

A note on JIT purchasing vs. EOQ with a price discount: An expansion of inventory costs

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Abstract

In a previous International Journal of Production Economics article, a comparative model was presented for inventory costs of purchasing under economic order quantity (EOQ) with quantity discounts and a just-in-time (JIT) order purchasing system. This paper expands the comparative model to include a relevant cost component not considered in the previous article. The results of the revised model show differences in the conclusions reached in the previous article, more specifically, the superiority of JIT in virtually any type of inventory ordering purchase decision. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

In Fazel et al. [1], a comparative cost function was presented that proposed to show where inventory ordering under an economic order quantity (EOQ) system or a just-in-time (JIT) system would be the most cost effective. They proposed a total cost difference function, Z that had two components given by

$$Z = TC_E - TC_J, \quad (1)$$

where TC_E is the total annual cost using an EOQ system for inventory ordering and TC_J is the total

annual cost using a JIT system for inventory ordering. The total EOQ cost function component, TC_E , for their proposed model was classically given by

$$\begin{aligned} TC_E = & \text{Annual ordering cost} \\ & + \text{Annual holding cost} \\ & + \text{Annual purchasing cost.} \end{aligned}$$

The resulting cost function formula was given by

$$TC_E = \frac{kD}{Q} + \frac{Qh}{2} + (P_E^0 - \pi_E Q)D \quad \text{for } Q \leq Q_{\max}, \quad (2)$$

where k is the cost of placing an order, D is annual demand, Q is order quantity, h is the annual inventory carrying cost per unit, P_E^0 is the purchasing cost per unit under an EOQ system, π_E is a constant representing rate at which the price of the item decreases with increase in order quantities, Q_{\max} is the maximum quantity that can be purchased and

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still receive a quantity discount whose rate is π_E . Taking the derivative of Eq. (2) with respect to Q , the resulting optimal order quantity, Q^* , is

$$Q^* = \sqrt{\frac{2kD}{h - 2\pi_E D}}. \quad (3)$$

For order quantities above Q_{\max} the minimum quantity purchase price of P_E^{\min} remains constant and the TC_E function becomes

$$TC_E = \frac{kD}{Q} + \frac{Qh}{2}(P_E^{\min})D \quad \text{for } Q > Q_{\max}. \quad (4)$$

Taking the derivative of Eq. (4) with respect to Q , the resulting optimal order quantity, Q^{**} , is

$$Q^{**} = \sqrt{\frac{2kD}{h}}. \quad (5)$$

The JIT total cost function, TC_J , for the proposed JIT portion of the model in Eq. (1), was limited to just the annual purchasing cost component in Eq. (2). The JIT total cost function is given by

TC_J = Annual purchasing cost, or

$$TC_J = P_J D, \quad (6)$$

where P_J is the product unit price under a JIT system. Here they assumed, based on their personal experience with JIT and a brief literature review [2], that the annual ordering costs and the annual holding costs were either negligible or transferred to suppliers, thereby being incorporated into the annual purchasing cost component of the function.

Since EOQ or large lot operations usually avail themselves of the quantity discount advantage, the focus of Fazel et al. [1] was on developing cost functions that would show where EOQ and JIT operations would be the least cost strategy. Specifically, a cost function that was valid for $Q \leq Q_{\max}$. The authors accomplished this by developing a total cost-difference function, Z , by substituting Eqs. (2) and (3) into Eq. (1), to derive

$$Z = kD \sqrt{\frac{h - 2\pi_E D}{2kD}} + \frac{h}{2} \sqrt{\frac{2kD}{h - 2\pi_E D}} + \left[P_E^0 - \pi_E \sqrt{\frac{2KD}{h - 2\pi_E D}} \right] D - P_J D. \quad (7)$$

For computed values of Z that are positive, the JIT system is less costly and for negative values of Z the EOQ is the least cost ordering system. Setting $Z = 0$ in Eq. (5) and solving for D , they derived the indifference point, D_{ind} , in which total annual cost of the EOQ system equals that of the JIT system, yielding

$$D_{\text{ind}} = \frac{2kh}{(P_J - P_E^0)^2 + 4k\pi_E}. \quad (8)$$

The economic value of D_{ind} is significant in that it denotes the point at which unit demand determines the superiority of the EOQ system over the JIT system. Fazel et al. [1] went on to demonstrate with an example, that at certain level of D , the EOQ system of ordering inventory was more cost-effective than a JIT system. It is important to note that the authors acknowledged that Q^* is real only when $(h - 2\pi_E D) > 0$.

