

How far should JIT vendor–buyer relationships go?

Israel David*, Moshe Eben-Chaime

Department of Industrial Engineering & Management, Ben-Gurion University of the Negev, P.O. Box 653, 84105 Beer-Sheva, Israel

Abstract

In pull production management systems such as JIT, deliveries must be made on an as-needed basis only, and production begins only when requested. It is supposed to match customer demand, i.e., producing only enough to replenish what the customer has used or sold. In this work, we argue that there should be a certain degree of independence between successive links of the supply chain, to allow flexibility in production management in individual links. We attempt to identify the degree of independence and level of flexibility in terms of lot sizing and delivery scheduling in a single-vendor–single-buyer system. Toward this aim, appropriate two-sided vendor–buyer inventory–production models are formulated, some interesting conclusions from their analysis are drawn, and a numerical study, which compares relevant policies, is discussed. It shows that imposing a lot-for-lot production on the JIT supplier is strikingly un-economical. On the other hand, delivery on demand can be met without intervening in the supplier's operations, and where deviation from the optimal of the resulting joint total costs is tolerable.

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1. Introduction

Modern manufacturing systems are pull systems, in which the trigger is in the buyers' hands, rather than push systems, in which the suppliers have more power. In pull systems, "deliveries must be made on an as-needed basis only", and even more, "production begins only when requested" (Nahmias, 1997). Further, production matches customer demand; i.e., only enough is produced to replenish what the customer has used or sold, rather than producing to a forecast of what the customer may require (Aalbregtse and Harmon, 1994). To cite a typical introductory text to JIT,

these systems "call for flexible schedules that pull material on demand and on short notice. This concept is, often mistakenly taken by suppliers as a request for buffer inventories. JIT considers buffer inventories a waste. It would not be healthy for a customer/supplier relationship to shift the need for a buffer inventory from the customer to the supplier. In the long run, the supplier is going to resent it and will make the customer pay for it. Buffer inventories will also suffer the quality problems associated with keeping material built ahead of time... For a supplier, the best approach is to reduce lead times. This will make the supplier lean and responsive..." (Hernandez, 1993).

Being "lean and responsive" sounds like good advice, but what is the hard-fact inventory management burden of that advice, even with moderate values of production setup costs? While

*Corresponding author. Tel.: +972-7-64-72-202; fax: +972-7-64-72-958.

E-mail address: idavid@black.bgu.ac.il (I. David).

the concept may be correct from the buyer's standpoint, is it really the best approach for the supplier? And what are the implications for the system as a whole?

A natural framework to deal with these questions is that of *integrated inventory models* (buyer–vendor coordination). A seminal work in the area is that of Banerjee (1986) who proposed the concept of a joint economic lot size (see also the extension Goyal, 1988). Even earlier anticipating works are those which focused on the potential savings for both parties (the vendor and the buyer), and the potential influence on the buyer's ordering pattern, of the vendor offering price quantity discounts. A comprehensive literature review of this early work is presented in Goyal and Gupta (1989). More recent contributions include Abad (1994), Parlar and Wang (1994), Aderohunmu et al. (1995), Lu (1995), Goyal (1995), Hill (1997), Viswanathan (1998), Bylka (1999) and Goyal and Nebebe (2000).

It is interesting to note that as early as in the review Goyal and Gupta (1989), the importance of integrated inventory models was primarily attributed to the increasing number of organizations that were implementing JIT. However, and oddly enough, almost none of the above referenced works specifically addresses JIT, or realistically reflects JIT environments. Abad (1994), and Parlar and Wang (1994), assume no production—the vendor, too, buys the product (from a third party). Banerjee (1986) assumes that the vendor manufactures each shipment independently ('lot-for-lot'), and Goyal (1988) assumes that delivery starts only upon completion of the production of a lot. Lu (1995) assumes that the production lot size makes an integral number of equal-size successive shipments and he develops methods to find the number of shipments in a multi-buyer environment. Goyal (1995) and Hill (1997) studied more intricate models in which successive shipments in a batch are inflated by a constant factor, such as the production to demand rate ratio. Goyal and Nebebe (2000) proposed a small initial shipment followed by a series of larger, equal-size shipments. They showed that these policies might reduce total costs—but the whole concept of uneven supply is clearly alien to the JIT realm.

