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# A genetic algorithm for optimizing defective goods supply chain costs using JIT logistics and each-cycle lengths

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ABSTRACT

The competitive environment of global markets has forced many manufacturers to select the most appropriate supply chain network (SCN) for reduction of total costs and wasted time. Cost reduction and selection of the appropriate length of each period are two important factors in the competitive market that are often not addressed comprehensively by researchers. In our study, we proposed genetic algorithms (GAs) for optimising a novel mathematical model of the defective goods supply chain network (DGSCN). In the proposed model, we assumed that all imperfect-quality products are not repairable, whereas those considered as scrap are directly sold to customers at a low price. The objective of the proposed model is to minimise the costs of production, distribution, holding and backorder. In addition to minimising the costs, the model can determine the economic production quantity (EPQ), the appropriate length of each cycle (ALOEC) and the quantities of defective products, scrap products and retailer shortages using Just-In-Time logistics (JIT-L). We used the GAs and a Cplex solver with probability parameters and various dimensions for validation of the studied model in real-life situations, and we compared the outputs to demonstrate the performance of the model. Additionally, to identify the appropriate length of each cycle (ALOEC), we needed to solve the model using exact parameters and same dimensions and prefer to use Lingo for this application.



## 1. Introduction

To survive in the current competitive environment of global markets, manufacturers have been forced to choose an appropriate supply chain network (SCN) for cost reduction in their inventory systems [1–5].One aspect of the supply chain (SC) is logistics, which is the process of planning, implementing and controlling the flow and storage of goods or services. The SC also involves the related information flow from a point of origin to a point of consumption for the purpose of conforming to customer requirements [6,7].

Furthermore, in recent years, one of the most important developments in manufacturing systems has been the Just-in-Time (JIT) system, which has been demonstrated to increase productivity. The JIT approach was initially used at Toyota in the 1960s and subsequently spread throughout the world. JIT has been used by many American companies (i.e., Xerox, GE, and Hewlett Packard) to decrease the level of inventory [8–10]. Kawtummachai and Van Hop [11] considered price and JIT delivery as objective programming parameters, and their results showed that the JIT delivery date lies in a different logical domain. Shyur and Shih [12] applied JIT to delivery, production quality, price/cost, facilities, technology, response to

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customer needs, and exit quality criteria. Guo and Tang [13] proposed a unified model for the optimisation of supply chains based on JIT and confirmed the accuracy of the cost optimisation. Amasaka [14] presented a new JIT method that is not limited only to TPS or TQM but also includes software and hardware systems. His findings showed that JIT had a positive impact on sales, design, improvement, and production in the Toyota factory. Additionally, Farahani and Elahipanah [15] solved the JIT distribution problem for a SCM by presenting a model with two target functions designed for a three-level distribution network in a supply chain.

Most of the classical production models assume that all goods generated by production machines are characterised by perfect quality. In reality, imperfect goods production is quite normal in manufacturers. Therefore, the costs of defective goods are considered essential in supply chain optimisation. In most cases, reworking of defective goods can be quite profitable if the cost of production is high or if raw materials are limited and costly [16–18].

Moreover, studies of inventory control models that oversee the basic concepts of warehouse needs and imperfect products after manufacturing have attracted researchers' attentions [19,20]. Lee and Kim [21] proposed a hybrid method for solving the optimisation problems of production–distribution planning in Supply Chain Management (SCM). In their model, the production and distribution capacities of a machine are limited. Flapper and Teunter [22] considered the defective products of single-item manufacturers that used the same facilities and production machines for reworking. Eroglu and Ozdemir [23] presented a model of defective goods and the ordered amount that is returned as shortage. To solve the proposed model, they used numerical examples that showed that defective and useless goods could reduce the overall profits. Another regularly adopted model is the economic production quantity model for defective goods [24]. However, these researchers did not consider the reworking function of defective goods.

Wahab and Jaber [25] presented an economic order quantity model for defective goods; it showed that defective goods and shortages might reduce the overall profit of the firm. Ramazani et al. [26] studied a three-objective stochastic model for a forward/reverse logistic network. The first objective was to maximise total profit, the second objective was to maximise the customer service level in both the forward and reverse networks, and the third objective was to minimise the total amount of raw material. Franca et al. [27] proposed a two-objective stochastic model for the supply chain to evaluate the relationship between profit and quality. The first objective was to maximise the profit, and the second was to minimise the defective raw material. Their numerical results showed that a decrement in the defective raw material led to an increment in the profit and a decrease in the financial risk.

Genetic Algorithms (GAs), which are search heuristics that mimic the process of natural evolution, have been routinely used to generate useful solutions for optimisation and search problems. Kannan et al. [28] proposed a mathematical model for a closed-loop battery-recycling SCN by solving with GAs and a GAMS optimiser. They evaluated their proposed model by comparing the results of GAs with that of a GAMS optimiser. Altiparmak et al. [29] proposed a mathematical model for a three-stage SCN with a single source and multiple products. In their model, they presented a solution approach based on steady-state genetic algorithms (ssGA). For evaluation of the ssGA, they compared their results with those obtained by Cplex. Abu Qudeiri et al. [30] investigated new GAs for a serial-parallel production line (S-PPL) to maximise production efficiency by optimising three decision variables: the buffer size between each pair of work stations, the machine numbers in each of the work stations, and the machine types.

According to the literature, cost, quality and time are three key elements that should be noted in the design of a SCN. Therefore, reducing costs, increasing the quality and selecting the appropriate length of each-cycle (ALOEC) are important aspects in a competitive market; however, less attention has been comprehensively focused on this issue. In this paper, we propose a mathematical model for the DGSCN. The proposed mathematical model can determine the ALOEC according to the importance of customer lead time using JIT logistics in addition to minimising the costs of production, maintenance, transportation, defective goods, scrap products, retailer shortage, and indirect costs and determining the economic production quantity. These factors are not fully addressed in other models. We used the GAs and a Cplex solver with probability parameters and various dimensions for validation of the studied model in real-life situations, and we compared the outputs to demonstrate the performance of the model. Additionally, to identify the appropriate length of each cycle (ALOEC), we needed to solve the model using exact parameters and same dimensions and prefer to use Lingo for this application. The remainder of the paper is organised as follows. Section 2 studies the model structure, assumptions, limitations, decision vari-

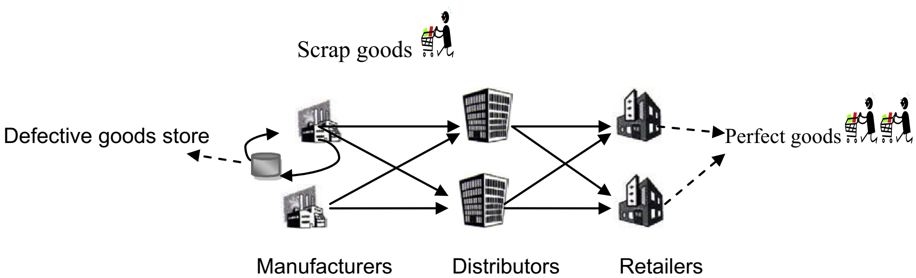


**Fig. 1.** The three level supply chain network (SCN).

ables, and parameters of the mathematical model. Section 3 presents the proposed GAs, and Section 4 investigates the results and discussion. Finally, Section 5 presents conclusions.

