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The vehicle routing problem with hard time windows and stochastic travel and service time

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ABSTRACT

In real-world environments, the variability is always present and influences the level and cost service offered to customers. In this scenario, the present work develops a strategy to solve the Vehicle Routing Problem with Time Windows (VRPTW) in which the travel time among the customers is known only probabilistically and the vehicles are not allowed to start the service before the earliest time windows. The fact there is waiting time brings a challenge to the model because the arrival time distribution at a customer can be truncated, affecting the arrival time in the following customers. A new method is developed to estimate the vehicle arrival time at each customer and to estimate the vehicle's probability to respect the customer's time window. The metaheuristic based on Iterated Local Search finds the best route with minimal expected cost, and it guarantees that certain levels of service are met. A benchmark is used to evidence the superior performance and accuracy of the proposed method.

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1. Introduction

Most scientific papers studying stochastic problems deal with problems in which the probabilistic parameters are the customer demand and/or the presence of the customer. A minority of studies in the literature investigate the routing problem with stochastic travel time (Jula, Dessouky, & Iannou, 2006) and the number of researches has increased in recent years. In practical situations, the travel time between two customers is subject to uncertainties, for example, due to the traffic conditions, making important to study the VRP with stochastic travel times.

While most articles have addressed the variant in which there is no waiting time for the vehicles, the variant with waiting time still represents a challenge in the literature, with many gaps to be covered and explored, which is related to the difficulty of calculating the probability of a vehicle arrives at the customer before the upper bound of the time windows.

The VRP with stochastic travel time in which there is waiting time is significantly more difficult than the case with no waiting time. The difficult comes from the fact that, given a route, the probability distribution of the arrival times at the customers is truncated due to the hard time windows, thus prohibiting the use of convolutions properties when summing the random variables (Gendreau, Jabali, & Rei, 2014).

A VRP with Time Windows (VRPTW) can be approached as soft time windows (with penalties on earliness and tardiness,) or as hard time windows (with earliness and tardiness generally forbidden). Specific problems with hard time windows include security patrol service, bank deliveries, postal deliveries, industrial refuse collection, grocery delivery, school bus routing, and urban newspaper distribution. Among the soft time windows problems, dial-a-ride problems is an important example (Desaulniers, Madsen, & Ropke, 2014). From the practical point of view, for situations in which the vehicles may not have any available place at customer locations to wait, the soft variant prevails.

Berhan, Beshah, and Kitaw (2014) presents a survey about stochastic formulation already proposed in the literature for vehicle routing problems. The authors present classification domains and attributes for the researches involving the Stochastic Vehicle Routing Problem (SVRP) that are used here to situate the current paper in the literature. Regarding the stochastic domain, the two stochastic variables are the travel and the service time (both normal distributed), the solution is given by an heuristic, the graph representation is symmetric, there is a multiple number of vehicles, the vehicles are capacitated, the objective function aims the minimization of the operational costs (objective domain) and it is used a Chance Constrained Model (applied model domain), where there is a required probability for the vehicle to visit the customer before the latest time windows (customer service level). Additionally, there is waiting time for the early arriving of the vehicles and the data for the experiments comes from instances from the literature.

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The main contributions of this paper are:

1. The development of a statistic model to sum a truncated non-normal with a normal variable.
2. The application of the developed statistic method to solve a difficult variant of the VRP with stochastic times: with waiting time and, additionally, stochastic service time.
3. It is performed a number of computational experiments with comprehensible analysis of the results.

Initially, a relevant literature in the subject is presented. Secondly, the mathematical formulation of the problem. Next, it is demonstrated that the existence of the waiting time as a random variable might result in situations where the probability density function of the vehicle arrival time at a customer is not well-modeled by a normal distribution, even assuming normality for the service time and for the travel time. After that, it is presented the proposed method to calculate the cumulative probability function of the arrival time along the customers of a given route. Experiments are performed to evaluate the proposed method by comparing its results with a benchmark (our implementation of the method described by Zhang, Lam, & Chen, 2013). In addition to that, the knowledge gained in the previous steps is applied to solve the Stochastic VRP with hard time windows and stochastic travel time and service time through the use of a metaheuristic based on the Iterated Local Search, with results and analyses obtained on Solomon's problem instances (Solomon, 1987). Finally, the main findings and conclusions are highlighted in Section 6.

2. Literature review

The SVRP introduces elements of uncertainty in the input data of the mathematical model. This means some properties of the VRP are not the same for the SVRP, and in general, the SVRP solution methods are more complicated. A literature review for the SVRP is given by Gendreau, Laporte, and Seguin (1996), Zeimpekis et al. (2007), and more recently by Ritzingera, Puchingera, and Hartlb (2016) including some of the most important publications in the field of Stochastic Vehicle Routing Problem. Specifically for the variant with stochastic demand, see Marinaki and Marinakis (2016) and Dror (2016). In order to consolidate a background for the present research, it's also mentioned Baldacci, Mingozzi, and Roberti (2012), Desaulniers et al. (2014) and Koç, Bektaş, Jabali, and Laporte (2016) and Wang and Zou (2016) for the VRP and VRPTW.

Regarding the classification of the VRPTW, especially in the context of stochastic travel times, presents some subtle details that might lead researches to certain misconceptions. According to Desaulniers et al. (2014), in the case of hard time windows, a vehicle that arrives too early at a customer must wait until the customer is ready to begin a service. Still according to the author, in general, waiting before the start of a time windows incurs no cost. In the case of the soft time windows, every time window can be violated barring a penalty cost. Similarly, according to Cordeau, Desaulniers, Desrosiers, Solomon, and Soumis (2002), the soft time windows can be violated at a cost, while a hard time windows do not allow for a vehicle to arrive at a customer after the latest time to begin service, and if it arrives before the customer is ready to begin service, it waits.

Considering the two definitions above, if the problem is modeled as hard time windows, for sure, there is waiting time. On the other hand, saying the problem is soft time windows, not necessarily means there is waiting time. For instance, Zhang et al. (2013) approaches a soft time windows problem with waiting time, and Tas, Dellaert, van Woensel, and de Kok (2014) approaches also a soft time windows but with no waiting time. Another observation is that for the deterministic version of the VRPTW, the soft time windows can also include the hard time windows by raising the penalty coefficients, however, this is not necessarily true when the travel time is stochastic. What matters is if the model is prepared or not to deal with the waiting time as a random variable. It is important to have this in mind, because the main subject of research of the present paper derives from the phenomenon raised by the existence of waiting time.

Models with no waiting time benefits from the use of convolutions properties while summing the random variables. The probability distribution of the sum of two or more independent random variables is the convolution of their individual distributions. The term is motivated by the fact that the probability density function of a sum of random variables is the convolution of their corresponding probability density functions respectively. Many well-known distributions have simple convolutions.

Models with waiting time cannot use these convolution properties. Even assuming a known probability distribution of the arcs and even assuming that the arcs are independent, there are difficulties inherent to this type of problem such as: determining the type of probability distribution of the arrival time at customers and the parameters of this distribution. Basically, the distribution of the arrival time changes after each customer along the route due to the truncation point given by the earliest time window.

In the context of VRPTW with stochastic travel time and no waiting time, some relevant studies were conducted by Russell and Urban (2008), Tas, Dellaert, van Woensel, and de Kok (2013), and Tas et al. (2014). Russell & Urban approached a VRP with soft time windows and Erlang Travel Times where is used a shifted gamma distribution to estimate the vehicle arrival time. Penalties are incurred for both early and later arrivals. A tabu search metaheuristic is used to solve the problem.

Tas et al. (2013) study the VRPTW with stochastic travel times including soft time windows using gamma distribution for the travel time. The service time is constant. The model considers both transportation costs (total distance traveled, number of vehicles used and drivers' total expected overtime) and service costs (early and late arrivals). A tabu search metaheuristic is used to solve the problem. According to the author, the distributions of the travel times most commonly applied so far are normal, log-normal, shifted gamma and gamma distributions.

Tas et al. (2014) study a vehicle routing problem with time-dependent and stochastic travel times, with soft time windows. Two metaheuristics are built: a Tabu Search and an Adaptive Large Neighborhood Search. The travel time is time-dependent, so the speed of vehicles vary throughout the day. The scheduling horizon is divided into intervals, each interval associated with a multiplier. These multipliers are used to specify that there is a different travel speed for different time intervals, where a larger multiplier indicates that more time is needed for traveling.

