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# A surcharge pricing scheme for supply chain coordination under JIT environment

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ABSTRACT

Just-In-Time (JIT) system involves frequent shipments of smaller batch sizes from the supplier to the buyer. For the buyer, it results in the reduction of the inventory holding cost. However, it is often accompanied by an increase in the set up cost for the supplier. Thus, the supplier may be reluctant to switch to the JIT mode unless he is assured of some form of compensation. In this paper, we introduce a pricing scheme where the buyer offers the supplier an increase in the wholesale price, to encourage the supplier to switch to the JIT mode. Such a pricing scheme may be termed as a surcharge. We develop the economics of surcharge pricing as a supply chain coordinating mechanism under JIT environment. We also establish the equivalence of surcharge pricing with other common coordination mechanisms like Quantity Discount (QD) and Joint Economic Lot Sizing (JELS) model.

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## 1. Introduction

Just-in-time (JIT) system is marked by frequent shipments from the supplier to the buyer in small batch sizes (Dong, Carter, & Dresner, 2001). The success of JIT in manufacturing firms like Toyota and Honda has resulted in many firms trying to move towards JIT. While it leads to lower inventories at the buyer's premises, implying lower inventory carrying costs and higher profitability for the buyer (Brox & Fader, 2002), it often increases the supplier's set up costs and the cost of transportation (Fazel, 1997). Due to this increase in cost to implement JIT, a supplier may be reluctant to participate unless he is guaranteed some form of compensation. Sometimes these increased costs are transferred to the buyer by changing the wholesale price (Fazel, 1997). According to Newman (1993) suppliers in Japan charge their JIT customers a premium of 5 percent in the wholesale price to switch to JIT mode. In practice, such compensations are decided in an ad hoc fashion and currently there is no systematic procedure to determine the same. We define such compensations as surcharge pricing. In this paper, we develop the economics behind this surcharge and establish how it can be used as a supply chain coordination mechanism in the JIT scenario. We also examine the equivalence of surcharge and other coordination mechanisms such as quantity discount and Joint Economic Lot Size Models (JELS) from a purely system-wide cost perspective.

Paying surcharge is a part of our everyday life. However, as buyers and suppliers we may not consciously acknowledge the process. For example, as consumers, we often pay a surcharge over the originally quoted wholesale price, while purchasing items in smaller quantities. This situation could arise because of budget and space constraints, and high holding costs. Similar situations arise in organizations as well, where buyers may be ready to pay a surcharge to offset the high set up costs of the supplier (Fazel, 1997; Voigt & Inderfurth, 2011). Suppliers prefer to operate at higher batch sizes because of high cost of set up (Esmaeili, Aryanezhad, & Zeephongsekul, 2009), or high transportation cost (Kelle, Al-khateeb, & Miller, 2003). Compensation from the buyer, in such cases, is supposed to offset the high set up or transportation cost incurred by the supplier if the supplier has to deliver more frequently. In real life, such compensation arises not only in the context of changing the batch sizes but also for other issues like bringing down the lead-time. In this paper, we are proposing this compensation as a surcharge on the wholesale price, rather than an ad hoc amount that is typically negotiated in practice.

While a lot of work has been done in the area of quantity discount (see Banerjee, 1986a; Monahan, 1984), literature has been relatively silent on surcharge pricing. The papers that employ the concept of surcharge pricing include Banerjee (1986b), Miller and Kelle, (1998), Lau, Lau, and Wang (2007) and Chakraborty and Chatterjee (2012). The objective of this paper is to establish surcharge pricing as a coordination mechanism and then to show

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its equivalence with other coordination models. We examine the feasibility of coordination that will maximize the buyer's resulting economic gains without altering the costs of the supplier. The model has also been extended to incorporate the case of information asymmetry.

The remainder of the paper is organized as follows: A brief review of relevant literature is presented in the next section. Problem statement is presented in Section 3, followed by Model development of the problem in Section 4. Section 5 extends the current model by relaxing the lot-for-lot and infinite production rate assumptions. Section 6 shows the equivalence of surcharge pricing with other coordination models like JELS and quantity discount. Section 7 presents the information asymmetry case for supplier's set up cost. Sensitivity Analysis results are covered in Section 8 and the concluding remarks are presented in Section 9.

## 2. Literature review

In a typical buyer-supplier or retailer-manufacturer scenario, a decision made in isolation, without focusing on the implications on the other partner and the entire supply chain, leads to supply chain inefficiencies (Cachon, 2003). Supply chain coordination assumes importance in this context (Cachon, 2003; Dudek, 2004). One of the most common ways to achieve supply chain coordination is by employing JELS models as developed by Goyal (1977, 88) and Banerjee (1986b). In JELS model, there is a central decision maker (can be either of the partners) who tries to achieve overall optimality in terms of costs for the entire supply chain. Such a scenario is idealistic and may not be realizable in practice (Vishwanathan and Piplani, 2001). Detailed reviews of such models can be found in (Goyal & Gupta, 1989) and (Sarmah, Acharya, & Goyal, 2006). Again, it is sometimes not in the best interests of the supply chain partners to be a part of any cooperative policy owing to differential benefits associated with central planning (Lu, 1995; Sucky, 2005, 2006; Darwish & Odah, 2010). Thus, one of the partners will always be reluctant to go for the JELS models without an assurance of some form of compensation (Goyal & Gupta, 1989). Hence, the need is felt for some decentralized mechanisms of coordination which can produce gains for both partners. Quantity discount mechanism falls under the category of decentralized mechanisms of coordinating the supply chain.

In a scenario where the buyer is stronger relative to the supplier, the buyer always prefers to order according to his own Economic Ordering Quantity (EOQ), which is also his optimal strategy. Discounting models by Monahan (1984), Lal and Staelin (1984), Banerjee (1986a), and Lee and Rosenblatt (1986) aim to find the optimal level of quantity discount to be offered to the buyer, asking him to revise his order size such that the supplier's total cost is minimized without making the buyer worse off. Any contract involving quantity discount which makes the buyer worse off is rejected, as the buyer has the power to reject the proposal. Detailed reviews of such work can be found in (Weng, 1995; Benton & Park, 1996; Munson & Rosenblatt, 1998; Sarmah et al., 2006) and (Zhang & Chen, 2013).

Unlike the quantity discount models, the current paper examines the case of a strong supplier, who due to his high set up cost, has the optimal strategy to set up the production process as less frequently as possible. Once the production is over, he would like the batch to be shipped to the buyer and hence will not keep any inventory. In such a case, the buyer has to carry the entire inventory. To reduce his inventory carrying costs, the buyer offers the supplier a proposal of increasing the wholesale price, to entice the supplier to reduce the batch sizes. The studies that recognize the possibility of such surcharge pricing process in the presence of a strong supplier include Banerjee (1986b) and Miller and Kelle (1998). However, these articles do not specifically define the term surcharge. Application of surcharge pricing can be found in (Lau et al., 2007 and (Chakraborty & Chatterjee, 2012). It may be noted that unlike our study, the focus of Lau et al. (2007) is on price dependent scenario to address the case of a Retailer-Stackelberg game. In our work we put greater emphasis on the working of surcharge pricing as a mechanism of achieving supply chain coordination and show its equivalence in overall supply chain cost with the other existing coordination mechanisms like JELS and quantity discount. As the equivalence of quantity discount and Vendor Managed Inventory (VMI) has been discussed in Chakraborty et al. (2015), we will not derive the results but will discuss the same in the numerical results section.

## 3. Problem statement

Consider a typical buyer-supplier scenario with the buyer facing a uniform deterministic demand. The relevant cost for the supplier is the set up cost per production run, while for the buyer it is the unit cost of purchase, a unit holding cost and ordering cost per order placed. Assume that backlogging is not allowed.

With the traditional assumption on the production cost structure of the supplier, it is understood that there is a fixed cost of set up every time production is undertaken and there is a variable cost per unit produced. Owing to high set up costs, the supplier undertakes a fixed number of set ups and delivers the goods as soon as they are produced. Thus, the production rate of the supplier is assumed to be infinite, and he follows a lot for lot policy in meeting the demand i.e. he will not keep any inventory.

For the buyer, on the other hand, the tradeoff between ordering costs and inventory costs may be such that it is optimal to have more shipments from the supplier. The lead time for procurement is assumed to be zero. The proposed model assumes that the buyer can ask the supplier to increase the number of set ups by offering him an increase in the wholesale price. Thus, he faces a tradeoff between an increase in the wholesale price and ordering costs, on the one hand, and a reduction in his inventory holding costs on the other hand. The supplier is benefitted from the additional per unit wholesale price from the buyer but has to incur extra costs through the increased number of set ups. It is to be noted that the proposal from the buyer's side will only be accepted by the supplier if he is not worse off as compared to his initial optimal plan. If the proposal from the buyer puts the supplier in a worse off position, he will reject such a proposal.

