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EOQ, JIT and fixed costs in the ready-mixed concrete industry

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Abstract

The successful implementation of just-in-time (JIT) purchasing policy in many industries has prompted many companies that still use the economic order quantity (EOQ) purchasing policy to ponder if they should switch to the JIT purchasing policy. Despite existing studies that directly compare the costs between the EOQ and JIT purchasing systems, this decision is, however, still difficult to be made, especially when price discount has to be considered. JIT purchasing may not always be successful even though plants that adopted JIT operations have experienced or can take advantage of physical space reduction. Hence, the objective of this study is to expand on a classical EOQ with a price discount model to derive the EOQ–JIT cost indifference point. The objective was tested and achieved through a survey and case study conducted in the ready-mixed concrete industry in Singapore.

Keywords: EOQ–JIT cost indifference point; Price discount; Inventory facility; Ready-mixed concrete

1. Introduction

The successful implementation of just-in-time (JIT) purchasing policy in various industries has prompted many companies that still use the economic order quantity (EOQ) purchasing system to ponder whether they should switch to the JIT purchasing policy (Chyr et al., 1990; D'Ouville et al., 1992; Norris, 1992; Grant, 1993; Cheng and Podolsky, 1996; Monden, 1998; Low and Chong, 2001; Wu and Low, 2003). This decision is, however, a difficult one, especially when price discount has to be considered. To help these companies make an informed decision, Fazel et al. (1998) developed a series of innovative mathematical models to directly compare the costs between the EOQ with a price discount purchasing system and the JIT purchasing system. While omitting the "fixed costs" such as rental, utilities and personnel salaries from the cost difference function between

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the EOQ and JIT system, Fazel et al. (1998) suggested that although the choice of either the EOQ or the JIT system depends on many parameters, an EOQ–JIT cost indifference point (i.e. the level of demand at which the costs were the same) existed between the EOQ system and the JIT purchasing system. Beyond the indifference point, JIT inventory purchasing was not preferred. Fazel et al. (1998) further suggested that JIT was only preferred when demand was low. In a recent paper, Schniederjans and Cao (2000) argued that those ''fixed costs'' items were not fixed and thus should not be left out from the EOQ–JIT cost difference function. Schniederjans and Cao (2000) suggested that in situations where plants adopting the JIT operations experienced or could take advantage of physical plant space square meter reduction, to include one single cost item, namely, the physical plant space factor, into the EOQ–JIT cost difference function would substantially increase the EOQ–JIT indifference point. Schniederjans and Cao (2000, p. 294) also suggested that there was a ''threshold point'' for an existing physical plant. The existing physical plant space thus might not be able to hold the substantially increased indifference point's amount of inventory. Hence, additional physical plant space had to be purchased when demand increased. The purchase of additional plant space would again provide one more opportunity for a further round of JIT square foot cost reduction (Schniederjans and Cao, 2000). Schniederjans and Cao (2000) then suggested that ''... the dynamic nature of a JIT system should continuously achieve a cost advantage over an EOQ system ...'' and the scenario ''... is much like a cat trying to catch its tail ...'' (Schniederjans and Cao, 2000, p. 294), Schniederjans and Cao (2000, p. 294) therefore concluded that ''... a JIT ordering system is preferable to an EOQ system at any level of annual demand and with almost any cost structure.'' Schniederjans and Cao (2000) also demonstrated their argument by analyzing an example, albeit, a figurative one.

JIT purchasing is not always successful. Many companies are still using the EOQ-based inventory ordering system to purchase their raw materials. This is despite the fact that the plants adopting JIT operations may experience or can take advantage of square footage reduction (Wu and Low, 2003). Fazel et al.'s (1998) models appear to be unable to clearly explain the wide adoption of the JIT policy in many companies. Likewise, Schniederjans and Cao's (2000) models seem to be unable to clearly explain the success achieved by the EOQ companies. This suggests that the real EOQ–JIT cost indifference point has not yet been derived. One possible reason is that the EOQ with a price discount model, from which Fazel et al. (1998) and Schniederjans and Cao (2000) developed their EOQ–JIT cost difference functions, was based on Harris' (1915) EOQ model. In Harris' (1915) EOQ model, namely the classical EOQ model, some inventory operating costs were assumed to be ''fixed''. This EOQ with a price discount model, which incorporated the classical EOQ model and a price discount scheme proposed by Fazel et al. (1998), is referred to as the classical EOQ with a price discount model in this study. Hence, the purpose of this study is to expand the classical EOQ with a price discount model to include those inventory operating costs that were left out. This is to derive the formula of the real EOQ–JIT cost indifference point, called the revised EOQ–JIT cost indifference point, as an extension of Fazel et al.'s (1998) and Schniederjans and Cao's (2000) works. This would be developed and tested through a survey and case study conducted in the ready-mixed concrete (RMC) industry in Singapore.

This paper will demonstrate that by including the ''physical plant space'' factor, as well as all other factors which were omitted by Fazel et al. (1998), it is still possible for an EOQ system to be more cost effective than a JIT system. The EOQ with a price discount model can be categorized into two scenarios where the optimal order quantity is below or above $Q_{\max}$, the maximum quantity that can be purchased and still receive a quantity discount rate π_{E}. Hence, the research aim will also be approached through covering these two scenarios. Before the main discussion, it will be helpful to revisit the EOQ with a price discount model to better understand the concept of the optimal order quantity, $Q_{\max}$, π_{E} and the two scenarios of the EOQ with a price discount model.

