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Performance evaluation of SCM in JIT environment

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A B S T R A C T

Under stochastic demand and deterministic processing times, we discussed a single-stage JIT production system with the production-ordering and supplier kanbans and derived a probability generating function (p.g.f.) of the stationary distributions of the backlogged demand. In this paper, we extend the system to supply chain management (SCM) in JIT environment with two kinds of kanbans under stochastic demand, deterministic processing times and withdrawals with lead time. These conditions are more realistic than the previous papers. We develop an algorithm for the exact performance evaluation of the SCM such as the stationary distributions of the inventory level, production quantities and total backlogged demand in each stage, using discrete-time Markov process. Optimal numbers of two kinds of kanbans in the system are determined by minimizing a general total cost function. Numerical examples are given to show the efficiency of the proposed approach.

1. Introduction

Supply chain management (SCM) plays an important role in various kinds of business environments. Taylor (2001) insists that the apparently simple concept such as low inventories with deliveries supplied just-in-time (JIT) for production process has far-reaching effects internal to the firm and externally throughout the supply chain. A JIT production system is a well-known successful model to design an effective supply chain management. It based on two concepts; one is JIT and the other is Autonomation that means defect free. These concepts include institutional aspects to manage the systems at the view point of manufacturing resources such as materials, labors and information. In the JIT production systems, the kanban is used as a tool to coordinate the both information and material flows (Monden, 1993).

Deleersnyder et al. (1989) have investigated effects of factors such as the number of kanbans, the machine reliability and the demand variability on the performance of a JIT production system with only the production-ordering kanban using a discrete-time Markov chain. Under stochastic demand and deterministic processing times, Ohno et al. (1995) discussed a single-stage JIT production system with the production-ordering and supplier kanbans and derived a probability generating function (p.g.f.) of the stationary distributions of the backlogged demand. In this paper, we expand the system to a multi-stage JIT production system with two kinds of kanbans under stochastic demand, deterministic processing times and withdrawals with lead time. These conditions are more realistic than those of Mitra and Mitrani (1990, 1991), Tayur (1993), Kirkavak and Dinçer (1996), and Berkley (1992).

We develop an algorithm for the exact performance evaluation of the JIT production system such as the stationary distributions of the inventory level, production quantities and total backlogged demand in each stage, by a discrete-time Markov process. Optimal numbers of two

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kinds of kanbans in the multi-stage JIT production system are determined by minimizing a general total cost function. Numerical examples show the efficiency of the proposed algorithm.

2. A multi-stage JIT production system

We expand the JIT production system with supplier and production-ordering kanbans (Ohno et al., 1995) into a multi-stage production system, which is shown in Fig. 1.

The preceding stage of stage 1 is a supplier. For simplicity, take the constant delivery cycle as one period. In this system, the demand is stochastic and the processing time is deterministic. The following notation is used:

n	the number of the stage
j	the index of the stage ($1\leqslant j\leqslant n$, n is the index of the final stage)
L_j	the delivery lead time of the parts at stage j
M_j	the number of production-ordering kanbans at stage j
N_j	the number of withdrawal kanbans at stage j
C_j	the production capacity of stage j
$D(k)$	the demand in period k
$B_j(k)$	the backlogged demand at stage j at the beginning of period k
$I_j(k)$	the inventory level of the part at stage j at the beginning of period k
$J_j(k)$	the number of the production-ordering kanban in the production-ordering kanban post at stage j at the beginning of period k
$P_j(k)$	the production quantity at stage j in period k
$Q_j(k)$	the delivery quantity of the part at stage j in period k
$X_j(k)=B_j(k)+J_j(k)$	the total backlogged demand at stage j at the beginning of period k

The order of parts consumed at stage j in period $k=1,2,\ldots$ is transmitted to stage ($j-1$) at the beginning of period ($k+1$), and they are delivered at the beginning of period ($k+L_j+1$). It is assumed that the demand of the product in each period is independent and identically distributed with distribution $\Pr\{D(k)=d\}=p_d,\ d=0,1,\ldots,D_{\max}\}$ and mean D, the order of the parts is instantly transmitted from stage j to stage ($j-1$), the excess demands of the products or parts are backlogged at each stage and the container capacity is equal to one.