In the context of VRPTW with stochastic travel time and with waiting time, important studies were conducted by Chang, Wan, and Tsang (2009), Errico, Desaulniers, Gendreau, Rei, and Russeau (2013, 2016), Jula et al. (2006), Li, Tian, and Leung (2010), Miranda (2011), Zhang et al. (2013) and Binart, Dejax, Gendreau, and Semet (2016)., Jula et al. (2006) has not investigated a VRPTW but a problem with similar characteristics, the Travelling Salesman Problem with Time Windows (TSPTW) with stochastic travel time in three different situations. Firstly, the service time and time windows are disregarded. Secondly, the service time is a random variable, being independent and identically distributed among customers. In the third situation, the existence of time windows and their effects are studied. In this last scenario, there is waiting time and it is the case of interest here. The authors suggest methods of estimating the mean and variance of the arrival time of the vehicle at customers using estimates obtained with first-order approximation of Taylor series. It is also found a lower bound for the confidence coefficient by applying Chebyshev and Chernoff bounds, assuming a normal distribution for all travel and service times.

Chang et al. (2009) studied a traveling salesman problem with time windows and stochastic and dynamic travel time, which is not a VRPTW but it also deals with the same phenomenon caused by the waiting time of the vehicle. While calculating the route service level, the first difficulty was the estimation of the mean and variance of the vehicle departure time. For this, they studied the relationship between arrival time and the time window and developed an approach to calculate the mean and variance of the arrival time with the assumption of normality for the vehicle's arrival time. The paper reports results for a problem with 12 nodes based on a real case of a distribution company.

Li et al. (2010) studied a VRPTW in which the travel time and service time are stochastic. The problem is formulated according two approaches: a Chance Constrained Program (CCP) model and a Stochastic Program with Recourse (SPR) model. They proposed a heuristic based on tabu search that considers the stochastic nature of the problem. The paper uses stochastic simulation (also referred to as Monte Carlo simulation) to calculate the probabilities involved. The simulation performed a statistical sampling and estimates the probability based on the law of large numbers. Each route was simulated at least 1000 times to collect data from the arrival time at each customer and then counted the number of violations for each customer, establishing the route's service level. Although this method produces good quality solutions, the computational cost is very high, and thus, even using heuristics, only small-scale problems can be solved in a non-prohibitive amount of time.

Miranda (2011) studies the VRPTW with normal travel time and hard time windows. The author analyses with clarity the effects of the truncation point at the earliest time windows in the arrival time distribution at the customers along a given route. The error for the normality assumption of the arrival travel time is investigated and linear multivariate regressions are used to estimate this error, then calculating the service level at the customer, the mean and the variance of the arrival time. A metaheuristic is developed to solve Solomon's instances.

Errico et al. (2013) proposed a formulation for the VRPTW with hard time windows, not with stochastic travel time but with stochastic service time. The travel distance is minimized and the global probability that the route plan is feasible with respect to customers' time windows is higher than a reliability threshold. Itś considered a symmetric triangular distribution with median equal to the deterministic value of the service time. An exact solution framework based on column generation is developed to solve the problem where the vehicle capacity and the customer demands are disregarded. As a continuation of his research with hard time windows and stochastic service times, Errico, Desaulniers, Gendreau, Rei, and Russeau (2016) studies the problem in the form of a chance-constrained model with a two-stage stochastic program and two recourse policies to recover operations feasibility when the first stage plan turns out to be infeasible. The problem is solved by exact branch-cut-and-price algorithms.

Zhang et al. (2013) study the problem where the vehicles are still not allowed to start the service before the earliest time windows. A penalty is applied for the waiting time (when the vehicle arrives before the earliest time windows and it has to wait) and for the tardiness time (when the vehicles arrive after the latest time window, violating the time window constraint). They adapted the method α-discrete from Miller-Hooks and Mahmassani (1998) to estimate the arrival time distribution at a customer. The method is applied to a normal distribution and log normal distribution for the travel time. It is used an Iterated Tabu search to solve the problem and the authors show that the assumption of normality for the arrival time is not appropriate. Considering the good results presented by the study, it was chosen as the benchmark for this current paper.

Ehmke, Campbell, and Urban (2015) present a stochastic variant of the vehicle routing problem with time windows where travel times are assumed to be stochastic. In their chance-constrained approach, restrictions are placed on the probability that individual time window constraints are violated, while the objective remains based on traditional routing costs. In addition they consider how to compute the start-service time and arrival time distributions for each customer. These distributions are then used to create a feasibility check that can be ‘‘plugged’ ’ into an algorithm for the VRPTW.

In Zhang, Lam, and Chen (2016) three probabilistic models are proposed to address on-time delivery from different perspectives to a vehicle routing problem with stochastic demands and time windows. The first one aims to search delivery routes with minimum expected total cost. The second one is to maximize the sum of the on time delivery probabilities to customers. The third one seeks to minimize the expected total cost, while ensuring a given on-time delivery probability to each customer. The authors discuss two approaches to deal with the three models: a preventive restocking policy and a detour-to-depot recourse policy. According to the authors, the first approach presents better numerical solutions.

Finally, Binart et al. (2016) solve a variant of the VRP where it's assumed that service as well as travel times are stochastic, both with discrete triangular distributions. It consists in determining vehicle routes in a single period to serve two types of customers: mandatory and optional. Only mandatory customers have to be served within a specified hard time window. The problem is tackled by a 2-stage solution method: the planning stage and the execution stage. Compared to the classical VRPTW, this variant presents some differences such as multiple depots, uncapacitated vehicles, priority within customers as well as stochastic travel and service times.

3. Problem statement

Let be $G = (V_0, A)$ a complete digraph, where $V_0 = \{0, \ldots, n\}$ is a set of vertices and $A = \{(i, j) : i, j \in V_0, \ i \neq j\}$ is a set of arcs. The vertex 0 represents the depot where m_0 vehicles with capacity Q are available. The set of customers is $V_0/\{0\} = \{1, \ldots, n\}$. Each customer $i \in V$ has a non-negative demand q_i, service time ST_i, and a time window $[e_i, l_i]$, where e_i is the earliest time windows and l_i is the latest time window. It is expected that the service start time is within the range $[e_i, l_i]$. If the vehicle arrives at customer i before e_i, it is necessary to wait until e_i. A time window $[e_0, l_0]$ is assigned to the depot. A travel time TT_{ij} is assigned to each arc $(i, j) \in A$. Both TT_{ij} and ST_i are random variables with known and independent probability density. Other assumptions are: $Q \geq q_i, i \in V$ (i.e., each vehicle has enough capacity to serve at least one customer,) and the fleet is big enough to serve all the customers. The notation is described as:

M	*large number*
m	*number of vehicles in a feasible solution,* $m \leq m_0$
$m*$	*number of required vehicles in the optimal solution,* $m* \leq m_0$
K	*the set of required vehicles in a feasible solution* $K = \{1, \ldots, m\}$.
x_{ijk}	*a boolean variable with value 1 when vehicle k serves the arc (i,j)*
AT_i	*arrival time at customer i*
SS_i	*service start time at customer i*
α_i	*required service level by customer i*

The model for the problem is described below:

$$\min M.m + \sum_{(i,j)\in A} \sum_{k\in K} E\left(TT_{ij}\right) x_{ijk} \tag{1.0}$$

Subject to:

$$\sum_{j\in V_0} \sum_{k\in K} x_{ijk} = 1, \quad \forall i \in V \tag{1.1}$$

$$\sum_{j\in V} x_{0jk} \ = 1, \quad \forall k \in K \tag{1.2}$$

$$\sum_{i\in V} x_{i0k} \ = 1, \quad \forall k \in K \tag{1.3}$$

$$\sum_{i\in V_0} x_{ijk} - \sum_{i\in V_0} x_{jik} \ = 0, \quad \forall j \in V, \ \ k \in K \tag{1.4}$$

$$\sum_{i\in V} q_i \sum_{j\in V_0} x_{ijk} \le Q, \quad \forall k \in K \tag{1.5}$$

$$P(AT_i \le l_i) \ge \alpha_i, \ \forall i \in V \tag{1.6}$$

$$e_i \le SS_i, \ \ \forall i \in V \tag{1.7}$$

The objective function (1) is formed by two parts: the vehicle fixed cost and the total mean travel time as the operating cost. The proposed model has a hierarchical optimization objective: the primary objective is to minimize the number of required vehicles to satisfy constraints (1.1) to (1.7); the secondary objective is to minimize the operating costs. This hierarchical optimization objective implies that one solution with fewer routes but higher operating costs is better than another with more routes but lower operating costs.

The Eq. (1.1) ensures each customer is visited only once by one vehicle. Eqs. (1.2) and (1.3) ensure each vehicle starts the route in the depot and also returns to it. The Eq. (1.4) ensures that each vehicle departs from a customer location after it visits the customer. The Eq. (1.5) is the capacity constraint. The Eq. (1.6) is the service level required by each customer. It ensure that the probability of a vehicle arrives at the customer before the latest time windows should be greater than a given threshold. The Eq. (1.7) ensure the service will only start after the earliest time window, therefore, if the vehicle arrives before e_i, it must wait until e_i. The service start time is given by the Eq. 1.8.

$$SS_i = max\{AT_i, e_i\}. \tag{1.8}$$

It is valid to notice that the departure time from depot is zero. A departure time greater than zero does not improve the objective function, because there is no penalty for the waiting time (common assumption for Hard Time Windows, Desaulniers et al., 2014).

4. The problem investigation and the development of the proposed method

This section will demonstrate clearly for the case with waiting time, how the earliest time windows as a truncation point, affects the arrival time distribution over the customers of a given route. The following section will then present in details the statistical method proposed to calculate the cumulative probability function of the arrival time, and consequently, the service level of the customer.

4.1. The effect of the waiting time in the arrival time distribution

In order to illustrate the phenomenon (effect of the existence of waiting time), one route is generated and the arrival times are obtained through simulation. In the Table 1, the values of the time windows of some customers were modified from the original instance (Solomon, 1987) to better illustrate the phenomenon under investigation. The vehicle speed is considered unitary, so the travel time and the travel distance have the same values. In this instance, it is assumed that the time is given in minutes and the service time for customers is zero.

The Table 1 shows data for a route with 6 customers. Let $E[X]$ and $Dev[X]$ be the mean and standard deviation of any random variable, respectively. TT_{ij} is a random variable with normal distribution that represents the travel time from the customer i to the customer j. AT_j and SS_j are random variables that represent the arrival time and service start time at customer j, respectively. SL_j and $P(WT_j)$ are the customer service level and vehicle waiting time probability at customer j, respectively.

The values for $E[AT_j], Dev[AT_j], E[SS_j]$ and $Dev[SS_j]$ were obtained from the stochastic simulation of the route (1 run of 20,000 replicas). An algorithm to perform the stochastic simulation is provided by Li et al. (2010). Fig. 1 shows the histograms of the arrival time of the vehicle for each of the six customers.

The Fig. 1 shows how the shape of the distribution changes over the customers of a route. At the first customer, the arrival time distribution is normal because the vehicle departs from the depot at the deterministic moment zero. Actually, is observed that the distribution is a little bit truncated, because all the travel times are left truncated at zero to not consider negative travel time. Due the fact there is no service time at the depot, the arrival time at the customer 1 is the travel time. The truncation point at this customer is the earliest time windows. Therefore the $P(N(16.63, 6.03^2) \le 21.84 = 81\%$, it means there is 81% chance the vehicle will have to wait until starting the service. The mean of the service start time is 22.48, because 81% of the arrivals are shifted to 21.84 and 19% are the left truncated part of the arrival, with a truncated average higher than the non-truncated arrival time.

Because there is no service time, the vehicle departs from the customer 1 at the moment 22.48 (in average) and travels 26.64 minutes (also in average) until arrive at the customer 2 with an average of 49.18. Then this process is repeated for the remaining customers.

Any route in which it is possible to have vehicle waiting time is subject to the phenomenon.

4.2. The statistical problem

This section introduces the statistical method proposed to calculate the cumulative probability function of the arrival time.

For the sake of simplicity, it is adopted a specific notation for the statistical problem. Let the random variable X_i be the arrival time (AT) at the customer i, and after its truncation at the earliest time windows e_i, it becomes the truncated variable X_i^{tr} with function f_x left-truncated at the point t, where $X_i^{tr} = t \ \ if \ \ x \le t$ and $X_i^{tr} = x \ \ if \ \ x > t$. The variable Y_i is the sum of the service time (ST) and travel time (TT), considering they are both normal: $ST_i + TT_{i,i+1} = N(\mu_{STi}, {\sigma_{STi}}^2) + N(\mu_{TTi,i+1}, {\sigma_{TTi,i+1}}^2) = N(\mu_{STi} + \mu_{TTi,i+1}, {\sigma_{STi}}^2 + {\sigma_{TTi,i+1}}^2) = Y_i$. By doing this, the problem becomes solving $X_{i+1} = X_i^{tr} + Y_i$ recursively from $i = 1$ to $i = n$, where n is the number of customers of a given route.

In the Fig. 2, X_1 is a random variable that after truncated to the left, becomes X_1^{tr} (a left-truncated normal distribution), and Y_1 is a non-truncated normal distribution. The sum $X_2 = X_1^{tr} + Y_1$ might be

Table 1
Example route.

	customer	1	2	3	4	5	6
Parameters	*ej*	21.84	46.91	52.44	73.65	88.14	89.76
	lj	31.47	63.85	65.83	99.23	102.88	105.29
	E[*Tij*]	16.63	26.64	6.84	29.43	9.60	9.06
	Dev[*Tij*]	6.03	6.95	0.73	10.28	1.35	0.96
Data from simulation	$E[AT_j]$	16.64	49.18	57.90	87.62	97.83	107.71
	$Dev[AT_j]$	6.02	7.22	5.06	11.51	10.58	9.63
	$E[SS_j]$	22.48	51.05	57.91	88.21	98.62	107.70
	$Dev[SS_j]$	1.83	5.00	5.06	10.49	9.59	9.63
	SL_j (%)	99.25	97.75	91.45	84.33	69.43	47.06
	$P(WT_j)$(%)	81.00	38.09	1.6	11.16	21.75	0.00

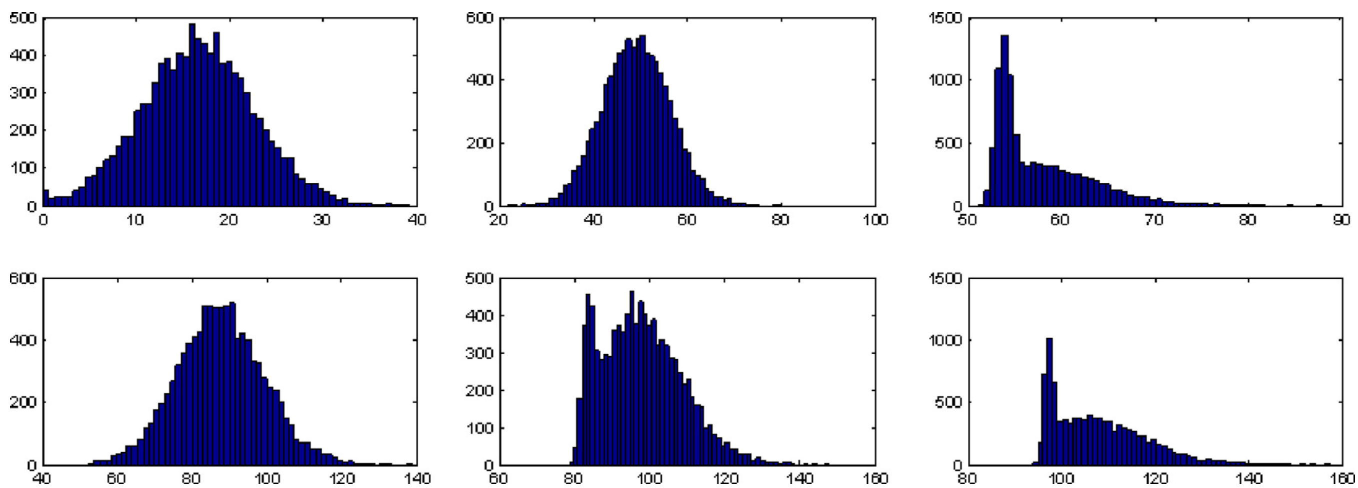

Fig. 1. Histograms of the vehicle arrival time.

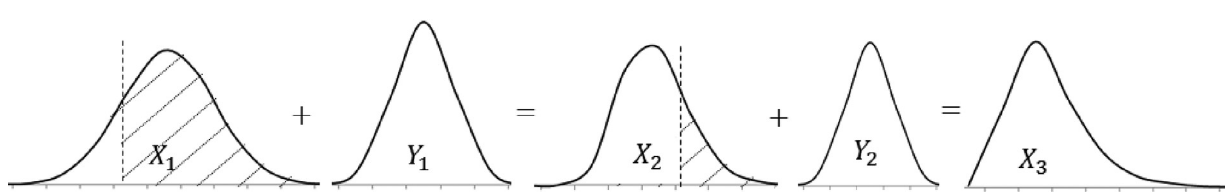

Fig. 2. Sum of variables.

