnier jokes!"
-Woody Allen

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Objectives

- Understand the use of the equilibrium constraint conditions
- Understand the development of the FBD in the solution of equilibrium problems

Tools

- Basic Trigonometry
- Pythagorean Theorem
- Algebra
- Visualization
- Position Vectors
- Unit Vectors

Review

- For a system to be in equilibrium, the three following constraint conditions must be satisfied

$$\sum F_x \vec{i} = 0$$

$$\sum F_y \vec{j} = 0$$

$$\sum F_z \vec{k} = 0$$

Review

- If we take the sign of the direction vector times the magnitude for each of the components we can then write

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

Review

- The free-body diagram is an isolation of an element and the identification of all forces which are acting on that element
- By an element, we mean a part of a mechanical system
- Forces can act on a system as either an applied force from some external source or
- Forces can act due to the connection of the system to some other system in response to the applied forces – these forces are known as reactions

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Ropes and Cables

- A rope **always pulls** on whatever it is connected to
- The force generated by a rope always has a line of action along the rope
- Whenever the same rope passes over, but not tied to, multiple connections, the force magnitude is the same through the length of the rope

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Problems

- In order to illustrate how the process works, we will work a couple of example problems
- The first will be a two-dimensional problem and the second will be a three-dimensional problem

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If the mass of cylinder C is 40 kg, determine the mass of cylinder A in order to hold the assembly in the position shown.

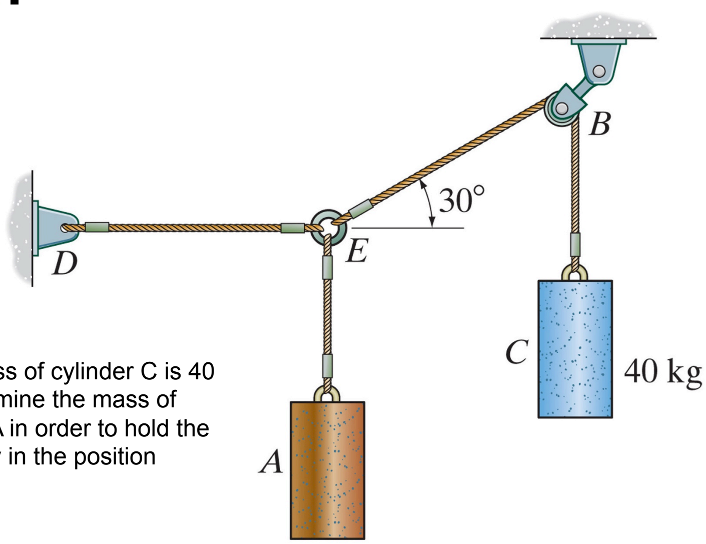

Figure: 03_FP005

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Determine the tension developed in cables AB, AC, and AD.

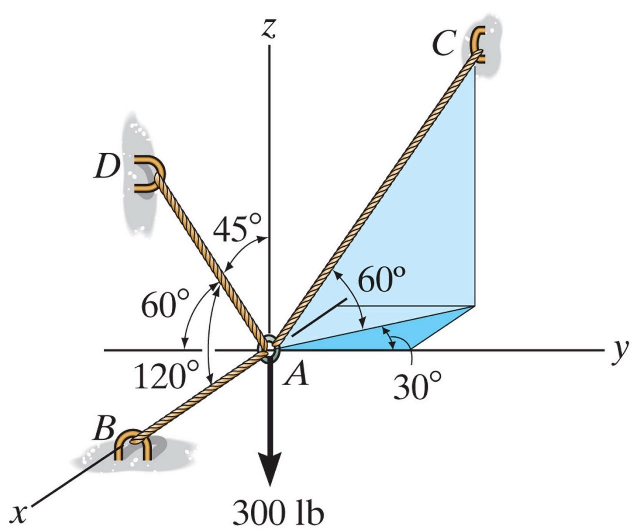

Figure: 03_FP010

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Homework

- Problem 3-29 (Hint: Set the force in each cable, one at a time, and solve for the forces in all the other cables)
- Problem 3-46
- Problem 3-71

- Read through 4.4

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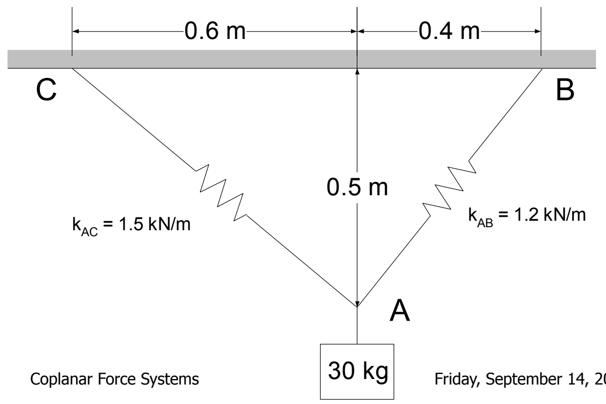
THE FOLLOWING SLIDES ARE EXAMPLE PROBLEMS I WORKED OUT DURING A PREVIOUS COURSE.

