Available online at www.sciencedirect.com
SCIENCE DIRECT®
Int. J. Production Economics 86 (2003) 1–10
international journal of production economics
www.elsevier.com/locate/dsw

A JIT lot-splitting model for supply chain management: Enhancing buyer–supplier linkage

Seung-Lae Kim[a,*], Daesung Ha[b]

[a] Department of Decision Sciences, Bennett S. LeBow College of Business, Drexel University, Philadelphia, PA 19104, USA
[b] Division of Management and Marketing, Lewis College of Business, Marshall University, Huntington, WV 25755, USA

Received 10 September 2001; accepted 19 August 2002

Abstract

This study develops a buyer–supplier coordination model to facilitate frequent deliveries in small lot sizes in a manufacturing supply chain. The proposed model, based on the integrated total relevant costs of both buyer and supplier, determines optimal order quantity, the number of deliveries/setups, and shipping quantity over a finite planning horizon in a relatively simple JIT single buyer single supplier scenario. Under deterministic conditions for a single product, we show that the optimal delivery policy adopted by both buyer and supplier in a cooperative manner can be economically beneficial to both parties. It is shown that the optimal delivery size can be unique, regardless of the order quantity and the number of deliveries. Numerical results are also presented.

Keywords: Just-in-time manufacturing; Lot-splitting strategy; Buyer–supplier linkage; Integrated inventory model

1. Introduction

Since the importance of just-in-time (JIT) was recognized in the early 1980s, there have been numerous studies discussing implementation of JIT and its effectiveness in US manufacturing firms from various dimensions. White et al. (1999) investigated both large and small US manufacturers to see which had greater differences and improvements in performance due to JIT implementation. Their study was conducted based on the set of ten JIT management practices identified by White et al. (1990). The ten JIT practices were: quality circles, total quality control, focused factory, total productive maintenance, reduced setup times, group technology, uniform workload, multifunction employees, Kanban, and JIT purchasing.

Although there has been a consensus on the notion that JIT is an overall organizational phenomenon and the greatest possible gains from JIT can be achieved when JIT practices operate as an integrated system (for example, see Sakakibara et al., 1997), the JIT purchasing practice has attracted more attention than any other practices from academicians and practitioners. As defined in White et al. (1999), the objective of

*Corresponding author. Tel.: +215-895-2181; fax: +215-895-2907.
E-mail addresses: kimsl@Drexel.edu (S.-L. Kim), ha@marshall.edu (D. Ha).

0925-5273/03/$ - see front matter
doi:10.1016/S0925-5273(03)00006-9

JIT purchasing is to improve quality, flexibility and levels of service from suppliers by developing a buyer–supplier long-term coordination based on mutual trust.

In recent years, researchers have extensively studied small lot sizing as a means of implementing successful JIT purchasing, with the buyer–supplier coordination focusing on material flows with an objective of minimizing supply chain costs. Small lot sizing improves the system's productivity by obtaining lower levels of inventory and scrap, lower inspection costs for incoming parts, and earlier detection of defects, etc., even though possible higher delivery costs and loss of discount rates may be incurred. In general, implementation of frequent deliveries in small lots requires firms to reduce the number of suppliers (even to a single supplier). Otherwise, the potential strength of relationship between buyer and supplier would be weakened. Here, the supplier is viewed as part of a team, providing certified quality material at lower costs, rather than as an opponent who is consistently seeking short-term price breaks in an adversarial bargaining process. The supplier and buyer work in a cooperative manner to synchronize supply with actual customer demand. In this scenario, it could be more reasonable to determine the order quantity and the delivery schedule based on their integrated total cost function, rather than using the buyer's or the supplier's individual cost functions.