See also Viswanathan (1998). Aderohunmu et al. (1995) is the only relevant study which we identified as being specifically JIT-oriented. It is based, however, on a restrictive model, as we describe later.

Indeed, in Section 2 below, we develop the models we adopt for describing the inventory pattern and the cost structure. In doing so we make more specific our critical reference to past literature. The objective of this study is to analyze and gain insight into the vendor–buyer relationships, and test some common JIT myths regarding optimal lot sizes or order/delivery quantities. Toward this aim, the models are analyzed in Section 3, while numerical illustrations of the significance of the findings are provided in Section 4. Conclusions are discussed in the closing section: in particular, it was found that imposing a lot-for-lot production on the JIT supplier is strikingly un-economical. On the other hand, the JIT delivery-on-demand imperative can be met without intervening in the supplier's operations, and in such a way that the incurred joint total costs deviate tolerably from the optimal.

2. Modeling a JIT single-product vendor–buyer relationship

We discuss a single-item, single-supplier and single-customer model, where the demand of the customer (JIT manufacturer) is continuous at fixed rate, D . The supplier (JIT vendor) production rate is P , where $P > D$. No shortages are allowed, and the cost structure is of an EOQ model plus delivery costs, as will be defined in Section 2.2 below.

2.1. The two-sided inventory patterns

Suppose the vendor produces in lots sizes Q , and delivers to the buyer n times per lot—the *delivery index*. Fig. 1 shows the inventory level pattern at the vendor, at the buyer, and that of the total system, respectively.

In the figure we let $r = D/P$, the demand-to-production ratio, and $q = Q/n$, the (constant) delivery quantity. The beginning of a cycle is the moment of production startup. $Q/(nD)$ are the

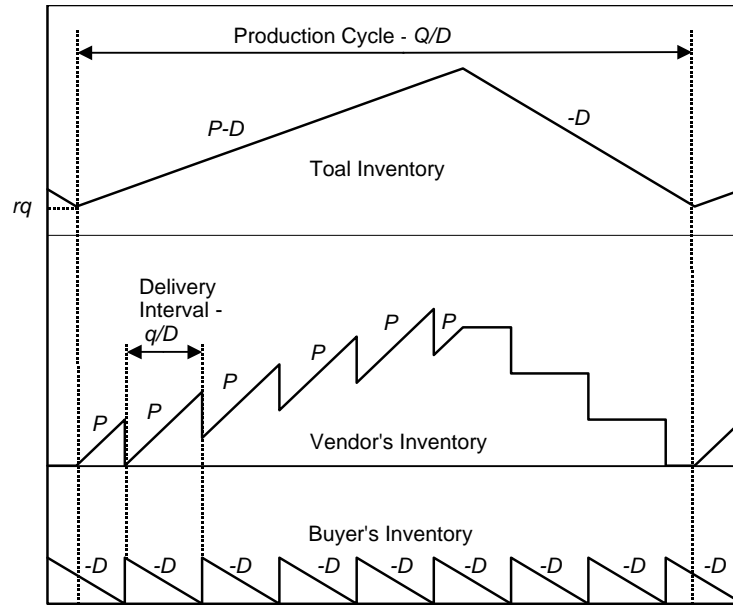


Fig. 1. Inventory patterns for the vendor and for the buyer.

delivery intervals, while $Q/(nP)$, the time needed to produce the delivery quantity, is, obviously, the time of first delivery after commencing production. Hence, $rq = (D/P)(Q/n)$ gives the initial inventory at the buyer (to suffice for the initial time $Q/(nP)$). From here the peak system inventory, I_{\max} is easily calculable, together with the following expressions (1)–(3) for the average inventory values \bar{I}_T , \bar{I}_B and \bar{I}_V (The subscripts B, V and T denote throughout the buyer, the vendor and the total system, respectively).

$$\bar{I}_T = \frac{QD}{nP} + \left(1 - \frac{D}{P}\right) \frac{Q}{2}, \quad (1)$$

$$\bar{I}_B = \frac{Q/n}{2}, \quad (2)$$

$$\begin{aligned} \bar{I}_V &= \frac{QD}{nP} + \left(1 - \frac{D}{P}\right) \frac{Q}{2} - \frac{Q}{2n} \\ &= \frac{Q}{2} \left(1 - \frac{D}{P}\right) + \frac{Q}{n} \left(\frac{D}{P} - \frac{1}{2}\right). \end{aligned} \quad (3)$$

Expression (3), which may be tedious to calculate directly from the vendor's variable pattern, is simply the difference between (1) and (2).