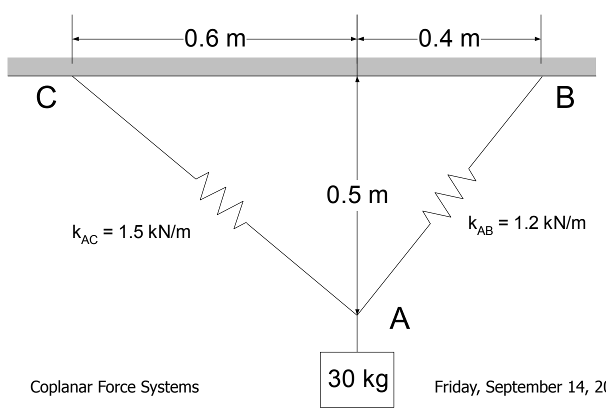
Problem Given

- We have the system as shown
- The mass of the block is 30 kg
- Both springs are stretched



Problem Required

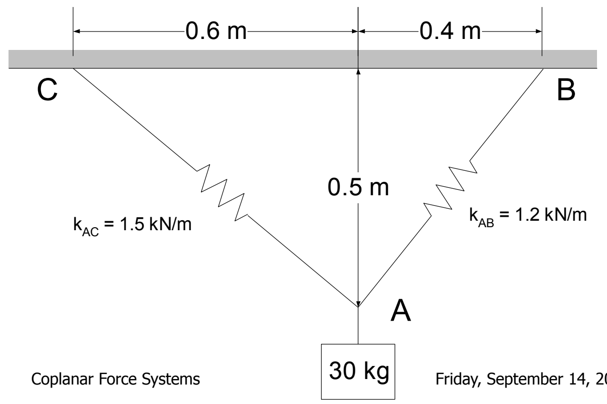
- The unstretched length of both springs, l_{0AB} and l_{0AC}



Problem Solution

- Since we know that both springs are stretched, we can calculate the change in length in each spring

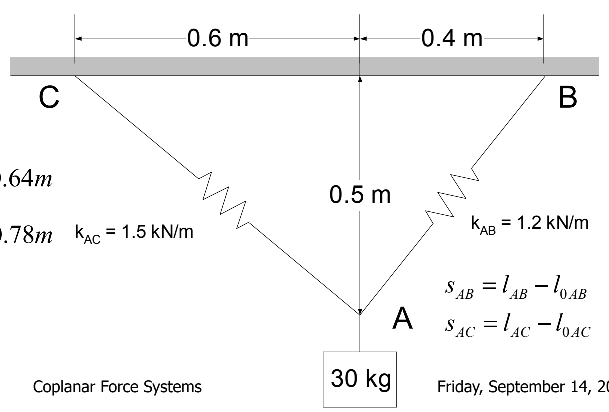
$$s_{AB} = l_{AB} - l_{0AB}$$
$$s_{AC} = l_{AC} - l_{0AC}$$



Problem Solution

- From the geometry of the system we can calculate the stretched lengths of the two springs, l_{AB} and l_{AC}

$$l_{AB} = \sqrt{(0.4m)^2 + (0.5m)^2} = 0.64m$$
$$l_{AC} = \sqrt{(0.6m)^2 + (0.5m)^2} = 0.78m$$

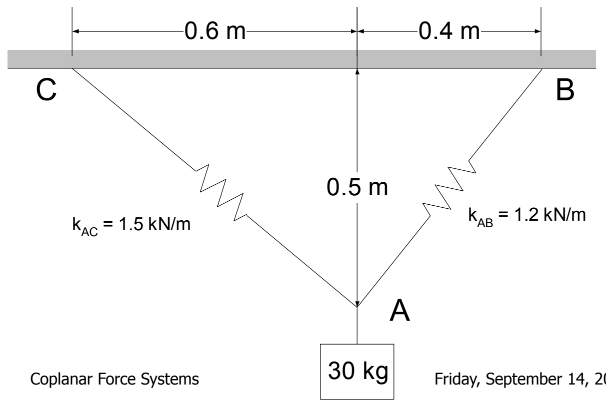


$$s_{AB} = l_{AB} - l_{0AB}$$
$$s_{AC} = l_{AC} - l_{0AC}$$

Problem Solution

- Substituting what we know and isolating what we want to know we have

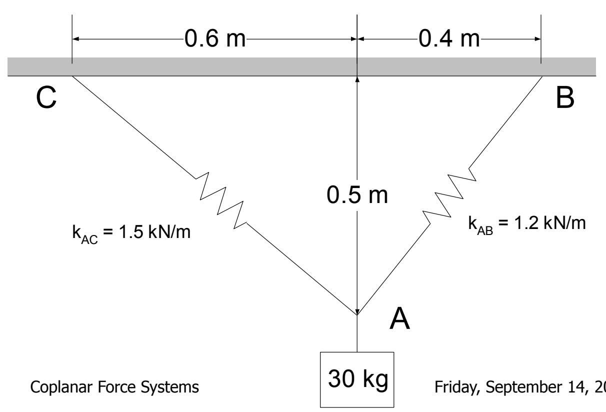
$$l_{0AB} = 0.64m - s_{AB}$$
$$l_{0AC} = 0.78m - s_{AC}$$



