



**Faculty of Engineering and Applied Science**

**SOFE 377U Design and Analysis of Algorithms**

**Assignment 2**

**Group Member 1**

**Name: Dhanushga Lionel**

**Student ID: 100616831**

**Group Member 2**

**Name: Thanushan Rameswaran**

**Student ID: 100462146**

**Date: 11/16/2018**

## Opening Comments:

For this assignment we used our UOIT issued laptops, running Windows 10. Our code is written in C++, compiled with Visual Studio.

## Pseudocode:

Algorithm hungarian

Input: cost matrix of jobs and cost

Output: optimal assignment of jobs to workers

For each row

    Subtract minimum row element from each row

For each column

    Subtract minimum column element from each column

Determine number of columns and rows that include all 0s in matrix (cover all zeros with minimum number of lines)

    While lines  $\neq$  sizeof.matrix{

        For each element not covered{

            Subtract minimum element from all elements}

        For each element covered by two lines{

            Add minimum element}

    }

For each zero determine corresponding position in original cost matrix

For each column{

    Job n = Worker (minimum element in column n)

}

## Big-O Analysis:

The original Hungarian/Munkres Algorithm worst-case running time was  $O(n^4)$ . The running time in recent years has decreased to  $O(n^3)$ .

Code:

```
#include <iostream>
#include <limits.h>

using namespace std;

//In each row of the matrix find the smallest element and subtract it from every element in the row
//When done go to step 2
void step_one(int matrix[5][5], int n, int& step)
{
    int minofRow;
    for (int r = 0; r < n; r++)
    {
        minofRow = matrix[r][0];
        for (int c = 0; c < n; c++)
        {
            if (matrix[r][c] < minofRow)
            {
                minofRow = matrix[r][c];
            }
        }
        for (int c = 0; c < n; c++)
        {
            matrix[r][c] -= minofRow;
        }
        step = 2;
    }
}

//Find a zero in the matrix. If no zeros found in row or column star, repeat for each element in matrix
//go to step 3;
void step_two(int matrix[5][5], int n, int* RowCover, int* ColCover, int** m, int& step)
{
    for (int r = 0; r < n; r++)
    {
        for (int c = 0; c < n; c++)
        {
            if (matrix[r][c] == 0 && RowCover[r] == 0 && ColCover[c] == 0)
            {
                m[r][c] = 1;
                RowCover[r] = 1;
                ColCover[c] = 1;
            }
        }
    }
    for (int r = 0; r < n; r++)
    {
        RowCover[r] = 0;
    }
    for (int c = 0; c < n; c++)
    {
        ColCover[c] = 0;
    }
    step = 3;
}

//Cover each column with the starred zero. If K number of columns are covered,
//the starred zeros describe set of unique assignments.
```

```

//If this is the case, go to step 7, else go to step 4
void step_three(int matrix[5][5], int n, int* ColCover, int** m, int& step)
{
    int ColCount;
    for (int r = 0; r < n; r++)
    {
        for (int c = 0; c < n; c++)
        {
            if (matrix[r][c] == 1)
            {
                ColCover[c] = 1;
            }
        }
        ColCount = 0;
        for (int c = 0; c < n; c++)
        {
            if (ColCover[c] == 1)
            {
                ColCount += 1;
            }
        }
        if (ColCount >= n || ColCount >= n)
        {
            step = 7;
        }
        else
        {
            step = 4;
        }
    }
}
//method to support step 4
void Finding_a_Zero(int matrix[5][5], int n, int* RowCover, int* ColCover, int row, int col)
{
    int r = 0;
    int c;
    bool done;
    row = -1;
    col = -1;
    done = false;
    while (!done)
    {
        c = 0;
        while (true)
        {
            if (matrix[r][c] == 0 && RowCover[r] == 0 && ColCover[c] == 0)
            {
                row = r;
                col = c;
                done = true;
            }
            c += 1;
            if (c > n || done)
            {
                break;
            }
        }
        r += 1;
    }
}

```

```

        if (r >= n)
        {
            done = true;
        }
    }
}
bool Star_in_Row(int** m, int n, int row)
{
    bool tmp = false;
    for (int c = 0; c < n; c++)
    {
        if (m[row][c] == 1)
        {
            tmp = true;
        }
    }
    return tmp;
}
void Find_Star_in_Row(int** m, int n, int row, int col)
{
    col = 1;
    for (int c = 0; c < n; c++)
    {
        if (m[row][c] == 1)
        {
            col = c;
        }
    }
}
//Find non-covered zero and prime it. If there is no starred zero in row containing primed zero
//Go to step 5.
//Else continue till no uncovered zeros remain.
//Save smallest value and go to step 6.
void step_four(int matrix[5][5], int n, int* RowCover, int* ColCover, int** m, int& path_row_0, int&
path_col_0, int& step)
{
    int row = -1;
    int col = -1;
    bool done;
    done = false;
    while (!done)
    {
        Finding_a_Zero(matrix, n, RowCover, ColCover, row, col);
        if (row == -1)
        {
            done = true;
            step = 6;
        }
        else
        {
            m[row][col] = 2;
            if (Star_in_Row(m, n, row))
            {
                Find_Star_in_Row(m, n, row, col);
                RowCover[row] = 1;
                ColCover[col] = 0;
            }
            else {