## 2. Modelling

We propose a mathematical model for a DGSCN that is able to determine the EPQ and ALOEC using JIT logistics in addition to minimising the costs of production, maintenance, transportation, rework of defective goods, scrap products and retailer shortages due to the production of defective goods and indirect costs. The proposed model is divided into three levels: level 1 indicates the manufacturers, level 2 represents the distributors and level 3 denotes the retailers. The supply chain network used in this study is shown in Fig. 1.

### 2.1. Assumptions and limitations of the model

In developing the proposed model, the following assumptions were established:

1. The amount of demand was assigned to the manufacturers at the beginning of the period.
2. The duration of each period was equal to the sum of the production and rework times.
3. Retailer shortage is allowed.
4. The model is designed for multiple manufacturers, distributors, retailers, products and multi-periods.
5. The inspection process for the products is perfect, and the inspection time is zero.
6. Rework of imperfect products is allowed.
7. We assumed that all imperfect quality products are not repairable, whereas those considered as scraps.
8. The locations of plants, distributors, retailers and suppliers are fixed.
9. The capacities of the manufacturers, the distributors and the retailers are known.
10. Products are temporarily stored at the distributors before delivery to the retailers.
11. There is no inventory at the distributors at the beginning or end of the planning horizon.

Furthermore, several limitations were considered in the proposed model:

I. The capacities of the manufacturer, distributor and retailers are limited.
II. The storage capacities for each perfect product are limited.
III. Store capacities and allocated storage capacities for defective goods are limited.
IV. All demands must be satisfied during the planning horizon.
V. The production and reworking times are limited.

### 2.2. Decision variables and parameters

The parameters and decision variables used in the cost optimisation are defined as follows:

#### 2.2.1. Parameters

$p_{ilt}$ : Percentage of defective goods $l$ produced by manufacturer $i$ during period $t$.
$\gamma_{ilt}$ : Rework cost per defective good $l$ by manufacturer $i$ during period $t$
$h_{jlt}$ : Holding cost of products $l$ at distributor $j$ during period $t$.
$h'_{ilt}$ : Holding cost of products $l$ for defective goods stored at manufacturer $i$ during period $t$.
$\theta_{ilt}$ : Time required to produce goods $l$ by manufacturer $i$ during period $t$.
$T\theta_t$ : Total production time during period $t$.
$\mu_{ilt}$ : Rework time required for goods $l$ by manufacturer $i$ during $t$.
$T\mu_t$ : Total rework time during period $t$.
$Pc_{ilt}$ : Production cost per item by manufacturer $i$ during period $t$.
$TC_{ijlt}$ : Shipping cost of each product $l$ from manufacturer $i$ to distributor $j$ during period $t$.
$T'C_{jklt}$: Shipping cost of each product $l$ from distributor $j$ to retailer $k$ during period $t$.
$T''C_{ilt}$ : Shipping cost of each defective goods $l$ at manufacturer $i$ during period $t$ (from production to defective goods storage and vice versa).
$\alpha_{ilt}$ : Percentage of scrap products$l$ produced by factory $i$ during period $t$.
$f_{ilt}$ : Discount cost per scrap goods $l$ for sale by factory $i$ during period $t$.
$\beta_{klt}$ : Shortage cost for each product $l$ at retailer $k$ during period $t$.

$Ind_t$: Indirect costs (such as stagnant capital and maintenance) during period$t$.
$In_{ljt}$ : Inventory of product $l$ at distributor $j$ during period $t$.
$d_{klt}$ : Demand of retailer $k$ for product $l$ during period $t$.
$S_{ilt}$ : Production capacity of factory $i$ for product $l$during period $t$.
$W_{jt}$ : Total storage capacity of distributor $j$ during period $t$.
$W_{jlt}$ : Storage capacity of distributor $j$ for product $l$ during period $t$.
$W'_{kt}$ : Total storage capacity of retailer $k$ during period $t$.
$W'_{klt}$ : Storage capacity of retailer $k$ for product $l$ during period $t$.

### 2.2.2. Variables

$X_{ijlt}$ : Amount of products $l$ transported from factory $i$ to distributor $j$ during period $t$.
$Q_{ilt}$ : EPQ of products $l$ by factory $i$ during period $t$.
$Def_{ilt}$ : Amount of defective goods $l$ produced by factory $i$ during period $t$.
$Sc_{ilt}$ : Amount of scrap goods $l$ produced by factory $i$ during period $t$.
$Co_{ilt}$ : Amount of perfect products $l$ produced by factory $i$ during $t$ before rework.
$TCo_{ilt}$ : Amount of perfect products $l$ produced by factory $i$ during $t$ after rework.
$Y_{jklt}$ : Amount of products $l$ transported from distributor $j$ to retailer $k$ during period $t$.
$B_{klt}$ : Amount of shortage of products $l$ in retailer $k$ during period $t$.
$T_t$ : ALOEC according to the importance of lead time by customers in period $t$.

## 2.3. Mathematical model

The objective function is presented as follows:

$$\begin{aligned} Z_{\min} = {} & \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} Pc_{ilt}.Q_{ilt} + \sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{t=1}^{T} h_{jlt}.\left(\sum_{\tau=1}^{t}\sum_{i=1}^{I} X_{ijl\tau} - \sum_{\tau=1}^{t}\sum_{k=1}^{K} Y_{jkl\tau}\right) + \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} h'_{ilt}.Def_{ilt} + \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} \gamma_{ilt}.Def_{ilt} \\ & + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{t=1}^{K} TC_{ijlt}.X_{ijlt} + \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{t=1}^{T} T'C_{jklt}.Y_{jklt} + \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} T''C_{ilt}.Def_{ilt} + \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} f_{ilt}.Sc_{ilt} \\ & + \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{t=1}^{T} B_{klt}.\beta_{klt} + \sum_{t=1}^{T} Ind_t.T_t. \end{aligned} \tag{1}$$