Since L_1 is the delivery lead time of the supplier, N_1 denotes the number of supplier kanbans and the number of supplier kanbans transmitted to the supplier at the beginning of periods k is $P_1(k-1)$, it holds that for $k=1,2,3,\ldots$

$$N_1=I_1(k)+\sum_{m=k-L_1}^{k-1}P_1(m), \tag{1a}$$

where $P_1(0), P_1(-1),\ldots,P_1(-L_1+1)$ are given. Similarly, N_j, $2\leqslant j\leqslant n$, withdrawal kanbans circulate between stages ($j-1$) and j, and

$$N_j=I_j(k)+P_j(k-1)+B_{j-1}(k)+\sum_{m=k-Lj+1}^{k-1}Q_{j-1}(m),\quad 2\leqslant j\leqslant n, \tag{1b}$$

where $Q_j(0), Q_j(-1),\ldots,Q_j(-L_{j+1}+2)$, $1\leqslant j\leqslant n-1$ are given. Since the inventory level of the part at the beginning of period ($k+1$) is changed from that of periods k by the difference between the delivery quantity from the preceding stage at the beginning of period k and the consumed quantity of the part in period k, it holds that

$$I_1(k+1)=I_1(k)+P_1(k-L_1)-P_1(k) \tag{2a}$$

and

$$I_j(k+1)=I_j(k)+Q_{j-1}(k-L_j+1)-P_j(k),\quad 2\leqslant j\leqslant n. \tag{2b}$$

In the JIT production system, the production quantity is determined by the minimum among the inventory level of the part, the production-order quantity and the production capacity. That is,

$$P_j(k)=\min(I_j(k),J_j(k),C_j),\quad 1\leqslant j\leqslant n. \tag{3}$$

The backlogged demand at stage j occurs at the beginning of period ($k+1$) if the sum of the backlogged demand at the beginning of period k and the demand from the subsequent stage or customers in period k exceeds the sum of the production quantity and the inventory level of produced parts, $M_j-J_j(k)$. Therefore,

$$B_j(k+1)=[B_j(k)+J_j(k)+P_{j+1}(k-1)-P_j(k)-M_j]^{+},\quad 1\leqslant j\leqslant n-1 \tag{4a}$$

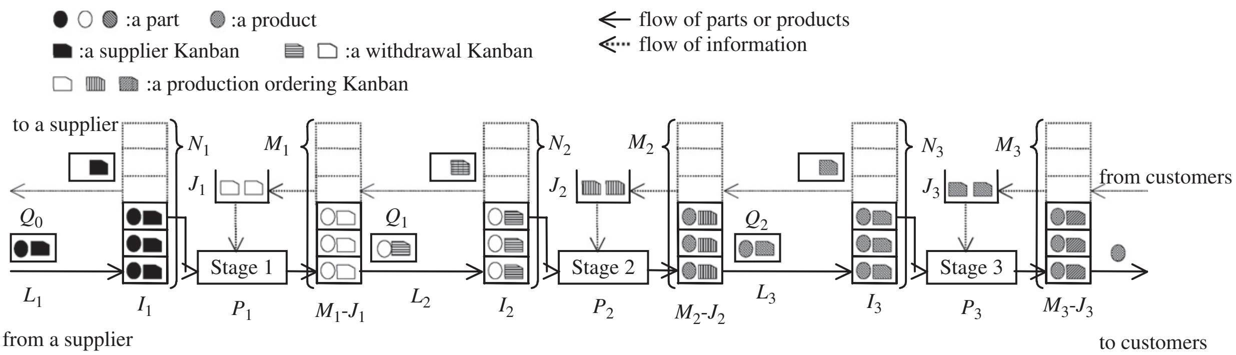

Fig. 1. A multi-stage JIT production system ($n=3$).

