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A revised EMQ/JIT production-run model: An examination of inventory and production costs

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Abstract

Production-runs are an important cost minimizing scheduling and production planning activity. Commonly used classic lot-sizing models (i.e., economic manufacturing quantity or EMQ models) do not reflect current just-in-time (JIT) lot-sizing cost realities. The purpose of this paper is to present a cost comparison of the classic EMQ model and a revised EMQ/JIT model to show efficacy of a more cost inclusive model.
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1. Introduction

The classic economic manufacturing quantity (EMQ) model seeks to determine the optimal production-run lot-size in manufacturing. EMQ modeling originated with the work of Harris (1915) and has become one of the most researched topics in the field of inventory management. More recent research has expanded the classic EMQ model to include cost and resource factors that may dominate a particular application setting. For instance, Chang and Hong (1994) developed an extended EMQ model to take failure prone equipment into consideration in the lot-sizing decision.

Unfortunately inventory models such as the EMQ model do not always include all of the relevant holding costs because they are either too complex of a cost component to represent in a simple quantity-based model or simply assumed away in the model development (Wacker, 1986). As observed by Heizer and Render (2001, p. 480) inventory holding costs, examples such as housing costs, building rent, and depreciation are often understated or just left out in cost data for inventory models, yet they often represents up to 40% of the total value of the inventory. Clearly there is a need to bring additional relevant holding costs into EMQ models that have not previously included.

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The EMQ and JIT cost-related models that currently appear in the literature focus on two types of costs: holding costs and setup (or run) costs. Studies by Grout and Seastrand (1987), Golhar and Sarker (1992), Jamal and Sarker (1993), Gunasekaran et al. (1993) presented lot-sizing formulas that provided a theoretical foundation for EMQ lot-sizing in a JIT environment. The variables in these models were all focused on the determination of lot-size, as a determining factor for total cost minimization. These models did not consider annual demand as a variable or that this variable could be used to make a choice between a JIT or an EMQ inventory system.

Hong et al. (1992) and Corbey and Jansen (1993) also analyzed the economic lot-size and setup costs and concluded that applying the EMQ-based models as lot-sizing rules was not correct because opportunity costs were neglected. Opportunity costs that were not fully considered in EMQ models included a differing variety of cost savings (i.e., physical facilities, setup costs, cost of capital, etc.) generated by smaller lot-sizes. In other words, the EMQ models for JIT environments did not fully take advantage of unique cost savings such as physical facility space reduction that normally occurs as JIT is implemented over a period of time.

Despite the prior research, Wolsey (1995) in a literature review research study found little of JIT cost elements being considered in most models. At the time, other researchers found that difficulties in estimating holding costs in EMQ models inhibited successful modeling. For example, Toelle (1996) suggested that estimating relevant holding costs in classic lot-sizing models might be inappropriate if the parameters could not be accurately estimated. Yet other researchers have proven that relevant factors in lot-sizing models unique to JIT systems must be considered in any modeling process. For example, Dave et al. (1996) developed an EMQ model, which takes into account the effect of varying marketing conditions on demand. Their results show that the inclusion of unique cost factors, such as advertising play a crucial role in determining lot-size and total costs. This study also illustrates how a single unique cost factor (i.e., advertising) can have a substantial impact on the variables (i.e., lot-size) sought in the model.

As the prior research reveals, one particular JIT cost element that is lacking in the EMQ models for JIT environments is the facility space reduction created when a firm adopts JIT. Yet other previous research on JIT suggests that cost savings of facility space reduction is one of the key benefits of the JIT system (Schonberger, 1982; Voss, 1990). Schonberger and Schniederjans (1984) observed many years ago that opportunity costs, including material control costs, uneven workload costs, work improvement (benefits foregone), and physical storage space costs are often omitted in the lot-sizing models based on classic economic order quantity (EOQ) models. They found that even when North American companies did include storage costs in inventory models they were likely to understate or omit relevant cost elements.

Why facility space reduction under a JIT system is so important is due to its potential size of impact in the cost considerations of any problem and that it is seen as an inevitable outcome of using JIT. There is long history of research connected to this particular cost component of JIT. According to Schonberger (1982) and Voss (1990), the reduction in facility space in a JIT environment is caused by the elimination of the space required to store incoming inventory, work-in-process inventory, and finished goods inventory. Many predominant JIT authors, such as Schonberger (1982) and Wantuck (1989), have long cited examples that prove conversion to JIT will reduce space in plants and factories. One example of a company that initiated a JIT operation saved the company 100,000 ft^2 (roughly 30% of the total facility space) of facility inventory storage and production area from their previous large-lot type of system (Chase et al., 1998). In the process of restructuring their layout to accommodate the JIT principles, they ended up renting the space to another company turning what would be a cost into a rental income. Other examples of the magnitude of impact of facility space reduction under a JIT system reported in the literature includes reducing floor space by 30% (Voss, 1990, p. 330), by 40% (Stasey and McNair, 1990, Chapter 13), and even 50% or more (Jones, 1991). Hay (1988, pp. 22–23) reported space reductions of up to 80%. These studies also revealed that the facility cost component was easily observed in JIT applications and could be accurately measured as a cost factor.