2. Classical EOQ with a price discount model

The cost incurred by suppliers is usually a decreasing function of the size of the delivery lot; the delivery price of an inventory is thus usually a decreasing function of the order quantity (Fazel et al., 1998). Goyal and Gupta (1989) concluded that there were three basic types of price discount models, namely, *Two-Part Tariff*, *Two-Block Tariff* and *All Unit Quantity Discount*. In the *Two-Part Tariff* price discount model, "the buyer is required to pay a fixed charge and a uniform price p for all units purchased. Although the buyer pays the same marginal unit price for all quantities, the average price paid is a monotonically decreasing function of quantity purchased" (Goyal and Gupta, 1989, p. 263). In the *Two-Block Tariff* price discount model, "the price of a unit, p_1, is maintained up to a quantity x, the per unit price p_2 is charged for all units in excess of quantity ($p_1 \succ p_2$) (Goyal and Gupta, 1989, p. 263). In the *All Unit Quantity Discount* model, when "a buyer buys more than a quantity x, the price of all units is decreased (Goyal and Gupta, 1989, p. 263). Britney et al. (1983a, b), Dolan (1987) and Wilcox et al. (1983) also suggested several variations within each of the three basic types of price quantity discount models. Please refer to Britney et al. (1983a, b), Dolan (1987) and Wilcox et al. (1983) for the details of the variations within each of the three basic types of price discount models.

Both Schniederjans and Cao's (2000) and Fazel et al.'s (1998) EOQ–JIT cost difference functions were based on a price discount scheme in which "… for quantities below a certain level ($Q_{\max}$) the delivery price is a decreasing, continuous and linear function of the order quantity. Beyond $Q_{\max}$, however, the price stays at its minimum ($P_E^{\min}$) which is the lowest price the supplier would charge no matter how large the order quantity is" (Fazel et al., 1998, p. 104). The discount functions are mathematically defined as (1) $P_E = P_E^0$ for $Q = 0$ where P_E^0 is the purchase price per unit when the order quantity equals zero and Q is the fixed order quantity;

(2) $$\frac{dP_E}{dQ} = -\pi_E \quad \text{or} \quad P_E = P_E^0 - \pi_E Q \quad \text{for } Q \leqslant Q_{\max},$$

where π_E is a constant representing the quantity discount rate (the rate at which the price of the item decreases with increase in order quantity); and (3) $P_E = P_E^{\min}$ for $Q \geqslant Q_{\max}$. This price discount scheme was one variation of the *All Unit Quantity Discount* model, as the buyer pays the same unit price for every unit purchased (Fazel et al., 1998). Since this study was developed based on the platform created by Fazel et al. (1998), the same price discount scheme is used in this study. It is important to note that P_E^0 is the initial condition for the first-order differential equation $dP_E/dQ = -\pi_E$ and may not be a feasible price, as the order quantity can never be zero in reality. If the inventory is not divisible, then the lowest order quantity ($Q_{\min}$) which one can order must be 1 with an unit price of P_E^1. In such a case, $P_E^0 = P_E^1 + \pi_E$. It is also important to note that the models in this paper were developed based on the *All Unit Quantity Discount* price scheme. The models in this paper may not therefore be applicable to the *Two-Part Tariff* and the *Two-Block Tariff* price discount schemes.

The classical EOQ with a price discount model aims to minimize the total of ordering and holding costs, while assuming that some inventory operating costs such as rental, utilities and personnel salary, etc. are "fixed" costs. The total annual cost of the classical EOQ system, TC_E, is the sum of the inventory ordering cost, inventory holding cost, and the cost of the actual purchased units, or

$$TC_E = \frac{kD}{Q} + \frac{Qh}{2} + (P_E^0 - \pi_E Q)D \quad \text{for } Q \leqslant Q_{\max}, \tag{1}$$

where h is the annual cost of holding one unit of inventory in stock, k is the cost of placing an order, D is the annual demand for the item, D/Q is the annual ordering frequency, $Q/2$ is the annual average inventory level in the inventory facility. The first ratio is the inventory ordering cost item. The second ratio is the inventory holding cost item. The last item is the annual purchasing cost component. Eq. (1) is for order quantities below $Q_{\max}$. For order quantities above $Q_{\max}$, as stated earlier, the minimum purchase price of $P_E^{\min}$ remains constant and the TC_E function thus becomes

$$\mathrm{TC_E} = \frac{kD}{Q} + \frac{Qh}{2} + P_\mathrm{E}^{\min} D \quad \text{for } Q \succ Q_{\max}. \tag{2}$$

Chalos (1992) suggested that k and h are the most subjective components in Eqs. (1) and (2). Nevertheless, k usually includes the inventory delivery charges and transaction costs relating to clerical paperwork. h often includes opportunity cost of the working capital tied up in purchased goods, taxes and insurance paid on inventory items, inventory spoilage cost and inventory obsolescence cost. The classical EOQ with a price discount model provides appropriate inventory ordering decisions only when its assumptions can be met. These assumptions are: (1) the inventory operating costs, rental, utilities and personnel salary, etc. are constant; and (2) h the annual cost of holding one unit of inventory in stock and k the cost of placing an order are constant. It should be noted that although the term "the total annual cost of an inventory item under an EOQ system" was used to refer to "$\mathrm{TC_E}$" in Eqs. (1) and (2) by Fazel et al. (1998) and Schniederjans and Cao (2000), "$\mathrm{TC_E}$" was not the actual total annual cost of an inventory item under an EOQ system. The actual total annual cost of an inventory item under an EOQ system should be the sum of "$\mathrm{TC_E}$" and the "fixed costs".

As mentioned earlier, the so-called "fixed costs", including "rental, utilities, and personnel salary" were excluded from the inventory holding cost item in Eqs. (1) and (2). This was an important assumption made by Fazel et al. (1998) and Schniederjans and Cao (2000) when they derived their EOQ–JIT cost indifference points. However, since (a) it is agreed that the so-called "fixed costs" were left out from the so-called "total annual cost of the EOQ system", and (b) Gaither (1996) suggested that the annual inventory holding cost should include the opportunity cost of the working capital tied up in purchased goods, taxes and insurance paid on inventory items, inventory spoilage cost and inventory obsolescence cost, together with the cost of physical storage, and (c) Schonberger (1982), Wantuck (1989), etc. proved that the so-called "fixed costs" would no longer be constant during JIT operations, and (d) Schniederjans and Olsen (1999) and Schniederjans and Cao (2000, 2001) observed that the saved inventory facilities can be rented out when the annual average inventory level dropped, then there is a reason to include all components of inventory holding costs into the holding cost item, when comparing an EOQ system with a JIT system. To sum up, one of the assumptions of the classical EOQ model, namely, the so-called "fixed" costs were excluded from the holding cost item, needs to be revised. Consequently, the classical EOQ with a price discount model needs to be expanded when comparing an EOQ purchasing system with a JIT purchasing system.

