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An integrated MRP and JIT framework☆

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Abstract

A new production planning and scheduling framework is developed to address the multi-stage, production-inventory system problem by integrating Material Requirements Planning (MRP) and Just-in-Time (JIT) production. The objective is to find detailed shop-floor schedules, which specify the quantity of an operation to be processed, at what time, and by which machine, so as to minimize total production cost. The integrated system gets rid of the major problems existing in MRP and JIT. First of all, the proposed integrated system incorporates both the scheduling and capacity planning aspects, simultaneously. Secondly, the integrated system eliminates the need to specify planned lead time. Thirdly, the integrated system, unlike MRP, provides detailed shop-floor schedules. Lastly, the integrated system does not have to operate in the level schedule case as in JIT, so it can handle a very general production environment.

Keywords: Material requirements planning; Just-in-time; Integrated system

1. Introduction

This article considers general, discrete, multi-stage, production-inventory system (MS-PIS) with multiple machines at each stage that manufacture multiple products. The MS-PIS is very important because it is by far the most common type of manufacturing systems (Goyal & Gunasekaran, 1990).

In the MS-PIS, each product or part requires one or more operations to be processed at some designated machines or work centers. Different machines can be used to perform the same operation; however, they may possess different setup times and per unit processing times, as well as different per unit time costs. A setup is required whenever there is a change of product or part processed in a machine; nonetheless, setup time is assumed to be sequence independent. It should be noted that machine capacity

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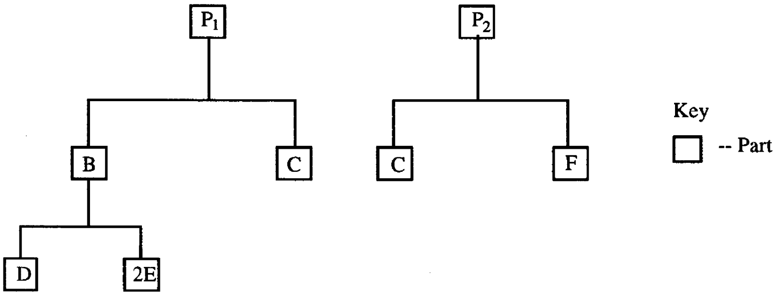

Fig. 1. BOM for P_1 and P_2.

is assumed to be finite. Demand is assumed to be time varying. Each product must be processed according to the precedence relationships as specified by the product structure (Bill of Materials (BOM)). The operations sequence of a part or product must follow its routing sheet specifications. Inventory holding cost is imposed on both work-in-process and finished goods.

A typical example of this type of manufacturing environment can be illustrated as follows. Suppose that a firm manufactures two products, P_1 and P_2. The BOM for the two products are shown in Fig. 1. According to the figure, product P_1 is made up of one unit of part B and one unit of part C, and part B is composed of one unit of part D and two units of part E, while product P_2 consisted of one unit of part C and one unit of part F.

Fig. 2 shows the operations required to process each part and product. For example, part B requires two operations: the first operation can only be processed by machine 2 (M_2); while the second operation