There is both a theoretical and a philosophical basis for the inclusion of a facility cost component in the studies by Schniederjans and Cao (2000, 2001). Comparing EOQ models, Fazel (1997) and Fazel et al. (1998) demonstrated how EOQ and JIT models could be theoretically compared on a cost basis to determine under what conditions of annual demand a decision maker should chose one system of inventory management over the other. Schniederjans and Cao (2000, 2001) extended Fazel's models by showing the mathematical derivation of Fazel's EOQ and JIT comparative cost functions with the same facility space cost reduction elements that are proposed in this paper. The formula derivation for the cost models from the original EOQ models were presented in these two papers. Since the EMQ model finds its origin in cost functions of EOQ models based on Harris's (1915) original work, the extension of the EOQ and JIT cost functions by Schniederjans and Cao's (2000, 2001) into EMQ is a theoretically logical extension. Also, philosophically, the advocacy of Harris's (1915) economic quantity modeling approach encouraged and suggested that unique cost elements should be identified and added to the EMQ-type models to better capture the true cost characteristics that differentiate firms in the same way as was done in Schniederjans and Cao (2000, 2001) and will be proposed in this paper.

Based on the prior research it is our contention that while EMQ models have considered the impact of a JIT manufacturing environment, they did not include the relevant JIT cost component of facility space cost reduction. We feel, as shown in Schniederjans and Cao (2000, 2001) that the facility space cost reductions are a significant, measurable, and an inherent outcome of adopting a JIT approach to managing inventory. Unlike prior EMQ modeling literature, this paper develops models that are specifically designed to make a choice between inventory systems based on a product's variable annual demand rate. The purpose of this paper, therefore, is to show the development and illustration of comparative theoretical EMQ and JIT production cost functions. Specifically to illustrate how including the facility space cost reduction will impact inventory and production cost functions and alter the decision to use a large-lot EMQ system or a small-lot JIT system for production planning.

The rest of this paper is organized as follows. Section 2.1 provides the model nomenclature for the paper. Section 2.2 briefly describes the classic EMQ model. Section 2.3 presents the development of the conventional JIT lot-sizing model for comparative purposes with the classic EMQ model. Section 2.4 presents the derivation of the revised EMQ/JIT production model (that includes facility space reduction costs) for comparison with the classic EMQ model. The further derivation of cost models used in the comparative analysis is presented in Section 2.5. An illustrative example is provided in Section 3.1 with a presentation of results of the comparison of the EMQ and conventional JIT model in Section 3.2 and a presentation of the results of the comparison of the EMQ and the revised EMQ/JIT model in Section 3.3. A discussion of the results of the problem and their implications for practicing mangers is presented in Section 4. Section 5 provides the summary and conclusions of the paper.

2. The model

2.1. Nomenclature

The following notations are employed in the derivation of the EMQ and JIT models for this paper:

D	annual demand in units of inventory
h	annual inventory holding cost per unit (\$/unit/year)
H	average inventory holding cost (\$)
k	setup cost for a production-run (an average \$ cost per run)
p	production rate (units)
q	JIT lot-size or manufacturing quantity (units)

Q	EMQ lot-size or manufacturing quantity (units)
Q^*	optimum EMQ lot-size or manufacturing quantity (units)
u	usage rate (units)
m	ratio of the number of production runs or setups under EMQ by the number under a JIT lot-size system
TC	total annual costs ($/year)
Z	cost difference between EMQ and JIT models ($)
P_{EMQ}	cost to produce a unit of product using an EMQ lot-sizing system in a finite production ($)
P_{EOQ}	cost to produce a unit of product using an EMQ lot-sizing system in an infinite production ($)
P_{J}	cost to produce a unit of product using a JIT lot-sizing system ($)
D_{ind}	indifference point of demand at which the total costs of comparative EMQ and JIT models is equal (units)
Subscripts	
EMQ	refers to the EMQ model
J	refers to the JIT model
rev	refers to the revised EMQ/JIT production model
JP	refers to the JIT model product cost
JC	refers to the JIT model setup cost