The constraints and their explanations are presented in the following:

$$Q_{ilt} \leqslant S_{ilt} \qquad \forall \quad i, l, t, \tag{2}$$

$$\sum_{i=1}^{I}\sum_{l=1}^{L} X_{ijlt} \leqslant W_{jt} \qquad \forall \quad j, t, \tag{3}$$

$$\sum_{i=1}^{I} X_{ijlt} \leqslant W_{jlt} \qquad \forall \quad j, l, t, \tag{4}$$

$$\sum_{j=1}^{J}\sum_{l=1}^{L} Y_{jklt} \leqslant W'_{kt} \qquad \forall \quad k, t, \tag{5}$$

$$\sum_{j=1}^{J} Y_{jklt} \leqslant W'_{klt} \qquad \forall \quad k, l, t, \tag{6}$$

$$Def_{ilt} = p_{ilt}.Q_{ilt} \qquad \forall \quad i, l, t, \tag{7}$$

$$Sc_{ilt} = \alpha_{ilt}.Def_{ilt} \qquad \forall \quad i, l, t, \tag{8}$$

$$Co_{ilt} = (1 - p_{ilt}).Q_{ilt} \quad \forall \quad i, l, t, \tag{9}$$

$$TCo_{ilt} = Co_{ilt} + Def_{ilt} - Sc_{ilt} \quad \forall \quad i, l, t, \tag{10}$$

$$\sum_{k=1}^{K} d_{klt} + \sum_{i=1}^{I} Sc_{ilt} \leqslant \sum_{i=1}^{I} Q_{ilt} \quad \forall \quad l, t \tag{11}$$

$$\sum_{k=1}^{K} d_{klt} \leqslant \sum_{i=1}^{I} TCo_{ilt} \quad \forall \quad l, t, \tag{12}$$

$$\sum_{i=1}^{I} Co_{ilt} \leqslant \sum_{k=1}^{K} d_{klt} \quad \forall \quad l, t, \tag{13}$$

$$\sum_{k=1}^{K} B_{klt} = \sum_{k=1}^{K} d_{klt} - \sum_{i=1}^{I} Co_{ilt} \quad \forall \quad l, t, \tag{14}$$

$$\sum_{j=1}^{J} Y_{jklt} = d_{klt} \quad \forall \quad k, l, t, \tag{15}$$

$$\sum_{k=1}^{K}\sum_{\tau=1}^{t} Y_{jkl\tau} \leqslant \sum_{i=1}^{I}\sum_{\tau=1}^{t} X_{ijl\tau} \quad \forall \quad j, l, t \neq T, \tag{16}$$

$$\sum_{i=1}^{I}\sum_{\tau=1}^{t} X_{ijl\tau} - \sum_{k=1}^{K}\sum_{\tau=1}^{t} Y_{jkl\tau} = \sum_{\tau=1}^{t} In_{lj\tau} \quad \forall \quad j, l, t \neq T, \tag{17}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{I} X_{ijlt} = \sum_{t=1}^{T}\sum_{k=1}^{K} Y_{jklt} \quad \forall \quad j, l, \tag{18}$$

$$\sum_{j=1}^{J} X_{ijlt} \leqslant TCo_{ilt} \quad \forall \quad i, l, t, \tag{19}$$

$$\sum_{i=1}^{I}\sum_{l=1}^{L} \theta_{ilt}.Q_{ilt} \leqslant T\theta_t \quad \forall \quad t, \tag{20}$$

$$\sum_{i=1}^{I}\sum_{l=1}^{L} \mu_{ilt}.Def_{ilt} \leqslant T\mu_t \quad \forall \quad t, \tag{21}$$

$$T\theta_t + T\mu_t \leqslant T_t \quad \forall \quad t, \tag{22}$$

$$X_{ijlt}, Y_{jklt}, Q_{ilt}, B_{klt}, Def_{ilt}, Sc_{ilt}, Co_{ilt}, TCo_{ilt} \geqslant 0 \quad \forall \quad i, j, k, l, t. \tag{23}$$

Eq. (1) represents an objective function that minimises the total costs of the supply chain, including the costs of production, holding at the distributor, defective good storage, rework of defective goods, transportation from the manufacturers to the distributors, transportation from the distributors to the retailers, transportation from the manufacturers to defective good storage and vice versa, the discount cost of scrap goods for sale, retailer shortages due to defective goods production, and indirect costs such as stagnant capital and maintenance.

Eq. (2) states the restriction of production capacity. Eqs. (3) and (4) denote the delivery capacity limitations of the distributors for all types of products and each type of product, respectively. Eqs. (5) and (6) consider the delivery capacity limitations of the retailers for all product types and each product type, respectively. Eq. (7) calculates the amount of defective goods, and Eq. (8) represents the amount of scrap goods. Eq. (9) calculates the amount of perfect goods before reworking. Eq. (10) expresses the total amount of perfect goods after reworking. Eq. (11) states that total production is equal to the sum of demand and scrap goods. Eq. (12) shows that the total demands are covered by the amount of perfect goods. Eqs. (13) and (14) investigate the amounts of shortage at the retailers due to defective goods. Eq. (15)explains how the total demands during the planning horizon are supplied. Eqs. (16) and (17) represent the inventory at the distributors; there is no inventory at the distributors in the beginning and at the end of the planning horizon. Eq. (18) shows the balance between the total inputs and outputs of goods moving to and from the distributors during the planning horizon. Eq. (19) assures that the total goods shipped from the manufacturers to the distributors are of perfect quality; in other words, the scrap goods are removed from the system. Eq. (20) explains the available time limitations of the production facilities for all production processes. Eq. (21) shows the available time limitations of the production facilities for rework processes. Eq. (22) shows the length of each period. Eq. (23) indicates that the production amount, deliveries to warehouses and retailers, retailer shortages, scrap goods, defective goods, and perfect goods before and after reworking should all have positive values.

## 3. Proposed GA

In this study, we chose the GA due to its many advantages: (i) Because the answer space is extensive, the number of infeasible answers is low and the probability of reaching the global optimum is high; (ii) The GA has the ability to work with many decision variables; (iii) The large number of dimensions in the proposed model requires along run-time because the GA has several starting points; therefore, it can simultaneously search the problem space from multiple directions, and if one fails to find the result, the other directions can continue. However, most other algorithms do not run in parallel and can only search the problem space from one direction at a time. Therefore, if the answer represents a local optimal answer or the result is a subset of the main answer, the algorithm must be started again from the beginning. The flowchart of the proposed GA is shown in Fig. 2.