The idea of joint optimization for buyer and supplier was initiated by Goyal (1976) and later reinforced by Monahan (1984), Lal and Staelin (1984), Lee and Rosenblatt (1986), Banerjee (1986a, b), Joglekar (1988), and Dada and Srikanth (1987). Goyal and Gupta (1989) provides a review of many integrated models for buyer–supplier coordination. While these studies focused on joint lot sizing and buyer–supplier coordination, the issue of frequent delivery in small quantities was overlooked. Taking a different path, Pan and Liao (1989), Larson (1989), and Ramaseshi (1990) developed EOQ-based models to discuss the effect of frequent deliveries on total costs. Their studies, however, failed to consider the issue of coordination from an integrated standpoint. Recently, Aderohunmu et al. (1995), Lu (1995), Banerjee and Kim (1995), and Hill (1997) discussed the benefits of multiple deliveries for a single order in an integrated inventory model, showing that a cooperative batching policy can significantly reduce total costs in a JIT environment. It should be noted that all these works assumed that the supplier could start shipping even before completing the entire lot as soon as the production quantity becomes greater than the shipping size.

The purpose of this study is to develop a JIT lot-splitting model that deals with buyer–supplier coordination. We limit our discussion to a simple JIT environment, i.e., single buyer and single supplier, under deterministic conditions for a single product. Comparing integrated total costs, we examine the benefits of the proposed JIT lot-splitting policy of facilitating multiple deliveries over the lot-for-lot delivery policy. We show that regardless of the size of order quantity, the delivery size converges to an optimal size that can be used as a basis for determining a standard transportation vehicle size.

The study is organized as follows: In Section 2, the rationale of the model, assumptions and notation are provided. Section 3 develops a lot-splitting (single setup, multiple deliveries) model and discusses how and when the optimal policy for buyer and supplier can be achieved. The conventional lot-for-lot model is also considered as a special case of multiple deliveries. In Section 4, numerical results are presented. Conclusions and implications of the results are summarized in Section 5.

2. Rationale of the model: Assumptions and notation

The total cost for an integrated inventory model includes all costs from both buyer and supplier. The buyer's total cost consists of ordering cost, holding cost, transportation and order receiving costs incurred due to multiple deliveries. The supplier's cost includes holding cost and setup and order handling costs. In our model, the buyer is assumed to pay transportation and order handling costs in order to facilitate frequent deliveries. In fact, the buyer's payment of transportation and order receiving costs can be viewed as an investment for the sake of streamlining inventory. When the buyer places an order, in a JIT environment, the supplier splits the order quantity into small lot sizes and delivers them over multiple periods. The supplier then needs to hold the inventory throughout the production of the order quantity.

As will be demonstrated, an integrated approach allows the buyer and the supplier to reduce their total costs as compared to non-integrated approach. Both parties in some equitable fashion can share savings resulting from cost reduction. Goyal (1976) suggested sharing it according to the ratio of the buyer's and supplier's total costs determined independently. Later, Joglekar and Tharthare (1990) presented the individually responsible and rational decision (IRRD) approach in which benefits are given in the form of cost savings. The supplier reduces its cost by imposing shipping and order handling costs on the buyer, and in turn, the buyer receives a unit price discount because of large order quantities over the contract period. Thus, both parties in the process of price negotiation share the savings in the total costs occurring in their model. However, Goyal and Srinivasan (1992) identified some conceptual issues in the IRRD approach. They stated that, unlike the joint integrated approach, the IRRD approach is likely to succeed only in certain situations. One such situation is when there is an initial error in the recognition of costs. The supplier can then dictate and offer a price reduction in return for the buyer's agreement to pay the cost of handling and processing the order.

We assume that the unit price is negotiated and fixed when the buyer and the supplier commit themselves to their long-term contract. With a fixed unit price, both parties cooperate as a team by exchanging necessary information, e.g., unit holding cost, demand rate, production rate and setup time. While this information is assumed to be fixed, it must be shared between parties to gain maximum benefit of the supply–chain relationship. The supplier, in this process, often takes a central decision making role with all necessary information for the purpose of a vendor-managed inventory system (VMI). See Schniederjans and Olson (1999). The savings obtained through the cooperation may be given in the form of one-time rewards or distributed uniformly to each party over the contract period.