Observe that the vendor's average inventory is the sum of two terms, and that the first of them, $(1 - D/P)Q/2$, is that of the classical, continuous-demand, *production–inventory* model (PI), see, e.g. Nahmias (1997). The two-term structure recurs in the literature we cited in the Introduction. Notably, the current extra term differs significantly from those of Lu (1995) and Aderohunmu et al. (1995). This is due to their redundant assumption that production starts immediately following the last delivery in a cycle. Since $P > D$, this is far from necessary and the vendor may enjoy periods of zero inventory, as in our Fig. 1. Consequently, the vendor's average inventory in the models of Lu and Aderohunmu et al. is always higher than in the classical PI model. The current model clearly demonstrates the possibility of *reduction* in the vendor's inventory. Thus, “the need for a buffer inventory” should not be necessarily shifted “from the customer to the supplier”. Moreover, the contrary sometimes holds.

Aderohunmu et al. assume further that the cycle length, Q/D , and the length of the production period, Q/P , are both integer multiplications of the delivery cycle length, Q/nD . However, because

the length of the production period is D/P times the cycle length, only rarely will it be the case that both lengths share a common denominator. While assuming an integral number of deliveries per cycle is reasonable, production would typically end in the course of the delivery cycle (Fig. 1).

2.2. The two-sided cost structure

We assume that the demand rate, the vendor's production cost and the buyer's purchasing cost do not change in time and that they are independent of the production/delivery quantities. Thus, the relevant annual-cost model includes a production setup cost, a delivery-associated cost and an inventory holding cost. We let C denote total costs, h the holding parameter and F the delivery cost, and get:

$$C_B = F_B \frac{nD}{Q} + h_B \frac{Q}{2n}, \quad (4)$$

$$C_V = K \frac{D}{Q} + F_V \frac{nD}{Q} + h_V \left(\frac{Q}{2} \left(1 - \frac{D}{P} \right) + \frac{Q}{n} \left(\frac{D}{P} - \frac{1}{2} \right) \right), \quad (5)$$

$$C_T = C_B + C_V = K \frac{D}{Q} + (F_B + F_V) \frac{nD}{Q} + (h_B - h_V) \frac{Q}{2n} + h_V \left(\frac{Q}{2} \left(1 - \frac{D}{P} \right) + \frac{QD}{nP} \right). \quad (6)$$

A fundamental advance in the current cost model is in recognizing that *delivery* associated costs apply to both sides, while commonly (e.g., Goyal, 1988; Aderohunmu et al., 1995; Lu, 1995), the buyer alone is charged with these costs. This does not mean necessarily that the transportation cost is divided between the two. (The responsibility for transportation costs is indeed an interesting issue in vendor–buyer relationships, but we do not touch it here.) It does mean, however, that there exist additional, unavoidable costs associated with each delivery on the vendor's side; e.g., preparing and, perhaps, loading the shipment, as well as on the *buyer's* side—unloading, and storing. The terms F include these costs plus the transportation cost.

3. Analytical considerations

Observe first that if we substitute $q = Q/n$ in the buyer's cost model (4) it takes the form of the classical EOQ model. It follows that if the buyer can, (s)he orders q^* units to be delivered every q^*/D units of time, where

$$q^* = \sqrt{\frac{2F_B D}{h_B}},$$

thereby reducing the buyer's annual cost to a minimum of

$$C_B^* = C_B(q^*) = \sqrt{2F_B h_B D}.$$

Therefore, so long as the cost parameters and demand rate remain constant, and the buyer is able to dictate the delivery quantity and schedule, and so long as buyer's inventory costs are at stake, (s)he is apathetic to the production lot size. The delivery index, n , is the communicating/adapting link and, consequently, is the one parameter to be negotiated.