### 3.1. Chromosomes

One of the important components of the GA is the selection of chromosomes. We attempted to select the best chromosomes in the proposed GAs that would give us good results and require a low run-time. In the studied mathematical model, the variable $Q_{ilt}$ has both direct and indirect relationships with the variables $Sc_{ilt}$, $Co_{ilt}$, $B_{klt}$, $X_{ijlt}$ and $Y_{jklt}$. Therefore, any change in variable $Q_{ilt}$ leads to certain changes in other variables, and thus, the variable $Q_{ilt}$ was chosen as the chromosome. The chromosomes are displayed in a matrix with the columns denoting the number of periods (T) and rows (I∗L). Each gene in the matrix was randomly created by Matlab software, as shown in Eq. (24).

$$Matrix\ Q_{(I*L)*T} = \begin{pmatrix} Q_{111} & \cdots & Q_{11T} \\ \vdots & \ddots & \vdots \\ Q_{IL1} & \cdots & Q_{ILT} \end{pmatrix}_{(I*L)*T} \tag{24}$$

### 3.2. Initial population

A certain number of chromosomes were randomly created according to:

Eq. (1) $Q_{ilt} \leqslant S_{ilt} \quad \forall\ i, l, t.$

And

Eq. (10)

$$\sum_{k=1}^{K} d_{klt} + \sum_{i=1}^{I} Sc_{ilt} \leqslant \sum_{i=1}^{I} Q_{ilt} \quad \forall \quad l$$

Next, the amount $Q_{ilt}$ should be expressed as follows [Eq. (25)]:

$$\sum_{k=1}^{K} d_{klt} < \sum_{i=1}^{I} Q_{ilt} \leqslant \sum_{i=1}^{I} S_{ilt} \qquad \forall \quad l, t \tag{25}$$

The initial population is randomly produced, and the fitness of each individual is subsequently evaluated. The individuals are sorted based on their fitness, and after crossover and mutation operations, the parents and produced children are placed in the pool. The value of the objective function of the children is calculated. Finally, the next generation of chromosomes is selected from among these chromosomes according to the population size. The chromosomes are arranged based on the value of the objective function and the best chromosomes are selected for the next generation.

### 3.3. Genetic operations

The following describes the main operations of the GA, which are crossover, mutation, and selection.

#### 3.3.1. Crossover

To perform the crossover, two chromosomes must be merged. First, the chromosomes to be combined should be identified and allowed to mix. The columns will be combined for each chromosome selected for the crossover, and the intersection

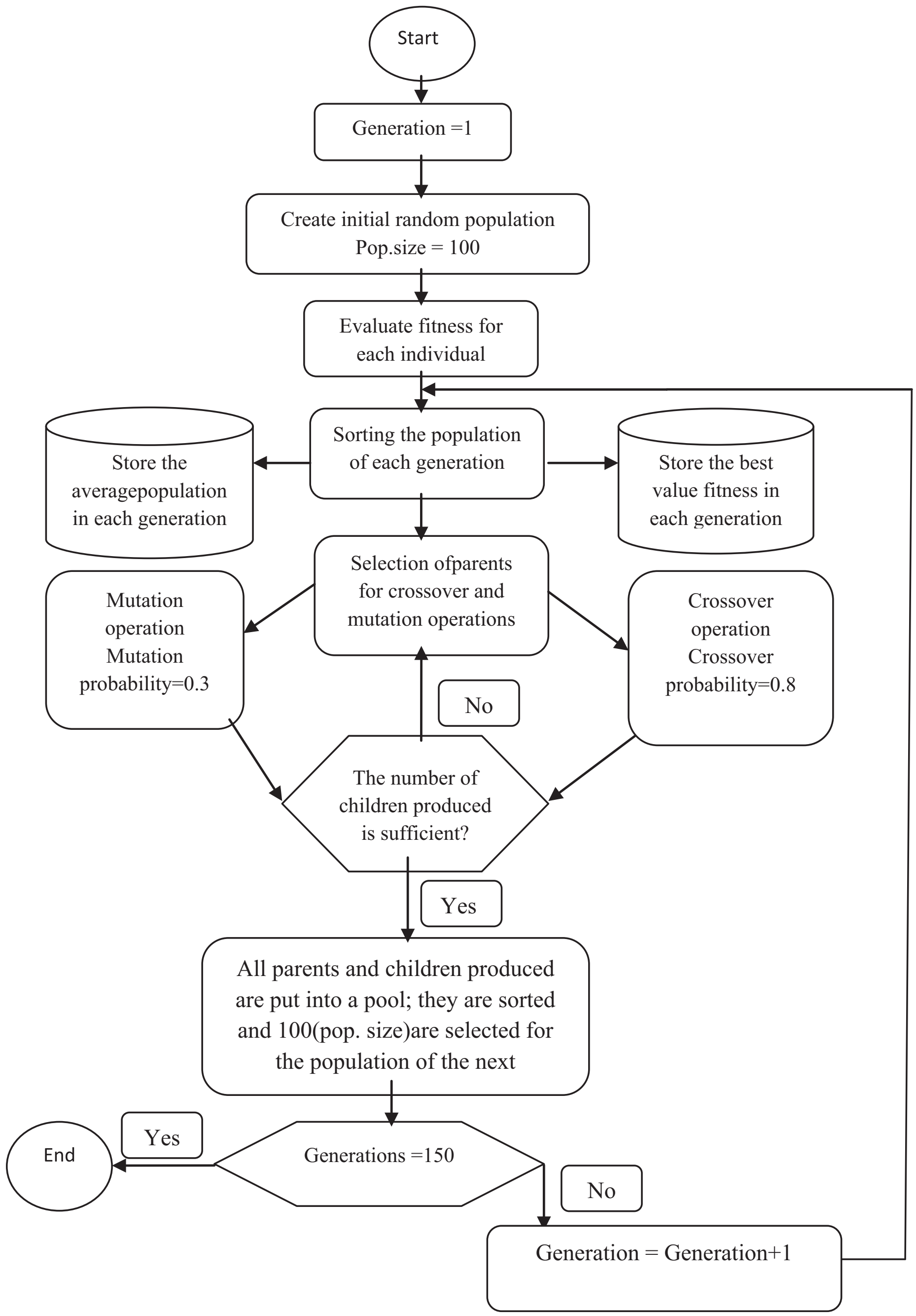


**Fig. 2.** Flowchart of the proposed GA.

point will be used to combine the chromosomes. For example, if we work with two chromosomes with four periods and three rows, the combination of the chromosomes will proceed as follows.

First, the intersection point is chosen. Thereafter, the values of both sides of the matrix are exchanged. The intersection point shown in this example denotes the first period (Fig. 3).