The decision problem is to choose the surcharge to be offered and the lot size to be proposed to the supplier by the buyer, to maximize the decrease in his overall costs, without making the supplier worse off from his optimal case.

The notations used in the model are presented below.

*Data*

| | |
|---|---|
| $w$ | Wholesale price charged initially by the supplier, i.e. unit cost of purchase for the buyer. |
| $H$ | The holding cost in dollar per unit per unit time |
| $C_O$ | The ordering cost of the buyer |
| $C_S$ | Set up cost of the supplier, includes the transportation costs. |
| $D$ | Known demand |

*Decision Variables*

| | |
|---|---|
| $Q$ | The new economic ordering quantity |
| $x$ | The amount of unit surcharge being paid |

## 4. Model development

There is no loss of generality in assuming that initially, the supplier sets up only once in the planning horizon and ships the entire quantity demanded in a single batch. For longer planning horizon,

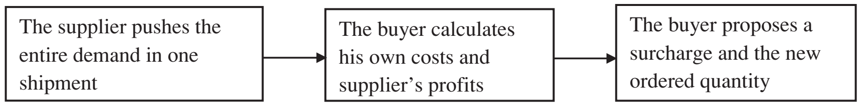


**Fig. 1.** The surcharge pricing process.

this assumption doesn't hold good and the supplier would like to deliver in a fixed number of shipments or by setting up a fixed number of times. In that case, we will concentrate on the time period between two consecutive set ups, which is identical to the case of a single set up. It is then followed by the buyer coming up with a proposal of increase in wholesale price. The surcharge pricing process is described below by means of a flowchart Fig. 1.

If the supplier ships the entire quantity demanded in a single batch, the resulting costs for the buyer and the supplier can be written as:

$$\text{Buyer's total cost} = B_1 = wD + \frac{HD}{2} + C_O \tag{1}$$

($\because$ Annual demand is deterministic)

$$\text{Supplier's total cost } = S_1 = C_S \tag{2}$$

When the buyer proposes a surcharge $x$ on the initially charged wholesale price $w$, asking the supplier to revise the shipped quantity to $Q$ units per batch, the resulting buyer's and supplier's cost can be rewritten as:

$$\text{Buyer's revised cost } = B_2 = (w+x)D + \frac{HQ}{2} + \frac{D}{Q}C_O \tag{3}$$

($\because$ Lead time is zero and no backorder allowed)

$$\text{Supplier's revised cost} = S_2 = \frac{D}{Q}C_S - xD \tag{4}$$

(Under lot-for-lot and infinite procurement rate assumption)

The change in buyer's and supplier's costs can be expressed as follows:

$$\begin{aligned}\Delta B = B_1 - B_2 &= wD + \frac{HD}{2} + C_O - (w+x)D - \frac{HQ}{2} - \frac{D}{Q}C_O \\ &= \frac{H}{2}(D-Q) - C_O\left(\frac{D}{Q} - 1\right) - xD \end{aligned}\tag{5}$$

$$\Delta S = S_1 - S_2 = C_S - \frac{D}{Q}C_S + xD = xD - C_S\left(\frac{D}{Q} - 1\right) \tag{6}$$

From (5) and (6), we must have $\Delta B \geq 0$ and $\Delta S \geq 0$ which gives the feasible range for $x$ as

$$\left[\left(\frac{D}{Q} - 1\right)\frac{C_S}{D},\ \left\{\frac{H}{2}(D-Q) - C_O\left(\frac{D}{Q} - 1\right)\right\}\Big/ D\right]$$

The buyer's decision problem is to choose the per unit surcharge $x$ to be offered and the new lot size $Q$ to be proposed to the supplier so that his additional gains i.e. decrease in overall costs from the initial case, is maximized. The supplier on the other hand, being in a stronger position, will not accept any proposal that makes him worse off from his optimal case, i.e. only if $\Delta S \geq 0$, the proposal will be acceptable to the supplier. The decision problem of the buyer can be expressed as:

$$\left.\begin{aligned} &\max_{x,\,Q} \quad \Delta B = \frac{H}{2}(D-Q) - C_O\left(\frac{D}{Q} - 1\right) - xD \\ &\text{S.T} \quad \Delta S = xD - C_S\left(\frac{D}{Q} - 1\right) \geq 0 \end{aligned}\right\} \tag{7}$$

The above optimization problem yields

$$Q^* = \sqrt{\frac{2(C_O + C_S)D}{H}} \quad \text{and} \quad x^* = C_S\left(\frac{1}{Q} - \frac{1}{D}\right) \tag{8}$$

The details have been worked out in Appendix A: Solution to the surcharge pricing model.

$Q^*$ and $x^*$ give respectively the optimal values of lot size and the surcharge that will be offered. If $Q_{\text{Buyer}} = \sqrt{\frac{2C_O D}{H}}$ denotes the lot size under buyer dominated scenario and $Q_{\text{Supplier}} = D$ denotes the lot size under supplier dominated scenario, then it is obvious that $Q_{\text{Supplier}} > Q^* > Q_{\text{Buyer}}$.

The above range of values of $x$ remains valid as long as $\frac{H}{2}(D-Q) - C_O(\frac{D}{Q} - 1) \geq (\frac{D}{Q} - 1)C_S$

$$\text{Alternatively, } Q \geq \frac{2(C_O + C_S)}{H}$$

From (8) we get $Q^* = \sqrt{\frac{2(C_O+C_S)D}{H}}$ i.e. the feasible range of $x$ will be valid when $D \geq \frac{2(C_O+C_S)}{H}$ or when the annual demand is sufficiently large. It may be noted that in industries with very high set up cost, there may not exist a feasible value of surcharge that could make the buyer better off from his initial position. In such a case, we need to relax the assumption of lot-for-lot delivery from the supplier and allow the supplier to carry inventory. Then the surcharge would act as a compensation for the supplier for his increased inventory carrying and set up costs. The details have been worked out in the next section.

## 5. Generalized surcharge pricing models

In this section, we relax two of the initially considered assumptions i.e. lot-for-lot and infinite production rate for the supplier.

### *5.1. Relaxation of lot for lot assumption*

We relax the lot for lot assumption implying that the supplier can now hold inventory. Buyer's and supplier's initial cost remain the same as in the earlier case. Further, buyer's revised cost also remains the same. It is the supplier's revised cost that is affected as a result of relaxation of the lot-for-lot assumption. It is shown by Lee and Rosenblatt (1986) that whenever the buyer orders $Q$ units after every equal time period, the supplier's Economic Production Quantity (EPQ) remains independent of $Q$ and is given by:

$$q = \sqrt{\frac{2DC_S}{H_S}} \tag{9}$$

where, $q$ is the EPQ of the supplier, $C_S$ is the set up cost and $H_S$ is the inventory carrying cost at the supplier's premises.

Thus, whenever the supplier is asked to deliver $Q$ units, assuming $q$ is a multiple of $Q$ ($q = n_1 Q$, $n_1$ is an integer); his total cost is given as: $S_2 = \frac{D}{q}C_S + \frac{q-Q}{2}H_S - xD$

Whenever the buyer wishes to decrease the order size by means of offering a surcharge, the supplier also decides on how much to produce each time, that will minimize his total costs. In such a case, the supplier will produce according to his EPQ given as $q = \sqrt{\frac{2DC_S}{H_S}}$. The supplier's change in cost becomes $\Delta S = S_2 - S_1 = C_S(\frac{D}{q} - 1) + \frac{q-Q}{2}H_S - xD$

Hence, the buyer's decision problem in this case, becomes

$$\left.\begin{aligned} &Max\ \Delta B = \frac{HD}{2} + C_O - xD - \frac{HQ}{2} - \frac{DC_O}{Q} \\ &S/T\ xD - C_S\left(\frac{D}{q} - 1\right) - \frac{q-Q}{2}H_S \geq 0 \end{aligned}\right\} \tag{10}$$

The solution to the above optimization problem yields $Q^*=\sqrt{\frac{2DC_O}{H-H_S}}$ (under the assumption $H > H_S$) and $x^*=\frac{C_S}{D}\left(\frac{D}{q}-1\right)+\frac{q-Q^*}{2D}H_S$

The above expressions give the revised order quantity and the surcharge that the buyer would like to offer. The details have been worked out in Appendix B.A: Solution to the generalized surcharge pricing model.