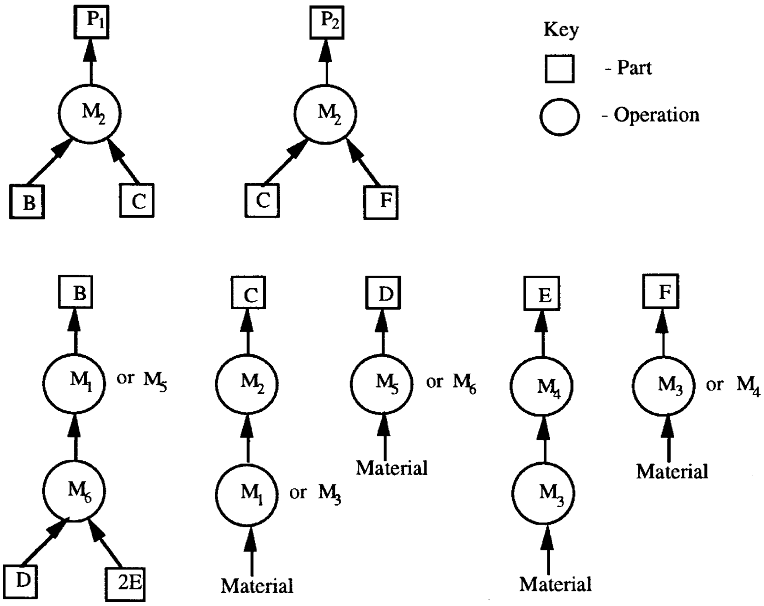

Fig. 2. Bill of operations for P_1 and P_2.

can be processed by M_1 or M_5. We call the diagrams shown in Fig. 2 as Bill of Operations, since they illustrate the operations and their precedent relationships required to produce a part or product. The demand for products P_1 and P_2, and parts B, C, D, E and F comes at some specific times for some specific quantities. This manufacturing environment is an example of the MS-PIS since it involves multiple products (Fig. 1) and multiple machines (Fig. 2). Also, each product or part may require one or more operations (Fig. 2).

The production planning and scheduling problem for the firm is to determine detailed shop-floor schedules, that specify which operation of which part to be processed by which machine at what time and in what quantity, so as to minimize the total production cost (TPC) while meeting jobs' due dates. The TPC consists of two major components: machine time cost (processing time cost and setup time cost) and inventory holding cost.

In this paper, we propose a framework which integrates both Material Requirements Planning (MRP) and Just-in-Time (JIT) concepts to provide better production planning, scheduling and control for the MS-PIS. It employs the logic of MRP and JIT, but it gets rid of some of the major problems in both systems. Next, we identify the major problems of MRP and JIT, since they motivate the development of the integrated framework.

2. Major problems of MRP

MRP is a computerized information system for managing dependent-demand inventory and scheduling stock replenishment orders (Krajewski & Ritzman, 1993). The general theme of MRP is to receive the right part, in the right quantity, and at the right time. The first major problem of MRP is the need to set planned lead time. Planned lead time represents the amount of time allowed for orders to flow through the production facility. It plays an important role in the phasing principle of MRP, that is, the planned order receipt date is offset by the planned lead time to determine the planned order release date. However, how should one determine the planned lead time? And what are the tradeoffs of setting a loose or tight planned lead time? These are two very important questions, since they should have a very significant impact on the performance of MRP.

The first question is very difficult. Generally, lead time is composed of processing time, setup time, waiting time in queue and a certain fixed idle time, such as the time needed to cool down a part. With the exception of the fixed idle time, the other components are very hard to pre-determine. The setup time primarily depends on the processing sequence of jobs, while the processing time primarily depends on the size of the batch. More importantly, the waiting time in queue is particularly difficult to estimate, since most orders spend most of their flowtimes in queues. Plossl and Welch (1979) and Huge (1979) found that the waiting time in queue can represent as much as 90 to 95% of the lead time. Hence, lead time is very much determined by how long it takes to obtain the required capacity, in other words, the congestion level of the shop. Therefore, setting optimal planned lead times for MRP is not a simple task.

Much research has been done to examine the lead time problem (Huge, 1979; Kanet, 1981, 1982, 1986; Melnyk & Piper, 1981, 1985; St. John, 1985). There are three main policies available to determine the planned lead time in MRP, which are constant (CON); number of operations (NOP) and total work (TWK). The CON policy uses the same planned lead time for any job, while the NOP and TWK policies set the planned lead time as a linear function of the NOP and total processing time required of the job, respectively. A simulation study (Conway, Maxwell & Miller, 1967) showed that NOP and TWK performed better than CON in terms of tardiness-oriented criteria; the same result was later concluded by other simulation studies (Baker & Bertrand, 1981). Although these are some generally good policies of setting the lead time, nonetheless, it remains a serious problem in MRP. As noted in Kanet (1986), “But total control (of lead time) is of no use — indeed it can be dangerous — without a corresponding measure of understanding”.