### *3.3.2. Mutation*

The mutation probability refers to the probability of change in any gene. In this study, each chromosome receives a certain number of genes that are assumed to change, and this number is derived by multiplying the total number of genes by the mutation probability. Accordingly, a number of genes must be selected to undergo mutation. For example, if the muta-

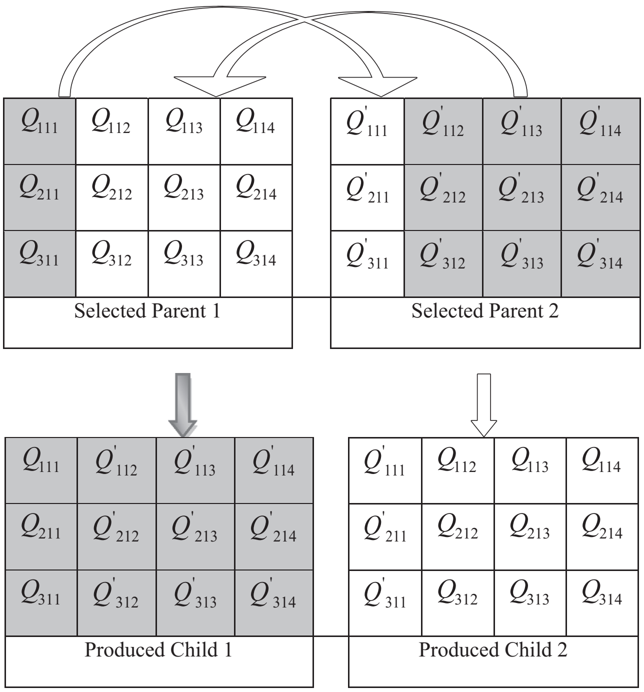


**Fig. 3.** Display of the crossover operation.

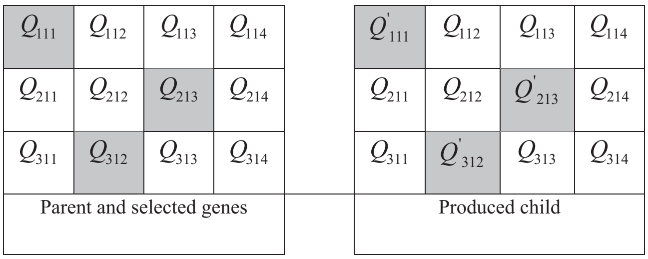


**Fig. 4.** Display of the mutation operation.

tion probability is 0.3 and I = 3, L = 1, and T = 4, the total number of genes required to undergo mutation will calculated as follows:

Round $(0.3 * 3 * 1 * 4 = 3.6) = 3$.

The resulting value is rounded off to the nearest whole number. Thus, in this example, three genes should undergo mutation. These three genes are randomly selected, and their values are changed (Fig. 4).

### *3.3.3. Selection*

Different strategies can be applied to perform the selection function, and the elite strategy was chosen in this study. First, the parents and produced children were placed in the pool, and the value of the objective function of the children was calculated. Finally, from among these chromosomes, the next generation of chromosomes was selected according to the population size. The chromosomes were arranged based on the value of the objective function and the best chromosomes were selected for the next generation.

## 4. Results and discussion

This study proposes a novel mathematical model for optimising the supply chain costs of imperfect products. The aim of the model is to optimise total costs, including production, holding, shipping, reworking of defective goods, scrap goods and retailer shortages due to defective goods production. Additionally, this model is able to determine the economic production quantity (EPQ), the appropriate length of each cycle (ALOEC) and the number of active manufacturers and distributors.

**Table 1**
Twelve sample problems with different dimensions.

| Sample problem | I | J | K | L | T |
|---|---|---|---|---|---|
| 1 | 2 | 2 | 2 | 2 | 2 |
| 2 | 3 | 2 | 3 | 2 | 3 |
| 3 | 4 | 5 | 3 | 1 | 3 |
| 4 | 3 | 6 | 2 | 2 | 2 |
| 5 | 4 | 3 | 4 | 1 | 4 |
| 6 | 3 | 4 | 4 | 3 | 3 |
| 7 | 4 | 3 | 6 | 1 | 4 |
| 8 | 6 | 2 | 4 | 1 | 3 |
| 9 | 2 | 3 | 2 | 2 | 4 |
| 10 | 6 | 4 | 5 | 1 | 7 |
| 11 | 10 | 9 | 8 | 1 | 6 |
| 12 | 8 | 9 | 10 | 1 | 10 |

**Table 2**
Parameters and values

| Parameters | Value |
|---|---|
| $p_{ilt}$ | Uniform (0,0.09) |
| $Pc_{ilt}$ | Normal (15,3) |
| $\gamma_{ilt}$ | Normal (6,2) |
| $h_{jlt}$ | Normal (10,3) |
| $h'_{ilt}$ | Normal (6,2) |
| $\theta_{ilt}$ | Normal (6,3) |
| $\mu_{ilt}$ | Normal (5,2) |
| $TC_{ijlt}$ | Normal (12,3) |
| $T'C_{jklt}$ | Normal (8,2) |
| $T''C_{ilt}$ | Normal (5,2) |
| $f_{ilt}$ | Normal (11,3) |
| $\beta_{klt}$ | Normal (6,1) |
| $\alpha_{ilt}$ | Uniform (0,0.9) |
| $d_{klt}$ | Normal (2000,1000) |
| $Ind_t$ | Normal (0,0.6) |