The next observation plays a key role in determining optimal production lot sizes:

Proposition 1. *When the delivery index is proportional to the production lot size, $n = \alpha Q$, both the vendor's optimal lot size and the jointly optimal lot size are that of the classical PI model:*

$$Q^* = \sqrt{\frac{2DK}{h_V \left(1 - \frac{D}{P} \right)}}. \quad (7)$$

Proof. We address the *vendor* model; the proof for the *total* (joint) model follows in a similar way. Substitute αQ for n in (5), to get

$$C_V = K \frac{D}{Q} + \alpha F_V D + h_V \frac{Q}{2} \left(1 - \frac{D}{P} \right) + \frac{h_V}{\alpha} \left(\frac{D}{P} - \frac{1}{2} \right).$$

Notice that the second term and the last term are independent of the lot size, while the other two terms are exactly those of PI, with $h = h_V$. \square

As an immediate application of Proposition 1, recall the first observation regarding the buyer, and assume that s/he selects the order quantity independently. In this case, as pointed out above,

$Q = nq^*$ and q^* is, of course, independent of Q and n . Thus, even assuming that the buyer determines the delivery frequency, by selecting the order quantity, independently, the optimal production lot size remains untouched!

Proposition 1 tells us even more. Differentiating (5) and (6) with respect to n and equating to zero, optimal delivery indices, in terms of the production-lot-size, are obtained:

$$n_V^*(Q) = Q \sqrt{\frac{h_V}{F_V D} \left(\frac{D}{P} - \frac{1}{2} \right)}, \quad (8)$$

$$n_T^*(Q) = Q \sqrt{\left(h_V \frac{D}{P} + \frac{h_B - h_V}{2} \right) / (F_B + F_V) D}. \quad (9)$$

Both of these terms are, again, of the form αQ . Ideally, then, Eq. (7) provides the optimal lot size for both the vendor's and the joint models. Still, because n is a whole number, $[n^*]$ or $[n^*] + 1$ should practically replace the results in (8) and (9), whichever gives a smaller value in (5)–(6), respectively. $[x]$ denotes the *integer value* of x . Finally, if the integer result is zero, the value 1 should be taken as the operative delivery index. It then also follows from (8) that when the production rate exceeds twice the demand rate, then $n_V^*(Q) = 1$, calling for a *lot-for-lot* policy.

The main conclusions so far are: (a) the buyer is quite indifferent with the vendor's production lot size, and (b) whether we take the buyer's, the vendor's, or the joint standpoint, the optimal production lot size is the same—that of the PI model as in Proposition 1. It should be noted that the routine, more rigorous way of optimizing a jointly convex function over the continuous Q and the discrete n , only obscures the picture unveiled by our own approach. (In the routine way one would fix n and find $Q^*(n)$, see Hill's equation (10), and then specify a double inequality for n^* , by plugging Q^* to the objective, and then minimizing over n . For not too-small an n^* , the round-up method would give the same n^* .)

We also observe that when q is fixed (by the buyer), both the vendor's and the joint cost model are reduced to single-variable models— Q is replaced by nq , and Eqs. (5) and (6) transform

into:

$$C_V = KD/nq + F_V D/q + h_V(nq/2(1 - D/P) + q(D/P - 0.5))$$

and

$$C_T = KD/nq + (F_B + F_V)D/q + (h_B - h_V)q/2 + h_V(nq(1 - D/P)/2 + qD/P).$$

This is another way of looking at the models, which makes clear the independence between the associated delivery cost and the production lot size.

