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Supply chain coordination through effective multi-stage inventory linkages in a JIT environment

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Abstract

We develop a mathematical model, with the objective of cost minimization, for coordinating the replenishment decisions for procurement, production and distribution inventories, associated with a single product, in a deterministic, multi-echelon supply chain environment. Such coordination is achieved by linking the inventories at the different echelons of the chain through integrated decision making. The supply chain structure examined here consists of a single manufacturer with multiple retailers and suppliers. For attaining the model objective, we suggest a heuristic two-phase solution algorithm, which is illustrated via a detailed numerical example. Furthermore, a number of additional problems, incorporating a variety of parametric configurations, are solved for examining and exploring some salient characteristics of the solutions obtained.

Keywords: Supply chain coordination; Inventory model; Heuristic algorithm

1. Introduction

In recent years, both academicians and practitioners have shown an increasing level of interest in finding ways of matching supply with demand, while maintaining minimum levels of inventories throughout the entire supply chain. Several researchers have examined many theoretical, as well as practical issues involving buyer–supplier coordination, as a means of attaining successful implementation of just-in-time (JIT)-based decision systems, focusing on material flows, in an effort to minimize the supply chain costs or maximize the entire chain's profitability. The suppliers of materials, producers of products, wholesalers and retailers in such an environment attempt to work as a team in a cooperative manner, in order to synchronize the supply of goods with their retail market demands. In this scenario, it is not uncommon to develop inter-linked, coordinated procurement, production and delivery schedules throughout the chain, with the goal of enhancing the performance of the entire supply chain, via joint optimization, rather than focusing on one or another individual party's performance objective(s).

Since Goyal (1976) advocated the notion of an integrated inventory model several years ago, a number of authors (e.g. Monahan, 1984; Banerjee, 1986a, b; Lee and Rosenblatt, 1986; Joglekar, 1988) have reinforced and embellished the concept, and many issues involving buyer–supplier cooperation have subsequently received a great deal of attention

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from a number of researchers. Interested readers may refer to Goyal and Gupta (1989) and Thomas and Griffin (1996) for a review of such integrated decision models for achieving buyer–supplier coordination. Some years ago, however, Joglekar and Tharthare (1990) have suggested that such cooperation is antithetical to the free enterprise system. They strongly argue in favor of the process in which each party independently implements its own optimal decisions over time. Although such faith in the free enterprise system may be admirable, many researchers believe that the idea of inter-firm cooperation and integration, for facilitating smooth operation of the entire supply chain as a unified entity, need not be incompatible or at odds with the tenets and legal framework of the free market system. Indeed, close buyer–supplier cooperation through strategic partnerships, where the various chain members equitably share the resultant net gain in system performance, is becoming a desirable and emulated trend (Li et al., 1996).

Early studies in this area conducted through the 1980s, including Goyal's (1976) pioneering work, focused only on joint lot sizing and buyer–supplier coordination for a single buyer and a single supplier based on a lot-for-lot approach. Subsequent researchers who followed this path have advanced the notion of integration through either novel lot splitting techniques (e.g. see Banerjee and Kim, 1995; Aderohunmu et al., 1995; Hill, 1997; Kim and Ha, 2003; David and Eben-Chaime, 2003), or by examining more complex structures involving multiple buyers and/or suppliers (see, e.g. Banerjee and Banerjee, 1992; Banerjee and Burton, 1994; Lu, 1995; Woo et al., 2001).

This paper is an attempt to fill a gap in the existing literature, much of which focuses on direct, two-echelon, buyer–supplier coordination. Major issues concerning procurement of materials, etc. from suppliers by the manufacturing stage, integrated with the coordinated decisions of production and distribution to retailers, have not received significant research attention to date. In view of this gap, we develop an integrated inventory model for coordinating the procurement of input materials, albeit in somewhat of a limited way, with the production schedule, which, in turn, is linked to the product distribution and delivery plan. As described later, we use the concept of integer lot size factors as potentially effective mechanisms for establishing linkages among inventories at various echelons of the supply chain for achieving coordination. Inventory integration through lot splitting, one of the techniques employed in our model developed here, is compatible and consistent with the principles of JIT. Thus, our approach is likely to be suitable for application in JIT environments.

It should be stated, however, that apart from the simplified treatment of the input inventories from supplier(s), as outlined later, our analysis is also limited by the assumption of deterministic conditions. Nevertheless, the results obtained here may provide useful insights towards the development of more complex approaches for supply chain integration under more realistic stochastic environments. In the next section of this study, we outline the development of our mathematical model and a suggested solution procedure, followed by a detailed numerical example, along with a discussion of some of the major characteristics of solutions to a set of 30 problems. Finally, some concluding remarks are presented in the last section.

