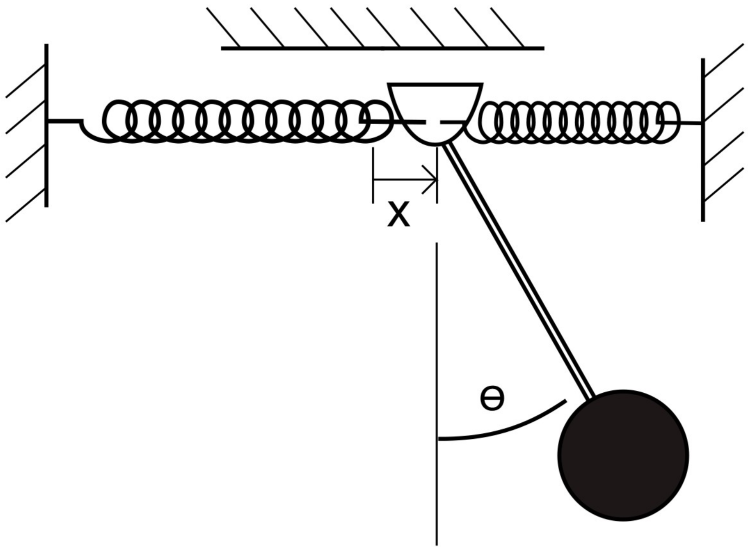
In [ ]:
```
import numpy as np
from numpy import sin,cos,pi
import matplotlib.pyplot as plt
plt.style.use('fivethirtyeight')
```

Homework #4

Problem 1



The pendulum bob of mass m, shown in the figure above, is suspended by an inextensible string from the point p. This point is free to move along a straight horizontal line under the action of the springs, each having a constant k. Assume that the mass is displaced only slightly from the equilibrium position and released. Neglecting the mass of the springs, show that the pendulum oscillates with a period of

$$P = 2\pi\sqrt{\frac{mg+2kr}{2kg}}$$

use a first-order Taylor series approximation for $\sin\theta \approx \theta$ and $\cos\theta \approx 1$

Solve for $\theta(t)$ if m=0.1 kg, r=1 m, $\theta(0)$=pi/6 rad, and $\dot{\theta}(0)$=0 rad/s for 2 cases:

a. k=20 N/m

b. k=∞ N/m

c. Plot the solutions of $\theta(t)$ for 2 periods on one figure

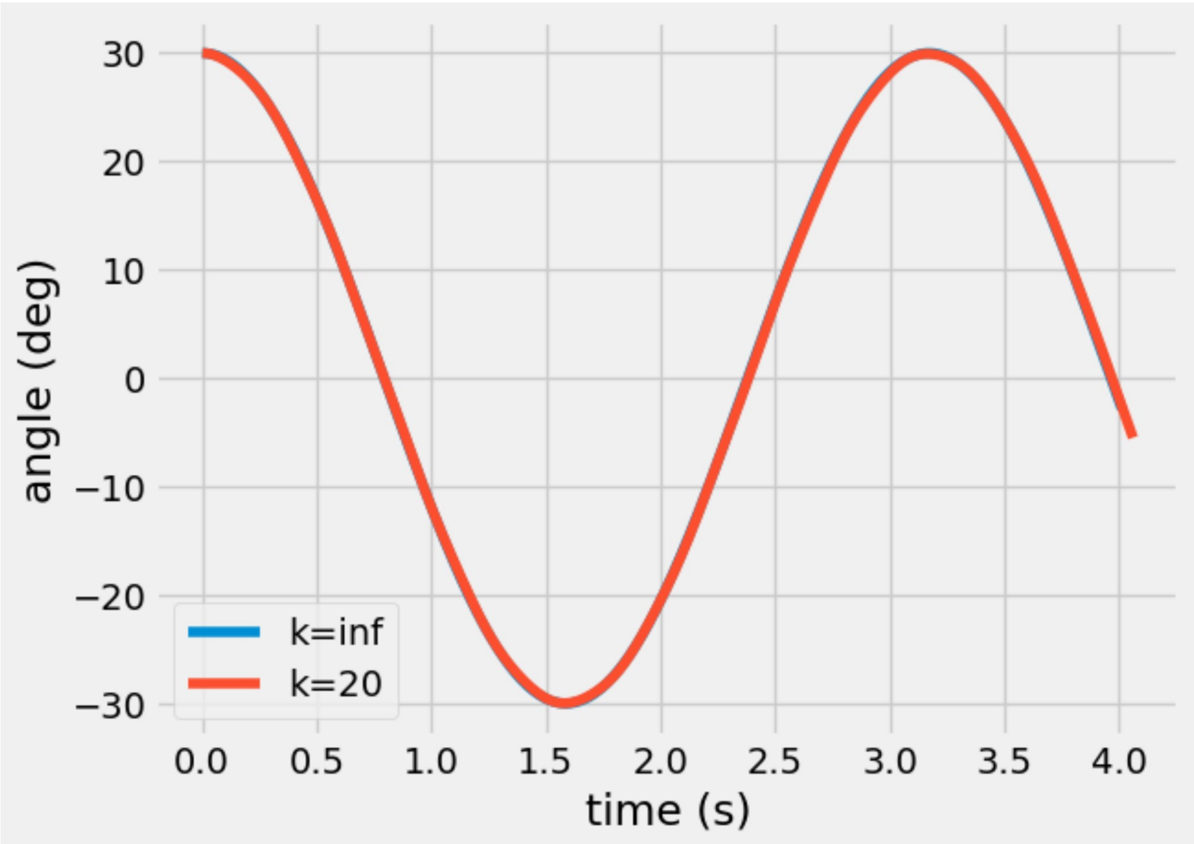
In [ ]:
```
from scipy.integrate import solve_ivp
```