### 5.2. Relaxation of infinite production rate

Whenever the supplier produces at a finite production rate, it takes some time for the inventory to build up before it could be shipped to the buyer. Further, like the previously studied models, shipping the entire lot to the buyer in a single shipment might not be optimal for the supplier, as the supplier will have to incur higher inventory carrying costs. If $q_{prod}$ denotes the production lot size for the supplier which is also the shipment lot size, then $q_{prod}=\sqrt{\frac{2DC_S}{H_S}}$ where $H_S$ is the per unit annual inventory carrying cost for the supplier [For details, see Appendix B.B: Derivation of production lot-size of the supplier]. At this shipped quantity, buyer's and supplier's initial costs are given as:

$$B_1 = \frac{D}{q_{prod}}C_O + \frac{q_{prod}}{2}H \quad \text{and} \quad S_1 = \frac{D}{q_{prod}}C_S + \frac{q_{prod}}{2}H_S$$

Whenever the buyer proposes a surcharge $x$ to the supplier in return to reduce the shipment batch size to Q, then with similar arguments as above, whenever the buyer orders Q units every time, and the supplier produces at a constant production rate, supplier's cost gets revised to $S_2=\frac{D}{q}C_S+\frac{q-Q}{2}H_S+\frac{q}{2}H_S-xD$ where $q$ is the new EPQ of the supplier given as $q=\sqrt{\frac{DC_S}{H_S}}$. The supplier's change in cost is thus given as

$$\Delta S = S_2 - S_1 = \left(\frac{1}{q} - \frac{1}{q_{prod}}\right)DC_S + \frac{2q - Q - q_{prod}}{2}H_S - xD$$

$$B_2 = \frac{D}{Q}C_O + \frac{Q}{2}H + xD$$

The buyer's problem thus becomes

$$Max\Delta_1 = \frac{D}{q_{prod}}C_O + \frac{q_{prod}}{2}H - \frac{D}{Q}C_O - \frac{Q}{2}H - xD$$

$$S/T \left(\frac{1}{q} - \frac{1}{q_{prod}}\right)DC_S + \frac{2q - Q - q_{prod}}{2}H_S - xD \leq 0 \tag{11}$$

Solving the resulting buyer's problem, we get $Q^* = \sqrt{\frac{2DC_O}{H-H_S}}$ and

$$x^* = \left(\frac{1}{q} - \frac{1}{q_{prod}}\right)C_S + \frac{2q - Q^* - q_{prod}}{2}H_S$$

The above expressions give the revised order quantity and the surcharge respectively that the buyer would like to order on offering the surcharge pricing contract.

## 6. Equivalence with other coordination mechanisms

In this section we establish the equivalence of surcharge pricing mechanism with other supply chain coordination mechanisms like Joint Economic Lot Size (JELS) model and Quantity discounting in terms of the total supply chain cost and the optimal lot size.

### 6.1. Equivalence with JELS model

In the Joint Economic Lot Size (JELS) models, there is a central decision maker who wishes to minimize the overall costs of the entire supply chain (Banerjee, 1986; Goyal, 1988). Under the lot-for-lot assumption, the total cost function is given as: $TC(Q) = \frac{D}{Q}C_S + \frac{D}{Q}C_O + \frac{Q}{2}H$

First order condition yields $\frac{dTC(Q)}{dQ} = -\frac{D}{Q^2}C_S - \frac{D}{Q^2}C_O + \frac{1}{2}H = 0$ giving

$Q^*_{JELS} = \sqrt{\frac{2D(C_O+C_S)}{H}}$ while the second order condition $\frac{d^2TC(Q)}{dQ^2} = \frac{2D}{Q^3}C_S + \frac{2D}{Q^3}C_O > 0$

This shows that the optimal order quantity under surcharge pricing equals that of JELS model.

Similarly, when lot-for-lot assumption is relaxed i.e. when the supplier also holds inventory, then $TC(Q_1, Q_2) = \frac{D}{Q_2}C_S + \frac{Q_2-Q_1}{2}H_S + \frac{D}{Q_1}C_O + \frac{Q_1}{2}H$

where $Q_1$ and $Q_2$ are respectively the EOQ and EPQ of the buyer and the supplier and $H_S$ is the inventory holding cost of the supplier

The first order conditions for optimization yield $\frac{\partial TC}{\partial Q_1} = -\frac{H_S}{2} - \frac{D}{Q_1^2}C_O + \frac{H}{2} = 0 \Rightarrow Q_1^* = \sqrt{\frac{2DC_O}{H-H_S}}$

$$\frac{\partial TC}{\partial Q_2} = -\frac{D}{Q_2^2}C_S + \frac{H_S}{2} = 0 \Rightarrow Q_2^* = \sqrt{\frac{2DC_S}{H_S}}$$

Second order conditions give $\frac{\partial^2 TC}{\partial Q_1^2} = \frac{2D}{Q_1^3}C_O > 0$, $\frac{\partial^2 TC}{\partial Q_2^2} = \frac{2D}{Q_2^3}C_O > 0$ and $\frac{\partial^2 TC}{\partial Q_1 \partial Q_2} = 0$ which shows that $Q_1{}^*$ and $Q_2{}^*$ are the points of minima for the objective function.

This again shows that the optimal order quantity in case of surcharge pricing equals that of JELS model with lot-for-lot relaxed, thereby yielding the same total supply chain costs.

### 6.2. Equivalence with quantity discount model

In the following paragraphs, we examine the equivalence of surcharge pricing model under lot-for-lot assumption with that of quantity discount model as proposed by Monahan (1984). In Monahan's quantity discount model, a buyer orders according to his EOQ, given by $Q_1 = \sqrt{\frac{2DC_O}{H}}$

Buyer's total costs are given as: $B_1 = wD + \frac{D}{Q_1}C_O + \frac{Q_1}{2}H$

Supplier's total cost in following a lot-for-lot policy is given as: $S_1 = \frac{D}{Q_1}C_S$

Whenever the supplier offers a discount $x_{dis}$ to the buyer and in return asks the buyer to revise the order to a new lot size $Q_{dis}$, supplier's and buyer's revised costs are given as:

$$S_2 = \frac{D}{Q_{dis}}C_S - x_{dis}D \quad \text{and} \quad B_2 = (w - x_{dis})D + \frac{D}{Q_{dis}}C_O + \frac{Q_{dis}}{2}H$$

The supplier wishes to maximize his change in costs subject to the condition of not making the buyer worse off. Thus, the supplier will solve the following optimization problem:

$$\text{Max} \quad \Delta S = S_2 - S_1 = -x_{dis}D + DC_S\left(\frac{1}{q} - \frac{1}{Q_{dis}}\right)$$

$$\text{S.T.} \quad \Delta B = B_2 - B_1 = -x_{dis}D - DC_O\left(\frac{1}{Q_1} - \frac{1}{Q_{dis}}\right) + \frac{H}{2}(Q_{dis} - Q_1) \leq 0$$

Solving it we get, $Q^*_{dis} = \sqrt{\frac{2(C_O+C_S)D}{H}}$ which is same as in the case for surcharge pricing under lot-for-lot policy. The corresponding value of discount offered is $x^*_{dis} = \frac{C_O}{Q_{dis}}\left[\sqrt{\frac{C_S+C_O}{C_O}} + \sqrt{\frac{C_O}{C_O+C_S}} - 2\right]$

This establishes that the revised order quantities in both surcharge pricing and quantity discount are the same when lot-for-lot policy is being followed. Similarly, when lot-for-lot assumption is relaxed, the optimal quantity discount and the revised order size are given as

$$x^*_{dis} = \frac{C_O}{Q}\left(\sqrt{\frac{z_1}{z_2}} + \sqrt{\frac{z_2}{z_1}} - 2\right) \text{ and } Q^*_{dis} = \sqrt{\frac{2DC_O}{H - H_S}}$$

$$z_1 = 2D\left(\frac{C_S}{Q_1} - \frac{C_O}{Q}\right) \text{ and } z_2 = (Q_1 - Q)H_S + QH$$

Hence, irrespective of the stronger partner, both the coordination processes lead to the same lot sizing decisions and overall cost savings, however, the benefits accrued to the different partners depend upon their relative strength. For the purpose of illustration, an example is presented in Appendix C: Example under full information. Even though from a purely cost perspective, we have shown these models to be equivalent, but from a supply chain structure perspective, the applicability of these models depend upon the relative strengths of the partners. It may be noted that these models are not interchangeable in nature.