Since it is often very hard to estimate the lead time accurately, the second question asks whether a firm should set a relatively loose or tight lead time. If the lead time is too tight, the MRP system may quickly break down, and the informal planning and scheduling system assumes control. The result is many past-due orders and expediting. On the other hand, if the due date is too loose, this may serve as a form of self-fulfilling prophecy; hence a long lead time may actually occur. Karmarker (1989) reported that an inflated lead time fosters poor performance. St. John (1985) investigated the cost of inflated planned lead times for the multi-product, multi-stage environment, where MRP system was employed. He found that total costs were significantly higher when the planned lead time was set to be long. Therefore, any deviation of the planned lead time from the actual lead time can create undesirable effects.

The second major issue in MRP deals with the question: how to decide the order quantity? This is generally called the lot-sizing decision. The lot-sizing issue has attracted a significant amount of research. For the uncapacitated case, Harris (1915) develops the well-known economic order quantity (EOQ) based on the uniform demand assumption. Wagner and Whitin (1958) introduce a dynamic programming procedure to handle the time varying demand case. Silver and Meal (1973) propose a heuristic that seeks to minimize the average cost of carrying and holding. Other popular heuristics are: lot for lot (LFL), period order quantity (POQ), part period algorithm (PPA) (DeMattais, 1968), part period balancing (PPB), least unit cost (LUC), least total cost (LTC), fixed order quantity (FOQ) and fixed period quantity (FPQ).

Although there exist more sophisticated lot sizing techniques, the average MRP system offers about three or four methods from which the user could choose from. The most common techniques are LFL, FOQ, FPQ, and POQ (Haddock & Hubicki, 1989). This is because the more realistic problems, such as the capacity constrained case, have been shown to be NP-hard (Bahl, Ritzman & Gupta, 1987). Hence, MRP systems usually opt for the simpler sub-optimal techniques.

The third major issue of MRP is that it does not produce a workable schedule for the shop-floor. The planned order release and the planned order receipt merely specify the start date and finish date of an order. Hence, MRP cannot determine the exact time period and workstation for processing each operation. When a job is released to the shop-floor, there is no guarantee that the job would be finished after the planned lead time has elapsed. This is because there has no capacity set aside for the job either in-house or from outside vendors. Depending on how busy the shop-floor is, the job may finish before or after the planned order receipt date. Therefore, it is too crude a tool to control the shop-floor scheduling and to ensure meeting the due dates objective.

The last major problem of MRP is capacity planning. When MRP is employed to perform capacity requirements planning, it assumes that the resource capacity (machine time) is utilized at the period that a job is released or at the middle period between planned order release and planned order receipt dates (Chase & Aquilano, 1995; Vollmann, Berry & Whybark, 1992). The arbitrariness is due to the fact that MRP only specifies the job release and completion dates. However, the actual processing of a job’s operation may span over more than one period, or it actually starts in a later period and spans over more than one period. Therefore, the capacity planning of MRP does not quite accurately reflect the real situation.

Table 1
MRP vs. JIT

Characteristic	MRP	JIT
Main focus	Computerized information system	Shop-floor physical operations
Main function	Parts scheduling without regard to capacity	Operations scheduling
Shop-floor work authorization	Push system	Kanban pull system
Rates of outputs	Variable or level schedule	Level, stable schedule
Capacity required	Capacity requirement planning	Visual
Forms of control	Middle management	Shop-floor, line workers

3. Major problems of JIT

The JIT approach was originated at the Toyota Motor Company by Taiichi Ohno and Shigeo Shingo. JIT is viewed by Ohno as, "In a nutshell, it is a system of production, based on the philosophy of total elimination of waste that seeks the utmost in rationality in the way we make things". Generally, the JIT approach requires the production of precisely the necessary parts in the necessary quantity at the right time. The main objective is to achieve zero inventory.