2.2. The EMQ model

The classic EMQ model (hereafter just referred to as the EMQ model) requires the development of a total annual production cost function (TC_{EMQ}). This function is the sum of the annual costs of machine or run setups, costs of carrying or holding produced inventory in stock, and the production costs (i.e., including materials purchased, manufacturing labor, subcontracted work, outsourced work, etc.) (Harris, 1915), or:

TC_{EMQ}= annual setup costs + annual carrying costs + annual production costs, or

$$\text{TC}_{\text{EMQ}} = \frac{kD}{Q} + h\frac{(p-u)Q}{2p} + P_{\text{EMQ}}D. \tag{1}$$

Only setup costs and carrying costs vary as function of Q units produced. Taking derivative of the setup costs and carrying costs in the TC_{EMQ} function and setting the derivative equal to zero leads to the EMQ lot-size formula or the optimum EMQ (Q^*):

$$Q^* = \sqrt{\frac{kD(2p)}{h(p-u)}}. \tag{2}$$

Now let

$$H = h\frac{(p-u)}{p}. \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1) yields the following total cost EMQ model at the optimum or cost minimized value of total production costs:

$$\text{TC}_{\text{EMQ}} = \sqrt{2kDH} + P_{\text{EMQ}}D. \tag{4}$$

While the optimal lot-size formula in Eq. (2) and it total cost function in Eq. (4) are the theoretical foundation for most current lot-sizing formulas, the focus of this paper in not on lot-sizing models, but their production cost functions for the purposes of a comparative cost analysis.

2.3. The conventional JIT production model

Mirroring the EMQ model, the total cost of a JIT production model also involves two cost components. One component is the total JIT production cost and the other is total JIT setup cost.

2.3.1. JIT production cost

The JIT total production cost function has been expressed by Fazel et al. (1998) as the product of the cost to produce a unit of product times the number of units of a single inventory item demanded annually, or:

$$\mathrm{TC_{JP}} = P_{\mathrm{J}}D. \tag{5}$$

2.3.2. JIT setup cost

In a JIT production system, the total setup costs ($\mathrm{TC_{JS}}$) can be represented as following ratio of setup times the cost of the setup, or:

$$\mathrm{TC_{JS}} = \frac{kD}{q}. \tag{6}$$

Adding the total setup cost in Eq. (5) to Eq. (6), we obtained the total cost for the conventional JIT production model:

$$\mathrm{TC_J} = P_{\mathrm{J}}D + \frac{kD}{q}. \tag{7}$$

Note, this model assumes that it is possible for JIT inventory costs to achieve a value of zero. While we recognize that this is not possible for all JIT products to achieve a zero inventory cost, we feel in such cases this model may be limited to providing only an approximation to the actual inventory cost and quantity values. In practice, JIT setup costs are higher than an EMQ model's in that $Q > q$. This is so since the JIT system has a greater setup change frequency than the large-lot EMQ system. This increased frequency is caused by the JIT production principle of more frequent but smaller lot-sizes. A value of m is used to reflect this cost factor that is in favor of the less-frequent and larger lot-size EMQ system.

Let

$$m = \frac{Q}{q} \quad \text{for} \quad m \geqslant 1. \tag{8}$$

The value of m depicts the ratio of the lot-size used in an EMQ system relative to the assumed smaller conventional JIT production model. The m ratio represents the number of setups required in the conventional JIT production model when compared with the number of setups in EMQ production model. Substituting Eq. (8) into Eq. (7) we have total cost function for the conventional JIT production model:

$$\mathrm{TC_J} = P_{\mathrm{J}}D + \frac{mkD}{Q}. \tag{9}$$

2.4. Revised EMQ/JIT production model

The JIT total production cost function in Eq. (9) leaves out the element of reduced facility space cost savings benefit. It is our contention that the $\mathrm{TC_{JP}}$ component of total production costs under a JIT system should be revised by subtracting the facility space cost savings from the production costs. Specifically, we feel the total JIT production cost function in Eq. (5) should be revised to include the annual facility space cost savings by

$$\mathrm{TC_{JP}} = P_{\mathrm{J}}D - CN, \tag{10}$$

where C is the annual cost to own and maintain a square foot of facility, and N is the number of square feet saved by initially adopting a JIT system. While the value of C can be determined by the facility overhead or leasing costs, the value of N is best determined by the result of changing from an EMQ system to a JIT system. In the ideal JIT situation there should be zero inventories, and hence, zero inventory space. This, of course, is not possible in most situations. Yet, as the prior research supports, there will be a space reduction in adopting JIT and the annual facility space cost reduction of CN is the amount that can be saved by implementing a JIT system. By subtracting CN from the total annual cost of a JIT system as stated in Eq. (10), it reflects our revision to the conventional JIT model. Adding the setup cost to Eq. (10), we obtain the revised EMQ/JIT production model

$$\mathrm{TC_J} = P_\mathrm{J} D + \frac{kD}{q} - CN. \tag{11}$$

Combining Eq. (11) and Eq. (9):

$$\mathrm{TC_J} = P_\mathrm{J} D + \frac{mkD}{Q} - CN. \tag{12}$$

Eq. (12) constitutes the revised EMQ/JIT model of total production costs under a JIT system.