## 7. Case of asymmetric cost information

While devising the surcharge pricing contract, it has been assumed that the buyer has full information about the cost structure of the supplier. As such information is critical for the supplier and could affect his competitive advantage; the supplier will not have an incentive to reveal it (Mukhopadhyay, Zhu, & Yue, 2008, Albrecht, 2010). Further, even if the supplier shares the information, he may convey an exaggerated value of his set up cost, as he stands to gain by the same. Thus the set up cost information is neither observable nor verifiable. This information asymmetry introduces the problem of *moral hazard* for the buyer (Corbett, DeCroix, & Ha, 2005) where the buyer acts as the principal while the supplier acts as the agent. Hence, under such a situation, the buyer will always be better off in using his own estimate rather than using the reported values from the supplier.

An analysis of the information asymmetry problem arising in the context of quantity discount has been studied previously by Corbett and De Groote (2000). In their problem, the supplier offers the buyer a menu of contracts, each having a price-quantity pair which is parameterized on the unknown value of buyer's inventory carrying cost. The buyer is asked to choose one from the contracts offered. Depending upon his *incentive compatibility*, the buyer chooses a price-quantity pair that minimizes his overall costs. The choice of the buyer actually reveals his cost structure. Thus, the said contract acts a screening mechanism. Similar studies that deal with information asymmetry problem through this screening mechanism using a *menu of contracts* include Ha (2000), Corbett and Tang (2003), Corbett, Zhou, and Tang (2004), Voigt and Inderfurth (2011), Voigt (2014). Sucky (2006) used a mathematical programming approach to solve the quantity discount problem under information asymmetry where the supplier offering the contract is unaware of both buyer's ordering and inventory carrying cost. In his work, the analysis involved only two unknown buyer types where they differ in ordering as well as inventory carrying costs. The solution approach not only maximizes the expected gains of the party offering the contract, but it also acts as a screening mechanism in the sense that a buyer will have no incentive to pretend to be another type of buyer while accepting the contract. Applying the approach developed by Sucky (2006) to our case we can formulate the problem as follows:

Supplier's cost after agreeing to an order quantity of $Q$ from the buyer is given as: $S(Q) = \frac{D}{Q}C_S$, whereas buyer's total cost is given as $B(Q) = \frac{D}{Q}C_O + \frac{QH}{2}$

Buyer's decision problem is thus to choose order quantity $Q$ and the surcharge $x$ to be offered to the supplier. The type of supplier is unknown to the buyer and suppliers' differ only with respect to the set up cost. Let there be two types of suppliers with their respective set up costs being $C_{S1}$ and $C_{S2}$ and each having a probability of occurrence of $\lambda_1$and $\lambda_2$ with $\lambda_1 + \lambda_2 = 1$. Further, let $x_1$ and $x_2$ are the surcharge prices being paid to the supplier of type I and type II respectively. The buyer wants to maximize the expected additional gain. Further, while devising the contract which has a price-quantity pair including the new order quantity and the surcharge offered, he has to screen the suppliers so that a supplier being of type I doesn't pretend to be of type II. The buyer in turn will provide a menu of contracts to the supplier to choose from.

Buyer's problem can be written as:

$$Max\ \frac{HD}{2} + C_O - [\lambda_1\{B(Q_1) + x_1\} + \lambda_2\{B(Q_2) + x_2\}]$$

which is equivalent to

$$Min\ \lambda_1\left\{\frac{D}{Q_1}C_O + \frac{Q_1H}{2} + x_1\right\} + \lambda_2\left\{\frac{D}{Q_2}C_O + \frac{Q_2H}{2} + x_2\right\} \tag{12}$$

$$S_1(Q_1) - x_1 \le C_{S1} \Rightarrow \frac{D}{Q_1}C_{S1} - x_1 \le C_{S1} \tag{13}$$

$$S_2(Q_2) - x_2 \le C_{S2} \Rightarrow \frac{D}{Q_2}C_{S2} - x_2 \le C_{S2} \tag{14}$$

$$S_1(Q_1) - x_1 \le S_1(Q_2) - x_2 \Rightarrow \frac{D}{Q_1}C_{S1} - x_1 \le \frac{D}{Q_2}C_{S1} - x_2 \tag{15}$$

$$S_2(Q_2) - x_2 \le S_2(Q_1) - x_1 \Rightarrow \frac{D}{Q_2}C_{S2} - x_2 \le \frac{D}{Q_1}C_{S2} - x_1 \tag{16}$$

$Q_1, Q_2 \ge 0$ and $x_1, x_2 \ge 0$

Constraints (13) and (14) are the incentive compatibility constraints for the suppliers of type I and type II respectively, i.e. the supplier will not accept the contract if it makes them worse off. Constraints (15) and (16) are used for screening purpose i.e. the supplier of type I will not accept a contract by pretending to be type II and vice-versa. The solution to the above problem yields the following three different outcomes.

$$\text{Contract I:} Q_1 = \sqrt{\frac{2D(C_O + C_{S1})}{H}} \quad \text{and}$$

$$Q_2 = \sqrt{\frac{2D(\lambda_2 C_O + C_{S2} - \lambda_1 C_{S2})}{\lambda_2 H}} \quad \text{with}$$

$$x_i = \left(\frac{D}{Q_i} - 1\right)C_{Si},\ i = 1, 2$$

$$\text{Contract II:} Q_1 = \sqrt{\frac{2D(\lambda_1 C_O + C_{S1} - \lambda_1 C_{S1})}{\lambda_1 H}} \quad \text{and}$$

$$Q_2 = \sqrt{\frac{2D(C_O + C_{S2})}{H}} \quad \text{with}$$

$$x_i = \left(\frac{D}{Q_i} - 1\right)C_{Si},\ i = 1, 2$$

$$\text{Contract III:} Q_1 = \sqrt{\frac{2D(C_O + C_{S1})}{H}} \quad \text{and}$$

$$Q_2 = \sqrt{\frac{2D(C_O + C_{S2})}{H}} \quad \text{with}$$
$$x_i = \left(\frac{D}{Q_i} - 1\right)C_{Si}, i = 1, 2$$

The details have been worked out in the Appendix D: Solution for asymmetric cost information.

The problem in applying this approach is the computational complexity. Even with two types of suppliers, the requirement is to solve sixteen different optimization sub-problems. For $k$ possible states, the problem will have $k$ different incentive compatibility constraints and $k(k-1)$ different screening constraints which leads to $k^2$ constraints in total. Further, for each constraint there will be one Lagrangian multiplier $\mu_i$. The total number of such combinations for zero or non-zero values of $\mu_i$ will be $2^{k^2}$. Thus, the solution process will be cumbersome. To tackle the issue of the presence of suppliers of multiple types or even the case of suppliers whose costs follow some continuous distributions, we present the following approach. It may be noted that our approach can provide the solution but it cannot handle the issue of screening.

In our problem, the supplier's incentive compatibility constraint is to set up only once in the planning horizon and not to keep any inventory. This choice is independent of supplier's set up cost which is unknown to the buyer. This independence restricts the usage of 'menu of contracts' approach to offer a surcharge as the 'price-quantity' pair cannot be parameterized on the set up cost. It is also to be noted that such a scenario is not restricted to the lot-for-lot case.

Hence in our analysis, we have considered a two-stage model (unlike Corbett and de Groote (2000), who considered a single stage problem). We confine ourselves to a single contract offered by the buyer, which the supplier can choose to accept or reject, based on his cost structure. Unlike the study by Corbett and de Groote (2000) for the information asymmetry case, the revelation principle is not required in our case. The design of the bargaining mechanism is such that, it is always in the best interests of the supplier to accept the contract offered once his *individual rationality* constraint is satisfied. Any contract that satisfies *individual rationality* constraint, if rejected, would mean that the solution is the initial *threat point*, where the supplier intends to set up once in the time horizon. The supplier does not gain anything extra and loses an opportunity to gain through the offer made by the buyer. Based on the above, we have developed models to take care of information asymmetry. It is obvious that the presence of information asymmetry affects the efficiency of the supply chain and the lot sizes are much higher compared to the case of full information.

As mentioned earlier, in the absence of full information about the cost structure of the partner, one can have an idea of the prior probability distribution of the same (Mukhopadhyay et al., 2008). In the following subsections, we discuss the model with respect to discrete and a continuous prior probability distribution of the supplier's set up cost.

### 7.1. Analysis with discrete probability distribution

Under the discrete case let $C_S$ follows the distribution P $\{C_S = C_{Si}\} = \lambda_i$, and $C_{Si} \le C_{Sj}$ if $i < j$ with $0 \le \lambda_i \le 1$, $\lambda_1 + \lambda_2 + \ldots + \lambda_n = 1$.

