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AN APPROXIMATE SOLUTION TO A JIT-BASED ORDERING SYSTEM

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Abstract: In this paper we consider a multi-stage multi-product production, inventory and transportation system including lot production processes and develop a goal programming model for a pull type ordering system based on the concept of a Just-In-Time(JIT) production system. We also present a pragmatic approach which reduces the required computational time for the formulated mixed integer goal programming problem using a mathematical programming modeling language. The proposed solution procedure is realized utilizing the post-optimal analysis which can be performed by the modeling language.

Key words: JIT, Inventory/Production, Integer Goal Programming, Mathematical Programming Modeling Language

1 INTRODUCTION

Much research has been done on the analysis of pull type ordering system, which is a way of implementing the JIT concept. Recently, mathematical programming approaches to the pull type ordering system, especially for determining the number of circulating kanbans in the kanban system, have been proposed[1]-[4]. Taking such research into account, we propose a new goal programming model for a pull type ordering system in a multi-stage multi-product production, inventory and transportation system. Our model improves insufficient aspects of the preceding models and also has the following characteristics: (1)it considers a manufacturing system in which the setup action is improved and can deal with the number of setups required; and (2)its objective includes not only replenishment inventory level but also cost objective. As it is essential to improve the setup action and reduce the setup time in the JIT implementation effort, our model describes the JIT concept more properly than past works.

When we apply a mathematical programming approach to ordering systems, it is very important to achieve the following: (1)to improve an interface for the model solution, as mathematical modeling requires transformations of the model form; and (2)to reduce the required computational time, as the mathematical programming model includes many integral variables. This paper proposes a pragmatic approach which tackles these problems at the same time using a mathematical programming package including a modeling language[5]. We present an approximate solution procedure which reduces the required computational time for the formulated mixed integer goal programming problem. The approximate procedure is realized utilizing the post-optimal analysis which can be performed by the modeling language.

2 MODEL DESCRIPTION

In this paper we consider a multi-stage multi-product production, inventory and transportation system which has an assembly-tree-structure. It consists of N stages and let $n \in \{1,2,\ldots,N\}$ index the stages with the understanding that n=1 stands for the final stage. Each stage, $n \in \{1,2,\ldots,N\}$, includes a production process, an immediately succeeding stock point and an on-hand stock point for the production process of the succeeding stage. Let $t \in \{0,1,\ldots,T\}$ index the time periods with the understanding that the planning horizon starts at the beginning of period 1 and finishes at the end of period T. Figure 1 shows an example of the conceptual diagram of the model presented in this paper.

We assume this system satisfies the following conditions:

(1) Demand for final products in each period is specified by customers and a backlog of orders is not permitted.
(2) Each stage produces M types of items and let $i \in \{1,2,\ldots,M\}$ index the items.
(3) At each stage, production and withdrawal quotas for the whole planning horizon are given. The quotas are determined by the effective demand imposed upon the stage.
(4) The raw material inventories supplied for most preceding stages are sufficient. But at other stages, production and withdrawal are restricted by inventories at each stock point.
(5) The lead time for production at stage n is LP^n , and that for withdrawal at stage n is LH^n .
(6) The setup time and processing time for each item at each stage are known and fixed in the planning horizon.
(7) The minimum inventory level at the end of each period is predetermined for each item at each stock point.
(8) The inventory carrying cost per unit, the setup cost per one time at each stage and the overtime labor cost for each stage in each period are known.
(9) When the production process requires setup, the size of sublot is given and the production is carried out based on the sublot.

The last condition is mainly due to the external setup. Without the corresponding minimum time span, we cannot switch items. As the preceding research works did not consider the setup itself or this aspect on the actual setup action, we introduced a minimum lot size in our model to take into account the setup constraint[4]. In the manufacturing system we considered in our past work, improvement of the setup action, especially the external setup, was insufficient. However, it is essential to improve the setup action and reduce the setup time in the JIT implementation effort. We therefore consider a manufacturing system in which the setup action is further improved to the extent that the production process can deal with the plural setup for each item in one period.

The objectives of the model are to minimize the sum of the replenishment level at each stock point and to minimize the sum of the inventory carrying cost, the setup cost and the overtime labor cost.