2. Model development and analysis

The major assumptions and inventory coordination procedures employed in the type of supply chain environment under consideration are as follows:

1. We consider the case of a single product treated independently under deterministic conditions. Its retail level demand rate is assumed to be constant.
2. This item is manufactured in a batch production facility and is delivered to n retailers at multiple locations for satisfying market demand. Consistent with many existing JIT systems, e.g. Dell computers, delivering a finished product to retail customers, without an elaborate distribution network of warehouses, etc., is not uncommon today. The supply chain structure under consideration consists of three echelons: input item supplier(s), a manufacturer with a single production facility and a set of retail locations.
3. The fundamental mechanism adopted here for achieving coordination in the delivery, production and procurement of various inventory items involved is a common delivery cycle time of t (common to all retailers). In other words, the retail inventories at all locations are replenished via a single shipment at regular intervals. Therefore, if the demand rate at retail location i is d_i units per time unit, a replenishment lot consisting

of $q_i = d_i t$ units of the product is delivered at that location every t time units. The total quantity (i.e. the aggregate lot), simultaneously delivered to all retailers, is Q (see Fig. 1, where, without loss of generality, all supply lead times are assumed to be zero).

4. At the production facility, the fixed setup cost of a production batch of the item is likely to be higher than the fixed delivery cost. Therefore, the post-production and retail inventories of the finished good are linked through a manufacturing batch size of KQ units, where K, a positive integer, is termed the production lot size multiplier (see Fig. 1, illustrating the case of $K = 3$).
5. The raw materials, parts, components, etc., necessary for manufacturing the product, are obtained from one or more external suppliers via single sourcing for each item, consistent with JIT practice. We suggest linking their procurement with the production process through the notion of a total ‘‘package’’ or ‘‘bundle’’ of materials (in the appropriate quantities, which can be accurately determined) required to produce a batch of the final product. Each input materials bundle, treated as a single composite item, is delivered to the production facility in L (a positive integer) equally split lots during each production cycle (see Fig. 1, which illustrates the case of $L = 2$). The integer L is defined as the materials delivery lot splitting factor. Of necessity, we assume that the various input items are unique to the product in question and are not used for making other products.
6. If the three echelons of the supply chain are owned by different organizational entities, the manufacturer of the item assumes the sole responsibility for making the inventory replenishment decisions at all stages. For JIT systems, such an arrangement is not uncommon in practice. Moreover, under strategic supply chain partnerships, advocated by many theoreticians and practitioners alike, the parties involved often agree to share cost, demand and other relevant information with total transparency, for gaining

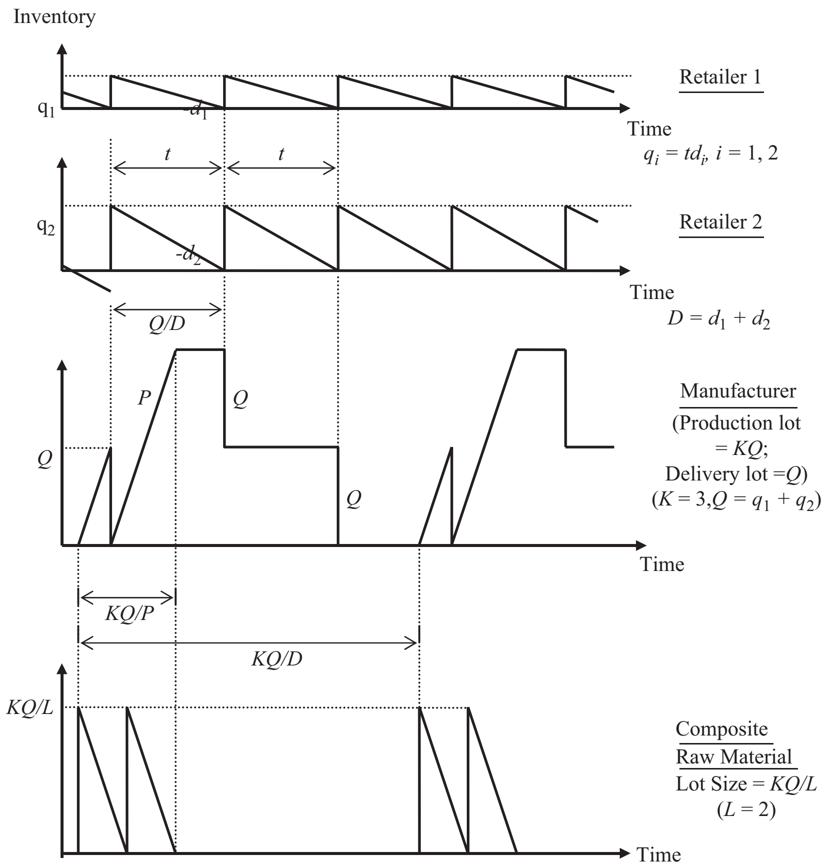

Fig. 1. Inventory–time plots for $n = 2$, $K = 3$ and $L = 2$ (all supply lead times are assumed to be 0).