Since there is a one-to-one correspondence between the supplier's set up cost and the surcharge offered, the surcharge will be higher for a higher value of $C_S$. Further, higher the surcharge offered, lower will be the additional gains for the buyer. On the other hand, offering a higher surcharge would increase the probability of acceptance from the supplier, as it has a higher chance of compensating him. The decision maker (i.e. the buyer) is forced with the choice of a surcharge offered corresponding to the particular value that $C_S$ may take. We assume that the buyer is risk-neutral and his problem is to find the value of surcharge that maximizes his expected additional gain. As mentioned in (8), the additional gains for the buyer for any choice $y$ of supplier's set up cost $C_S$ is given by substituting $C_S = y$ and using the optimal value of $x^*$ in (7)

$$\Delta B(y) = \frac{HD}{2} - \frac{HQ}{2} - \left(\frac{D}{Q} - 1\right)(C_O + y) \tag{17}$$

which holds only if $x \ge C_S(\frac{1}{Q} - \frac{1}{D})$ else $\Delta B(y) = 0$

So for the various available states of the set up cost, the buyer's decision problem and the expected gains may be depicted in a tabular form as in Table 1.

**Table 1**
Decision structure of the buyer.

| Assumed state of $C_S$ | True state of $C_S$ | | | | Expected gains |
|---|---|---|---|---|---|
| | $C_{S1}$ | $C_{S2}$ | … | $C_{Sn}$ | |
| $\boldsymbol{C_{S1}}$ | $\Delta_1(C_{S1})$ | 0 | … | 0 | $\lambda_1 \Delta_1(C_{S1})$ |
| $\boldsymbol{C_{S2}}$ | $\Delta_1(C_{S2})$ | $\Delta_1(C_{S2})$ | … | 0 | $(\lambda_1+\lambda_2)\, \Delta_1(C_{S2})$ |
| … | … | … | … | … | … |
| $\boldsymbol{C_{Sn}}$ | $\Delta_1(C_{Sn})$ | $\Delta_1(C_{Sn})$ | … | $\Delta_1(C_{Sn})$ | $\Delta_1(C_{Sn})$ |

Each value in the $n \times n$ matrix denotes the gain that the buyer can get for various assumptions on the states of $C_S$ and its true value. The risk neutral buyer's decision problem will be to choose that value of $C_S$ that maximizes his expected gain. Further, any choice of surcharge offered that does not correspond to any of the possible values of $C_S$ is sub-optimal.

### 7.2. Analysis with continuous probability distribution

In this case, the buyer holds a prior continuous probability distribution of supplier's $C_S$ with density function *f(x),* distribution function *F(x)* and support $[\underline{C_S}, \overline{C_S}]$ with $\overline{C_S}$ and $\underline{C_S}$ being the upper and lower bounds of $C_S$. An assumption regarding the density and distribution function is needed here (in accordance with Corbett & de Groote, 2000). We assume that the distribution of $C_S$ follows a decreasing hazard rate: i.e. $D_{C_S}[f(C_S)/F(C_S)] \le 0$ where $D_{C_S}$ denotes the first order derivative w.r.t. $C_S$. Distributions like uniform, normal, logistic, chi-square etc. follow this property. For further analysis on decreasing hazard rate one may refer to Shaked and Shanthikumar (1994).

In this case, the buyer tries to find his expected gain corresponding to every value of $C_S$ lying in the interval $[\underline{C_S}, \overline{C_S}]$ and then select the value of surcharge depending on the value of $C_S$ chosen that gives him the maximum expected gain. The decreasing hazard rate property of the distribution ensures the existence of unique point of optimum (Lariviere, 2006).

The buyer chooses a value $y \in [\underline{C_S}, \overline{C_S}]$ and calculates the expected gain corresponding to that chosen value. Here $y$ is the decision variable that determines the surcharge to be offered. Hence the gain for the buyer is:

$$\Delta B(y) = \begin{cases} G(y) & \text{when } X \le y \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

where $X$ is a random variable denoting the assumed value of

$$CS \text{ and } G(y) = \frac{HD}{2} - \sqrt{2HD(C_O + y)} + C_O + y$$

$$(\text{Substituting Q} = \sqrt{\frac{2(C_O + C_S)D}{H}} \text{ in (17)}) \tag{19}$$

The buyer thus, solves the following problem of maximizing the expected additional gains given by $E(\Delta B(y)) = \int_{\underline{C_S}}^{y} G(y)f(x)dx + 0(\int_{y}^{\overline{C_S}} f(x)dx) = G(y)F(y)$ where *F(x)* and *f(x)* are the distribution and density functions of $C_S$ respectively.

First order condition for expected profit maximization gives

$$\frac{d}{dy}E(\Delta B(y)) = G(y)F'(y) + G'(y)F(y) = 0 \tag{20}$$

Substituting $G'(y) = 1 - \frac{HD}{\sqrt{2HD(C_O+y)}}$ in the above equation we get

$$\left\{\frac{HD}{2} - \sqrt{2HD(C_O+y)} + C_O + y\right\}F'(y) + \left\{1 - \frac{HD}{\sqrt{2HD(C_O+y)}}\right\}F(y) = 0 \tag{21}$$

Uniqueness of the optimum value is ensured by the assumption of decreasing hazard rate of the distribution (Lariviere, 2006).

In particular, if $C_S$ follows uniform distribution between [*a, b*] (in accordance with Corbett et al. (2004)) then we have $F(y) = \frac{y-a}{b-a}$ and $f(y) = \frac{1}{b-a}$ with:

$$G'(y) = 1 - \sqrt{\frac{2HD}{C_O + y}} \tag{22}$$

Substituting these values in (21) we get

$$\left(\frac{HD}{2} - \sqrt{2HD(C_O+y)} + C_O + y\right) + \left(1 - \sqrt{\frac{2HD}{C_O+y}}\right)(y-a) = 0 \tag{23}$$

Solving for $y$ will give the optimum value at which the expected gain of the buyer becomes the highest. Since the above equation in $y$ is cumbersome in nature, we will limit ourselves to numerical analysis for getting the insights. Further, it is to be noted that in the case when $C_S$ is uniformly distributed in [*a, b*], the optimal value of surcharge offered does not depend upon the value of upper bound $b$. If the value of $y$ exceeds that of $b$, then in that case $y$ should be replaced by $b$. The respective values of $x$ and $Q$ can be found by substituting the value of $y$ in (8) in place of $C_S$.

The above section shows how the buyer as a decision maker would make decisions in the presence of information asymmetry. It is also evident that the lot sizes will deviate from the full information case and this creates inefficiency of the entire supply chain. For the purpose of illustration, an example is presented in Appendix E: Example under Information Asymmetry.

## 8. Sensitivity analysis

The results on the analysis of the effect of Supplier's set up cost, Buyer's ordering cost, and Buyer's inventory carrying cost on surcharge pricing process under full cost information are first presented. This is followed by an analysis of the effect of prior probability distribution on surcharge pricing scheme under information asymmetry.

### 8.1. Analysis under full cost information

In this subsection, we analyze the effect of Supplier's set up cost, Buyer's ordering cost, and Buyer's inventory carrying cost, on surcharge pricing process.

(a) Effect of Supplier's set up cost, $C_S$ on surcharge pricing process.

We fix the parameters like Annual Demand, $D = 10000$, Buyer's ordering cost, $C_O = 100$ dollar per order, Buyer's annual inventory carrying cost, $H = 2$ dollar per unit, and perform a sensitivity analysis by varying supplier's set up cost, $C_S$. The following table presents the variation of optimal order size (Q), buyer's and supplier's overall costs (TCB and TCS respectively) and optimal level of surcharge offered Table 2.

**Table 2**
Sensitivity of buyer's costs with respect to supplier's set up cost $C_S$.

| *Cs* | 1000 | 1250 | 1500 | 1750 |
|---|---|---|---|---|
| Q | 3316.62 | 3674.23 | 4000.00 | 4301.16 |
| TCB | 5633.25 | 6098.47 | 6500.00 | 6852.33 |
| TCS | 1000.00 | 1250.00 | 1500.00 | 1750.00 |
| TSCC | 6633.25 | 7348.47 | 8000.00 | 8602.33 |
| Surcharge | 0.202 | 0.215 | 0.225 | 0.232 |

It is clear from the above table that if the supplier's set up cost increases, the buyer's total cost also increases. With the increase in supplier's set up cost, buyer will have to pay more to the supplier in order to compensate him for carrying out the extra number of production set ups. The compensation is through the surcharge paid by the buyer and the same is also reflected in the increase in per unit surcharge paid by the buyer. Further, higher set up cost also affect the batch sizes, as with the increase in set up cost, the buyer would want the supplier to deliver less frequently as more production set ups will be accompanied by greater compensation and eventually buyer's overall cost will also be affected.