Once the long-term contract is set up, the demand information and inventory position of the buyer are given to the supplier. The total demand rate, production rate, and delivery times are assumed to be constant and deterministic. It is also assumed that all cost parameters including unit price are known and constant, and no quantity discount is assumed. Backordering is not allowed, and the following notations are adopted:

A	ordering cost for buyer
C	supplier's hourly setup cost
D	annual demand rate for buyer
F	fixed transportation cost per trip
H_B	holding cost/unit/year for buyer
H_S	holding cost/unit/year for supplier, $H_B > H_S$
N	number of deliveries per batch cycle (integer value)
P	annual production rate for supplier, $P > D$
Q	order quantity for buyer
q	delivery size per trip, $q = Q/N$
S	setup time/setup for supplier
V	unit variable cost for order handling and receiving

3. Single-setup-multiple-delivery (SSMD) model

In this section, we consider the single-setup-multiple-delivery (SSMD) model under which the buyer's order quantity is manufactured at one setup and shipped in equal amounts over multiple deliveries. This approach of splitting the order quantity into multiple small lots is consistent with the JIT implementation.

We first develop total cost functions and determine the order quantity and the number of deliveries that minimize the integrated total cost of the SSMD policy. Second, we show that the delivery size converges to a unique optimal delivery size even when the order quantity and the number of deliveries vary.

Without loss of generality, we assume the multiple deliveries are to be arranged in such a way that each succeeding delivery arrives at the time that all inventories from the previous delivery have just been depleted. In order to get a feel for the number of deliveries necessary to complete the order quantity, we consider a typical case of the SSMD policy. In this example, the production time, Q/P, is longer than three times the depletion time, q/D, but shorter than four times it, i.e., $3q/D < Q/P < 4q/D$. Fig. 1 depicts this scenario.

The top half of Fig. 1 shows the buyer's inventory level, while the bottom half displays the supplier's.

The total cost for the buyer is composed of ordering cost, holding cost, and transportation and order receiving cost:

$$\mathrm{TC}(Q,N)_{\text{Buyer}} = \frac{D}{Q}A + \frac{Q}{2N}H_{\mathrm{B}} + \frac{DN}{Q}\left(F + V\frac{Q}{N}\right). \tag{1}$$

The supplier's total cost consists of setup cost and holding cost:

$$\mathrm{TC}(Q,N)_{\text{Supplier}} = \frac{D}{Q}CS + \frac{QH_{\mathrm{S}}}{2N}\left\{(2-N)\frac{D}{P} + N - 1\right\}, \tag{2}$$

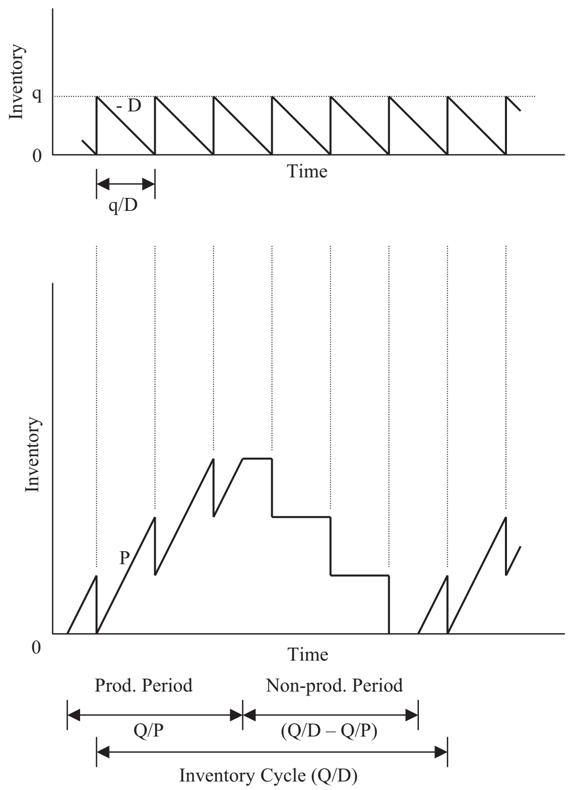

Fig. 1. Inventory time plot for SSMD model (one setup six deliveries).

where the expression for the holding cost was derived by Joglekar (1988). Adding Eqs. (1) and (2) yields the aggregate total cost function for buyer and supplier as follows:

$$\mathrm{TC}(Q,N)_{\mathrm{Aggregate}} = \frac{D}{Q}(A+CS) + \frac{Q}{2N}\left[H_{\mathrm{B}} + H_{\mathrm{S}}\left\{\frac{(2-N)D}{P} + N - 1\right\}\right] + \frac{DN}{Q}F + DV. \tag{3}$$

Notice that $N=1$ changes Eqs. (1)–(3) to the single delivery policy, which is a special case of the SSMD policy. It can easily be shown that the Hessian matrix of Eq. (3) is positive definite. This ensures that the total cost function in Eq. (3) is jointly convex.

