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JIT production planning approach with fuzzy due date for OKP manufacturing systems[1]

Wei Wang[a,*], Dingwei Wang[a], W.H. Ip[b]

[a] Department of Systems Engineering, College of Information Science and Engineering, Northeastern University, Shenyang, 110006, People's Republic of China

[b] Department of Manufacturing Engineering, The Hong Kong Polytechnic University, Hong Kong

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Abstract

In general practice, most customers use a fuzzy due date rather than exact date when operating their one-of-a-kind product (OKP) manufacturing systems. They usually have different degrees of satisfaction with the due date. In order to clearly describe the practical problems, two kinds of models with different types of fuzzy due dates for OKP manufacturing systems are built to control production by the just-in-time (JIT) philosophy. In addition, we have developed an algorithm for JIT production planning with a fuzzy due date. The proposed method extends the traditional production planning approaches that have a precise nature to those with a case imprecise nature. The computational results have shown that the algorithm has the potential to be applied to practical OKP manufacturing systems. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Master production scheduling (MPS) plays a key role in the manufacturing resource planning (MRP-II) system [1]. This provides a medium-term production planning to meet the marketing requirements and to utilize the available production capacity of the manufacturing systems. The traditional production planning models are used to minimize the total production cost or to maximize the product output [2,3]. Ever since JIT manufacturing systems were successfully developed in Japan [4], earliness/tardiness scheduling and planning problems have been a very active research area [5–7].

Since economic reform in China, a number of manufacturing enterprises have begun to change their goal from high output to high-quality service. Production time has become an important object of interest to many enterprise managers. The OKP

* Corresponding author. Tel.: + 86 24 391 4141; fax: + 86 24 390 6300; e-mail: dwwang@mail.neu.edu.cn.

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manufacturing system involves the production of non-standard items in a one-off mode by general-purpose machinery. Its production characteristics include production on the basis of customer's orders, many kinds of products, complex composition, and a lengthy production period. In particular, the due date is usually determined as a duration time by the manufacturer and customer, that is an uncertain or fuzzy due date. If the product is delivered in the duration time, the customer should accept no conditions. Otherwise, there are earliness/tardiness penalties for the manufacturer.

In the presented papers, the models of earliness/tardiness production planning are built under the assumption that the due date is an exact date [6,7]. It is clear that there are differences between the theory and the actual production. Due to the complexity of the fuzzy due date, the original earliness/tardiness production planning models with a due date are inefficient. The JIT production control cannot be met by the manufacturer. Now, the problems of a fuzzy due date have attracted many researchers [8–11]. However, the research presented here focuses on single-machine and parallel multi-machine scheduling problems with earliness and tardiness penalties [10,11]. The processing capacity of the single-machine and parallel multi-machine is constant in these problems. When we face the production planning problems of a whole manufacturing system in a planning horizon longer than the scheduling horizon, the processing capacity is an aggregated capacity of many distinct facilities. Due to the maintenance and renewals of facilities, it cannot be taken as a constant. Therefore, the manufacturing capacity constraints have to be considered in the earliness/tardiness production planning problems. This paper focuses on earliness/tardiness production planning models with a fuzzy due date for the OKP manufacturing system.

This paper is organized into six sections. First, we give the problem description for the OKP manufacturer. In order to control production by JIT philosophy, we built two models of earliness/tardiness production planning with different types of fuzzy due dates as shown in Section 3. In Section 4, based on the proofs of theorems presented, the algorithm of production planning with a fuzzy due date can be achieved. This extends the traditional production planning approaches with a due date into a fuzzy due date. Finally, we also give a computational example and some concluding remarks.

2. Problem description

Assuming that an OKP manufacturer is going to produce n products in the coming planning horizon $[1, T]$, these products have differences in the standard, type and sale prices, etc. All types of products are not replaced by each other. According to the customers' orders, the following is known: the contract price of the order i is q_i, $i = 1, 2, \dots, n$. The order i asks the earliest due date e_i^L and the latest due date time d_i^U, and the best due date is within a time duration (e_i^U, d_i^L), where $(e_i^U, d_i^L) \subset (e_i^L, d_i^U)$, $i = 1, 2, \dots, n$. Based on the processing technique, we know each product has to pass m processing stages. The manufacturing timespan of the product i is p_i, and the capacity requirement of product i for stage j in the kth production period is $a_i^j(k)$, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$, $k = 1, 2, \dots, p_i$. In the planning horizon the available capacity of stage j in period t is $R_j(t)$, $t = 1, 2, \dots, T$, $j = 1, 2, \dots, m$. Because the ordered products have to pass the product design and technique preparation before production, they cannot be processed until the earliest starting time z_i, $i = 1, 2, \dots, n$. It is assumed that the system is the type of continuous production that means once production has begun, it cannot be interrupted until completed.