advantage over competing supply chains. If, on the other hand, the echelons are owned by a single firm, these decisions can be easily centralized at the production stage. In any event, a fundamental purpose of such centralization is to improve the performance of the whole supply chain, instead of attempting to focus solely on the gains of any of its stages.

Based on items 2–4 above, a total of K shipments (each consisting of Q units and the ith retailer receiving q_i units per delivery) are made directly to the n retailers at fixed intervals of t time units, through the production of a single batch of KQ units of the item. This approach (see Fig. 1) is easy to implement and a number of researchers to date have concluded that this can be an effective means of achieving buyer–supplier coordination (see, for example, Lee and Rosenblatt, 1986; Joglekar, 1988; Banerjee and Burton, 1994). In a JIT environment, the various facilities are generally in close proximity to each other. Also, the delivery lot for each retailer is likely to be relatively small in size, so that supplying all retail locations simultaneously every cycle through a single shipment via a single transport vehicle of sufficient capacity may be feasible. For simplicity, we, thus, assume that the retail locations receive relatively frequent less-than-truckload shipments through "milk-run" supply trips. The vehicle routing problem associated with such milk-run trips is not considered in this paper.

To illustrate the notion of combining all relevant input materials into a single composite item, outlined under item 5 above, suppose that to manufacture each unit of a product, 10 lb of material X (at a cost of \$1/lb) and 3 units of component Y (each costing \$10) are required. Treating these items as a composite single input, we define a "unit of materials input" as 10 lb of X in conjunction with 3 units of Y, at a cost \$40 per "unit" ($\$1 \times 10 + \$10 \times 3$). Thus, if the product's manufacturing batch size is, say, 100 units, 100 units of the composite input materials would constitute a materials package or bundle (i.e. 1000 lb of material X and 300 units of component Y). Furthermore, if, for instance, the inventory carrying cost rate on the part of the manufacturer is \$0.20/\$/year, the holding cost of each unit of this materials package would be \$8 per "unit" per year ($\$40 \times 0.2$) at the pre-manufacturing stage of the supply chain.

The approach involving the delivery of an input materials bundle in equally split lots has been shown to be an effective and easy to implement coordinating mechanism for manufacturers and suppliers, especially in JIT environments (see Banerjee and Kim, 1995; Kim and Ha, 2003). For simplicity, we assume that all the suppliers' facilities operate as make-to-stock systems and, hence, do not take into account the suppliers' costs related to their own inventory replenishment policies. Thus, if KQ is the manufacturing batch size, as mentioned above, each input materials bundle is delivered to the production facility in L (a positive integer) equally split lots. It is clear that, consistent with the JIT approach, KQ/L units of the composite materials input are delivered L times at regular intervals during each production cycle (see Fig. 1, which illustrates the case of $L = 2$).

The restrictive assumption under item 5 above, stating that the input materials are unique to the product in question, implying that multiple products are made from completely different materials, may represent serious problems in coordinating production and supply. Nevertheless, there are at least some instances where such an assumption may be somewhat realistic. For example, to supply regional clusters of stores with standard sized, economical replacement residential windows, large home improvement retail chains, such as Home Depot, often use local vendors. Some of these vendors fabricate this type of standard inexpensive windows, at their own respective facilities, in batches from purchased pre-cut materials. At such a facility, windows of a certain size may require a specific set of pre-cut wood and/or metal frame materials, as well as pre-cut insulated glass sheets, both of which are relatively high-cost items, unique to this product. In this case, only some of the relatively low-cost hardware items may be common to windows of various sizes. Therefore, the supply of the more important materials unique to a particular manufactured product can be gainfully linked with the latter's production schedule, while other less important input supplies can be controlled by conventional uncoordinated procedures, so that possible savings may still be realized through bulk purchase price discounts and/or full truckload shipments. The ability to coordinate at least a substantial subset of input items may, thus, somewhat mitigate the practical difficulties alluded to above in linking supply and production.

