FITE7405: Assignment 3
Group 7
Final Report

Group information

Member	Contributions
Lee Chung Ho (3036030037)	Asian option pricing, Basket option pricing, GUI development
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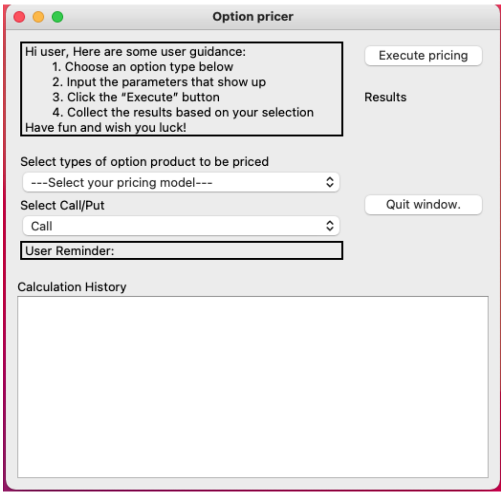
Wong Chun Ming (3036034851)	European option pricing, American option pricing, GUI development

User Interface Description

(a)


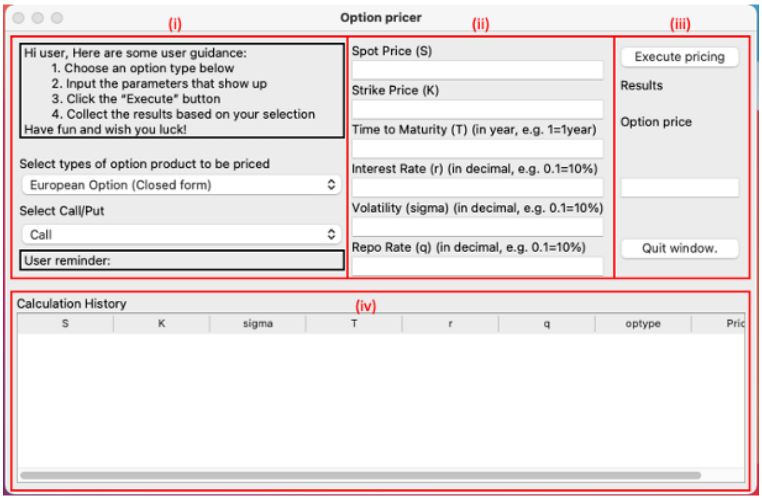
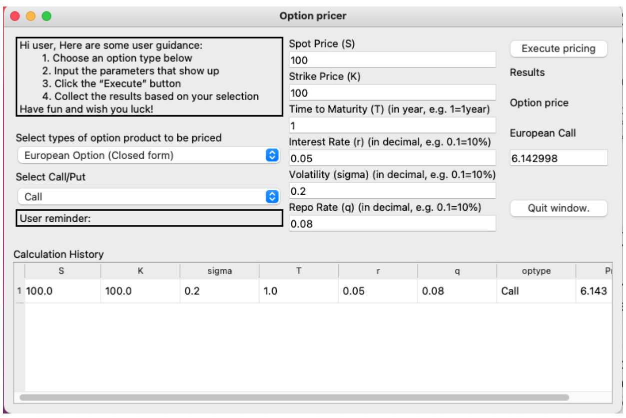
(b)


Figure 1 GUI (a) initial page, (b) after option selection

When any user runs the option pricer code, the user would see a GUI initial page like figure 1(a). If any option product is selected, input parameter boxes would show up. Illustrated in figure 1(b), there are 4 parts in the GUI:
(i): for selection of option type
(ii): for user input parameters
(iii): for the result display
(iv): for storing calculation history
Steps for using the option pricer:

1. Choose an option type in (i)
2. Input the parameters that show up in (ii)
3. Click the "Execute" button in (iii)
4. Collect the option price, confidence interval, or implied volatility depending on your selection in step one
5. A history of successful calculations is displayed in the table. Figure 2 (a) shows an example of it. Both input parameters and results are reported, which allows users to compare the pricing results for the same option product with different input parameters conveniently. The records are erased every time another option product is selected.