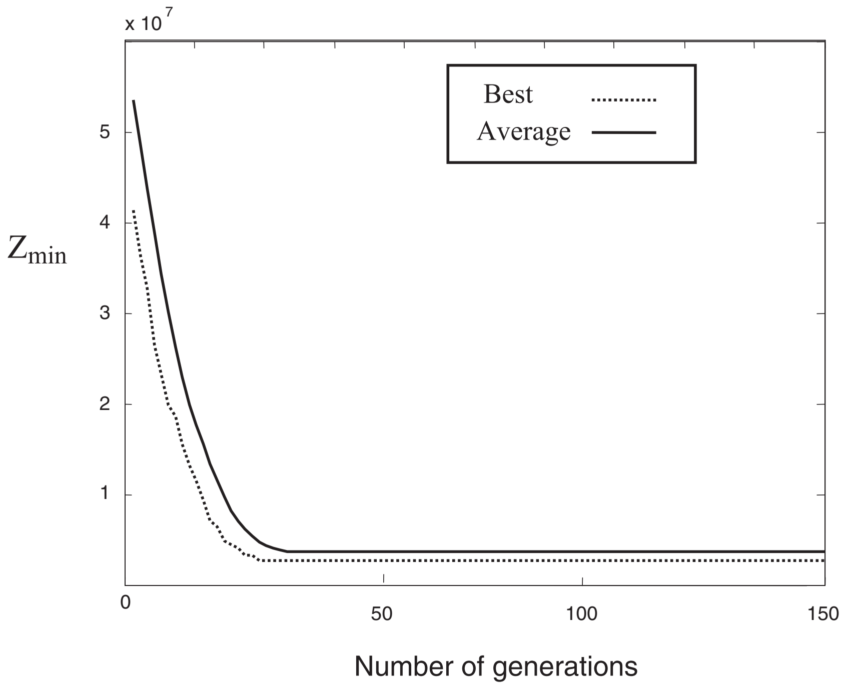


**Fig. 5.** GA output for $Z_{\min}$ (best and average) of sample problem 11.

**Table 3**
Results of nine runs for twelve sample problems solved with the GA.

| Sample problem | $Z_{min}$ (Run 1) | $Z_{min}$ (Run 2) | $Z_{min}$ (Run 3) | $Z_{min}$ (Run 4) | $Z_{min}$ (Run 5) | $Z_{min}$ (Run 6) | $Z_{min}$ (Run 7) | $Z_{min}$ (Run 8) | $Z_{min}$ (Run 9) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 358400 | 365020 | 351474 | 342600 | 355289 | 370550 | 340958 | 360840 | 341649 |
| 2 | 875600 | 866700 | 857654 | 873840 | 853500 | 860200 | 880090 | 854990 | 860500 |
| 3 | 338280 | 319010 | 347800 | 358650 | 329300 | 347030 | 350790 | 320500 | 352340 |
| 4 | 428500 | 408040 | 459270 | 416080 | 425120 | 412034 | 445006 | 436090 | 420830 |
| 5 | 743100 | 740980 | 752780 | 739930 | 724310 | 720100 | 742700 | 722770 | 725840 |
| 6 | 1588000 | 1587410 | 1591460 | 1521860 | 1579210 | 1532080 | 1529660 | 1523820 | 1590040 |
| 7 | 1133600 | 1125980 | 1110320 | 1137060 | 1101520 | 1112600 | 1133030 | 1127070 | 1135540 |
| 8 | 583300 | 620560 | 603500 | 632930 | 618070 | 595900 | 584883 | 615610 | 604380 |
| 9 | 685800 | 690600 | 681030 | 688500 | 693090 | 701230 | 696710 | 691928 | 684200 |
| 10 | 1667000 | 1659540 | 1671580 | 1688010 | 1669210 | 1707000 | 1657800 | 1651390 | 1672160 |
| 11 | 2024000 | 2110000 | 2090830 | 2180340 | 2040700 | 2125870 | 2140650 | 2010230 | 2060890 |
| 12 | 3388000 | 3278060 | 3350800 | 3409850 | 3275970 | 3360320 | 3343860 | 3389480 | 3275006 |

**Table 4**
Results of nine runs for ten sample problems solved with Cplex.

| Sample problem | $Z_{min}$ (Run 1) | $Z_{min}$ (Run 2) | $Z_{min}$ (Run 3) | $Z_{min}$ (Run 4) | $Z_{min}$ (Run 5) | $Z_{min}$ (Run 6) | $Z_{min}$ (Run 7) | $Z_{min}$ (Run 8) | $Z_{min}$ (Run 9) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 350418 | 367402 | 341049 | 350712 | 345510 | 349100 | 351034 | 335900 | 339240 |
| 2 | 862530 | 868921 | 854004 | 863490 | 848900 | 855609 | 848300 | 859100 | 849231 |
| 3 | 333880 | 329040 | 340076 | 315009 | 318781 | 351500 | 318007 | 322110 | 316903 |
| 4 | 429609 | 403490 | 439007 | 417540 | 410500 | 414007 | 410005 | 396843 | 420087 |
| 5 | 733679 | 740006 | 743006 | 741800 | 716660 | 719993 | 732005 | 718002 | 725008 |
| 6 | 1522900 | 1590707 | 1510600 | 1532000 | 1566008 | 1540088 | 1511866 | 1515700 | 1570600 |
| 7 | 1134000 | 1123000 | 1127098 | 1136000 | 1090660 | 1080900 | 1141000 | 1082707 | 1088970 |
| 8 | 575000 | 601590 | 613530 | 642956 | 572009 | 580400 | 583787 | 573009 | 581098 |
| 9 | 674900 | 690000 | 680003 | 677450 | 690600 | 675909 | 701006 | 692300 | 683600 |
| 10 | 1648908 | 1658980 | 1647890 | 1692006 | 1648000 | 1661009 | 1650775 | 1657000 | 1646211 |
| 11 | 2027005 | 2050500 | 2091800 | 2006750 | 2102000 | 2077098 | 2000650 | 2008590 | 2121008 |
| 12 | 3288840 | 3267006 | 3288000 | 3268900 | 3303000 | 3351089 | 3310090 | 3267980 | 3270098 |

**Table 5**
GA and CPLEX results for 12 sample problems with different dimension.

| Sample problem | GA | | CPLEX | | Gap% = [(GA − CPLEX)/ CPLEX] ∗ 100 |
|---|---|---|---|---|---|
| | $Z_{min}$ (least) | Run time (sec) | $Z_{min}$ (least) | Run time (sec) | |
| 1 | 340958 | 161 | 335900 | 0.008 | 1.5 |
| 2 | 853500 | 254 | 848300 | 0.015 | 0.6 |
| 3 | 319010 | 281 | 315009 | 0.015 | 1.27 |
| 4 | 408040 | 301 | 396843 | 0.016 | 2.82 |
| 5 | 720100 | 296 | 716660 | 0.016 | 0.48 |
| 6 | 1521860 | 396 | 1510600 | 0.017 | 0.74 |
| 7 | 1101520 | 284 | 1080900 | 0.016 | 1.9 |
| 8 | 583300 | 234 | 572009 | 0.014 | 1.97 |
| 9 | 681030 | 274 | 674900 | 0.015 | 0.9 |
| 10 | 1651390 | 561 | 1646211 | 0.018 | 0.31 |
| 11 | 2010230 | 1372 | 2000650 | 0.021 | 0.48 |
| 12 | 3275006 | 2586 | 3267006 | 0.024 | 0.24 |

Twelve sample problems with different dimensions were used to validate the correctness and fine functions of the model, as shown in Table 1. For solution of the model and accuracy of the results, we did not rely only on an exact optimiser such as Cplex. Instead, we proposed a GA, and we compared the results of both methods. The studied GA was tested with different generation sizes. The appropriate generation size was 150; increasing the generation size increased the run-time of the GA but resulted in minimal changes in the answers. To mimic a real-life situation, statistical distributions were used to select the values of the parameters, as shown in Table 2.

In the proposed GA, the population size is 100. Therefore, in each generation, 100 answers ($Z_{min}$) were produced, and the average of 100 answers was reported together with the best answer in each generation. The $Z_{min}$ (best) is the smallest amount of answers, and the $Z_{min}$ (average) is the average of 100 answers in each generation. The curves for $Z_{min}$ (best) and $Z_{min}$ (average) for Sample Problem 11 are shown in Fig. 5. This figure shows the fine function of the proposed GAs. Additionally, to demonstrate the performance accuracy of the model and the GA, each sample problem was solved nine times by