3. Revised EOQ with a price discount model

The revised EOQ model was identified from the RMC industry in land-scarce Singapore. The expensive land rental in Singapore promoted the RMC suppliers to reduce the size of their inventory facilities to save on inventory holding costs. "An inventory facility", in this study, is defined as a physical plant place where raw materials, goods or merchandise are stored. An inventory facility can be a storehouse, a warehouse, an aggregates depot, or a cement terminal. The total cost under the revised EOQ with a price discount model is thus

$$\mathrm{TC_{Er}} = \frac{kD}{Q} + \frac{QH}{2} + (P_\mathrm{E}^{0} - \pi_\mathrm{E} Q)D \quad \text{for } Q \leqslant Q_{\max}, \tag{3}$$

$$\mathrm{TC_{Er}} = \frac{kD}{Q} + \frac{QH}{2} + P_\mathrm{E}^{\min} D \quad \text{for } Q \succ Q_{\max}. \tag{4}$$

$\mathrm{TC_{Er}}$ is the sum of the inventory ordering cost, the expanded inventory holding cost, and the cost of the actual purchased units. $\mathrm{TC_{Er}}$, with the inclusion of the so-called "fixed costs", is the actual total cost of the EOQ ordering system and is thus greater than $\mathrm{TC_E}$ in Eqs. (1) and (2). H is the expanded annual cost of holding one unit of inventory in stock. "H", with the inclusion of the additional inventory holding costs, is thus significantly greater than "h" in Eqs. (1) and (2). The revised EOQ model assumes that the so-called "fixed costs", including rental, utilities and personnel salary are proportion to the annual average inventory level. This assumption is possible, particularly when the square meter area of an inventory facility is designed in proportion to its annual average inventory level and the rental, utilities and personnel salary are in proportion to the size of the inventory facility. The revised EOQ with a price discount model is particularly suitable for the scenarios in which the so-called "fixed costs" are not fixed, for example during the feasibility study stage and design stage of an inventory facility, or the excess inventory facility space can be rented out when the annual average inventory level drops, as observed by Schniederjans and Olsen (1999) and Schniederjans and Cao (2000, 2001).

By taking the first-order derivative with respect to Q of Eq. (3) and setting it to equal to zero, the optimum order quantity under the revised EOQ with a price discount system, Q_r^*, can be derived as

$$Q_r^* = \sqrt{\frac{2kD}{H - 2\pi_E D}}. \tag{5}$$

Note that Q_r^* is the optimum order quantity for the revised EOQ system only when $Q_r^* \leqslant Q_{max}$. For an order quantity above Q_{max}, as mentioned earlier, the price will be at a fixed level, P_E^{min}. The optimum order quantity, Q_r^{**}, minimizing the sum of ordering and holding costs of the revised EOQ system, is thus given by taking the first-order derivative with respect to Q of Eq. (4) as

$$Q_r^{**} = \sqrt{\frac{2kD}{H}}. \tag{6}$$

Q_r^{**} is the optimum order quantity for the revised EOQ with a price discount system if Q_r^{**} is greater than Q_{max}. Otherwise, the total cost associated with an order quantity that equals Q_r^{**} should be calculated and compared with the total cost for an order quantity that equals Q_{max}. The quantity that yields the lower cost is the optimum order quantity for the revised EOQ system. The optimum order quantity for the revised EOQ with a price discount model may thus be Q_r^* (Eq. (5)), Q_r^{**} (Eq. (6)), or Q_{max}. Nevertheless, it can be proved that the optimal order quantity for the revised EOQ with a price discount system is governed by Eq. (5) when

$$D \prec \frac{Q_{max}^2 H}{2\pi_E Q_{max}^2 + 2k},$$

equal to Q_{max} when

$$\frac{Q_{max}^2 H}{2\pi_E Q_{max}^2 + 2k} \leqslant D \leqslant \frac{H}{2k} Q_{max}^2$$

and governed by Eq. (6) when

$$D \succ \frac{H}{2k} Q_{max}^2.$$

Similarly, the optimal order quantity for the classical EOQ with a price discount model derived from Eq. (1) is

$$Q^* = \sqrt{\frac{2kD}{h - 2\pi_E D}}. \tag{7}$$

The optimal order quantity for the classical EOQ with a price discount model derived from Eq. (2) is

$$Q^{**} = \sqrt{\frac{2kD}{h}}. \tag{8}$$

Q^* is the optimum order quantity for the classical EOQ with a price discount model only when $Q^* \leqslant Q_{max}$. Q^{**} is the optimum order quantity for the classical EOQ with a price discount model if Q^{**} is greater than Q_{max}. Otherwise, the total cost associated with an order quantity that equals Q^{**} should be calculated and compared with the total cost for an order quantity that equals Q_{max}. The quantity that yields the lower cost is the optimum order quantity. The optimum order quantity for the classical EOQ with price discount model may thus be Q^* (Eq. (7)), Q^{**} (Eq. (8)), or Q_{max} (Fazel et al., 1998).

The optimum order quantity of the revised EOQ with a price discount model, Q_r^* and Q_r^{**} are significantly less than the optimum order quantity of the classical EOQ with a price discount model, Q^* and Q^{**}, respectively, as H the annual cost of holding one unit of inventory in the revised EOQ with a price discount model is substantially greater than h the annual cost of holding one unit of inventory in the classical EOQ with a price discount model, supposing the values of the other parameters, namely, D, k and π_E are the same.

