RSVPs *or how I learned how many people will attend your wedding*

♡ Let θ be the true proportion of people that will attend your wedding out of N total invitations that you send.

♡ Let y be the yes's out of n RSVPs you have received to date.

♡ We want to know the posterior density of θ given the data y.

♡ Given θ, the likelihood of getting y yes's and $n-y$ no's is

$$p(y|\theta) \propto \theta^y(1-\theta)^{n-y}.$$

♡ A conjugate prior for θ is

$$p(\theta) \propto \theta^{\alpha-1}(1-\theta)^{\beta-1}, \text{ where } \theta \sim \text{Beta}(\alpha, \beta).$$

♡ Using Bayes' rule, the posterior density for θ is

$$\begin{aligned} p(\theta|y) &\propto p(y|\theta)p(\theta) \\ &= \theta^{y+\alpha-1}(1-\theta)^{n-y+\beta-1} \\ &= \text{Beta}(\alpha+y, \beta+n-y). \end{aligned}$$

♡ The posterior mean of θ is an average of the prior mean $(\frac{\alpha}{\alpha+\beta})$ and the sample mean $(\frac{y}{n})$:

$$\text{E}(\theta|y) = \frac{\alpha+y}{\alpha+\beta+n}.$$

♡ The posterior variance of θ is

$$\text{var}(\theta|y) = \frac{(\alpha+y)(\beta+n-y)}{(\alpha+\beta+b)^2(\alpha+\beta+n+1)} = \frac{\text{E}(\theta|y)[1-\text{E}(\theta|y)]}{\alpha+\beta+n+1}.$$

♡ The prior density is equivalent to having received $\alpha-1$ previous yes's and $\beta-1$ previous no's. As y and $n-y$ grow large compared to α and β, the posterior mean approaches the sample mean and the posterior variance goes to 0 at rate $\frac{1}{n}$.

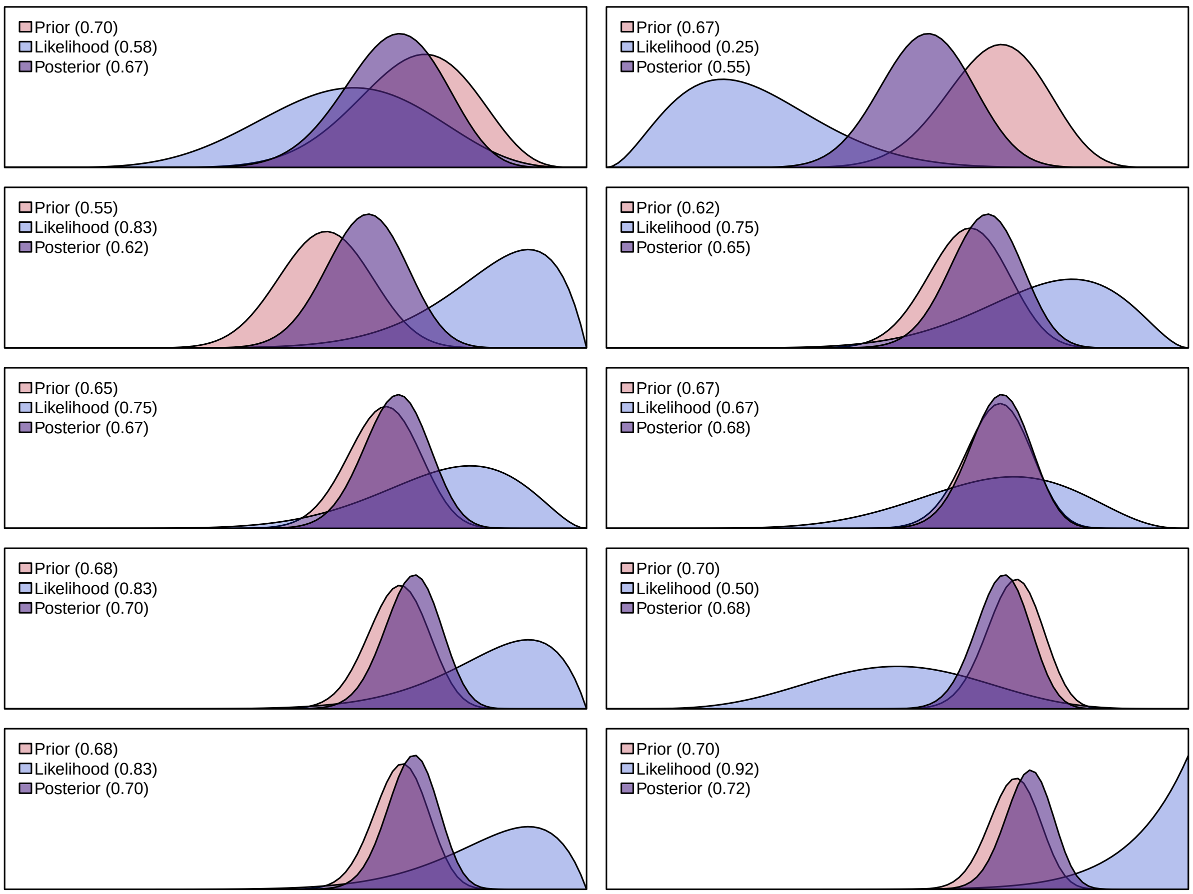
♡ After each response we can update the prior with the previous posterior.

♡ Below is an example of how the posterior will update as you receive responses.

♡ Suppose you believe that $\theta = 0.7$, but you want your prior to be relatively weak compared to the data, such that the mean of the posterior after receiving all of the RSVPs will be close to $\frac{y}{N}$. (Maybe your prior can account for people that respond yes but do not attend the wedding, or forget to respond but still attend). For example, if $N = 100$, let $\theta \sim \mathrm{Beta}(\alpha = 14,\ \beta = 6)$ such that the prior is the same weight as 20 invitiations and $\mathrm{E}(\theta) = 0.7$.

♡ If you receive responses in groups of 10, where each row of y is the number of yes's out of the 10 responses, here is how your posterior will update:

$$y_i = [6,\ 2,\ 9,\ 8,\ 8,\ 7,\ 9,\ 5,\ 9,\ 10].\ \frac{y}{N} = 0.73.$$



$$\mathrm{E}(\theta|y_i) = [0.67,\ 0.55,\ 0.62,\ 0.65,\ 0.67,\ 0.68,\ 0.70,\ 0.68,\ 0.70,\ 0.72].$$

$$\mathrm{E}(\theta|y) = 0.72.\ \mathrm{var}(\theta|y) = 9e-04.\ 95\%\ \text{confidence interval: } [0.64,\ 0.80].$$