Owing to the fact that the availability capacity cannot meet the customer's requirements at all times, there are always capacity shortages in some periods and surplus in other periods. When a capacity shortage occurs, the plant has to either produce early or make deliveries tardily. One reason for this is that the completion of product i is beyond the due window (e_i^L, d_i^U), $i = 1, 2, \dots, n$. Since the products produced early must be held in finished goods inventories until their due time, the production cost of the items increases. While late delivery may involve some type of compensation for the customer. Therefore, earliness/tardiness penalties exist for the manufacturer. Another salient factor is that the due date is within the due window (e_i^{L}, d_i^{U}) but beyond the best due window (e_i^{U}, d_i^{L}) which the customer feels to be the most satisfactory delivery time. Although there are no earliness/tardiness penalties for the manufacturer, the customer has a low degree of satisfaction with the due date. The manufacturer also gains an unsatisfactory reputation. Therefore, all of the above scenarios are undesirable. Our JIT production planning with a fuzzy due date aims to find the optimal production planning in the horizon [1, T], in order to maximize the customers' satisfaction level and minimize the total earliness/tardiness penalties, subject to the manufacturing capacity constraints.

In general, we can assume that the earliness and tardiness penalties are both proportional to the contract prices. This is because inventory cost and tardiness compensations are both dependent upon the contract prices. Let α and β be the penalty rates of earliness and tardiness of a period. Then, for product i, the earliness and tardiness penalty costs, α_i and β_i, can be determined by

$$\alpha_i = \alpha q_i, \quad \beta_i = \beta q_i, \qquad i = 1, 2, \ldots, n, \tag{1}$$

where q_i is the contract price of product i.

3. Two models of JIT production planning

Assuming that a product completion of period t is at the end of period t, and the starting of production t is at the beginning of period t.

The definition of the model's variable for the completion time is

$$x_i(t) = \begin{cases} 1, & \text{if product } i \text{ is completed at } t, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

3.1. Due window – the interval fuzzy number

For all i: when $e_i^{\mathrm{L}} = e_i^{\mathrm{U}}$, $d_i^{\mathrm{U}} = d_i^{\mathrm{L}}$ in the orders, the due window is the same as the best due window when the customers are satisfied with the due date. Its membership function for the due date and degree of satisfaction of the customer can be seen in Fig. 1a. Thus, our problem becomes how to find the optimal production planning to minimize the total earliness/tardiness penalties within a due window, subject to the constraints of limited manufacturing capacity.

If product i is finished before the due date e_i^{L}, there are some additional earliness penalties and the amount is

$$\alpha q_i \sum_{t=1}^{e_i^{\mathrm{L}}-1} (e_i^{\mathrm{L}} - t)x_i(t).$$

If product i is finished after the latest delivery time d_i^{U}, its production cost should include the additional tardiness penalties, which is

$$\beta q_i \sum_{t=d_i^{\mathrm{U}}+1}^{T} (t - d_i^{\mathrm{U}})x_i(t).$$

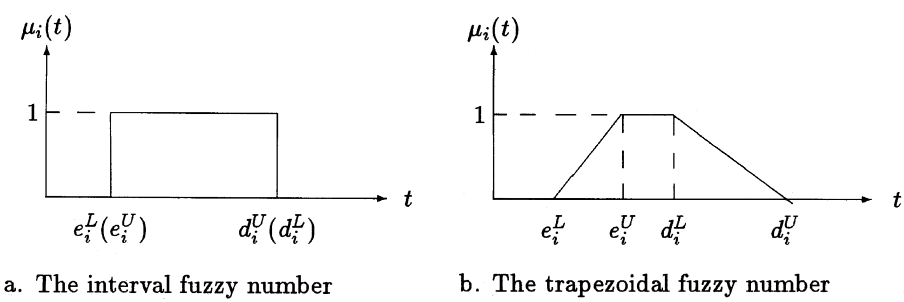

Fig. 1. The membership function for the due date and degree of satisfaction.