Again, the relatively small distances and lot sizes in a typical JIT scenario may allow the simultaneous pickup of the required amounts of the various unique input items from the associated suppliers, and delivery of the fraction $1/L$ of the input materials bundle to the production facility when needed, perhaps via a single transport vehicle, similar to the distribution of the finished product to the n retailers, as discussed earlier. The embedded vehicle scheduling problems involved in the pickup of materials from the suppliers as well as delivery of the product to retailers are not dealt with in this paper for the sake of simplicity.

Without loss of generality we assume that the unit inventory holding cost per time unit, i.e. H_r, is the same at all locations. Therefore, if the fixed cost of delivering an aggregate lot of Q units to all the retailers is A_r, this stage's total relevant cost (TRC) (i.e. the sum of the fixed delivery and the variable inventory holding costs) per time unit incurred collectively by the retailers is given by

$$\begin{aligned}\mathrm{TRC}_r(Q) &= \frac{DA_r}{Q} + H_r\sum_{i=1}^{n} q_i/2 \\ &= \frac{DA_r}{Q} + \frac{QH_r}{2},\end{aligned} \tag{1}$$

where $D = \sum_{i=1}^{n} d_i$ represents the product's aggregate or total market demand rate. At the manufacturing stage, if A_m is the fixed setup cost per manufacturing batch of the product, H_m is its variable unit holding cost per time unit, P is the item's production rate per time unit and K is the production lot size multiplier, the TRC per time unit incurred at this stage is

$$\begin{aligned}\mathrm{TRC}_m(Q,K) = \frac{DA_m}{KQ} + \frac{QH_m}{2} \\ \times \left\{(2-K)\frac{D}{P} + K - 1\right\}\end{aligned} \tag{2}$$

(see Joglekar, 1988 for a detailed derivation of the above expression).

For the pre-production stage at the manufacturing facility, treating the appropriate raw materials, purchased parts, components, etc. as a composite input and employing the concept of an input materials package for each production lot of the product, procured from the suppliers in L equally split lots (as discussed above), the corresponding TRC per time unit at this stage is

$$\mathrm{TRC}_s(Q,K,L) = \frac{DLA_s}{KQ} + \frac{KQDH_s}{2PL}, \tag{3}$$

where A_s is the fixed cost of delivery of a lot of the composite input materials and H_s is its unit inventory carrying cost per time unit. A complete derivation of the cost function (3) is provided by Banerjee and Kim (1995).

We obtain the aggregate TRC per time unit for the manufacturing, retail and the pre-production supply stages of the entire chain under study, by adding the right-hand sides of expressions (1)–(3) above and collecting terms, i.e.

$$\begin{aligned}\mathrm{TRC}(Q,K,L) = \frac{D}{Q}\left[A_r + \frac{1}{K}(A_m + A_sL)\right] \\ + \frac{Q}{2}\left[H_r + H_m\left\{(2-K)\frac{D}{P} + K - 1\right\}\right. \\ \left. + \frac{KDH_s}{PL}\right].\end{aligned} \tag{4}$$

Note that K and L in the cost function (4) above are restricted to positive integers, whereas Q, for all practical intents and purposes, can be treated as a continuous variable. Also, it can be easily shown that (4) is strictly convex in Q, K and L.

As mentioned before, our objective is to attempt to optimize the performance of the total system (i.e. the manufacturer and all the retailers as a whole), rather than focus on any one stage or member of the supply chain. Therefore, our effort is directed at obtaining optimal values of Q, K and L, in (4), resulting in the minimization of the aggregate TRC, TRC(Q,K,L). This is not an easy or straightforward task, in view of the integrality requirements stated earlier. Hence, we adopt a heuristic approach in the hope of finding a near optimal solution. Towards this end, we relax the integer requirements on K and L in (4) and obtain the first order optimality conditions shown below, by equating, respectively, $\partial\mathrm{TRC}/\partial Q$, $\partial\mathrm{TRC}/\partial K$ and $\partial\mathrm{TRC}/\partial L = 0$, i.e.

$$Q = \sqrt{\frac{2D[A_r + (1/K)(A_m + A_sL)]}{H_r + H_m\{((2-K)D/P) + K - 1\} + KDH_s/PL}}, \tag{5}$$

$$K = \frac{1}{Q}\sqrt{\frac{2D(A_m + A_sL)}{H_m(1 - D/P) + H_sD/PL}}, \tag{6}$$

$$L = KQ\sqrt{\frac{H_s}{2PA_s}}. \tag{7}$$

It is to be noted that in each of the above expressions, an unknown variable is expressed as a function of two other unknowns, which indicates the need for employing an iterative process for the simultaneous solution of Eqs. (5)–(7).

