**3.4 Mathematical Model**

*Parameters:*

𝐾: set of orders received during the planning period.

: quantity of ground spherical glass for grinding in order .

: revenue from order , if delivered on time.

: delivery time of the order, .

: penalty rate for late delivery of the order; the penalty incurred for each week of delay is 1% of the order's revenue.

: working days of the first and second stages of work, . When *j* = 0, the working days of the first stage of work is set to 3 days; otherwise (*j* = 1), the working days of the second stage of work is set to 5 days.

: daily grinding capacity of each set of grinding axis, one set is 2 axis, which can grind 100 surfaces of glass lens.

: a large integer for modeling.

: The actual date corresponding to Day l. Day 1 reflects to the actual date. Since no jobs are dispatched on holidays, day 1 of the planning cycle for August 2024 is August 1st, day 2 of the planning cycle for August 2024 is August 2nd, and so on.

𝐷 = {0,1}: machine types; 0 refers to the 660-machine model, 1 the 770-machine model.

𝐸𝑑: set of machines of different models, 𝑑∈𝐷; 𝐸0 is the set of 660 machines, 𝐸1 the set of 770 machines**. [用軸去編碼]**

𝑃𝑑: set of machine axis numbers corresponding to the model, with each 2 axes as a group, 𝑑∈𝐷; 660-machine model has 4 axes, 𝑃1 = {0, 1}; 770-machine model has 6 axes, 𝑃2 = {2, 3, 4}.

𝐼: set of glass surfaces to be ground, 𝐼 = {0, 1}, 0 for surface A, 1 for surface B.

𝐽: set of production stages, 𝐽 = {0, 1, 2}, 0 for the first adjustment stage, 1 for the second adjustment stage, 2 for the official small batch production stage.

𝐿: set of planning calendar days with holidays excluded. For example, in August 2024, there are only 22 working days excluding holidays, so 𝐿 = {1, 2, …, 20, 22}.

Decision variables:

: Binary variable to indicate whether the *j*th stage of the *i*th surface of order *k* starts its grinding operation on the *p*th axis of machine *e* on day *l*, for *l* ∈ *L*, *k* ∈ *K*, *i* ∈ *I*, *e* ∈ *Ed*, *p* ∈ *Pd*, *d* ∈ D, *j* ∈ {0, 1}. Note that the third stage of small-batch production does not have a defined start time because grinding can start on multiple machines at the same time

: For stage *j* on *i* of order *k*, is grinding work performed on machine *e* on axis *p* on day *l*, *, , , .*

: Number of days of delayed delivery for the *i*th surface of order *k*,.

: Number of weeks of delayed delivery for the *i*th surface of order *k*, .

: Number of weeks of delayed delivery for order *k*, .

Objective: Minimizing the total penalty incurred by late delivery of orders

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|  |  |  | (1) |

Constraints:

1. At any given time, a single order can be in the first, second, or third grinding stage on each axis of each machine

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|  |  |  | (2) |

1. At any given time, each set of axes on every machine can perform only one grinding operation from either the first, second, or third stage of a single order

(3)

1. For each side of each order, only one set of axes of one machine starts the first- and second-stage grinding

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|  |  |  | (4) |

1. The second stage of each side of each order must use the same set of axes as the first stage of the same machine.

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|  |  |  | (4) |

1. The start time of the second grinding stage for each order, which must immediately follow the end time of the first grinding phase.

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|  |  |  | (5) |

1. The grinding time for the first and second stages of the order is calculated, which must be equal to the set time (3 days, 5 days)

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|  |  |  | (6) |

1. Grinding tasks for orders during the planning period are executed on a machine only when a corresponding start time is defined

(7)

1. The third stage of grinding on each side of each order, which must end later than the end time of the second stage.

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|  | ,  ; |  | (7) |

1. The third stage of grinding on each side of each order, which must meet the required quantity of the order

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|  |  |  | (8) |

1. Only 100 axles can operate in a day, as 2 axles form a group, so at most 50 groups of axles can be in operation.

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|  |  |  | (9) |

1. Calculation of delayed delivery days per side per order.

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| --- | --- | --- | --- |
|  |  |  | (10) |

1. Calculation of the number of weeks of late delivery for each side of each order

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|  |  |  | (11) |

1. Calculation of the number of weeks of late delivery for each order.

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|  |  |  | (12) |