3.1. Optimization

From the aggregate total cost in Eq. (3), we now determine the optimal order quantity and the optimal number of deliveries. By taking the first derivatives of Eq. (3) with respect to N and Q, setting them equal to zero, and solving for N and Q simultaneously, we obtain the following formulas:

$$N^* = \sqrt{\frac{(A+CS)\{P(H_{\mathrm{B}} - H_{\mathrm{S}}) + 2DH_{\mathrm{S}}\}}{F(P-D)H_{\mathrm{S}}}}$$

and

$$Q^* = \sqrt{\frac{2D(A+CS)}{H_{\mathrm{S}}(1-D/P)}}. \tag{4}$$

Since the number of deliveries is an integer greater than or equal to one and the order quantity is positive, optimization results in Eq. (4) indicate that the supplier's capacity P must be greater than D. Thus, the buyer's annual demand can be considered the lower boundary of the supplier's capacity. Let us assume that $H_{\mathrm{B}} > H_{\mathrm{S}}$, since cost (and value) is added as a product is moved downstream, leading to higher holding costs.

If N^* in Eq. (4) is not an integer, we choose N, which yields $\min\{\mathrm{TC}(N^+), \mathrm{TC}(N^-)\}$ in Eq. (3), where $N^+$and N^- represent the nearest integers larger and smaller than the optimal N^*. Substituting $N^*$and Q^* into Eq. (3), the minimum annual aggregate total cost is obtained as

$$\mathrm{TC}(N^*, Q^*)_{\mathrm{Aggregate}} = \sqrt{2D(A+CS)H_{\mathrm{S}}\left(1-\frac{D}{P}\right)} + \sqrt{2DF\left(H_{\mathrm{B}} - H_{\mathrm{S}} + \frac{2H_{\mathrm{S}}D}{P}\right)} + DV. \tag{5}$$

3.2. Minimum order quantity

We now look for the required order quantity that makes the SSMD policy superior to the single-delivery policy. Any savings from implementing the SSMD policy over the single-delivery policy can be obtained by subtracting Eq. (3) from Eq. (3) in which N is replaced by 1 as

$$\mathrm{SV}(Q,N) = \frac{Q}{2}\left\{H_{\mathrm{B}} + H_{\mathrm{S}}\left(\frac{2D}{P} - 1\right)\right\}\left(1 - \frac{1}{N}\right) + \frac{D}{Q}F(1-N). \tag{6}$$

Note that when $N=1$, the savings vanish. It can be shown that Eq. (6) is concave and increasing at a diminishing rate over the entire range of order quantity. This implies that the larger the order quantity is, the more benefit both parties can get through their long-term contract. The minimum order quantity $Q_{\min}$ that makes the SSMD policy favorable over the single-delivery policy is found by solving

$\mathrm{SV}(Q,N) \geqslant 0$ for Q as

$$Q \geqslant \sqrt{\frac{2DFNP}{P(H_B - H_S) + 2DH_S}} = Q_{\min}. \tag{7}$$

As shown in Eq. (7), $Q_{\min}$ is non-linear but monotonically increasing in N.

For a given order quantity ($Q \geqslant Q_{\min}$), savings increase as N reaches the optimal number of deliveries N_{opt} and then decrease until N arrives at the maximum number of deliveries $N_{\max}$ at which any savings from the SSMD policy completely vanish. From Eq. (6), $N_{\max}$ can be obtained as $(N_{\mathrm{opt}})^2$, where $N_{\mathrm{opt}} = Q/q^*$. (The q^* is the optimal delivery size defined in Eq. (8). And the N_{opt} is the same as N^* defined in Eq. (4) if the order quantity is Q^*.)