(b) Effect of Buyer's ordering cost, $C_O$ on surcharge pricing process

We fix the parameters like Annual Demand, $D = 10000$, Supplier's set up cost, $C_S = 1000$ dollar per production set up, Buyer's annual inventory carrying cost, $H = 2$ dollar per unit, and perform a sensitivity analysis by varying buyer's ordering cost, $C_O$. The following table presents the variation of optimal order size, buyer's and supplier's overall costs and optimal level of surcharge offered Table 3.

**Table 3**
Sensitivity of buyer's costs with respect to buyer's ordering cost $C_O$.

| *Co* | 100 | 125 | 150 | 175 |
|---|---|---|---|---|
| Q | 3316.6 | 3354.1 | 3391.2 | 3427.8 |
| TCB | 5633.3 | 5708.2 | 5782.3 | 5855.7 |
| TCS | 1000.0 | 1000.0 | 1000.0 | 1000.0 |
| TSCC | 6633.3 | 6708.2 | 6782.3 | 6855.7 |
| Surcharge | 0.20 | 0.20 | 0.19 | 0.19 |

It is clear from the above table that as buyer's ordering cost goes up, the optimal replenishment quantity also goes up. With the increase in ordering cost, a buyer is keener to order in higher batch sizes to enable him to order less frequently.

(c) Effect of Buyer's inventory carrying cost, $H$ on surcharge pricing process.

We fix the parameters like Annual Demand, $D = 10000$, Buyer's ordering cost, $C_O = 100$ dollar per order, Supplier's set up cost, $C_S = 1000$ dollar per set up and perform a sensitivity analysis by varying buyer's inventory carrying cost, $H$. The following table presents the variation of optimal order size, buyer's and supplier's overall costs and optimal level of surcharge offered Table 4.

From the above table, it can be concluded that as buyer's inventory carrying cost goes up, the optimal batch size goes down. Higher inventory holding cost will prompt the buyer to place the order more frequently with lower batch size.

### 8.2. Analysis under information asymmetry

(a) Effect of prior discrete distribution, on the optimum batch sizes and the surcharge offered.

**Table 4**
Sensitivity of buyer's costs with respect to buyer's inventory carrying cost $H$.

| $H$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Q | 3316.62 | 2708.01 | 2345.21 | 2097.62 |
| TCB | 5633.25 | 7124.04 | 8380.83 | 9488.09 |
| TCS | 1000.00 | 1000.00 | 1000.00 | 1000.00 |
| TSCC | 6633.25 | 8124.04 | 9380.83 | 10488.09 |
| Surcharge | 0.20 | 0.27 | 0.33 | 0.38 |

**Table 5**
Sensitivity of expected gains prior probability distribution.

| $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $x^*$ | Expected gains | Targeted supplier segment |
|---|---|---|---|---|---|
| 0 | 0.1 | 0.9 | 0.8569461 | 246.8254 | Higher |
| 0 | 0.2 | 0.8 | 0.8569461 | 246.8254 | Higher |
| 0 | 0.3 | 0.7 | 0.8569461 | 246.8254 | Higher |
| 0 | 0.4 | 0.6 | 1.1426968 | 264.4156 | Middle |
| 0 | 0.5 | 0.5 | 1.1426968 | 330.5195 | Middle |
| 0.1 | 0.1 | 0.8 | 0.8569461 | 246.8254 | Higher |
| 0.1 | 0.2 | 0.7 | 0.8569461 | 246.8254 | Higher |
| 0.1 | 0.3 | 0.6 | 1.1426968 | 264.4156 | Middle |
| 0.1 | 0.4 | 0.5 | 1.1426968 | 330.5195 | Middle |
| 0.1 | 0.5 | 0.4 | 1.1426968 | 396.6234 | Middle |
| 0.2 | 0.1 | 0.7 | 1.2086305 | 299.4615 | Lower |
| 0.2 | 0.2 | 0.6 | 1.2086305 | 299.4615 | Lower |
| 0.2 | 0.3 | 0.5 | 1.1426968 | 330.5195 | Middle |
| 0.2 | 0.4 | 0.4 | 1.1426968 | 396.6234 | Middle |
| 0.2 | 0.5 | 0.3 | 1.1426968 | 462.7273 | Middle |
| 0.3 | 0.1 | 0.6 | 1.2086305 | 449.1922 | Lower |
| 0.3 | 0.2 | 0.5 | 1.2086305 | 449.1922 | Lower |
| 0.3 | 0.3 | 0.4 | 1.2086305 | 449.1922 | Lower |
| 0.3 | 0.4 | 0.3 | 1.1426968 | 462.7273 | Middle |
| 0.3 | 0.5 | 0.2 | 1.1426968 | 528.8312 | Middle |
| 0.4 | 0.1 | 0.5 | 1.2086305 | 598.923 | Lower |
| 0.4 | 0.2 | 0.4 | 1.2086305 | 598.923 | Lower |
| 0.4 | 0.3 | 0.3 | 1.2086305 | 598.923 | Lower |
| 0.4 | 0.4 | 0.2 | 1.2086305 | 598.923 | Lower |
| 0.4 | 0.5 | 0.1 | 1.2086305 | 598.923 | Lower |

**Table 6**
Sensitivity of expected gains with $a$.

| $a$ | Expected gains | $x^*$ | $Q^*(y)$ | $y*$ |
|---|---|---|---|---|
| 100 | 5.095 | 0.399 | 78.154 | 142.70 |
| 105 | 4.761 | 0.387 | 79.076 | 146.32 |
| 110 | 4.444 | 0.375 | 79.990 | 149.96 |
| 115 | 4.143 | 0.363 | 80.892 | 153.59 |
| 120 | 3.859 | 0.350 | 81.780 | 157.20 |
| 125 | 3.590 | 0.337 | 82.668 | 160.85 |

We set the values of parameters $C_O = 25, H = 10, D = 1000$. We take the possible values of $C_S$ as 1000, 2000 and 3000 respectively and perform the sensitivity analysis on their prior probability distribution to find which segment of the supplier is actually being targeted by the buyer in offering the surcharge. In doing so, we also calculate the resulting expected gains and the amount of surcharge being offered Table 5.

The above table shows how the change in the prior probability distribution affects the decision making of the buyer. Even when all the other parameters remain fixed, a change in the prior distribution changes the expected gains, offered value of surcharge and the segment of the supplier targeted.

(b) Effect of prior continuous distribution on the optimum batch sizes and the surcharge offered

We set the values of parameters $C_O = 50, H = 2.5, D = 1000$. We assume that $C_S$ varies uniformly between [$a$, 1000]. Finally, we parameterize $a$ and check for the results Table 6.

From the table, it is clear that the expected gains decrease with an increase in the lower bound of the interval of the uniform distribution. An increase in the value of $a$ implies that the assumed value of $C_S$ also increases, leading to a further increase in the offered amount of compensation, indicating a decrease in buyer's expected profits.

## 9. Concluding remarks

It is clear from the previous sections that the optimal batch size (Q) and the total supply chain cost (TSCC) remain the same for Surcharge Pricing (SP), Quantity Discount (QD) and JELS models. It is also evident that it is not in the best interests of the stronger partner to be part of the JELS agreement which will make him worse off. To achieve JELS solution purely in terms of cost perspective, either of the decentralized coordination mechanisms i.e. QD or SP can be employed depending upon the strength of the partner. QD will be preferable for a strong-buyer scenario whereas SP will be preferable in a strong-supplier scenario. The respective share of the cost for the buyer and the supplier also differ across the three models of SC coordination. The individual's cost share in the arrangement reflects his power. For e.g. in a surcharge pricing process, where the supplier enjoys the power to dictate the batch size, the buyer's share of cost is higher as compared to the cost in JELS model which doesn't take into account the power of the partner. On the other hand, in case of quantity discount model, the buyer enjoys the power to dictate the batch size, and the same is reflected in a higher cost for the supplier as compared to his share of cost in JELS model.

In the case of information asymmetry, the decision making of the buyer is contingent on the prior probability distribution that the buyer holds of the supplier's cost. Different prior probability distributions of the supplier's set up cost lead to different solutions. Even for the same probability distribution, the different parameter values of the distribution lead to different lot size, different expected gain and different values of surcharge offered. It is also evident that the presence of information asymmetry results in a deviation of lot size from the optimal one, leading to inefficiencies in the supply chain. However, even in the presence of information asymmetry, some part of inefficiency can be removed and the overall supply chain costs can be reduced. Such a coordination mechanism in the presence of information asymmetry can be referred to as 'weak' form of coordination, unlike 'strong' form of coordination as seen in surcharge pricing with full information (Albrecht, 2010).