In order to find an integrated, feasible inventory replenishment policy for coordinated manufacturing, distribution and procurement activities (i.e. with integer K and L values), we develop a simple two-phase algorithm, using the results shown above. We choose this algorithm, with the practitioner in mind, such that the procedure is simple to understand and can be implemented easily. The first phase of our algorithm relaxes the integrality requirements and, using an iterative technique, determines the theoretically optimal, but not necessarily feasible, values of Q, K and L, that simultaneously satisfy the first order optimality conditions (5)–(7) obtained earlier. The second phase employs a heuristic neighborhood search procedure, based on the results of the first phase, for arriving at an acceptable feasible solution, with integer values of K and L.

Phase 1:

Step 1: Initialize Q with the aggregate EOQ $= \sqrt{2DA_r/H_r d}$ at the retail stage and set L to an arbitrary value, say 1.
Step 2: Compute K, using the current values of Q and L in condition (6).
Step 3: Substitute the current values of K and L in (5) and recalculate Q.
Step 4: Use the current values of Q and K to compute the new value of L from (7).
Steps 2–4 are repeated until convergence occurs. Note that if Q^*, K^* and L^* are the terminal values of, respectively, Q, K and L at the conclusion of this phase, TRC(Q^*,K^*,L^*) obtained from (4) constitutes a lower bound on the feasible optimal solution.

Phase 2: The values of K^* and L^* obtained in phase 1 are likely to be non-integers and, thus, the surrounding integers need to be examined. Suppose that $[K]^-$ is the largest integer that is less than or equal to K^* and $[K]^+$ is the smallest integer that is greater than or equal to K^*. Similarly, let $[L]^-$ and $[L]^+$ be the integers surrounding L^*. Using the various possible combinations of these integer values of K and L, calculate the TRC values using (4). Clearly, under the worst case, when both $K^*>1$ and $L^*>1$, the TRC has to be determined for four possible combinations of these variables. Suppose that the integer pair $[K]^*$ and $[L]^*$ yields the minimum value of TRC from (4). These are then substituted into (5) to obtain a feasible value of Q, say Q^{**}, the aggregate delivery lot size.

The quality of the feasible solution thus obtained can be compared to the lower bound, by means of the solution effectiveness measure, E, defined as

$$E = \frac{\mathrm{TRC}(Q^{**}, [K]^*, [L]^*)}{\mathrm{TRC}(Q^*, K^*, L^*)}. \tag{8}$$

Finally, in order to get a feel for the relative benefits of our suggested coordination procedure compared with the situation without such coordination, where each supply chain stage attempts to minimize the relevant costs incurred locally at that stage only, given the replenishment policy at the immediately succeeding stage. With the simplifying assumption that the retail outlets are owned by a single firm, the optimal aggregate delivery lot size, say Q_r^*, is the standard EOQ derived from (1). Given this value of Q_r^*, and since (2) is strictly convex in K, the resulting optimal lot size multiplier at the production stage, say K_m^*, can be determined by successively substituting $K = 1, 2, \ldots$, etc. into (2) until the TRC(Q_r^*, K) value starts to increase. Similarly, assuming that all supplies are obtained from a single source and given the values of Q_r^* and K_m^*, the optimal materials delivery lot splitting factor, say L_s^*, can be easily obtained from (3). Supposing that the TRC for the entire supply chain, as a result of successively optimizing the performance of the various stages without coordination, is TRC(Q_r^*, K_m^*, L_s^*), obtained from (4), we define the relative advantage of coordination through inventory linkages to be the cost reduction factor (CRF) as

$$\mathrm{CRF} = \mathrm{TRC}(Q**, [K]*, [L]*)/\mathrm{TRC}(Q_r*, K_m*, L_s*). \tag{9}$$

This ratio, CRF, serves as an approximate measure of the relative advantage of inventory coordination over individual stage optimization assuming single retail and supply entities.

3. Numerical example and computational results

Consider an example where a manufacturer supplies a product to three retailers. Two raw materials (A and B procured from different suppliers) are needed to produce this product. The producer's inventory carrying cost rate is \$0.25/\$/year. The following data apply:

Retailers:

$d_1 = 5000$, $d_2 = 3000$, $d_3 = 4000$ units/year.

Aggregate demand rate: $D = 5000 + 3000 + 4000 = 12{,}000$ units/year.
$A_r = \$100$/delivered lot and $H_r = \$20$/unit/year.

Manufacturer: Production rate: $P = 60{,}000$ units/year, $A_m = \$750$/setup, $H_m = \$15$/unit/year.
Raw materials:

A: 2 sq. ft at \$5/sq. ft required to produce a unit of the product.
B: 5 lb at \$6/lb required to produce a unit of the product.
Thus, a unit of composite material input consists of 2 sq. ft of A and 5 lb of B, for which $H_s = 0.25[2(5) + 5(6)] = \$10$/composite input materials unit/year.
Fixed material procurement cost: $A_s = \$40$/delivered lot.