Problem Solution

- Now we have a new subproblem
- We have to find s_{AB} and s_{AC}

$$l_{0AB} = 0.64m - s_{AB}$$
$$l_{0AC} = 0.78m - s_{AC}$$

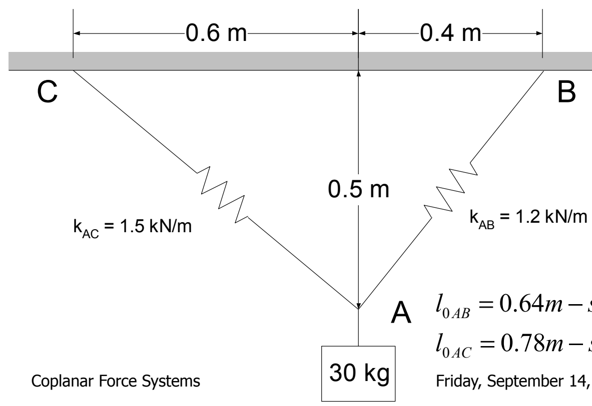


Problem Solution

- We know that in a spring, the force is proportional to the elongation

$$F_{AB} = k_{AB} s_{AB}$$

$$F_{AC} = k_{AC} s_{AC}$$



$$l_{0AB} = 0.64m - s_{AB}$$

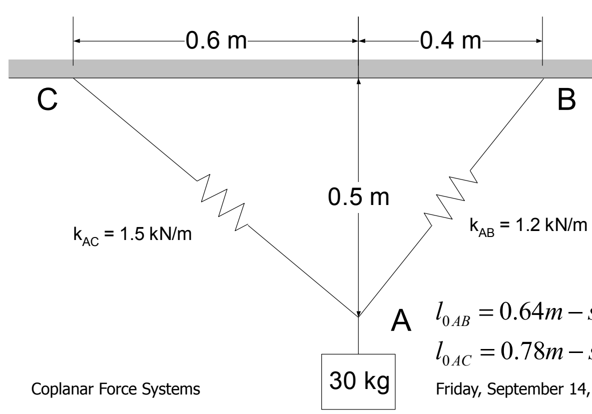
$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- We know the spring constants k_{AB} and k_{AC} so we can isolate what we are looking for

$$\frac{F_{AB}}{1.2\frac{kN}{m}} = s_{AB}$$

$$\frac{F_{AC}}{1.5\frac{kN}{m}} = s_{AC}$$



$$l_{0AB} = 0.64m - s_{AB}$$

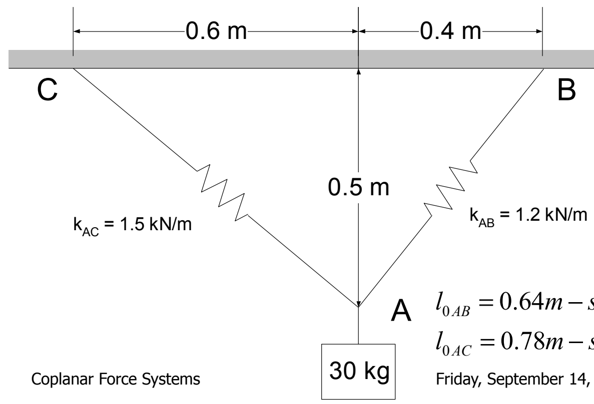
$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- Finally we are at a point where we need to solve for forces, F_{AB} and F_{AC}

$$\frac{F_{AB}}{1.2\dfrac{kN}{m}} = s_{AB}$$

$$\frac{F_{AC}}{1.5\dfrac{kN}{m}} = s_{AC}$$



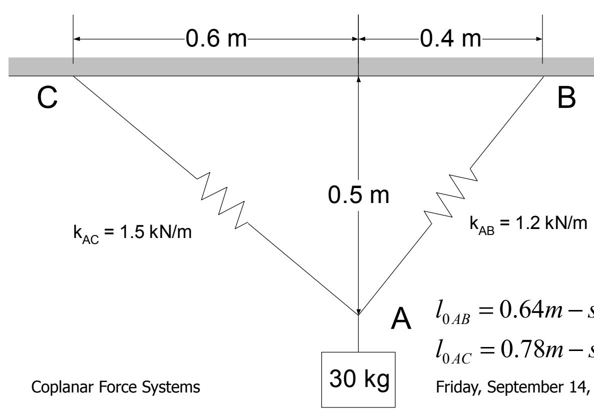
$$l_{0AB} = 0.64m - s_{AB}$$
$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- We will select a system that F_{AB} and F_{AC} both act on if possible