Let $F(x)$ be the amount of additional penalty. Then, JIT production planning with a due window can be described as follows:

(WN)

$$\min_x F(x) = \sum_{i=1}^{n} \left[\alpha q_i \sum_{t=1}^{e_i^L - 1} (e_i^L - t) x_i(t) + \beta q_i \sum_{t=d_i^U+1}^{T} (t - d_i^U) x_i(t) \right], \tag{3}$$

s.t.

$$\sum_{t=1}^{T} x_i(t) = 1, \quad i = 1, 2, \dots, n, \tag{4}$$

$$\sum_{i=1}^{n} \sum_{k=t}^{t+p_i-1} a_i^j(k - t + 1) x_i(k) \leqslant R_j(t),$$
$$j = 1, 2, \dots, m, \quad t = 1, 2, \dots, T, \tag{5}$$

$$\sum_{t=1}^{T} t x_i(t) \geqslant z_i + p_i - 1, \quad i = 1, 2, \dots, n, \tag{6}$$

$$x_i(t) = 0 \text{ or } 1, \quad i = 1, 2, \dots, n, \quad t = 1, 2, \dots, T. \tag{7}$$

Comparing with the model in the literature [5], there are several differences in the model (WN). (1) Its objective function is to minimize the total earliness and tardiness penalties within the due window. (2) The due window is a duration time that can indicate two kinds of fuzzy information about the customer's requirement. (3) Available capacity and the product's requirement capacity are all considered as variables. (4) The model (WN) is a linear 0–1 programming problem solved by common branch and bound methods [12]. Since the performance of the model depends on the way to search for the optimal, it is very important to build the model [13].

3.2. Fuzzy due date – the trapezoidal fuzzy number

For all i, when $e_i^L \neq e_i^U$, $d_i^U \neq d_i^L$ in the order, the membership function for the due date and degree of satisfaction of the customer can be seen in Fig. 1b. Thus, our problem becomes how to find the optimal production planning with a fuzzy due date to maximize the satisfaction level of the customers subject to the constraints of limited manufacturing capacity.

Let $\mu_i(c_i)$ be the membership function for the due date c_i and the customer's degree of satisfaction for the product i.

$$\mu_i(c_i) = \begin{cases} 0, & c_i \leqslant e_i^L, \\ \dfrac{(c_i - e_i^L)}{e_i^U - e_i^L}, & e_i^L < c_i < e_i^U, \\ 1, & e_i^U \leqslant c_i \leqslant d_i^L, \\ \dfrac{(d_i^U - c_i)}{d_i^U - d_i^L}, & d_i^L < c_i < d_i^U, \\ 0, & c_i \geqslant d_i^U. \end{cases} \tag{8}$$

After introducing the model's variable, the degree of satisfaction of the product i can be determined from the following formula:

$$\mu_i(t x_i(t)) = \sum_{t=e_i^L+1}^{e_i^U-1} \frac{(t - e_i^L) x_i(t)}{e_i^U - e_i^L} + \sum_{t=e_i^U}^{d_i^L} x_i(t) + \sum_{t=d_i^L+1}^{d_i^U-1} \frac{(d_i^U - t) x_i(t)}{d_i^U - d_i^L}. \tag{9}$$

Let λ be used to aggregate the satisfaction level, that is

$$\lambda = \min_i \{\mu_i(t x_i(t))\}. \tag{10}$$

Thus, JIT production planning with a fuzzy due date can be described as follows:

(FZ)

$$\max \lambda \tag{11}$$

$$\text{s.t.} \quad \sum_{t=e_i^L+1}^{e_i^U-1} \frac{(t - e_i^L) x_i(t)}{e_i^U - e_i^L} + \sum_{t=e_i^U}^{d_i^L} x_i(t) + \sum_{t=d_i^L+1}^{d_i^U-1} \frac{(d_i^U - t) x_i(t)}{d_i^U - d_i^L} \geqslant \lambda, \quad i = 1, 2, \dots, n, \tag{12}$$

$$\sum_{t=1}^{T} x_i(t) = 1, \quad i = 1, 2, \dots, n, \tag{13}$$

$$\sum_{i=1}^{n} \sum_{k=t}^{t+p_i-1} a_i^j(k - t + 1) x_i(k) \leqslant R_j(t),$$
$$j = 1, 2, \dots, m, \quad t = 1, 2, \dots, T. \tag{14}$$

$$\sum_{t=1}^{T} t x_i(t) \geqslant z_i + p_i - 1, \quad i = 1, 2, \dots, n, \tag{15}$$

$x_i(t) = 0$ or 1, $\lambda \in [0, 1]$,

$$i = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, T. \tag{16}$$

Comparing the model (WN) with the model (FZ), there are several differences. (1) It is a fuzzy or max–min programming problem caused by applying Bellman and Zadeh's max–min decision [14,15]. Its objective is to maximize the satisfaction level. (2) Fuzzy due date is a duration time that can indicate a lot of fuzzy information about the customer's requirements. (3) The model (FZ) is actually a linear mixture 0–1 programming problem, and the optimal solution can be obtained by a simple method [15].