Solution: The following result from the application of our two-phase algorithm described earlier:
Phase 1:

Step 1: $Q = \text{EOQ} = [2(12{,}000)(100)/20]^{1/2} = 346.41$ and $L = 1$.
Step 2: Substituting $Q = 346.41$ and $L = 1$ into (6) results in $K = 3.36$.
Step 3: Using $K = 3.36$ and $L = 1$ in (5), we obtain new $Q = 372.30$.
Step 4: L is updated to 1.81 using $Q = 372.30$ and $K = 3.36$ in (7).
Repeating steps 2–4 with successively revised values of these variables results in convergence, yielding $K^* = 2.64$, $Q^* = 465.45$ and $L^* = 1.77$, after 16 iterations. Using these in (4), the lower bound, $\text{TRC}(Q^*,K^*,L^*)$, is found to be \$21,220.72.

Phase 2: Evaluating the TRC function (4) using the 4 possible (K, Q) pairings of (2,1), (2,2), (3,1) and (3,2), the minimum TRC of \$21,260.29 and $Q^{**} = 425.21$ are yielded by the (3,2) pair. Finally, the effectiveness measure of this solution, E, from (8) is 1.0019.

The solution obtained above implies that, in practical terms, the producer ships an aggregate lot of 425 units of the product at regular intervals of 0.03541667 of a year (i.e. 425/12,000), such that 177, 106 and 142 units are delivered in each shipment cycle to retail locations 1, 2 and 3, respectively. Since $[K]^* = 3$, the manufacturing batch size is 1275 units with a replenishment cycle time of 0.10625 year and a production cycle length of 0.02125 year. A bundle of input materials for this production lot consists of 2550 sq. ft of material A and 6375 lb of material B. Thus, with $[L]^* = 2$, 1275 sq. ft of A and 3187.5 lb of material B are delivered together at the production facility, twice during each production cycle. The annual TRC resulting from this coordinated production, procurement and distribution policy is estimated to be about \$21,260. Instead of this procedure, if each stage's performance is optimized based on the ordering policy of the immediately succeeding stage, as outlined above, the resulting Q_r^*, K_m^* and L_s^* values are, respectively, 346.41 units, 4 and 2, resulting in a supply chain TRC of about \$21,564, or a CRF value of 0.9898 using (9).

The solution obtained for this hypothetical example appears to be of high quality, as indicated by the fact that the resulting TRC is only about 0.19% more than the lower bound established. Compared to individual stage optimization with single supply and retail entities, the coordinated policy yields a slightly more than 1% cost reduction. However, with multiple retailers and suppliers, this cost reduction is likely to significantly higher (i.e. lower CRF).

In order to examine some major characteristics of our proposed mechanism for achieving supply chain coordination, using the concepts of a production lot size multiplier and a delivery lot size factor, we have solved additional 29 problems with different combinations of the relevant fixed and variable cost parameters, as well as for various values of the demand to production rates ratio (D/P), as shown in Table 1. This table also shows the summary results, i.e. the $[K]^*$, $[L]^*$ and E values obtained for each of these 30 problems through the application of our suggested algorithm.

We limit the number of test problems deliberately by selecting appropriate combinations of these parameters so that a wide variety of representative operating conditions are depicted. Keeping D fixed three P values are chosen to represent an extensive range of very high to fairly low relative production rates. Similarly, H_s is fixed at 10 while the H_m and H_r values are chosen to be 15 and 20, respectively, representing relatively low holding costs at successive echelons. Another holding cost scenario is examined through setting the latter two parameters at 30 and 60, respectively, depicting a scenario of higher and more divergent holding costs at successive stages. In a similar vein, keeping in mind that