```

```

        done = true;
        step = 5;
        path_row_0 = row;
        path_col_0 = col;
    }
}

//methods that will support step 5;
void Find_Star_in_Col(int n, int** m, int c, int& r)
{
    r = -1;
    for (int i = 0; i < n; i++)
    {
        if (m[i][c] == 1)
        {
            r = i;
        }
    }
}

void Find_Prime_in_Row(int n, int** m, int r, int& c)
{
    for (int j = 0; j < n; j++)
    {
        if (m[r][j] == 2)
        {
            c = j;
        }
    }
}

void Augment_Path(int** m, int path_count, int** path) {
    for (int p = 0; p < path_count; p++)
    {
        if (m[path[p][0]][path[p][1]] == 1)
        {
            m[path[p][0]][path[p][1]] = 0;
        }
        else
        {
            m[path[p][0]][path[p][1]] = 1;
        }
    }
}

void Covers_Clear(int n, int* RowCover, int* ColCover) {
    for (int r = 0; r < n; r++)
    {
        RowCover[r] = 0;
    }
    for (int c = 0; c < n; c++)
    {
        ColCover[c] = 0;
    }
}

void Primes_Erase(int n, int** m)
{
    for (int r = 0; r < n; r++)
    {

```

```

        for (int c = 0; c < n; c++)
        {
            if (m[r][c] == 2)
            {
                m[r][c] = 0;
            }
        }
    }

    //Constructing series of alternating primed and starred zeros
    //Z0 represents uncovered primed zero found in step 4
    //Z1 denotes starred zero in column of Z0 (if any).
    //Z2 denotes primed zero in row of Z1. Continue until the series terminates at a primed zero that
    //has no starred
    //zero in its column. Unstar each of starred zeros and star each primed zeros of series
    //erase all primes and uncover every line in matrix,
    //when finished return to step 3;
    void step_five(int n, int* RowCover, int* ColCover, int** m, int& path_row_0,
        int& path_col_0, int& path_count, int** path, int& step)
    {
        bool done;
        int r = -1;
        int c = -1;
        path_count = 1;
        path[path_count - 1][0] = path_row_0;
        path[path_count - 1][1] = path_col_0;
        done = false;
        while (!done) {
            Find_Star_in_Col(n, m, path[path_count - 1][1], r);
            if (r > -1) {
                path_count += 1;
                path[path_count - 1][0] = r;
                path[path_count - 1][1] = path[path_count - 2][1];
            }
            else
                done = true;
            if (!done) {
                Find_Prime_in_Row(n, m, path[path_count - 1][0], c);
                path_count += 1;
                path[path_count - 1][0] = path[path_count - 2][0];
                path[path_count - 1][1] = c;
            }
        }
        Augment_Path(m, path_count, path);
        Covers_Clear(n, RowCover, ColCover);
        Primes_Erase(n, m);
        step = 3;
    }

    //method to support step 6
    void Find_Smallest(int matrix[5][5], int n, int* RowCover, int* ColCover,
        int& minVal)
    {
        for (int r = 0; r < n; r++)
        {
            for (int c = 0; c < n; c++)
            {
                if (RowCover[r] == 0 && ColCover[c] == 0)

```

```

        {
            if (minVal > matrix[r][c])
            {
                minVal = matrix[r][c];
            }
        }
    }
}

//Add value found in step 4 to every column in covered row and subtract from every element
//in uncovered column.
//Return to step 4 without altering stars, primes or covered lines
void step_six(int matrix[5][5], int n, int* RowCover, int* ColCover, int& step)
{
    int minVal = INT_MAX;
    Find_Smallest(matrix, n, RowCover, ColCover, minVal);
    for (int r = 0; r < n; r++)
        for (int c = 0; c < n; c++) {
            if (RowCover[r] == 1)
                matrix[r][c] += minVal;
            if (ColCover[c] == 0)
                matrix[r][c] -= minVal;
        }
    step = 4;
}

void runHungarian(int matrix[5][5], int n)
{
    int* RowCover = new int[n];
    int* ColCover = new int[n];
    int** m = new int*[n];
    int path_row_0 = 0;
    int path_col_0 = 0;
    int path_count = 0;
    for (int i = 0; i < n; i++) {
        RowCover[i] = ColCover[i] = 0;
        m[i] = new int[n];
        for (int j = 0; j < n; j++) {
            m[i][j] = 0;
        }
    }
    int** path = new int*[n * 2];
    for (int i = 0; i < n * 2; i++) {
        path[i] = new int[2];
    }
    bool done = false;
    int step = 1;
    while (!done)
    {
        switch (step)
        {
            case 1:
                step_one(matrix, n, step);
                break;
            case 2:
                step_two(matrix, n, RowCover, ColCover, m, step);
                break;
        }
    }
}

```



```

        case 3:
            step_three(matrix, n, ColCover, m, step);
            break;
        case 4:
            step_four(matrix, n, RowCover, ColCover, m, path_row_0, path_col_0, step);
            break;
        case 5:
            step_five(n, RowCover, ColCover, m, path_row_0, path_col_0,
                path_count, path, step);
            break;
        case 6:
            step_six(matrix, n, RowCover, ColCover, step);
            break;
        case 7:
            done = true;
            break;
    }


}

}
int main()
{
    int matrix[5][5] = {
        { 10, 5, 13, 15, 16 },
        { 3, 9, 18, 13, 6 },
        { 10, 7, 2, 2, 2 },
        { 7, 11, 9, 7, 12 },
        { 7, 9, 10, 4, 12 }
    };

    runHungarian(matrix, 5);
    for (int x = 0; x < 5; x++)
    {
        cout << x << ", " << assignment[x] << "\t";
        cout << "\ncost: " << cost << endl;
    }
    system("pause");
    return 0;
}

```

Output:

 C:\Users\100616831\source\repos\Project10\Debug\Project10.exe

```
0,1    1,0    2,4    3,2    4,3
```

```
cost: 23
```

```
Press any key to continue . . .
```