JIT production can be viewed in a colloquial fashion as consisting of 'Big JIT' and 'Little JIT' (Chase & Aquilano, 1995). Big JIT is more of a management philosophy that encompasses every aspect of a firm's production activities. On the other hand, Little JIT is focused on 'Kanban' pull production scheduling and control method. Our emphasis here is on the Little JIT; that is, the production and inventory control tools used in JIT. The Kanban pull system suggests that production should be triggered by a pull signal from a downstream work center when it has demand for component parts. That is, the downstream work center serves as a customer for its upstream work center. The result is that the upstream work center will not produce unless there is a demand or 'pull' from its customer (downstream work center). On the other hand, a 'push' system, driven by the upstream workstation, pushes out the parts (that later become work-in-process inventory) without regard to the demand of its downstream workstation. The excessive WIP inventory is one of the major disadvantages of push systems.

One of the major requirements for JIT production is a stable environment so as to obtain a level schedule. A JIT production system requires a uniform flow of goods through the system to achieve a good coordination of the different operations and the movement of goods and materials from the supplier to the final output. Therefore, production schedules must be fixed over a time horizon in order for production and purchasing schedules to be established. Once plans are set, they generally are not allowed to change. Therefore, Kanban system is more suitable to companies that produce repetitive products. In fact, the simplicity principle employed in JIT reinforces the level schedule idea. Furthermore, JIT is more of a planning concept rather than a planning and scheduling technique. Hence, it does not generate a formal shop-floor schedule.

Table 1 compares and contrasts some key characteristics of MRP and JIT. Two of them are worth emphasizing here. First, MRP's main focus is information system; while JIT's main focus is shop-floor physical operations. Second, the main function of MRP is product/part scheduling; while the main function of JIT is operations scheduling.

4. The proposed integrated MRP–JIT approach

A number of papers have appeared which discuss the possible integration of MRP–JIT. Nonetheless, most of them are focused on the conceptual level of JIT philosophy and are more concerned with combining rather than integrating MRP and JIT. Belt (1987) suggests that companies should use MRP as a framework to focus on JIT techniques, such as reducing lead times and lot sizes. Blackburn (1985) suggests that combining BOM and routing information on one document facilitates the JIT implementation in the job shop. Cook and Muinch (1984) discuss some issues and concepts to integrate MRP and JIT. Discenza and MacFadden (1988) believe that MRP and JIT should be integrated through software unification. They argue that the use of computer can greatly enhance the capability of JIT without the loss of the pull system concept.

Flapper, Miltenburg and Wijngaard (1991) propose one of the most rigorous frameworks for integrating MRP and JIT. Their three-step framework makes use of MRP's backflushing and phantom features, and allows JIT principles to be utilized to the fullest extent. Flapper et al. (1991) suggest that MRP is an ideal mechanism for planning and control propose, while JIT is the best tool for reducing cost and lead times. By taking advantage of the two, one can obtain the best of both worlds.

The essence of the proposed framework is to incorporate the pull element of operations scheduling in JIT into MRP. It is generally agreed among researchers that MRP is a push system. However, it is very important to make the distinction between materials (or parts) planning level and operations scheduling levels when we say MRP is a push system.

We believe that MRP is a pull system at the materials planning level. Since the product explosion logic of MRP suggests that the parent item pulls the material requirements from its predecessors. The parent part here is parallel to the downstream operation in JIT. Therefore, only when there is demand from the parent item (downstream operation), it would pull the production of its child item(s) (upstream operation(s)). On the contrary, this is not true at the shop-floor operations scheduling level. Once a production order is released to shop-floor, it is pushed through the system as fast as possible without regard to whether there is demand from the downstream operations. Hence, we conclude that MRP is a push system at the operation scheduling level.