To sum up, the revised EOQ with a price discount model is different from the classical EOQ with a price discount model on four counts. Firstly, the so-called "fixed costs", such as rental, utilities, personnel salaries, etc., are considered in the inventory holding cost item in the revised EOQ with a price discount model, thus $H \succ h$. Secondly, the so-called "fixed costs" are also included into the total annual inventory costs in the revised EOQ with a price discount model, thus $TC_{Er} \succ TC_E$. Thirdly, the revised EOQ with a price discount model prefers small lot sizes and frequent deliveries. Last but not least, the revised EOQ with a price discount model aims to reduce the actual total inventory ordering and holding cost, while the classical EOQ with a price discount model aims to reduce the sum of the inventory ordering cost and a part of the inventory holding cost. The last point makes it very clear that the revised EOQ with a price discount model is more suitable than the classical EOQ with a price discount model in representing the total cost under the EOQ with a price discount system when comparing the EOQ system with the JIT system.

4. Revised EOQ–JIT cost indifference point

The earlier discussion indicates that the EOQ with a price discount model can roughly be categorized into two scenarios: the optimal order quantity is below $Q_{\max}$ or the annual demand is below

$$\frac{Q_{\max}^2 H}{2\pi_E Q_{\max}^2 + 2k}$$

and the optimal order quantity is above $Q_{\max}$ or the annual demand is above $(H/2k)Q_{\max}^2$. The revised EOQ–JIT cost indifference points for the two scenarios are developed as below.

4.1. Order quantity below $Q_{\max}$

It was stated earlier that when order quantities is less than $Q_{\max}$, the optimal order quantity for the revised EOQ with a price discount model is governed by Eq. (5). Eq. (5) results in a total annual optimal cost under the EOQ purchasing approach of

$$TC_{Er} = kD\sqrt{\frac{H - 2\pi_E D}{2kD}} + \frac{H}{2}\sqrt{\frac{2kD}{H - 2\pi_E D}} + \left[P_E^0 - \pi_E\sqrt{\frac{2kD}{H - 2\pi_E D}}\right]D. \tag{9}$$

It is essential to note that Eq. (9) is valid only when two conditions are concurrently satisfied. Firstly, k the cost of placing one order and H the expanded annual cost of holding one unit of inventory are constant. Secondly, the inventory is ordered at its optimal order quantity and the optimal order quantity is below $Q_{\max}$.

Under the JIT system, the ordering cost and holding cost, including the so-called "fix costs" are mainly transferred to the supplier. The total annual cost under the JIT system, TC_J, thus, is the annual purchase cost (Fazel et al., 1998), given by

$$TC_J = P_J D, \tag{10}$$

where P_J is the unit price under the JIT system. P_J is greater than P_E^0. This is to partially reflect the holding costs and ordering costs that have been transferred to the materials suppliers (Fazel et al., 1998). To make a comparison between the total costs under the EOQ system and the JIT system, a Z_r model that combines the total annual optimal cost under the EOQ system in Eq. (9) and the total annual cost under the JIT system in Eq. (10) can be presented as

$$Z_r = kD\sqrt{\frac{H - 2\pi_E D}{2kD}} + \frac{H}{2}\sqrt{\frac{2kD}{H - 2\pi_E D}} + \left[P_E^0 - \pi_E\sqrt{\frac{2kD}{H - 2\pi_E D}}\right]D - P_J D. \tag{11}$$

Z_r represents the cost difference between an EOQ purchasing system and a JIT purchasing system. Setting Z_r equal to zero, the root of Eq. (11) is the revised EOQ–JIT cost indifference point, D_{indr}

$$D_{\text{indr}} = \frac{2kH}{(P_{\text{J}} - P_{\text{E}}^{0})^2 + 4\pi_{\text{E}}k}. \quad (12)$$

4.2. Order quantity above $Q_{\max}$

When order quantities are above $Q_{\max}$, the optimal order quantity for the revised EOQ with a price discount model is governed by Eq. (6). Eq. (6) results in $\text{TC}_{\text{Er}}^{**}$ the total annual optimal cost under the EOQ purchasing approach of

$$\text{TC}_{\text{Er}}^{**} = \sqrt{2kDH} + P_{\text{E}}^{\min}D. \quad (13)$$

The Z_{r}^{**} model that combines the total annual optimal cost under the EOQ system in Eq. (13) and the total annual cost under the JIT system in Eq. (10) can be presented as

$$Z_{\text{r}}^{**} = \sqrt{2kDH} + P_{\text{E}}^{\min}D - P_{\text{J}}D. \quad (14)$$

Setting Z_{r}^{**} equal to zero, the root of Eq. (14) is the revised EOQ–JIT cost indifference point, D_{indr}^{**}, when order quantities are above $Q_{\max}$, given by

$$D_{\text{indr}}^{**} = \frac{2kH}{(P_{\text{J}} - P_{\text{E}}^{\min})^2}. \quad (15)$$

5. Discussion

5.1. Comparing the present study with that of Fazel et al. (1998)

D_{indr} in Eq. (12) is similar to D_{indF}, the EOQ–JIT cost indifference point proposed by Fazel et al. (1998), which is given by

$$D_{\text{indF}} = \frac{2kh}{(P_{\text{J}} - P_{\text{E}}^{0})^2 + 4\pi_{\text{E}}k}.$$

Since H is significantly greater than h, D_{indr} the revised EOQ–JIT cost indifference point is significantly greater than D_{indF} the EOQ–JIT cost indifference point proposed by Fazel et al. (1998), supposing the values of the other parameters, namely, k, P_{J}, P_{E}^{0} and π_{E} are the same. This is because some inventory operating costs including rental, utilities, personnel salary, etc. were not accounted for in D_{indF} (Schniederjans and Cao, 2000). This finding seems to suggest that the JIT system can still remain cost effective even at a high level of annual demand, thus modifying Fazel et al.'s (1998) conclusion that JIT was cost effective only at low level of annual demand.