$$\frac{F_{AB}}{1.2\dfrac{kN}{m}} = s_{AB}$$

$$\frac{F_{AC}}{1.5\dfrac{kN}{m}} = s_{AC}$$



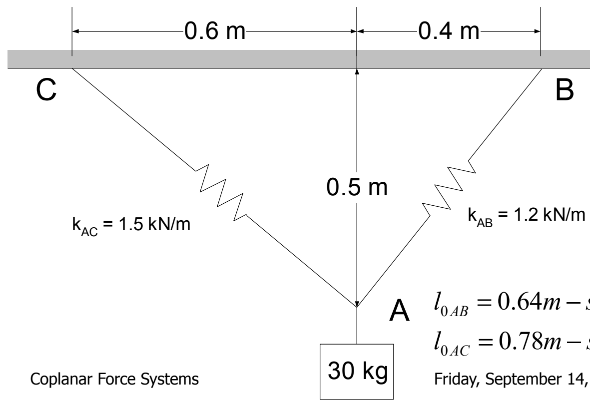
$$l_{0AB} = 0.64m - s_{AB}$$
$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- In this case it is the point A where the two springs and the box are connected

$$\frac{F_{AB}}{1.2\frac{kN}{m}} = s_{AB}$$

$$\frac{F_{AC}}{1.5\frac{kN}{m}} = s_{AC}$$



$$l_{0AB} = 0.64m - s_{AB}$$

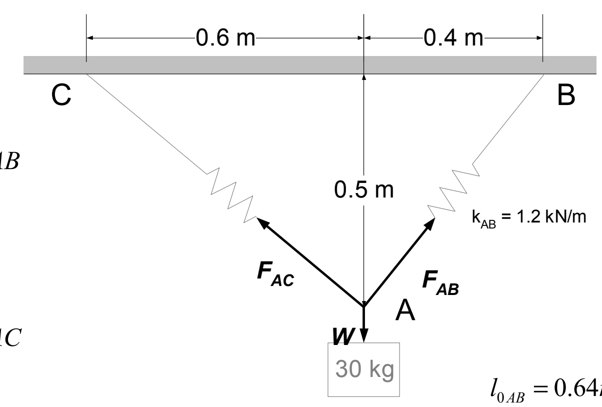
$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- We can draw the FBD of the point

$$\frac{F_{AB}}{1.2\frac{kN}{m}} = s_{AB}$$

$$\frac{F_{AC}}{1.5\frac{kN}{m}} = s_{AC}$$

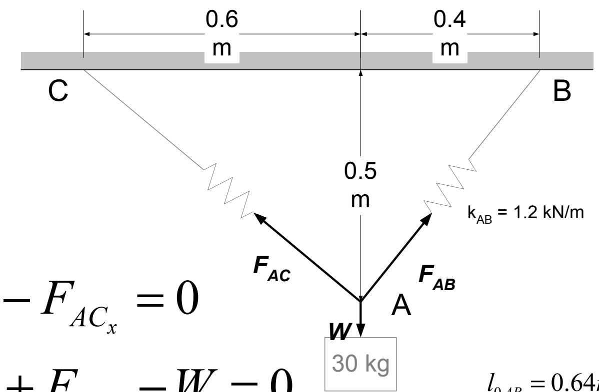


$$l_{0AB} = 0.64m - s_{AB}$$

$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- And using the constraint equations, develop two expressions



$$\sum F_x = F_{AB_x} - F_{AC_x} = 0$$

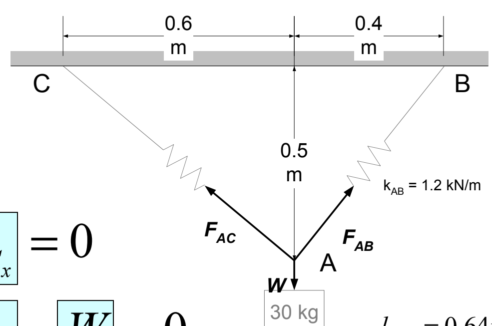
$$\sum F_y = F_{AB_y} + F_{AC_y} - W = 0$$

$$l_{0AB} = 0.64m - s_{AB}$$

$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- We have five unknowns and two equations so we need to extract more information from the problem to proceed



$$\sum F_x = \boxed{F_{AB_x}} - \boxed{F_{AC_x}} = 0$$

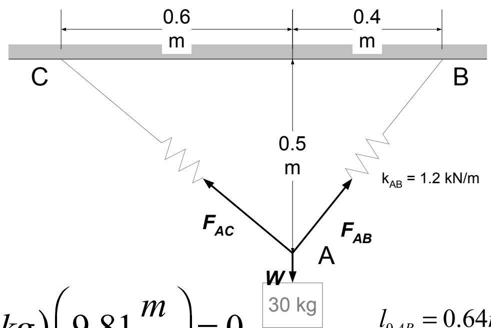
$$\sum F_y = \boxed{F_{AB_y}} + \boxed{F_{AC_y}} - \boxed{W} = 0$$

$$l_{0AB} = 0.64m - s_{AB}$$

$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- Since we have the mass of the block, finding W is just a conversion problem