4. The algorithm of JIT production planning with a fuzzy due date

Owing to the JIT production of the manufacturer it is necessary that the models are changed quickly between each other. In order to obtain an algorithm of earliness/tardiness production planning with a fuzzy due date for OKP manufacturing systems, we first give the following necessary proofs.

Definition 4.1 (The solution of the model (FZ)). If $\lambda^* > 0$, or $\lambda^* = 0$ and for all i: when $x_i^*(t) = 1$, $e_i^{\mathrm{L}} \leqslant tx_i^*(t) \leqslant d_i^{\mathrm{U}}$, then the model (FZ) is solvable.

Theorem 1. *If, and only if, x^* is a solution of model (FZ), x^* is an optimal solution of model (WN) and $F(x^*) = 0$.*

Proof. *Sufficient*: If x^* is a solution of model (FZ), then $\lambda^* > 0$, or $\lambda^* = 0$ and for all i: when $x_i^*(t) = 1$, $e_i^{\mathrm{L}} \leqslant tx_i^*(t) \leqslant d_i^{\mathrm{U}}$.

According to Eqs. (10) and (8), we know: when $\lambda^* > 0$, $e_i^{\mathrm{L}} < tx_i^*(t) < d_i^{\mathrm{U}}$, where $x_i^*(t) = 1$.

Therefore, a solution x^* of model (FZ) establishes the following formula.

$$e_i^{\mathrm{L}} \leqslant tx_i^*(t) \leqslant d_i^{\mathrm{U}} \quad \text{where } x_i^*(t) = 1, \quad i = 1, 2, \ldots, n. \tag{17}$$

Comparing the constraints of the model (FZ) with model (WN), we know that an optimal solution $x^*(t)$ of model (FZ) is feasible for the model (WN).

Owing to the objective function of the model (WN) $F(x) \geqslant 0$, by (17) and (3) we can easily know that x^* is an optimal solution of the model (WN) and $F(x^*) = 0$.

Necessary: If a solution x^* of the model (WN) makes $F(x^*) = 0$, then by (3) we get: for all i,

$$e_i^{\mathrm{L}} \leqslant tx_i^*(t) \leqslant d_i^{\mathrm{U}} \quad \text{where } x_i^*(t) = 1. \tag{18}$$

From Eq. (9) we get, for all i, $\mu_i(tx_i^*(t)) \geqslant 0$. Therefore, according to Eq. (11), it is easy to determine: $\lambda^* \geqslant 0$.

Let $\lambda^* = 0$. Then, for all i, x^* establishes the formula (12).

Owing to the same constraints between the model (WN) and the model (FZ) on formulae (13)–(15), we find that x^* is a solution of the model (FZ). □

Theorem 2. *If x^* is an optimal solution of the model (WN) and $F(x^*) > 0$, then there is no solution for the model (FZ).*

Proof. If x^* is an optimal solution of the model (WN) and $F(x^*) > 0$, according to formula (3) it is easy to calculate: for all i, when $x_i^*(t) = 1$, the following formula are all established

$$z_i + p_i - 1 \leqslant tx_i^*(t) < e_i^{\mathrm{L}} \quad \text{or} \quad d_i^{\mathrm{U}} < tx_i^*(t) \leqslant T. \tag{19}$$

Thus, x^* in formula (19) makes formula (12) become

$$\sum_{t=e_i^{\mathrm{L}}+1}^{e_i^{\mathrm{U}}-1} \frac{(t - e_i^{\mathrm{L}})x_i(t)}{e_i^{\mathrm{U}} - e_i^{\mathrm{L}}} + \sum_{t=e_i^{\mathrm{U}}}^{d_i^{\mathrm{L}}} x_i(t) + \sum_{t=d_i^{\mathrm{L}}+1}^{d_i^{\mathrm{U}}-1} \frac{(d_i^{\mathrm{U}} - t)x_i(t)}{d_i^{\mathrm{U}} - d_i^{\mathrm{L}}} = 0.$$

According to formula (16), we can obtain the following:

(1) If let $\lambda^* = 0$, then x^* establishes formula (12). But according to the Definition 4.1 and formula (19), x^* is not a solution in the model (FZ).