2.5. Cost differences

To demonstrate the costs differences between the EMQ model and its comparisons with both the conventional JIT model and the revised EMQ/JIT model, two total cost comparison functions are developed. Based on the same theoretical costs comparison functions between EOQ and JIT systems as in Fazel (1997) and Fazel et al. (1998), the total cost difference between the EMQ model and the conventional JIT model [i.e., Eqs. (4) and (9)] can be subtracted from one another to create a Z function, or

$$Z = \mathrm{TC_{EMQ}} - \mathrm{TC_J} = \sqrt{2kDH} - \frac{m}{2}\sqrt{2kDH} + (P_\mathrm{EOQ} - P_\mathrm{J})D. \tag{13}$$

Similarly the cost difference between EMQ and the revised EMQ/JIT production model [i.e., Eqs. (4) and (12)] can be subtracted to create a similar but revised Z_rev function:

$$Z_\mathrm{rev} = \sqrt{2kDH} - \frac{m}{2}\sqrt{2kDH} + (P_\mathrm{EOQ} - P_\mathrm{J})D + CN. \tag{14}$$

The interpretation of both of these Z and Z_rev values are the same: if the resulting Z or Z_rev values are positive, then a small-lot JIT system of lot-sizing is less costly than a large-lot EMQ system. Alternatively, if the Z or Z_rev values are negative, the EMQ system is less costly than a JIT system.

2.5.1. Infinite production

One important scenario of EMQ worth of mentioning is when the production rate is much larger than demand rate or infinite production (Jaber and Bonney, 1999). In this situation we can modify Eqs. (13) and (14) to include infinite production. Substituting $H = h(p-u)/p$ into Fazel's Eq. (13), we then take the limit of Eq. (14) as p approaches infinity, H will approach h, and Eq. (13) can be written as

$$Z = \sqrt{2kDh} - \frac{m}{2}\sqrt{2kDh} + \left(\lim_{p\to\infty} P_\mathrm{EMP} - P_\mathrm{J}\right)D. \tag{15}$$

By the same token, this paper's proposed model can be modified from Eq. (14) for the infinite production situation as following:

$$Z_\mathrm{rev} = \sqrt{2kDh} - \frac{m}{2}\sqrt{2kDh} + \left(\lim_{p\to\infty} P_\mathrm{EMP} - P_\mathrm{J}\right)D + CN. \tag{16}$$

Since the focus of this paper is on the finite production of EMQ situation, development of Eqs. (15) and (16) is suggested as an extension for future research.

2.6. The indifference point equations

For analytical purposes production managers might find it helpful to know the exact indifference point at which the total costs of EMQ and JIT models are equal (i.e., both the conventional JIT and the revised EMQ/JIT models), from the perspective of the total cost of production. This indifference point defines the level of annual demand where we would switch from using the smaller lot-sized JIT system to a larger lot-size EMQ system. To aid managers in making the system choice two equations are developed to determine the indifference points with respect to annual demand.

2.6.1. EMQ vs. conventional JIT production model

Where $Z = 0$ determines the indifference point where EMQ and conventional JIT model's total annual production costs are equal. The indifference point, D_{ind} is derived as follows by first setting $Z = 0$ in Eq. (13), or

$$\sqrt{2kDH} - \frac{m}{2}\sqrt{2kDH} + (P_{\mathrm{EOQ}} - P_{\mathrm{J}})D = 0, \tag{17}$$

then simplifying Eq. (17) and subtracting from both sides to arrive at

$$\sqrt{2kDH}\left(1 - \frac{m}{2}\right) = (P_{\mathrm{J}} - P_{\mathrm{EOQ}})D. \tag{18}$$