(a)


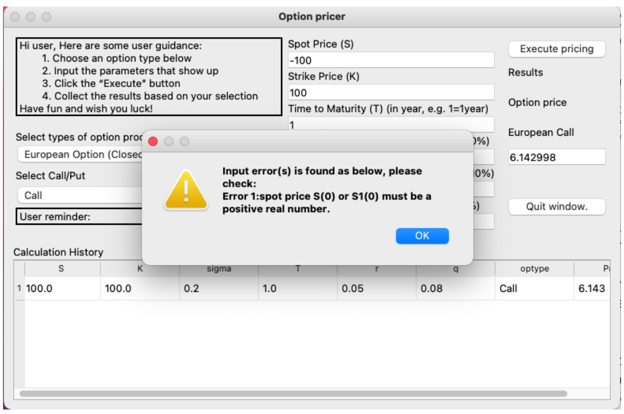
(b)


Figure 2 Example of (a) a successful calculation, (b) a failed calculation

There are some restrictions put on the input parameters. Figure 2(b) shows an example of a failed calculation. If these restrictions are violated, the GUI will generate a message box to users listing the needed amendments by points. Due to the length limit on this report, for detailed restrictions, please check the Annex submitted along with this report.

Functionalities of each class/function

Class MainWindow is inherited from class QMainWindow in PyQt5 to create a GUI for the option pricer. Figure 3 summarises the objectives and associated functions in each step. It consists of the following methods listed in the table below:

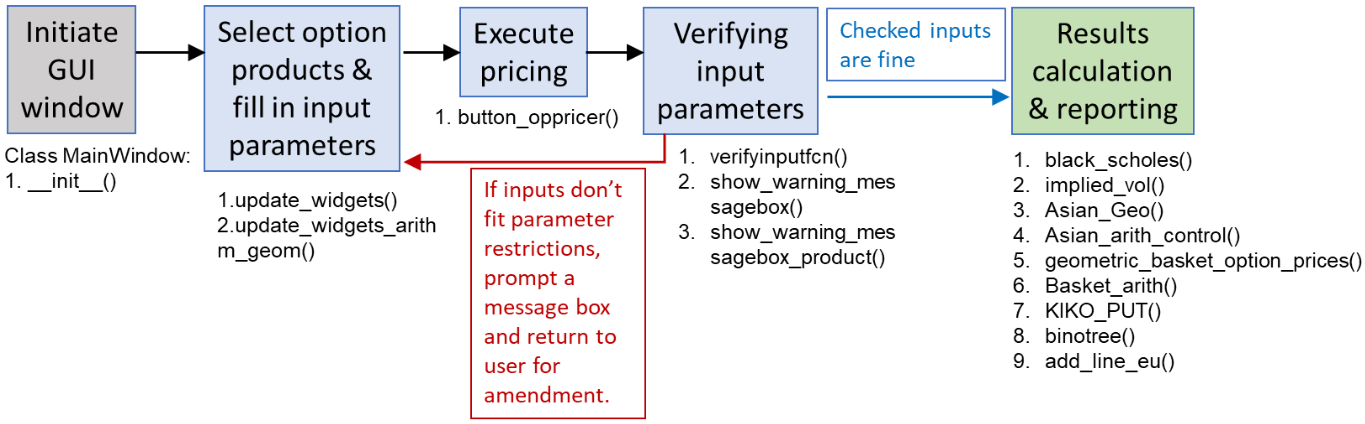


Figure3: Diagram showing the steps and corresponding functions in the minipricer

Method	description
__init__	It initialises the class. It also contains all widgets and layouts needed for the pricer.
show_warning_messagebox	It collects error message content from verifyinputfcn(), and shows the message in a popup message box.
show_warning_messagebox_product	It shows an error message in a popup message box when no option is selected.
update_widgets	It reads the current state of the option selection in the combo box widget, and shows only the necessary input widgets needed.
update_widgets_arithm_geom	For Asian options and Basket options pricing with Monte Carlo methods, this controls the displaying of two checkboxes related to Monte Carlo methods options.
button_oppricer	It collects all the input by the user and calls the option pricing methods below if the inputs are verified correct.
binotree	It executes the binomial tree method for American option pricing.
black_scholes	It executes the black scholes formula for European option pricing.
implied_vol	It executes Newton's method for implied volatility numerical estimation. It takes in the option price of an European option.
Asian_Geo	It executes the closed form formula of geometric asian option for n observation periods in a closed form analytical approach.
Asian_arith_control	It executes the Monte Carlo method of asian option for n observation periods. Both the Arithmetic and geometric pricing calculations are implemented. For the geometric asian options, user can only select a simple Monte Carlo method for the pricing. For the arithmetic asian options, user can select either the simple Monte Carlo method or the Monte Carlo method with control variate. Geometric basket option is used as the control variate. np.random.seed (7) is applied here.
geometric_basket_option_prices	It executes the closed form formula of geometric basket option for 2 assets in a closed form analytical approach.
Basket_arith	It executes the Monte Carlo method of basket option for 2 assets. Both the Arithmetic and geometric pricing calculations are implemented. For the geometric basket options, user can only select a simple Monte Carlo method for the pricing. For the arithmetic basket options, user can select either the simple Monte Carlo method or the Monte Carlo method with control variate. Geometric basket option is used as the control variate. np.random.seed (7) and (17) are applied here.
KIKO_PUT	It executes the quasi-Monte Carlo method of knock-in-knock-out put option with cash rebate. np.random.seed (1000) is applied here.
add_line_eu	It collects the input parameters and the results from different pricing and stores them in the output table for every calculation from the same option product classes.
verifyinputfcn	It verifies that the input of the user is in correct form. If not, it returns error messages back to show_warning_messagebox(self, msgcontent). The message box will list details for users to amend the input parameters in order to carry out appropriate calculations. A number of input parameter restrictions are added for checking and they are listed in Annex.

Test cases and analysis
In the cases for asian and basket options below, assume r = 0.05, T = 3, and S(0) = 100. The number of paths in Monte Carlo simulation is M = 100, 000. The first row is the base case, and the parameters bolded in blue is those differ from base case:

Asian options:

σ	K	n	Type	Price (Geom) Closed form	Price (Geom) MC	Conf. Int. (Geom)	Price (Arithm, w cov)	Conf. Int. (Arithm, w cov)	Price (Arithm, w/o cov)	Conf. Int. (Arithm, w/o cov)
0.3	100	50	Put	8.482705	8.461034	(8.387904 , 8.534163)	7.804651	(7.800230 , 7.809072)	7.784277	(7.715382 , 7.853173)
0.3	100	**100**	Put	8.431080	8.461173	(8.388045 , 8.534301)	7.749334	(7.744932 , 7.753735)	7.777594	(7.708778 , 7.846409)
0.4	100	50	Put	12.558769	12.534248	(12.437690 , 12.630806)	11.288150	(11.280384 , 11.295917)	11.265456	(11.175754 , 11.355157)
0.3	100	50	**Call**	13.259126	13.294320	(13.164325 , 13.424314)	14.734997	(14.723836 , 14.746158)	14.773983	(14.629550 , 14.918415)
0.3	100	**100**	**Call**	13.138779	13.150089	(13.021836 , 13.278341)	14.601504	(14.590559 , 14.612449)	14.614040	(14.471455 , 14.756626)
0.4	100	50	**Call**	15.759820	15.811000	(15.636123 , 15.985876)	18.220085	(18.198561 , 18.241610)	18.279179	(18.076115 , 18.482244)

Analysis: We can observe that with the same number of simulated paths, the confidence interval of covariate-controlled arithmetic option price is smaller than that of price without control variate, i.e. a fastest converging rate. Moreover, the geometric option price from closed form formula and Monte Carlo simulation are very close. When the number of observation periods (n) increases, the prices of both call and put, geometric and arithmetic asian options decrease. When the volatility of the asset (σ) increases, the prices of both call and put, geometric and arithmetic asian options increase.