Table 1
Summary of solutions to test problems

$D=12{,}000$		$P=24{,}000$ $(D/P=0.5)$				$P=60{,}000$ $(D/P=0.2)$				$P=600{,}000$ $(D/P=0.02)$			
Selected cost parameter values													
A_s, A_m, A_r	H_s, H_m, H_r	$[K]^*$	$[L]^*$	E	CRF	$[K]^*$	$[L]^*$	E	CRF	$[K]^*$	$[L]^*$	E	CRF
10, 750, 75	10, 15, 20	5	7	1.0001	0.9998	3	4	1.0003	0.9898	2	1	1.0001	0.9647
	10, 30, 60	6	5	1.0003	0.9995	4	3	1.0006	0.9957	3	1	1.0011	0.9841
40, 750, 100	10, 15, 20	5	4	1.0015	0.9981	3	2	1.0019	0.9859	2	1	1.0067	0.9629
	10, 30, 60	6	3	1.0016	0.9988	4	1	1.0024	0.9950	3	1	1.0079	0.9859
25, 250, 50	10, 15, 20	4	3	1.0016	0.9993	2	1	1.0026	0.9863	2	1	1.0236	0.9634
	10, 30, 60	5	2	1.0015	0.9985	3	1	1.0002	0.9901	2	1	1.0214	0.9836
50, 100, 75	10, 15, 20	3	2	1.0029	0.9994	2	1	1.0069	0.9879	1	1	1.0481	0.9500
	10, 30, 60	3	1	1.0013	0.9996	2	1	1.0126	0.9944	2	1	1.0487	0.9606
100,100,100	10, 15, 20	2	1	1.0042	0.9983	1	1	1.0568	0.9113	1	1	1.1797	0.8713
	10, 30, 60	2	1	1.0207	0.9959	2	1	1.0717	0.9926	2	1	1.1738	0.9501

A_m is likely to be the highest amongst the fixed cost parameters and that A_s is unlikely to exceed A_r, these cost values are chosen to represent a large range of possible conditions, ranging from very low to very high relative production setup costs and identical to quite divergent fixed costs at successive echelons. Thus, we feel that in spite of the use of a limited number of test problems, these are sufficient for detecting notable patterns in the solutions obtained under a wide range of possible conditions.

An examination of Table 1 reveals that the production lot size multiplier, K, generally tends to be larger than or equal to the procurement lot size factor, L, except when A_m is very large compared to A_s, in conjunction with relatively low stage holding costs and a moderate to low production rate. Also, both K and L generally tend to increase as both the production setup cost and the D/P ratio increase. However, there appears to be a slight tendency towards a decline in the value of K, coupled with an even lesser increase in L, if there is a concurrent increase in inventory carrying costs, as the production rate approaches the demand rate. In such circumstances, the values of K and L tend to move in opposite directions, appearing to counterbalance each other's effects on the relevant cost components.

Another interesting observation that emerges from Table 1 is that when the production rate is very high, i.e. when the D/P value approaches zero, as well as when the differences between the various fixed costs get smaller, the corresponding values of L^* (obtained in phase 1 of the solution algorithm) get closer and closer to zero. Since $[L]^*$ is then set to 1, for forcing feasibility, the lower bounds tend to become less tight (yielding relatively low, albeit infeasible, TRC figures, resulting from fractional L values) in comparison to those obtained under other parametric combinations. In fact, in one such case, the TRC obtained is almost 18% larger than its lower bound. Otherwise, under most of the parametric conditions examined, the lower bounds appear to remain fairly tight, with the feasible solutions yielding only slightly higher TRC values.

Finally, the CRF values in Table 1 indicate that the benefits of inventory coordination through the suggested linkage techniques appear to be the highest under relatively high production rates, in conjunction with low holding costs and fixed stage replenishment costs that are close to each other. Also for moderate to high production rates, the cost savings yielded by coordination tend to diminish as the holding costs in successive stages increase more substantially.

4. Summary and conclusions

In this paper, we suggest a procedure for coordinating the inventory replenishment, production and shipping decisions for a single product in a supply chain, consisting of a single producer with multiple customers (i.e. retailers) and suppliers. Fundamentally, such coordination is achieved through establishing effective linkages between, raw materials, finished goods and distribution inventories within the chain. The linking mechanism involves regular, coordinated shipments to suppliers, an integer valued production lot size multiplier and another integer valued materials procurement lot splitting factor, coupled with the concept of a package or bundle of raw materials parts, components, etc., treated as a single composite input. One drawback of this approach is that this coordination mechanism is limited to input materials that are used exclusively for producing the product in question. The supply of materials that are common to multiple products cannot be linked to the production batches through this technique. Consequently, this limits the widespread application of our technique for multi-stage inventory coordination. Nevertheless, our work may still prove to have at least some value in situations where the relevant assumptions may be at least approximately true.

The development of a deterministic mathematical model, with the objective of total relevant supply chain cost minimization, is outlined for simultaneously determining a set of coordinated replenishment decisions involving production, procurement and retail inventories. As in vendor managed inventory systems, we suggest that the responsibility for making such decisions lie solely with the manufacturer of the product. In view of the complexity of the model structure, we suggest a two-phase heuristic algorithm for obtaining an acceptable feasible solution. The solution technique is illustrated in detail through a numerical example and further 29 problems, representative of a wide variety of parametric configurations, are solved, in order to explore some major characteristics of the solutions obtained under different conditions.