2. The revised model

We agree with Fazel et al. [1] that Eqs. (2)–(5) represent the classic “quantity discount model” from the family of EOQ models originally proposed by Harris [3]. This same basic model is found in virtually every inventory management textbook. Yet, it should be noted that the quantity discount model gives the EOQ system the advantage of a price break that is not available to the JIT system. No equally obvious cost advantage was given to the JIT cost function when, in practice, one very obvious cost advantage does exist for a JIT system. This cost advantage is the facility size reduction that occurs in the inventory storage and production areas as a result of adopting a JIT system.

Past and present research on JIT system has clearly documented the inevitable reduction in facility square feet. The reduction in facility square footage is caused by the elimination of the space required to store incoming inventory, work-in-process inventory, and finished-goods inventory. JIT experts, such as Schonberger [4, pp. 121–122] and Wantuck [5, p. 16], have long cited examples that prove that conversion to JIT will reduce space in

plants and factories. A more recent example occurred at Federal Signal, an emergency signal manufacturer in Illinois [6]. Federal Signal initiated a JIT operation that saved them 100,000 square feet (roughly 30% of the total facility space) of facility inventory storage and production area from their previous large-lot type of system. In the process of restructuring their layout to accommodate the JIT principles, they ended up renting the space to another company turning what would be a cost into a rental income.

Even fairly small plants have experienced the reduction of square feet when converting to JIT. Tristate Industries, Inc., an Indiana-based manufacturer of industrial piping, applied JIT principles in their 37,000 square feet operations and reported saving 25% of their operating space, which they promptly reallocated to a new product line [7]. Other examples of facility space reduction reported in literature include reports of reducing floor space by 30% [8, p. 330], 40% [9, Chapter 13], and even 50% or more [10]. Hay [11, pp. 22–23] reported space reductions of up to 80%.

With regards to the Fazel et al. [1] model, we suggest a change by expanding Eq. (6) representing the cost function for the JIT component of the model. It is our contention that the TC_J component should be revised to reduce the cost of storage by the saving of square feet brought on by reduced inventory in a JIT system. Specifically, we feel that the total JIT cost function should be revised to include the annual facility cost-reduction, given by

TC_J = Annual purchasing cost

– Annual facility cost reduction,

$$TC_J = P_J D - CN \quad (9)$$

where C is the annual cost of owning and maintaining a square foot of facility, and N is the number of square feet saved by initially adopting a JIT system. The value of C is commonly determined for purposes of overhead costing. The value of N is best determined when a change is made from a large lot system (like EOQ) to a JIT system. Based on the history of JIT implementation, the estimation of N is not difficult. Since the annual facility cost reduction CN can be saved by adopting a JIT system, it should be subtracted from the total an-

nual cost of a JIT system as stated in Eq. (9). This would result in a new total cost-difference function, Z (new), as given by

$$\begin{aligned} Z(\text{new}) = & kD \sqrt{\frac{h - 2\pi_E D}{2kD}} + \frac{h}{2} \sqrt{\frac{kD}{h - 2\pi_E D}} \\ & + \left[P_E^0 - \pi_E \sqrt{\frac{2kD}{h - 2\pi_E D}} \right] D \\ & - (P_J D - CN). \end{aligned} \quad (10)$$

3. Discussion and results

To illustrate the difference between the original results of Fazel et al. [1] and when using Eqs. (9) and (10), we revisited their example problem. In that problem the annual demand was allowed to vary, to examine the impact on the total cost-difference function. The JIT purchasing cost per unit was \$50.50, the EOQ purchasing cost was \$50 per unit, but decreased by \$0.0004 per unit up to a maximum order quantity of 2500 units. Cost per unit for order quantities beyond 2500 units was fixed at \$49 per unit. The annual holding cost per unit was \$15 and the ordering cost per order was \$60. Using the notation in Eq. (7), $P_J = \$50.50/\text{unit}$, $P_E^0 = \$50/\text{unit}$, $P_E^{\min} = \$49/\text{units}$, $\pi_E = 0.0004$, $Q_{\max} = 2,500$ units, $h = \$15/\text{unit}/\text{year}$, and $k = \$60/\text{order}$. The total cost-difference results based on Fazel et al. [1] are depicted in Fig. 1 as the Z function and actual values in the table listed at the bottom of the figure. Note that at the bottom of Fig. 1 the Z -axis numbers change from a positive value (i.e., 69.0465) to a negative value (i.e., –290.02) at a demand level between 5000 and 6000 units. The actual resulting D_{ind} indifference point from Eq. (8) occurs at a D annual demand level of approximately 5202 units.