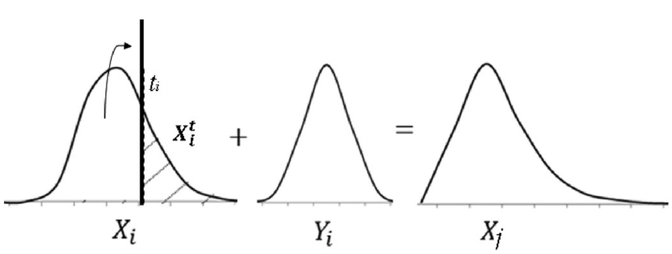

Fig. 3. Sum of a truncated non-normal and a normal random variable.

a non-normal distribution. It is desired to calculate $X_3 = X_2^{tr} + Y_2$, where Y_2 is a known normal variable and X_2^{tr} is a left-truncated non-normal. How to calculate the cumulative probability function of X_3?

Now, considering Y_i for $i = 1, \ldots, n$ is a known normal variable, also considering $X_i = X_{i-1}^{tr} + Y_{i-1}$ where X_{i-1}^{tr} is left-truncated, let's assume a sequence of sum of variables, such as $X_4 = X_3^{tr} + Y_3, \ \ldots, \ X_n = X_{n-1}^{tr} + Y_{n-1}$. How to calculate the cumulative probability function of X_i for $1, \ldots, n$? Fig. 3 illustrates the case and it also highlights the fact that $X_i^{tr} = t \ if \ x \leq t$.

4.3. The solution framework for the statistical problem

The cumulative distribution function of $X_j = X_i^t + Y_i$ (in the context of the Fig. 3) is obtained as follows:

$$P\left\{X_j \leq c\right\} = F_{X_i^{tr}+Y_i}(c) = P\left\{X_i^t + Y_i \ \leq c\right\}$$
$$= \int_{-\infty}^{\infty} F_{X_i^{tr}}(c - y_i) \ \ f_y(y_i) \ dy \tag{2}$$

Eq. 2 is known as the convolution of the marginal distributions and it gives the distribution of sums of variables. This equation can be approximated according to the Eq. 3, through a discrete function with I intervals, where y_f and y_0 are the upper and lower bounds of the integration, respectively.

$$\int_{-\infty}^{\infty} F_{X_i^{tr}}(c - y_i) \ \ f_y(y_i) \ dy \cong \frac{y_f - y_0}{I} * \sum_{k=1}^{I} \left(\frac{dy_{k-1} + dy_k}{2} \right) \tag{3}$$

Where $dy_k = F_{X^{tr}}(c - y_k) \ \ f_y(y_k)$.

The cumulative function $F_{X^{tr}}(c - y_k)$ needs to be calculated for a set of values $(c - y_k)$. In order to do it, the cumulative function

```
1  function  [prob]=Conv(t,MeanY,DevY,c,mprob,vidx,I)
2  y0= MeanY-3.5*DevY; yf= MeanY+3.5*DevY;
3  minX=mprob(1,1);maxX=mprob(vidx(4),1);
4  x0=max(minX,t) +max(y0,0);
5  xf=max(maxX,t) +yf;
6  if c > xf
7      prob=1;  return
8  elseif c < x0
9      prob=0;  return
10 end
11 if yf > c-t
12     yf= c-t;
13 end
14 inc=(yf-y0)/I;
15 yf= yf+ inc/1000;
16 den=(sqrt(2*pi)* DevY);
17 y=y0; tot=0; dyconv2=0;
18 while y < yf
19     cy=c-y;
20     if cy <=vidx(1)
21         Fx=0;
22     elseif cy >= vidx(3)
23         Fx=1;
24     else
25         q=ceil(((cy)-vidx(1))/vidx(2));
26         den2=(mprob(q+1,1) - mprob(q,1));
27         Fx =((cy)* (mprob(q+1,2) - mprob(q,2))
28             + mprob(q,2) * mprob(q+1,1)
29             - mprob(q+1,2) * mprob(q,1))/den2;
30     end
31     dyconv=Fx * (exp(-0.5 * ((y - MeanY)/DevY)^2)/den);
32     tot= tot+ (dyconv+dyconv2)/2; dyconv2=dyconv;
33     y=y+inc;
34 end
35 prob=tot*inc;
36 end
```

Fig. 4. Algorithm to solve the convolution.

is calculated in the previous iteration (i) for each $(c - y_q)$ where $q = \{1, \ldots nint\}$. This information is stored in a matrix $mprob_i$ as following:

$$mprob_i = \begin{bmatrix} c - y_1 & c - y_2 & \ldots & c - y_{nint} \\ F_{X^{tr}}(c - y_1) & F_{X^{tr}}(c - y_2) & \ldots & F_{X^{tr}}(c - y_{nint}) \end{bmatrix}$$

Where $F_{X^{tr}}(c - y_1) \cong 0$ and $F_{X^{tr}}(c - y_{nint}) \cong 1$.

If $(c - y_{q-1}) < (c - y) < (c - y_q)$, then $F_{X^{tr}}(c - y)$ is calculated through a linear interpolation of the adjacent points. For the calculation of the next iteration $(j + 1)$, it is necessary firstly to compute $mprob_j$, solving the Eq. 3 for each $c - y_q$ in $q = \{1, \ldots nint\}$. Note that for the first iteration, the random variable X_1 is normal distributed, so $mprob_1$ is calculated directly using the equation for the normal cumulative function.

Observe that the Eq. 2 does not lend itself to a precise analytical characterization. In this context, the probability distribution X does not have a known shape; even with a known equation for F_X and f_y, the product $F_X(c - y)\ f_y$ leads to complex equations for which an integration cannot be solved analytically. Due to this fact, an efficient algorithm to solve Eq. 3 was developed, and it will be presented in the next section.

4.4. The convolution function

It is desired to calculate the probability $P\{X^{tr} + Y \le c\}$ given by the Eq. 3. In the context of the VRP, this is used to obtain the service level at a given customer. It is intended to solve this convolution as fast as possible, considering it will be applied for a *NP*-hard problem.

The complete algorithm is given in the Fig. 4. The function "Conv" describes how to solve the convolution of the Eq. 3. The inputs are listed in the Table 2. The algorithms presented in the paper might have some unnecessary declaration of variables for the sake of clarity.

The function "Conv" receives the parameters of the Table 2 and returns the value for the probability $P\{X^{tr} + Y \le c\}$.

The line 3 gets the values of x for which the probability is $\cong 0$ and $\cong 1$, respectively, where $\min X = F_X^{-1}(\cong 0)$ and $\max X = F_X^{-1}(\cong 1)$. The lines 4 and 5 estimate the values $x_0 = F_{X+Y}^{-1}(\cong 0) = Max\{\min X, t\} + Max\{MeanY - 3.5DevY, 0\}$ and $x_f = F_{X+Y}^{-1}(\cong 1) = Max\{\max X, t\} + (MeanY + 3.5DevY)$. These values are important to create an exit condition in the lines 6 to 10 that saves computational effort for this case, avoiding the loop of the lines 18 to 34.

Table 2
Notation for the inputs of the algorithm.

MeanX	Mean of X_i
DevX	Standard Deviation of X_i
t_i	Truncation point of X_i
MeanY	Mean of Y_i
DevY	Standard Deviation of Y_i
c	Desired value to calculate the probability $P\{X^{tr}+Y \le c\}$
$mprob_i$	Cumulative probability matrix F_X, with the values of X_i in the column 1 and the column 2 with values of $P\{X_i \le c\}$ in the space of the cumulative probability [0,1], with *nint* points (number of intervals), $q = \{1, \ldots nint\ \}$.
vidx	Auxiliary vector with $vidx(1) = mprob_i(1,1)$; $vidx(2) = increment\ \delta\ of\ values\ of\ X\ in\ mprob_i$; $vidx(3) = mprob_i(1,n)$;
I	Number of intervals to calculate Eq. 3.

The lines 2 and 12 define the lower bound and the upper bound of the integration. These bounds are given by the Eqs. 4 and 5.

$$y_0 = (\mu - 3.5\ \sigma)\ \text{for}\ Y \sim N\left(\mu, \sigma^2\right) \tag{4}$$

$$y_f = Min\{(\mu + 3.5\ \sigma), (c-t)\}\ \text{for}\ Y \sim N\left(\mu, \sigma^2\right) \tag{5}$$

For $Y \sim N(\mu, \sigma^2)$, $P\{Y \le (\mu - 3.5\sigma)\} \cong 0$ and $P\{Y \ge (\mu + 3.5\sigma)\} \cong 0$. In the Eq. 5, if $(\mu + 3.5\ \sigma) > (c-t)$ then $y_f = (c-t)$. It happens because, in the Eq. 3, for $f_y(y)$, when y reaches is maximum value $(\mu + 3.5\ \sigma)$, for $F_X(c-y)$, the value $(c-y)$ reaches its minimum value that must be greater than t.