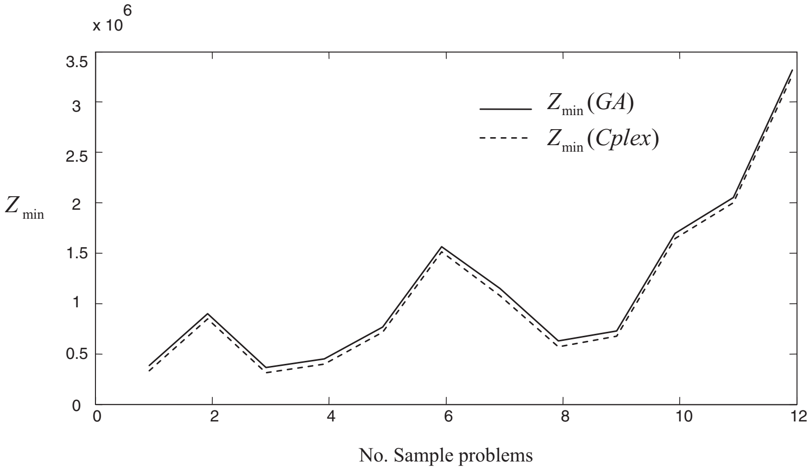


**Fig. 6.** Comparison of the GA and Cplex results.

**Table 6**
Outputs of the model for the best amount of sample problem 4.

| $Sc_{ilt}$ | $Def_{ilt}$ | $Co_{ilt}$ | $TCo_{ilt}$ | $B_{klt}$ | $Q_{ilt}$ | $X_{ijlt}$ | $Y_{jklt}$ |
|---|---|---|---|---|---|---|---|
| $Sc_{111} = 76$ | $Def_{111} = 291$ | $Co_{111} = 4190$ | $TCo_{111} = 4406$ | $B_{111} = 0$ | $Q_{111} = 4482$ | $X_{1111} = 0$ | $Y_{1111} = 0$ |
| $Sc_{112} = 16$ | $Def_{112} = 191$ | $Co_{112} = 2585$ | $TCo_{112} = 2760$ | $B_{112} = 174$ | $Q_{112} = 2777$ | $X_{1112} = 397$ | $Y_{1112} = 397$ |
| $Sc_{121} = 0$ | $Def_{121} = 0$ | $Co_{121} = 0$ | $TCo_{121} = 0$ | $B_{121} = 108$ | $Q_{121} = 0$ | $X_{1121} = 0$ | $Y_{1121} = 0$ |
| $Sc_{122} = 262$ | $Def_{122} = 335$ | $Co_{122} = 4307$ | $TCo_{122} = 4379$ | $B_{122} = 0$ | $Q_{122} = 4642$ | $X_{1122} = 0$ | $Y_{1122} = 0$ |
| $Sc_{211} = 0$ | $Def_{211} = 0$ | $Co_{211} = 0$ | $TCo_{211} = 0$ | $B_{211} = 215$ | $Q_{211} = 0$ | $X_{1211} = 4406$ | $Y_{1211} = 0$ |
| $Sc_{212} = 0$ | $Def_{212} = 0$ | $Co_{212} = 0$ | $TCo_{212} = 0$ | $B_{212} = 0$ | $Q_{212} = 0$ | $X_{1212} = 2362$ | $Y_{1212} = 0$ |
| $Sc_{221} = 256$ | $Def_{221} = 373$ | $Co_{221} = 5195$ | $TCo_{221} = 5303$ | $B_{221} = 0$ | $Q_{221} = 5568$ | $X_{1221} = 0$ | $Y_{1221} = 0$ |
| $Sc_{222} = 0$ | $Def_{222} = 0$ | $Co_{222} = 0$ | $TCo_{222} = 0$ | $B_{222} = 72$ | $Q_{222} = 0$ | $X_{1222} = 4379$ | $Y_{1222} = 0$ |
| | | | | | | $X_{2111} = 0$ | $Y_{2111} = 2050$ |
| | | | | | | $X_{2112} = 0$ | $Y_{2112} = 0$ |
| | | | | | | $X_{2121} = 0$ | $Y_{2121} = 2703$ |
| | | | | | | $X_{2122} = 0$ | $Y_{2122} = 1958$ |
| | | | | | | $X_{2211} = 0$ | $Y_{2211} = 2355$ |
| | | | | | | $X_{2212} = 0$ | $Y_{2212} = 2362$ |
| | | | | | | $X_{2221} = 5303$ | $Y_{2221} = 2600$ |
| | | | | | | $X_{2222} = 0$ | $Y_{2222} = 2421$ |

**Table 7**
Values of the parameters for the considered sample problem in Fig. 7.

| Parameters | Value of parameters | | |
|---|---|---|---|
| $\alpha_{ilt}$ | $\alpha_{111} = 0.3$ | $\alpha_{211} = 0.35$ | $\alpha_{311} = 0.27$ |
| $\beta_{klt}$ | $\beta_{111} = 10$ | $\beta_{211} = 14$ | $\beta_{311} = 12$ |
| $\gamma_{lit}$ | $\gamma_{111} = 4$ | $\gamma_{211} = 6$ | $\gamma_{311} = 2$ |
| $\theta_{ilt}$ | $\theta_{111} = 4$ | $\theta_{211} = 6$ | $\theta_{311} = 5.5$ |
| $\mu_{ilt}$ | $\mu_{111} = 1.3$ | $\mu_{211} = 2$ | $\mu_{311} = 2.5$ |
| $f_{ilt}$ | $f_{111} = 7$ | $f_{111} = 9$ | $f_{111} = 10$ |
| $Pc_{ilt}$ | $Pc_{111} = 30$ | $Pc_{211} = 25$ | $Pc_{311} = 27$ |
| $p_{ilt}$ | $p_{111} = 0.07$ | $p_{211} = 0.05$ | $p_{111} = 0.08$ |
| $T''C_{ilt}$ | $T''C_{111} = 3$ | $T''C_{211} = 6$ | $T''C_{311} = 2$ |
| $d_{klt}$ | $d_{111} = 3000$ | $d_{211} = 6000$ | $d_{311} = 8000$ |
| $Ind_t$ | $Ind_1 = 0.5$ | | |
| $h_{jlt}$ | $h_{111} = 8$ | $h_{211} = 7$ | |
| $h'_{ilt}$ | $h'_{111} = 2$ | $h'_{211} = 4$ | $h'_{311} = 7$ |
| $TC_{ijlt}$ | $TC_{1111} = 6$ | $TC_{2111} = 4$ | $TC_{3111} = 8$ |
| | $TC_{1211} = 5$ | $TC_{2211} = 3$ | $TC_{3211} = 9$ |
| $T'C_{jklt}$ | $T'C_{1111} = 6$ | $T'C_{2111} = 3$ | $T'C_{1211} = 6$ |
| | $T'C_{2211} = 4$ | $T'C_{1311} = 9$ | $T'C_{2311} = 5$ |