In this example the use of the Z functions in Eqs. (7) and (10) must meet the condition ($h - 2\pi_E D > 0$). This means that the D annual demand in the Z functions in Eqs. (7) or (10) must be:

$$D < h/2\pi_E. \quad (11)$$

This also means that D must be less than 18,750 units (i.e., $15/(2 \cdot 0.0004)$). Fortunately, the problem

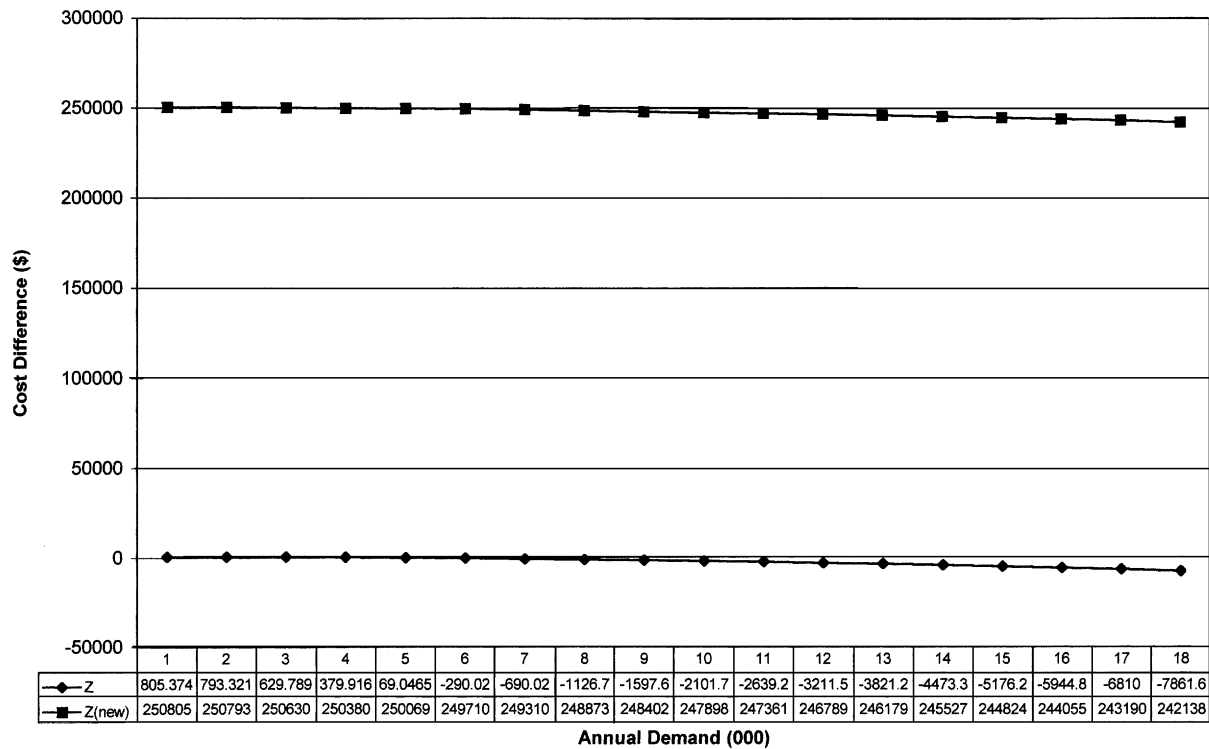


Fig. 1. Cost difference values for Z and Z(new).

example had its D_{ind} occur at a demand level (i.e., 5202 units) well below the 18,750 units level because the cost structure of the problem favored the EOQ side of the cost-difference function.