In [ ]:
```
def my_ode_a(t,r,):
    """
     input is time, t (s) and r=[position p (m), angle (rad), velocity p (m/s), ang
     output is dr=[velocity p (m/s), angle velocity (rad/s), accel p (m/s/s), angle
     the ODE is defined by:

     dr = f(t,r)"""
    l=1
    m=0.1
    k=20
    g=9.81
    dr=np.zeros(np.size(r))
    dr[0]=r[2]
    dr[1]=r[3]
    x, a, v, w = r
    M = np.array([[m, m*l/2],
                 [m*l/2, m*l**2/4*5]])
    rhs = np.array([m*l/2*w**2*a- 2*k*x,
                     -m*g*l/2*a])
    dr[2:] = np.linalg.solve(M, rhs)
    return dr

def my_ode_b(t,r,):
    """
     input is time, t (s) and r=[position p (m), angle (rad), velocity p (m/s), ang
     output is dr=[velocity p (m/s), angle velocity (rad/s), accel p (m/s/s), angle
     the ODE is defined by:

     dr = f(t,r)"""
    l=1
    m=0.1
    k=999999
    g=9.81
    dr=np.zeros(np.size(r))
    dr[0]=r[2]
    dr[1]=r[3]
    x, a, v, w = r
    M = np.array([[m, m*l/2],
                 [m*l/2, m*l**2/4*5]])
    rhs = np.array([m*l/2*w**2*a- 2*k*x,
                     -m*g*l/2*a])
    dr[2:] = np.linalg.solve(M, rhs)
    return dr
```

In [ ]:
```
l=1
m=0.1
k_a=20
k_b= 999999
g=9.81

P_a=2*pi*np.sqrt((2*k_a*l+m*g)/(2*k_a*g))
P_b=2*pi*np.sqrt((2*k_b*l+m*g)/(2*k_b*g))

t_a=np.linspace(0,P_a*2, 1000)
t_b=np.linspace(0,P_b*2, 1000)
```

In [ ]:
```
a_20 = solve_ivp(my_ode_a,[0,2*P_a],[0, pi/6,0,0], t_eval=t_a); # create solution f
a_inf = solve_ivp(my_ode_b,[0,2*P_b],[0, pi/6,0,0], t_eval=t_b); # create solution

plt.plot(a_inf.t,a_inf.y[1]*180/pi,'-',label='k=inf')#conver rad to deg
plt.plot(a_20.t,a_20.y[1]*180/pi,'-',label='k=20')

plt.xlabel('time (s)')
plt.ylabel('angle (deg)')
plt.legend()
```

Out[ ]: <matplotlib.legend.Legend at 0x200a1466e80>



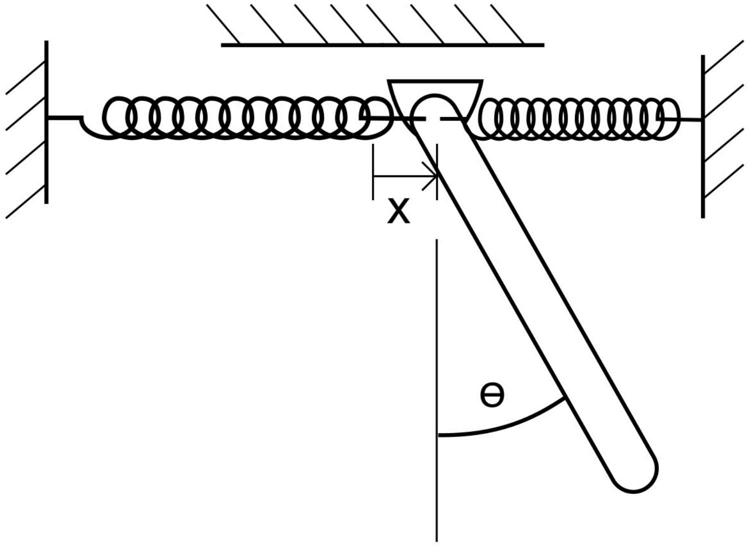
In [ ]:
```
from scipy.linalg import *
from scipy.optimize import fsolve,root
```