Line 14 calculates the increment for the value y in the interval $y_0 \le y \le y_f$. Line 15 just adds a very small value to the upper bound *UB* in order to ensure the integration will be calculated up till $y = y_f$ preventing numerical errors. Line 16 calculates the denominator of the normal probability density function $f_{(x)} = \frac{1}{\sigma\sqrt{2\pi}} e^{-0.5\frac{(x-\mu)^2}{\sigma^2}}$.

Line 17 initializes the values to perform the integration loop in the lines 18 to 34. Lines 19 to 30 calculate the value of $F_X(c-y)$. The matrix *mprob* contains the values of F_X. Line 21 and 23 calculates $F_X(c-y)$ when $cy = (c-y)$ is without the bounds of *mprob* and line 27 calculates $F_X(c-y)$ for the values of *cy* within the range. The first column of the matrix *mprob* is a vector with the values of $X = (x_1, x_2, \ldots,\ x_{nint}\ )$. For a given value $x = cy = (c-y)$ it is necessary to find the values x_q and x_{q+1} where $x_q < x < x_{q+1}$. The index q is calculated in the line 25, then the line 27 calculates $F_X(c-y)$ through a linear interpolation of $F_X(x_q)$ and $F_X(x_{q+1})$. The bigger *nint*, the smaller the gap between two consecutive points, then the smaller the error of the linear interpolation. It will be discussed later. The equation in the line 25 is key in the algorithm because it allows to find the index q with computational cost $O(1)$.

$$q = \frac{x - F_X(x_1)}{\delta} = \frac{cy - vidx(1)}{vidx(2)} \tag{6}$$

$$F_X(x_q) = \frac{x\left((F_X\left(x_{q+1}\right) - F_X(x_q)\right) + F_X(x_q)x_{q+1} - F_X\left(x_{q+1}\right)x_q}{x_{q+1} - x_q} \tag{7}$$

In the Eq. 6, δ is the increment or step size for the values x_1 to x_{nint} where $q = 1, \ldots n\,int$. In the Eq. 7, $F_X(x_q) = mprob(2,q)$, $x_q = mprob(1,q)$ and $x = cy$ in the algorithm.

The value of $F_X(c-y)\ f_y(y)$ is calculated in the line 31. The lines 32 and 33 stores the cumulative value and increments the variable, respectively. Finally, the line 35 calculates the value of the integral.

Note that the presented method might work for any situation where the variable Y_i has a known density function. It is enough to use the equation of such function as $f_y(y_i)$ in the Eq. 2, and consequently in the line 31 of algorithm described in the Fig. 4. Studies using other type of distributions for the travel time will be subject of future research.

At this point, it was showed how to calculate $P\{X^{tr} + Y \le c\}$. The next step shows how to obtain the cumulative probability function of X^{tr}.

4.5. The cumulative probability function

The strategy is to use the convolution algorithm described previously to calculate $n\,int$ discrete points in the space of the cumulative probability [0, 1] of the random variable X.

The function "F" (algorithm of the Fig. 5) describes how to obtain the cumulative probability matrix (*mprob*) with the values of X_i in the column 1 and with values of $P\{X_i \le c\}$ in the column 2, in the space of the cumulative probability [0, 1] with *nint* points, where $q = \{1, \ldots nint\ \}$. The inputs are the same already described in the Table 2.

The algorithm firstly calculates the initial value (x_1) and the final value x_{nint} for which the cumulative function F_X will be computed. The second step is the loop to calculate the values $F_X(x_q)$ for each one of the *nint* values x_q between x_1 and x_{nint}. The algorithm also calculates the mean and standard deviation of the sum of variables $X + Y$.

Lines 2 to 7 are the same already described in the function "Conv" (Fig. 4), calculating a lower and upper bounds for the space of probability [0, 1]. The lines 4 and 5 gets the values of x for which the probability is ≅0 and ≅1, respectively, where $min\,X = F_X{}^{-1}(\cong 0)$ and $max\,X = F_X{}^{-1}(\cong 1)$. Alternatively, the use of $min\,X = F_X{}^{-1}(0.05)$ and $max\,X = F_X{}^{-1}(0.95)$ helped to obtain *mprob* more centralized within the space of probability [0, 1]. So, instead of getting the first and last value of *mprob*, it might get $min\,X$ and $max\,X$ from the indices closest to 0.05 and 0.95, respectively.

These bounds are used in the line 9 to calculate the increment of values for which the probability $P\{X + Y \le t\}$ will be computed. The values for which $P\{X + Y \le t\}$ is calculated and its respective cumulative probability are stored in the matrix *mprobN*.

Line 10 starts the values to perform the loop in the lines 11 to 44. Lines 11 and 12 give 3 conditions to keep looping: the first is only to ensure the first interaction of the loop, in the second condition it is ensured it will keep looping while the last probability calculated is less than 99.9% and the difference between 2 consecutive probability calculations (dy) is greater than 0.0001. The third condition guaranties the loop at least until $F_x = 0.98$. The conditions 2 and 3 are justified due to the asymptotic behavior of the cumulative function.

Lines 16 and 18 get the number of intervals that will be used to calculate $P\{X^{tr} + Y \le t\}$ using the function "Conv" in the line 28 (already presented in the Fig. 3). While $P\{X^{tr} + Y \le t\} < 5\%$, it is used $I = 5$, because few steps are enough to solve the convolution, for the other cases, $I = I2$ (values are discussed later).

Between the lines 19 to 25 there is a specific condition to speed up the algorithm. It is observed that after $F_x = 0.70$ approximately, the variation rate of the cumulative probability curve starts to decrease quickly because it is asymptotic. After this point, if the difference between two consecutive probabilities is less than 3**%** (from preliminary tests), the algorithm will jump the current point and move to the next point. The skipped point will have its probability calculated in the line 33 as the linear interpolation of its adjacent points, considering the increment for x is constant, it coincides with the mean. This is illustrated by Fig. 6a and b, where the solid line is the actual cumulative distribution from simulation and the circles are the calculated points by the method.

The Fig. 6a shows a common behavior of the cumulative function which derivation reduces dramatically at the right side. Fig. 6b

```
function[mprobN,vidxN]=F(t,MeanY,DevY,mprob,vidx)
y0= MeanY-3.0*DevY;
yf= MeanY+3.0*DevY;
minX=mprob(1,1);
maxX=mprob(vidx(4),1);
x0=max(minX,t) +max(y0,0);
xf=max(maxX,t) +yf;
idx=0;dy=1;
inc=(xf-x0)/(nint);
vidxN(1)=x0; vidxN(2)=inc; ping1=0; gapF=0.03;
while idx==0|| (mprobN(idx,2)<0.999 && abs(dy)> 0.0001 )
            || mprobN(idx,2)< 0.98
    idx=idx+1;
    mprobN(idx,1)=x0+(idx-1)*inc;
    if idx==1 || mprobN(idx-1,2) < 0.05
        I=I1;
    else
        I=I2;
        if mprobN(idx-1,2) > 0.7
            if  (mprobN(idx-1,2)-mprobN(idx-2,2))<gapF && ping1==0
                idx=idx+1;
                mprobN(idx,1)=x0+(idx-1)*inc;
                ping1=1;
            end
        end
    end
    c=mprobN(idx,1);
    [mprobN(idx,2)]=Conv(t,MeanY,DevY,c,mprob,vidx,I);
    if idx==1
        dy=mprobN(idx,2);
    else
        if ping1==1
            mprobN(idx-1,2)=0.5*mprobN(idx-2,2)+0.5*mprobN(idx,2);
            ping1=0;
            dy=(mprobN(idx-1,2) - mprobN(idx-2,2));
        end
        dy=(mprobN(idx,2) - mprobN(idx-1,2));
    end
end
vidxN(3)=mprobN(idx,1); vidxN(4)=idx;
end
```

Fig. 5. Function "F" – Calculation of the Cumulative Probability Function.