Basket options:

S1(0)	S2(0)	K	σ1	σ2	ρ	Type	Price (Geom) Closed form	Price (Geom) MC	Conf. Int. (Geom)	Price (Arithm, w cov)	Conf. Int. (Arithm, w cov)	Price (Arithm, w/o cov)	Conf. Int. (Arithm, w/o cov)
100	100	100	0.3	0.3	0.5	Put	11.491573	11.490518	(11.393118 , 11.587918)	10.569526	(10.557294 , 10.581758)	10.568513	(10.474165 , 10.662862)
100	100	100	0.3	0.3	**0.9**	Put	12.622350	12.611416	(12.505710 , 12.717122)	12.428131	(12.425381 , 12.430882)	12.417266	(12.312189 , 12.522343)
100	100	100	**0.1**	0.3	0.5	Put	6.586381	6.593876	(6.527541 , 6.660210)	5.515702	(5.507094 , 5.524309)	5.522084	(5.464941 , 5.579228)
100	100	**80**	0.3	0.3	0.5	Put	4.711577	4.706982	(4.649071 , 4.764893)	4.252905	(4.245263 , 4.260546)	4.248568	(4.193371 , 4.303765)
100	100	**120**	0.3	0.3	0.5	Put	21.289105	21.302720	(21.166674 , 21.438766)	19.876759	(19.860403 , 19.893114)	19.890021	(19.756487 , 20.023556)
100	100	100	**0.5**	**0.5**	0.5	Put	23.469148	23.483821	(23.333890 , 23.633752)	21.068674	(21.040364 , 21.096983)	21.082708	(20.936538 , 21.228878)
100	100	100	0.3	0.3	0.5	**Call**	22.102093	21.857435	(21.633547 , 22.081322)	24.502348	(24.471017 , 24.533679)	24.243756	(24.005051 , 24.482462)
100	100	100	0.3	0.3	**0.9**	**Call**	25.878826	25.559873	(25.291193 , 25.828553)	26.359500	(26.353096 , 26.365904)	26.037164	(25.765556 , 26.308772)
100	100	100	**0.1**	0.3	0.5	**Call**	17.924737	17.803099	(17.649932 , 17.956265)	19.449661	(19.430026 , 19.469295)	19.314047	(19.142155 , 19.485939)
100	100	**80**	0.3	0.3	0.5	**Call**	32.536256	32.288058	(32.035316 , 32.540800	35.397788	(35.365384 , 35.430193)	35.137970	(34.871416 , 35.404525)
100	100	**120**	0.3	0.3	0.5	**Call**	14.685466	14.455477	(14.263798 , 14.647156)	16.596618	(16.566957 , 16.626278)	16.351105	(16.144347 , 16.557862)
100	100	100	**0.5**	**0.5**	0.5	**Call**	28.449387	27.942405	(27.543963 , 28.340847)	35.022166	(34.913252 , 35.131079)	34.445892	(33.980077 , 34.911708)

Analysis: We can observe that with the same number of simulated paths, the confidence interval of covariate-controlled arithmetic option price is smaller than that of price without control variate, i.e. a fastest converging rate. Moreover, the geometric option price from closed form formula and Monte Carlo simulation are very close. When the price correlation between the two assets(ρ) increases, the prices of both call and put, geometric and arithmetic basket options increase. When the strike price of the basket option decreases, the prices of geometric and arithmetic basket PUT options decrease, but the prices of geometric and arithmetic basket CALL options increase. When the strike price of the basket option increases, the prices of geometric and arithmetic basket

PUT options increase, but the prices of geometric and arithmetic basket CALL options decrease. When the volatilities of the assets (σ1 and σ2) increase, the prices of both call and put, geometric and arithmetic basket options increase.