5.2. Comparing the present study with that of Schniederjans and Cao (2000)

The concept of the holding capacity of an inventory facility can assist to compare the present study with that of Schniederjans and Cao (2000). The "holding capacity of an inventory facility" is defined as the number of inventory that can be held by the inventory facility at a specific time. Assuming each unit of an inventory item takes up α m^2 inventory facility, the holding capacity of an inventory facility can be derived by dividing the number of square meters of an inventory facility by the square meters occupied by a unit of inventory, or: $N_{\text{E}}/\alpha = Q_{\text{h}}$, where Q_{h} is the holding capacity of an inventory facility and N_{E} is the square meter area of an inventory facility under an EOQ system. To allow for flexibility, the size of the inventory facility, in practice, is usually designed to be greater than the size needed to hold the exact amount of optimal order quantity of inventory. It is reasonable to assume that the size of the inventory facility is b times the size which holds the optimal order quantity amount of inventory, or $Q_{\text{h}} = bQ_{\text{r}}^{*}$ for $Q \leqslant Q_{\max}$, or $Q_{\text{h}} = bQ_{\text{r}}^{**}$ for $Q \succ Q_{\max}$, where b, called the stock flexibility parameter, is greater than or equal to 1. Within the EOQ-based model, where the safety stock is not considered and the consumption of the inventory is regular and predictable, it is possible for b to approach the value of 1.

5.2.1. For scenarios where $Q \leqslant Q_{\max}$

Substituting $Q_{\text{h}} = bQ_{\text{r}}^{*}$ into $N_{\text{E}}/\alpha = Q_{\text{h}}$, would result in $N_{\text{E}} = \alpha bQ_{\text{r}}^{*}$, namely, the formula of the square meter area of an inventory facility governed by its optimal order quantity when order quantities are below $Q_{\max}$. Substituting D_{indr} in Eq. (12) into Eq. (5), Q_{rind}^{*} the optimal order quantity at the revised EOQ–JIT cost indifference point can be derived as

$$Q^{*}_{\mathrm{rind}} = \frac{2k}{P_{\mathrm{J}} - P^{0}_{\mathrm{E}}}.$$

Substituting Q^{*}_{rind} for Q^{*}_{r} in $N_{\mathrm{E}} = \alpha b Q^{*}_{\mathrm{r}}$, would result in N_{Eind} the minimum square meter area of the inventory facility that can accommodate the EOQ–JIT cost indifference point's amount of inventory as

$$N_{\mathrm{Eind}} = \frac{2\alpha bk}{P_{\mathrm{J}} - P^{0}_{\mathrm{E}}}.$$

Hence, once the inventory space reaches $2\alpha bk/(P_{\mathrm{J}} - P^{0}_{\mathrm{E}})$ and the annual demand reaches

$$\frac{2kH}{(P_{\mathrm{J}} - P^{0}_{\mathrm{E}})^2 + 4\pi_{\mathrm{E}}k},$$

and if

$$\frac{2kH}{(P_{\mathrm{J}} - P^{0}_{\mathrm{E}})^2 + 4\pi_{\mathrm{E}}k}$$

is below

$$\frac{Q^{2}_{\max}H}{2\pi_{\mathrm{E}}Q^{2}_{\max} + 2k}$$

or the order quantity is below $Q_{\max}$, the physical inventory plant space under the EOQ system can accommodate D_{indr} amount of inventory. The total cost under the EOQ system, where the cost of the physical inventory plant space under the EOQ system has been included, will be equal to the total cost under the JIT system.

5.2.2. For scenarios where $Q \succ Q_{\max}$

Substituting $Q_{\mathrm{h}} = bQ^{**}_{\mathrm{r}}$ into $N_{\mathrm{E}}/\alpha = Q_{\mathrm{h}}$, would result in $N_{\mathrm{E}} = \alpha b Q^{**}_{\mathrm{r}}$, namely, the formula of the square meter area of an inventory facility governed by its optimal order quantity when order quantities are above $Q_{\max}$. Substituting the D^{**}_{indr} in Eq. (15) into Eq. (6), Q^{**}_{rind} the optimal order quantity at the revised EOQ–JIT cost indifference point can be derived as

$$Q^{**}_{\mathrm{rind}} = \frac{2k}{P_{\mathrm{J}} - P^{\min}_{\mathrm{E}}}.$$

Substituting Q^{**}_{rind} for Q^{**}_{r} in $N_{\mathrm{E}} = \alpha b Q^{**}_{\mathrm{r}}$, would result in N^{**}_{Eind} the minimum square meter area of the inventory facility that can accommodate the EOQ–JIT cost indifference point's amount of inventory, when order quantities are above $Q_{\max}$, as

$$N^{**}_{\mathrm{Eind}} = \frac{2\alpha bk}{P_{\mathrm{J}} - P^{\min}_{\mathrm{E}}}.$$

Hence, once the inventory space reaches $2\alpha bk/(P_{\mathrm{J}} - P^{\min}_{\mathrm{E}})$ and the annual demand reaches $(2kH/(P_{\mathrm{J}} - P^{\min}_{\mathrm{E}})^2)$, and if $(2kH/(P_{\mathrm{J}} - P^{\min}_{\mathrm{E}})^2)$ is above $(H/2k)Q^{2}_{\max}$ or the order quantity is above $Q_{\max}$, the physical inventory plant space under the EOQ system can accommodate D^{**}_{indr} amount of inventory. The total cost under the EOQ system, where the cost of the physical inventory plant space under the EOQ system has been included, will be equal to the total cost under the JIT system.

The cost of the physical inventory plant space under the EOQ system can be balanced by the JIT system. For example, if F represents the annual cost to own and maintain a square meter of physical inventory plant space, $2\alpha bF$ is then a component of H. It seems that Schniederjans and Cao (2000) overlooked that it was possible for an inventory facility to hold the EOQ–JIT cost indifference point's amount of inventory once the square meters of an inventory facility reach N_{Eind} or N^{**}_{Eind}. Hence, the implication of this finding is that an EOQ-based system can be more cost effective than a JIT system when (1) the size of the inventory facility is above N_{Eind} and the magnitude of the annual demand is above the EOQ–JIT cost indifference point D_{indr}, if the optimal order quantity is below $Q_{\max}$, or the annual demand is below

$$\frac{Q^{2}_{\max}H}{2\pi_{\mathrm{E}}Q^{2}_{\max} + 2k};$$

or (2) the size of the inventory facility is above N^{**}_{Eind} and the magnitude of the annual demand is above the EOQ–JIT cost indifference point D^{**}_{indr}, if the optimal order quantity is above $Q_{\max}$, or the annual demand is above $(H/2k)Q^{2}_{\max}$.