At this point a qualification on m must be made. According to Waters-Fuller (1996), the purchasing price in JIT (P_{J}) is usually large than the purchasing price (P_{EOQ}) in an EMQ situation at least at the initial stage when a company wants to switch from EMQ model to a JIT production model, that is, $P_{\mathrm{J}} > P_{\mathrm{EOQ}}$. If the right-hand side of Eq. (18) is positive, then the left-hand side of Eq. (18) also needs to be positive. Since $\sqrt{2kDH} > 0$, then $1 - (m/2) > 0$, and we obtain

$$m < 2. \tag{19}$$

Combining both Eqs. (8) and (19), we get

$$1 \leqslant m < 2. \tag{20}$$

The EMQ will always be more cost-effective than that of the conventional JIT production system unless the lot-size used in EMQ model is approaching the JIT lot-size of $1 < m < 2$. When $m \geqslant 2$, the Z value will be always negative, therefore the EMQ is more cost effective than the conventional JIT production system. We cannot square both sides of Eq. (18) when $m \geqslant 2$, because both sides are not positive simultaneously. In other words, there will never be an indifference point when $m \geqslant 2$. Now we can continue the derivation of the indifference point equation given the condition in Eq. (20). By squaring both sides of Eq. (18):

$$2kDH\left(1 - \frac{m}{2}\right) = (P_{\mathrm{J}} - P_{\mathrm{EOQ}})^2 D^2 \tag{21}$$

then, solving for the demand indifference point (D_{ind}) we have

$$D_{\mathrm{ind}} = \frac{2kH(1 - m/2)^2}{(P_{\mathrm{J}} - P_{\mathrm{EOQ}})^2}, \tag{22}$$

then, substituting H from Eq. (3) into Eq. (22), we have

$$D_{\mathrm{ind}} = \frac{kh(1 - u/p)(2 - m)^2}{2(P_{\mathrm{J}} - P_{\mathrm{EOQ}})^2}. \tag{23}$$

Eq. (23) depicts the cost-indifference point D_{ind} which is the level of demand for which the total cost of EMQ and conventional JIT production model become equal, that is, $Z = 0$ when $1 \leqslant m < 2$.

2.6.2. EMQ vs. revised EMQ/JIT production model

Where $Z_{\text{rev}} = 0$ determines the cost-indifference point where EMQ and the revised EMQ/JIT production costs are equal. The derivation of the revised indifference point $D_{\text{ind(rev)}}$ where facility cost reductions is included in the model is similar to the conventional JIT model. We start again by setting $Z_{\text{rev}} = 0$:

$$\sqrt{2kDH} - \frac{m}{2}\sqrt{2kDH} + (P_{\text{EOQ}} - P_{\text{J}})D + CN = 0 \tag{24}$$

then simplifying Eq. (24) and subtracting from both sides to arrive at

$$\sqrt{2kDH}\left(1 - \frac{m}{2}\right) = (P_{\text{J}} - P_{\text{EOQ}})D - CN. \tag{25}$$

Different from the derivation of the indifference point equation for EMQ vs. conventional JIT production model in Eq. (22), we need not worry about the possibility of negativity. Indeed, after subtracting CN at the right-hand side of the equation, the right-hand side of the equation can be either positive or negative. Continuing, by squaring both sides of Eq. (25) we have

$$2kDH\left(1 - \frac{m}{2}\right)^2 = (P_{\text{J}} - P_{\text{EOQ}})^2 D^2 - 2(P_{\text{J}} - P_{\text{EOQ}})CND + (CN)^2, \tag{26}$$

then changing Eq. (26) into the standard formula $ax^2 + bx + c = 0$:

$$(P_{\text{J}} - P_{\text{EOQ}})^2 D^2 - 2\left[(P_{\text{J}} - P_{\text{EOQ}})CN + kH\left(1 - \frac{m}{2}\right)^2\right]D + C^2N^2 = 0 \tag{27}$$

and then using the standard formula:

$$X = \frac{-b \pm \sqrt{\mathrm{b}^2 - 4ac}}{2a}$$

we can derive the formula for $D_{\text{ind(rev)}}$:

$$D_{\text{ind(rev)}} = \frac{\left[(P_{\text{J}} - P_{\text{EOQ}})CN + kH\left(1 - \frac{\mathrm{m}}{2}\right)^2\right] \pm \sqrt{2(P_{\text{J}} - P_{\text{EOQ}})CNkH(1 - m/2)^2 + (1 - m/2)^4 k^2 H^2}}{(P_{\text{J}} - P_{\text{EOQ}})^2}, \tag{28}$$

then, substituting H from Eq. (3) into Eq. (28), we have

$$D_{\text{ind(rev)}} = \frac{[(P_{\text{J}} - P_{\text{EOQ}})CN + kh(p-u)/p(1 - m/2)^2] \pm \sqrt{2(P_{\text{J}} - P_{\text{EOQ}})CN(kh(p-u))/p(1 - m/2)^2 + k^2h^2(p-u)^2/p^2(1 - m/2)^4}}{(P_{\text{J}} - P_{\text{EOQ}})^2}. \tag{29}$$