4. Numerical illustrations

The aforementioned *lot-for-lot* policy—producing in lot sizes as ordered by the buyer, will be labeled below as “JIT”, to highlight its connection to widespread JIT sermons as cited in the Introduction. By contrast, we label by “DOD” a policy of *delivery-on-demand*, delivering in quantities q as demanded by the buyer. Obviously, then, delivery-on-demand can be met, independent of vendor's production scheduling and there seems to be no justification to intervene in her/his operation. Evidently, there are dramatic differences between these two approaches in terms of the annual cost. Changing only the delivery index as required for DOD is much less expensive for the vendor than producing in lot sizes as ordered by the buyer. To illustrate, we start by considering two pointwise examples. Let the demand rate be 24,000 units/yr, the production rate be 60,000 units/yr, $K = \$470/\text{setup}$, $F_V = \$30/\text{delivery}$, $h_V = \$1/\text{unit yr}$, $F_B = \$10/\text{delivery}$ and $h_B = \$3/\text{unit yr}$. The buyer's optimal order quantity is of 400 units with annual cost of \$1200. Since $D/P = 0.4 < 0.5$, the optimal policy for the vendor is a lot-for-lot delivery of 7746 units a lot, with annual cost of \$3098.4. If the vendor is forced to deliver 400 units per delivery, the optimal lot size is that of Proposition 1—6131 units. Rounding down to 6000 units gives $n = 15$ and an annual vendor's cost of \$5440—a 75.6% increase from the minimum. While this is a significant increase, it is less dramatic than the increase in the buyer's annual cost, which results from taking deliveries of 7746

units. The buyer's cost in this case rises almost 10-fold, to \$11,622. A similar change—by a factor of about 10—results from forcing the vendor to produce JIT, in lots of 400 units. It goes up to \$30,080 annually!

As another example, reduce now the production rate to 40,000 units/yr. This reduction does not affect the buyer, but D/P increases to $0.6 > 0.5$. Consequently, the vendor's optimal policy is to produce in lots of about 7500 units, which are delivered in three shipments, with annual cost of \$3542. For DOD of 400 units, the lot size increases to 7600 units and the annual cost to \$4844—about 37%. Producing in lots of 400 units results in \$30,120 annual cost for the vendor, that is, 8.5 times the minimum.

Tables 1 and 2, below, generalize these results, covering a number of combinations of the parameter values. Both tables list the total cost, C_T , under optimality (= OPT), under DOD, and under JIT—producing the buyer's *daily* demand. They

Table 1
Efficiency of “JIT” and “DOD” policies. Basic combination 1

h_V	$C_{T\text{opt}}$	$C_{T\text{dod}}$	$C_{T\text{jit}}$	jit/opt	dod/opt
1	952.1	3213.1	65008.0	68.3	3.4
5	2128.9	4111.1	65040.0	30.6	1.9
10	3010.7	4796.1	65080.0	21.6	1.6
50	6732.1	7800.0	65400.0	9.7	1.2
$F_V + F_B$					
1	1754.3	1861.1	62790.0	35.8	1.1
5	1968.4	2861.1	63790.0	32.4	1.5
10	2128.9	4111.1	65040.0	30.6	1.9
50	2805.9	14111.1	75040.0	26.7	5.0
100	3313.2	26611.1	87540.0	26.4	8.0
P					
10000	2424.3	4518.0	65034.0	26.8	1.9
6000	2308.8	4349.1	65036.7	28.2	1.9
4000	2128.9	4111.1	65040.0	30.6	1.9
2000	712.1	2555.0	65050.0	91.3	3.6
K					
250	2128.9	4111.1	65040.0	30.6	1.9
200	1961.9	3944.2	52540.0	26.8	2.0
150	1772.5	3754.7	40040.0	22.6	2.1
100	1547.7	3530.0	27540.0	17.8	2.3
10	864.0	2846.2	5040.0	5.8	3.3

Table 2
Efficiency of “JIT” and “DOD” policies. Basic combination 2

h_V	$C_{T\text{opt}}$	$C_{T\text{dod}}$	$C_{T\text{jit}}$	jit/opt	dod/opt
1	692.2	2953.2	27508.0	39.7	4.3
5	1547.7	3530.0	27540.0	17.8	2.3
10	2188.8	3974.2	27580.0	12.6	1.8
50	4894.3	5962.3	27900.0	5.7	1.2
$F_V + F_B$					
1	1173.2	1280.0	25290.0	21.6	1.1
5	1387.3	2280.0	26290.0	19.0	1.6
10	1547.7	3530.0	27540.0	17.8	2.3
50	2224.7	13530.0	37540.0	16.9	6.1
100	2732.1	26030.0	50040.0	18.3	9.5
P					
10000	1689.2	3782.9	27534.0	16.3	2.2
6000	1637.7	3678.0	27536.7	16.8	2.2
4000	1547.7	3530.0	27540.0	17.8	2.3
2000	710.3	2553.2	27550.0	38.8	3.6
K					
250	2128.9	4111.1	65040.0	30.6	1.9
200	1961.9	3944.2	52540.0	26.8	2.0
150	1772.5	3754.7	40040.0	22.6	2.1
100	1547.7	3530.0	27540.0	17.8	2.3
10	864.0	2846.2	5040.0	5.8	3.3