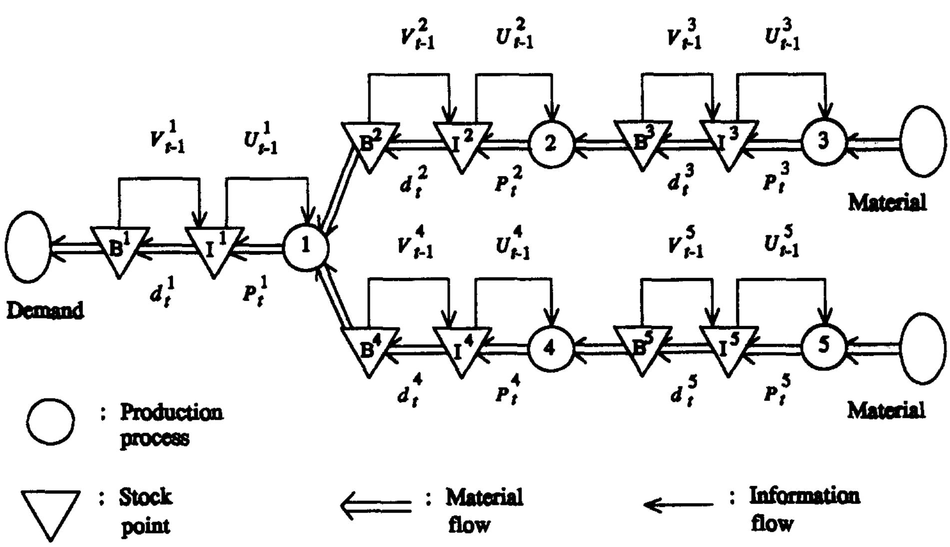

Fig.1 A Conceptual Diagram of the Presented Model (N=5)

We propose the following mixed integer goal programming model for a pull type ordering system.

$$\text{lexicographically minimize} \quad f = \{ (z_1^+ - z_1^-), (z_2^+ - z_2^-) \} \tag{1}$$

subject to

$$\sum_{n=1}^{N}\sum_{i=1}^{M} \left(I_0^{n(i)} + \sum_{j=1}^{LP^n} P_{j-LP^n}^{n(i)} + U_0^{n(i)} + B_0^{n(i)} + \sum_{j=1}^{LB^n} d_{j-LB^n}^{n(i)} + V_0^{n(i)} \right) + (z_1^- - z_1^+) = 0 \tag{2}$$

$$\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{M} \left(CI^{n(i)} I_t^{n(i)} + CB^{n(i)} B_t^{n(i)} + CS^{n(i)} X_t^{n(i)} \right) + \sum_{t=1}^{T}\sum_{n=1}^{N} CO_t^n O_t^n + (z_2^- - z_2^+) = 0 \tag{3}$$

$$I_t^{n(i)} = I_{t-1}^{n(i)} + P_{t-LP^n}^{n(i)} - d_t^{n(i)} \qquad (i=1,2,\ldots,M;\ n\in J;\ t=1,2,\ldots,T) \tag{4}$$

$$B_t^{1(i)} = B_{t-1}^{1(i)} + d_{t-LB^1}^{1(i)} - D_t^{(i)} \qquad (i=1,2,\ldots,M;\ t=1,2,\ldots,T) \tag{5}$$

$$B_t^{n(i)} = B_{t-1}^{n(i)} + d_{t-LB^n}^{n(i)} - e^{sn(i)} P_t^{sn(i)} \qquad (i=1,2,\ldots,M;\ n\in J1;\ t=1,2,\ldots,T) \tag{6}$$

$$U_t^{n(i)} = U_{t-1}^{n(i)} - P_t^{n(i)} + d_t^{n(i)} \qquad (i=1,2,\ldots,M;\ n\in J;\ t=1,2,\ldots,T) \tag{7}$$

$$V_t^{1(i)} = V_{t-1}^{1(i)} - d_t^{1(i)} + D_t^{(i)} \qquad (i=1,2,\ldots,M;\ t=1,2,\ldots,T) \tag{8}$$