In Eq. (29), use the (+) sign when $1 \leqslant m < 2$ and use the (−) sign when $m > 2$. When $m = 2$ the formula for indifference-point becomes

$$D_{\text{ind(rev)}} = \frac{(P_{\text{J}} - P_{\text{EOQ}})CN}{(P_{\text{J}} - P_{\text{EOQ}})^2}. \tag{30}$$

3. Illustrative example

3.1. Problem and cost information

To illustrate the use of the models we provide a hypothetical example. In this example we assume that a company manufactures single item product X. The production cost per unit of X under an EMQ system is $P_{\mathrm{EOQ}} = \$20$, while under a JIT system the production cost is higher at $P_{\mathrm{J}} = \$20.20$. Using the same logic as Fazel (1997) and Fazel et al. (1998), we assume that P_{J} is larger than P_{EOQ} as a result of JIT-unique requirements of more frequent deliveries. Daily production rate is $p = 100$ units and daily usage rate is $u = 50$ units. Cost for a production setup is estimated at $k = \$100$. Annual carrying cost is $h = \$5$ per unit. The values for the annual cost to own and maintain a square foot of facility (C) and the number of square feet saved by adopting a JIT system (N) have to be estimated. Let us assume a cost per square foot of facility is only $C = \$10/\mathrm{ft}^2/\mathrm{year}$ and the facility in this example has a total square footage of 500,000. Let us further assume that a reduction due to adopting a JIT system is only a 10% reduction in square footage. That would mean that the number of square feet saved by adopting JIT is $N = 50{,}000$ (i.e., $500{,}000 \times 0.10$). Thus, $CN = \$500{,}000$. Based on the previously cited research on JIT improvements, all of these assumptions can be considered very conservative.

The value of m can be used as a variable in either Eqs. (23) or (29) and provides an opportunity to conduct a type of sensitivity analysis for lot-sizing. For illustration purposes, we simulate four m scenarios in both the comparison of the EMQ vs. the conventional JIT model and the EMQ vs. the revised EMQ/JIT model. The four m scenarios (i.e., $m = 1, 1.5, 2$, and 8) will be used to examine four modeling assertions:

1. a conventional JIT model can be more cost effective than the EMQ production model when $1 \leqslant m < 2$ for given annual demand level,
2. an EMQ production model is always more cost effective than the conventional JIT model when $m \geqslant 2$,
3. the inclusion of facility space cost reduction under a revised EMQ/JIT model will substantially increase the desirability of using a JIT-based production system over the EMQ production system, and
4. that there exists an inverse relationship between the size of m and the annual demand, such that as m increases annual demand will decrease.

3.2. Problem results: EMQ vs. conventional JIT production model

By plugging the four values of m into the Z and Z_{rev} cost functions, we can explore the resulting impact on indifference points of total annual demand using Eqs. (23), (29) and (30). Table 1 presents the computational results of the EMQ vs. conventional JIT production model. All of the values in Table 1 are the annual demand values necessary to achieve the indifference points at a particular m ratio of lots per year.

For both $m = 1$ and 1.5, the indifference points exist at 3125 units and 781 units, respectively. This supports the first assertion that the conventional JIT production can be less costly than the EMQ model when $1 \leqslant m < 2$ in the situations where annual demand is less than 3125 units for $m = 1$ and 781 units for $m = 1.5$.

In the second assertion an EMQ production model should always be more cost effective than the conventional JIT model when $m \geqslant 2$. As can be seen in Fig. 1, when $m = 2$ and 8, there are no indifference points. Indeed it is easy to extrapolate in Fig. 1 all total cost functions of $m \geqslant 2$ and see that they will not have indifference points. This means that when $m \geqslant 2$, that is, when the lot-size of an EMQ model is larger than 2 times of the lot-size that is used by a conventional JIT operation, the cost structure of the EMQ model is always more cost-effective than that of the conventional JIT production model.

Table 1
Indifference points for differing levels of m

Variable m	EMQ vs. conventional JIT (D_{ind})	EMQ vs. revised EMQ/JIT ($D_{\mathrm{ind(rev)}}$)
1	3125	2589965
1.5	781	2544587
2	N/A	2500000
8	N/A	2025589