Using the same data used in Section 4, Fig. 2 depicts the savings function for $H_B > H_S$. As shown, when $H_B > H_S$, the SSMD policy warrants more frequent deliveries and thus more benefits than in the single delivery case. This is intuitively true because when the buyer has a higher holding cost than the supplier, the buyer might request that the supplier deliver the order quantity in small lots more often. As a result, both sides can minimize the system's on-hand inventories and maximize their savings as a whole. The preceding discussion can be summarized as follows:

Fact 1. *(a) For a given $Q \geqslant Q_{\min}$, the SSMD policy always yields less aggregate total cost for any lot size than the single-delivery policy for any number of deliveries $N < N_{\max}$. (b) The SSMD policy is more beneficial over single-delivery policy when $H_B > H_S$, all else held equal.*

3.3. Delivery size

The optimal delivery size q^*, which remains the same over multiple deliveries, is obtained by dividing Q^* by N^* from Eq. (4), as follows:

$$q^* = \sqrt{\frac{2DFP}{P(H_B - H_S) + 2DH_S}}. \tag{8}$$

To examine the properties of q^*, we first look at the relationships among the variables Q, N, and q. Given q defined in Eq. (8), the order quantity, Q, should vary within a range to make the number of deliveries an

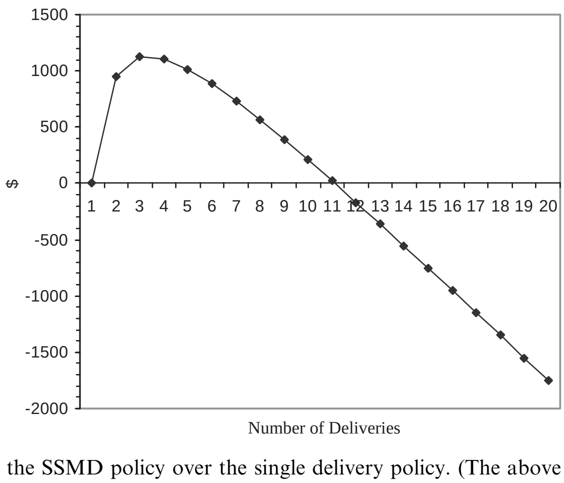

Fig. 2. Savings from implementing the SSMD policy over the single delivery policy. (The above savings graph shows that when the buyer's holding cost is \$7 and the supplier's holding cost is \$6.)

integer. Hence the number of deliveries $N = Q/q$ is an integer, and as such, the range of the order quantity that justifies an integer N is defined as

Theorem 1. $q^*\sqrt{N(N-1)} \leqslant Q \leqslant q^*\sqrt{N(N+1)}$, *for* $N \geqslant 2$.

The proof of Theorem 1 is provided in Appendix A. It can easily be seen from the theorem that the minimum order quantity for the number of deliveries of $N+1$ is greater than the upper limit for the number of deliveries of N by one unit, i.e., $q^*\sqrt{N(N+1)}+1$, and remains unchanged until the order quantity becomes large enough for $N+2$. Similarly, the maximum order quantity for the number of deliveries of $N-1$ is $q^*\sqrt{N(N-1)}-1$, and remains the same until the order quantity becomes small enough for $N-2$. Since the order quantity varies within the range as in Theorem 1, the delivery size will also vary. By dividing each term of the theorem by the number of deliveries, we obtain

$$q^*\sqrt{1-\frac{1}{N}} \leqslant \frac{Q}{N} \leqslant q^*\sqrt{1+\frac{1}{N}} \tag{9}$$

for $N \geqslant 2$. It is important to note from Eq. (9) that as N increases, both the upper and lower bounds for Q/N converge to q^*. Remember that Q increases within a range defined in *Theorem 1* as N increases. Thus the delivery size Q/N varies, but by the squeezing theorem (Ellis and Gulick, 1982, p. 73), it converges to the unique optimal q^* as N increases. We now summarize the preceding discussion on q^* into the following corollary without a formal proof.

Corollary 1. *The delivery size, q, converges to the unique optimal delivery size,* q^*, *in Eq.* (8), *as the order quantity, Q, and the number of deliveries, N, increase.*

In order to see the behavior of the delivery size Q/N, we present the results of numerical analysis in Fig. 3 based on the data used in the Section 4.

The saw tooth diagram in Fig. 3 shows different ranges of delivery size appropriate to a different number of deliveries. Starting from two deliveries ($N=2$), each vertical line segment indicates a transition of a delivery frequency to the next (higher) delivery frequency, such as N to $N+1$. Given the assumed parameter values, we searched for the optimal values of N and Q by simultaneously changing their values in small quantities. Our results show that the optimal number of deliveries varies from 2 to 14 as the order

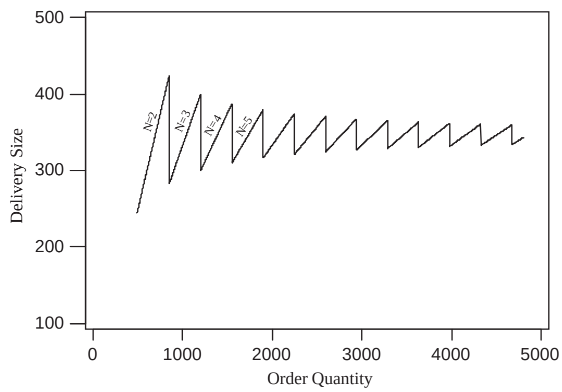

Fig. 3. Optimal delivery sizes evaluated at different order quantities. (When annual demand is 4800 units, annual production capacity is 19,200 units, buyer's holding cost is \$7, supplier's holding cost is \$6 and fixed transportation cost is \$50, the optimal delivery size is 343 units, the number of deliveries is between 2 and 14, the minimum delivery size is 245 units, the maximum delivery size is 424 units, the minimum order quantity is 490 units and the maximum order quantity is 4800 units.)

quantity increases to the annual demand level. For example, provided two deliveries are necessary to make the SSMD policy superior to the single-delivery policy, the delivery size varies from 245 units to 424 units before three deliveries are justified. Refer to the leftmost diagonal line segment of the diagram. When the order quantity gets large enough to justify three deliveries ($N=3$), the delivery size drops to its lowest level of 283 units and then grows to its highest level of 400 units before the higher level of order quantity Q justifies four deliveries, and so on.

As shown in Fig. 3, when the order quantity Q increases, the lower and upper bounds of the range of delivery size q converge to the optimal delivery size, q^*. The results of our numerical analysis also show the same converging pattern over the various levels of production capacity and annual demand as long as the former exceeds the latter.

Convergence in delivery size to an optimal level can shed some light on the issue of the standardization of the transportation vehicle size. Since the capacity of the transportation vehicle is normally an integer multiple of the optimal delivery size, the vehicle size can be standardized easily. Consequently, material flows could be smoothed and better synchronized to improve customer service without increasing inventories. This could also be applied to the determination of Kanban size in JIT manufacturing.

4. Numerical example

Consider a buyer who is currently using an EOQ policy with a single delivery assumption, seeking short-term price breaks. The buyer wants to change the current ordering practice toward a development of a long-term relationship with a supplier for successful JIT implementation. The buyer currently has the annual demand of 4800 units and the order cost is \$25 per order. For order shipments, the buyer pays the fixed transportation cost of \$50 per trip as well as the unit variable cost for order handling and receiving of \$1.00/unit. We further assume that the supplier uses 25% of its annual production capacity of 19,200 units in order to fulfill the buyer's order requirements. The supplier currently spends 6 hours with five workers to set up the production system. With the hourly wage of \$20 per worker, the one time setup cost is \$600 (\$20/hour × 5 workers × 6 hours).

We assume that the current H_B and H_S are \$7 per unit per year and \$6 per unit per year. The optimal order size and the optimal number of deliveries, applying the procedure developed in Section 3.1, are 1155 units and $N = 3$. Thus, the delivery size is 385 units per delivery. The integrated aggregate total costs from Eq. (3) with $N = 3$ (or from Eq. (5)) are \$11,389.53 per year. However, if the single delivery policy ($N=1$) were used, we would have obtained in Eq. (3) the optimal order quantity of 873 units and the integrated aggregate total cost of \$12,221.59 per year. This indicates that the SSMD represents a cost-saving ordering policy as it can benefit both buyer and supplier total savings of \$832.06 over the single delivery policy. Notice that the order quantity has to be greater than 600 units as defined in Eq. (7). Otherwise, the single delivery policy would be preferable. The results of the analyses are summarized in Table 1.