The revised EOQ–JIT cost indifference point and the minimum square meter area of the inventory facility that can accommodate the EOQ–JIT cost indifference point's amount of inventory, and the impact of the annual demand on the adoption of appropriate purchasing systems are further illustrated by a survey and a case study below. The survey covered the RMC industry in Singapore. The case study was from the cement division of one RMC supplier in Singapore.

6. A survey

RMC is a product that is widely used in the construction of building and civil works in the construction industry. The production of RMC is a highly repetitive manufacturing process. Cement, one of the raw materials of RMC may be purchased either using the EOQ system (Tommelein and Li, 1999) or the JIT system (Wu and Low, 2003).

To understand the procurement systems of cement adopted by the RMC suppliers in Singapore, a survey and intensive site studies were conducted by the authors from April 2002 to July 2002 and from May 2003 to December 2003, respectively. The survey covered all the 15 registered members of the Ready Mixed Concrete Association of Singapore (RMCAS). These 15 RMC suppliers were the major players in the RMC market in Singapore. They were RDC Concrete Pvt. Ltd., Hanson Concrete Pvt. Ltd., Eastern Concrete Pvt. Ltd., Island Concrete Pvt. Ltd., G&W Readymix Pvt. Ltd., Lafarge Concrete Pvt. Ltd., Pan United Concrete Pvt. Ltd., Topmix Concrete Pvt. Ltd., Jurong Readymix Concrete Pvt. Ltd., Oriental Concrete Pvt. Ltd., Trimix Pvt. Ltd., Sunway Concrete Products Pvt. Ltd., Goodmix Investment Pvt. Ltd., WP Conc-pact Pvt. Ltd., and Bilicon Readymix Pvt. Ltd.

The survey showed that among all the RMC suppliers surveyed, approximately 60 percent of them purchased cement using the JIT system and 40 percent of them ordered their cement using the EOQ-based system. The survey results are summarized in Table 1. Company names are not specified in Table 1 in order to preserve their anonymity. Nevertheless, the information in Table 1 suffices to provide an overall picture of the purchasing system of cement adopted by RMC suppliers in Singapore. It is interesting to note that the cement division of RMC supplier company D, E, F, G and I also sold cement to the other RMC suppliers.

Table 1
Cement purchasing systems practiced by RMC suppliers in Singapore

	RMC suppliers														
Material	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Cement	○	○	○	☻	☻	☻	☻	○	☻	○	○	○	○	○	○

Notes: "○" denotes that cement is purchased using the JIT system. "☻" denotes that the cement is purchased using the EOQ system.

Most of the cement consumed by Singapore RMC suppliers was imported from overseas, mainly Japan, Taiwan and China. The ordering costs were thus high. These made the small and frequent orders of cement impracticable. Furthermore, only five RMC suppliers had their own cement divisions who were engaged in the cement business. Hence, the demand volume of each cement division in these five RMC suppliers was quite high. In addition, the holding cost of cement was also high. For these reasons, the cement divisions of these five RMC suppliers have built their own huge silos and procured their cement in an EOQ-based system.

7. A case study

The cement division of RMC supplier I used an EOQ system to procure cement from Japan. The cement division of this RMC supplier built two silos on Pulau Damar Laut island. The designed holding capacity of each of the silo was approximately 25,000 ton. The average area occupied by one silo was about 2800 m^2. The sum of the holding capacities of the two silos was approximately 40,000 ton (where safety and flexibility factor had been considered. Stock flexibility parameter was $b = 1.25$). As stated previously, most of the cement consumed in the Singapore RMC industry was imported from overseas. Cement imported by the cement division of supplier I was mainly from Japan using 40,000 ton cement carriers in the year 2003. Supplier I raised an order approximately once per month for about 40,000 ton of cement. The annual demand, D, was 520,000 ton in 2003. The order was often raised 2 or 3 months before the departure of a cement carrier from Japan. The Japanese cement manufacturer offered a few alternative pricing strategies. One of the pricing strategies offered was as follows. The delivery price started at $P_E^0 = \text{S\$ } 45/\text{ton}$. For every additional ton ordered, the price would decrease by $\pi_E = 2.5 \times 10^{-4}$ dollar for the entire order lot. The discount was valid for order quantity up to $Q_{max} = 20,000$ ton, when the price per unit became $P_E^{min} = \text{S\$ } 40/\text{ton}$. Beyond this level, the price remains the same. The annual cost of holding 1 ton of cement was $H = \text{S\$ } 344$/year per ton. The holding cost can be broken down into $h_{checkin}$ cement check in cost, $h_{storage}$ cement storage costs, and $h_{checkout}$ cement check out cost. The cement check in cost was the depreciation and operating cost of the facilities to unload cement from a cement carrier to a silo and $h_{checkin} = \text{S\$ } 199$/year per ton. The cement storage costs were $h_{storage} = \text{S\$ } 45$/year per ton. The cement storage costs include the depreciation cost of the silo facilities, utilities, personnel salary, property tax, insurance, cement spoilage cost, opportunity cost of the working capital tied up with the purchased cement and the land rental. The cement check out cost was the depreciation cost and operating cost of the facilities, mainly cement trucks, to deliver cement from the silo to a RMC batching plant and $h_{checkout} = \text{S\$ } 100$/year per ton. The cost of placing an order was $k = \text{S\$ } 432{,}000$/order for transportation alone. Each ton of cement took up $\alpha = 0.112\,\text{m}^2/\text{ton}$ inventory facility space. The annual cost to rent a square meter of inventory facility was $F = \text{S\$ } 84$/year per m^2. If cement was purchased under a JIT system in Singapore, the cost was $P_J = \text{S\$ } 69$/ton. It is essential to note that the data for this case study were collected by interviewing the overseas investment manager, the financial manager, the production manager and the customer service supervisor of the cement division of supplier I. These data are real-life data and are unlike those demonstrated in the hypothetical examples given by Fazel et al. (1998, p. 107) and Schniederjans and Cao (2000, p. 291).