The surcharge pricing process as proposed in the work could be very useful in case of firms operating in JIT environment in the presence of a strong supplier. JIT is accompanied by frequent shipments to reduce the overall level of inventory carrying costs. To make the supplier a part of JIT, a proposal of unit price increase on the wholesale price from the buyer can be beneficial to both the partners involved in the transaction. Such a surcharge pricing contract helps to compensate the supplier for having a higher number of production set ups and increases the efficiency of the supply chain. Surcharge pricing essentially benefits the supply chain through a tradeoff between the supplier's set up cost and buyer's inventory carrying cost. When the buyer has full information of the supplier's cost, there will be an optimal outcome for the supply chain. However, under information asymmetry, there is again a tradeoff between a rejection of proposal from the supplier's side which ultimately converges to the case of no coordination and an opportunity loss from the buyer's side in offering more in the form of a surcharge. Further, the presence of information asymmetry causes a deviation in the lot sizes from the efficient outcome and increases inefficiency in the solution. But contracting under information asymmetry is more efficient than the initial case of no coordination.

The current work contributes in the following ways: first, it finds an optimal surcharge to be offered by the buyer to the supplier, unlike the ad hoc surcharge being paid by the buyer. Secondly, it establishes surcharge pricing as a mechanism for achieving supply chain coordination. Thirdly, it brings the equivalence of surcharge pricing with other supply chain coordination mechanisms like quantity discount and JELS purely on the basis of cost and optimal batch size. Fourthly, it extends the scope of the surcharge pricing mechanism in the presence of information asymmetry about supplier's set up cost, where the problem has been solved both for discrete and continuous prior distribution for supplier's set up cost.

The present work explores a very niche area of supply chain with a strong supplier under JIT environment, who likes to operate by setting only once in the planning horizon. The same analysis can be carried over to incorporate the case of stochastic demands. Further, the individual rationality constraint for the supplier can be modified to some participation constraint, in which case, the supplier will only participate in the negotiation process if assured of some external benefits. The present work can be extended by relaxing the assumption of a risk-neutral buyer.

## Appendix A. Solution to the surcharge pricing model.

The problem in (7)

$$\underset{x,Q}{Max}\quad \Delta B = \frac{H}{2}(D-Q) - C_O\left(\frac{D}{Q}-1\right) - xD$$

$$\text{S.T.}\quad \Delta S = xD - C_S\left(\frac{D}{Q}-1\right) \geq 0$$

can be expressed as:

$$Max\Delta B = \frac{HD}{2} - \frac{HQ}{2} + \left(1-\frac{D}{Q}\right)(C_O + C_S)$$

since the expression $\frac{HD}{2} + C_O - xD - \frac{HQ}{2} - \frac{DC_O}{Q}$ will attain its maximum value at the minimum value for $x$ given by $x^* = C_S(\frac{1}{Q} - \frac{1}{D})$ . There is no reason for the buyer to leave any surplus to the supplier, thus, making the participation constraint binding for optimality. Thus, the problem reduces to an unconstrained optimization problem $Max\Delta B = \frac{HD}{2} - \frac{HQ}{2} + (1 - \frac{D}{Q})(C_O + C_S)$

The first order condition for maximization gives:

$$\frac{d\Delta B}{dQ} = \frac{DC_S}{Q^2} + \frac{DC_O}{Q^2} - \frac{H}{2} = 0$$

Solving the above equation we get $Q = \sqrt{\frac{2(C_O+C_S)D}{H}}$

Further, the second order condition gives, $\frac{d^2(\Delta B)}{dQ^2} = \frac{-2D}{Q^3}(C_O + C_S) < 0$

Thus, the values of $Q^*$ and $x^*$ give a point of maxima.

## Appendix B.A. Solution to the generalized surcharge pricing model.

The problem (10) reduces to the following unconstrained maximization problem

$$Max\ \Delta BQ = \frac{H(D-Q)}{2} - C_O\left(\frac{D}{Q}-1\right) - C_S\left(\frac{D}{q}-1\right) - \frac{(q-Q)H_S}{2}$$

First order condition for profit maximization gives: $\frac{d\Delta B}{dQ} = \frac{DC_O}{Q^2} - \frac{(H-H_S)}{2} = 0$

which implies $Q^* = \sqrt{\frac{2DC_O}{H-H_S}}$ (under the assumption $H>H_S$) and $x^* = \frac{C_S}{D}(\frac{D}{q} - 1) + \frac{q-Q}{2D}H_S$

Second order condition gives $\frac{d^2\Delta B}{dQ^2} = -\frac{2DC_O}{Q^3} < 0$

Thus, the values of $Q^*$ and $x^*$ give the point of maxima.

## Appendix B.B. Derivation of production lot-size of the supplier.

Let $r$ be the finite production rate.

The first shipment from the supplier to the buyer is made at time $t = 0$ and it reaches the buyer instantaneously. The production for making such a shipment starts $t_1$ time units before the shipment. Let $q_s$ be the shipment quantity in every lot which is also equal to the production lot size. Thus, $q_{prod} = \text{rt}_1$. The total supplier's cost is $S_1(q_{prod}) = \frac{q_{prod}}{2}H_s + \frac{D}{q_{prod}}C_S$

First order condition yields $q^*_{prod} = \sqrt{\frac{2DC_S}{H_s}}$ thereby making supplier's initial cost as $\sqrt{2DC_SH_s}$.

Whenever the surcharge is offered along with a proposal of a revised batch size of $Q$, we assume that the supplier decides on a manufacturing quantity q and starts production of $q$ units. Once the production of q units is finished, the supplier starts making the shipments. Hence the total inventory costs incurred by the supplier is given by $\frac{q-Q}{2}H_S + \frac{q}{2}H_S$

## Appendix C. Solution for asymmetric cost information problem (16).

Lagrangian function $L\ (Q_1, Q_2, x_1, x_2)$

$$\begin{aligned} L = {} & \lambda_1\left\{\frac{DC_O}{Q_1} + \frac{Q_1H}{2} + x_1\right\} + \lambda_2\left\{\frac{DC_O}{Q_2} + \frac{Q_2H}{2} + x_2\right\} \\ & + \mu_1\left(\frac{DC_{S1}}{Q_1} - x_1 - C_{S1}\right) + \mu_2\left(\frac{DC_{S2}}{Q_2} - x_2 - C_{S2}\right) \\ & + \mu_3\left(\frac{DC_{S1}}{Q_1} - x_1 - \frac{DC_{S1}}{Q_2} + x_2\right) + \mu_4\left(\frac{DC_{S2}}{Q_2} - x_2 - \frac{DC_{S2}}{Q_1} + x_1\right) \\ & - \omega_1x_1 - \omega_2x_2 - \omega_3Q_1 - \omega_4Q_2 \end{aligned}$$

$\mu_1, \mu_2, \mu_3, \mu_4, \omega_1, \omega_2, \omega_3, \omega_4$ are the Lagrangian multipliers

KKT conditions are:

$$\left.\begin{aligned} & \frac{\partial L}{\partial Q_1} = \frac{\lambda_1 H}{2} - \lambda_1\frac{D}{Q_1^2}C_O - \mu_1\frac{D}{Q_1^2}C_{S1} - \mu_3\frac{D}{Q_1^2}C_{S1} \\ & + \mu_4\frac{D}{Q_1^2}C_{S1} - \omega_3 = 0 \\ & \frac{\partial L}{\partial Q_2} = \frac{\lambda_2 H}{2} - \lambda_2\frac{D}{Q_2^2}C_O - \mu_2\frac{D}{Q_2^2}C_{S2} - \mu_4\frac{D}{Q_2^2}C_{S2} \\ & + \mu_3\frac{D}{Q_2^2}C_{S2} - \omega_4 = 0 \\ & \frac{\partial L}{\partial x_1} = \lambda_1 - \mu_1 - \mu_3 + \mu_4 - \omega_1 = 0 \\ & \frac{\partial L}{\partial x_2} = \lambda_2 - \mu_2 - \mu_4 + \mu_3 - \omega_2 = 0 \end{aligned}\right\}\text{(Stationarity)}$$

$$\begin{aligned} & \frac{DC_{S1}}{Q_1} - x_1 - C_{S1} = 0, \frac{DC_{S2}}{Q_2} - x_2 - C_{S2} = 0, \\ & \frac{DC_{S1}}{Q_1} - x_1 - \frac{DC_{S1}}{Q_2} + x_2 = 0, \\ & \frac{DC_{S2}}{Q_2} - x_2 - \frac{DC_{S2}}{Q_1} + x_1 = 0 \end{aligned}$$