5. Summary and conclusions

In this study, we investigate the effects of a JIT lot-splitting strategy on the integrated buyer–supplier total relevant costs by examining the optimal order quantity, the number of deliveries, and the shipping size over a finite planning horizon. Our results show that the integrated buyer–supplier strategy of facilitating multiple deliveries in small lots can save cost over the conventional single-delivery policy. The savings in total costs could be shared by both parties according to the importance of their contribution to the improved performance of the system.

Table 1
Analysis of the example

Policy	Single delivery	SSMD
Order quantity, Q^*	$Q^*=873$ units/order. Result of optimization of Eq. (3) w.r.t. Q, with N being replaced by 1	$Q^*=1{,}155$ units/order by Eq. (4). Range of order quantity for $N=3$ is (849–1,200) units by Theorem 1
No. of orders, D/Q^*	5.50 ($\cong 6$)	4.16 ($\cong 4$) times/yr.
No. of deliveries, N	$N=1$ time/order	$N=3$ times/order, by the procedure in Section 3.1. $N^*=3.33$ by Eq. (4).
Delivery size, $q=Q^*/N$	$q=873/1=873$ units	$q=1{,}155/3=385$ units. Range of delivery size for $N=3$ is (282.84–400) units by Eq. (9). $q^*=346.41$ units, by Eq. (8).
Aggregate total costs, $\mathrm{TC}(Q^*, N)_{\mathrm{Agg}}$	\$12,221.59/yr by Eq. (3) with Q^* and $N=1$	\$11,389.53/yr by Eq. (3), using $N=3$
$\mathrm{TC}(Q^*, N^*)_{\mathrm{Agg}}$	N/A	\$11,381.79/yr by Eq. (5), or by Eq. (3) using $N^*=3.33$
Savings, SV (Q^*, N)	\$0	\$832.06/yr by Eq. (6), using $N=3$
SV (Q^*, N^*)	\$0	\$839.80/yr by Eq. (6), using $N^*=3.33$

Our study shows that the delivery size converges to an optimal level as the size of order quantity increases. Such convergence in delivery size can offer insights on the issue of the standardization of the transportation vehicle size. The optimal delivery size or its multiple should be key in determining the right size of capacity of the standard transportation vehicle. Once the standardization is made, material flows between buyer and supplier could be easily smoothed and synchronized to improve efficiency of the system without increasing inventory on either side.

The knowledge of the convergence of the optimal delivery size would reasonably be advantageous to intra-firm production units where there is full information exchange. Within a firm's micro production level, such a property could also be applicable to the determination of Kanban size. In practice, most JIT plants optimize only the number of Kanbans without much consideration of a right size of container. However, simultaneous optimization of the number and size of the container would be a more efficient way of keeping the system's material flow smooth. This would eventually help establish low inventories to improve performance.

Although our work is limited by the assumptions that were made in our analyses, it may provide valuable insights for future studies in more realistic and complex situations. The proposed model can be extended to the case of multiple numbers of products, buyers and suppliers, where setup reduction is incorporated.

Acknowledgements

The authors are grateful to the anonymous referees for their comments and suggestions. Seung-Lae Kim acknowledges that the research was partially supported by the Summer Research Grant of Lebow College of Business, Drexel University.