The proposed integrated system is a pull system in both materials planning and operations scheduling levels. We will present the integrated system in the following order. First, we discuss the logic of the integrated system. Second, we describe the major inputs and outputs of the integrated MRP and JIT system. Third, we discuss the performance criteria used to evaluate the goodness of a schedule.

The integrated system, as in MRP, is driven by the demand of a part or product derived from the master production schedule (MPS). The demand of a part triggers or pulls the production of the last operation of that item; hence, heuristics (Chang & Ho, 2001a) embedded in the integrated system would be required to perform the operations scheduling. The scheduling of the last operation triggers the need to schedule its preceding or upstream, if any, operation. This pull-oriented scheduling process continues until all the necessary operations are scheduled to manufacturing the part or product.

If the part or product under consideration (called parent item) consists of other parts (called child items), then the scheduling of the first operation of the parent item triggers the need to schedule the last operation(s) of the child item(s). Because the production of the parent item requires the production of the child item(s), that is, product explosion. Again, this pull-oriented scheduling process continues until all the necessary child items are scheduled.

We use the following example to illustrate how the proposed integrated framework captures the pull

Table 2
Routing file

Part no.	Operation no.	Machine no.	Setup time	Processing time
P_1	1	2	5	0.5
P_2	1	2	2	0.6
B	1	6	3	0.8
B	2	1, 5	6	0.2
C	1	1, 3	7	0.7
C	2	2	3	0.4
D	1	5, 6	6	0.8
E	1	3	4	0.3
E	2	4	5	0.5
F	1	3, 4	5	0.7

element through its operations scheduling process. Suppose that there are 10 and 25 units of demand for product P_1 in periods 8 and 10, respectively, and that there are 20 and 5 units of demand for product P_2 in periods 6 and 7, respectively. Furthermore, Table 2 gives the machine or resource routing information, for example, it shows that the second operation of part B can be performed by either machine 1 or machine 5 and that the setup time and unit processing time for both machines are 6 and 0.2 h, respectively. Also, a capacity of 40 h is assumed for each machine in each period. It should be noted that the example routing file assumes, without the loss of generality, that the same setup time and unit processing time are used for those routing with multiple machines, and that the machine cost per unit for each machine is identical.

The proposed integrated system has been modeled as an integer linear program and solved by using integer programming software (Chang & Ho, 2001b). Chang and Ho (2001a) also propose forward and backward heuristics to implement the proposed framework (see Appendix A for descriptions of the proposed heuristics). We will use a forward heuristic from Chang and Ho (2001a) to demonstrate the example. The forward heuristic starts at the beginning of the planning horizon, working forward towards the end of the planning horizon. The earliest demand in the planning horizon is 20 units of P_2 in period 6, so the integrated system first finds the last operation of P_2, i.e. the most downstream operation, that is triggered by customers' demand. Since P_2 requires only one operation, the integrated system can schedule this operation at machine 2 in period 6, or earlier. The forward heuristic employs an objective criterion, such as least unit total cost (LUTC), to determine the operations scheduling. To demonstrate the principle of the proposed framework, we assume that the heuristic schedules the operation in period 6 at machine 2, as shown in machine 2 of Table 3. This triggers or pulls the production of parts C and F.

The integrated system will now schedule the second operation of part C and the first operation of part F in period 5 or earlier, since no consecutive operations of a part can be scheduled in the same period. Suppose that the forward heuristic schedules operation 2 of part C and operation 1 of part F in period 5 at machines 2 and 3, respectively (see Table 3). This triggers the need to schedule operation 1 of part C in period 4 or earlier and signals that the material requirements of part F should arrive by period 5. Table 3 indicates the scheduling of operation 1 of part C in period 4 at machine 1. Now, the integrated framework schedules the operations derived from the 5 units of demand of P_2 in period 7, and this process continues until all demands are covered. It should be noted that if the heuristic determines that the benefit of setup elimination is larger than the cost of holding inventory, it may schedule the operation in an earlier period