$$Q_1, Q_2 \geq 0 \quad \text{and} \quad x_1, x_2 \geq 0 \qquad \text{(Primal Feasibility)}$$

$$\mu_1, \mu_2, \mu_3, \mu_4, \omega_1, \omega_2, \omega_3, \omega_4 \geq 0 \qquad \text{(Dual Feasibility)}$$

$$\mu_1\left(\frac{DC_{S1}}{Q_1}-x_1-C_{S1}\right)=0,\ \mu_2\left(\frac{DC_{S2}}{Q_2}-x_2-C_{S2}\right)=0,$$
$$\mu_3\left(\frac{DC_{S1}}{Q_1}-x_1-\frac{DC_{S1}}{Q_2}+x_2\right)=0,\ \mu_4\left(\frac{DC_{S2}}{Q_2}-x_2-\frac{DC_{S2}}{Q_1}+x_1\right)=0,$$
$$\omega_1 Q_1=0,\ \omega_2 Q_2=0,\ \omega_3 x_1=0,\ \omega_4 x_2=0$$
(Complementary Slackness)

Clearly $Q_1 \neq 0$, $Q_2 \neq 0$, $x_1 \neq 0$, $x_2 \neq 0$ hence $\omega_1 = \omega_2 = \omega_3 = \omega_4 = 0$

For $\mu_i$'s we have 16 possible permutations whether $\mu_i = 0$ or $\mu_i \neq 0$. Around 8 combinations will be ruled out by using the stationarity conditions and we are left with only 8 possible combinations. For e.g. if all $\mu_i$'s are zero which means $\lambda_1 = \lambda_2 = 0$ which is ruled out. The only feasible cases are: (i) $\mu_1 = 0$, $\mu_2 \neq 0$, $\mu_3 \neq 0$, $\mu_4 = 0$ (ii) $\mu_1 = 0$, $\mu_2 \neq 0$, $\mu_3 \neq 0$, $\mu_4 \neq 0$ (iii) $\mu_1 \neq 0$, $\mu_2 = 0$, $\mu_3 = 0$, $\mu_4 \neq 0$ (iv) $\mu_1 \neq 0$, $\mu_2 = 0$, $\mu_3 \neq 0$, $\mu_4 \neq 0$ (v) $\mu_1 \neq 0$, $\mu_2 \neq 0$, $\mu_3 = 0$, $\mu_4 = 0$ (vi) $\mu_1 \neq 0$, $\mu_2 \neq 0$, $\mu_3 = 0$, $\mu_4 \neq 0$ (vii) $\mu_1 \neq 0$, $\mu_2 \neq 0$, $\mu_3 \neq 0$, $\mu_4 = 0$ and (viii) $\mu_1 \neq 0$, $\mu_2 \neq 0$, $\mu_3 \neq 0$, $\mu_4 \neq 0$

On solving for these eight feasible cases we will get the desired contracts. It is to be noted that some of these cases will lead us to identical contracts. As an instance for Case I when $\mu_1 = 0$, $\mu_2 \neq 0$, $\mu_3 \neq 0$, $\mu_4 = 0$ we have $\lambda_1 - \mu_3 = 0$ and $\lambda_2 - \mu_2 + \mu_3 = 0$ implies $\mu_3 = \lambda_1$ and $\mu_2 = 1$

$$\frac{\lambda_1 H}{2}-\frac{\lambda_1 D}{Q_1^2}C_O-\frac{\lambda_1 D}{Q_1^2}C_{S1}=0 \Rightarrow Q_1^2=\frac{2D(C_O+C_{S1})}{H} \text{ and}$$
$$\frac{\lambda_2 H}{2}-\frac{\lambda_2 D}{Q_2^2}C_O-\frac{D}{Q_2^2}C_{S2}+\frac{\lambda_1 D}{Q_2^2}C_{S2}=0 \Rightarrow Q_2^2 =\frac{2D(\lambda_2 C_O+C_{S2}-\lambda_1 C_{S2})}{\lambda_2 H}$$

Similarly for the remaining seven cases we can get the solution.

## Appendix D. Example under full information.

(a) Under lot for lot assumption.

Let Annual Demand, $D = 10000$, Buyer's ordering cost, $C_O = 100$ dollar per order, Buyer's annual inventory carrying cost, $H = 2$ dollar per unit, Supplier's set up cost, $C_S = 1500$ dollar per set up

(i) In surcharge pricing (SP) model, the supplier prefers to ship the entire demand of 100000 units in one shipment (supplier acting optimally). The buyer's cost under such a policy will be (using (1)) 10100 dollar and the supplier's cost will be 1500 dollar. Once the buyer proposes a surcharge, the optimal values for surcharge and optimal batch size are respectively 0.225 per unit and 4000 units. The respective buyer's and supplier's costs are given as 6500 and 1500 dollar.
(ii) In quantity discount (QD) model, the buyer will prefer to order according to his EOQ given by $Q = \sqrt{\frac{2DC_O}{H}}$ (buyer acting optimally). The buyer's cost under such a policy will be (using (1)) 2000 dollar and the supplier's cost will be 15000 dollar. Once the supplier proposes a quantity discount, the optimal values for discount and optimal batch size are respectively 0.225 per unit and 4000 units. The respective buyer's and supplier's costs are given as 2000 and 6000 dollar.
(iii) In JELS model, both the supplier and buyer will act together for the optimality of the entire supply chain. In such a case the optimal order quantity will be 4000 units. The respective costs of the buyer and supplier are 4250 and 3750 dollar.

**Table E.5**
Expected gains for the buyer under various states of $C_S$.

| $Y$ | $P$ | $\Delta(y)$ | $E(\Delta(y))$ |
|---|---|---|---|
| 1250 | 0.3 | 4001.53 | 1200.46 |
| 1500 | 0.4 | 3600.00 | 2520.00 |
| 1750 | 0.3 | 3247.68 | 3247.68 |

(b) Lot for lot relaxed

For analyzing the lot for lot relaxation case, we will consider the data from the above example. In addition, we will take the supplier's annual holding cost, $H_S = 1$ dollar per unit.

(i) In surcharge pricing model, the supplier prefers to ship the entire demand of 100000 units in one shipment (supplier acting optimally). The buyer's cost under such a policy will be (using (1)) 10100 dollar and the supplier's cost will be 1500 dollar. Once the buyer proposes a surcharge, the optimal values for surcharge and optimal batch size are respectively 0.327 per unit and 1414 units. The respective buyer's and supplier's costs are given as 5391.4 and 1500 dollar.
(ii) In quantity discount (QD) model, the buyer will prefer to order according to his EOQ given by $Q = \sqrt{\frac{2DC_O}{H}}$ (buyer acting optimally). The buyer's cost under such a policy will be 2000 dollar and the supplier's cost will be 4977 dollar. Once the supplier proposes a quantity discount, the optimal values for discount and optimal batch size are respectively 0.0005 per unit and 1414 units. The respective buyer's and supplier's costs are given as 2000 and 4891 dollar.
(iii) In JELS model, both the supplier and buyer will act together for the optimality of the entire supply chain. In such a case the optimal order quantity will be 1414 units. The respective costs of the buyer and supplier are 2121.3 and 4770.12 dollar.

## Appendix E. Example under information asymmetry.

(a) Discrete type space of supplier's set up cost.

Annual Demand, $D = 10000$, Buyer's ordering cost, $C_O = 100$ dollar per order, Buyer's annual inventory carrying cost, $H = 2$ dollar per unit while the buyer also holds prior probability distribution of supplier's set up cost given as: $P\{C_S = 1250\} = 0.3$, $P\{C_S = 1500\} = 0.4$ and $P\{C_S = 1750\} = 0.3$

Table E.5.

It is known that the buyer will choose the value of $y$ that will give him maximum expected gain and will design his SP contract corresponding to that particular value of $y$.

In this case $y^* = 1750$, $Q^* = 4301$, $x^* = 0.23$.

(b) Continuous Type space of Supplier's set up cost

Annual Demand, $D = 100$, Buyer's ordering cost, $C_O = 10$ dollar per order, Buyer's annual inventory carrying cost, $H = 5$ dollar per unit while the buyer also holds prior probability distribution of supplier's set up cost given as U [100, 200]. In this case the optimum value for $Q^*$ is 78 occurring for $y^* = 142.7$. The buyer offers a surcharge of 0.39 per unit to the supplier.

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