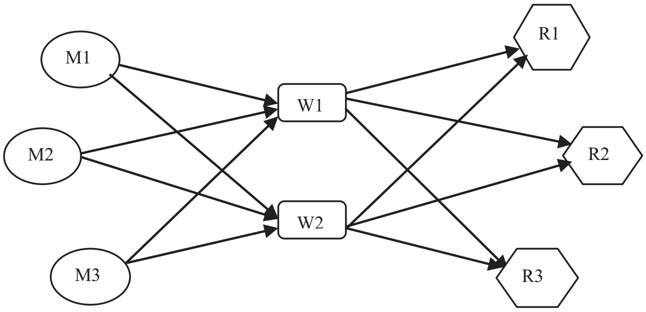


**Fig. 7.** Scheme of a sample problem with dimensions $i = 3$, $j = 2$, $k = 3$, $l = 1$ and $t = 1$.

**Table 8**
Results of nine sample problems with the same dimensions.

| Sample problem | $T\theta_1$ | $T\mu_1$ | $T_1$ | $\sum sc$ | $\sum Def$ | $\sum Co$ | $\sum TCo$ | $\sum B$ | $\sum Q$ | $Z_{min}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 69500 | 1600 | 71100 | 364 | 1215 | 16149 | 17000 | 851 | 17364 | 731339 |
| 2 | 77000 | 1800 | 78800 | 356 | 1183 | 16173 | 17000 | 827 | 17356 | 715358 |
| 3 | 85000 | 2000 | 87000 | 347 | 1143 | 16204 | 17000 | 796 | 17347 | 699035 |
| 4 | 93000 | 2200 | 95200 | 339 | 1103 | 16235 | 17000 | 764 | 17339 | 682712 |
| 5 | 100418 | 2448 | 102866 | 332 | 1080 | 16251 | 17000 | 748 | 17332 | 666132 |
| 6 | 108000 | 2650 | 110650 | 332 | 1080 | 16251 | 17000 | 748 | 17332 | 670023 |
| 7 | 115000 | 2900 | 117900 | 332 | 1080 | 16251 | 17000 | 748 | 17332 | 673648 |
| 8 | 123000 | 3100 | 126100 | 332 | 1080 | 16251 | 17000 | 748 | 17332 | 677748 |
| 9 | 130000 | 3300 | 133300 | 332 | 1080 | 16251 | 17000 | 748 | 17332 | 681348 |

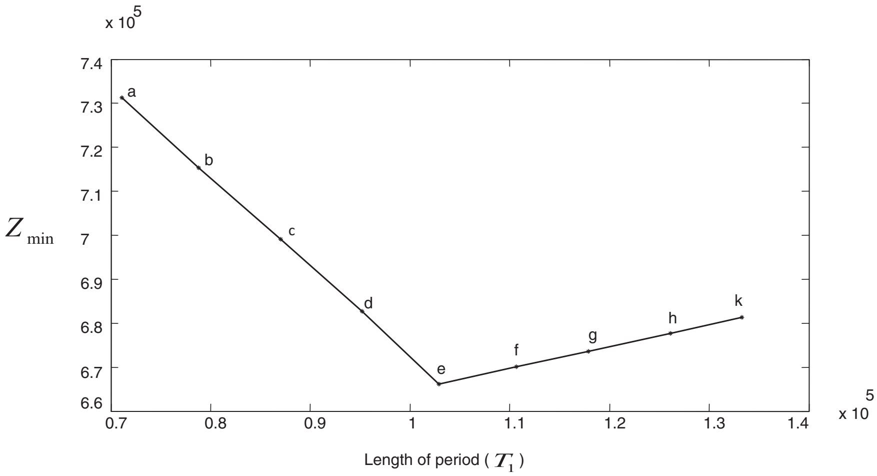


**Fig. 8.** Relationship between the length of the period and $Z_{\min}$.

the GA and Cplex using the parameters of Table 2. Due to the statistical distributions chosen for the values of the parameters, the outputs are different. The best result of each run from the GA and Cplex are illustrated in Tables 3 and 4. In addition, the best result of each sample problem (out of nine runs, from Tables 3 and 4) is selected and presented in Table 5 together with its run time. The results from Fig. 5 and 6 and Table 5 show the correctness and fine function of the model and the GA. The findings for the proposed model (i.e., the total costs, amounts of EPQ, defective goods, scrap goods, retailer shortage, perfect goods and amounts of products) that should be transferred between the nodes of the supply chain network for sample problem 4 are shown in Table 6.

We used the GA and Cplex together with probability parameters for validation of the studied model in a real-life situation and compared the outputs to demonstrate the good performance of the model. Table 5 and Fig. 6 show the fine function of the presented model.

Another objective of the proposed model was to find the appropriate length of each cycle (ALOEC).To reach this answer; we needed to solve the model using the exact parameters. In this case, we prefer to use Lingo.

According to the restrictions of Eqs. (20)–(22), any change in the $T\theta$ and $T\mu$ has a direct effect on the variables $Q_{ilt}$ and $Def_{ilt}$ and consequently, on $Sc_{ilt}$, $Co_{ilt}$, $B_{klt}$ and $Z_{\min}$. Additionally, because $T\theta_t + T\mu_t = T_t \quad \forall \quad t$, $T_t$ has a direct effect on the mentioned variables. We illustrate this scenario using an example.

We considered a supply chain network with dimensions $i = 3, j = 2, k = 3, l = 1, t = 1$ and the exact parameters shown in Table 7. According to the dimensions, there are 18 routes from the manufacturers to the retailers in the network (see Fig. 7).

If no time limitation exists, the model searches for the best path using the parameters in Table 7 to obtain the minimum cost (see sample problem 5 in Table 8 and point e in Fig. 8). Additionally, the smallest time for production is needed to supply the retailer demands (see sample problem 1 in Table 8 and point a in Fig. 8). The total costs decreased with increments in the production and repair times (see sample problems 1,2,3,4 and 5 in Table 8 and points a, b, c, d and e in Fig. 8). Point e represents the minimum, and any further increment in the times leads to increase in the total costs due to indirect costs such as stagnant capital and maintenance. The relationship between the total costs and the length of the period is shown in Fig. 8, and the best length for the period lies in the interval$[T_a, T_e]$. This period is dependent on the significance of the customer order time.

## 5. Conclusions

In this paper, we have proposed a new mathematical model for a three-echelon defective goods supply chain network (DGSCN) using JIT logistics that not only minimises the costs of production, holding, transport, defective goods, scrap products, retailer shortages and indirect costs (i.e., stagnant capital and maintenance) but also determines the economic production quantity (EPQ) and the appropriate length of each period (ALOEP) according to the importance of customer lead time (see Tables 5,6 and 8). These factors are not addressed comprehensively in other models.

The GA and Cplex techniques were used to solve the proposed mathematical model. To model the real states accurately, the values of the parameters were treated as probabilities with normal and uniform distributions. The calculated gap between the best results of the GA and Cplex demonstrates the accuracy and fine function of the proposed model. Additionally, Table 5 shows that the results and run time from Cplex are better than those of the GA solver.

Another objective of the investigated model was to identify the appropriate length of each cycle (ALOEC).To obtain this answer, we were required to solve the model using the exact parameters and prefer to use Lingo in this case. This model can also determine the number of active manufacturers and distributors and is applicable for all producers faced with problems of defective goods.

## Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.apm.2013.08.023.

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