(2) If let $0 < \lambda^* \leqslant 1$, then x^* does not establish formula (12). Thus, x^* is not a solution in the model (FZ).

Therefore, according to (1) and (2) above it is clear that there is no solution in the model (FZ). □

Owing to the fact that the model (WN) and model (FZ) are of linear programming problem types, we can obtain the algorithm of production planning with a fuzzy due date. The produce of the algorithm is described as follows:

Algorithm

Step 1. According to the orders and the available capacity of manufacturing system, rewrite the data in the form of the input file of model (WN).

Step 2. Call the subprogram of branch-and-bound to compute x^* and $F(x^*)$ of the model (WN).

Step 3. If $F(x^*) = 0$, go to the next step; else, go to the step 10.

Step 4. For all i, if $tx_i^* = e_i^{\mathrm{L}}$ or d_i^{U}, go to the step 9; else, go to the next step.

Step 5. According to the orders and the available capacity of the manufacturing system, rewrite the gathered data in the form of the input file of model (FZ).

Step 6. Call the subprograms of linear programming to compute x^* and λ^* of the model (FZ).

Step 7. For all i: if $x_i^*(t) = 0$ or 1, go to the next step; else, after the branch-and-bound, go to the step 6.

Step 8. Output the result of $x_i^*(t)$, λ^* and $F(x^*) = 0$.

Step 9. Output the result of $x_i^*(t)$, $\lambda^* = 0$ and $F(x^*) = 0$.

Step 10. Output the result of $x_i^*(t)$, $F(x^*)$ and no solution for the model (FZ).

5. Computational results

The above algorithm was programmed in FORTRAN77 and run on a PC/586. By the computation of a number of examples, satisfactory results have been achieved. In fact, the approach was specifically designed for the Master Production Schedule Module of a CIM in an actual Blower Works. The results show us that it has potential for practical applications. Due to space limitations, we can only give a short example here.

An OKP manufacturer is going to produce 6 products in the next planning horizon [16,9], and each product has to pass through 2 processing stages. According to the inventory cost and tardiness compensation, we take $\alpha = 0.5$ and $\beta = 1.0$. The known production period p_i, the earliest starting time z_i of product i, and the capacity requirement $a_i^j(k)$ of product i for stage j in the production period are shown in Table 1.

The available capacities $R_j(t)$ of stages j in period t is shown in Table 2. The customer's requirements of orders are shown in Table 3. By the above algorithm, after a run time of 52 s we obtain the optimal results shown in Table 3.

After analysis of the computational results, we discover that the production planning utilizes enough of the available capacity of each stage to make the completion time as close to the customer's requirement as possible.

Table 1
Capacity requirements of products

i	p_i	z_i	$j\backslash a_i^j(k)\backslash k$	1	2	3	4
			1	4	4	5	
1	3	2	2	4	5	4	
			1	4	5		
2	2	1	2	7	6		
			1	5	3	3	4
3	4	1	2	4	2	3	2
			1	2	3	2	
4	3	1	2	1	2	1	
			1	3	4	4	3
5	4	1	2	4	2	4	5
			1	3	2	3	
6	3	2	2	2	2	2	

Table 2
Available capacities of stages $R_j(t)$

$j\backslash t$	1	2	3	4	5	6	7	8	9	10
1	12	12	12	12	10	12	12	12	12	12
2	12	12	10	10	12	12	12	12	12	12

Table 3
Order quantities and results

The data of orders						Computational results
i	q_i	e_i^L	e_i^U	d_i^L	d_i^U	Production time
1	50	5	6	7	10	6, 7, 8
2	40	4	4	7	8	4, 5
3	30	6	7	8	10	6, 7, 8, 9
4	50	4	7	8	9	3, 4, 5
5	20	5	6	7	10	6, 7, 8, 9
6	30	4	5	6	9	3, 4, 5
Minimum of earliness/tardiness						$F(x^*) = 0$
Maximum of satisfactory levels						$\lambda^* = 0.3333$

6. Conclusions

The JIT production planning problems with a fuzzy due date discussed in this paper are extensions of various earliness/tardiness production planning problems with a due date that have attracted much attention in recent years. Existing studies with a fuzzy due date are focused on the production scheduling without capacity constraints. Our approach extends it from the shop scheduling level to the aggregated planning level of manufacturing systems. Since the job completion date is always acceptable without any penalty for a time duration rather than the exact due date, the method is useful for improving the service quality of manufacturing enterprises by JIT delivery in the environment of a free market system. In fact, it provides an efficient way to improve the production planning method of MRP-II systems via the philosophy of JIT.

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