Though we do not provide a strict proof of convergence, the first phase of the proposed algorithm terminates in less than 20 iterations for every instance examined. The use of any available computational software package, such as MathCAD, or even a basic spreadsheet software (e.g. Microsoft Excel, which is utilized here), renders such computational requirements almost trivial. Moreover, the simplicity and speed of our solution procedure and its relatively easy implementation via a spreadsheet application should have considerable appeal for practicing managers in enhancing supply chain profitability through inventory coordination. Also, the results of our numerical experiments attempt to identify the operating conditions under which the benefits of such coordination are likely to be substantial.

In spite of the major strength of our decision model in its ability to achieve supply chain coordination with relative ease and effectiveness, it suffers from one major deficiency, in that the inventory replenishment decisions on the part of the raw materials suppliers are not incorporated in the analysis in any significant way. Furthermore, our analysis is limited by the assumption of deterministic conditions. Nevertheless, this paper is not totally without merit. It is our hope that this work will encourage future work in this and related topic areas, involving further embellishments of the model structure, in negating its existing shortcomings. Future research in this direction should also incorporate more real-world complexities, such as stochastic environments, more supply chain stages (or echelons), etc. and should attempt to develop more refined solution methodologies. Although the resulting analyses are likely to be far more complex, we hope that our suggested framework may be able to provide some useful guidelines, including heuristic approaches, towards developing acceptable solutions. Finally, from the standpoint of practitioners, operating under real-world uncertainties, our suggested coordination mechanism, in conjunction with heuristically derived safety stocks, is likely to be superior to an uncoordinated supply chain without such inventory linkages.

References

Aderohunmu, R., Mobolurin, A., Bryson, N., 1995. Joint vendor–buyer policy in JIT manufacturing. Journal of Operational Research 46 (3), 375–385.

Banerjee, A., 1986a. A joint economic-lot-size model for purchaser and vendor. Decision Sciences 17, 292–311.

Banerjee, A., 1986b. On a quantity discount pricing model to increase vendor profits. Management Sciences 32 (11), 1513–1517.

Banerjee, A., Banerjee, S., 1992. Coordinated, order-less inventory replenishment for a single supplier and multiple buyers through electronic data interchange. International Journal of Technology Management 7, 328–336.

Banerjee, A., Burton, J., 1994. Coordinated versus independent inventory replenishment policies for a vendor and multiple buyers. International Journal of Production Economics 35, 215–222.

Banerjee, A., Kim, S.L., 1995. An integrated JIT inventory model. International Journal of Operations and Production Management 15 (9), 237–244.

David, I., Eben-Chaime, M., 2003. How far should JIT vendor–buyer relationships go? International Journal of Production Economics 81–82, 361–368.

Goyal, S.K., 1976. An integrated inventory model for a single supplier–single customer problem. International Journal of Production Research 15 (1), 107–111.

Goyal, S.K., Gupta, Y., 1989. Integrated inventory models: the buyer–vendor coordination. European Journal of Operational Research 41, 261–269.

Hill, R.M., 1997. The single-vendor single-buyer integrated production inventory model with a generalized policy. European Journal of Operational Research 97, 493–499.

Joglekar, P.N., 1988. Comments on 'a quantity discount pricing model to increase vendor profits'. Management Sciences 34 (11), 1391–1398.

Joglekar, P.N., Tharthare, S., 1990. The individually responsible and rational decision approach to economic lot sizes for one vendor and many purchasers. Decision Sciences 21, 492–506.

Kim, S.L., Ha, D., 2003. A JIT lot-splitting model for supply chain management: enhancing buyer–supplier linkage. International Journal of Production Economics 86, 1–10.

Lee, H., Rosenblatt, M., 1986. A generalized quantity discount pricing model to increase supplier's profits. Management Sciences 32 (9), 1177–1185.

Li, Susan X., Huang, Z., Ashley, A., 1996. Improving buyer–-seller system cooperation through inventory control. International Journal of Production Economics 43, 37–46.

Lu, L., 1995. A one-vendor multi-buyer integrated inventory model. European Journal of Operational Research 81, 312–323.

Monahan, J.P., 1984. A quantity discount model to increase vendor profit. Management Sciences 30 (6), 720–726.

Thomas, D.J., Griffin, P.M., 1996. Coordinated supply chain management. European Journal of Operational Research 94, 1–15.

Woo, Y.Y., Hsu, S., Wu, S., 2001. An integrated inventory model for a single vendor and multiple buyers with ordering cost reduction. International Journal of Production Economics 73, 203–215.