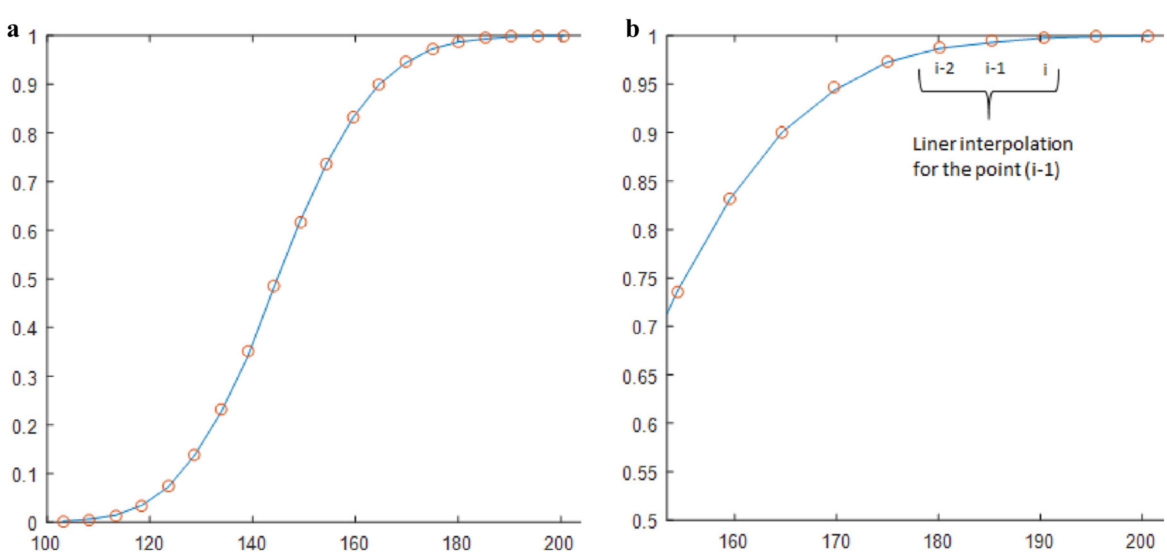

Fig. 6. (a) Cumulative Function. (b) Linear interpolation.

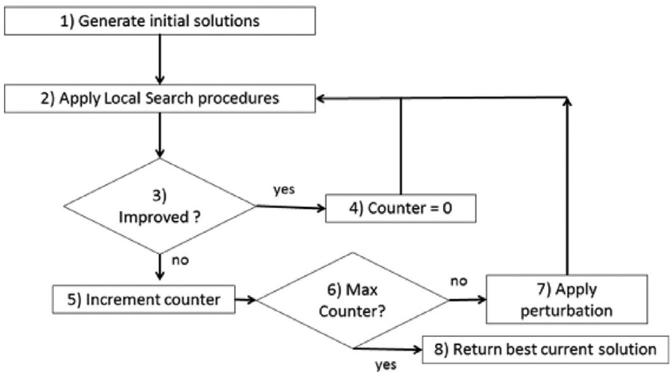

Fig. 7. Metaheuristic for the Stochastic VRP.

zooms in the right side to show a situation where the point $i-1$ is linear interpolated by the points $i-2$ and i.

There are two main parameters for the proposed method in this paper: *nint* (number of intervals or the size of the vector with the cumulative probability function) and *I*2 (number of increments used to solve the convolution). Preliminary tests to adjust the parameters were performed and the parameters used in this paper are $nint = 40$ and $I2 = 25$.

4.6. Metaheuristic to solve the stochastic VRP

This section presents the metaheuristic in which the statistical method proposed in the previous section will be used. Any heuristic can be adapted to use the algorithms described in the Figs. 4 and 5. The main point of this topic is not to evaluate the metaheuristic but evaluate the proposed method to assess the service level of the customers and analyze the influence of different stochastic models in the solution of the VRP. This is the reason the metaheuristic is not detailed in this paper but its main structure is given in the Fig. 7.

The Fig. 7 presents the main loop of the metaheuristic used in the experiments to solve the VRPTW with normal travel and service time. The general framework is an Iterated Local Search (Lourenço, Martin, & Stüzle, 2003) procedure, so when the algorithm is stuck in a local minimum a perturbation mechanism is triggered.

The step 1 generates initial solutions for the problem. The insertion heuristic *I1* given in Solomon (1987) is extended by considering the expected violations of the time windows where travel/service times are stochastic. The employed parameters lead to 10 runs in total. The solution best evaluated in the objective function is selected for the next step.

The step 2 consists of a set of local searches based on 2-Opt∗, Or-Opt (Potvin & Robillard, 1995), *Cross Exchange* (Taillard, Badeau, Gendreau, Guertin, & Potvin, 1997) and Edge assembly crossover (Nagata, 2007), this last one, specifically designed to reduce the number of routes. If this step finds a better solution, the counter is reset to zero and the step 3 is repeated. If not, the counter of iterations is incremented and the stop criterion (maximum number of iterations) is checked (step 6). While the stop criterion is not triggered, a perturbation procedure based on the Natural Cross-over (Potvin, 2007) is applied.

The vehicle routes obtained by the metaheuristic depart from the depot at time 0. Considering the mathematical formulation has no penalties associated with waiting time or delay time, shifting the starting time from the depot would not improve the service levels at the customers or change the evaluation of the objective function.

Next step will illustrate how the search for better solutions can be affected by the stochastic nature of the problem.

4.7. Feasibility test in a local search procedure

Basic movements performed by the local search procedures include the insertion and the removal of customers from the routes. As an example, let's assume a route r_a where a vehicle visits the following sequence of customers, $r_a = \{1, 2, 3, 4, \ldots\}$ with n_a customers, and another route $r_b = \{5, 6, 7, 8, \ldots\}$ with n_b customers. Lr_a and Lr_b are the current load of the vehicles, given by the sum of the customer demands visited by each vehicle. The cumulative probability array for the customer i is $mprob_i$ (notation from Table 2). And the corresponding auxiliary array is $vidx_i$ with the first value of $mprob_i$, the value of the increment, and the last value of $mprob_i$. In the algorithm, $mprob_i$ and $vidx_i$ are stored and updated for each customer. For a certain movement, the local search will consider to interchange the customers 2 and 6, removing 6 from r_b and inserting it in r_a in the position of 2. Analogously, 2 is moved to r_b.

The Algorithm 1 tests the feasibility of one interchange move, where customers 2 and 6 from different routes are being switched. Line 1 initializes a boolean variable indicating if the movement is feasible ($ok = 1$) or not feasible ($ok = 0$). Line 2 tests if this movement improves or not the objective function (here considering only the travel time as an example). If it improves, line 3 tests the capacity constraint. If it holds, line 4 calculates the service level for the inserted customer 6, using the function "Conv" described in the Fig. 4.The insertion of a new customer does not affect the arrival time in the previous customers. Here, to calculate the arrival time of the new customer it is necessary the information of the truncation point and the arrival time in the previous customer, the parameters of the normal distribution given by the sum of the travel time with the service time, and the latest time windows of the inserted customer.

In the line 6, if the service level for the first inserted customer is feasible, it is tested in the next lines the feasibility of the other inserted customer. Line 13 initializes the index with the position of the inserted customer in the route r_a. The loop 14–24 calculates the array with the discrete cumulative function for all the customers from the inserted position on. With this information calculated in the line 16 (function "F" described in the Fig. 5), the line 17 obtains the service level for the next customer. The line 25 initializes the index with the position of the inserted customer in the route r_b and another loop is performed to check the feasibility of this route.

At any moment of the algorithm, if the service level of a customer is violated, the algorithm is finished and returns $ok = 0$.

5. Experiments and results

This section describes the tests where the proposed method is compared with the α-discrete method described in Zhang et al. (2013) (the benchmark method). The accuracy of the estimates for the non-tardiness probability and the computational time are the metrics used to compare the methods. The first experiment tests the method on a set of 9037 routes in order to validate the consistency of the proposed method. In the second experiment the methods are integrated to a heuristic to solve a VRP using the same 7 Solomon's instances from the benchmark, but with 100 customers (Zhang et al., 2013 solved the 7 instance in a version with 20 customers).

The error of each method is given by their difference from the results obtained by stochastic simulation. An algorithm for how to perform this simulation is provided in the work of Li et al. (2010).

Algorithm 1. Feasibility test for an interchange move.

```
Set; ok = 0;
If (TT_{1,6} + TT_{6,3}) + (TT_{5,2} + TT_{2,7}) ≤ (TT_{1,2} + TT_{2,3}) + (TT_{5,6} + TT_{6,7}) then
    If (Lr_a − q_2 + q_6 ≤ Q) And (Lr_b − q_6 + q_2 ≤ Q) then
        SL_6 = Conv(e_1, E[TT_{1,6}] + E[ST_1], sqrt(Dev[TT_{1,6}]^2 + Dev[ST_1]^2), l_6, mprob_1, vidx_1);
        If SL_6 ≤ α then
            SL_2 = Conv(e_5, E[TT_{5,2}], Dev[TT_{5,2}], l_2, mprob_5, vidx_5);
            If SL_2 ≤ α then
                ok = 1;
            Else
                ok = 0;
            End
        End
        Set; p = 2;
        While (ok = 1)And(p ≤ n_a) do
            i = r_a(p − 1); j = r_a(p); k = r_a(p + 1);
            [mprob_j, vidx_j] = F(e_i, E[TT_{i,j}] + E[ST_i], sqrt(Dev[TT_{i,j}]^2 + Dev[ST_i]^2), mprob_i, vidx_i);
            SL_k = Conv(e_j, E[TT_{j,k}] + E[ST_j], sqrt(Dev[TT_{j,k}]^2 + Dev[ST_j]^2), l_k, mprob_j, vidx_j);
            If SL_k ≤ α then
                ok = 1;
            Else
                ok = 0;
            End
            p = p + 1;
        End
        Set; p = 2;
        While (ok = 1)And(p ≤ n_b) do
            i = r_b(p − 1); j = r_b(p); k = r_b(p + 1);
            [mprob_j, vidx_j] = F(e_i, E[TT_{i,j}] + E[ST_i], sqrt(Dev[TT_{i,j}]^2 + Dev[ST_i]^2), mprob_i, vidx_i);
            SL_k = Conv(e_j, E[TT_{j,k}] + E[ST_j], sqrt(Dev[TT_{j,k}]^2 + Dev[ST_j]^2), l_k, mprob_j, vidx_j);
            If SL_k ≤ α then
                ok = 1;
            Else
                ok = 0;
            End
            p = p + 1;
        End
    End
End
return ok
```

The experiments are based on the well-known Solomon's benchmark problems (Solomon, 1987) in which the geographical data are randomly generated in problem sets R, clustered in problem sets C, and a mix of random and clustered structures in problem sets by RC. The tests were performed using a computer with Windows 8.1, 16 GB RAM, I7 2.6 GHz. The software Matlab, version 15, was used to implement the algorithms.