and

$$B_n(k+1) = [B_n(k) + J_n(k) + D(k) - P_n(k) - M_n]^+, \tag{4b}$$

where $[x]^+ = \max(0, x)$. Since the number of production-ordering kanbans at the production-ordering kanban post at the beginning of period (k+1) is the minimum between M_j and the total backlogged demand at the beginning of period (k+1)

$$J_j(k+1) = \min(M_j, B_j(k) + J_j(k) + P_{j+1}(k-1) - P_j(k)), \quad 1 \leqslant j \leqslant n-1 \tag{5a}$$

and

$$J_n(k+1) = \min(M_n, B_n(k) + J_n(k) + D(k) - P_n(k)). \tag{5b}$$

The delivery quantity in period k is determined by the minimum between the sum of the demand from the subsequent stage or customers in period k and the backlogged demand at the beginning of period k and the sum of the production quantity in period k and the inventory level of the product at the stage at the beginning of period k. That is,

$$Q_j(k) = \min(P_{j+1}(k-1) + B_j(k), P_j(k) + [M_j - X_j(k)]^+), \quad 1 \leqslant j \leqslant n-1, \tag{6a}$$

and

$$Q_n(k) = \min(D(k) + B_n(k), \quad P_n(k) + [M_n - X_n(k)]^+). \tag{6b}$$

Since the total backlogged demand at the beginning of period (k+1) is changed from that of period k by the difference between the demand from the subsequent stage or customer in period k and the production quantity at the stage in period k, it holds that

$$X_j(k+1) = X_j(k) + P_{j+1}(k-1) - P_j(k), \quad 1 \leqslant j \leqslant n-1, \tag{7a}$$

and

$$X_n(k+1) = X_n(k) + D(k) - P_n(k). \tag{7b}$$

3. The stationary distribution of the JIT production system

Define the state space in period k as

$$S(k) = [I_j(k), X_j(k), Q_{j-1,m}(k), 1 \leqslant j \leqslant n, k - L_j + 1 \leqslant m \leqslant k-1],$$

where $Q_{0,m}(k) = P_1(m-1)$ and $Q_{j-1,m}(k)$ means the undelivered quantity of the part at stage $j-1$ in period k when it was ordered at the beginning of period m. Let $RI_j(k)$, $x_j(k)$, and $q_{j-1,m}(k)(1 \leqslant j \leqslant n,\ k-L_j+1 \leqslant m \leqslant k-1)$ denote realizations of the corresponding random variables $I_j(k)$, $X_j(k)$, and $Q_{j-1,m}(k)$, respectively. Since $D(k)$, $k = 1,2,3, \ldots$, are independent and identically distributed, and $[I_j(k)$, $X_j(k)$, $Q_{j-1,m}(k)$, $1 \leqslant j \leqslant n$, $k-L_j+1 \leqslant m \leqslant k-1]$ determines all states of the JIT production system by Eqs. (1)–(7), $S(k)$ is a Markov chain with the following transition probabilities:

$$Pr\{S(k+1) = s(k+1) | S(k) = s(k)\}$$

$$= Pr\{D(k) = d\} = \begin{cases} p_d; & \text{if conditions are true,} \\ 0; & \text{otherwise,} \end{cases}$$

where conditions mean that

$$RI_j(k+1) = RI_j(k) + q_{j-1}(k - L_j + 1) - \min\{RI_j(k), x_j(k), M_j, C_j\}, \quad 1 \leqslant j \leqslant n,$$

$$x_j(k+1) = x_j(k) + N_{j+1} - RI_{j+1}(k) - \sum_{m=k-L_{j+1}+1}^{k-1} q_{j,m}(k) - [x_j(k) - M_j]^+ - \min\{RI_j(k), x_j(k), M_j, C_j\}, \quad 1 \leqslant j \leqslant n-1,$$