Now suppose we factor in the annual facility cost reduction as defined by Eqs. (9) and (10). Let us assume a plant consisting of only 200,000 square feet of facility space and a very conservative percent reduction caused by the adoption of JIT of only 5%. This would result in an N annual reduction of 10,000 square feet (i.e., $200,000 \times 0.05$). Assume that the C annual cost per square foot of facility space is only \$25. This results in the addition of \$250,000 (i.e., CN) of cost reduction to TC_j for all levels of D . The resulting revised $Z(\text{new})$ values are presented in Fig. 1. As can be seen at the bottom of Fig. 1, all of these values are positive. Clearly, for any value of D in the quantity discount range of $Q \leq Q_{\max}$, JIT has a favorable total cost difference.

To examine the sensitivity of the model to changes in EOQ costs, the values for h and k were reduced by 50% and the discount rate of was π_E doubled. Making these changes in the model did not change the results. For all values of D in the quantity discount range of $Q \leq Q_{\max}$, JIT still had the favorable total cost difference. Since the revision of the JIT component costs makes the selection of a JIT system significantly preferable to an EOQ system for the range of $Q \leq Q_{\max}$, the issue of the quantity discount rate, π_E becomes mute in this example problem. This requires the use of Eq. (4) for the remainder of the range or $Q > Q_{\max}$. By substituting Eq. (5) into Eq. (4), an alternative $Z(\text{alt})$ function of the total annual cost difference is given by

$$Z(\text{alt}) = kD\sqrt{\frac{h}{2kD}} + \frac{h}{2}\sqrt{\frac{2kD}{h}} + (P_E^{\min})D - (P_JD - CN). \tag{12}$$

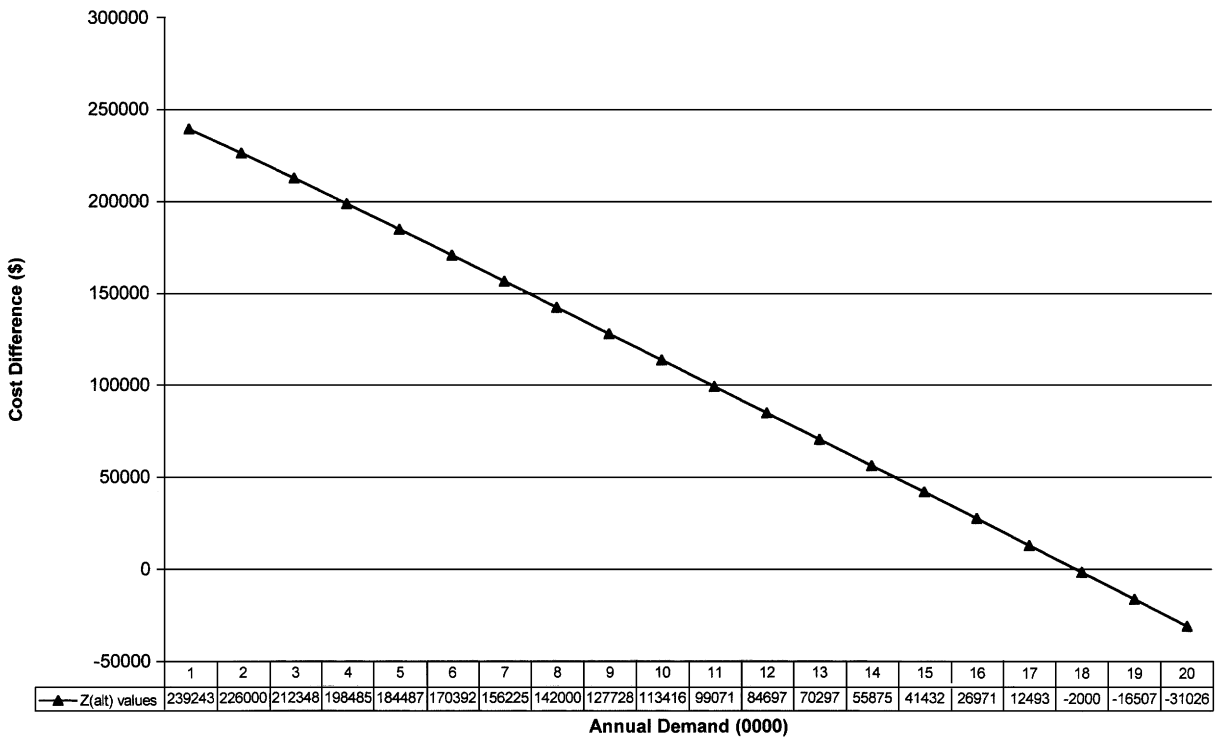


Fig. 2. Cost difference values for Z(alt).