The routine order quantity, 40,000 ton/order, was above Q_{max}. Eq. (6) was therefore suitable for calculating the optimal order quantity. Based on Eq. (6), the economic order quantity was $Q_r^{**} = 36,139\,\text{ton/order}$. Q_r^{**} was close to the routine order quantity 40,000 ton/order. Eq. (15) thus can be used to derive the EOQ–JIT cost indifference point. According to Eq. (15), D_{indr}^{**} the EOQ–JIT cost indifference point was 353,408 ton.

$$N_{Eind}^{**} = \frac{2\alpha bk}{P_J - P_E^{min}}$$

can be used to derive N_{Eind}^{**} the minimum square meter area of the inventory facility that can accommodate the EOQ–JIT cost indifference point's amount of inventory. N_{Eind}^{**} was derived as 4171 m^2. Since the areas in square meters of the two silos, 5600 m^2, were greater than N_{Eind}^{**}, and since the annual demand, 520,000 ton was greater than D_{indr}^{**}, the EOQ system was found to be more cost effective than the JIT system.

The batching capacity of the widely used batching plant was 90 m^3/h in Singapore. The average demand for cement of the 90 m^3/h batching plant was approximately 40,500 ton/year during 2003 as estimated by the production manager of the RMC batching plant division of RMC supplier I. It was found that the annual demand of one batching plant was significantly less than the break-even point. Hence, 60 percent of the RMC suppliers surveyed ordered their cement under a JIT system. These RMC suppliers used a number of 100 ton silos to store their buffer stock. The 100 ton silos were filled on a daily basis. To reap economies of scale, the cement divisions of approximately 40 percent of the RMC suppliers surveyed, namely, Supplier D, E, F, G, together with I, have however, built a number of huge multi-cells silos on the Pulau Damar Laut island. The cement received at the Pulau Damar Laut Island was then delivered to their RMC batching plant divisions and batching plants of other RMC suppliers. It is important to highlight that the holding capacity of a square meter of the physical plant space where 100 ton silos are used is 0.115. The holding capacity of a square meter of the physical plant space where 25,000 ton silos are used is 0.109. Hence it is reasonable to assume that

α, the holding capacity of a square meter of the physical plant space, is a constant.

The practices of supplier D, E, F, G and I appear to suggest that when the demand is greater than the break-even point, the EOQ system can be more cost effective than the JIT system. More importantly, by operating in an EOQ based system, supplier I could avoid the expensive out-of-stock costs, which may happen during the typhoon season under the JIT system. Japan, Taiwan and China, where the cement is imported from, are all subject to the vagaries of typhoons during the spring and autumn seasons. In addition, supplier I can even influence cement prices in the Singapore cement market, thus placing it in a better position to compete against those RMC suppliers who purchased cement in a JIT system. During the prolonged economic slump in the construction industry of Singapore, JIT RMC supplier O was bought over by EOQ supplier I in January 2003. JIT company RMC supplier K went into liquidation in August 2003. The quality engineer of JIT RMC supplier K divulged that its revolving trucks, which were bought at about S$ 100,000 per unit were sold at only S$10,000 per unit. Its 90 m^3/h batching plant, which was bought at about S$ 400,000 per unit, was sold at about S$ 80,000 per unit. JIT RMC supplier B, ranked No. 4 in terms of production capability was bought over. The overseas investment manager of the cement division of the EOQ supplier I divulged that JIT RMC supplier B was established through the purchase of the then PN RMC Pvt. Ltd. at S$ 35 million and the then RT RMC Pvt. Ltd. at S$ 14 million in year 1999. JIT supplier B was sold at only S$ 4 million in November 2003. The selling price of RMC supplier B was less than 10 percent of its initial purchase price. RMC supplier B was purchased by others in less than 4 years!

In a more recent paper, Schniederjans and Cao (2001) quoted Pan and Liao's (1989) study to support their argument that JIT was virtually always the preferable alternative for inventory purchasing decision. In Pan and Liao's (1989) study, the EOQ model was converted into a series of JIT purchasing models that can be used to determine inventory deliveries and cost savings and demonstrated that there was no limitation on the cost advantage of using JIT based on the model parameter of annual demand. This raises the question whether it was possible or not to use 2500 ton cement carriers, rather than 40,000 ton cement carriers, to conduct frequent deliveries, thus significantly reducing the total annual cost of cement purchasing. This question was raised to the production manager of the cement division of supplier I. The production manager explained that the transportation of bulk Portland cement must use specialized transportation vehicles, such as cement trucks or cement carriers. The transportation cost of bulk Portland cement from Japan to Singapore by a 40,000 ton cement carrier was about S$ 10.8/ton. The transportation cost of bulk Portland cement from Japan to Singapore by a 2500 ton cement carrier was about S$ 20.0/ton. The transportation cost of bulk Portland cement in Singapore by a cement truck was as high as S$ 0.3/ton per kilometer. In addition, as indicated by π_E the purchase price can be increased if cement was ordered in small lot sizes. The difference between the selling price, P_J and the purchase price, P_E^0, was only S$ 24. The average delivery cost of cement was around S$ 4.0/ton in Singapore, where it was assumed that the average transportation distance was between 10 and 20 km, because Singapore is a relatively small island. In addition, the expensive operating and depreciation costs of the cement silos and cement check-in facilities must be paid. To sum up, it was not economically justifiable for the cement division of supplier I to break down its order size from 40,000 to 2500 ton to match the available cement carriers and to deliver in a JIT pattern.