KIKO Put Option with Rebate:

Row	S(0)	T	σ	K	r	n	L	U	CR	Price	Conf. Int.	Effect on price (cf. base)
1 (base)	100	2	0.2	100	0.05	24	80	125	1.5	6.009210	(5.945353 , 6.073067)	/
2	**110**	2	0.2	100	0.05	24	80	125	1.5	3.705766	(3.658416 , 3.753116)	↓
3	100	**3**	0.2	100	0.05	24	80	125	1.5	6.688497	(6.620122 , 6.756873)	↑
4	100	2	**0.4**	100	0.05	24	80	125	1.5	14.053902	(13.932055 , 14.175749)	↑
5	100	2	0.2	**110**	0.05	24	80	125	1.5	8.435391	(8.349922 , 8.520861)	↑
6	100	2	0.2	100	**0.10**	24	80	125	1.5	3.542101	(3.498425 , 3.585778)	↓
7	100	2	0.2	100	0.05	**100**	80	125	1.5	6.170768	(6.107108 , 6.234428)	↑
8	100	2	0.2	100	0.05	24	**100**	125	1.5	6.958510	(6.895335 , 7.021685)	↑
9	100	2	0.2	100	0.05	24	80	**150**	1.5	5.726914	(5.661511 , 5.792318)	↓
10	100	2	0.2	100	0.05	24	80	125	**3.0**	6.638689	(6.576433 , 6.700945)	↑

Analysis: Similar to a vanilla put option, an increase in the interest rate and the underlying price will cause the KIKO put's price to decrease. An increase in time to maturity, volatility, and strike price will lead to an increase in the option price. An increase in the no. of observation period does not cause a significant change in the option price because as the sample sizes increase, the option price should converge to the true value. When the lower barrier increases, the option price increases because it is easier for the option to knock in and become effective. When the upper barrier increases, the option price decreases.When the cash rebate increases, the option value increases because the option holder can receive more when the option knocks out.

Implied Volatility (Only cases with option prices within theoretical range between intrinsic value and discounted asset/strike value). Theoretical range: Call options: $C>max(Se^{-q(T-t)}-Ke^{-r(T-t)},0)$ and $C<Se^{-q(T-t)}$, Put options: $P>max(Ke^{-r(T-t)}-Se^{-q(T-t)},0)$ and $P<Ke^{-r(T-t)}$

Row	S(0)	K	T	r	q	Call Price	Implied volatility (Call)	Effect (cf. base)	Put Price	Implied volatility (Put)	Effect (cf. base)
1 (base)	100	100	1	0.05	0.1	10	0.3305	/	10	0.2017	/
2	**105**	100	1	0.05	0.1	10	0.2660	↓	10	0.2629	↑
3	100	**105**	1	0.05	0.1	10	0.3766	↑	10	0.0907	↓
4	100	100	**2**	0.05	0.1	10	0.2877	↓	10	0.0911	↓
5	100	100	1	**0.08**	0.1	10	0.2998	↓	10	0.2490	↑
6	100	100	1	0.05	**0.15**	10	0.3920	↑	10	0.1086	↓
7	100	100	1	0.05	0.1	**20**	0.6102	↑	**20**	0.4796	↑

Note1: In the case of S(0)=K with r=q, due to a zero initial sigma derived, Newton-Raphson method cannot converge to a solution.
Note2: The option price should not be out of the theoretical range due to non-arbitrage principle.
Analysis:When the underlying asset price increases, the implied volatility of a call option will decrease while that for a put option will increase. When the strike price increases, the implied volatility of a call option will increase while that for a put option will decrease. When the time to maturity increases, the implied volatility for both call and put option will decrease. The interest rate has a favorable effect on the implied volatility of the put option but not a call option. In contrast, the repo rate has a favorable effect on the implied volatility of the call option but not a put option. When both the call and put prices are double, the implied volatility of the put option will record a larger extent of increase.