Table 3
Operations scheduling

	1	2	3	4	5	6	7	8	9	10
Machine #1										
Part (operation) no.:				C(1)		C(1)	B(2)		B(2)	
number of units				20		10	10		25	
Part (operation) no.:				C(1)						
number of units				5						
Machine #2										
Part (operation) no.:					C(2)	P_2(1)	P_2(1)	P_1(1)	C(2)	P_1(1)
number of units					20	20	5	10	25	25
Part (operation) no.:					C(2)		C(2)			
number of units					5		10			
Machine #3										
Part (operation) no.:				E(1)	F(1)	E(1)		C(1)		
number of units				20	20	50		25		
Part (operation) no.:					F(1)					
number of units					5					
Machine #4										
Part (operation) no.:					E(2)		E(2)			
number of units					20		50			
Part (operation) no.:										
number of units										
Machine #5										
Part (operation) no.:					D(1)		D(1)			
number of units					10		25			
Part (operation) no.:										
number of units										
Machine #6										
Part (operation) no.:						B(1)		B(1)		
number of units						10		25		
Part (operation) no.:										
number of units										

to avoid a setup. For instance, after scheduling 5 units of operation 1 of P_2 in period 7, the forward heuristic schedules 5 units of operation 1 of part F in period 5 rather than period 6 to save a setup.

Table 3 summarizes the operations scheduling results. Unlike MRP, the table shows that the lead time for the same product may differ. For example, the lead time for the demand of product P_2 in period 6 is 2; while the lead time for the demand of product P_2 in period 7 is 3. MRP generally uses loose and fixed lead time, which inflates the actual lead times and leads to very poor performance.

The integrated system is similar to MRP in the following respects. They both are computerized information systems. Moreover, the integrated system employs both time-phasing and product explosion principles of MRP. Time-phasing refers to the scheduling to produce or receive an appropriate amount of material so that it will be available in the time periods when needed; while product explosion refers to the breaking down of parent items into component parts that can be individually planned and scheduled.

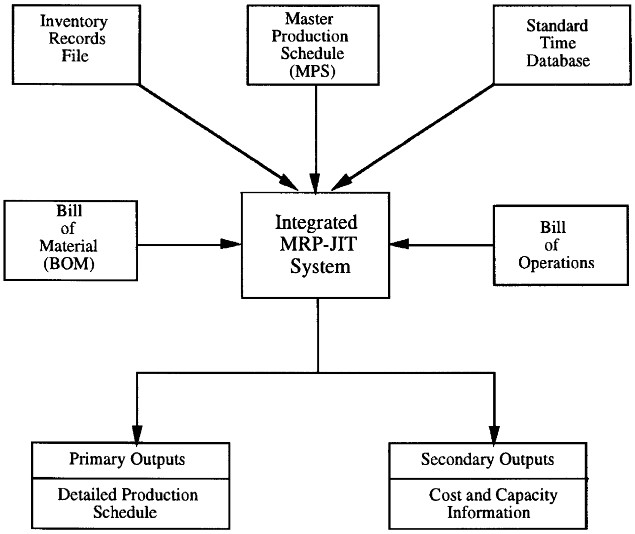

Fig. 3. Basic structure of the integrated system.

More importantly, the time-phasing concept under the integrated system phases out the actual operations time instead of the planned lead time (which includes expected operations time and expected waiting time) in MRP.