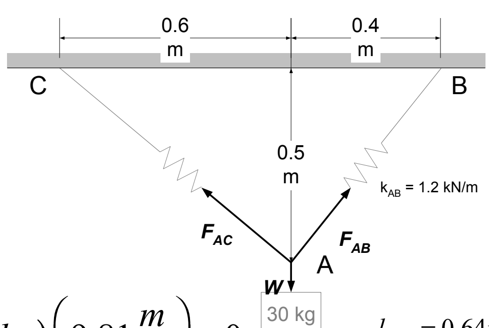
$$\sum F_x = \boxed{F_{AB_x}} - \boxed{F_{AC_x}} = 0$$

$$\sum F_y = \boxed{F_{AB_y}} + \boxed{F_{AC_y}} - (30kg)\left(9.81\frac{m}{s^2}\right) = 0$$

$$l_{0AB} = 0.64m - s_{AB}$$
$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- We still have four unknowns and two equations, we need to look at the forces



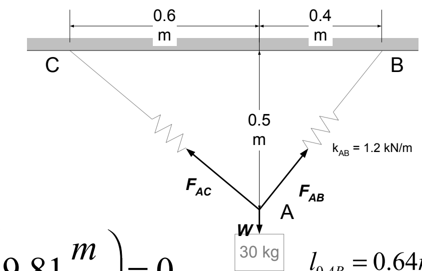
$$\sum F_x = \boxed{F_{AB_x}} - \boxed{F_{AC_x}} = 0$$

$$\sum F_y = \boxed{F_{AB_y}} + \boxed{F_{AC_y}} - (30kg)\left(9.81\frac{m}{s^2}\right) = 0$$

$$l_{0AB} = 0.64m - s_{AB}$$
$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- We actually know the line of action of each of the two forces because we know that each force acts along the axis of the spring



$$\sum F_x = \boxed{F_{AB_x}} - \boxed{F_{AC_x}} = 0$$

$$\sum F_y = \boxed{F_{AB_y}} + \boxed{F_{AC_y}} - (30kg)\left(9.81\frac{m}{s^2}\right) = 0$$

$$l_{0AB} = 0.64m - s_{AB}$$
$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

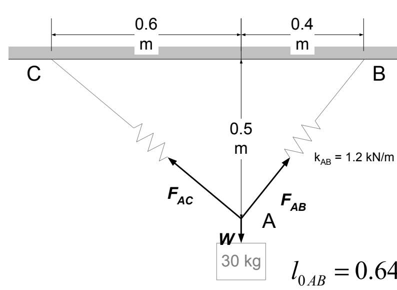
- Earlier in the problem we found the l_{AB} and l_{AC} so we can use these distances, the hypotenuse of each triangle to get more information

$$F_{AB_x} = \frac{0.4m}{0.64m}F_{AB}$$

$$F_{AC_x} = \frac{0.6m}{0.78m}F_{AC}$$

$$F_{AB_y} = \frac{0.5m}{0.64m}F_{AB}$$

$$F_{AC_y} = \frac{0.5m}{0.78m}F_{AC}$$



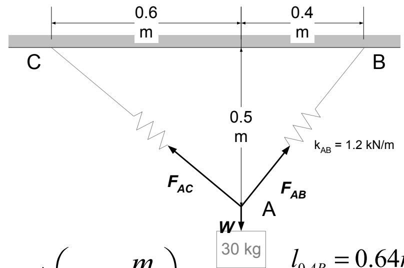
$$l_{0AB} = 0.64m - s_{AB}$$
$$l_{0AC} = 0.78m - s_{AC}$$

$$\sum F_x = \boxed{F_{AB_x}} - \boxed{F_{AC_x}} = 0$$

$$\sum F_y = \boxed{F_{AB_y}} + \boxed{F_{AC_y}} - (30kg)\left(9.81\frac{m}{s^2}\right) = 0$$

Problem Solution

- If we substitute this information we have two equations with two unknowns



$$\sum F_x = \frac{0.4m}{0.64m}F_{AB} - \frac{0.6m}{0.78m}F_{AC} = 0$$

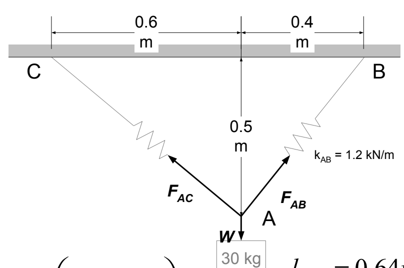
$$\sum F_y = \frac{0.5m}{0.64m}F_{AB} + \frac{0.5m}{0.78m}F_{AC} - (30kg)\left(9.81\frac{m}{s^2}\right) = 0$$

$$l_{0AB} = 0.64m - s_{AB}$$

$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- Solving the top equation for F_{AC}



$$F_{AC} = \frac{0.4m}{0.64m}F_{AB}\frac{0.78m}{0.6m} = 0.812F_{AB}$$

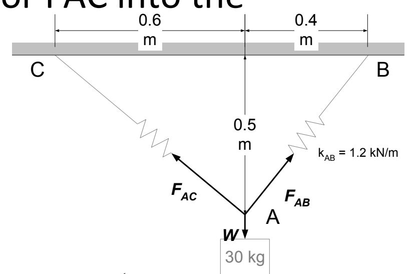
$$\sum F_y = \frac{0.5m}{0.64m}F_{AB} + \frac{0.5m}{0.78m}F_{AC} - (30kg)\left(9.81\frac{m}{s^2}\right) = 0$$

$$l_{0AB} = 0.64m - s_{AB}$$

$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- Substituting the value for FAC into the second equation



$$\sum F_y = \frac{0.5m}{0.64m}F_{AB} + \frac{0.5m}{0.78m}(0.812F_{AB}) - (30kg)\left(9.81\frac{m}{s^2}\right) = 0$$

$$F_{AB} = \frac{(30kg)\left(9.81\frac{m}{s^2}\right)}{\frac{0.5m}{0.64m} + \frac{0.5m}{0.78m}(0.812)}$$