Table 3
Results for the experiment 1.

N	Metric	Proposed	L=10	L=20
1	Mean Error (p.p.)	0.250	0.614	0.323
2	Std. Dev. Error (p.p.)	0.472	1.057	0.521
3	95th Percentile (p.p.)	1.139	2.925	1.450
4	Time (seconds)	115.1	256.4	418.5

5.1. Validation of the statistical method

The experiment in this section aims to validate the proposed method, comparing its accuracy and running time with a benchmark. In order to give more robustness to the test, all the 56 well known Solomon's benchmark problems with 100 customers were adapted to generate instances with different levels of waiting time. Approximately 160 routes were generated for each instance, using the constructing heuristic *I1* from Solomon (1987), totalizing a database formed by 9037 routes with 3 to 55 customers. All the instances used in the experiment have the same features of the original instances of Solomon, such as vehicle capacity, customer location, time windows and service time. Only a standard deviation for the travel time and service time were added.

Let $dis(i, j)$ the distance between two customers, the travel time is a normal distribution with average $E[TT_{i,j}] = dis(i, j)$ and standard deviation $ev[TT_{i,j}] = U[0.1; 0.6] * dis(i, j)$ where U is a uniform distribution. Considering the vehicle velocity is unitary then the value of mean travel time is equal to the value of the distance travel $dis(i, j)$. The service time is also a normal distribution with average $E[ST_i]$ and standard deviation $[ST_i] = U[0.1; 0.6] * E[ST_i]$. The total number of customers in the data base is 101,701 (not considering the first customer where the distribution is perfectly normal). It is performed 2 probability calculations per customer (earliness and tardiness) then the number of calculations to test the accuracy of the method is 203,402.

The Table 3 displays the results. The second column describes the 4 metrics used to compare the methods. The first 3 metrics give descriptive information for the accuracy in percentage points for 203,402 calculations performed: the mean, standard deviation of the error and the 95th percentile. The metric number 4 gives the computational time in seconds (average of 5 runs). The running time is the amount of time spent by the method to sweep all the routes and calculate the probabilities. The third column of the table has the results for the proposed method, the last two columns refers to the benchmark using 2 configurations for the discrete parameter (L).

The Table 3 shows that the proposed method obtained better results in all the four metrics when compared with the benchmark using discrete parameter L with value 10. The computational time of the proposed method was 2.2 times faster. The comparison

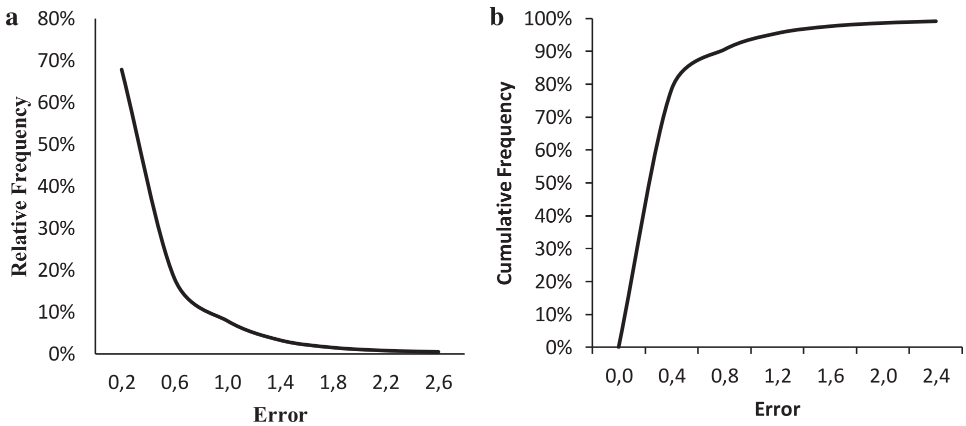

Fig. 8. (a) Error-Frequency Distribution. (b) Error-Cumulative Distribution.

Table 4
Feature of the instances (100 customers).

Instance	Geographic type	$min(l_i - e_i)$	$max(l_i - e_i)$	Total Demand	$E[ST_i]$
R105	Uniform	30	30	1458	10
R109	Uniform	37	83	1458	10
C101	Clustered	37	89	1810	90
C106	Clustered	29	387	1810	90
RC101	Mix	30	30	1724	10
RC106	Mix	60	60	1724	10
RC107	Mix	41	155	1724	10

of the proposed method with the α-discrete method using $L = 20$ shows that the proposed method had slightly better results for the accuracy (metrics 1 to 3) and it was much faster (about 3.6 times).

In order to better illustrate the behavior of the errors, the error distribution is presented in the Fig. 8a and b.

The Fig. 8a shows that the frequency distribution has a positive skew with a right longer tale and it is significantly close to an exponential distribution. Errors close to zero have high relative frequency evidencing the quality of the results. In the Fig. 8b, it is seen the cumulative function, and it gives a good idea of the percentiles. For instance, there is approximately 90% probability of having an error less than 1%.

5.2. Application in a stochastic VRP

The same 7 Solomon's instances used by the benchmark were applied in this experiment to demonstrate the performance of the proposed model. The features of the database can be seen in the Table 4.

Considering $dis(i, j)$ the distance between two customers, the travel time is a normal distribution with average $E[TT_{i,j}] = dis(i, j)$ and standard deviation $Dev[TT_{i,j}] = U[0.1; 0.6] * dis(i, j)$. The vehicle velocity is unitary. The service time is also a normal distribution with mean $E[ST_i]$ equal to the deterministic service time, and standard deviation $Dev[ST_i] = U[0.1; 0.6] * E[ST_i]$. The required service level of the customers is 0.8 (α_i), it means the required probability of the vehicles arriving before the latest time windows for all the customers must be greater or equal to 80%.

The Table 5 gives the results for these experiments over 10 runs. The first columns is the Solomon instance, the second is the method (the proposed method, α-discrete method with L=10, and the α-discrete method with L=20). The others columns are, respectively: the number of vehicles, the travel time given by the total of the average travel time of the active arcs ($\Sigma E(T_{ij})$); the lowest service level of a customer in the solution (given by simulation), the average service level of the customers in the solution (also given by simulation), the average error and the maximum error (both in percentage points) for the service level given by the method, the computational time in seconds. In the bottom of the table, it is calculated the average of the results for the 7 instances, for each method.

In the Table 5, it is clear that the method influences the VRP solution. In all instances, different methods obtained different values for the objective function. In some situations the service level constraint did not hold by a significant margin, for example, in the instance R105, method L10, the customer with the lowest service level had 76.42% while the required is 80%. In this case, the errors for the service level calculation were significantly higher than the others, in the worst case reaching 5.26 percentage points.

In the bottom of the Table 5, the total of the results shows the proposed method was superior for the average and maximum errors of the service level. For the average error, the proposed method got a total error of 0.18 percentage points against 0.69 and 1.42 for L20 and L10 respectively. For the maximum error, the difference is bigger: 1.05, 2.58 and 5.22 for the proposed, L20 and L10 respectively.

From the Table 5, it possible to see that, in terms of accuracy, the proposed method is slightly better than L20 and in terms of running time, approximately 5 times faster. The results make clear that the statistical method to evaluate the feasibility of the solutions impacts significantly the running time.

The Table 6 intends to show the influence of the required service level in the objective function. It is given a summarized data with the average of the results for the 7 instances. It was obtained using the metaheuristic with the proposed statistical method.