$$V_t^{n(i)} = V_{t-1}^{n(i)} - d_t^{n(i)} + e^{sn(i)} P_t^{sn(i)} \qquad (i=1,2,\ldots,M;\ n\in J1;\ t=1,2,\ldots,T) \tag{9}$$

$$P_t^{n(i)} \le U_{t-1}^{n(i)} \qquad (i=1,2,\ldots,M;\ n\in J;\ t=1,2,\ldots,T) \tag{10}$$

$$d_t^{n(i)} \le V_{t-1}^{n(i)} \qquad (i=1,2,\ldots,M;\ n\in J;\ t=1,2,\ldots,T) \tag{11}$$

$$P_t^{n(i)} = L^{n(i)} X_t^{n(i)} \qquad (i=1,2,\ldots,M;\ n\in K;\ t=1,2,\ldots,T) \tag{12}$$

$$\sum_{i=1}^{M} a^{n(i)} P_t^{n(i)} + \sum_{i=1}^{M} S^{n(i)} X_t^{n(i)} \le W_t^n + O_t^n \qquad (n\in K;\ t=1,2,\ldots,T) \tag{13}$$

$$\sum_{i=1}^{M} a^{n(i)} P_t^{n(i)} \le W_t^n + O_t^n \qquad (n\in J\text{-}K;\ t=1,2,\ldots,T) \tag{14}$$

$$\sum_{j=1}^{T} P_j^{n(i)} \ge Q^{n(i)} \qquad (i=1,2,\ldots,M;\ n\in J) \tag{15}$$

$$\sum_{j=1}^{T} d_j^{n(i)} \ge R^{n(i)} \qquad (i=1,2,\ldots,M;\ n\in J) \tag{16}$$

where,

$$R^{1(i)} = \max \{ 0,\ \sum_{j=1}^{T} D_j^{(i)} - B_0^{1(i)} + SB_T^{1(i)} \} \qquad (i=1,2,\ldots,M) \tag{17}$$

$$Q^{n(i)} = \max \{ 0,\ R^{n(i)} - I_0^{n(i)} + SI_T^{n(i)} \} \qquad (i=1,2,\ldots,M;\ n\in J) \tag{18}$$

$$R^{n(i)} = \max \{ 0,\ e^{sn(i)} Q^{sn(i)} - B_0^{n(i)} + SB_T^{n(i)} \} \qquad (i=1,2,\ldots,M;\ n\in J1) \tag{19}$$

$$B_t^{1(i)} \ge SB_t^{1(i)} \qquad (i=1,2,\ldots,M;\ t=1,2,\ldots,T) \tag{20}$$

$$B_t^{n(i)} \geq SB_t^{n(i)} \qquad (i=1,2,...,M;\ n \in J1;\ t=1,2,...,T) \qquad (21)$$

$$I_t^{n(i)} \geq SI_t^{n(i)} \qquad (i=1,2,...,M;\ n \in J;\ t=1,2,...,T) \qquad (22)$$

$$0 \leq O_t^n \leq A_t^n \qquad (i=1,2,...,M;\ n \in J;\ t=1,2,...,T) \qquad (23)$$

$$X_t^{n(i)} \quad : \text{non-negative integer,} \qquad (i=1,2,...,M;\ n \in K;\ t=1,2,...,T) \qquad (24)$$

$$P_t^{n(i)},\ d_t^{n(i)} \quad : \text{non-negative integer,} \qquad (i=1,2,...,M;\ n \in J;\ t=1,2,...,T) \qquad (25)$$

$$U_0^{n(i)},\ V_0^{n(i)} \quad : \text{non-negative integer,} \qquad (i=1,2,...,M;\ n \in J\ ) \qquad (26)$$