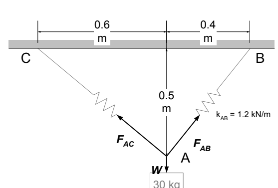
$$l_{0AB} = 0.64m - s_{AB}$$
$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- Substituting into the top equation the value for F_{AB} and solving for F_{AC}

$$F_{AB} = 226N$$

$$F_{AC} = 0.812F_{AB} = 183.5N$$



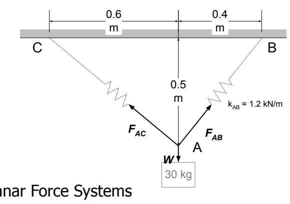
$$l_{0AB} = 0.64m - s_{AB}$$
$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- Using the force in each spring and the spring constant we can determine the elongation of each spring

$$s_{AB} = \frac{F_{AB}}{k_{AB}} = \frac{226N}{1200\,{}^{N}\!/\!{}_{m}} = 0.19m$$

$$s_{AC} = \frac{F_{AC}}{k_{AC}} = \frac{183.5N}{1500\,{}^{N}\!/\!{}_{m}} = 0.12m$$



$$l_{0AB} = 0.64m - s_{AB}$$

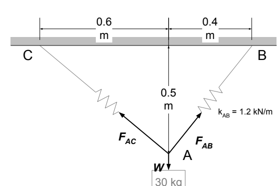
$$l_{0AC} = 0.78m - s_{AC}$$

Problem Solution

- Finally, substituting in to our length equations

$$l_{0AB} = 0.64m - 0.19m = 0.45m$$

$$l_{0AC} = 0.78m - 0.12m = 0.66m$$



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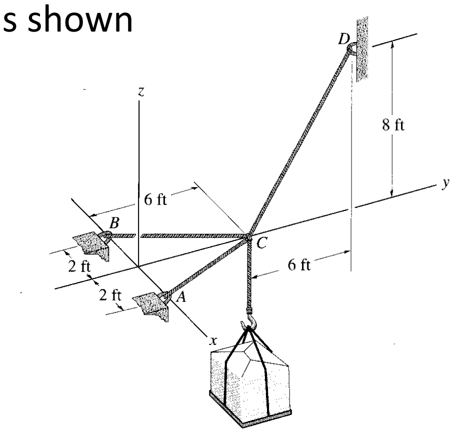
Problem Given

- Load is 500 lb
- System as shown



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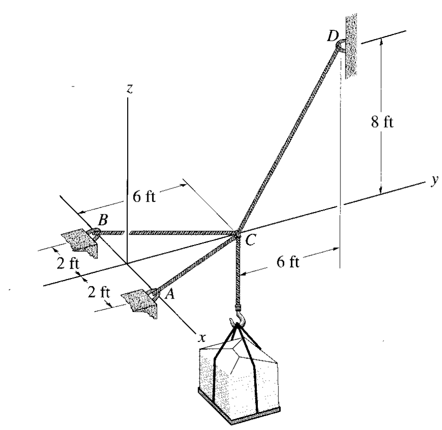
Problem Required

- Force in each cable



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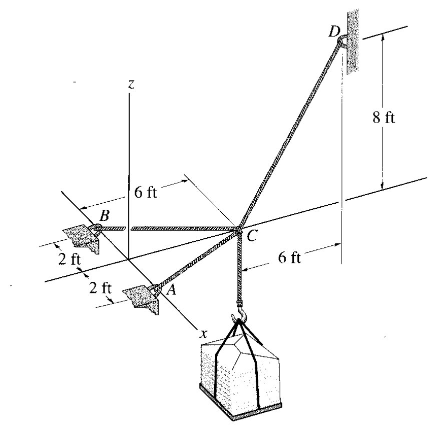
Problem Solution

- Forces are $\boldsymbol{F_{CB}}$, $\boldsymbol{F_{CA}}$, and $\boldsymbol{F_{CD}}$
- All the forces are generated by ropes



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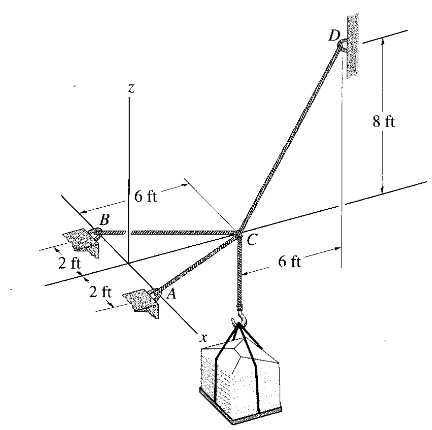
Problem Solution