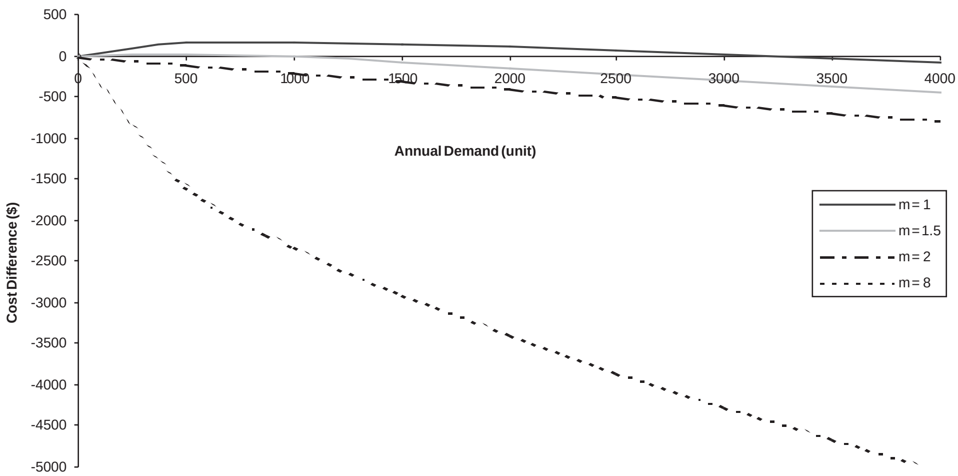

Fig. 1. EMQ vs. conventional JIT.

3.3. Problem results: EMQ vs. revised EMQ/JIT production model

Table 1 also presents the computational results of the EMQ vs. revised EMQ/JIT production models. When the relatively small facility space-saving costs are included in the total costs, the results in Fig. 2 show that indifference points exist for all m scenarios and require substantially larger annual demand values than the EMQ model. Note the increase where $m = 1$ of 3125 units under the conventional JIT model in Fig. 1 that changes to an annual demand of 2,589,965 units in Fig. 2 due to the small facility cost savings included in the revised EMQ/JIT model. This means that when $m = 1$ the revised EMQ/JIT production model is superior to the EMQ model, as long as the annual demand is less than 2,589,965 units. Clearly the magnitude of required annual demand from Figs. 1 and 2 shows the dramatic increase in demand that would be necessary to justify using EMQ model over the more cost efficient revised EMQ/JIT model. This we feel supports our third assertion that the cost performance of the revised EMQ/JIT production model is better than the EMQ model in most m situations except those where annual demand is extremely large. Figs. 1 and 2 illustrate that "extremely large" annual demand would have to be an increase in the order of 1000 times the normal annual demand levels (i.e., the addition of the JIT physical space costs required the annual demand threshold to jump from the thousands in Fig. 1 to the millions in Fig. 2). While operations can have very large annual product demand, it was observed by Schniederjans and Cao (2001) that the increased unit demand on a similar operation would most likely force it to change from a lot-size operation to a JIT-type continuous production operation. This is so because larger annual demand requires more physical space, thereby creating a further advantage to the JIT costs savings opportunity found in the

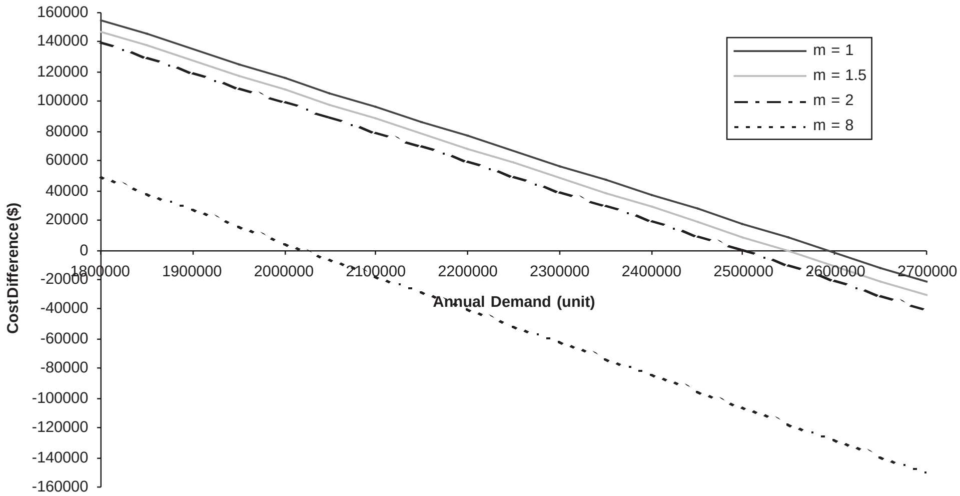

Fig. 2. EMQ vs. revised EMQ/JIT.

EMQ/JIT production model. As demand is increased, either by a customer demand serge or normal growth in market sales, the annual demand threshold whereby the EMQ/JIT model would suggest a change to an EMQ model would be further increased. Like a "leap-frog" situation, the EMQ/JIT model would continually suggest a JIT-type operation as annual demand became continuously ever larger.