also give the respective ratios of $C_{T\text{jit}}$ and $C_{T\text{dod}}$ relative to the minimum value $C_{T\text{opt}}$. Each table centers around one basic parameter combination. In the four sections of each table one parameter varies at a time.

Table 1 has $D = 2000$ units/yr, $P = 4000$ units/yr, $h_V = \$5$, $h_B = 1.5h_V$, $F_V + F_B = \$10$, $K = \$250$ and 250 days in a year; i.e., daily demand of 8 units, as the basic combination, while Table 2 has the same basic combination, only K is reduced to \$100. The ensuing repetition of the fourth section in Table 2 has been kept for clarity.

The difference between JIT and DOD as reflected in the tables is prominent: in all but a couple of cases the JIT/OPT ratio is in an order of magnitude larger than the DOD/OPT ratio. One exception where the two ratios come closer is when the inventory holding costs are high— $h_V = \$50$, in which case $\text{DOD/OPT} = 1.2$ while JIT/OPT equals 9.7 in Table 1, and 5.7 in Table 2. Another exception is when the setup cost is very low— $K = \$10$. Here $\text{JIT/OPT} = 5.8$, and $\text{DOD/OPT} = 3.3$.

Indeed, modern production management focuses strongly on these cost components and the balance between them. Thus, the exceptional behavior should be anticipated. However, there is a vital difference between these cases. In the first case—high holding cost, the ratios come closer due to large increases in both the optimal cost and the DOD cost, while in the second case—low setup cost, the cause is a dramatic decrease of the JIT cost. Again, setup cost reduction is a major goal of, as well as a mean toward lean and efficient production. Yet, reducing the setup cost is a long-term effort that may require significant capital investments, which are not always feasible and/or justified. Tables 1 and 2 indicate that, otherwise, forcing JIT production can be financially destructive for the vendor.

5. Conclusions

In this work we have examined and analyzed some aspects of the relationships between a buyer and her/his manufacturer vendor in an attempt to answer how far should they go, in terms of determining lot sizes and delivery frequency. To this aim, inventory and cost models for each party and for the joint systems were formulated and analyzed. The inventory models correct some errors, relax some restrictive assumptions and clear up a degree of ambiguity that prevails in previous works. The cost models recognize that delivery-associated costs are charged against both parties, even if only one of them pays for the transportation.

Analysis of these models indicates a high degree of independence between parties in terms of quantities/lot sizes. The communicating factor between the vendor's and the buyer's models is the delivery index. This allows flexibility in choosing the order quantity by the buyer and production lot size by the vendor. Furthermore, the optimal production lot size remains that of the classical production–inventory model with continuous demand, be it in view of the vendor's optimization, the joint optimization or the independent selection of the order/delivery quantity by the buyer. Realizing that the delivery index has

only a partial effect on the total cost for each party, a policy of compromising on this issue and avoiding profound interventions, seems sufficient. Numerically, it has been found that a double-digit cost inflation from the joint-optimal is encountered by forcing the vendor to “JIT”—produce and deliver the buyer's order quantity. In contrast, the cost is inflated by a single digit factor when the vendor is free to choose the production lot size and is only forced to deliver on demand.