- A FBD of the connection at C would include all the unknown forces we are looking for



Problem Solution

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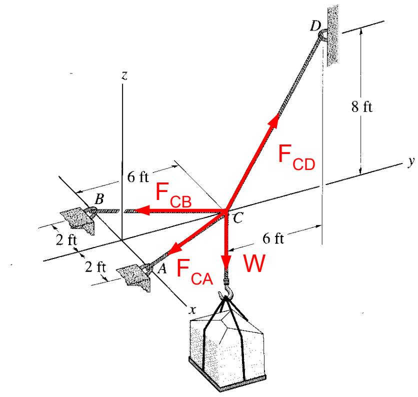
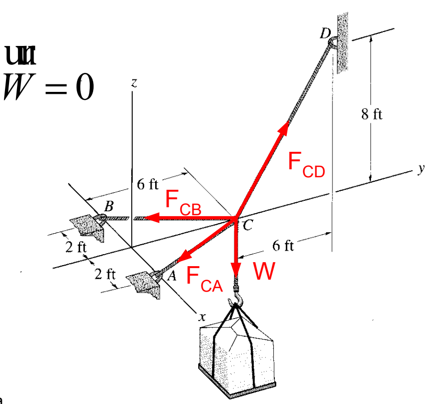


Problem Solution

- We can write our constraint for equilibrium

$$\vec{F} = 0$$

$$\vec{F} = \vec{F}_{CA} + \vec{F}_{CB} + \vec{F}_{CD} + \vec{W} = 0$$



Problem Solution

- Now we need to find the Cartesian vector representation for each force

$$\vec{F}_{CA} = F_{CA}\vec{u}_{CA}$$

$$\vec{u}_{CA} = \frac{\vec{P}_{CA}}{P_{CA}}$$

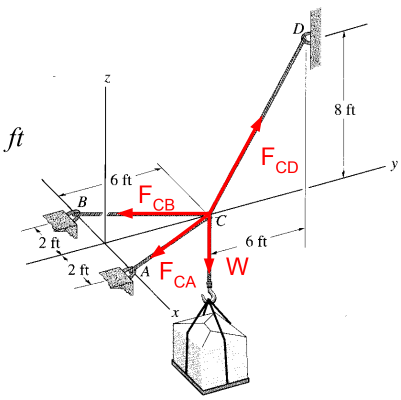
$$\vec{P}_{CA} = \{A\} - \{C\} = \{2,0,0\}\,ft - \{0,6,0\}\,ft$$

$$\vec{P}_{CA} = \left(2\vec{i} - 6\vec{j}\right) ft$$

$$P_{CA} = 6.32\,ft$$

$$\vec{u}_{CA} = \frac{\left(2\vec{i} - 6\vec{j}\right) ft}{6.32\,ft} = 0.32\vec{i} - 0.95\vec{j}$$

$$\vec{F}_{CA} = F_{CA}\left(0.32\vec{i} - 0.95\vec{j}\right)$$



Problem Solution

- Now we need to find the Cartesian vector representation for each force

$$\vec{F}_{CB} = F_{CB}\vec{u}_{CB}$$

$$\vec{u}_{CB} = \frac{\vec{P}_{CB}}{P_{CB}}$$

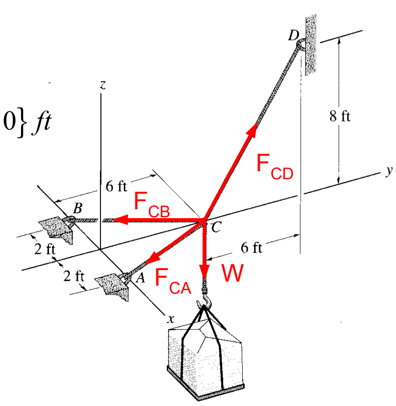
$$\vec{P}_{CB} = \{B\} - \{C\} = \{-2,0,0\}\,ft - \{0,6,0\}\,ft$$

$$\vec{P}_{CB} = \left(-2\vec{i} - 6\vec{j}\right) ft$$

$$P_{CB} = 6.32\,ft$$

$$\vec{u}_{CB} = \frac{\left(-2\vec{i} - 6\vec{j}\right) ft}{6.32\,ft} = -0.32\vec{i} - 0.95\vec{j}$$

$$\vec{F}_{CB} = F_{CB}\left(-0.32\vec{i} - 0.95\vec{j}\right)$$



Problem Solution

- Now we need to find the Cartesian vector representation for each force

$$\vec{F}_{CD} = F_{CD}\vec{u}_{CD}$$

$$\vec{u}_{CD} = \frac{\vec{P}_{CD}}{P_{CD}}$$

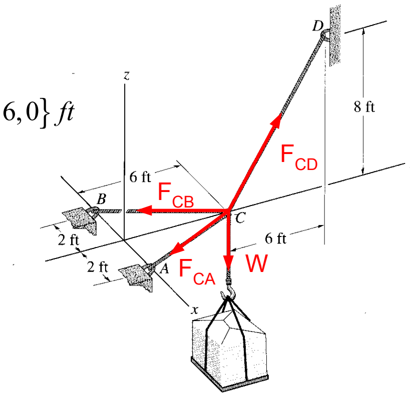
$$\vec{P}_{CD} = \{D\}-\{C\} = \{0,12,8\}\,ft - \{0,6,0\}\,ft$$

$$\vec{P}_{CD} = \left(6\vec{j}+8\vec{k}\right) ft$$

$$P_{CD} = 10.00\,ft$$

$$\vec{u}_{CD} = \frac{\left(6\vec{j}+8\vec{k}\right) ft}{10.00\,ft} = 0.6\vec{j}+0.8\vec{k}$$