$$x_n(k+1) = x_n(k) + d - \min\{RI_n(k), x_n(k), M_n, C_n\},$$

$$q_{j,m}(k) = \min\{N_{j+1} - RI_{j+1}(k) - \sum_{m=k-L_{j+1}+1}^{k-1} q_{j,m}(k), \times \min\{RI_j(k), x_j(k), M_j, C_j\} + [M_j - x_j(k)]^+\}, \quad 1 \leqslant j \leqslant n-1,$$

$$q_{0,m}(k) = N_1 - RI_1(k) - \sum_{m=k-L_1+1}^{k-1} q_{0,m}(k) \quad \text{and}$$

$$RI_j(k) + \sum_{m=k-L_j+1}^{k-1} q_{j-1,m}(k) \leqslant N_j, \quad 1 \leqslant j \leqslant n. \tag{8}$$

Under stability condition (Ohno et al., 1995), for $1 \leqslant j \leqslant n$, $k-L_j+1 \leqslant m \leqslant k-1$ and integers i_j, x_j, $q_{j-1,m}$, the limiting probabilities $\Pr\{I_j(\infty) = i_j,\ X_j(\infty) = x_j,\ Q_{j-1,m}(\infty) = q_{j-1,m},\ 1 \leqslant j \leqslant n, 1 - L_j \leqslant m \leqslant -1\} = \lim_{k\to\infty} Pr\{I_j(k) = i_j,\ X_j(k) = x_j,\ Q_{j-1}(i) = q_{j-1,m},\ 1 \leqslant j \leqslant n,\ k-L_j+1 \leqslant m \leqslant k-1\}$ exist. Denote by π the limiting distribution. That is, $\pi = (\Pr\{I_j(\infty) = i_j,\ X_j(\infty) = x_j,\ Q_{j-1,m}(\infty) = q_{j-1,m},\ 0 \leqslant i_j \leqslant N_j,\ 0 \leqslant x_j \leqslant M_j+N_{j+1},\ 0 \leqslant q_{j-1,m} \leqslant N_j,\ 1 \leqslant j \leqslant n,\ 1-L_j \leqslant m \leqslant -1\})$. Then π can be obtained by solving the following balance equations of the Markov chain:

$$\pi \boldsymbol{M} = \pi \text{ and } \pi \boldsymbol{e}^{\mathrm{T}} = 1, \tag{9}$$

where $\boldsymbol{e}$ is a row vector with all elements equal to one and $\boldsymbol{M}$ is the transition probability matrix.

4. An algorithm for the exact performance evaluation

We consider the total backlogged demand, inventory level and production quantities as performance measures in the JIT production system and devised an algorithm for the exact performance evaluation of the system as follows:

Step 1. For L_j, M_j, N_j, and C_j $(1 \leqslant j \leqslant n)$, define the state space S(k) and calculate the transition probability matrix $\boldsymbol{M}$ by Eq. (8).

Step 2. Compute the limiting distribution π of S by solving the balance Eq. (9).

Step 3. Compute the limiting distribution of the total backlogged demand $\{\Pr\{X_j(\infty) = x_j\}\}$ and that of inventory levels $\{\Pr\{I_j(\infty) = i_j\}\}$ from $\pi = (\Pr\{I_j(\infty) = i_j,\ X_j(\infty) = x_j,\ Q_{j-1,m}(\infty) = q_{j-1,m},\ 0 \leqslant i_j \leqslant N_j,\ 0 \leqslant x_j \leqslant M_j+N_{j+1},\ 0 \leqslant q_{j-1,m} \leqslant N_j,\ 1 \leqslant j \leqslant n,\ 1-L_j \leqslant m \leqslant -1\})$, and the limiting distribution of production quantity $\{\Pr\{P_j(\infty) = p_j\}\}$ by Eqs. (3), (5), (7) and π.