$$z_1^+,\ z_1^-,\ z_2^+,\ z_2^- \geq 0 \qquad (27)$$

Equation (2) indicates the goal constraint, minimizing the sum of the replenishment level at each stock point, with priority level 1. Equation (3) indicates the goal constraint, minimizing the sum of the inventory carrying cost, the setup cost and the overtime labor cost, with priority level 2. Equations (4)-(6) show the conservation of material flow at each stock point. Equations (7)-(9) describe the conservation of ordering flow for production and withdrawal. Constraints (10) and (11) indicate that quantities of production and withdrawal are restricted by ordering quantities. Constraint (12) denotes the relation between production and sublot. Constraints (13) and (14) indicate that production is restricted by the production capacity. Constraints (15) and (16) show that the sum of production and withdrawal must satisfy the quota for the whole planning horizon respectively. Equations (17)-(19) show the effective demand at each stage. Constraints (20)-(22) indicate that the inventory quantity at the end of each period must be kept at least the minimum inventory level. Constraint (23) shows the overtime labor available at each stage. The non-negative integrality of the number of setups, quantities of production and withdrawal, and initial ordering quantities, is enforced by constraints (24)-(26). Constraint (27) shows deviation variables to be minimized.

3 MODEL SOLUTION

3.1 Framework of the solution procedure

When applying a mathematical programming approach to ordering systems, the problem to be solved is two-fold. One part is to improve an interface for the model solution, as mathematical modeling requires transformations of the model form. The other is to reduce the computational time required, as the mathematical programming model includes many integral variables. This paper proposes a pragmatic approach which tackles both problems at the same time. In order to do this we do the following:

(1) Reformulate the presented model in order to reduce the number of integral variables and describe the reduced mixed integer goal programming problem by a mathematical programming modeling language.
(2) Adopt a sequential mixed integer goal programming[6],[7] to solve the resulting mixed integer goal programming problem.
(3) Develop an approximate solution procedure to rapidly obtain a suboptimal solution of the single objective mixed integer goal programming problem.

In this paper, we use XPRESS-MP[5], which consists of a modeling language, mp-model, and an optimizer, mp-opt. The modeling language algebraically describes the resulting mixed integer goal programming problem and automatically generates a matrix file which is the input form to the optimizer. The solution procedure presented in this paper cannot be realized only by the individual functions of the standard package. The development of a solution system utilizing and coordinating the functions is also required.

3.2 Approximate procedure to the single objective problem

The optimizer we utilize in order to solve the mixed integer goal programming problem is based on the "branch-and-bound" method. In the branch-and-bound method, rules for branching and bounding operations influence the required computational time. Working within the parameters on the optimizer, we can control those rules according to the characteristics of the problems to be solved. We propose to use an approximate solution procedure by controlling the following: (1)the selection of branching variables; (2)the selection of nodes; and (3)the upper bound which is used at the bounding operation. The "approximate procedure" we propose in this paper is based on the following rules: (1)Select branching variables according to a priority ordering policy; (2)Use a depth-first search; and (3)Use $(1 - \alpha)$ times of the incumbent value as the upper bound at the bounding operation. Using this approximate procedure, we can obtain a suboptimal solution which guarantees that the relative error based on the incumbent value is less than or equal to α (see Eq.(28)).

The priority ordering policy is in this order; (1) $X_t^{n(l)}$: variables for the number of setups, (2) $U_0^{n(l)}$, $V_0^{n(l)}$: variables for initial orders, and (3) $P_t^{n(l)}$, $d_t^{n(l)}$: variables for production and withdrawal quantities.

The upper bound used at the bounding operation is as follows;

$$\text{CUTOFF} = \text{IPOBJ} \times (1 - \alpha), \tag{28}$$

where CUTOFF : upper bound at the bounding operation,
IPOBJ : incumbent value at integer programming (IP) step,
α : relative error (ex. $\alpha = 0.01$).

The approximate procedure is realized utilizing the post-optimal analysis which can be performed by the modeling language. In the branch-and-bound search, the optimizer uses the upper bound, CUTOFF, calculated by utilizing the post-optimal analysis.

4 CONCLUSION

We considered multi-stage, multi-product production systems and developed a goal programming model for a pull type ordering system based on the concept of a JIT production system. Our model considers a manufacturing system in which the setup action is improved and can deal with the number of setups required. As it is essential to improve the setup action and reduce the setup time in the JIT implementation effort, our model describes the JIT concept more properly than past works. This paper also proposed a pragmatic approach, using a mathematical programming modeling language, in order to solve the problems on a mathematical programming approach to ordering systems.

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