Appendix A. Proof of Theorem 1

In order to find the range of order quantity Q, which justifies the delivery frequency of $N=i$, we solve $TC(Q, N=i) - TC(Q, N=i+1) \leqslant 0$ for the upper bound of Q, and $TC(Q, N=i) - TC(Q, N=i+1) \geqslant 0$

for its lower bound. Successive forward substitutions of values of N from $N = 2$ and generalizations for N can lead us to the following range of Q:

$$\sqrt{\frac{2DFP}{P(H_{\mathrm{B}} - H_{\mathrm{S}}) + 2DH_{\mathrm{S}}}}\sqrt{N(N-1)} \leqslant Q \leqslant \sqrt{\frac{2DFP}{P(H_{\mathrm{B}} - H_{\mathrm{S}}) + 2DH_{\mathrm{S}}}}\sqrt{N(N+1)}.$$

Notice that the common factor of two bounds of the range is nothing more than q^* of Eq.(8). Thus the range of order quantity Q can be rewritten in terms of q^*, for $N \geqslant 2$.

$$q^* \sqrt{N(N-1)} \leqslant Q \leqslant q^* \sqrt{N(N+1)}.$$

This completes the proof.

References

Aderohunmu, R., Mobolurin, A., Bryson, N., 1995. Joint vendor–buyer policy in JIT manufacturing. Journal of Operational Research 46 (3), 375–385.

Banerjee, A., 1986a. A joint economic-lot-size model for purchaser and vendor. Decision Sciences 17, 292–311.

Banerjee, A., 1986b. On a quantity discount pricing model to increase vendor profits. Management Sciences 32 (11), 1513–1517.

Banerjee, A., Kim, S.L., 1995. An integrated JIT inventory model. International Journal of Operations and Production Management 15 (9), 237–244.

Dada, M., Srikanth, K.N., 1987. Pricing policies and quantity discounts. Management Science 33 (10), 1247–1252.

Ellis, R., Gulick, D., 1982. Calculus with Analytic Geometry. Harcourt Brace Jovanovich, Inc., New York.

Goyal, S.K., 1976. An integrated inventory model for a single supplier–single customer problem. International Journal of Production Research 15 (1), 107–111.

Goyal, S.K., Srinivasan, G., 1992. The individually responsible and rational decision approach to economic lot sizes for one vendor and manu purchases: A comment. Decision Sciences 23, 777–784.

Goyal, S.K., Gupta, Y., 1989. Integrated inventory models: The buyer–vendor coordination. European Journal of Operational Research 41, 261–269.

Hill, R.M., 1997. The single-vendor single-buyer integrated production inventory model with a generalized policy. European Journal of Operational Research 97, 493–499.

Joglekar, P.N., 1988. Comments on 'A quantity discount pricing model to increase vendor profits'. Management Science 34 (11), 1391–1398.

Joglekar, P.N., Tharthare, S., 1990. The individually responsible and rational decision approach to economic lot sizes for one vendor and many purchasers. Decision Sciences 21, 492–506.

Lal, R., Staelin, R., 1984. An approach for developing an optimal discount pricing policy. Management Science 30 (12), 1524–1539.

Larson, P.D., 1989. An inventory model which assumes the problem away: A note on Pan and Liao. Production and Inventory Management Journal, 4th Quarter, 73–74.

Lee, H., Rosenblatt, M., 1986. A generalized quantity discount pricing model to increase supplier's profits. Management Science 32 (9), 1177–1185.

Lu, L., 1995. A one-vendor multi-buyer integrated inventory model. European Journal of Operational Research 81, 312–323.

Monahan, J.P., 1984. A quantity discount model to increase vendor profit. Management Science 30 (6), 720–726.

Pan, A.C., Liao, C. 1989. An inventory model under just-in-time purchasing agreement. Production and Inventory Management Journal (1st Quarter), 49–52.

Ramasesh, R.V., 1990. Recasting the traditional inventory model to implement just-in-time purchasing. Production and Inventory Management Journal (1st Quarter), 71–75.

Sakakibara, S., Flynn, F., Schroeder, R., Morris, W., 1997. The impact of just-in-time manufacturing and its infrastructure on manufacturing performance. Management Science 43 (9), 1246–1257.

Schniederjans, M.J., Olson, J., 1999. Advanced Topics in Just-In-Time Management. Quorum Books, pp. 81–83.

White, R., Pearson, J., Ruch, W., 1990. The composition and scope of jit. Operations Management Review 7 (3&4), 9–18.

White, R., Pearson, J., Wilson, J., 1999. JIT manufacturing: A survey of implementations in small and large US manufacturers. Management Science 45 (1), 1–15.