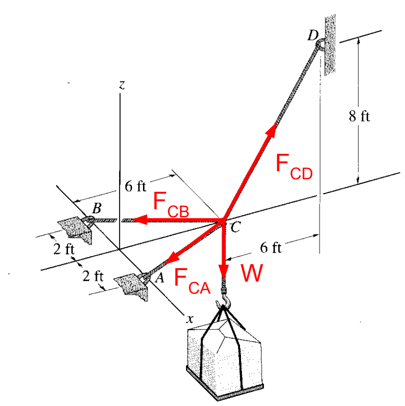
$$\vec{F}_{CD} = F_{CD}\left(0.6\vec{j}+0.8\vec{k}\right)$$



Problem Solution

- Now we need to find the Cartesian vector representation for each force

$$\vec{W} = 500lb\left(-\vec{k}\right)$$



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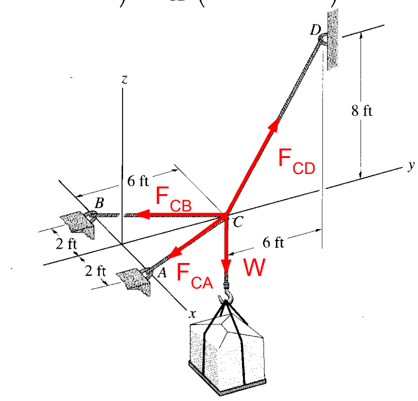
Problem Solution

- Summing the Cartesian vector representations for all the forces

$$F_{CA}\left(0.32\vec{i}-0.95\vec{j}\right)+F_{CB}\left(-0.32\vec{i}-0.95\vec{j}\right)+F_{CD}\left(0.6\vec{j}+0.8\vec{k}\right)+500lb\left(-\vec{k}\right)$$



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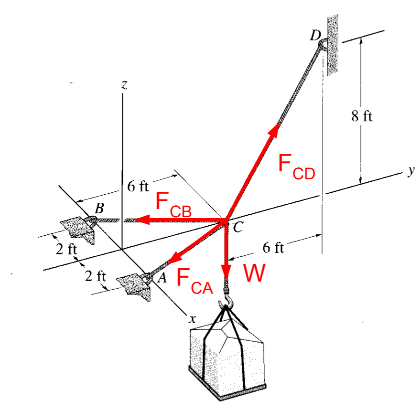
Problem Solution

- Collecting terms with like unit vectors

$$\left(0.32F_{CA}-0.32F_{CB}\right)\vec{i}+\left(-0.95F_{CA}-0.95F_{CB}+0.6F_{CD}\right)\vec{j}+\left(0.8F_{CD}-500lb\right)\vec{k}$$



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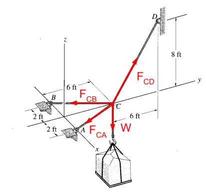
Problem Solution

- The coefficient of each unit vector must be equal to 0
- If it is not then we have an acceleration along that axis and we no longer have equilibrium
- This gives us three equations in three unknowns



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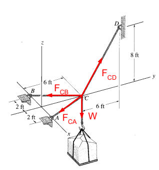
Problem Solution

- The equations are

$$\left(0.32F_{CA} - 0.32F_{CB}\right) = 0$$

$$\left(-0.95F_{CA} - 0.95F_{CB} + 0.6F_{CD}\right) = 0$$

$$\left(0.8F_{CD} - 500lb\right) = 0$$



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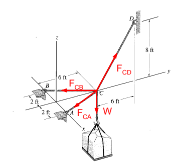
Problem Solution

- From the first equation

$$F_{CA} = F_{CB}$$
$$\left(-0.95F_{CA} - 0.95F_{CB} + 0.6F_{CD}\right) = 0$$
$$\left(0.8F_{CD} - 500lb\right) = 0$$



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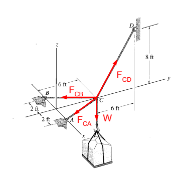
Problem Solution

- From the third equation

$$F_{CA} = F_{CB}$$
$$\left(-0.95F_{CA} - 0.95F_{CB} + 0.6F_{CD}\right) = 0$$
$$F_{CD} = \frac{500lb}{0.8} = 625lb$$



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Problem Solution

- Substituting the information from the first and third equation in the second equation

$$F_{CA} = F_{CB}$$

$$-0.95F_{CA} - 0.95F_{CA} + 0.6(625lb) = 0$$

$$F_{CA} = \frac{375lb}{1.9} = 197.37lb$$

$$F_{CB} = 197.37lb$$