5. Optimization of the numbers of kanbans

The JIT production system adapts to variable demands at a small cost by production smoothing (Monden, 1993). Through production smoothing, the stages can reduce idle time or overtime costs of workers or machines. We call these costs related to production quantities production fluctuation costs and include them in a cost function of the JIT production system.

Suppose that the ordered quantities, $Q_j(0),\ldots, Q_j(-L_{j+1}+2)$, $0\leqslant j\leqslant n-1$ are given. Then, a standard cost function over K periods is as follows:

$$
\begin{aligned}
&A(M_j, N_j, 1\leqslant j\leqslant n, K)\\
&\quad= E\left[\sum_{j=1}^{n}\left\{\sum_{k=1}^{K}\left\{A_{I_j}(I_j(k)-P_j(k)/2)\right.\right.\right.\\
&\qquad + B_{I_j}(M_j - J_j(k)) + A_{B_j}B_j(k)\\
&\qquad + A_{O_j}P_j(k-1) + A_{W_j}Q_{j-1}(k-L_j)\\
&\qquad + \sum_{i=0}^{\min(M_j,C_j)} A_{P_j}(i)Pr\{P_j(k)=i\} + C_{B_j}H\{B_j(k)>0\}\\
&\qquad + C_{\mathrm{OW}_j}(M_j,N_j)\} + A_{S_j}(I_j(K)-P_j(K))\\
&\qquad \left.\left. + \sum_{i=1}^{L_j} A_{E_j}(i)P_j(K-i)\right\}\right],
\end{aligned}
\tag{10}
$$

where $H\{Z\}$ is the indicator function of event Z, that is, $H\{Z\}=1$ if Z occurs; $=0$, otherwise. In addition, A_{I_j}, the inventory cost of one part in stage j per period; B_{I_j}, the inventory cost of one product in stage j per period; A_{B_j}, the backlogged cost of one product in stage j per period; A_{O_j}, the ordering cost of one part in stage j; A_{W_j}, the withdrawing cost of one part in stage j; $A_{P_j}(i)$, the production fluctuation cost per period when the production quantity at stage j is i; C_{B_j}, the backlogged cost at stage j per once; A_{S_j}, the salvage cost of one part in stage j at the end of period K; $A_{E_j}(i)$, the salvage cost of one part in stage j elapsed i periods after the ordering at the end of period K and $C_{\mathrm{OW}_j}(M_j,N_j)$, the fixed cost per period of storage space at stage j, ordering and withdrawing when the numbers of kanbans are M_j and N_j.

We consider the average cost per periods over an infinite planning horizon in this research. The average costs per period, $A(M_j,N_j,1\leqslant j\leqslant n)$ is defined by

$$A(M_j,N_j,1\leqslant j\leqslant n) = \limsup_{K\to\infty} A(M_j,N_j,\quad 1\leqslant j\leqslant n,K)/K. \tag{11}$$

Under the stability condition, distributions of $B_j(k)$, $I_j(k)$, and $J_j(k)$ also converge to their own stationary distributions, as k tends to infinity, and denote by $B_j(\infty)$, $I_j(\infty)$, and $J_j(\infty)$ random variables with the stationary distributions. Then it follows from (10) and (11) that

$$
\begin{aligned}
&A(M_j,N_j,1\leqslant j\leqslant n)\\
&\quad=\sum_{j=1}^{n}\{A_{I_j}(E[I_j(\infty)]-E[P_j(\infty)]/2)\\
&\qquad + B_{I_j}(M_j - E[J_j(\infty)]) + A_{B_j}E[B_j(\infty)]\\
&\qquad + (A_{O_j}+A_{W_j})E[P_j(\infty)] + \sum_{i=0}^{\min(M_j,C_j)} A_{P_j}(i)Pr\{P_j(\infty)=i\}\\
&\qquad + C_{B_j}Pr\{B_j(\infty)>0\} + C_{\mathrm{OW}_j}(M_j,N_j)\}.
\end{aligned}
\tag{12}
$$

Thus, if we obtain the stationary distributions and expectations of random variables in (12) by the algorithm devised in Section 4, we can calculate the value of (12) and can determine optimal numbers of kanbans, M_j^* and N_j^* $(1\leqslant j\leqslant n)$ that minimize $A(N_j, M_j, 1\leqslant j\leqslant n)$.