European option with repo rate:

Row	S(0)	T	Sigma	K	r	q	Call Price	Effect on call price (cf. base)	Put price	Effect on put price (cf. base)
1 (base)	100	3	0.3	100	0.05	0.1	11.0827	/	23.0717	/
2	**120**	3	0.3	100	0.05	0.1	19.3809	↑	16.5535	↓
3	100	3	0.3	**110**	0.05	0.1	8.8376	↓	29.4337	↑

4	100	3	0.3	100	**0.08**	0.1	13.4668	↑	18.0478	↓
5	100	3	**0.4**	100	0.05	0.1	16.1889	↑	28.1779	↑
6	100	**4**	0.3	100	0.05	0.1	11.071	↓ (mixed)	25.9121	↑
7	100	**2**	0.3	100	0.05	0.1	10.5662	↓ (mixed)	19.1769	↓
8	100	**3**	0.3	100	0.05	**0.2**	3.7385	↓	34.9281	↑
9	100	**1**	0.3	100	0.05	**0.2**	5.2067	(mixed) (cf. row 8)	18.4566	↓ (cf. row 8)
10	100	**4**	0.3	100	0.05	**0.2**	2.9468	(mixed) (cf. row 8)	39.887	↑ (cf. row 8)

Analysis: For European options with non-zero repo rate, conditional on holding other input parameters constant, the effect of a change in S(0) (or Sigma, or K, or r) on CALL (PUT) option prices will be similar to those as in the case of European options with no repo rate (see rows (2) to (5) in above Table). For repo rate q, holding other parameters constant, higher q will decrease CALL option price but increase PUT option prices (row(8)). For time-to-maturity T, as shown in rows (6)-(7) and (8)-(10), holding other parameters constant, the effect of increasing T on CALL option price will depend on the difference between r and q, so there will be no clear direction about the effect on the CALL option prices when T increases or decreases. For PUT option, holding other parameters constant, increasing T will tend to increase put option prices.

American Option

Row	S(0)	T	Sigma	K	r	N-step	Type	Put price	Effect on put price (cf. base)
1 (base)	50	2	0.4	50	0.1	200	Put	7.4676	/
2	50	2	0.4	**70**	0.1	200	Put	20.8314	↑
3	50	2	0.4	50	0.1	**10**	Put	7.3286	less precise
4	50	2	0.4	50	0.1	**100**	Put	7.4622	less precise
5	50	2	0.4	50	0.1	**1000**	Put	7.4724	more precise
6	**60**	2	0.4	50	0.1	200	Put	4.7657	↓
7	50	**3**	0.4	50	0.1	200	Put	8.3200	↑
8	50	2	**0.5**	50	0.1	200	Put	9.8621	↑
9	50	2	0.4	50	**0.2**	200	Put	5.4103	↓

Analysis: For American CALL options without dividend paying, it will be the same as vanilla European CALL options without dividend since it is not optimal to exercise CALL options earlier than at maturity. Therefore, we will test and analyse only American PUT options without dividend paying with respect to different parameters. Based on rows (1) to (2), a higher strike price K will increase the American PUT option prices. Based on row (1), (3) to (5), as the number of tree steps N-steps increases, the PUT option price will be more precise. The marginal improvement of increasing N-steps on PUT option price becomes smaller when N-steps reach 100 or above. Similar to vanilla European PUT options, higher S(0) (row (6)) or higher r (row (9)) will lower American PUT option prices, while higher T (row (7)) and Sigma (row (8)) will increase American PUT option prices.

Further analysis on KIKO put option prices: As the payoff of KIKO function is not smooth, the change of option price over some parameters must not be monotonic. We plotted the price change against the upper barrier in figure 4 below. Parameters other than U, T, CR are held as the base case. The intuition of this analysis is that If the cash rebate is low, we do not want the option to get knocked out, but if the CR is high, we want it knocked out more. The figure shows that under different CR, the price of a KIKO can be strictly decreasing/ increasing/ initially increasing then decreasing. But they converge to some point, where the option never get knocked out. Although they converge to some point of "never knocked out". But if the time to maturity is increased, it is still possible for it to get knocked out. So, we take into account the Time to maturity in the plot. And it shows that the convergence happens much faster with a shorter T. We do not consider the lower barrier, because when the lower barrier decreases, the price decreases and converges to some point where the option may never get knocked in. So, the profit only comes from asset paths that got knocked out. Eventually, the effect goes back to the upper barrier.