Nevertheless, there are a number of differences between them. First of all, MRP does not provide a daily production schedule. However, the integrated system gives the detailed day-to-day schedule. Second, the lead time is fixed and pre-determined in MRP. On the other hand, the lead time of the integrated system can be varied, and it is derived from the schedule. It should be noted that a certain known lead time can easily be included in the integrated system, for example, the cooling down time of a part. Third, a period defined in the integrated system, usually a shift or a day, is generally shorter than that of MRP. Lastly, the integrated system determines the TPC as part of its output. In contrast, MRP does not consider the total cost, since it essentially specifies the planned order releases and receipts dates. Therefore, the integrated system provides an important avenue to evaluate the effectiveness of different solutions based on different heuristics.

The integrated system is similar to JIT because its operations scheduling process is driven by the pull from downstream operations. However, JIT does not provide a pre-determined daily formal schedule. Also, JIT production follows a more rigid scheduling of products since it requires more or less level production, which is more suitable to repetitive manufacturing environment. On the other hand, the integrated system does not require a level production, hence, it can be employed in both repetitive or batch manufacturing environments. Furthermore, the integrated system employs heuristics to construct schedules to minimize the TPC, while JIT production does not.

The five major inputs of the integrated system are: MPS, BOM, inventory records file, bill of operations and standard time database. The first three inputs are the same as those in MRP. The MPS specifies the independent demand by period for each part. The BOM provides the parts and their quantities required to produce a (parent) product. The inventory records file provides the on-hand inventory and the portion of fixed lead time for each part.

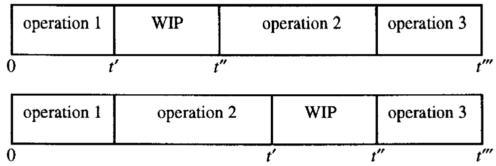

Fig. 4. WIP under two different production schedules.

The bill of operations gives the NOP and their processing precedent relationships. Also, it designates the machine(s) that can process the operations. Finally, the standard time database provides information, such as machine capacity, unit machine time cost, and operation unit processing time and setup time information.

The primary output of the integrated system is the detailed production schedule, which specifies for each work center which operation of which part is to be processed at what time period. Moreover, the objective function value, i.e. TPC, is provided. The secondary outputs include the detailed cost and capacity utilization reports. Fig. 3 illustrates the basic structure of the integrated system.

The TPC consists of three components, which are total machine time cost, holding cost and penalty cost. Total machine time cost includes the costs of setup time and unit processing time. Different machines may require different setup time and per unit processing time and they may have different per unit time cost. Hence, depending on the machine selection and whether a setup is required, the operations scheduling can have a significant impact on the total machine time cost.

Holding cost is assessed whenever a part, work-in-process or finished goods, is waiting to be processed or sold. The Kanban pull concept is designed to minimize inventory as much as possible. Some of the problems associated with inventory include high carrying costs, risks of obsolescence and hiding the underlying problems in production, including quality. Generally, the degree of the associated inventory problems intensifies as the degree of the completeness of inventory increases. For example, finished goods (automobiles) are more expensive to carry than raw materials (steel and glass), and they have a greater chance of obsolescence than raw materials. Therefore, it is more desirable to hold raw materials or WIP of the early stage than WIP of the later stage for the same amount of time.

Fig. 4 shows two different processing timing of a product (with three operations) using Gantt charts. The first Gantt chart shows that operation 1 WIP is held for $t'' - t'$, while the second Gantt chart shows that operation 2 WIP is held for the same period of time, $t'' - t'$. Since the WIP held in the first Gantt chart is of an earlier stage (stage 1) than the WIP held in the second Gantt chart (stage 2), the processing timing of the first Gantt chart is preferred to that of the second Gantt chart.

Hence, the integrated system assumes that per unit holding cost of WIP increases according to the cost of additional processing (value-added). This causes the later stage WIP to be more expensive to carry than the early stage WIP, as there is more value added into the product in the later stage. Therefore, the integrated production planning and scheduling framework drives inventory to its least processed form as much as possible.