6. Numerical examples

The algorithm devised in Section 4 is applied to the 3-stage JIT production system with the average demand $D=2$, the lead time $(L_1, L_2, L_3)=(1, 1, 1)$ and the production capacity $(C_1, C_2, C_3)=(4, 4, 4)$. The distribution of the demand $D(k)$, $k=1,2,3,\ldots$ is a binomial distribution with mean D. The truncated backlogged demand level is 10.

Fig. 2 shows the distributions of the total backlogged demand at each stage with $(M_1, M_2, M_3)=(4, 4, 4)$ and $(N_1, N_2, N_3)=(5, 6, 7)$.

The cost parameters in (12) are set as follows:

$$
\begin{aligned}
&(A_{I_1},A_{I_2},A_{I_3})=(1,3,5),(B_{I_1},B_{I_2},B_{I_3})\\
&\quad=(3,5,10),(A_{B_1},A_{B_{12}},A_{B_{13}})=(0,0,0),\\
&(A_{O_1}+A_{W_1},A_{O_2}+A_{W_2},A_{O_3}+A_{W_3})\\
&\quad=(1,2,3),(C_{B_1},C_{B_2},C_{B_3})=(0,0,1000),\\
&(C_{\mathrm{OW}_1}(M_1,N_1),C_{\mathrm{OW}_2}(M_2,N_2),\\
&\quad C_{\mathrm{OW}_3}(M_3,N_3))=(0,0,0)
\end{aligned}
$$

and

$$A_{P_j}(i)=\begin{cases}50(i-3) & 4\leqslant i\\ 0 & 0\leqslant i\leqslant 3\end{cases},\quad j=1,2,3.$$

Then the optimal numbers of production-ordering kanbans and withdrawal kanbans are $(M_1^*, M_2^*, M_3^*)=(3, 3, 4)$ and $(N_1^*, N_2^*, N_3^*)=(5, 6, 6)$, respectively.

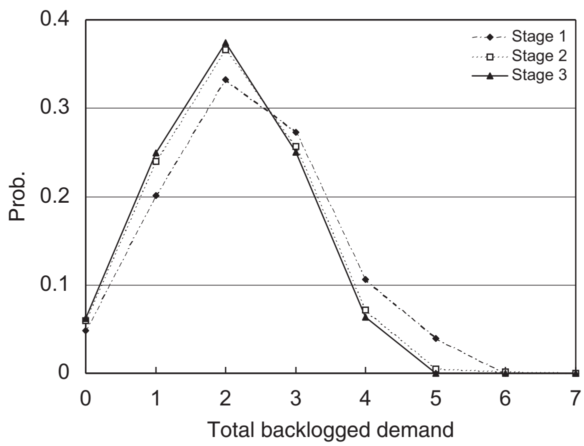

Fig. 2. Probability distributions of the total backlogged demand for $(M_1, M_2, M_3)=(4, 4, 4)$ and $(N_1, N_2, N_3)=(5, 6, 7)$.

7. Conclusion

In this paper, we deal with a multi-stage production and distribution supply chain in JIT environment with two kinds of kanbans. Under the stochastic demand and deterministic processing times and withdrawals with lead time, an algorithm for the exact performance of the JIT production system is devised based on the Markov process. We can compute optimal numbers of two kinds of kanbans, which are the tools to synchronize the SCM using the proposed algorithm. Numerical results show the exact performance of the multi-stage SCM in JIT environment and the optimal numbers of two kinds of kanbans that give the substantial impacts on the SCM.

Acknowledgments

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