Based on $Z(\text{alt})$, the value for the indifference point between an EOQ system and a JIT system, $D_{\text{ind}(\text{alt})}$, becomes

$$D_{\text{ind}(\text{alt})} = \frac{[(P_J - P_E^{\min})CN + kh] + \sqrt{2(P_J - P_E^{\min})CN kh + k^2 h^2}}{(P_J - P_E^{\min})^2}. \quad (13)$$

The computed values for the $Z(\text{alt})$ function over the quantity discount range $Q > Q_{\max}$ and a fixed respective annual demand of 10,000–200,000 units are presented in Fig. 2. The resulting $D_{\text{ind}(\text{new})}$, indifference point from Eq. (13) now occurs at a D annual demand level of 178,620 units.

While the slope of the $Z(\text{alt})$ function is greater than the $Z(\text{new})$ function, the substantially larger D value creates a situation that causes the EOQ system to behave like a JIT system. Using Eq. (5) with an annual demand of 178,620 units, Q^{**} is 1,195 units. Assuming 261 working days in a year

(i.e., 365 – 104 weekend days) a firm would have to have an order quantity of 1195 units arrive every 1.75 days to meet the annual demand of 178,620 units. This is consistent with a JIT philosophy where small, frequent order quantities are recommended. Hence, the EOQ system has to become a JIT system whenever very large annual demand levels are reached. Also, we have shown in the computations of Fig. 2 that the cost adjustment for a very small reduction in square feet of facility can substantially move the cost indifference point between the two systems. We know that more realistic JIT-caused reductions in square feet of facility (i.e., 30%, 40%, etc.) will have an even more dramatic impact on shifting the cost indifference point to extremely high levels of annual demand, forcing even more frequent JIT-like deliveries.

For annual demand levels in the example problem above the 178,620 units (where supposedly EOQ is cost-preferable to JIT), the order frequency only increases as it would be expected to under

a JIT system. More over, the very large annual demand levels would almost certainly necessitate increasing the plant square footage to handle larger volumes of work. There would be a threshold point where new facilities would have to be acquired with increased holding costs for the EOQ side of the Z cost comparison equation. At the same time, the costs to cover the increase in facility for the EOQ-holding component would be going up, the plant square footage increase would provide an opportunity for a further round of JIT square foot cost reductions. As suggested by Schonberger and [12], there are dynamic forces of continuous improvement for inventory reduction when a JIT system is adopted in place of an EOQ system. Much like a cat trying to catch its tale, the dynamic nature of a JIT system should continuously achieve a cost-advantage over an EOQ system while increasing production activity to meet increasing demand.

4. Summary and conclusions

In this paper, a previous mathematical model was expanded to more fairly represent the realistic JIT cost environment that manufacturers face when implementing a JIT order purchasing system. This addition involved the consideration of the cost savings caused by the reduced square footage in facilities that adopt a JIT ordering system. This study finds that by including the reduced square footage cost-advantage to the JIT side of the comparison model, a JIT system is preferable to an EOQ system for manufacturers whose annual demand is at fairly low levels. The results of the example problem have also shown that the cost indifference point between the two systems occurs at a demand level so substantially large that it would also always require a substantial additional cost in plant facilities. Such an additional cost would again result in a favoring of the JIT system. Also, at such high levels of demand, JIT manufacturers will almost always qualify for P_E^{\min} discounted price per unit, resulting in still further cost advantages in favor of a JIT system. It is our conclusion that a JIT ordering system is preferable to an EOQ system at any level

of annual demand and with almost any cost structure.

As Fazel, Fischer and Gilbert [1] and others [13] have pointed out, the theoretical nature of the economic costing models do not consider many of the other advantages and disadvantages that a JIT system can offer its users. Flexibility, quality and a host of other advantages are common place in JIT systems. And while these advantages can decrease manufacturing costs, other disadvantages, like stock outs caused by a JIT ordering policy can add to the costs. These cost considerations may impact, to a greater or lesser degree, for different manufacturers and, therefore, represent a limitation on the results of this paper.

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