It is commonly suggested that to make JIT-production economically justified, one has to reduce setup costs. Indeed, our model suggests that the cost inflation decreases as the setup cost decreases. However, consider the scale of the required decrease. To make a single digit cost deflation, the setup cost has to go down to less than 4% of its initial value! In general, if, for a given value of setup cost, the optimal production lot size is n times the order/delivery quantity, a reduction by a factor $1/n^2$ is required in order to make JIT production justified. Alternatively, our results suggest that focusing on delivery associated cost reduction (while leaving the vendor to run her/his operation) might be a preferable strategy. The incurred out-of-pocket cost reduction cannot be outweighed by other considerations, important as they may be. Indeed, JIT advocates claim that its advantages stem from a quality perspective, arguing that it enables early detection and correction of quality problems. But “DOD” practice provides an adequate response in this regard, too, as units are delivered on a first-manufactured-first-delivered basis. Moreover, DOD may even be superior to the small batch production implied by JIT, in terms of quality, since changeovers are less frequent.

In sum, systems of vendors and suppliers contain mechanisms for self-control and there seems to be no need to intervene in their autonomous operation. This, of course, should not prevent continuous efforts to reduce setup costs and improve the systems as a whole. It seems, however, that a sweeping dictation of supplier operations only adds to a list of JIT myths, as described in Chapter 16 of Silver et al. (1998). In contexts where suppliers (“vendors”) and customers (“buyers”) belong to the same firm (i.e. are

under the same accounting system), the lesson is that departments need not try too hard to jointly schedule operations. This would be news of particular value to internal suppliers with more than one internal “customer”—a subject for further research.

Acknowledgements

An equivalent of Fig. 1 may be found, in a somewhat condensed form, in Hill's paper. Also, we took the liberty of transcribing the writing of one of the referees. The work has been partially supported by the Paul Ivanier Center of Robotics and Production Management, Ben-Gurion University of the Negev, Israel.

References

- Aalbrechtse, R.J., Harmon, R.L., 1994. Production and manufacturing resource planning in a just-in-time environment. In: Robeson, J.F., Copacino, W.C. (Eds.), *The Logistics Handbook*, The Free Press, MacMillan, Inc., New York.
- Abad, P.L., 1994. Supplier pricing when the buyer's annual requirements are fixed. *Computers & OR* 21 (2), 155–167.
- Aderohunmu, R., Ayodele, M., Bryson, N., 1995. Joint vendor–buyer policy in JIT manufacturing. *Journal of the OR Society* 46, 375–385.
- Banerjee, A., 1986. A joint economic lot size model for purchaser and vendor. *Decision Science* 17, 292–311.
- Bylka, S., 1999. A dynamic model for the single-vendor, multiple-buyer problem. *International Journal of Production Economics* 59, 297–304.
- Goyal, S.K., 1988. A joint economic lot size model for purchaser and vendor: A comment. *Decision Science* 19, 236–241.
- Goyal, S.K., 1995. A one-vendor multi-buyer integrated inventory model: A comment. *European Journal of Operational Research* 82 (1), 209–210.
- Goyal, S.K., Gupta, Y.P., 1989. Integrated inventory models: The buyer–vendor coordination. *European Journal of Operational Research* 41 (3), 261–269.
- Goyal, S.K., Nebebe, F., 2000. Determination of economic production-shipment policy for a single-vendor–single-buyer system. *European Journal of Operational Research* 121 (1), 175–178.
- Hernandez, A., 1993. *Just in Time Quality, A Practical Approach*. Prentice-Hall, Englewood Cliffs, NJ.
- Hill, R.M., 1997. The single-vendor single-buyer integrated production–inventory model with a generalized policy. *European Journal of Operational Research* 97, 493–499.
- Lu, L., 1995. A one-vendor multi-buyer integrated inventory model. *European Journal of Operational Research* 81 (2), 312–323.
- Nahmias, S., 1997. *Production and Operations Analysis*, 3rd ed.. Irwin, Homewood, IL.
- Parlar, M., Wang, Q., 1994. Discounting decisions in supplier–buyer relationship with a linear buyer's demand. *IIE Transactions* 26 (2), 34–41.
- Silver, E.A., Pyke, D.F., Peterson, R., 1998. *Inventory Management and Production Planning and Scheduling*, 3rd ed.. Wiley, New York.
- Viswanathan, S., 1998. Optimal strategy for the integrated vendor–buyer inventory model. *European Journal of Operational Research* 105, 38–42.