The survey also showed that almost all the RMC suppliers purchased their aggregates and sand using the JIT system. Aggregates and sand consumed by the RMC industry were mainly imported from Indonesia by 2500 or 4000 ton barges. The transportation distance is much shorter than that of cement, hence, rendering the small and frequent orders and deliveries of aggregates and sand economically feasible. Furthermore, as suggested by the aggregates and sand supervisor of RMC supplier B, more than twenty companies were engaged in the aggregates and sand business and they have rented depots in Pasir Ris, Tuas, or

Sungei Kadut. Hence, this rendered the demand volume of each of the RMC suppliers to be relatively limited. In addition, the land rental of the aggregates and sand depot in Pasir Ris, Tuas, or Sungei Kadut was not cheap. These appear to be the reasons why almost all the aggregates and sand consumed by the RMC industry in Singapore were purchased in a JIT fashion. The purchase of aggregates and sand from Indonesia and the procurement of cement from Japan by the RMC industry in Singapore seem to confirm Pan and Liao's (1989) conclusion. They suggested that the annual demand would not affect the adoption of inventory purchasing system when it was assumed that order splitting did not affect the ordering cost.

The survey and the case study seem to suggest that although JIT may be the better alternative in most cases, the EOQ system will remain to be advantageous, especially during an economically slump period and where inventory order size is very large. JIT companies dominated where liquidation was concerned in the RMC industry of Singapore. One possible reason for this is that the large EOQ companies, who also supply raw materials to the small and medium sized JIT companies, can control the market prices of some raw materials to some extent. These EOQ companies, who consumed large amount of raw materials and also sold large volume of raw materials to the JIT companies, can therefore intentionally, by mutual agreement, raise the raw material prices, or at least maintain the prices of the raw materials. The selling price of a product will usually drop during an economic slump, thus causing the JIT companies into liquidation. The EOQ companies can still make a profit from selling raw materials. Once the JIT companies go into liquidation, the EOQ companies, with fewer competitors, can then ''unite'' again to increase the product's selling price to make more profit. This industrial practice of the RMC industry in Singapore seems to suggest that JIT may not always be preferred over an EOQ system. More importantly, the case study is able to demonstrate that the concept of the revised EOQ–JIT cost indifference point can withstand examination from industry practices.

8. Conclusions and further study

JIT purchasing of raw materials is an important technique of the JIT philosophy. However, JIT purchasing is not always more cost effective than the EOQ purchasing system. By expanding a classical EOQ with a price discount model, conducting a survey and a case study in the RMC industry in Singapore, this study suggests that it is possible for the EOQ purchasing system to be more cost effective than the JIT purchasing system. By including all the cost items that were omitted by Fazel et al. (1998) and suggested by Schniederjans and Cao (2000), this study suggests that Fazel et al. (1998) tended towards the EOQ system and Schniederjans and Cao (2000) tended towards the JIT system. The survey and case study also suggested that JIT was probably a better alternative in most cases. However, the EOQ system can be more cost effective than a JIT system, especially when the order size cannot be cost effectively split into a number of smaller-sized and more frequent order deliveries.

The EOQ with price discount models had three basic types (Goyal and Gupta, 1989). The models in this study were developed based on the price discount scheme proposed by Fazel et al. (1998) which was a variation of the *All Unit Quantity Discount* scheme. To complete the cost comparison between the JIT purchasing and the EOQ with price discount inventory management systems, further investigations to study the *Two-Part Tariff* and *Two-Block Tariff* price discount schemes are necessary. In addition, the models developed in the paper assumed that the ''fixed'' inventory operating costs including rental, utilities and personnel salary were adjustable. Hence, the revised EOQ model is applicable only when the ''fixed'' inventory operating costs are not fixed, for example, during the feasibility study stage and design stage of an inventory facility, or for scenarios in which the redundant inventory facilities can be rented out when the annual average inventory level drops. These scenarios can be encountered, particularly when an inventory item can alternatively be purchased by a JIT fashion, as observed by Schniederjans and Olsen (1999) and Schniederjans and Cao (2000, 2001). Nevertheless, this assumption of the revised EOQ model limits the application of the models developed in this paper.

Fazel (1997), Fazel et al. (1998), Schniederjans and Olsen (1999), and Schniederjans and Cao (2000, 2001) had developed a series of innovative mathematical models to derive the EOQ–JIT cost indifference point to help manufacturers who were still using the EOQ purchasing system to consider switching over to the JIT purchasing system. However, the out-of-stock risks inherent in a JIT purchasing system was excluded from their studies. Further investigations to study the impact of the out-of-stock risks on the selection of an inventory purchasing system are therefore warranted. More insightful understanding of the impact of these factors on the EOQ–JIT cost indifference point may then be achieved by those working towards the full implementation of the JIT philosophy in the industry.

In addition, Cao and Schniederjans (2004) also presented a cost comparison of the classic economic manufacturing quantity (EMQ) model and a revised JIT model to show the efficacy of the JIT production-runs when a JIT company can capitalize on the physical space reduction. The classic EMQ model, which originated from the work of Harris (1915), seeks to determine the optimal production-run lot size in manufacturing. Unfortunately, unlike this present study, Cao and Schniederjans' (2004) model treated the inventory space reduction as a constant and ignored the out-of-stock cost that result from JIT production-runs. The difference between the approach adopted in this study and Cao and Schniederjans' (2004) model is also recommended for further research.

Based on the practices of the cement business in the RMC industry in Singapore, a new production model was proposed in this paper. This model proposed that larger companies whose inventory ordering sizes cannot be economically broken down tended to adopt an EOQ system and smaller and medium-sized companies tended to adopt a JIT system. The large companies, acting as a wholesaler, and apart from managing their own production, can reap the benefit of economies of scale and control the raw materials market. The small and medium-sized companies can operate faster, eliminate waste (Zhu et al., 1994), achieve continuous improvement (Low and Chan, 1997), improve customer service, and build organizational competitiveness (Cheng and Podolsky, 1996). This production model can result in a "win–win" situation for both the large companies and the small and medium-sized companies. Hence, it will be useful to further study how and in what conditions this production model can be transferred to other industries or other countries.

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