Finally, penalty cost occurs whenever the system fails to schedule all the necessary operations to meet the demand on time (meeting all due dates) due to insufficient capacity. In this case, a penalty is levied, as the firm may have to subcontract the products outside or employ other means of production to manufacture them, which can increase the cost considerably. Hence, the penalty cost should be set sufficiently high. For example, a per unit processing cost that is twice as expensive as it would be, produced in-house using the most inefficient or expensive machine.

5. Conclusions

An integrated MRP and JIT framework is proposed to solve the MS-PIS. The essence of the integrated system is that it incorporates the pull element of JIT into MRP. The integrated system is important because it gets rid of the major problems existing in MRP and JIT. There are many avenues for future research in this area. First of all, efficient heuristics need to be developed to solve the MS-PIS problem under the proposed framework. Moreover, it is interesting to determine the factors, such as the length of planning horizon and the size of a time period, that have significant impacts on the performance of heuristics. Lastly, the relationship between due dates and TPC warrants further investigation.

Appendix A

We define the following notation for the proposed heuristics.

m	number of machines
n	number of parts or products
T	number of periods in the planning horizon
p_i	NOP for part i
k	subscript for machines, $k = 1, \dots, m$
i	subscript for parts or products, $i = 1, \dots, n$
t	subscript for time period, $t = 1, \dots, T$
j	subscript for operations of part i, $j = 1, \dots, p_i$
D_{ijt}	the derived demand for operation j of part i that must be completed in period t or earlier
Q_{ijkt}	the quantity of operation j of part i scheduled on machine k at time period t
Z_{ijkt}	unit total cost (processing, setup, and holding costs) if a particular demand for operation j of part i processed by machine k at period t

The steps of the backward and forward procedures based on LUTC criterion are given as follows:
Backward procedure

Step 1 Set $\alpha = \{D_{ijt'} | D_{ijt'} > 0$, for i is the end product or part with independent demand, j is the last operation of part i and $t' = 1, \dots, T\}$.
Step 2 Set $t = T$.
Step 3 Get $\beta = \{D_{ijt'} | D_{ijt'} \in \alpha$ and $t' = t\}$.
Step 4 If $\beta = \emptyset$, then go to Step 9, else enter Step 5.
Step 5 For all D_{ijt} in β, determine $Q_{ijkt'}$ such that $\min\{Z_{ijkt'}\}$, i.e. LUTC criterion, where $t' \le t$.
Step 6 Schedule $Q_{ijkt'}$ and update α and β accordingly.
Step 7 If any predecessor of $Q_{ijkt'}$ exists, then derive the corresponding demands for the immediate predecessors and update them in α.
Step 8 Go to Step 4.
Step 9 Set $t = t - 1$. If $t > 0$, then go to Step 3; otherwise, stop.

Forward procedure

Step 1 Set $\alpha = \{D_{ijt'} | D_{ijt'} > 0$, for i is the end product or part with independent demand, j is the last operation of part i and $t' = 1, \dots, T\}$.

Step 2 Set $t = 1$.
Step 3 Get $\beta = \{D_{ijt'} | D_{ijt'} \in \alpha \text{ and } t' = t\}$.
Step 4 If $\beta = \emptyset$, then go to Step 9, else enter Step 5.
Step 5 For all D_{ijt} in β, determine $Q_{ijkt'}$ such that $\min\{Z_{ijkt'}\}$, i.e. LUTC criterion, where $t' \leq t$.
Step 6 Schedule $Q_{ijkt'}$ and update α and β accordingly.
Step 7 If any predecessor of $Q_{ijkt'}$ exists, then derive the corresponding demands for the immediate predecessors and update them in α.
Step 8 If $\beta \neq \emptyset$, then go to Step 5.
Step 9 If $D_{ijt'} = \emptyset$ for $\forall i, j$ and $t' < t$, then go to Step 10; else, let t be the earliest time such that $D_{ijt} \neq 0$, and go to Step 3.
Step 10 Set $t = t + 1$. If $t \leq T$, then go to Step 3; otherwise, stop.

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