Our final assertion stated that there would be an inverse relationship between the size of m and the annual demand. As can be seen in Fig. 2, when m increases the indifference point of annual demand decreases. This illustrates that as the lot-size of EMQ system approaches that of a JIT lot-size, the better the cost performance of the JIT production system. This observation is supported by JIT literature. Schonberger (1982, 1986) and Wantuck (1989) repeatedly explain and show that production cost efficiency is at its best when a firm's production lot-size approaches a JIT smaller sized production level.

4. Discussion

There are several implications that can be drawn from the assertions in the previous example for practicing production managers. One implication is that if your production facility is currently operating under an EMQ production cost structure and your existing lot-size (Q) is fairly small and might be the same size under a possible JIT-type operation lot-size (q), then you will minimize your total production costs by converting to a JIT-type operation since $m = Q/q = 1$.

A second implication is that if your production facility is currently operating under an EMQ production cost structure and your existing lot-size is fairly large relative to a possible JIT-type operation lot-size, then you might want to explore possible lot-size scenario's using relevant comparative cost equations as developed in this paper to seek to minimize your total production costs. It is note worthy to observe in the illustrative problem that the resulting $D_{\text{ind(rev)}}$ value of 2,589,965 units that would be necessary to make the switch to the EMQ approach is not a possibility in a problem where the actual annual demand stated in the problem could only be at most 18,250 units (i.e., 50 units daily $\times$ 365 days). The EMQ production cost structure would only end up being the best cost minimizing approach if a cost factor like facility space reduction is not that relevant in a particular situation and the annual demand is extremely large.

A possible third implication observed in this paper's example was that if a firm has a very large lot-size and conversion to a JIT system will not reduce it, perhaps because of some technology constraint, and if the same firm has a very large annual demand, they will probably need to continue using the EMQ production cost system. We recommend though, that such a situation should be confirmed using as a basis the equations presented in this paper, with necessary adjustments for the unique cost circumstances of the organization in question.

5. Summary and conclusions

In this paper, a classic EMQ lot-sizing production-run cost model is compared with a conventional JIT production-run cost model based on similar logic of that of a JIT purchasing model proposed by Fazel (1997) and Fazel et al. (1998). The findings suggest that in most cases the EMQ model is superior to the conventional JIT production model. This was a questionable result in light actual practice that favors a JIT production system (Chase et al., 1998). Based on Schniederjans and Cao (2000, 2001), the validity of the conventional JIT production model was questioned in light of the exclusion of cost information relevant to JIT models. A revised JIT production model was developed that more inclusively captured the dynamic cost of facility space savings in a JIT production situation. The results of an example used to illustrate the production cost models indicated that when using a very small space cost saving figure, the revised EMQ/JIT production model would be superior to the EMQ model in production operations in all but the very extreme levels of annual demand requirements. This is same result as was shown in Schniederjans and Cao's (2000, 2001) studies on EOQ models.

One important scenario of EMQ worth of mentioning is the fact that it is easier to shift to a JIT system from an EMQ system when production rate is much larger than the demand rate or in the situation of infinite production (Jaber and Bonney, 1999).

One of the contributions of this paper lies in the indifference point equations derived, which were demonstrated as being able to provide a cost sensitive analysis. Their ease of use to simulate different scenarios by changing different m values—ratio between Q (manufacturing quantity of EMQ) and q (manufacturing quantity of JIT) was shown in an example. The sensitivity analysis example also supported the assertion that the closer the lot-size of EMQ is to JIT lot-size, the better the cost performance of the production system, which is in line with the findings of current JIT literature.

One of the limitations of our paper might be the fact that no actual revised EMQ/JIT model that could be used to determine the optimal EMQ (Q or q) was provided. The authors feel that this was unnecessary since the ideal value for EMQ in a JIT environment is a value Schonberger (1982) suggested as always approaching one. We also feel that reality of lot-sizing demands some flexibility in actually determining a true lot-size. Indeed, we proposed the use of the equations in this paper based on the possible estimation of m or the assumed lot-sizes of Q and q. Since the JIT value of q is universally set at a theoretical optimal value of one in the literature, it is more realistic and practical to allow users to set their own theoretically possible values for Q and q in the use of this paper's equations. Indeed, our equations permit the value of q to be as large as is need be to correctly model a JIT operation.

Another limitation of this paper is the fact that only one additional cost component (i.e., facility space cost reduction) was added to the revised EMQ/JIT production model. As Schniederjans and Cao (2001) demonstrated with a EOQ pricing model, such cost components as economies of scale, work improvement, opportunity costs and the like can be included easily as extensions of the theoretical models presented in this paper. Indeed, it is hoped that this paper provides a much needed foundation for other theoretical model extensions that continue to show the benefits of adopting JIT-type cost structures that are so commonly observed in actual industry practice.

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