In order to make easier the analysis of the Table 6, the graphs 9 and 10 are provided. The graphs give the average number of vehicles (NV) obtained by the stochastic model for a required service level of 0%, 20%, 40%, 60%, 70%, 80% and 90%. The lines "NV Det" and "TT Det" are the average number of vehicles given by a deterministic approach (11.57 vehicles) and the average travel time

Table 5
Results for instances with 100 customers.

Instance	Method	NV	TT	SL (min)	SL (avg)	SL error (avg)	SL error (max)	Seconds
R105	Proposed	17.67	1615.75	81.66%	99.31%	0.18%	1.81%	18.40
R105	L = 20	18.33	1610.01	81.39%	95.75%	0.72%	3.13%	91.90
R105	L = 10	18.00	1621.42	76.42%	95.41%	2.03%	5.26%	29.36
R109	Proposed	15.00	1488.81	79.53%	95.45%	0.17%	0.89%	6.36
R109	L = 20	15.33	1420.59	81.80%	94.73%	0.67%	3.07%	151.71
R109	L = 10	15.00	1355.76	78.59%	93.82%	0.76%	5.29%	91.01
C101	Proposed	17.00	2284.77	83.82%	95.52%	0.22%	0.87%	29.49
C101	L = 20	16.67	2227.70	81.64%	95.46%	0.71%	2.66%	150.91
C101	L = 10	17.00	2396.18	85.55%	97.31%	1.59%	6.14%	17.97
C106	Proposed	14.67	1722.32	81.33%	98.65%	0.18%	0.70%	5.56
C106	L = 20	14.67	1947.99	78.21%	94.44%	0.54%	2.93%	50.93
C106	L = 10	15.00	1621.80	80.76%	94.48%	1.28%	6.58%	27.53
RC101	Proposed	19.67	2012.76	78.32%	100.22%	0.16%	1.42%	15.18
RC101	L = 20	20.67	2064.79	77.61%	97.02%	0.49%	2.09%	31.19
RC101	L = 10	20.00	2062.91	79.28%	94.25%	1.66%	3.69%	69.85
RC106	Proposed	14.67	1584.63	82.26%	95.57%	0.16%	0.82%	24.43
RC106	L = 20	14.67	1684.96	79.19%	96.51%	1.27%	2.10%	105.15
RC106	L = 10	15.00	1719.23	80.22%	99.45%	1.52%	4.86%	66.77
RC107	Proposed	14.00	1569.10	83.03%	96.23%	0.16%	0.86%	34.99
RC107	L = 20	13.67	1466.84	78.83%	99.02%	0.47%	2.07%	105.26
RC107	L = 10	14.00	1463.49	76.73%	93.71%	1.11%	4.74%	133.44
Average	**Proposed**	**16.10**	**1754.02**	**81.35%**	**97.28%**	**0.18%**	**1.05%**	**19.20**
Average	**L = 20**	**16.29**	**1774.70**	**79.81%**	**96.13%**	**0.69%**	**2.58%**	**98.15**
Average	**L = 10**	**16.18**	**1748.68**	**79.65%**	**95.49%**	**1.42%**	**5.22%**	**62.28**

Table 6
Results for different required service levels.

Service Level	NV	TT	SL (min)	SL (avg)	SL error (avg)	SL error (max)	Seconds
0%	11.57	1187.50	00.00%	59.46%	0.16%	1.23%	13.58
20%	12.29	1353.09	20.44%	74.39%	0.32%	1.45%	33.67
40%	13.29	1399.01	43.10%	81.89%	0.26%	1.26%	14.16
60%	14.52	1516.91	60.60%	90.16%	0.24%	1.16%	21.19
70%	15.52	1608.84	71.11%	93.99%	0.20%	1.55%	20.55
80%	16.10	1754.02	81.28%	97.28%	0.18%	1.05%	19.20
90%	16.90	1842.56	90.90%	98.64%	0.14%	1.38%	25.02

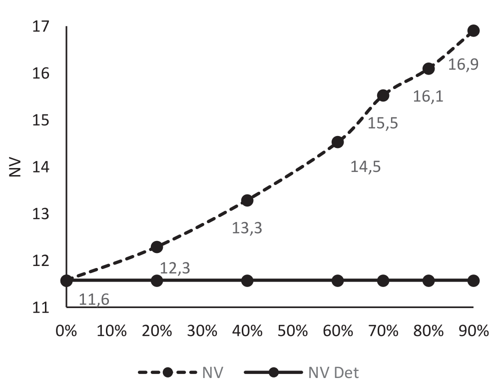

Fig. 9. SL vs Vehicles.

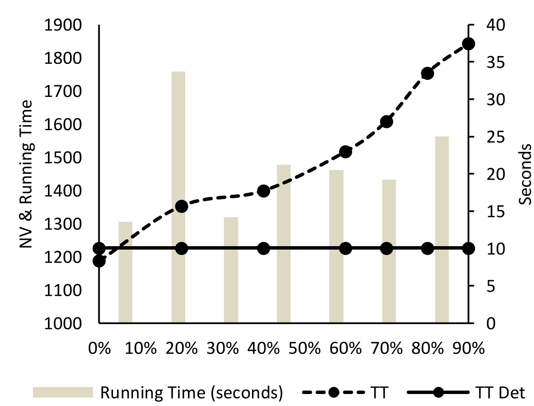

Fig. 10. SL vs (Travel Time & Running Time).

(1225.98). The line "TT" is the travel time and the line "Running Time" is the average time to solve the 7 instances, as shown in Figs. 9 and 10.

The graph 9 shows that the number of vehicle decreases as long as the required service level also decreases. The graph 10 shows similar behavior for the travel time. For SL=0 the average number of vehicles and the travel time were smaller than the deterministic solution. It is plausible because in this extreme situation, the latest time window is disabled, making easier to find best solutions, while the deterministic approach, although allowing poor service levels, still has the latest time window enabled. The graph 10 also shows that the running time is not clearly affected but the required service level.

The Table 7 is related to one route of the optimal solution of the instance RC106 for the deterministic version of the problem with hard time windows (Transportation Optimization Portal, 2016).

The example above shows the service levels of a solution obtained by a deterministic model (in the scope of the standard deviations here considered). The first 3 customers have a service level of 100%, and the customers 28 and 89 a service level of only 39.6% and 47.1% respectively. Planning a route where the vehicle has only

Table 7
Example from a deterministic route.

Customer j	33	31	29	27	28	26	89
e_j	51	50	52	57	55	100	117
l_j	111	110	112	117	115	160	177
$E[TT_{ij}]$	51.48	10.44	2.00	5.00	5.83	3	37.54
$Dev[TT_{ij}]$	7.29	4.09	0.51	0.73	2.94	1.71	8.18
$E[ST_{ij}]$	10	10	10	10	10	10	10
$Dev[ST_{ij}]$	1.68	3.21	1.3	4.1	4.37	3.65	4.49
$SL_j(\%)$	100.0%	100.0%	99.9%	97.2%	39.6%	99.7%	47.1%

39.6% chance of arriving before the latest time windows may have impact on business, evidencing the importance of the stochastic model over the deterministic model.

6. Concluding remarks and further research

This paper approaches a VRP - Hard Time Windows with a normal distribution for the travel time and also for the service time. The research shows that the existence of a waiting time due to the truncation of the arrival time at the earliest time window demands a specific statistical method to obtain the cumulative probability distribution of the vehicles over the customers. In the case of this paper, it was showed that, even with both travel and service time following a normal distribution, the arrival time is not well-modeled by a normal distribution.

Both experiments, one involving a specific data base of routes, and the other with a metaheuristic solving Solomon's instances, evidenced that the proposed method outperformed the benchmark in terms of accuracy and running time. With a slight better quality, the proposed method was 3.6 times faster than the benchmark in the first experiment, and 5 times faster in the second experiment. In the second experiment, using exactly the same metaheuristic, only changing the method to check the time windows feasibility, it was showed that the method used to check this constraint impacts significantly the running time. Considering the problem is *NP*-hard, it is important the development of efficient methods to check if the service level constraint holds.

A detailed algorithm for the statistical method is presented, and also another algorithm showing how the method can be used for a basic movement involving the insertion and removal of a customer from the route. Such movements are the foundation of local search procedures utilized in any metaheuristic solving the VRP.

It is also studied how the required service level influences the solution for the VRP, having a clear relationship with the number of vehicles and the travel time, but not with the computational time. It is also given an example with a route from an optimal solution obtained by a deterministic model, showing that this approach is subject to planning route with low service level for the customer, which may be relevant for business purposes.

The applicability of the method for other probability distributions such as gamma and log-normal is seen as an opportunity for future researches. Another opportunity is to study a variant with time-dependent travel time since speeds of vehicles vary throughout the day. It is believed that the foundation to explore such opportunities was given by the present paper.

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