In [ ]:
```
from scipy.integrate import solve_ivp # import the ordinary differential equation i
```

Problem 2

In [ ]:
```
from IPython.display import YouTubeVideo
YouTubeVideo('eOvwiYRroso')
```

Out[ ]:



The pendulum arm of mass m, shown in the figure above, is held in place by two springs. This point is free to move along a straight horizontal line under the action of the springs, each having a constant k. Assume that the mass is displaced only slightly from the equilibrium position and released. Neglecting the mass of the springs, solve for the nonlinear equations of motion and use the `solve_ivp` to determine $\theta(t)$

Solve for $\theta(t)$ if m=1 kg, L=1 m, $\theta(0)$=pi/6 rad, and $\dot{\theta}(0)$=0 rad/s for

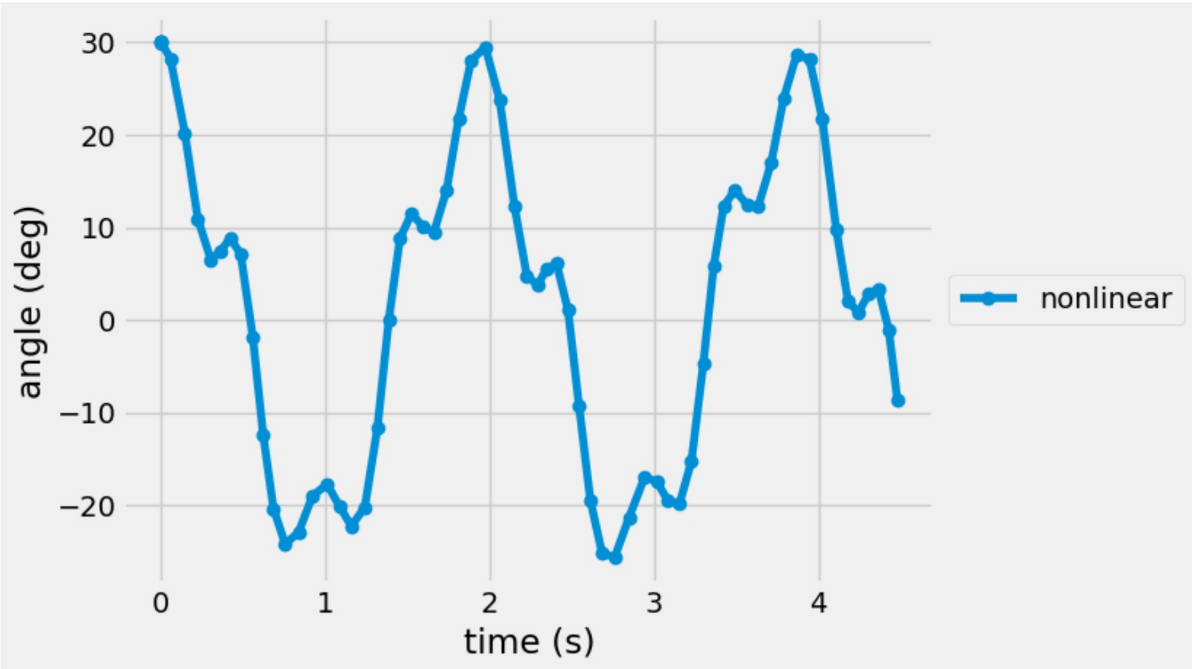
k=20 N/m

Plot the nonlinear solutions of $\theta(t)$ for 2 periods on one figure

In [ ]:
```
def my_ode(t,r,):
    """
     input is time, t (s) and r=[position p (m), angle (rad), velocity p (m/s), ang
     output is dr=[velocity p (m/s), angle velocity (rad/s), accel p (m/s/s), angle
     the ODE is defined by:

     dr = f(t,r)"""
    l=1
    m=1
    k=20
    g=9.81
    dr=np.zeros(np.size(r))
    dr[0]=r[2]
    dr[1]=r[3]
    x, a, v, w = r
    M = np.array([[m, m*l/2*np.cos(a)],[m*l/2*np.cos(a), m*l**2/3]])
    rhs = np.array([m*l/2*w**2*np.sin(a) - 2*k*x,-m*g*l/2*np.sin(a)])
    dr[2:] = np.linalg.solve(M, rhs)
    return dr
```

In [ ]:
```
l=1
m=1
k=20
g=9.81
P=2*pi*np.sqrt((2*k*l+m*g)/(2*k*g))
r=solve_ivp(my_ode,[0,2*P],[0, pi/6,0,0]); # default = 'RK45'
plt.plot(r.t,r.y[1]*180/pi,'-o',label='nonlinear') # <-------------- your new plot,
plt.legend(loc='center left', bbox_to_anchor=(1, 0.5))
plt.xlabel('time (s)')
plt.ylabel('angle (deg)')
```

Out[ ]: Text(0, 0.5, 'angle (deg)')



In [ ]: